Problems on Fundamentals of A.C

1. The equation for alternating current is given by

$$i = 28.28 \sin(314t + 30^\circ).$$

Calculate the RMS value, frequency & phase angle.

Solⁿ:

$$i = 28.28 \sin(314t + 30^\circ)$$

This is of the form

$$i = I_m \sin(\omega t + \phi).$$

(i) RMS value

$$I = \frac{I_m}{\sqrt{2}} = \frac{28.28}{\sqrt{2}} = 20A$$

(ii) Frequency

$$\omega = 2\pi f = 314$$

$$\Rightarrow f = \frac{314}{2\pi} = 50Hz$$

(iii) Phase angle

$$\phi = 30^\circ$$

2. Find the root-mean-square value of the resultant current in a wire which carries simultaneously a direct current of 10A and a sinusoidally alternating current with a peak value of 10A.

Solⁿ:

$$I_{dc} = 10A, \quad I_{m(ac)} = 10A$$

$$\Rightarrow I_{rms(ac)} = \frac{I_{m(ac)}}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7.07A.$$

Resultant current is,

$$I = \sqrt{I_{dc}^2 + I_{rms(ac)}^2} = \sqrt{10^2 + 7.07^2} = 12.5A$$

3. An alternating current of frequency 60Hz has a peak value of 120A. Write its equation for instantaneous value. Reckoning the time from the instant the current is zero and is becoming positive, find

a) the instantaneous value after 1/360 seconds

b) the time taken to reach 96A for the first time.

Sol^n

Given $f = 60\,Hz$, $I_m = 120A$

$$\Rightarrow \omega = 2\pi f = 120\pi$$

Hence, the equation for current is,

$$\boxed{i = 120 \sin(120\pi t)}$$

a) at $t = \frac{1}{360}$ seconds,

$$i = 120 \sin\left(120\pi * 1/360\right)$$

$$\boxed{i = 103.923\,A}$$

Note:- Since 'ω' is in radians/second, you must keep the calculator in radians mode to find the current value. If kept in degree mode then the answer would be 2.193A, which is wrong.

b) when $i = 96A$,

$$96 = 120 \sin(120\pi t)$$

$$\Rightarrow \sin(120\pi t) = 96/120 = 0.8$$

$$\Rightarrow 120\pi t = \sin^{-1}(0.8) = 0.9272 \text{ radian}$$

$$\therefore t = \frac{0.9272}{120\pi}$$

$$\Rightarrow \boxed{t = 0.002459\,s}$$

4. An alternating current varying sinusoidally has an RMS value of 20A. If the frequency is 50Hz,

a) write down the equation for its instantaneous value

b) calculate the value of current at 0.0025 seconds and at 0.0125 seconds after passing through a positive maximum value.

c) at what time, measured from a positive maximum value, will the instantaneous current be 14.14A ?

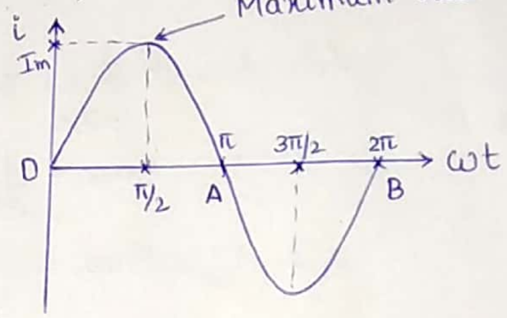
Solⁿ

$$I = 20A \Rightarrow I_m = 20\sqrt{2} = 28.28A.$$

$$f = 50Hz = \omega = 2\pi f = 100\pi \text{ radians/sec.}$$

$$\Rightarrow T = 20ms$$

a) The equation for the sinusoidal current with reference point 'O' as zero time point is,



$$i = I_m \sin \omega t$$

$$\boxed{i = 28.28 \sin(100\pi t)} \text{ A.}$$

To calculate b) and c), we are supposed to find the values after the wave crosses $\omega t = \pi/2$ radians. Hence the current equation must be written as,

