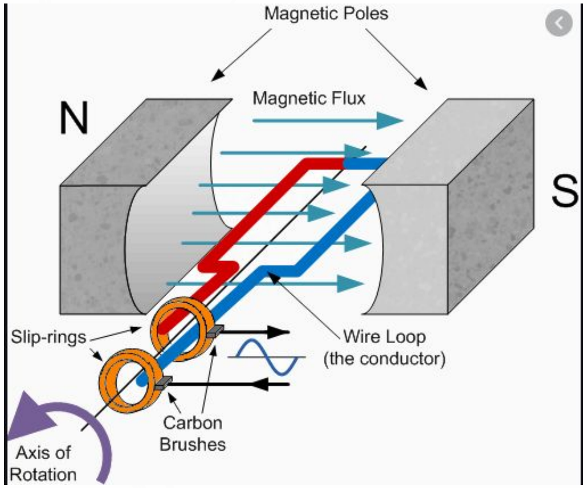
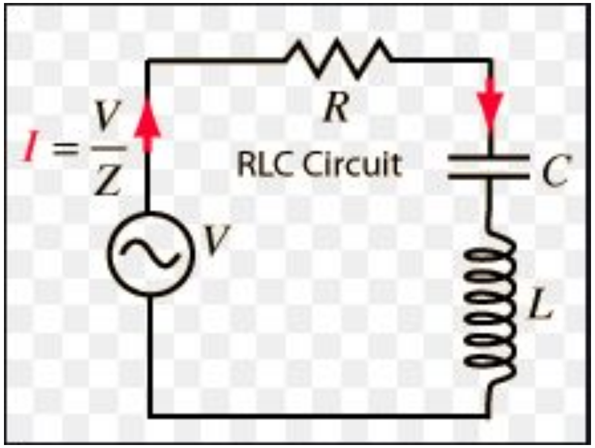
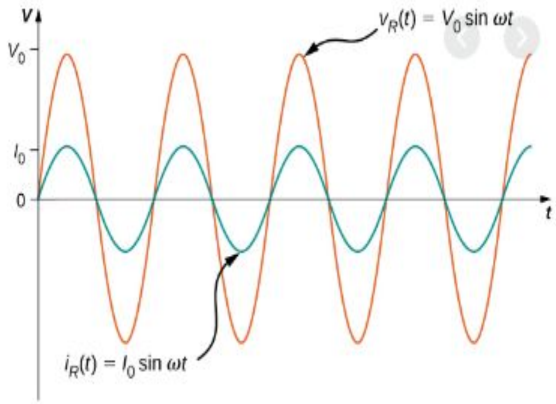
Elements of Electrical Engineering (21EE13)

AC Fundamentals
UNIT 2

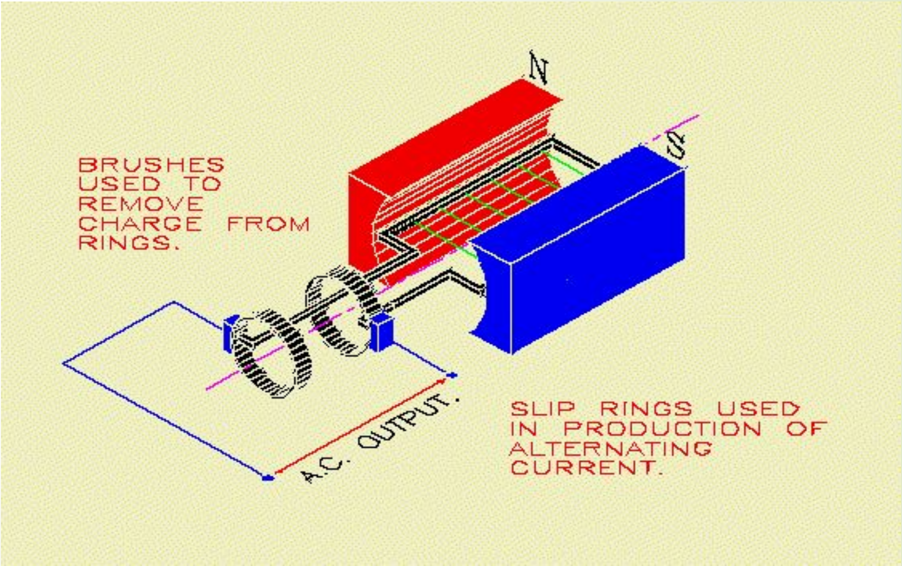






Alternating Voltage is a voltage that:

1. **Continuously varies in magnitude**
2. **Periodically reverses in polarity**



Symbol for a sinusoidal voltage source.

A wave is a disturbance.

- Unlike water waves, electrical waves cannot be seen directly but they have similar characteristics.
- <u>All</u> periodic waves can be constructed from **sine waves**, which is why sine waves are fundamental to alternating current.

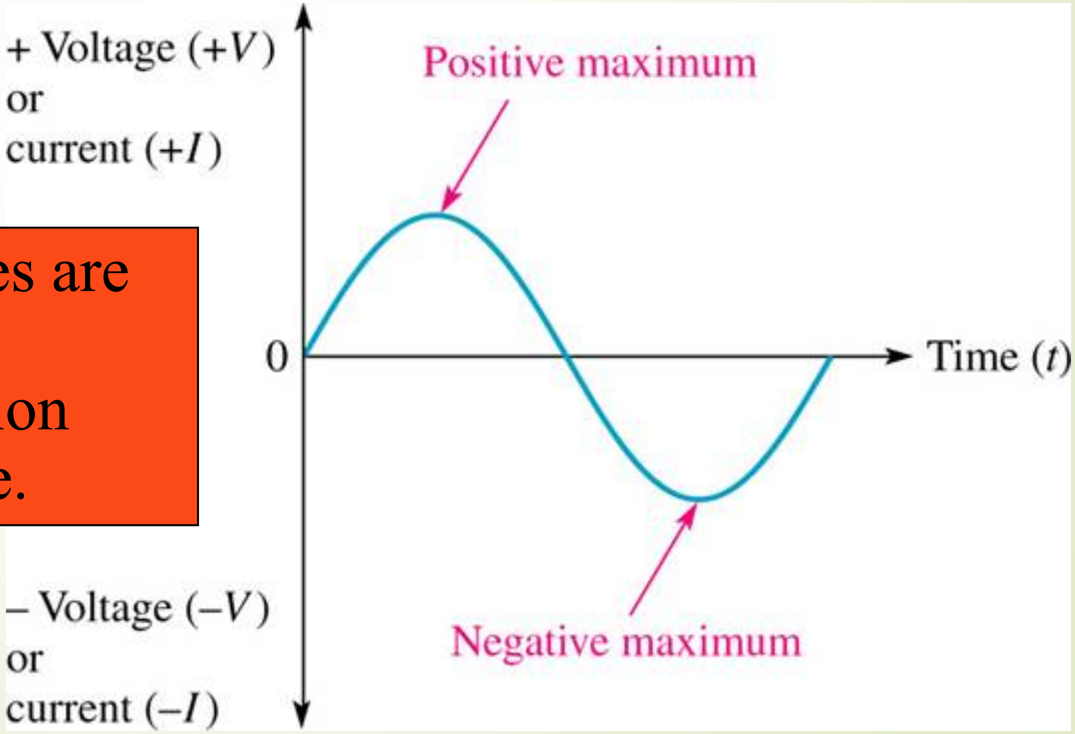
Advantages of AC:

- The voltages in AC system can be raised or lowered with the help of device called Transformer.
- As Voltages can be raised, electrical transmission at high voltages is possible.
- The construction and cost of AC generators is economical.
- AC machines require less maintenance.
- Whenever required AC can be easily converted to DC.

Sine waves

The sinusoidal waveform (sine wave) is the fundamental alternating current (ac) and alternating voltage waveform.

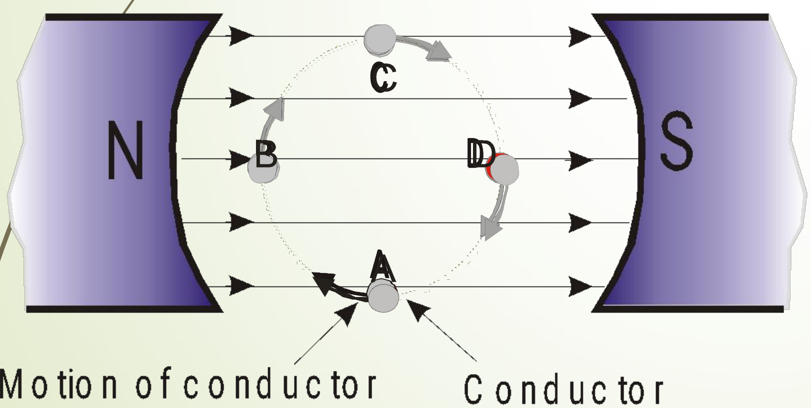
Electrical sine waves are named from the mathematical function with the same shape.

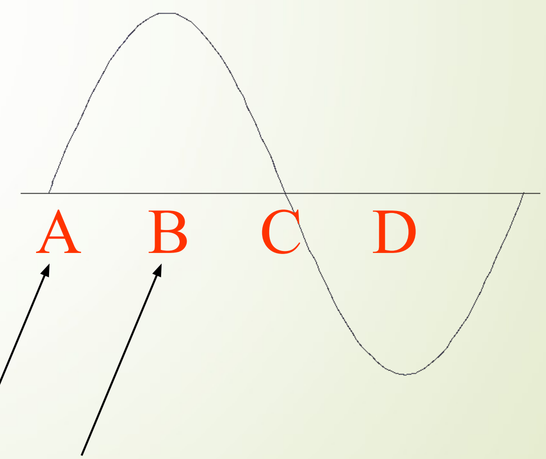


Sinusoidal voltage sources

Sinusoidal voltages are produced by ac generators and electronic oscillators.

When a conductor rotates in a constant magnetic field, a sinusoidal wave is generated.





When the conductor is moving parallel with the lines of flux, no voltage is induced.

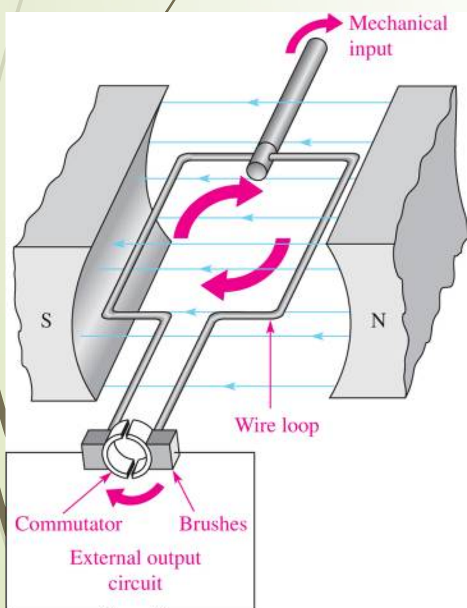
When the loop is moving perpendicular to the lines of flux, the maximum voltage is induced.

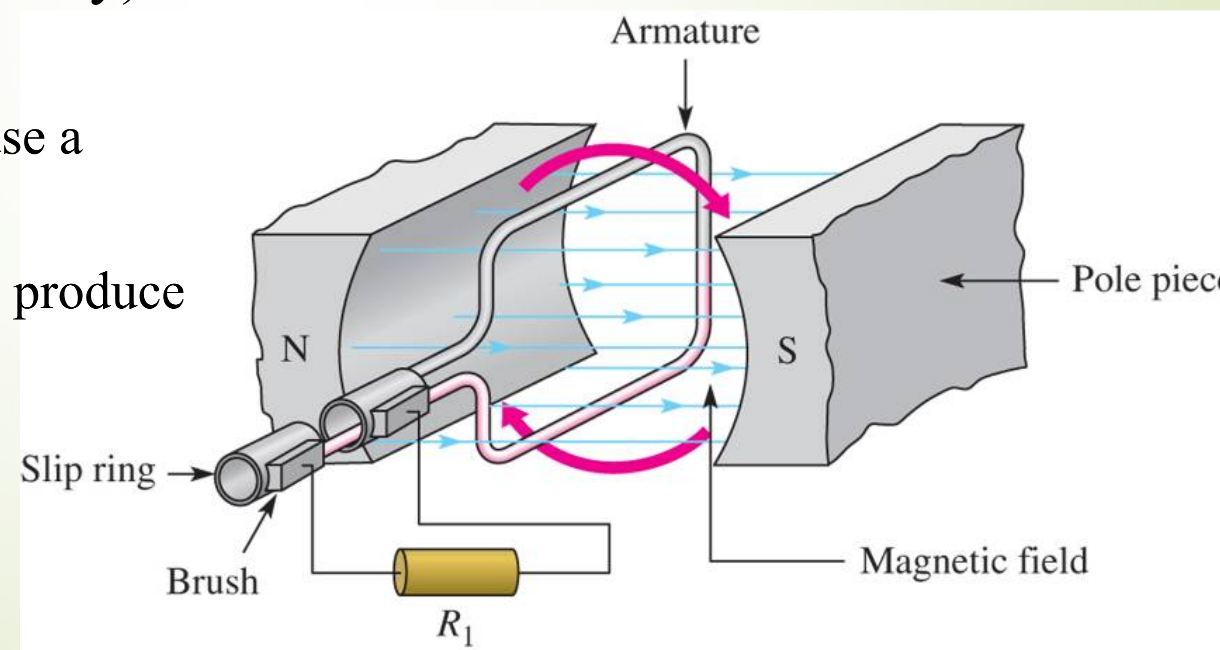
AC generator (alternator)

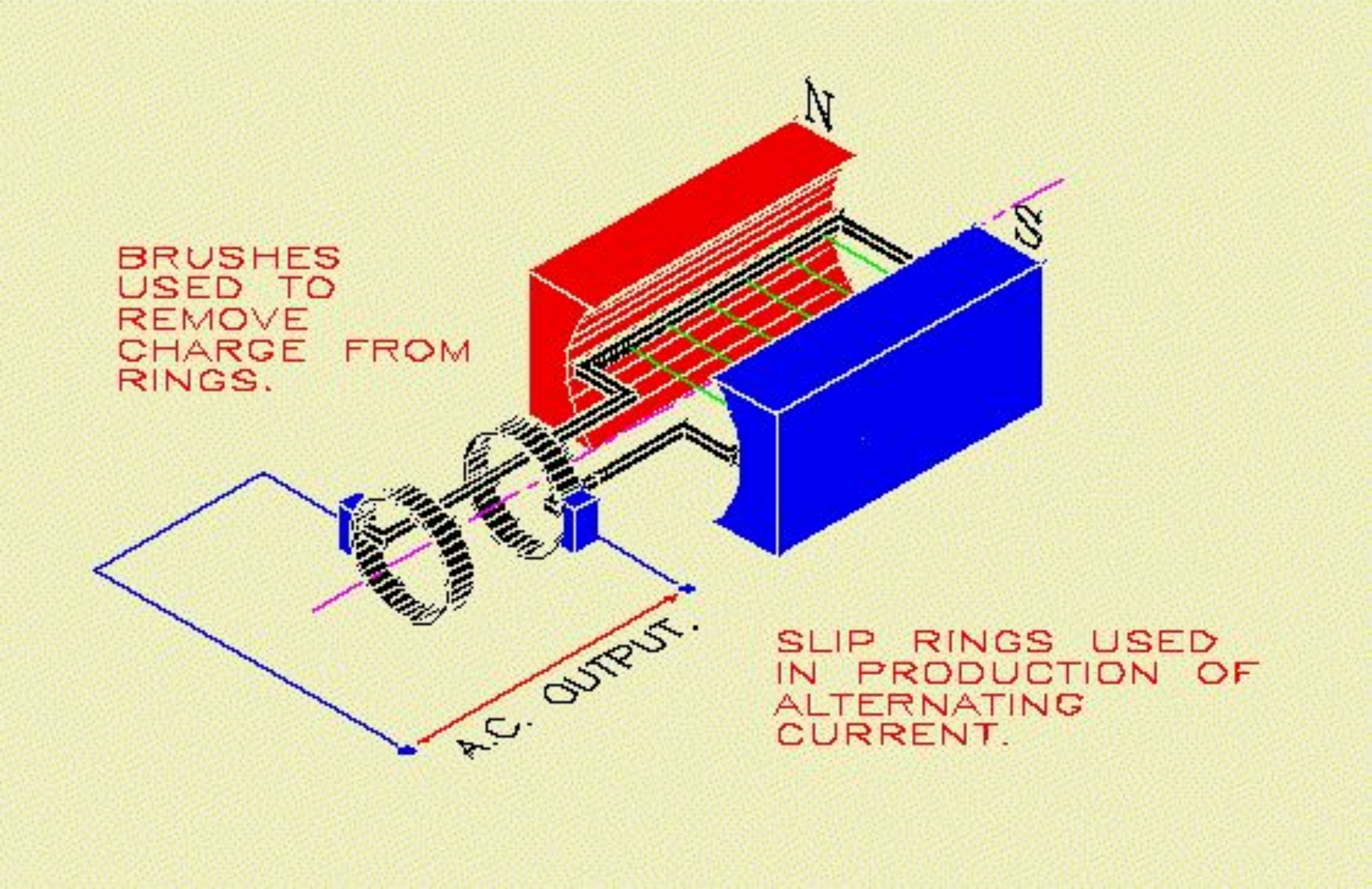
- Generators convert rotational energy to electrical energy.
- The armature has an induced voltage, which is connected through slip rings and brushes to a load.
- The armature loops are wound on a magnetic core (not shown for simplicity).

y use a

to produce

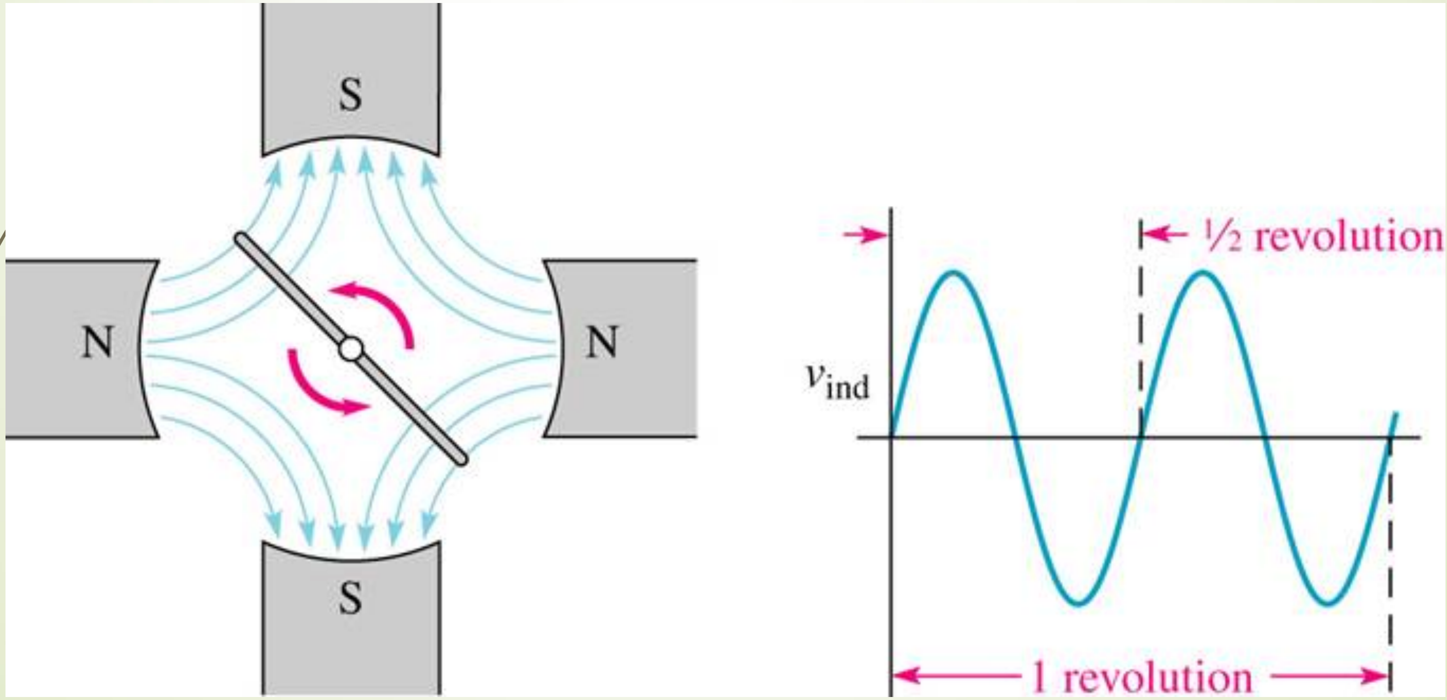





N
S
BRUSHES USED TO REMOVE CHARGE FROM RINGS.
A.C. OUTPUT.
SLIP RINGS USED IN PRODUCTION OF ALTERNATING CURRENT.

