DEPARTMENT OF MATHEMATICS

Academic year 2023-2024 (Even Sem)

NUMBER THEORY VECTOR CALCULUS AND COMPUTATIONAL METHODS
MA221TC
QUESTION BANK
UNIT-I
NUMBER THEORY

PART - A

1. Check whether the linear congruence $2x + 3y = 3$ has solutions. If yes, then the solution is __________.
2. Multiplicative inverse of 5 in congruence modulo 3 is ________.
3. The solution of the linear congruence $7x \equiv 3 \pmod 5$ is ________.
4. The last digit of the number 3^{403} is ________,
5. The remainder obtained when 4^{362} is divided by 7 is ______.
6. Multiplicative inverse of 10 in congruence modulo 11 is ______.
7. The number of divisors of the integer 3424 is _____.
8. The sum of divisors of the integer 1008 is _____.
9. The prime factorisation of the integer 45682 is ____.
10. The number of multiplicative inverses in $\mathbb{Z}_{148}$ is ______.
11. The value of the Euler – totient function at 120 is ____.
12. Suppose $(n, e) = (77, 7)$ is the public key, then the cipher text for the plain text $M = 12$ is ______.
13. Let $(n, e) = (91, 5)$ be the public key. Then the private key is _____.
14. Suppose $(n, e) = (33, 3)$ is the public key and $d = 7$ is the private key, then the plain text of the cipher text 31 is ______.

PART – B

1. Show that 8 divides $n^2 - 1$ for any odd number n.
2. Find the gcd of the integers 252 and 198 and express $\gcd(252, 198)$ as a linear combination of 252 and 198.
3. Use Euclidean algorithm to find $\gcd(12345, 54321)$, hence find integer solution for the linear equation $12345x + 54321y = \gcd(12345, 54321)$.
4. Obtain all solutions of the linear congruence $144x \equiv 4 \pmod{35}$.
5. Solve the congruence $89x \equiv 2 \pmod{232}$.
6. Find last two digits of the number 36^{233}
7. Find the remainder when 3^{1288} is divided by 642.

8. Obtain the remainder when 20^{245} is divided by 21.
9. Determine all solutions of the linear congruence $15x \equiv 35 \pmod{760}$.
10. Given the public key $(n, e) = (85, 7)$, encrypt plain text HCM, where the alphabet $A, B, C, \dots X, Y, Z$ are assigned the numbers $3, 4, 5, \dots 27, 28$. Give the cipher text. Find the private key.
11. In a RSA cryptosystem, a participant A uses two prime numbers $p = 5$ and $q = 11$ to generate the public and private keys. If the public key of A is $(n, e) = (55, 7)$, then determine
    i) The private key of A.
    ii) Cipher text for the plain text $M = 15$.