$$i = 28.28 \sin(100\pi t + \pi/2)$$

b) At $t = 0.0025\,s$ after the positive maxima,

$$i = 28.28 \sin\left[(100\pi * 0.025) + \pi/2\right] =$$

$$(100\pi * 0.0025 + \pi/2) = 3\pi/4 \Rightarrow \sin(3\pi/4) = 0.707.$$

$$\therefore i = 28.28 * 0.707 \Rightarrow \boxed{i = 20A}$$

at $t = 0.125s$ after the positive maxima,

$$i = 28.28 \sin\left[(100\pi * 0.125) + \pi/2\right]$$

$$(100\pi * 0.00125) + \pi/2 = 7\pi/4 \Rightarrow \sin(7\pi/4) = -0.707$$

$$\therefore \; i = 28.28 * (-0.707)$$

$$\Rightarrow \boxed{i = -20A}$$

c) $i = 14.14A$ after positive maxima will be at

$$14.14 = 28.28 \sin(100\pi t + \pi/2)$$

$$\Rightarrow \sin\left(100\pi t + \frac{\pi}{2}\right) = 0.5$$

$$\Rightarrow 100\pi t + \frac{\pi}{2} = 0.5236$$

$$\therefore 100\pi t = -1.0471.$$

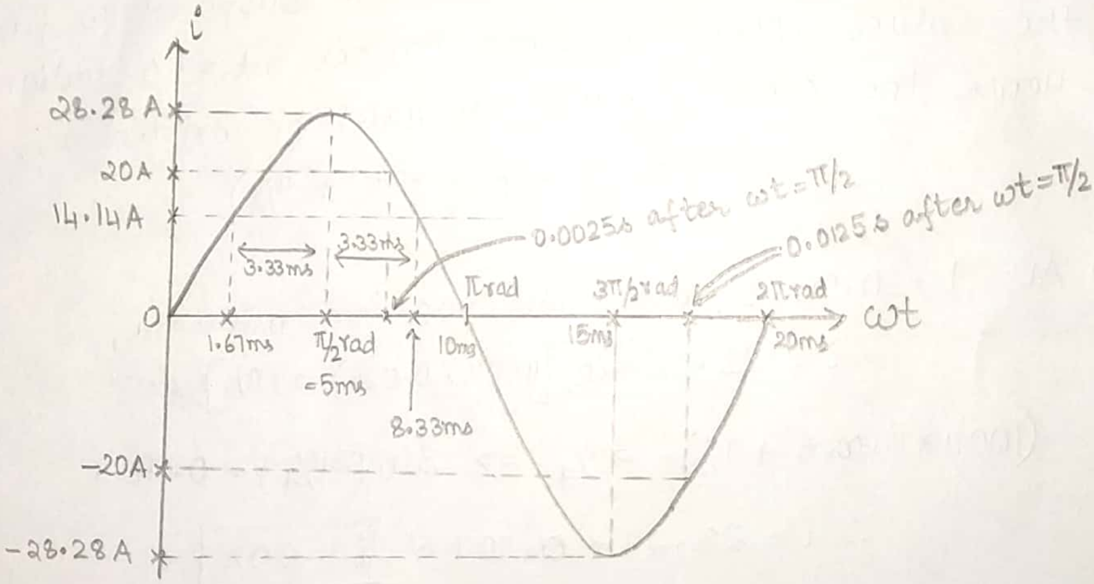
$$\therefore \boxed{t = \frac{1}{300} \text{ seconds}} = 3.3ms$$

for $t = \pm \frac{1}{300s}$ from the positive maxima, i.e., at

$$t_1 = 5ms + \frac{1}{300} = 8.33 ms$$

$$t_2 = 5ms - \frac{1}{300} = 1.67 ms$$

the value will be 14.14A.



5. An alternating current of frequency 50Hz has an amplitude of 100A. Calculate,

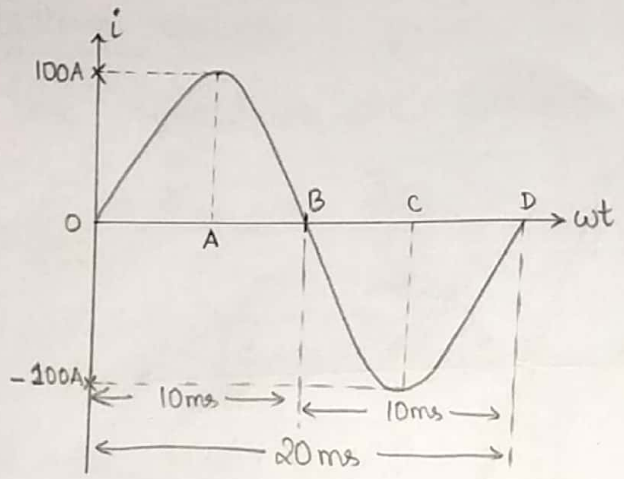
a) its value 1/600 seconds after the instant the current is zero and its value of decreasing thereafter/wards.

b) the time after the instant the current is zero and increasing thereafterwoards when the current reaches the value 86.6A.

Solⁿ:

Given $I_m = 100A$

$f = 50Hz \Rightarrow \omega = 100\pi$ rad/sec.



The current has an equation of the form

$$i = I_m \sin \omega t$$

$$\Rightarrow i = 100 \sin (100\pi t)$$

a) To calculate the value, it is required to measure the time when the current reaches zero and reduces thereafter. i.e., time is measured from point B.

$$\therefore t = 10ms + \frac{1}{600}s = 0.01167s.$$

The value at this point is,

$$i = 100 \sin (100\pi * 0.01167)$$

$$\Rightarrow \boxed{i = -50A}$$

b) To calculate the value at $i = 86.6A$, it is given that the value is measured starting from current zero and when the current is increasing. i.e., the reference point is 'O'. Therefore,