AC generator (alternator)

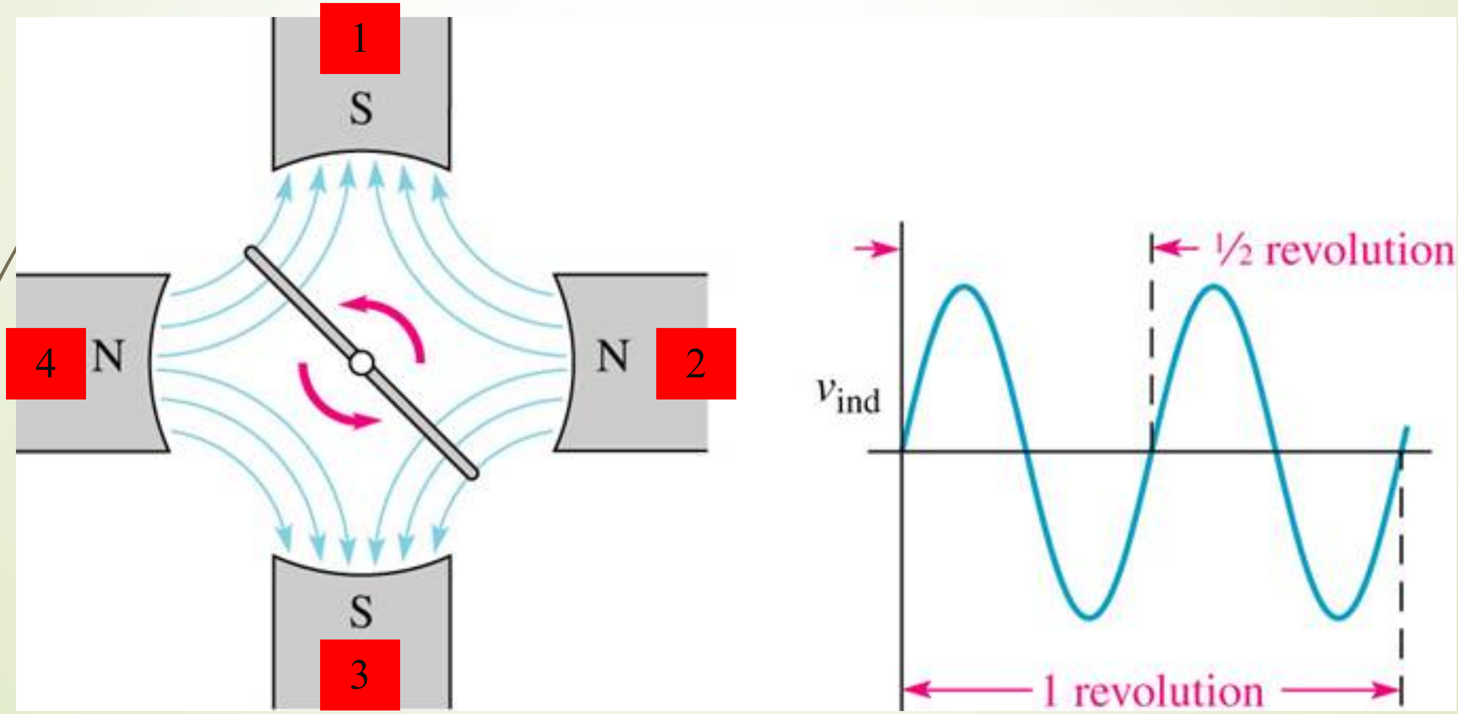
- Increasing the number of poles increases the number of cycles per revolution.
- A four-pole generator will produce two complete cycles in each revolution.



Output Frequency of an AC Generator

$$f = \frac{Ns}{120}$$

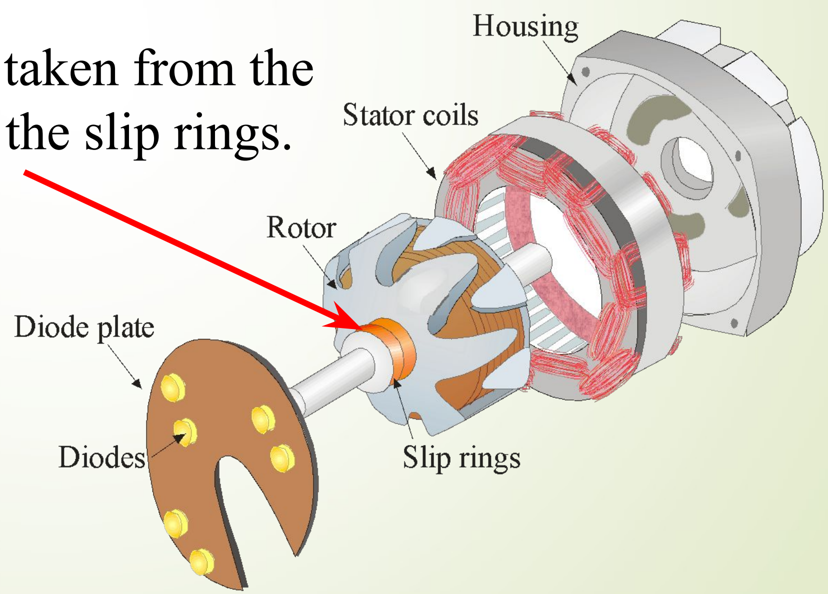
f **– frequency (Hz)**
N – number of poles
s **- speed in RPM**



Alternators

- In vehicles, alternators generate ac, which is converted to dc for operating electrical devices and charging the battery.
- AC is more efficient to produce and can be easily regulated, hence it is generated and converted to DC by diodes.

The output is taken from the rotor through the slip rings.



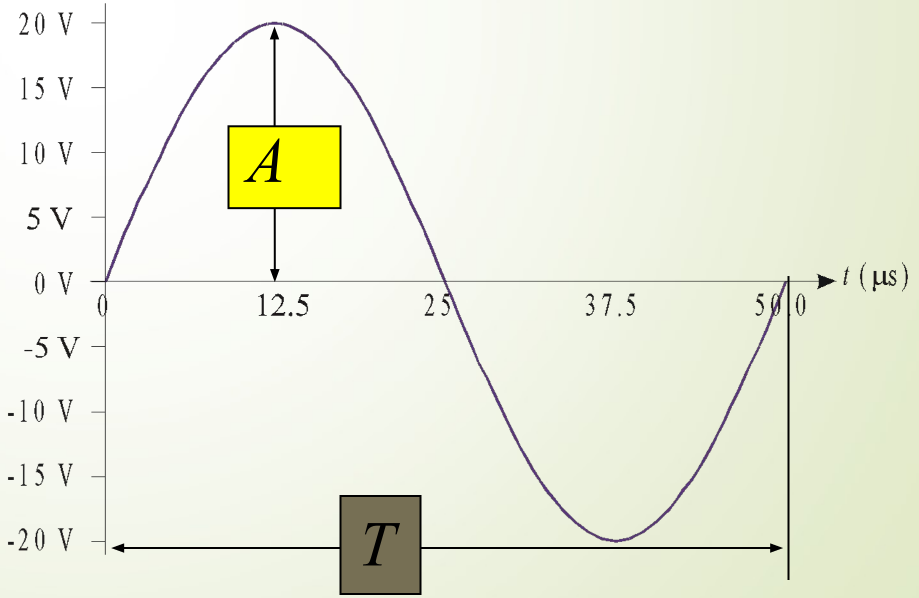
Sine waves

Sine waves are characterized by the amplitude and period.

1. The **amplitude** is the maximum value of a voltage or current
2. The **period** is the time interval for one complete cycle.

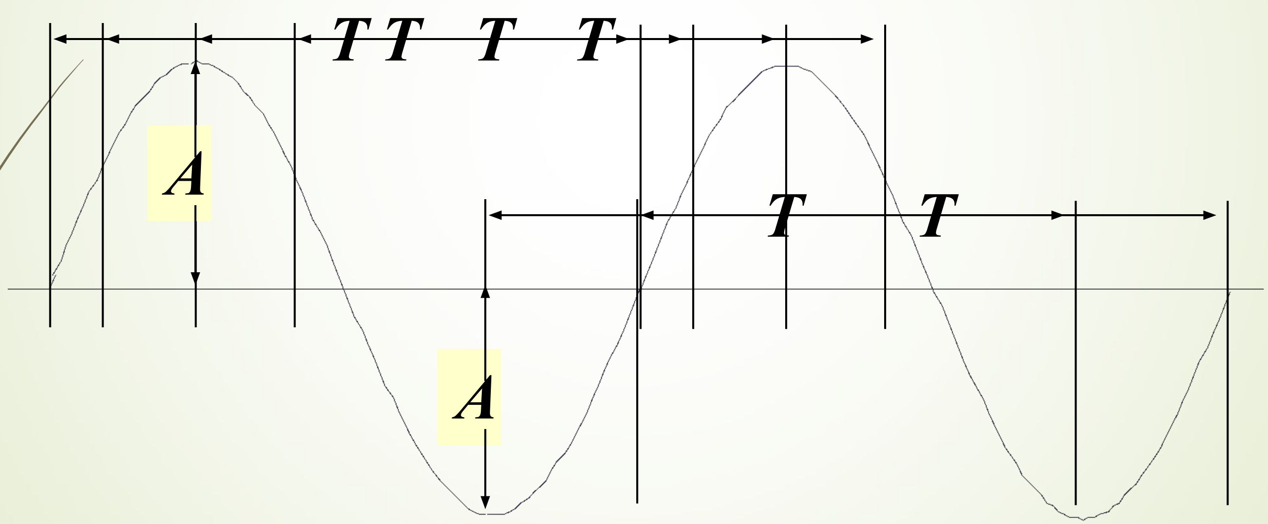
The amplitude (A) of this sine wave is 20 V

The period is 50.0 μs

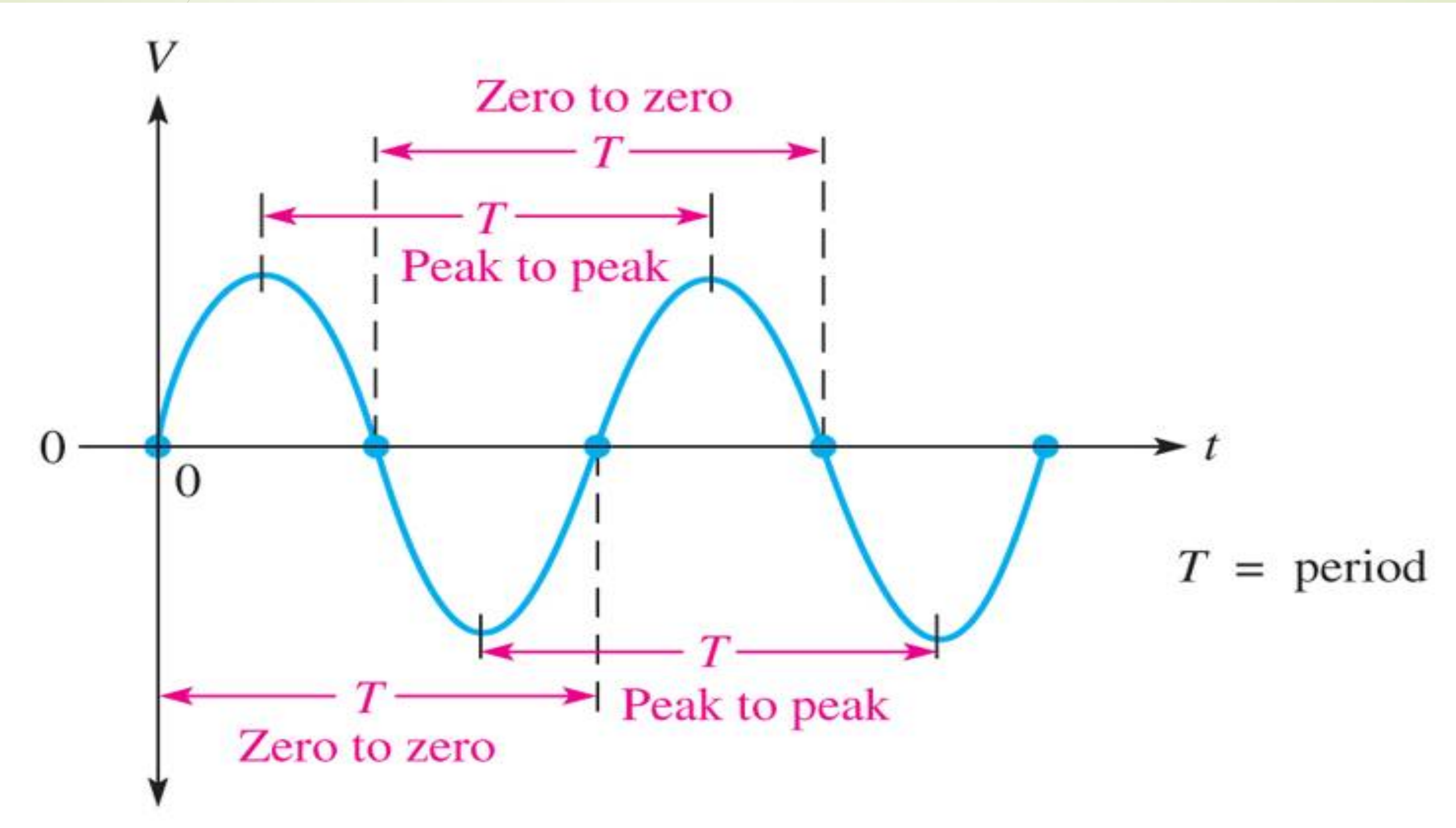


Sine waves

The period (T) of a sine wave can be measured between any two corresponding points on the waveform.



By contrast, the amplitude of a sine wave is only measured from the center to the maximum point.

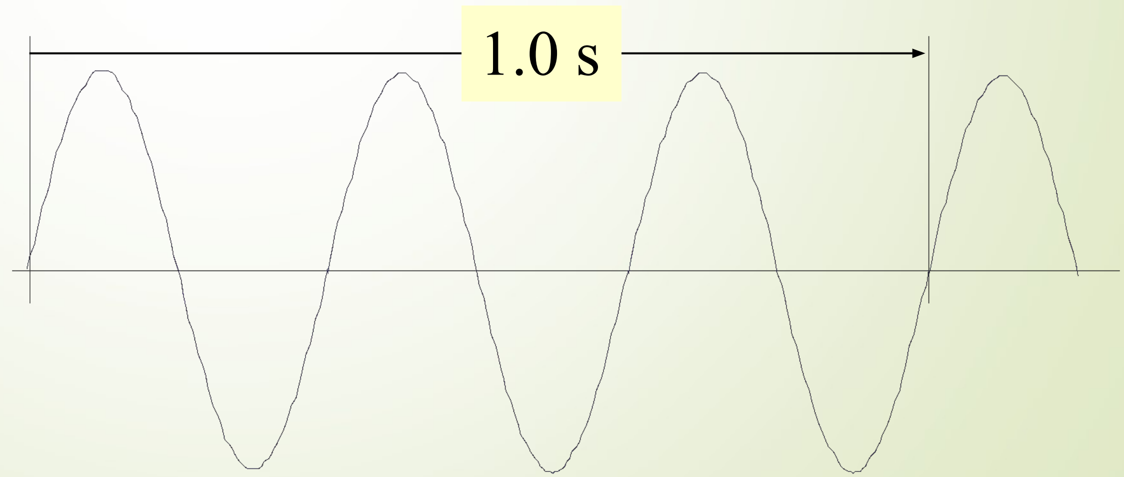

V
Zero to zero
T
T
Peak to peak
0
0
t
T = period
T
Peak to peak
T
Zero to zero

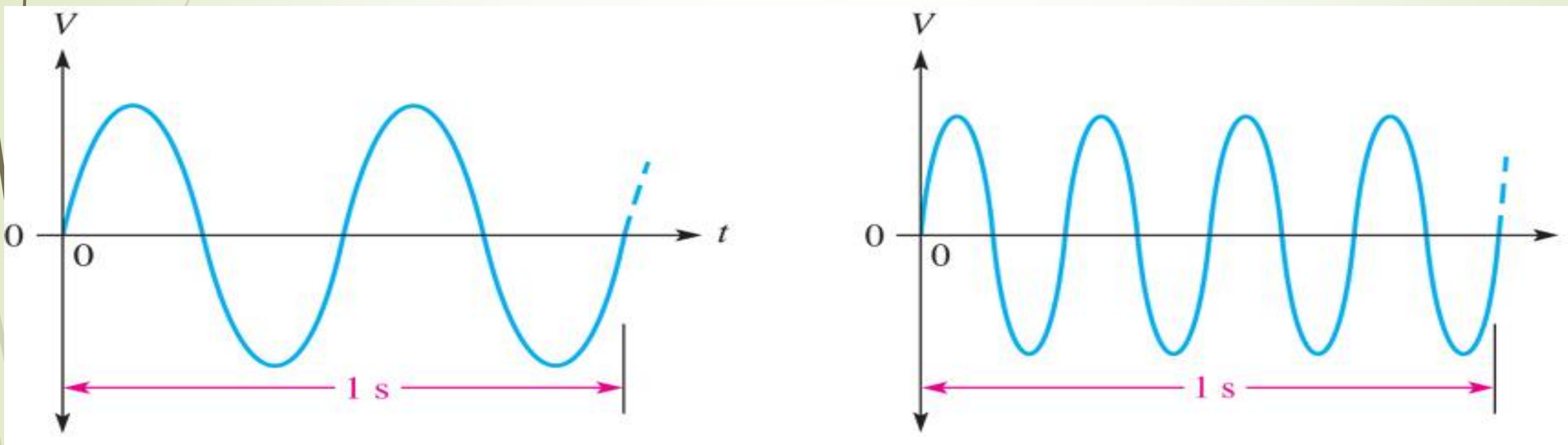
Frequency

Frequency (*f*) is the number of cycles that a sine wave completes in one second.