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UNIT-II

VECTOR DIFFERNTIATION

Part A

1. Find $grad\phi$, if $\phi = \log(x^2 + y^2 + z^2)$
2. If $f(x, y, z) = 3x^2y - y^3z^2$, find ∇f and $|\nabla f|$ at $(1, -2, -1)$.
3. If $f = x^2y\, z$ and $g = xy - 3z^2$, calculate $\nabla(\nabla f . \nabla g)$
4. Find the unit vector normal to the surface $x^3 + y^3 + 3xyz = 3$ at the point $(1, 2, -1)$.
5. Find the maximum directional derivative of $\phi(x, y\,, z) = x^3y^2z$ at $(1, -2\,, 3\,)$.
6. If $\phi(x, y, z) = c_1,\ \ \psi(x, y, z) = c_2$ are two surfaces, then the orthogonality condition in terms of normals to the surfaces is______________________.
7. If $\vec{F} = (bx^2y - yz)\hat{\imath} + (xy^2 + xz^2)\hat{\jmath} + (2xyz - 2x^2y^2)\hat{k}$ is solenoidal then b = ________.
8. A particle is moving along the curve $x = a\cos t\,, y = a\sin t, z = at\ \tan\alpha$. Then the magnitude of velocity at $t = 0$ is _________.
9. If the vector field $\vec{F} = (x + 2y + az)\boldsymbol{i} + (2x - 3y - z)\boldsymbol{j} + (4x - y + 2z)\boldsymbol{k}$ is irrotational then the value of a is _________.
10. If the vector field $\vec{A} = (x + 3y)\boldsymbol{i} + (y - 2z)\boldsymbol{j} + (x + mz)\boldsymbol{k}$ is solenoidal, then the value of m is _______.
11. Given the curve $x = t^2 + 2, y = 4t - 5, z = 2t^2 - 6t$ find the unit tangent vector at the point $t = 2$.
12. If $\phi = 2x^2yz^3$, then find $\nabla^2\phi$ at $(1, 1,1\,)$.
13. The experiments show that the heat flows in the direction of maximum decrease of temperature. Find this direction when the temperature $T = x^2 + y^2 + 4z^2$ at the point $(2, -1\,, 2)$.
14. If $\phi = x^3 + y^3 + z^3 - 3xyz$, then $div(grad\ \phi)$ is ________.
15. Find $div\vec{F}$ and $curl\vec{F}$, where $\vec{F} = grad(x^3 + y^3 + z^3 - 3xyz)$.

Part B

1. Determine the angle between the tangents to the curve $\vec{r} = t^2\hat{\imath} + 2t\hat{\jmath} - t^3\hat{k}$ at the points $t = \pm 1$.
2. A particle moves along the curve $\vec{r} = 2t^2\hat{\imath} + (t^2 - 4t)\hat{\jmath} - (3t - 5)\hat{k}$. Find the components of velocity and acceleration in the direction of vector $\vec{c} = \hat{\imath} - 3\hat{\jmath} + 2\hat{k}$ at $t = 1$.
3. Obtain the directional derivative of $\phi(x, y\,, z) = \frac{1}{x^2+y^2+z^2}$ at the point $(1, 2, -3)$ in the direction of $2\hat{\imath} - 3\hat{\jmath} + \hat{k}$.
4. Determine the angle between the normals to the surface $2x^2 + 3y^2 = 5z$ at the points $(2, -2, 4)$ and $(-1, -1, 1)$.
5. Determine the angle between the surfaces $x^2 + y^2 + z^2 = 16$ and $x^2 + y^2 - z = 4$ at the point $(2, -1, 2)$ common to them.

6. Determine the constants a and b so that the surface $3x^2 - 2y^2 - 3z^2 + 8 = 0$ is orthogonal to the surface $ax^2 + y^2 = bz$ at the point $(-1,\ 2, 1)$.
7. If $\vec{r} = x\,\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and $r = |\,\vec{r}|$ then show that div $(r^n\vec{r}) = (n+3)r^n$
8. If $\vec{r} = x\,\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and $r = |\,\vec{r}|$ then show that $\frac{\vec{r}}{r^3}$ is solenoidal.
9. If $\vec{r} = x\,\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and $r = |\,\vec{r}|$ then show that $\nabla^2 f(r) = f''(r) + \frac{2}{r}f'(r)$.
10. If $\vec{r} = x\,\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and $r = |\,\vec{r}|$then show that $\nabla^2(r^{n+1}) = (n+1)(n+2)r^{n-1}$.
11. Obtain the directional derivative of $\phi(x,y) = x^3 - 3xy + 4y^2$ along the vector $u = \cos\frac{\pi}{6}\ i + \sin\frac{\pi}{6}\ j$ and also find $D_u\phi(1,2)$.
12. Determine the value of the constant ‘a’ such that: $\overrightarrow{F} = (axy - z^3)\hat{\imath} + (a-2)x^2\,\hat{\jmath} + (1-a)\,xz^2\hat{k}$ is irrotational and hence find a scalar function ϕ such that $\vec{F} = \nabla\phi$.
13. Show that $\overrightarrow{F} = (2xy^2 + yz)\hat{\imath} + (2x^2y + xz + 2yz^2)\,\hat{\jmath} + (2y^2z + xy)\hat{k}$ is a conservative force field. Find its scalar potential.
14. Obtain the curl and divergence of the vector field $\vec{\boldsymbol{F}} = xyz\,\boldsymbol{i} + x^2y^2z\,\boldsymbol{j} + yz^3\boldsymbol{k}$ and hence find scalar potential ϕ if $curl\ \vec{F} = 0$.
15. Show that the vector field
$$\vec{\mathbf{F}} = (x^2 - yz)\mathbf{i} + (y^2 - xz)\mathbf{j} + (z^2 - xy)\mathbf{k}$$
is irrotational. Obtain the scalar potential ϕ such that $\vec{\mathbf{F}} = \nabla\phi$.
16. Suppose that the temperature at a point (x,y,z) in space is given by $T(x,y,z) = \frac{80}{1+x^2+2y^2+3z^2}$, where T is measured in degree Celsius and x, y, z meters. In which direction does the temperature increases fastest at the point $(1,1,-2)$? What is the maximum rate of increase?