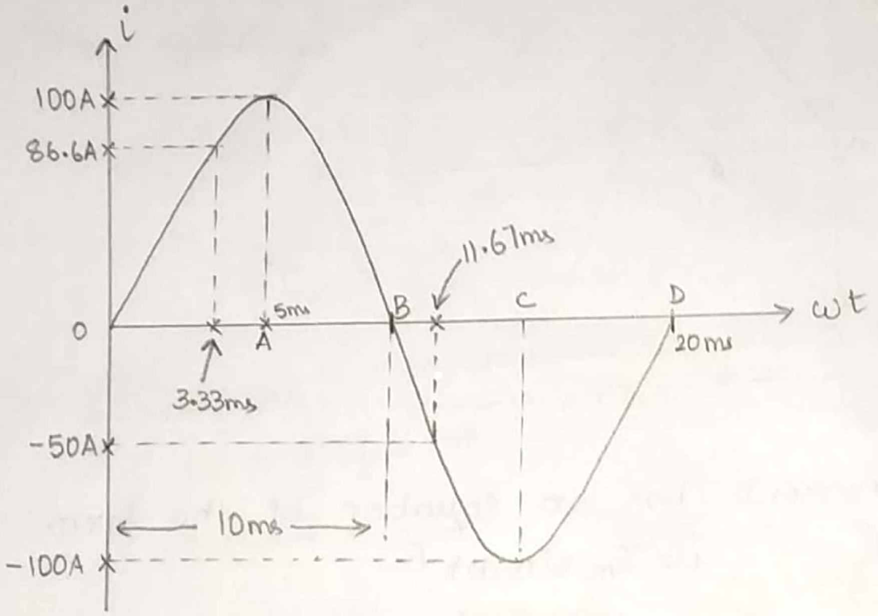
$$i = I_m \sin \omega t$$

$$86.6 = 100 \sin(100\pi t)$$

$$\Rightarrow \sin(100\pi t) = 0.866$$

$$\Rightarrow 100\pi t = 1.047 \text{ radians}$$

$$\Rightarrow \boxed{t = 3.33\text{ ms}}$$



6. The mathematical expression for the instantaneous value of an alternating current is

$$i = 7.071 \sin\left(157.08\, t - \frac{\pi}{4}\right) \text{ amperes.}$$

Calculate its effective value, periodic time & the instant at which it reaches its positive maximum value. Sketch the waveform from $t = 0$ over one complete cycle.

Solⁿ.

The given current is

$$i = 7.071 \sin\left(157.08t - \frac{\pi}{4}\right) \text{ amps}$$

comparing with

$$i = I_m \sin(\omega t - \phi),$$

$I_m = 7.071\,A$

$\omega = 157.08$ rad/sec

$\phi = \frac{\pi}{4}$ radians

a) Effective value

The effective or RMS value is

$$I_{RMS} = \frac{I_m}{\sqrt{2}} = \frac{7.071}{\sqrt{2}}$$

$$\boxed{I_{RMS} = 5\,A}$$

b) Periodic time

$$\omega = 2\pi f = 157.08$$

but $f = \frac{1}{T}$

$$\therefore \frac{2\pi}{T} = 157.08$$

$$\Rightarrow T = \frac{2\pi}{157.08} = 0.04\,s$$

or $\boxed{T = 40\,msec}$

c) time at which $i = I_m$

The positive maximum is

$I_m = 7.071A$.

Substituting in the given equation,

$$7.071 = 7.071 \sin\left(157.08t - \frac{\pi}{4}\right)$$

$$\Rightarrow \sin\left(157.08t - \frac{\pi}{4}\right) = 1$$

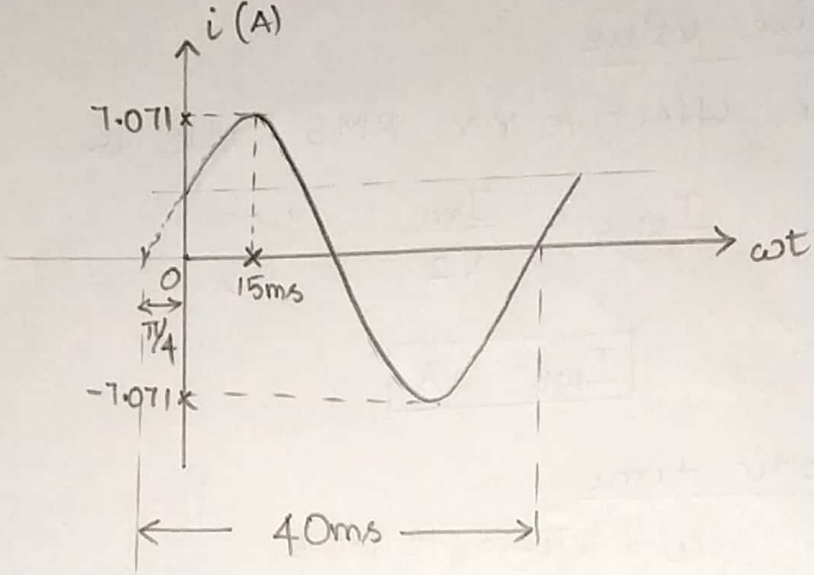
$$\Rightarrow 157.08t - \frac{\pi}{4} = 1.5707 \text{ rad}$$

$$\Rightarrow 157.08t = 2.3561 \text{ rad}$$

$$\Rightarrow t = 0.015\,s$$

$$\Rightarrow \boxed{t = 15\,ms}$$

d) <u>Waveform</u>



7. A 50Hz sinusoidal voltage applied to a 1ϕ circuit has its RMS value of 200V. Its value at t=0 is $(\sqrt{2} * 200)$V positive. The current drawn by the circuit is 5 A (RMS) and lags behind the voltage by one-sixth of a cycle. Write the expressions for the instantaneous values of voltage & current. Sketch their waveforms, and find their values at t = 0.0125 sec.

Sol^n:

$$f = 50Hz \Rightarrow \omega' = 2\pi f = 100\pi$$

$$V_{rms} = 200V.$$

The equation of voltage is

$$v = V_m \sin(\omega t + \phi)$$

At $t = 0$, $v = (200\sqrt{2})$ volts

$$\therefore 200\sqrt{2} = 200\sqrt{2} * \sin([\omega(0)] + \phi)$$

$$\Rightarrow \phi = \frac{\pi}{2} \text{ rad} = 1.5707 \text{ rad}.$$

$\therefore$ The expression for voltage is

$$\boxed{v = 282.842 \sin(100\pi t + 1.5707 \text{ rad}) \text{ V}}$$

$$T = \frac{1}{f} = \frac{1}{50} = 0.02s.$$

Current lags voltage by $1/6^{th}$ of cycle i.e., $\frac{T}{6}$ i.e.