Frequency is measured in **hertz** (Hz).

If 3 cycles of a wave occur in one second, the frequency is 3.0 Hz





(a) Lower frequency: fewer cycles per second

(b) Higher frequency: more cycles per second

Period and frequency

The period and frequency are reciprocals of each other.

$$f = \frac{1}{T}$$

and

$$T = \frac{1}{f}$$

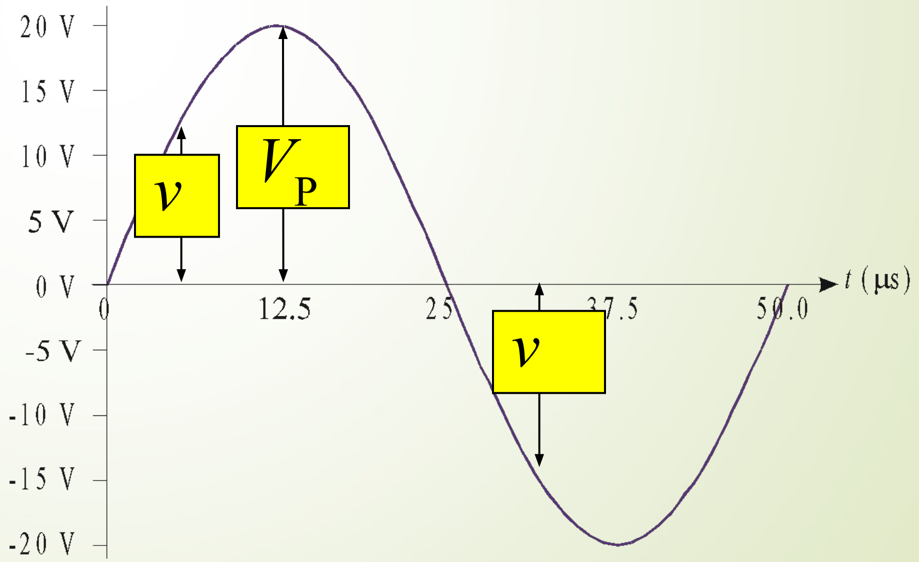
If the period is 50 μs, the frequency is 0.02 MHz = 20 kHz.

(The $\boxed{1/x}$ key on your calculator is handy for converting between f and T.)

Sine wave voltage and current values

- **Instantaneous value (*v*): Voltage or current at any point on the curve.**
- **Peak value (V_P for voltage): The amplitude of a sine wave.**

The peak voltage of this waveform is 20 V.

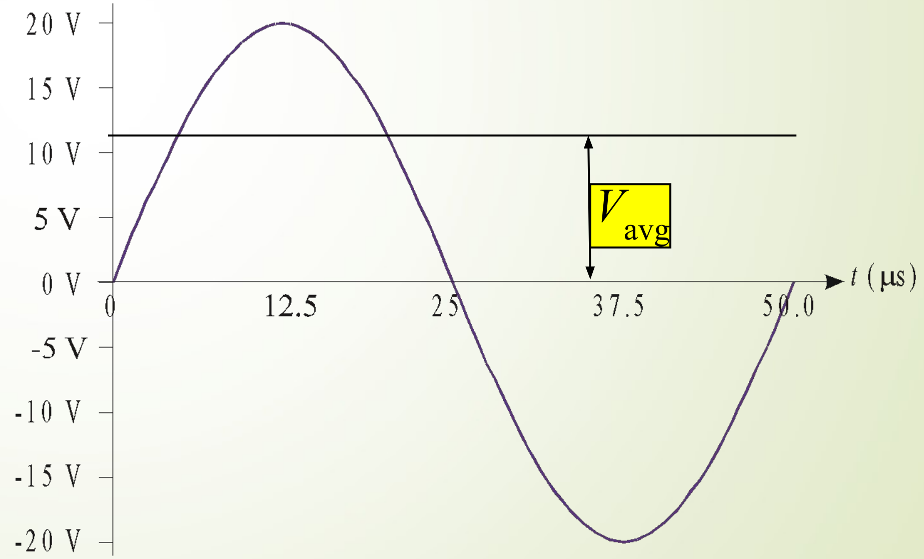


Sine wave voltage and current values

The average value (actually the half-wave average) is used when comparing power supplies.

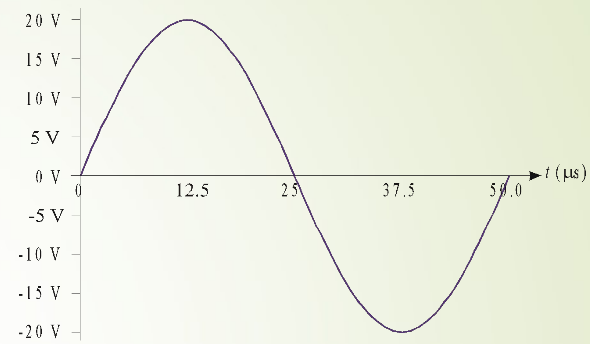
$$V_{avg} = 0.637 V_p \qquad I_{avg} = 0.637 I_p$$

The average value for the sinusoidal voltage is **12.7 V.**



Sine wave voltage and current values

Peak to peak value: Value from positive peak to negative peak. Equation =



$$V_{PP} = 2V_P \qquad I_{PP} = 2I_P$$

RMS (root mean squared) value: Is the sinusoidal wave with the same heat value as a DC voltage source (known as the effective value)

$$V_{rms} = 0.707V_P \qquad I_{rms} = 0.707I_P$$

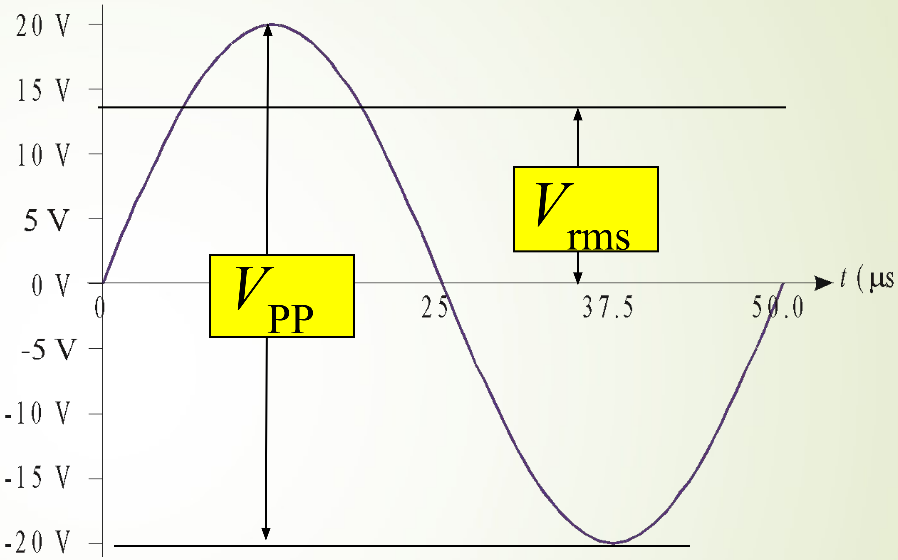
$$V_p = 1.414V_{vms} \qquad I_p = 1.414I_{vms}$$

Sine wave voltage and current values

$V_P = 20$ volts

The peak-to-peak voltage is **40 V.**

The rms voltage is **14.1 V.**



$$V_{PP} = 2.828 V_{rms}$$

$$V_P = 1.414 V_{rms}$$

This is magnitude V_{pp}

Angular Measurement

Degree:

Angular measurement equal to 1/360 of the circumference of a circle.

Radian (rad):

Angle formed when the distance along the circumference of a circle equal the radius of the circle.

PI (π):

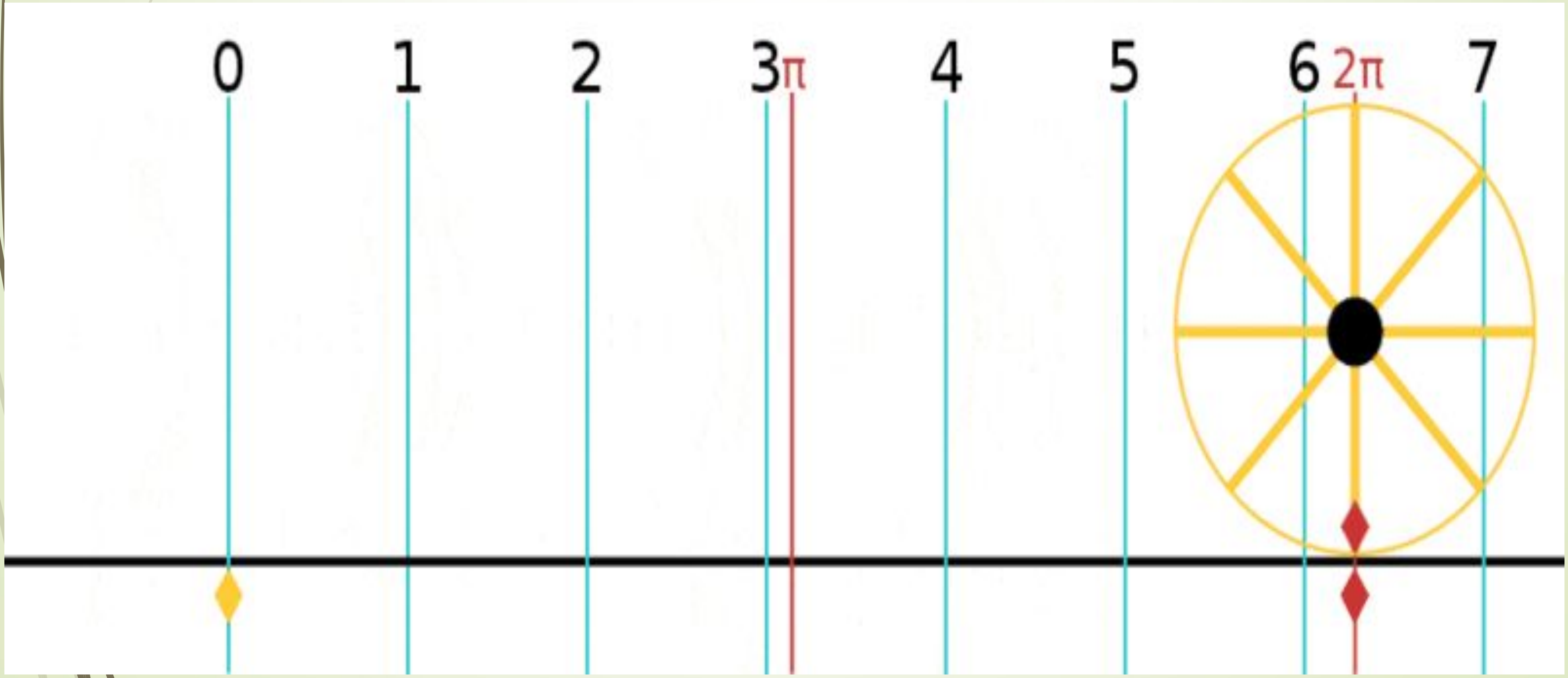
The ratio of the circumference of a circle to its diameter.

Has a constant value ≈ 3.1416

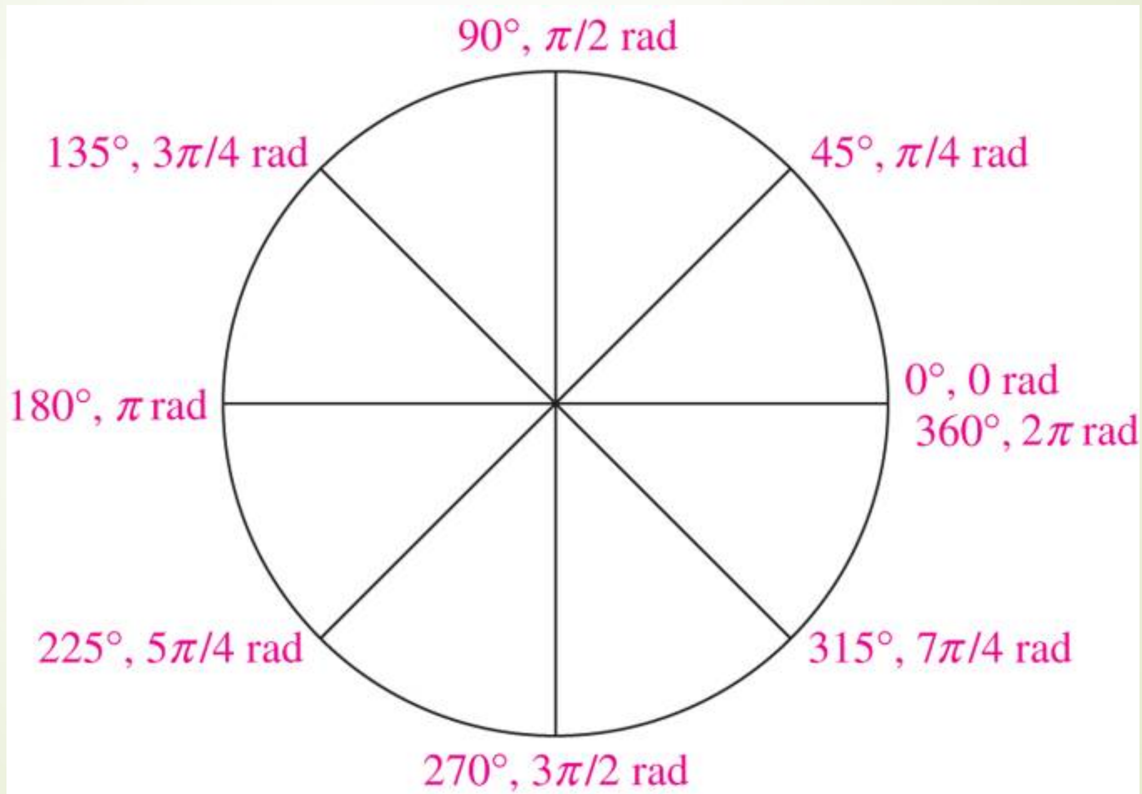
Angular measurement

Angular measurements can be made in degrees (^{o}) or radians.

There are 360^o or 2π radians in one complete revolution.