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Unit-III

VECTOR INTEGRATION

PART A

1. If $curl\,\vec{F} = 0$, then for any closed curve C $\int_C \vec{F} \cdot d\vec{r}$=_____________.
2. If R is the projection of the surface $x = 4$ on to the yz-plane, then $ds =$__________.
3. If R is the projection of the surface $x = \sqrt{4 - z^2 - y^2}$ on to the yz-plane, then $ds =$__________.
4. If R is the projection of the surface $y = \sqrt{16 - z^2 - x^2}$ on to the xz-plane, then $ds =$__________.
5. If C is the curve $y = 2$ from $x = 0$ to $x = 2$ in the XY-plane, then =__________.
6. If S is the surface of the sphere $x^2 + y^2 + z^2 = 16$, then for $\vec{F} = x\hat{\imath} + y\hat{\jmath} - z\hat{k}$ $\iint_S \vec{F} \cdot \hat{n}\, ds =$__________.
7. If S is the surface of the parallelepiped bounded by the planes $x = 0$, $x = 2$, $y = 0$, $y = 3$, $z = 0$ and $z = 4$, then for $\vec{F} = x\hat{\imath} + y\hat{\jmath} - z\hat{k}$ $\iint_S \vec{F} \cdot \hat{n}\, ds =$__________.
8. If S is the surface of the sphere $x^2 + y^2 + z^2 = 4$, then for $\vec{F} = x^2\hat{\imath} + y\hat{\jmath} - z\hat{k}$ $\iint_S \vec{F} \cdot \hat{n}\, ds =$__________.
9. If S is the surface of the parallelepiped bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 2$, $z = 0$ and $z = 3$, then for $\vec{F} = x\hat{\imath} + y^2\hat{\jmath} - z\hat{k}$ $\iint_S \vec{F} \cdot \hat{n}\, ds =$__________.
10. If C is the boundary of the rectangle bounded by $x = 1$, $y = 1$, $x = 3$, $y = 4$, then $\oint_C (xdy - ydx) =$__________.
11. If C is the circle bounded by $x^2 + y^2 = 8$, then $\oint_C (xdy - ydx) =$__________.
12. If C is the boundary of the triangle bounded by $y = 0$, $x - y = 0$ and $x + y = 2$, then $\oint_C (xdy - ydx) =$__________.
13. If C is the boundary of the triangle bounded by $y = 0$, $x = 0$ and $x + y = 2$, then $\oint_C (xdy - ydx) =$__________.
14. If C is the boundary of the triangle with vertices $(0,0)$, $(2,0)$ and $(1,1)$, then $\oint_C (xdy - ydx) =$__________.
15. If C is the boundary of the triangle with vertices $(0,0)$, $(2,0)$ and $(0,2)$, then $\oint_C (xdy - ydx) =$__________.