$$\frac{0.02}{6} = 3.33*10^{-3} s.$$

$\therefore$ The extra lag angle is,

$$\theta = \omega t = 2\pi f t = 100\pi * 3.33*10^{-3}$$

$$\theta = 1.04709 \text{ rad}$$

Thus, the total lag angle is

$$\phi - \theta = 1.5707 - 1.04709 = 0.5236 \text{ rad}.$$

$$I_{rms} = 5A \Rightarrow I_m = 5\sqrt{2} = 7.071A.$$

$\therefore$ The expression for current is,

$$\boxed{i = 7.071 \sin(100\pi t + 0.5236) \text{ A}}$$

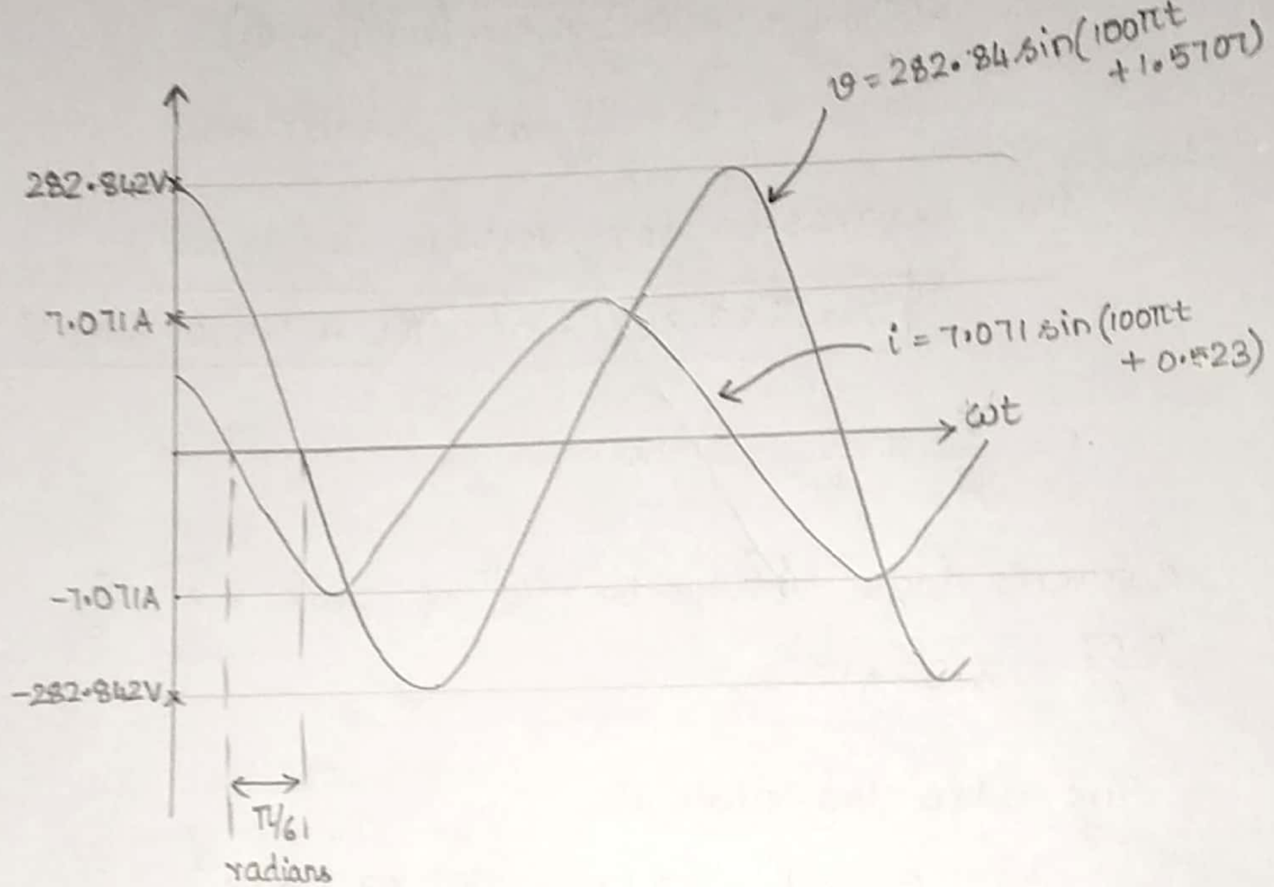
At $t = 0.0125s$,

$$V = 282.842 \sin\left([100\pi * 0.0125] + 1.5707\right)$$

$$\boxed{V = -200V}$$

$$i = 7.071 \sin\left([100\pi * 0.0125] + 0.5236\right)$$

$$\boxed{i = -6.83A}$$



8. An alternating current varying sinusoidally with a frequency of 50Hz has an RMS value of 20A. At what time, measured from the negative maximum value, instantaneous current will be $10\sqrt{2}$ A ?

Solⁿ

$I_{rms} = 20A$.