Angular measurements



Angular measurement

There are 2π radians in one complete revolution and 360^{o} in one revolution. To find the number of radians, given the number of degrees:

$$\text{rad} = \frac{2\pi\ \text{rad}}{360°} \times \text{degrees}$$

This can be simplified to:

$$\boxed{\text{rad} = \frac{\pi\ \text{rad}}{180°} \times \text{degrees}}$$

To find the number of degrees, given the number of radians:

$$\boxed{\text{deg} = \frac{180°}{\pi\ \text{rad}} \times \text{rad}}$$

Angular measurement

How many radians are in 45°?

$$\text{rad} = \frac{\pi \ \text{rad}}{180^\circ} \times \text{degrees}$$

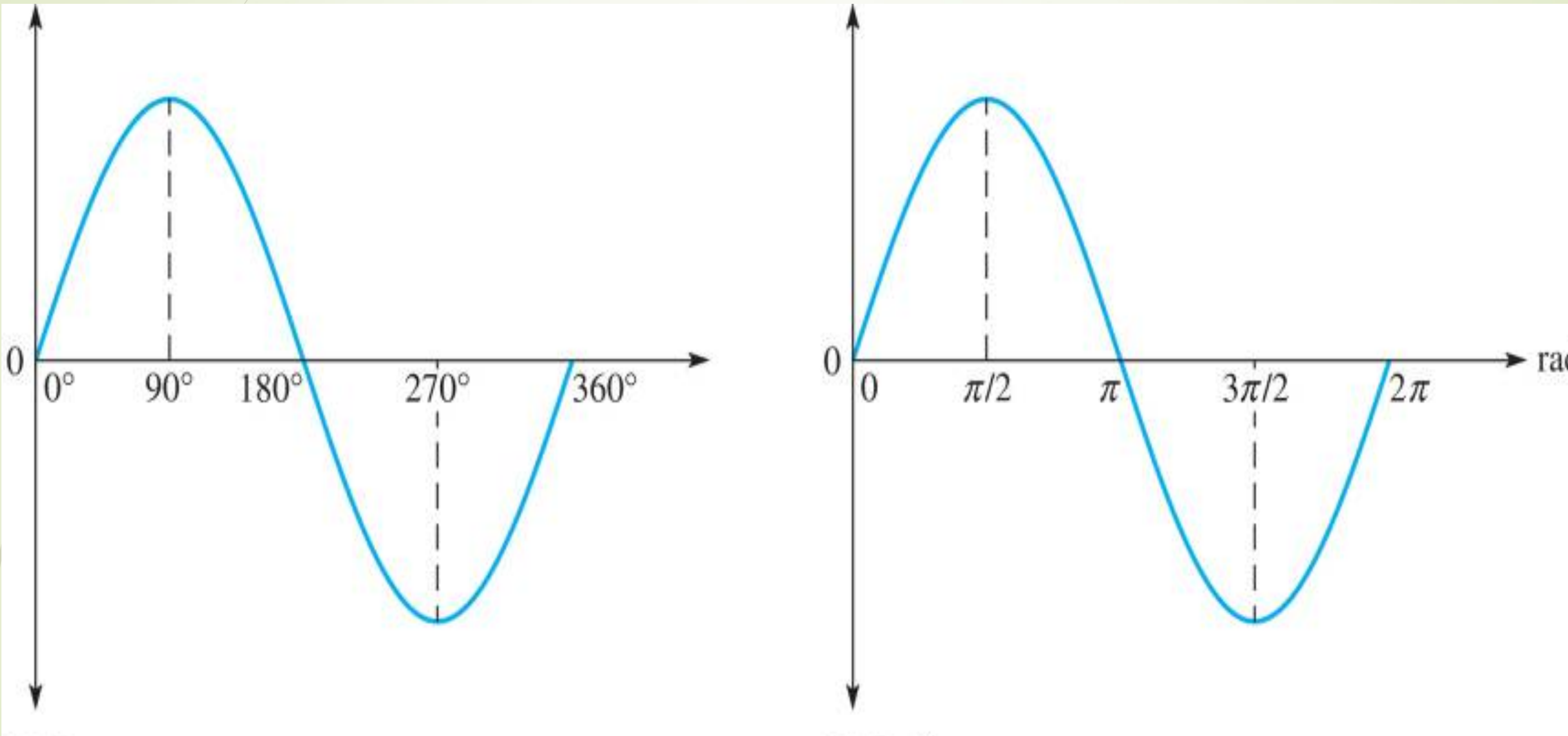
$$=$$

How many degrees are in 1.2 radians?

$$\deg = \frac{180^\circ}{\pi \ \text{rad}} \times \text{rad}$$

$$=$$

Sine wave angles

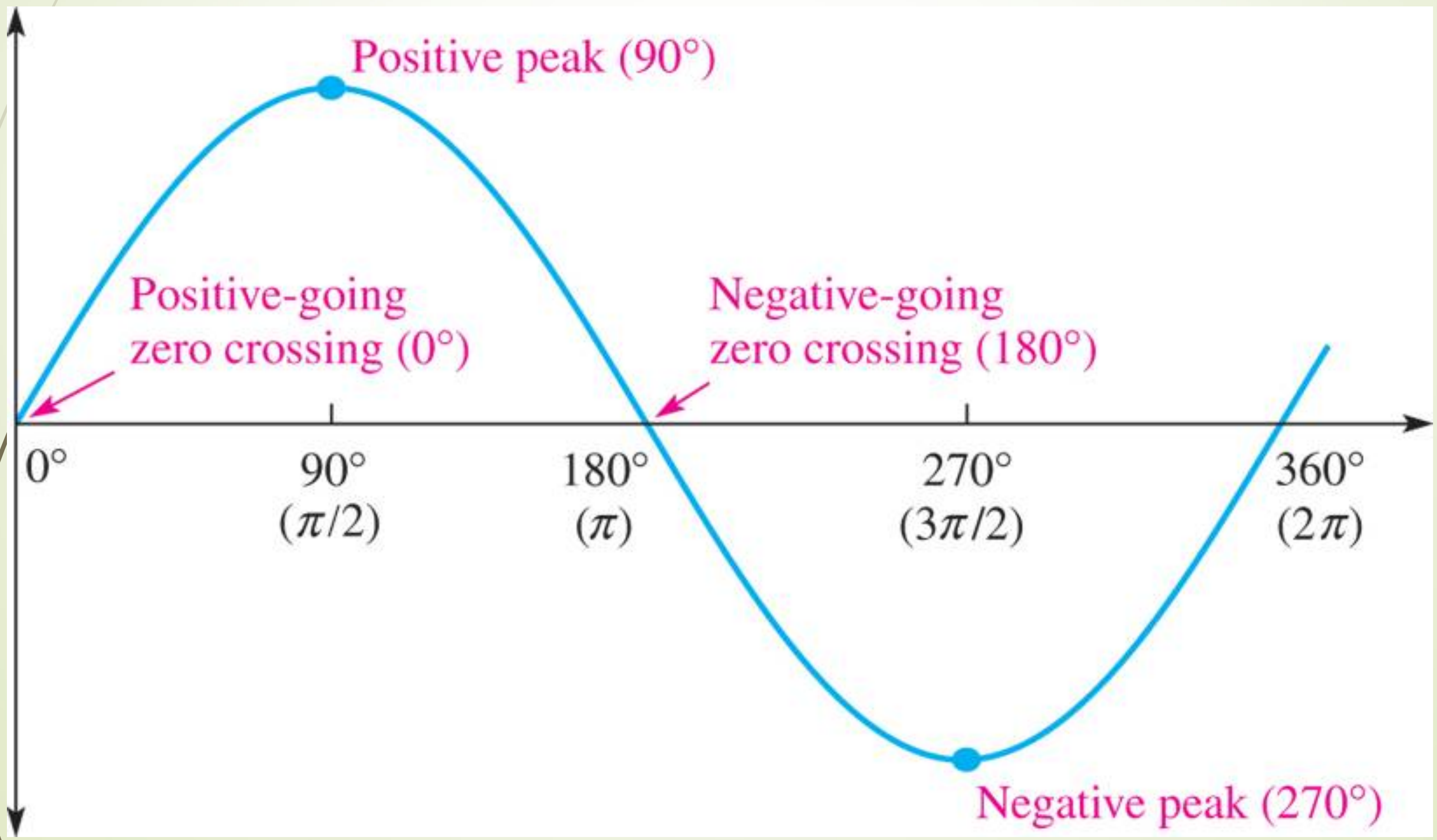


(a) Degrees (b) Radians

Phase of a Sine Wave

Phase:

Angular measurement that specifies the position on the sine wave relative to a reference point.



Phase shift

Phase Shift of a sine wave is an angular measurement that specifies the position of a sine wave relative to a reference wave.

- Sine wave shifted right (lags) – ϕ
- Sine wave shifted left (leads) + ϕ

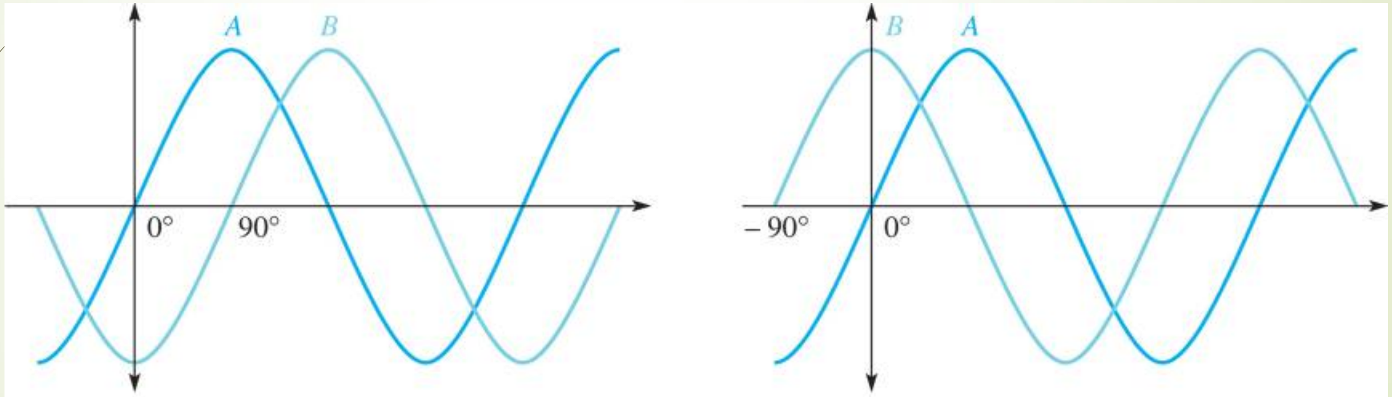
$$\boxed{v = V_{\mathrm{P}} \sin(\theta \pm \phi)}$$

where

$\varphi =$ Phase shift

Phase shifts

Occurs when a sine wave is shifted right or left in relation to the base/reference sine wave.

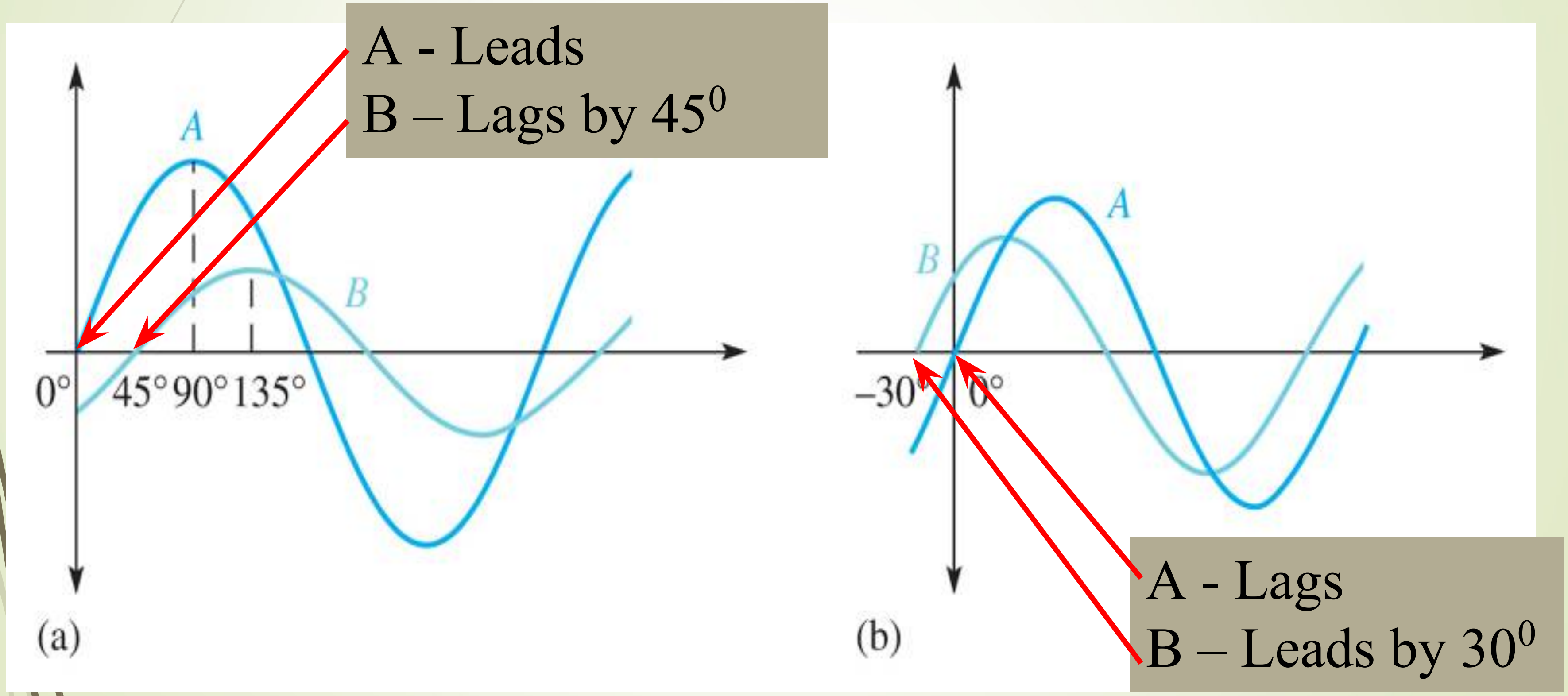


(a) *A* leads *B* by 90°, or *B* lags *A* by 90°.

(b) *B* leads *A* by 90°, or *A* lags *B* by 90°.

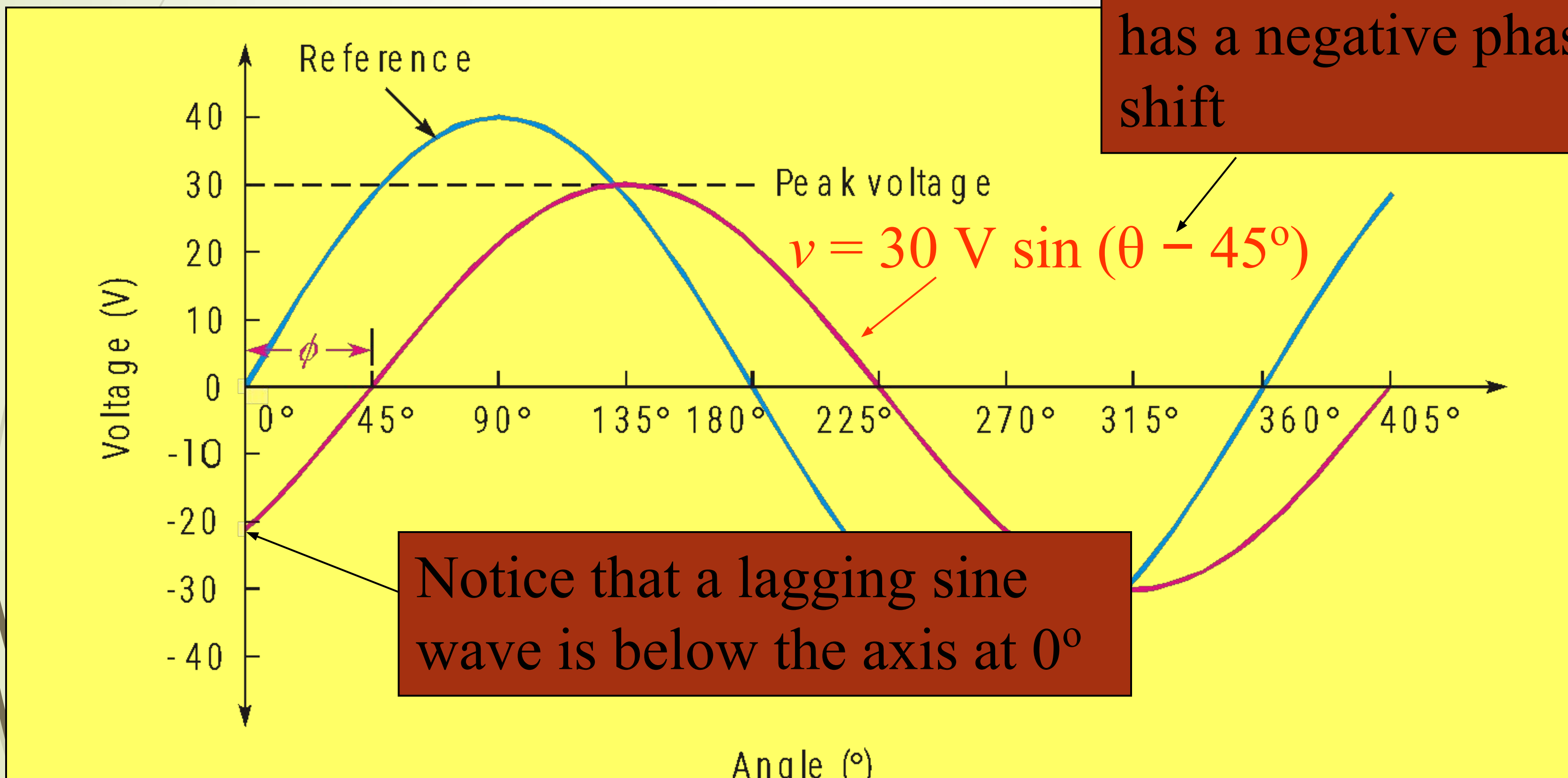
Phase shift – Lead/Lag

Occurs when a sine wave is shifted right or left in relation to the base/reference sine wave.