PART B

1. If $\vec{F} = (2x + y^2)\hat{\imath} + (3y - 4x)\hat{\jmath}$ evaluate $\oint_C \vec{F} \cdot d\vec{r}$ around the triangle in the xy-plane with vertices $(0,0)$, $(2,0)$, $(2,1)$ (a) in counterclockwise direction (b) in clockwise direction.
2. If $\vec{A} = (y - 2x)\hat{\imath} + (3x + 2y)\hat{\jmath}$, compute the circulation of $\vec{A}$ about a circle c in the xy-plane with centre at the origin and radius 2, if c is traversed in the positive direction.
3. Find the total work done in moving a particle in a force field $\vec{A} = 3xy\hat{\imath} - 5z\hat{\jmath} + 10x\hat{k}$ along the curve $x = t^2 + 1$, $y = 2t^2$, $z = t^3$ from $t = 1$ to $t = 2$.
4. Calculate the flux of water through the parabolic cylinder $y = x^2$, between the planes $x = 0$, $z = 0$, $x = 3$, $z = 2$ if the velocity vector is $\vec{A} = y\hat{\imath} + 2\hat{\jmath} + xz\hat{k}$ m/sec.

5. Find the flux across the surface of the parabolic cylinder $y^2 = 8x$ in the first octant bounded by the planes $y = 4$ and $z = 6$ when the velocity vector $\vec{V} = 2y\hat{\imath} - z\hat{\jmath} + x^2\hat{k}$.
6. Use Green's theorem to evaluate the line integral $\oint_C(-y^3dx + x^3dy)$, where C is boundary of the cicle $x^2 + y^2 = 4$.
7. Use Green's theorem to evaluate the line integral $\oint_C\left(\frac{e^y}{x}dx + (e^y \ln y + 2x)dy\right)$, where C is the boundary of the region bounded by $y = 2$, $y = x^4 + 1$.
8. Verify Stokes' theorem for $\vec{A} = xz\hat{\imath} - y\hat{\jmath} + x^2y\hat{k}$ where S is the surface of the region bounded by $x = 0$, $y = 0$, $z = 0$, $2x + y + 2z = 8$ which is not included in the xz-plane.
9. Verify Stokes' theorem for $\vec{A} = y^2\hat{\imath} + xy\hat{\jmath} - xz\hat{k}$ where S is the hemisphere $x^2 + y^2 + z^2 = a^2$, $z \geq 0$.
10. Evaluate $\oint_C(\sin z\ dx - \cos x\, dy + \sin y\, dz)$ where C is the rectangle $0 \leq x \leq \pi$, $0 \leq y \leq 1, z = 3$.
11. Evaluate the surface integral $\iint_S \vec{F} \cdot \hat{n}\, ds$ for $\vec{F} = ax\hat{\imath} + by\,\hat{\jmath} + cz\hat{k}$ and S is unit sphere centered at origin.
12. Verify the divergence theorem for $\vec{A} = 2x^2y\hat{\imath} - y^2\hat{\jmath} + 4xz^2\hat{k}$ taken over the region in the first octant bounded by the cylinder $y^2 + z^2 = 9$ and the plane $x = 2$.
13. Verify the divergence theorem for $\vec{A} = 4x\hat{\imath} - 2y\hat{\jmath} + z^2\hat{k}$ taken over the region bounded by $x^2 + y^2 = 4$, $z = 4$, $z = 0$ and $z = 3$.
14. Verify the divergence theorem for $\vec{A} = 2xy\hat{\imath} + yz^2\hat{\jmath} + xz\hat{k}$ and S is the total surface of the rectangular parallelopiped bounded by the coordinate planes, $x = 1$, $y = 2$ and $z = 3$.
15. Verify the divergence theorem for $\vec{A} = x^2\hat{\imath} + y^2\hat{\jmath} + z^2\hat{k}$ and S is surface of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