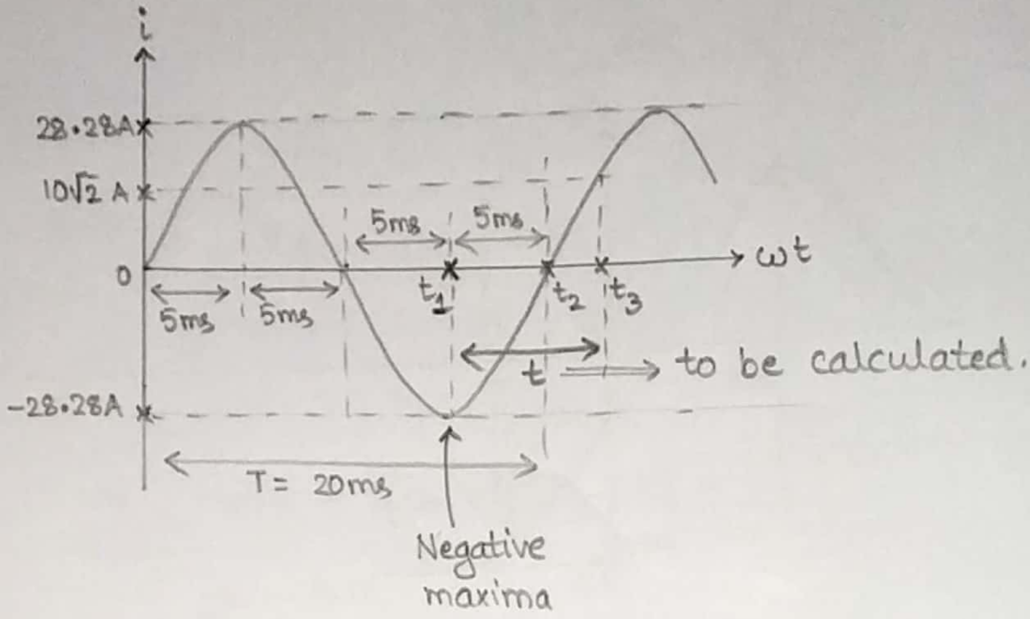
$\Rightarrow I_m = \sqrt{2}\, I_{rms} = \sqrt{2} * 20 = 28.28 A$.

$f = 50Hz \Rightarrow T = \frac{1}{f} = 20ms$.

∴ The equation of the current is

$$i = I_m \sin(\omega t) = I_m \sin(2\pi f t)$$

$$i = 20\sqrt{2} \sin(100\pi t)$$



The value $10\sqrt{2}$ A occurs when

$$10\sqrt{2} = 20\sqrt{2} \sin(100\pi t)$$

$$\Rightarrow t = 1.66\text{ms}.$$

This indicates that 1.66ms after the waveform crosses $\omega t = 0$ axis, a value of $10\sqrt{2}$ occurs.

From the above waveform, this means that the time

$$(t_3 - t_2) = 1.66\text{ms}.$$

Now, we are supposed to consider time from negative maximum. i.e., from t_1. The time from t_1 to t_3 is to be calculated. This is,

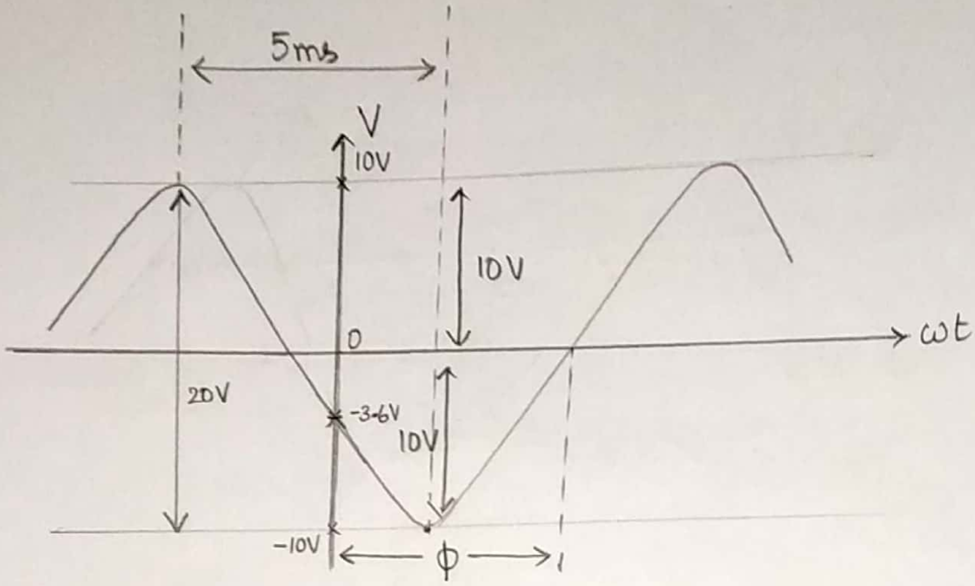
$$t = (t_1 \text{ to } t_2) + (t_2 \text{ to } t_3)$$

$$= 5\text{ms} + 1.66$$

$$\boxed{t = 6.66\text{ms}}$$

9. A sinusoidal voltage is 20V peak-to-peak, has a time of 5ms between consecutive peak & trough, and at t=0 is -3.6V and decreasing. Find the equation for the instantaneous value of the voltage, and the value at t = 12ms.