Phase shift

Example of a wave that lags the reference (not on guided notes)



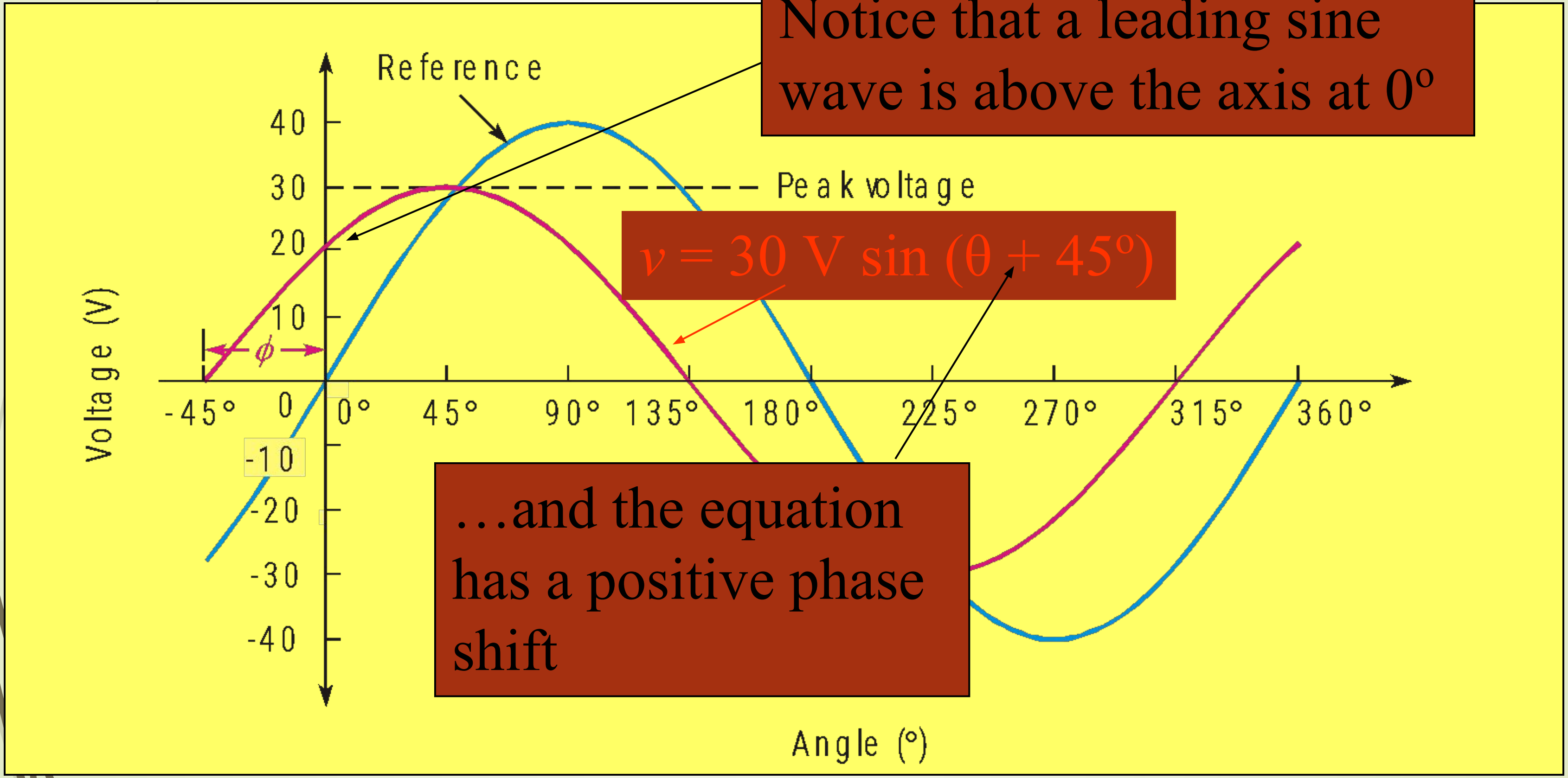
$v = 30\text{ V} \sin(\theta - 45^{o})$

…and the equation has a negative phase shift

Notice that a lagging sine wave is below the axis at 0^{o}

Phase shift

Example of a wave that leads the reference (not on guided notes)



Notice that a leading sine wave is above the axis at 0°

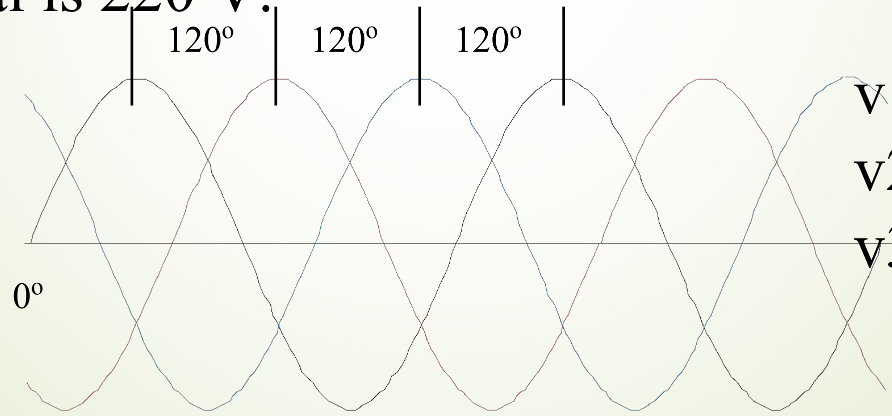
$v = 30\text{ V} \sin(\theta + 45^{\circ})$

…and the equation has a positive phase shift

PolyPhase power

An important application of phase-shifted sine waves is in electrical power systems.

- Electrical utilities generate ac with three phases that are separated by 120°.
  - 3-phase power is delivered to the user with three live lines plus neutral. The voltage of each phase, with respect to neutral is 220 V.



$v1 = Vm \sin\theta$

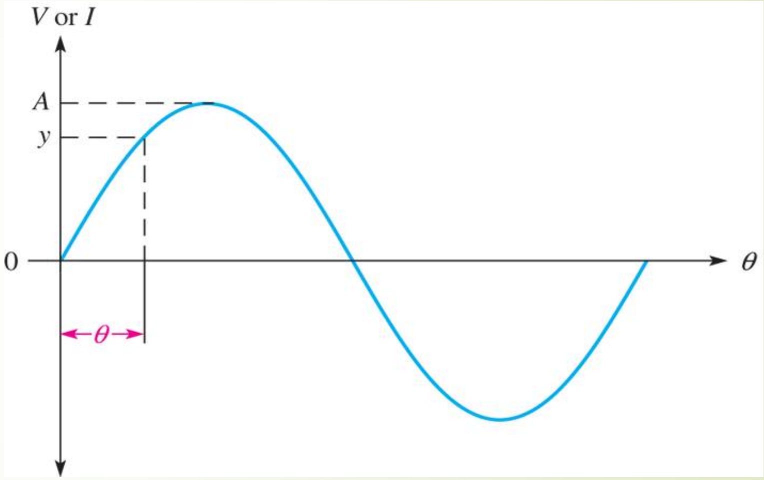
$v2 = Vm \sin(\theta - 120)$

$v3 = Vm \sin(\theta + 120)$

Sine wave equation

Instantaneous values of a wave are shown as *v* or *i*.

The equation for the instantaneous voltage (*v*) of a sine wave is



$$v = V_p \sin\theta$$

where

V_p = Peak voltage

θ *(theta)* = Angle in rad or degrees

If the peak voltage is 25 V, the instantaneous voltage at 50 degrees is 19.2 V

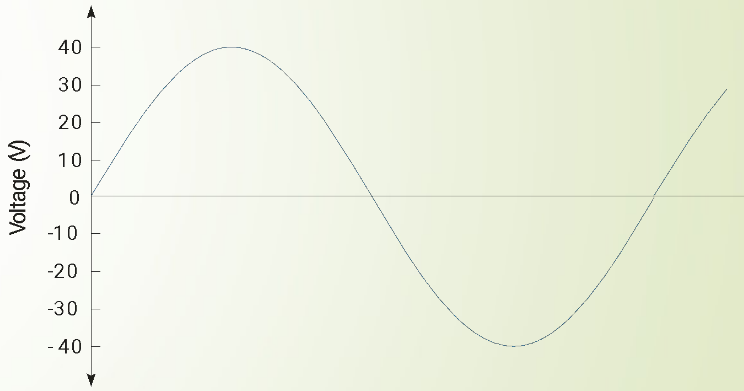
Sine wave equation

A certain sine wave has a positive-going zero crossing at 0° and an peak value of 40V. Calculate its instantaneous voltage for the degrees listed below for the sine wave below.

$$\boxed{v = V_p \sin\theta}$$

V_p = Peak voltage

θ *(theta)* = Angle in rad or degrees



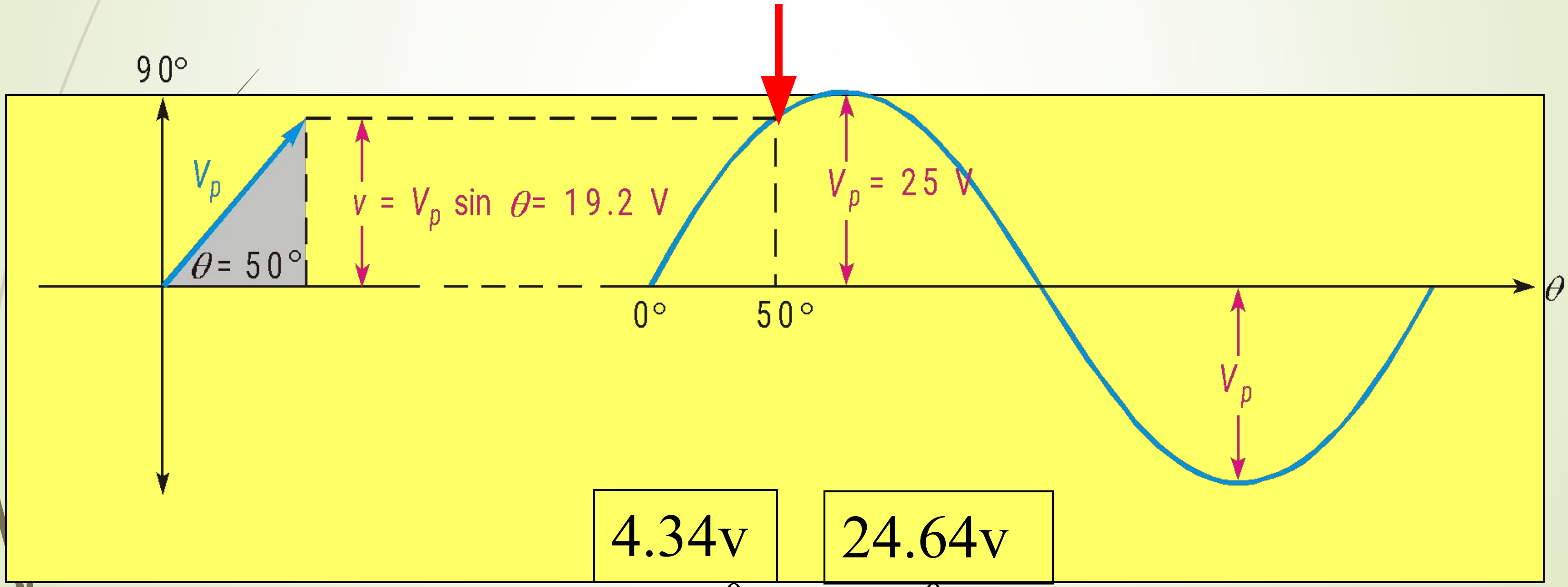
45°, 125°, 180°, 220°,325°

Sine wave equation

- Instantaneous Values

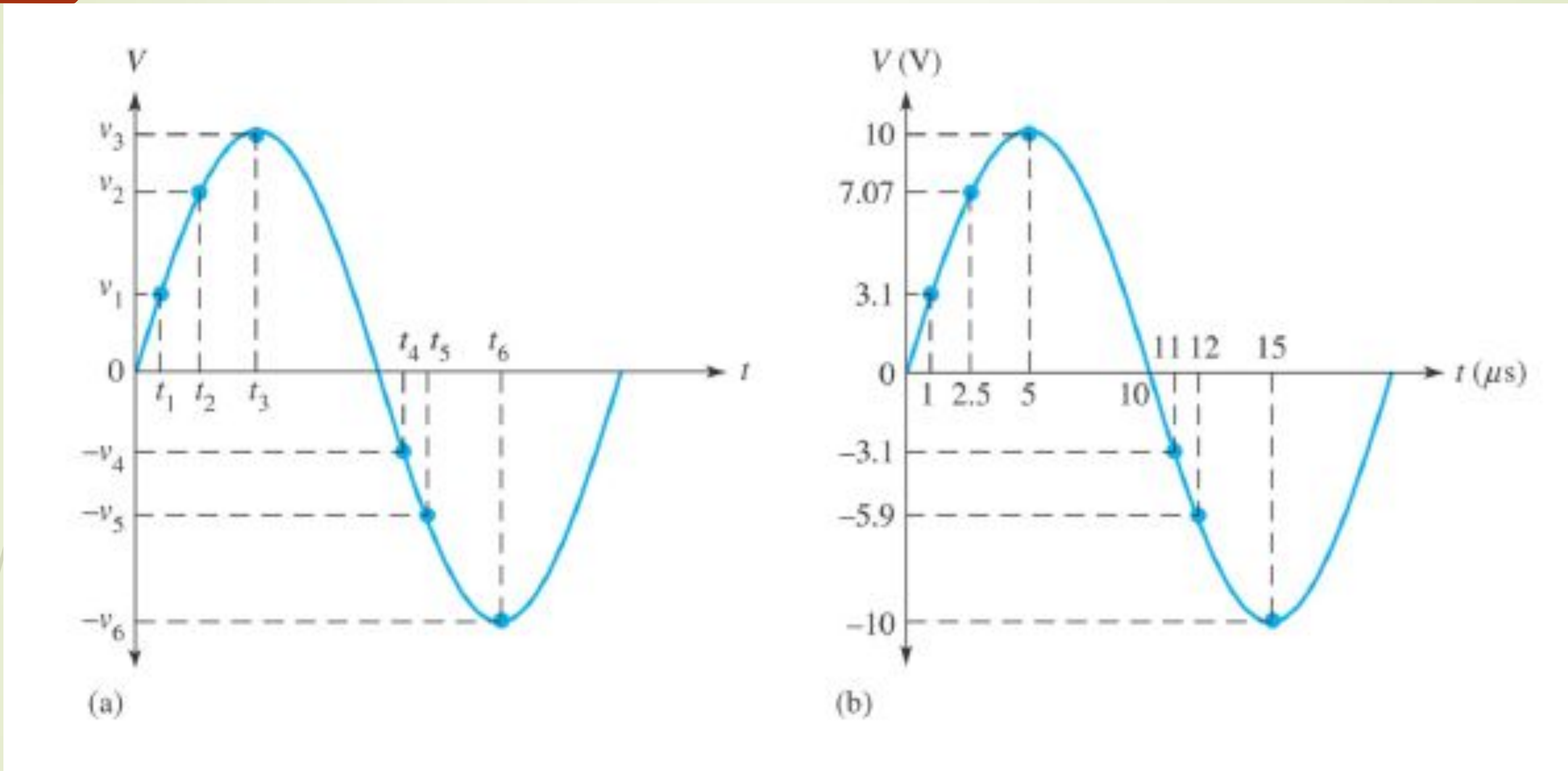
A plot of the example in the previous slide (peak at 25 V).

The instantaneous voltage at 50° is 19.2 V as previously calculated.



What is the voltage at 10^0 and 80^0?

Instantaneous Values

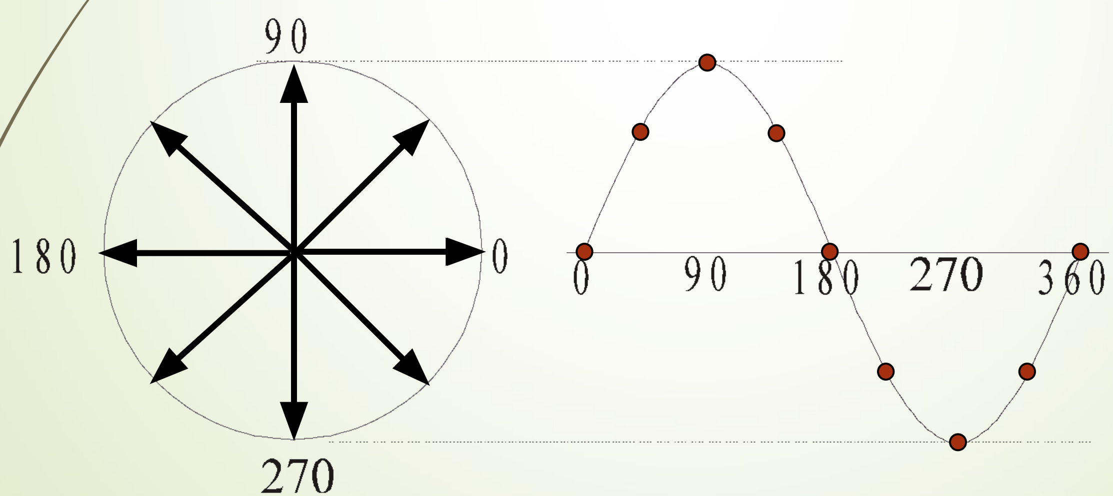


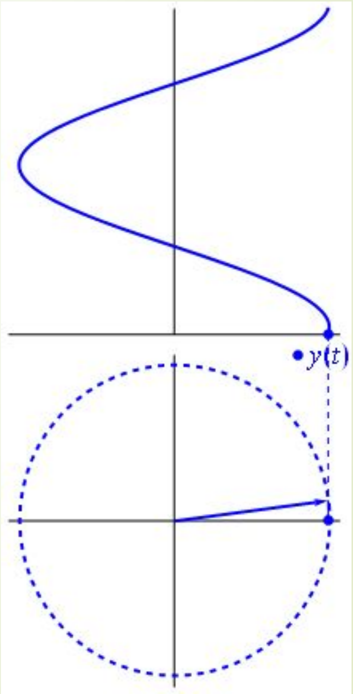
v_1: 3.1 V @ t_1: 1 μs
v_2: 7.07 V @ t_2: 2.5 μs
v_5: -5.9 V @ t_5: 12 μs

Phasors

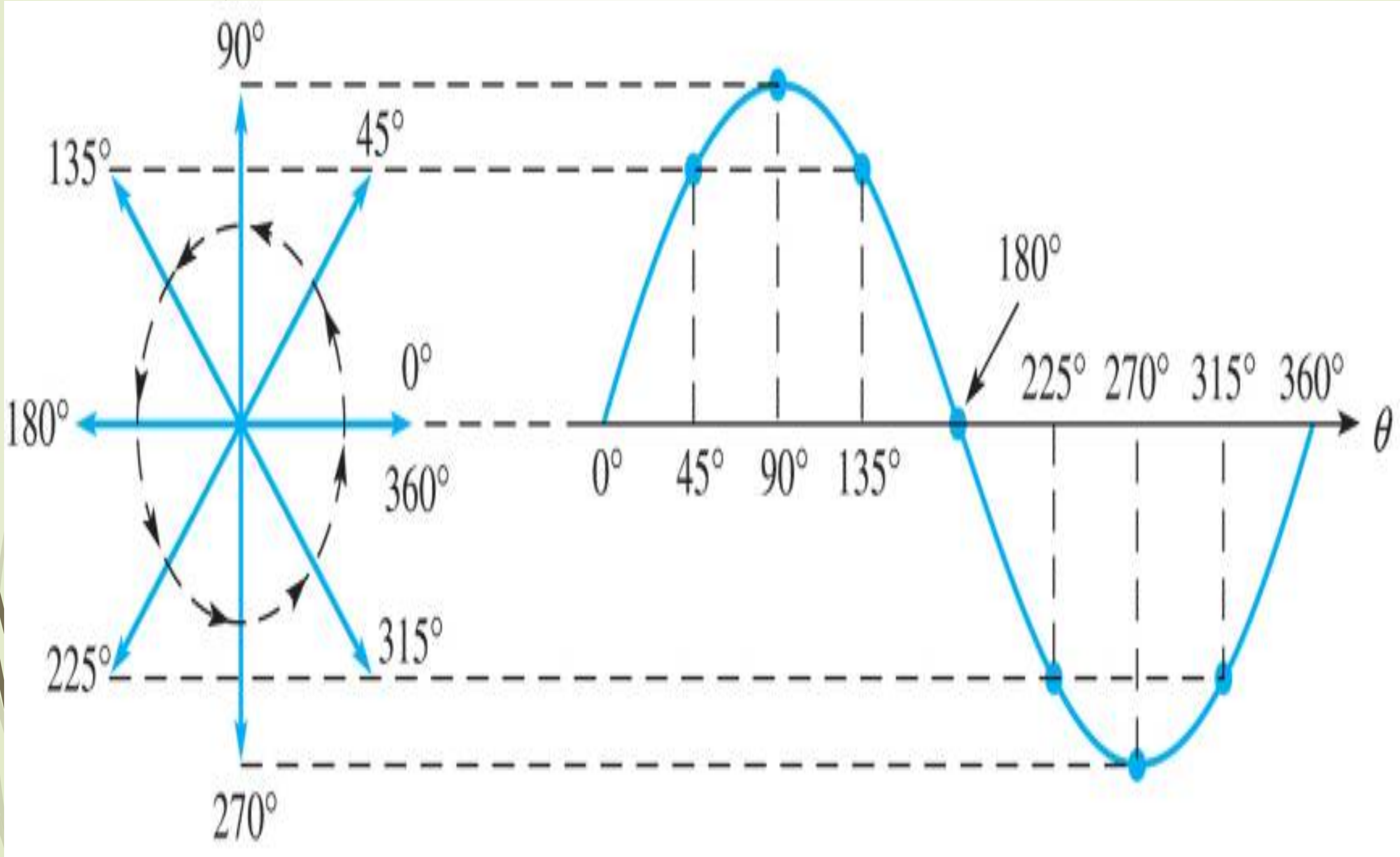
Phasor (Phase Vector):

Representation of a sine wave whose amplitude (**A**) and angular frequency (**ω -** omega) are changing at a constant rate.