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UNIT-IV

LINEAR ORDINARY DIFFERENTIAL EQUATIONS OF HIGHER ORDER

PART A

1. If $y = e^{-t}$ is the solution of the equation $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + py = 0$ then the value of p is________________
2. If $y = e^{-3t}$ is the solution of the equation $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + py = 0$ then the value of p is________________
3. The solution of the differential equation $\frac{d^2y}{dt^2} + \omega^2 y = 0$ is_____________
4. If the general solution of the differential equation is $y = e^{2x}(Acos5x + Bsin5x)$, the value of A and B are____________ given that $y(0) = 0 \; and \; y'(0) = 15$
5. Find the P.I of $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 3e^{-2x}$.
6. Find the P.I of $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 4$.
7. The solution of the differential equation $(D^2 - 4)y = e^x$
8. Solve $(D^3 - 1)y = 0$
9. The complimentary function of $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = logx$ is _______.
10. What is the constant coefficient differential equation corresponding to $x\frac{d^2y}{dx^2} + \frac{dy}{dx} = \frac{1}{x}$.
11. The homogeneous linear differential equations whose auxiliary equation roots are 0,-1,-1 _____.
12. The solution of $x^2y'' + xy' = 0 \; is$_____.
13. The solution of $y'' + y = 0$ with $y(0) = 0$, $y\left(\frac{\pi}{2}\right) = 2$is _________.
14. The periodic time of the motion described by the differential equation $\frac{d^2x}{dt^2} + 4x = 0$ is________.
15. The complimentary function of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x sinx$ is _______.

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PART B

1. Solve the initial value problem $y'' - 4y' + 5y = 0$ subject to the conditions $y(0) = 1$, $y'(0) = 2$.
2. Solve$(D^4 - 2D^3 + 2D^2 - 2D + 1)y = 0$.
3. Solve $x^2y'' + xy' + 9y = 3x^2 + \sin(3\log x)$.
4. Solve $x^4y''' + 2x^3y'' - x^2y' + xy = \sin(\log x)$.
5. Solve $(D^2 - 3D + 2)y = \cos(e^{-x})$ by the method of variation parameters.
6. A particle undergoes forced vibrations according to the law $x'''(t) + 25x(t) = 21\cos(2t)$. If the particle starts from rest at $t = 0$,find the displacement at any time $t > 0$.
7. The differential equation of simple pendulum is $x''(t) + \omega^2 x(t) = F\sin(\omega t)$, where ω and F are constants. If $x = 0$, $x' = 0$ at $t = 0$, determine the motion x (t).
8. Solve $(D^2 + 1)y = \frac{1}{1+sinx}$ by the method of variation parameters.
9. Solve$(D^2 - 1)y = sinx\,(1 + x^2)e^x$.
10. Solve$(D^2 + 1)y = sin^2x$.
11. Solve$(D^2 + 2D + 1)y = xe^{-x}cosx$.
12. Solve$(D^2 - 1)y = x^2cosx$.
13. Solve $(D^6 - D^4)y = x^2$.
14. Solve$(D^2 - 4D + 1)y = cosxcos2x + sin^2x$.
15. Solve the boundary value problem$y'' - 2y' - 3y = 0, y(0) = 0, y(1) = e^3 - \frac{1}{e}$.

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UNIT-V

NUMERICAL METHODS

PART A

1. For the following data, find $\Delta^2 y_0$ =________________.

u	1	1.4	1.8	2.2
y=$f(u)$	3.49	4.82	5.96	6.5

2. In terms of second order forward difference $\nabla y_5 - \nabla y_4$ is________.
3. In terms of first order backward difference $\Delta^2 y_4$ is ___________.
4. From a difference table it is found that second differences are 3,5,9,-7. If $\Delta^2 y_1 = 5$, then $\nabla^2 y_3$=_________________.
5. Construct forward difference table for the following data:

v	-2	0	2	4
$f(v)$	20	24	29	36

6. Given

x	0	2	4	6
$f(x)$	7	13	43	145

The value of $f'(2)$=___________________.

7. Given

x	0	2	4	6
$f(x)$	7	13	43	145

The value of $f'(6)$=___________________.