Sol^n



$$\text{Peak value} = \frac{\text{Peak-to-peak Value}}{2}$$

$$\therefore V_m = \frac{20}{2} = 10V.$$

Time between consecutive peak & trough = 5ms

$\therefore$ Total time period is

$$T = 2.5 + 5 + 2.5$$

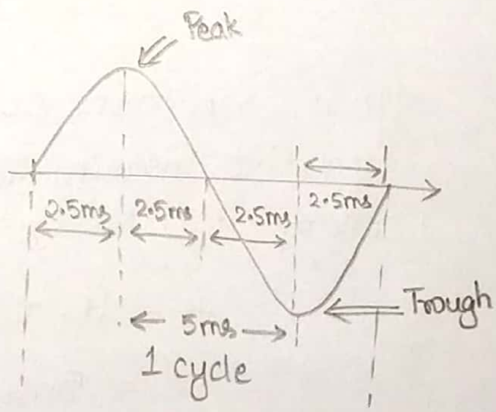
$$T = 10ms$$

Then the frequency is

$$f = \frac{1}{T} = \frac{1}{10ms}$$

$$= 0.1 \text{ kHz}$$

$$f = 100Hz$$



Since $V \neq 0$ at $t = 0$, there is a phase angle.
Thus, the voltage equation is

$$v = V_m \sin(\omega t + \phi)$$

$$\therefore \quad v = 10 \sin(\omega t + \phi)$$
$$= 10 \sin(2\pi f t + \phi)$$
$$v = 10 \sin(628.3t + \phi)$$

At $t = 0$, $v = -3.6V$

$$\Rightarrow \quad -3.6 = 10 \sin(628.3(0) + \phi)$$
$$\Rightarrow \sin(\phi) = -0.36$$
$$\Rightarrow \phi = -158.9^\circ \text{ or } 338.9^\circ.$$

As seen from the waveform, the voltage wave is shifted to the right by an angle less than 180°. Also, the waveform is lagging. Hence,

$$\phi = -158.9^\circ.$$

Thus, the voltage equation is,

$$\boxed{v = 10 \sin(628.3t - 158.9^\circ)} \text{ volts}$$

At $t = 12ms$,

converting degree to radians

$$v = 10 \sin\left[628.3(20 * 10^{-3}) - \left(\frac{158.9 * \pi}{180}\right)\right]$$

$$\boxed{v = -9.985V}$$

10. A 50Hz sinusoidal current has a peak factor of 1.4 and form factor of 1.1. Its average value is 20A. The instantaneous value of current is 15A at t = 0. Calculate the peak value and phase angle. Write down its equation & draw the waveform.

Sol^n

$f = 50Hz$

$$k_p = \frac{I_m}{I_{rms}} = 1.4 \qquad k_f = \frac{I_{rms}}{I_{av}} = 1.1$$

$$\Rightarrow I_m = 1.4(I_{rms}) \qquad \Rightarrow I_{rms} = 1.1(I_{av})$$

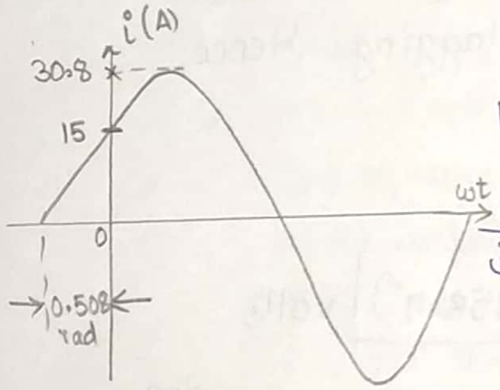
$$= 1.1 * 20$$

$$I_{rms} = 22A$$

$$\therefore I_m = 1.4 * 22$$

$$\boxed{I_m = 30.8A} \text{ - - - - peak value}$$

Given $i = 15A$ at $t = 0$. The equation of the form $i = I_m \sin(\omega t + \phi)$ becomes,



$$i = 30.8 \sin(100\pi t + \phi)$$

$$15 = 30.8 \sin[100\pi(0) + \phi]$$

$$\frac{15}{30.8} = \sin\phi$$

$$\Rightarrow \boxed{\phi = 29.144^\circ}$$

Converting to radians,

$$\phi = 29.144 * \frac{\pi}{180} = 0.50866 \text{ rad}$$

$\therefore$ The equation is,

$$i = 30.8 \sin(\omega t + \phi)$$

$$\boxed{i = 30.8 \sin(100\pi t + 0.50866)} \text{ volts}$$

11. An alternating waveform has a peak factor of 1.5 and a form factor of 1.15. If the maximum value is 4500V, calculate the average & RMS values.

Sol^n:

$$K_f = \frac{V_{rms}}{V_{av}} = 1.15 \qquad K_p = \frac{V_m}{V_{rms}} = 1.5$$

$\therefore$ ~~$V_m = K_p * V_{rms}$~~

$$\therefore V_{rms} = \frac{V_m}{K_p} = \frac{V_m}{1.5} = \frac{4500}{1.5}$$

$$\Rightarrow \boxed{V_{rms} = 3000V}$$

$$K_f = \frac{V_{rms}}{V_{av}} = 1.15 \Rightarrow V_{av} = V_{rms}/1.15$$

$$= 3000/1.15$$

$$\boxed{V_{av} = 2608.69\ V}$$

12. An alternating voltage is given by $v = 141.1 \sin 314t$. Find the following:

(i) frequency (ii) RMS & average values
(iii) instantaneous value at $t = 3ms$
(iv) time taken for the voltage to reach 100V for the first time after passing through zero value.