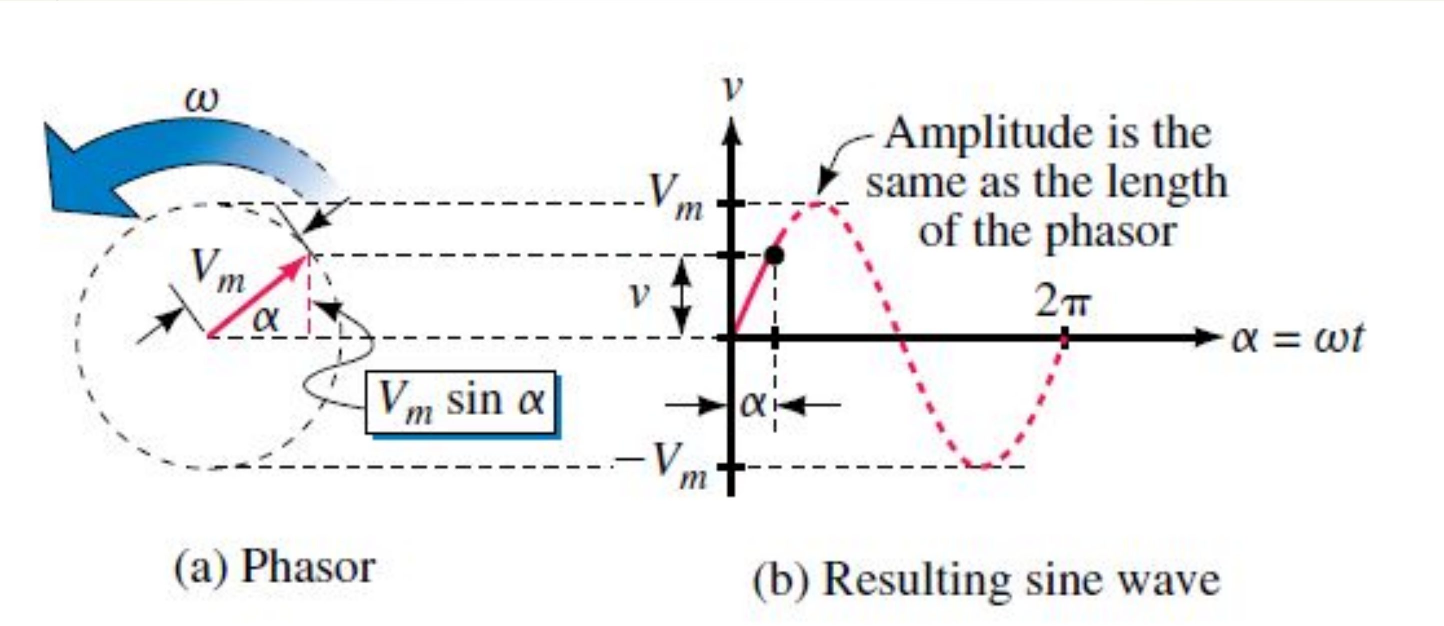


Phasors



Phasors

- **Rotating vectors whose projection onto a vertical or horizontal axis can be used to represent sinusoidally varying quantities**

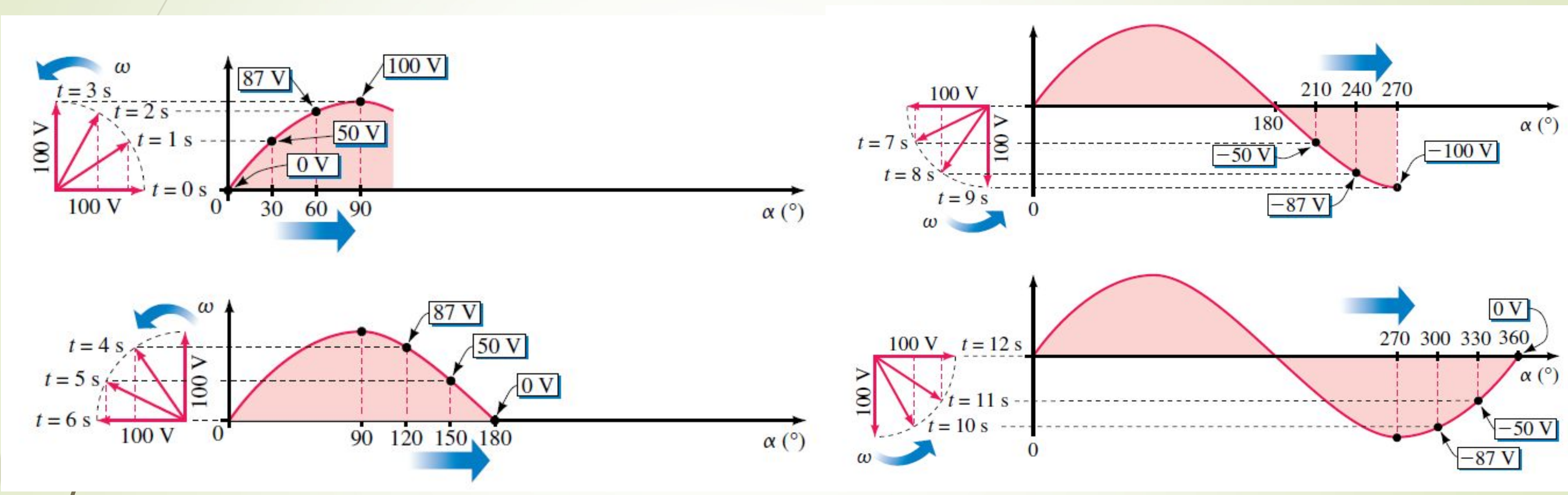


A sinusoidal waveform

Produced by plotting vertical projection of a phasor that rotates in the counterclockwise direction at a constant angular velocity ω

Phasors

- **Phasors apply only to sinusoidally varying waveforms**



Angular Velocity

- The rate that the generator coil rotates is called its **angular velocity** (**ω**).
- Angular position can be expressed in terms of angular velocity and time.

$$\theta = \omega t \quad \textit{(radians)}$$

- Rewriting the sinusoidal equation:

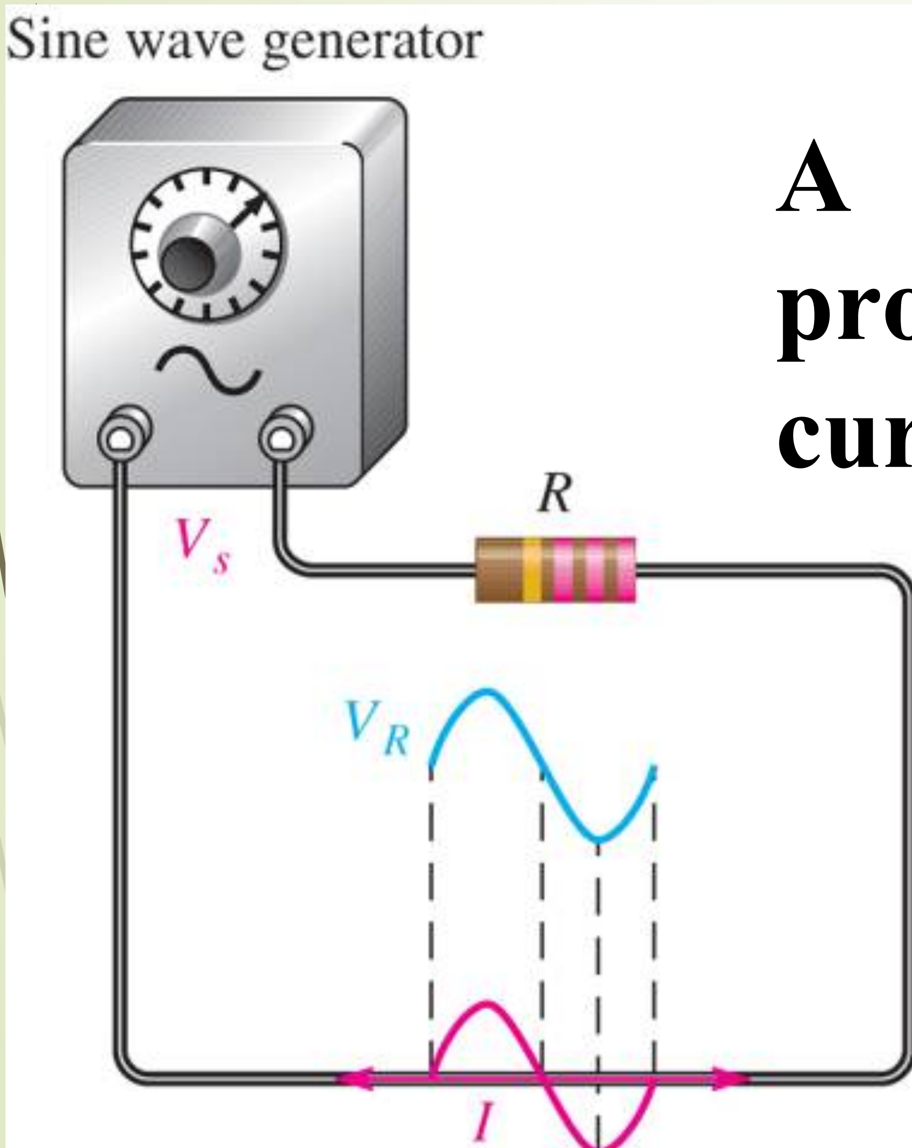
$$e(t) = E_m \sin \omega t \quad \text{(V) or } e(t) = E_m \sin \theta \text{ (V)}$$

- Conversion from frequency (***f***) in **Hz** to angular velocity (**ω**) in **radians per second**

$$\omega = 2\pi f \quad \text{(rad/s)}$$

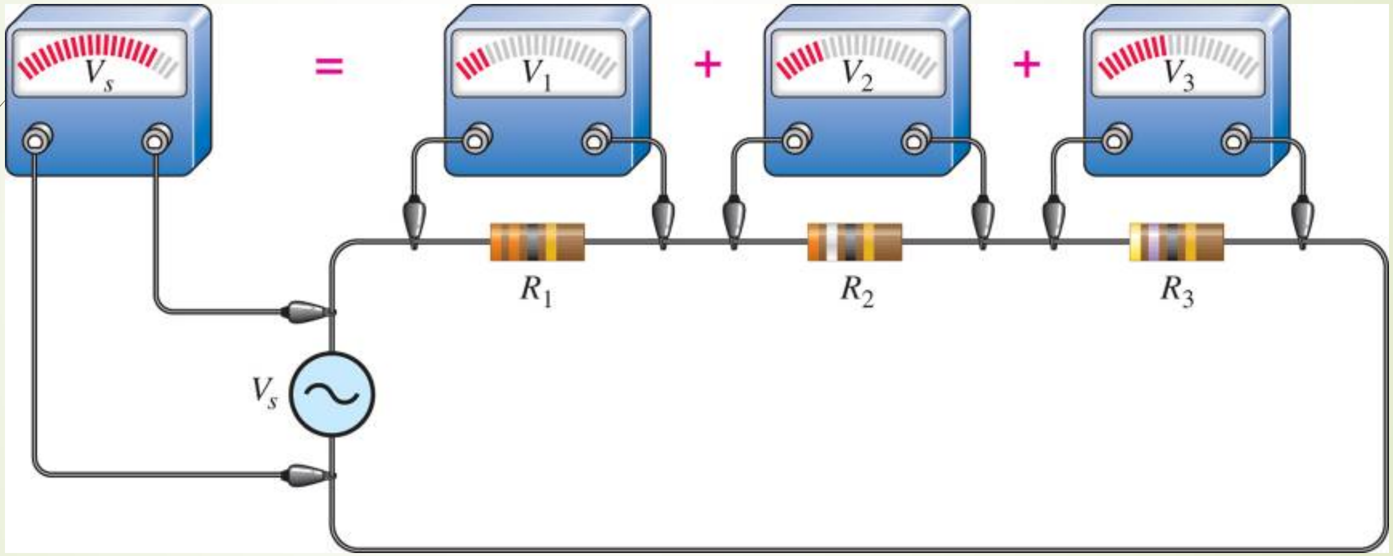
- In terms of the period (***T***)

$$\boxed{\omega = 2\pi f = \frac{2\pi}{T} \quad \text{(rad/s)}}$$



A sinusoidal voltage produces a sinusoidal current.

Kirchhoff's voltage law applies to AC circuits just like DC circuits



Power in AC circuits is calculated using **RMS** values for voltage and current.

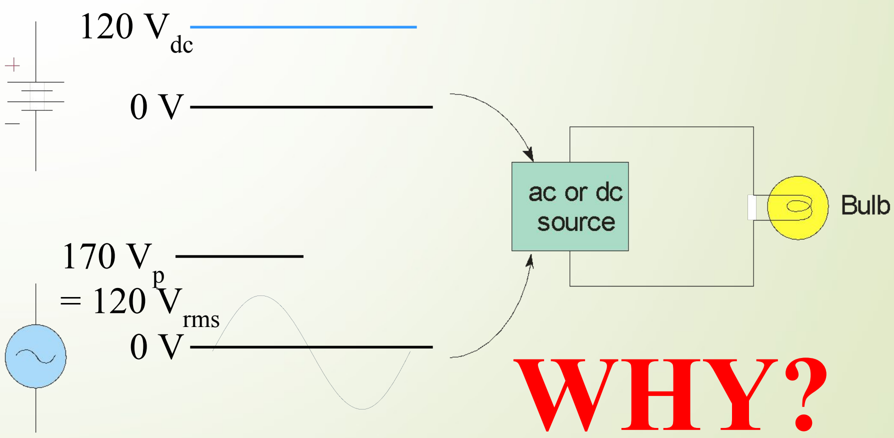
The power formulas are:

$$P = V_{\text{rms}} I_{\text{rms}}$$

$$P = \frac{V_{rms}^2}{R}$$

$$P = I_{\text{rms}}^2 R$$

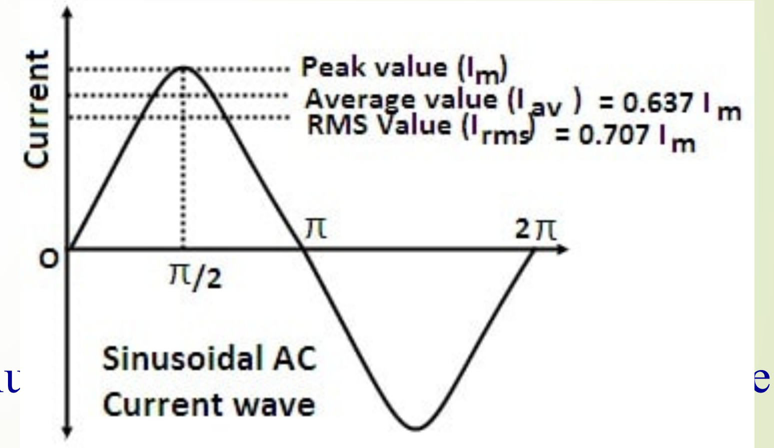
The dc and the ac sources produce the same power to the bulb:



WHY?

