Affine Hybrid Systems

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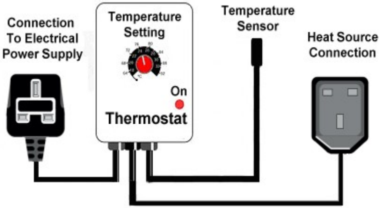
Motivation

- Hybrid System: A dynamical system that exhibits both continuous and discrete behaviors
- Examples
    - Thermostat
    - Dubins car
    - Robot navigation
    - Aircraft flight control systems
    - Medical devices such as pacemakers

Motivating Examples

Thermostat

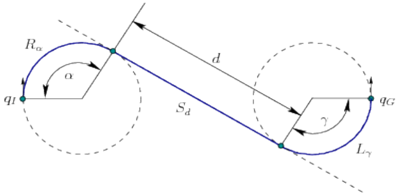
- Theromostat has two discrete modes: *ON*, *OFF*;
- Temprature sensor senses the temperature of the thermostat;
- Heat source connection is the source of generating heat.



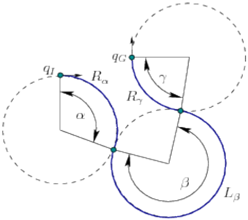
Motivating Examples

Dubins car

- Dubins car has three discrete modes: *R*(clock wise), *L*(anti-clock wise), *S* (straight line);
- When dubins car changes its mode, the angular velocity is reset by the controller;
- It drives on a curve which is a mixed sequence of paths *R*, *L*,*S*.



$R_\alpha S_d L_\gamma$

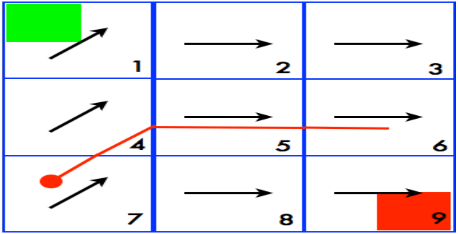


$R_\alpha L_\beta R_\gamma$

Motivating Examples

Robot navigation

- Robots neet to navigate on the floor without reaching bad region (red color);



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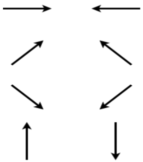

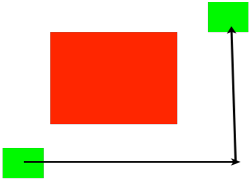

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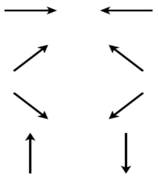

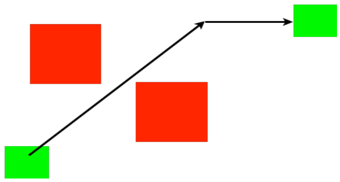

Affine Dynamical Systems (ADS)

ADS

This is a linear system where system matrix $A \in \mathbb{R}^{n \times n}$ is a constant matrix and $B \in \mathbb{R}^{n}$ is a constant vector i.e.

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B, \qquad \mathbf{x}(0) \in X_0 \subseteq \mathbb{R}^n$$

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Solution of ADS

$x(t)$ is a solution starting from the initial point $x(0)$ at time t if $x(t) = \Phi(\mathbf{x}(0), A, B, t)$, where

$$\Phi(\mathbf{x}(0), A, B, t) = e^{At}\mathbf{x}(0) + Bt.$$

Affine Hybrid Systems (AHS)

AHS

AHS is a hybrid system $\mathcal{H} = (\mathcal{Q}, q_0, X, X_0, F, I, E, R)$ where

- $\mathcal{Q}$ is a set of modes,
- $q_0 \in \mathcal{Q}$ is an initial mode,
- $X = \mathbb{R}^n$ is a continuous state space, for some n,
- $X_0 \subseteq X$ is an initial set of states,
- $F : \mathcal{Q} \times \mathbb{R}_{\geq 0} \to X$ is a flow function,
- $I : \mathcal{Q} \to 2^X$ is an invariant function,
- $E \subseteq \mathcal{Q} \times \mathcal{Q}$ is a set of edges, and
- $R : E \to 2^{X \times X}$ is a switching relation.

Transitions of AHS

Continuous transitions

$(q, \mathbf{x}) \xrightarrow{t}_C (q, \mathbf{x}')$ is a continuous transition if $\mathbf{x}'$ satisfies the differential equation $\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B$, and all the solutions starting from $\mathbf{x}$ within the time t satisfy the invariant condition $I(q)$.

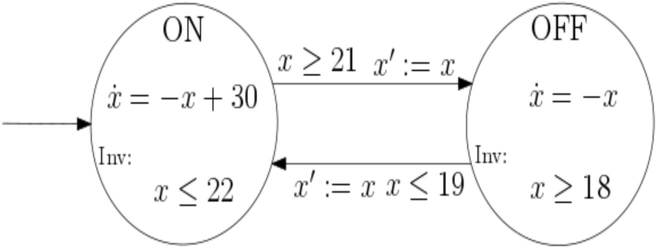
Discrete transitions

$(q, \mathbf{x}) \longrightarrow_D (q', \mathbf{x}')$ is a discrete transition if $(\mathbf{x}, \mathbf{x}')$ belongs to the switching relation $R(e)$, where $e = (q, q')$.

Execution of AHS

An execution of AHS is an alternating sequence of continuous and discrete transitions i.e.
$(q_0, \mathbf{x}_0) \xrightarrow{t_0}_C (q_0, \mathbf{x}_1) \longrightarrow_D (q_1, \mathbf{x}_2) \xrightarrow{t_1}_C \ldots.$

Formal model of thermostat

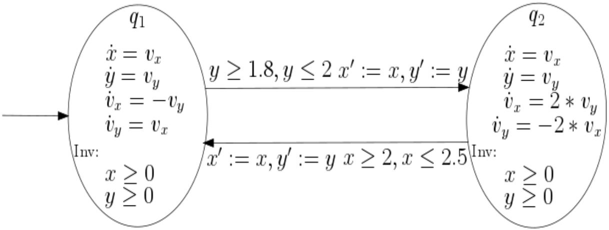


Formal model of dubins car

Differential equation of circular motion

- When angular velocity ω becomes zero, the following equation represents a differential equation of a straignt line.

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{v}_x \\ \dot{v}_y \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\omega \\ 0 & 0 & \omega & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ v_x \\ v_y \end{bmatrix}$$



Summary

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- We have seen the applications of hybrid systems;
- We have understood the formal model of hybrid systems;
- Next, we will see the simulation of hybrid systems through the tool "Beaver".