Sol^n:

$$v = 141.1 \sin(314t)$$

$\therefore V_m = 141.1V.$ (comparing with $v = V_m \sin\omega t$)

$\omega = 314$ rad/sec

$\therefore 2\pi f = 314$

$$\Rightarrow f = \frac{314}{2\pi} \Rightarrow \boxed{f = 50Hz}$$

The RMS value is

$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{141.1}{\sqrt{2}} \Rightarrow \boxed{V_{rms} = 99.77V}$$

The average value is

$$V_{av} = V_m * 0.637$$

$$= 141.1 * 0.637$$

$$\boxed{V_{av} = 89.88V}$$

When $t = 3ms$,

$$v = 141.1 \sin(314 * 3*10^{-3})$$

$$\boxed{v = 114.11V}$$ (keep the calculator in radian mode).

to reach 100A for the first time,

$$100 = 141.1 \sin(314t)$$

$$\Rightarrow \sin(314t) = \frac{100}{141.1} = 0.7087$$

$$\Rightarrow 314t = \sin^{-1}(0.7087)$$

$$t = 45.13^{\circ} = 0.7876 \text{ rad}$$

$\cancel{t = 45.18}$

$$\therefore 314t = 0.7876 \text{ rad}$$

(note that $\omega = 314$ rad/sec $\Rightarrow \omega t =$ rad/sec * sec = rad).

$$\therefore t = \frac{0.7876}{314}$$

$$= 0.002508 s$$

$$\boxed{t = 2.508 ms}$$

13. Draw a phasor diagram showing the following voltages:

$v_1 = 100 \sin 500t$ $\qquad$ $v_2 = 200 \sin(500t + \pi/3)$

$v_3 = -50 \cos 500t$ $\qquad$ $v_4 = 150 \sin(500t - \pi/4)$

Solⁿ:

$$v_1 = 100 \sin(500t) \longrightarrow (1)$$

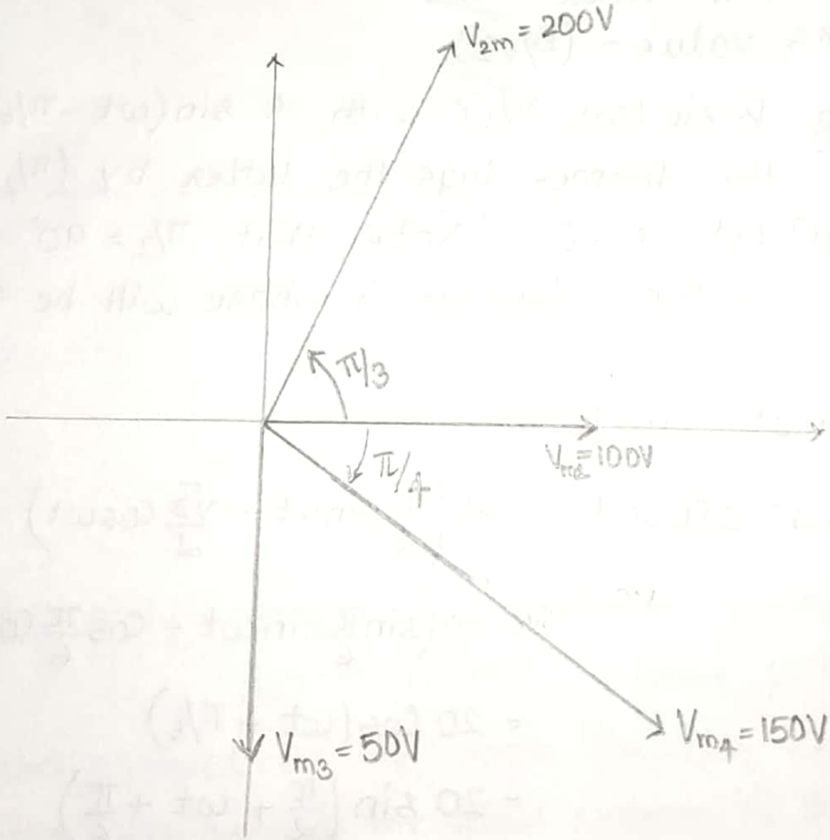
$$v_2 = 200 \sin\left(500t + \frac{\pi}{3}\right) \longrightarrow (2)$$

$$v_3 = -50 \cos(500t)$$
$$= -50 \sin\left(\frac{\pi}{2} - 500t\right)$$

$$v_3 = 50 \sin\left(500t - \frac{\pi}{2}\right) \longrightarrow (3) \quad [\because -\sin\theta = \sin(-\theta)]$$

$$v_4 = 150 \sin(500t - \pi/4) \longrightarrow (4)$$

Phasors for (1) to (4), taking (1) as reference is shown below.



14. Calculate the maximum & RMS values of the following quantities:

(i) $40 \sin \omega t$ (ii) $B \sin(\omega t - \pi/2)$

(iii) $10 \sin \omega t - 17.3 \cos \omega t$

Draw the phasor diagram showing the phase difference with respect to $A \sin(\omega t - \pi/6)$

Sol^n.

(i) For $40 \sin \omega t$,

Maximum value = $\underline{40}$

RMS value = $\frac{\text{max. value}}{\sqrt{2}} = \frac{40}{\sqrt{2}} = \underline{\underline{28.28}}$.

When we compare $40 \sin(\omega t)$ with $A \sin(\omega t - \pi/6)$, we see that the given waveform leads by $\pi/6$ rad i.e. $30°$.

(ii) for $B \sin(\omega t - \pi/2)$

Maximum value = $\underline{B}$

RMS value = $\underline{\underline{(B/\sqrt{2})}}$

Comparing $B \sin(\omega t - \pi/2)$ with $A \sin(\omega t - \pi/6)$, we see that the former lags the latter by $(\pi/2 - \pi/6)$ rad i.e., 1.047 rad or $60°$. (Note that $\pi/2 = 90°$ while $\pi/6 = 30°$, & the difference in phase will be $90 - 30 = 60°$).