RMS VALUE

- **The RMS value of AC current is equal to that amount of DC current which produces the same heating effect flowing through the same resistance for the same time. OR**
- **The RMS value of AC current is equal to that amount of DC current which produces the same heating effect flowing through the same resistance for the same time.**



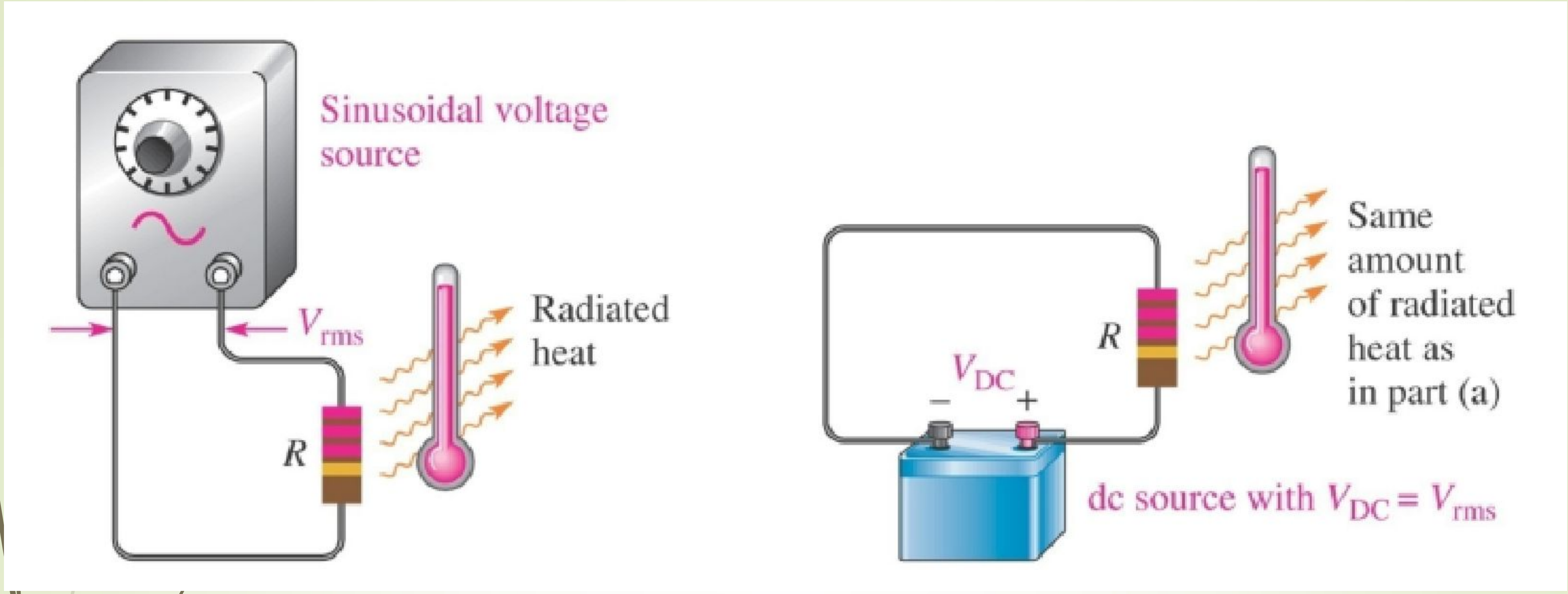
- **I_{RMS} = 0.707 x peak value of current**
- **V_{RMS} = 0.707 x peak value of voltage**

- **For example,** if an **AC** sine wave has a **RMS** valu e energy to a circuit as a **DC** supply of **240 volts**.

OR

- 5A alternating current is flowing through a circuit, it means the RMS value of AC current flowing through the circuit is 5A. And it will produce the same amount of heat (energy) as will be produced by 5A DC current.

Root - Mean - Square (RMS)



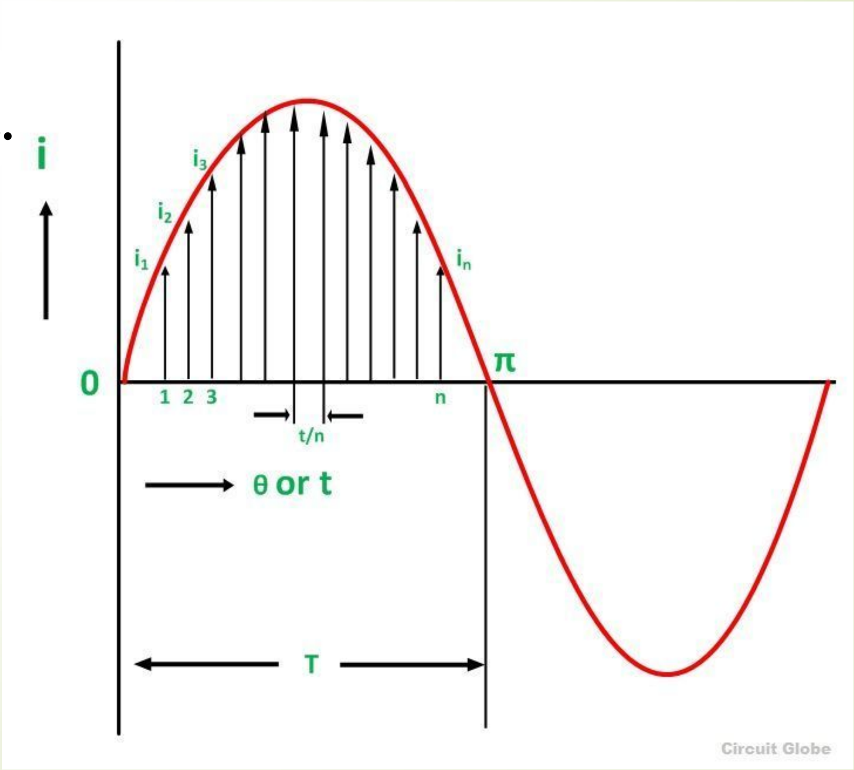
When the same amount of heat is being produced by the resistor in both setups, the sinusoidal voltage has a RMS value equal to the DC voltage.

Derive an Expression for RMS Value of AC

- Let $i = I_m \sin\omega t$ be the alternating current flowing through a resistance of R ohms for time t seconds and produces the same heat as produced by I_{eff} (a direct current).
- The base of one alteration is divided into n equal parts so that each interval is of t/n seconds as shown in the figure below.

Let i1, i2, i3 ……….in be the mid-ordinates.

Then, heat produced in
first interval $= i1^2Rt/n$ joules
Second interval $= i2^2Rt/n$ joules
Third interval $= i3^2Rt/n$ joules
.
.
.
nth interval $= in^2Rt/n$ joules



RMS Value of AC or Effective Value of an AC

- Total heat produced in time t

  $= Rt[(i_1 2 + i_2^2 + i_3^2 + \ldots\ldots + i_n^2)/n]$ …..(1)
- Since I_{eff} is considered as the effective value of this current.
  Then heat produced by this current in time t = $I_{eff}^2 Rt$ …….(2)

  By definition, equations (1) and (2) are equal. Therefore,

  $I_{eff}^2 Rt = Rt[(i_1^2 + i_2^2 + i_3^2 + \ldots\ldots + i_n^2)/n]$

  Or $I_{eff}^2 = [(i_1^2 + i_2^2 + i_3^2 + \ldots\ldots + i_n^2)/n]$

  Or $I_{eff} = [(i_1^2 + i_2^2 + i_3^2 + \ldots\ldots + i_n^2)/n]^{1/2}$

  Or $I_{eff} = I_{RMS}$ = square root of the mean of the squares of the instantaneous current.

Using the integral calculus RMS or effective value

The term *rms* stands for (square) root of the mean of the squares of the instantaneous current values.

$$\therefore \quad I_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} i^2 \, . \, d\theta}$$

$$I_{rms}^2 = \frac{1}{\pi}\int_0^{\pi} I_m^2 \sin^2\theta \, d\theta = \frac{I_m^2}{\pi}\int_0^{\pi} \sin^2\theta \, . \, d\theta$$

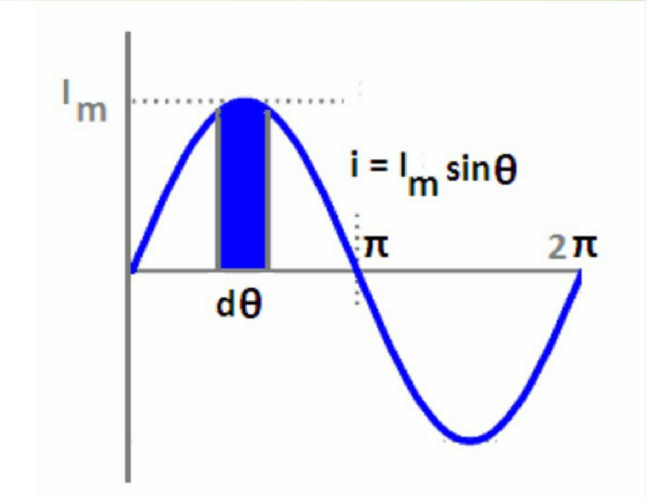
$$= \frac{I_m^2}{\pi}\int_0^{\pi}\left(\frac{1-\cos 2\theta}{2}\right) . \, d\theta = \frac{I_m^2}{2\pi}\int_0^{\pi}(1-\cos 2\theta) \, . \, d\theta$$

$$= \frac{I_m^2}{2\pi}\left|\theta - \frac{1}{2}\sin 2\theta\right|_0^{\pi}$$

$$= \frac{I_m^2}{2\pi}\left|\left(\pi - \frac{1}{2}\sin 2\pi\right) - \left(0 - \frac{1}{2}\sin 2\times 0\right)\right|$$

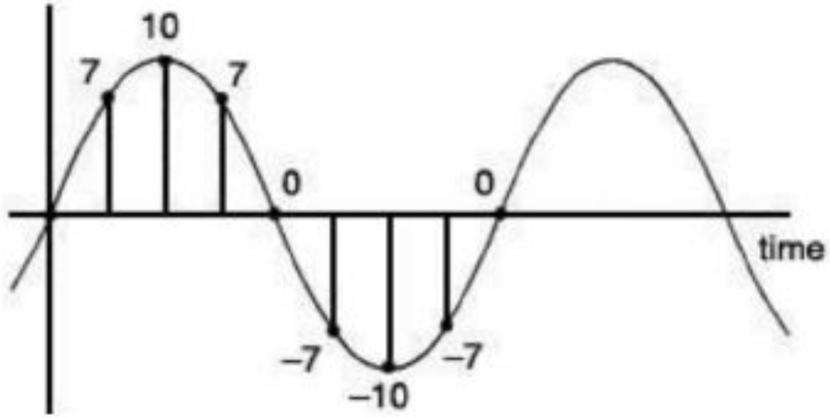
$$= \frac{I_m^2}{2\pi}\,|\,\pi - 0 - 0 + 0\,| = \frac{\pi}{2\pi} I_m^2 = \frac{I_m^2}{2}$$

$$\therefore \quad I_{rms} = \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}} = 0.707 \, I_m$$



$$i = I_m \sin\theta$$

RMS Value



Values	7	10	7	0	-7	-10	-7	0
Squares	49	100	49	0	49	100	49	0

Sum of squares = 396

Average of squares = 396/8 = 49.5

Square root ~ 7

$V_{rms} = 0.707 V_P \quad = 0.707\ (10) = 7.07$

$V_{rms} \sim 7$

Average Value

The average of all the instantaneous values of an alternating voltage and currents over one complete cycle is called Average Value. OR

- The arithmetic average of all the instantaneous values considered in an alternating quantity current over one cycle is called the average value of AC Current (I_{av}).
- It is that steady current (DC) that transfers **the same charge** as transferred by the AC current in same time across any circuit.
- In the case of symmetrical waves like sinusoidal current or voltage wave, the positive half is exactly equal to the negative half, therefore, the average value over a complete cycle is zero.
- Since work is being done by the current in the positive as well as in the negative half cycle, therefore, the average value is determined regardless of signs, or only the positive half is considered.

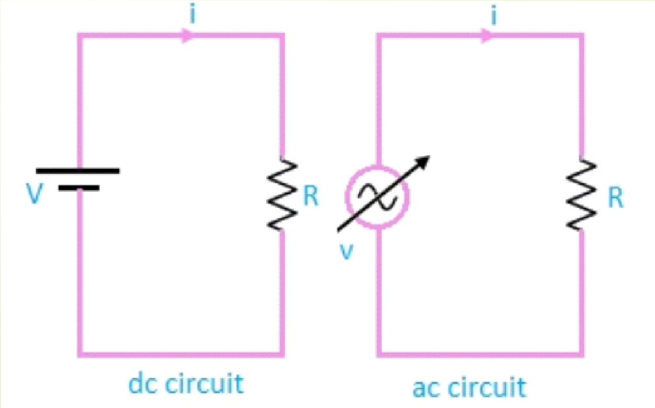
$I = V/R;$ $\qquad$ $i = v/R$

$I = Q/t,$ $\qquad$ $Q_{ac} = i \times t$

$Q_{dc} = I \times t$

As per definition,

$Q_{dc} = Q_{ac}$



Derive an Expression for Average Value of Alternating Current

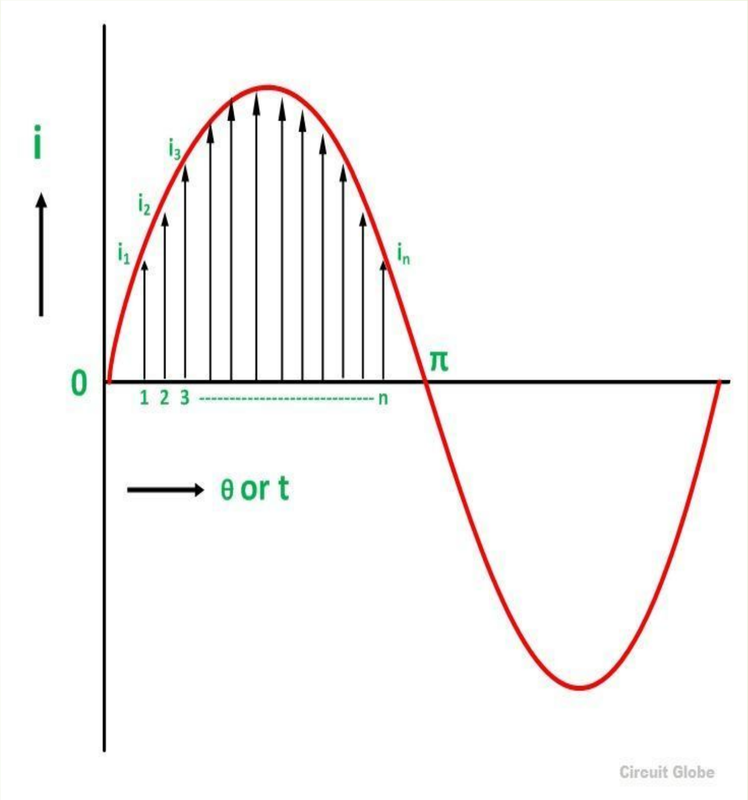
- To determine, the average value of alternating current, consider only in the positive half cycle and divide it into n number of equal parts.
- Let i_1, i_2, i_3 ……in be the mid-ordinates.

Now, average value of current,

- I_{av} = Mean of mid-ordinates

$= (i_1 + i_2 + i_3 + \ldots\ldots.+i_n)/n$

= Area of half cycle/length of base of half cycle



Using Integral Calculus Average Value of Alternating Current

We know that the standard equation of alternating current is

$$i = I_m \sin \omega t = I_m \sin \theta$$

- **Average value of complete cycle:**
- Let $i = I_m \sin \omega t = I_m \sin \theta$

$$I_{AV} = \frac{1}{2\pi}\int_0^{2\pi} i\,d\theta = \frac{1}{2\pi}\int_0^{2\pi} i_m \sin\theta\,d\theta$$

$$= \frac{I_m}{2\pi}\left[-\cos\theta\right]_0^{2\pi} = \frac{I_m}{2\pi}(\cos 2\pi - \cos 0)$$

$$= \frac{I_m}{2\pi}(1-1) = 0\ ; \quad I_{AV} = 0$$

- Thus, the average value of a sinusoidal wave over a complete cycle is zero.

$$V_{AV} = \frac{2V_M}{\pi} = 0.637 \times V_M$$

★ **Average Current Value**

$$I_{AV} = \frac{2I_M}{\pi} = 0.637 \times I_M$$

Average value of current over a half cycle

Let $i = I_m \sin \omega t = I_m \sin \theta$

$$I_{AV} = \frac{1}{\pi}\int_0^{\pi} i\,d\theta$$

$$= \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta.d\theta \quad [\therefore i = I_m \sin\theta]$$

$$= \frac{I_m}{\pi}\left[-\cos\theta\right]_0^{\pi} = \frac{I_m}{\pi}\left[-\cos\pi - (-\cos 0)\right]$$

$$= \frac{I_m}{\pi}[(+1) - (-1)] = \frac{I_m}{\pi}(+2)$$

$$I_{AV} = \frac{2}{\pi} I_m = 0.637\ A$$

Peak Factor and Form factor

- Peak Factor is defined as the ratio of maximum value to the R.M.S value of an alternating quantity.

$$\text{Peak Factor} = \frac{I_m}{I_{r.m.s}} \text{ or } \frac{E_m}{E_{r.m.s}}$$

$$\text{Peak Factor} = \frac{I_m}{I_{r.m.s}} = \frac{I_m}{I_m/\sqrt{2}} = \sqrt{2} = 1.4142$$

The value of Peak Factor is **1.4142**

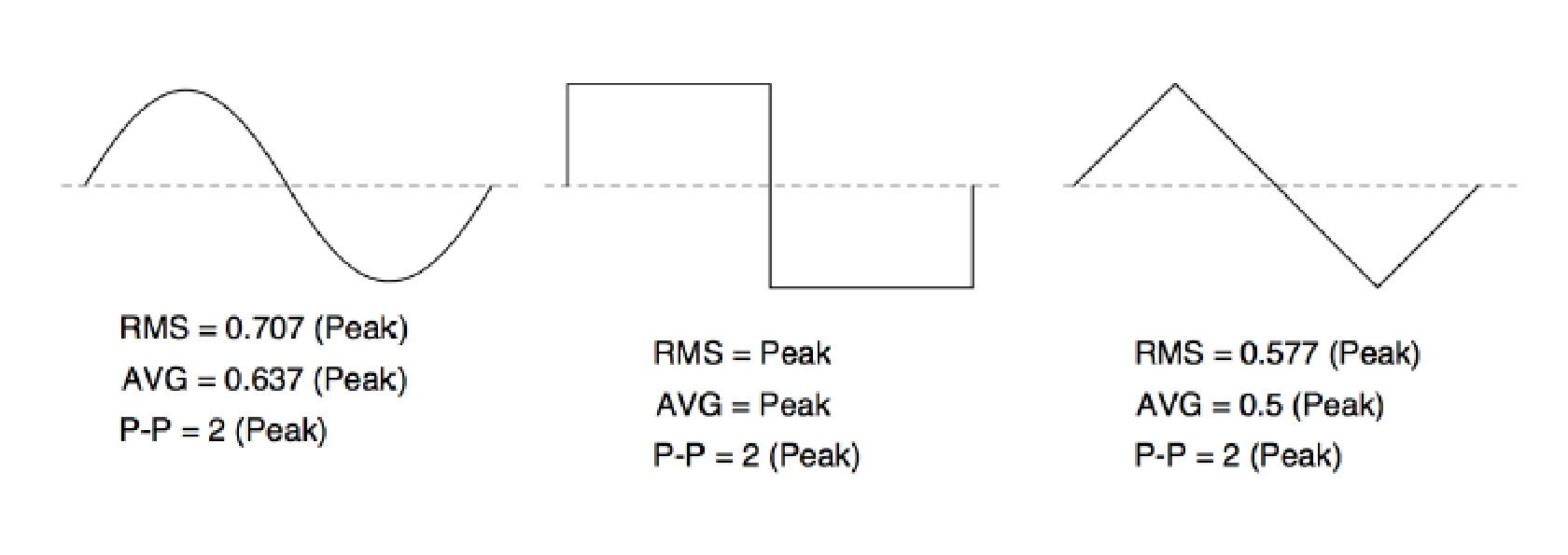
The ratio of the root mean square value to the average value of an alternating quantity (current or voltage) is called Form Factor.

$$\text{Form Factor} = \frac{I_{r.m.s}}{I_{av}} \text{ or } \frac{E_{r.m.s}}{E_{av}}$$

$$\text{Form Factor} = \frac{I_{r.m.s}}{I_{av}} = \frac{I_m/\sqrt{2}}{2I_m/\pi} = \frac{\pi I_m}{2\sqrt{2}I_m} = 1.11$$

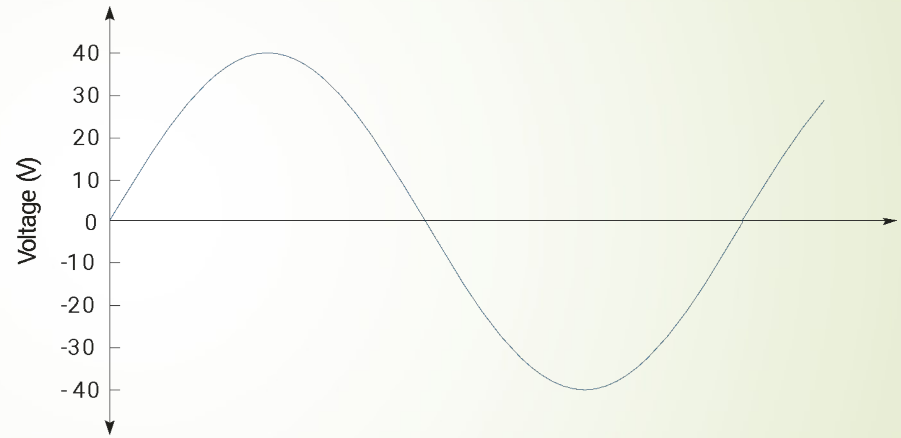
The value of Form Factor is **1.11**

Waveform Parameters



Power in resistive AC circuits

Assume a sine wave with a peak value of 40 V is applied to a 100 Ω resistive load. What power is dissipated?