8. The n^{th} differences of a polynomial of degree n is __________.
9. The value of $\Delta^3[(1+3x)(1-7x)(1-5x)]$ taking the interval of differencing h=1 is _______.
10. Given

x	4	6	8	10
y	1	3	8	16

Find $\frac{d^2y}{dx^2}$ at $x = 4$.

11. Given

x	4	6	8	10
y	1	3	8	16

Find $\frac{d^2y}{dx^2}$ at $x = 10$.

12. The Lagrange polynomial that passes through three data points $y(1) = 3,\ y(2) = 4,\ y(5) = 5$ is given by $p(x) = L_0(x)\ 3\ + L_1(x)\ 4 + L_2(x)\ 5$. The value of $L_0(x)$ at 3 is _______.
13. Roots of the equation $f(x) = 0$ is __________, if given

x	-1	0	2
$f(x)$	0	1	2

14. The coefficient of y_2 in the Lagrange's interpolation formula to fit a polynomial for the following data is__________.

x	-1	0	2	3
y	y_0	y_1	y_2	y_3

15. Find the interpolating polynomial for the following data

x	0	1	2
y	0	5	2

PART B

1. Find the number of men getting wage of ₹ 10,000 from the following table:

Wages (in ₹, in thousands)	5	15	25	35
No of Men	9	30	35	42

2. From the following table find the value of θ at $x = 43$.

x	40	50	60	70	80	90
θ	184	204	226	250	276	304

3. Estimate from the following table $f(3.8)$

x	0	1	2	3	4
$f(x)$	1	1.5	2.2	3.1	4.6

4. The following table gives the population of a town during the last six censuses. Estimate the increase in population during the period from 1956 to 1982:

Year	1941	1951	1961	1971	1981	1991
Population(in thousands)	12	15	20	27	39	52

5. The following table gives the viscosity of a liquid at different temperatures. Calculate its viscosity at $T = 170$.

Temperature(degree)	110	130	160	190
Viscosity(γ)	10.8	8.1	5.5	4.8

6. From the following table, estimate the number of students who obtained marks between 45 and 65:

Marks (x):	30-40	40-50	50-60	60-70	70-80
No. of students(y):	33	44	53	37	32

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7. Use Lagrange's interpolation to find the cubic polynomial that interpolates the following data points: $\{(-2,-3),(-1,1),(0,-1),(1,-3)\}$
8. The following are the measurements T made on a curve recorded by oscillograph representing a change of current I due to changes in the conditions of an electric current. Find I at $T = 1.6$

T	1.2	2.0	2.5	3.0
I	1.36	0.58	0.34	0.20

9. If $f(1) = -3, f(3) = 9, f(4) = 30, f(6) = 132$, then find the real root of the equation $f(x) = 0$.
10. If $y(1) = 4, y(3) = 12, y(4) = 19$ and $y(x) = 7$, find x using Lagrange' s interpolation formula.
11. The distance covered by an athlete for 50 meters is given in the following table.

Time (sec)	0	1	2	3	4	5	6
Distance (meter)	0	2.5	8.5	15.5	24.5	36.5	50

Determine the speed of the athlete at t = 5sec.

12. A rod is rotating in a plane. The following table gives the angle θ(radians) through which the rod has turned for various values of the time t second.

t	0	0.2	0.4	0.6	0.8	1.0	1.2
θ	0	0.12	0.49	1.12	2.02	3.20	4.67

Calculate the angular velocity and the angular acceleration of the rod, when $t = 0.4$ second.

13. The table below reveals the velocity v of a body during the specific time t, find its acceleration at $t = 1.1$

t	1.0	1.1	1.2	1.3	1.4
v	43.1	47.7	52.1	56.4	60.8

14. Find $y'(0)$ and $y''(5)$ from the following table:

x	0	1	2	3	4	5
y	4	8	15	7	6	2

15. The following data gives corresponding values of pressure and specific volume of a superheated stream.

v	2	4	6	8	10
p	105	42.7	25.3	16.7	13

Find the rate of change of pressure with respect to volume when $v = 2$.