(iii) For $10 \sin \omega t - 17.3 \cos \omega t$

$$10 \sin \omega t - 17.3 \cos \omega t = 20\left(\frac{1}{2} \sin \omega t - \frac{\sqrt{3}}{2} \cos \omega t\right)$$

$$= 20\left(\sin\frac{\pi}{6} \sin \omega t - \cos\frac{\pi}{6} \cos \omega t\right)$$

$$= 20 \cos(\omega t + \pi/6)$$

$$= 20 \sin\left(\frac{\pi}{2} + \omega t + \frac{\pi}{6}\right)$$

$\therefore$ Maximum value = $\underline{\underline{20}}$

RMS value = $20/\sqrt{2} = \underline{\underline{14.14 A}}$.

comparing this with the waveform $A \sin(\omega t - \pi/6)$, we see that the phase difference is

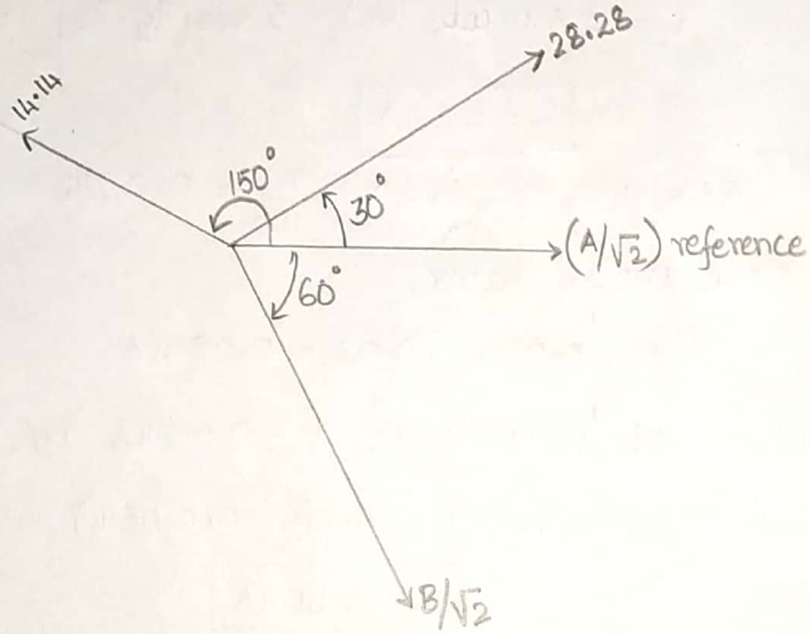
$$\frac{\pi}{2} + \frac{\pi}{6} - (-\pi/6) = \frac{\pi}{2} + \frac{2\pi}{6} = \frac{\pi}{2} + \frac{\pi}{3} = \frac{5\pi}{6}$$

$\therefore$ phase difference $= \frac{5\pi}{6}$ radians

$$= \frac{5\pi}{6} * \frac{180^\circ}{\pi} = \frac{5 * 180}{6} = 5 * 30$$

$$= 150^\circ$$

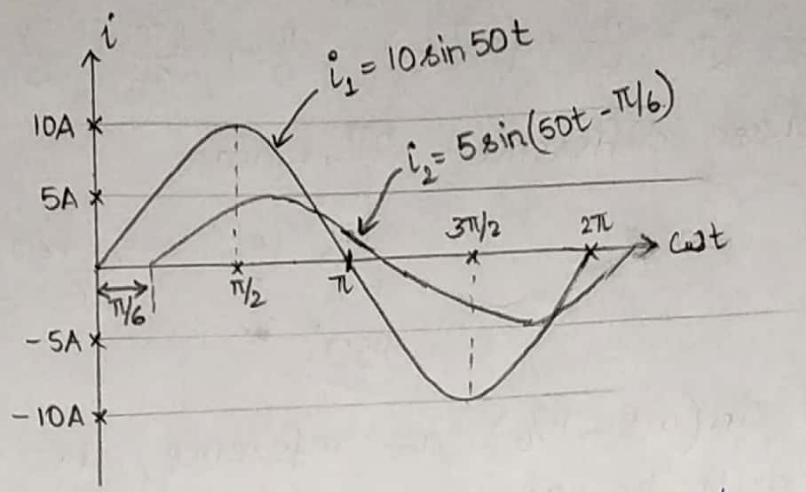
Taking $A \sin(\omega t - \pi/6)$ as reference, the phasor diagrams will be as shown below.



15. An alternating current is represented by the equation $i_1 = 10 \sin 50t$, where 't' is in seconds.

A second current of the same frequency but half the amplitude lags behind the first current by 30°. Find the value of the second current when the first current is at the positive peak, and also the values of both currents 0.02s later. Sketch the waveform of both the currents.

Sol^n:



30° when converted to radians is $30\pi/180 = \pi/6$ radians

When i_1 is at its peak, $\omega t = 50t = 90^\circ = \frac{\pi}{2}$ radians. At this point, magnitude of second current is,

$$i_2 = 5\sin(50t - \pi/6) = 5\sin(\pi/2 - \pi/6) = 5\sin(\pi/3)$$

$$\Rightarrow \boxed{i_2 = 4.33A}$$

$$\omega t = 5t = 90^\circ = \pi/2 \Rightarrow t = \frac{\pi}{100} = 0.0314\,s$$

At a time 0.02s later,

$$t = 0.0314 + 0.02 = 0.0514\,s.$$

The values of currents at $t = 0.0514s$ are,

$$i_1 = 10\sin 50t = 10\sin(50*0.0514)$$

$$\Rightarrow \boxed{i_1 = 5.409A}$$

and $i_2 = 5\sin(50t - \pi/6)$

$$= 5\sin[(50*0.0514) - \pi/6]$$

$$\boxed{i_2 = 4.45A}$$