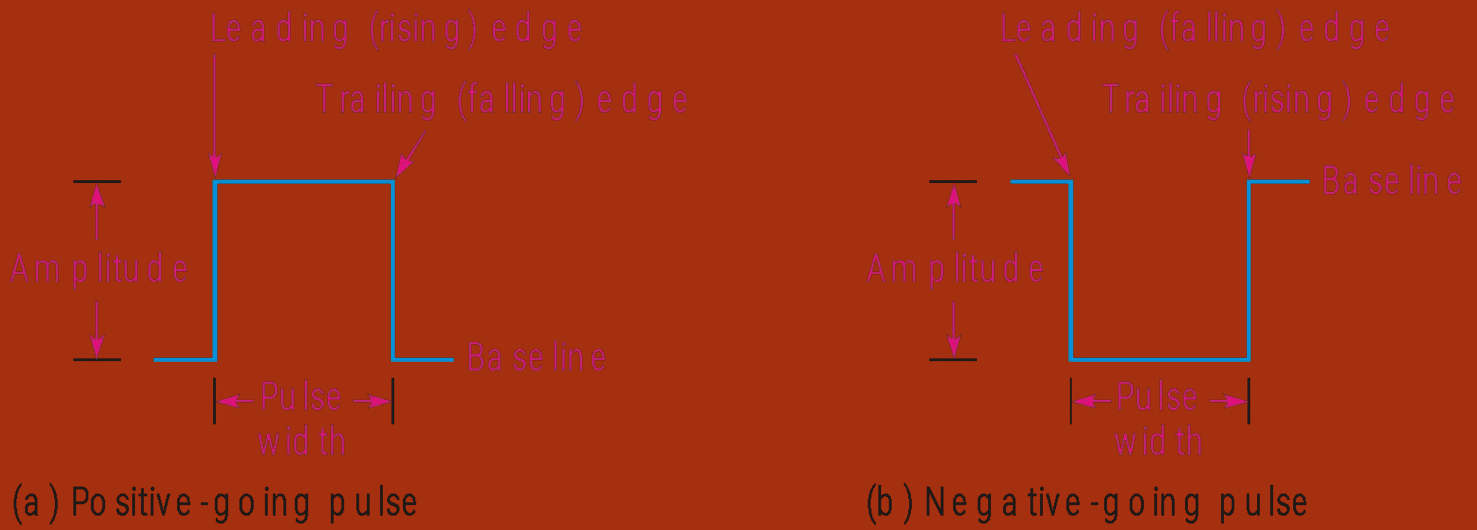


$$V_{\text{rms}} = 0.707 \times V_{\text{p}} = 0.707 \times 40\ \text{V} = 28.3\ \text{V}$$

$$P = \frac{V_{rms}^2}{R} = \frac{28.3\ \text{V}^2}{100\ \Omega} = 8\ \text{W}$$

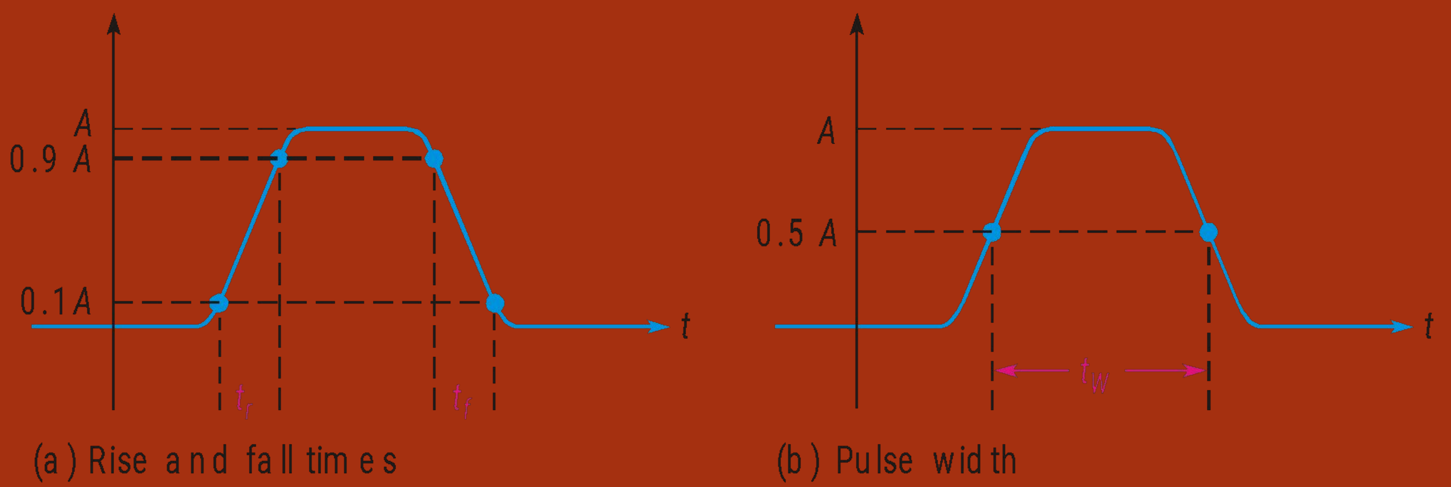
Pulse definitions

Ideal pulses



Pulse definitions

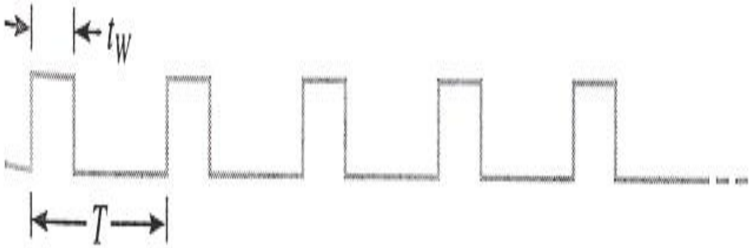
Non-ideal pulses

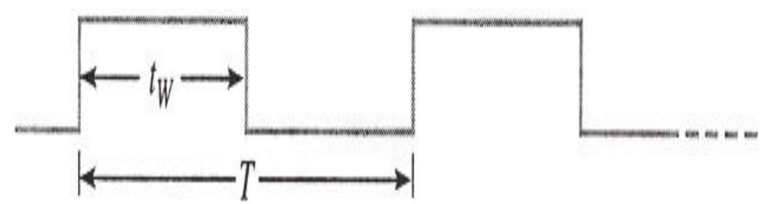


Notice that rise and fall times are measured between the 10% and 90% levels whereas pulse width is measured at the 50% level.

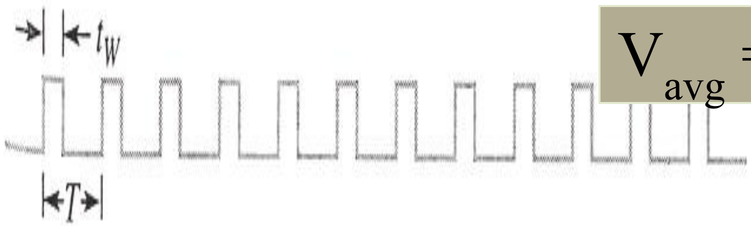
Repetitive pulse waveforms

- **Periodic waveforms repeat at fixed intervals.**
- **Pulse repetition frequency: Rate at which the pulses repeat.**
- **Duty Cycle – Ratio of pulse width (t_w) to the period (T)**





$$\text{Percent Duty Cycle} = \left(\frac{tw}{T}\right)100$$

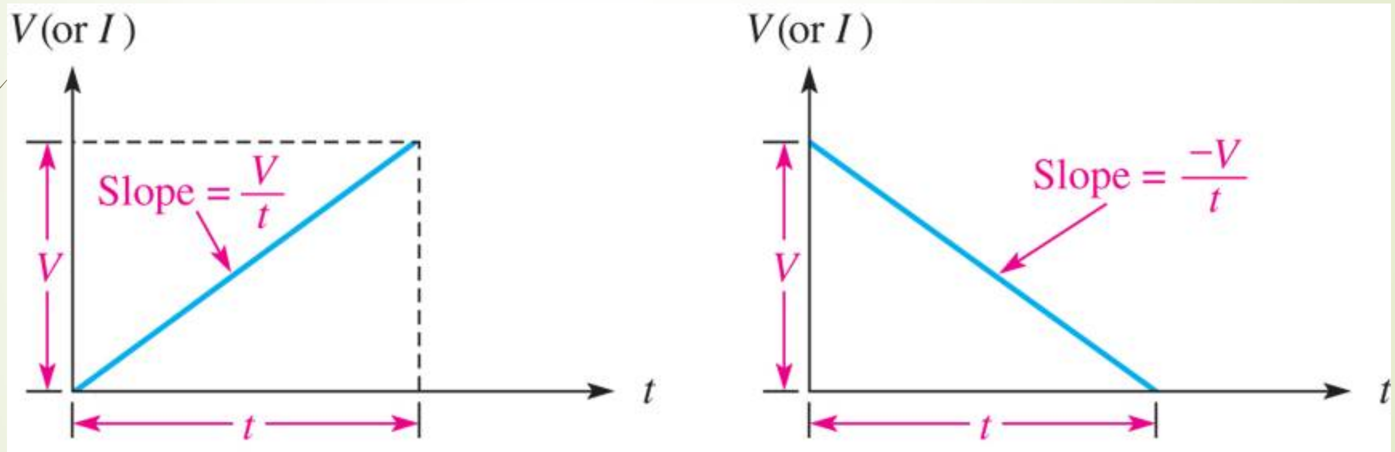


$$V_{avg} = \text{baseline} = (\text{duty cycle})(\text{ amplitude})$$

Voltage ramps

Ramp – Linear increase or decrease in voltage or current.

$$\text{Slope} = \frac{Yaxis}{Xaxis} = \frac{\pm V}{t} \, or \, \frac{\pm I}{t}$$



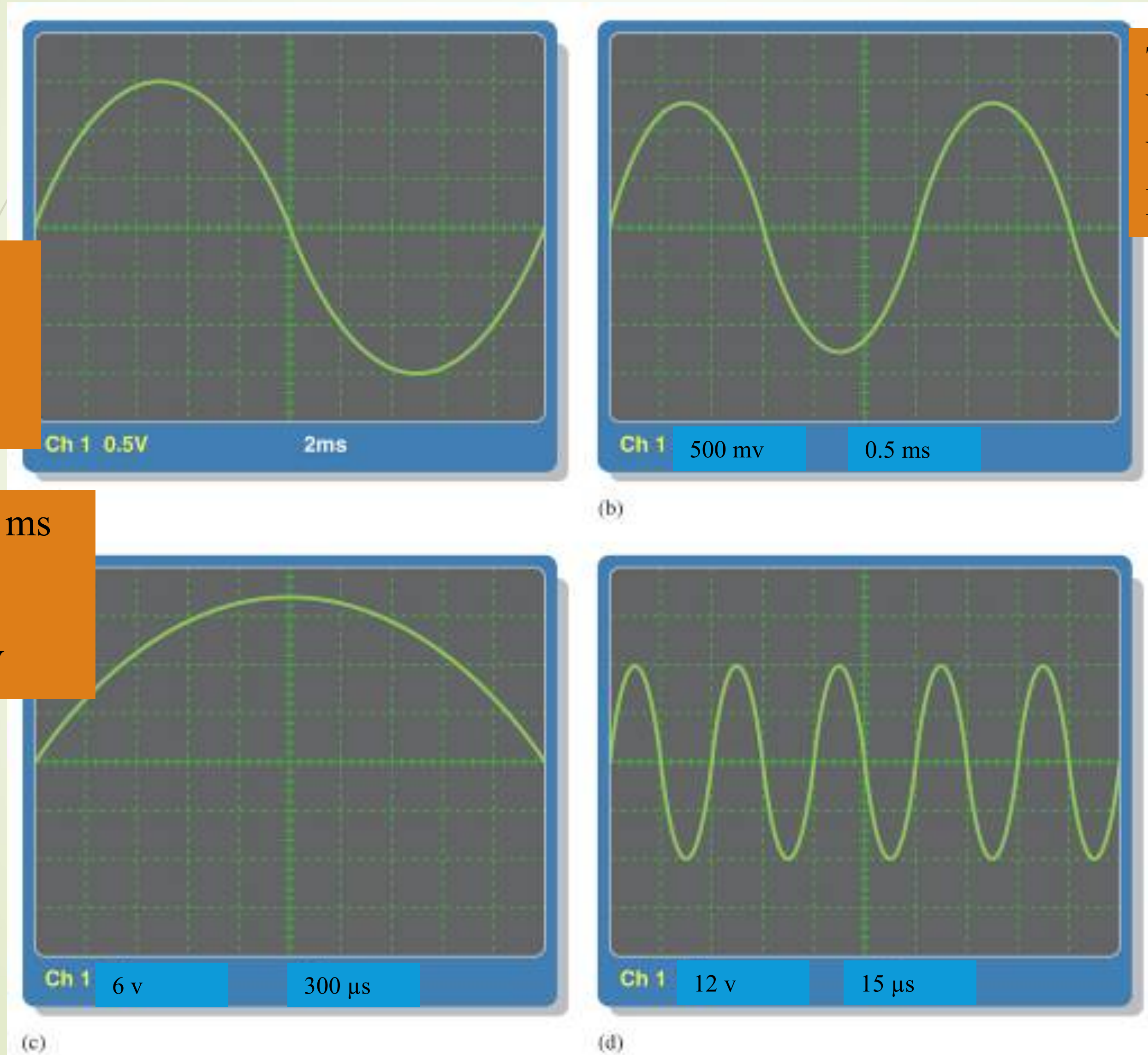
(a) Positive ramp

(b) Negative ramp

Oscilloscope

A device that traces the graph of a measured electrical signal on its screen.

Calculate for each wave: Period, Peak, Peak to Peak, RMS



T= 20 ms
Vp = 1.5 v
Vpp = 3.0 v
RMS = 1.06 V

T= 3.0 ms
Vp = 1250 mv
Vpp = 2500 mv
RMS = 833.8 mV

T= 6000 µs; 6 ms
Vp = 20.4 v
Vpp = 40.8 v
RMS = 14.4 V

T= 30 µs
Vp = 24 v
Vpp = 48 v
RMS = 16.97 V

Selected Key Terms

Sine wave A type of waveform that follows a cyclic sinusoidal pattern defined by the formula $y = A \sin \theta$.

Alternating current Current that reverses direction in response to a change in source voltage polarity.

Period (T) The time interval for one complete cycle of a periodic waveform.

Frequency (f) A measure of the rate of change of a periodic function; the number of cycles completed in 1 s.

Hertz The unit of frequency. One hertz equals one cycle per second.

Selected Key Terms

Instantaneous value The voltage or current value of a waveform at a given instant in time.

Peak value The voltage or current value of a waveform at its maximum positive or negative points.

Peak-to-peak value The voltage or current value of a waveform measured from its minimum to its maximum points.

rms value The value of a sinusoidal voltage that indicates its heating effect, also known as effective value. It is equal to 0.707 times the peak value. *rms* stands for root mean square.

Quiz

1. In North America, the frequency of ac utility voltage is 60 Hz. The period is

   a. 8.3 ms

   b. 16.7 ms

   c. 60 ms

   d. 60 s

Quiz

2. The amplitude of a sine wave is measured

   a. at the maximum point

   b. between the minimum and maximum points

   c. at the midpoint

   d. anywhere on the wave

Quiz

3. An example of an equation for a waveform that lags the reference is

a. $v = -40 \text{ V} \sin(\theta)$

b. $v = 100 \text{ V} \sin(\theta + 35^\circ)$

c. $v = 5.0 \text{ V} \sin(\theta - 27^\circ)$

d. $v = 27 \text{ V}$

Quiz

4. In the equation $v = V_p \sin\theta$, the letter v stands for the

   a. peak value

   b. average value

   c. rms value

   d. instantaneous value

Quiz

8. For the waveform shown, the same power would be delivered to a load with a dc voltage of

a. 21.2 V

b. 37.8 V

c. 42.4 V

d. 60.0 V