①

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & k \\ 3 & 2 & k^2 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 4-k \\ 0 & 1 & 6-k^2 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 4-k \\ 0 & 0 & k^2-k-2 \end{bmatrix}$$

$$k^2 - k - 2 = 0 \implies k = 2, -1$$

① ⓑ $u_1 = \dfrac{11 \cdot 2 - u_2 + u_3}{10}$, $u_2 = \dfrac{20 \cdot 1 - u_1 - u_3}{10}$

$$u_3 = \dfrac{35 \cdot 6 + u_1 - u_2}{10}$$

	u_1	u_2	u_3
1	1.12	1.898	3.4822
2	1.2784	1.5339	3.5344
3	1.3200	1.5245	3.5395

②

$$\begin{bmatrix} 1 & -1 & 1 & 2 & : & 6 \\ 2 & 1 & 2 & -1 & : & 6 \\ 4 & 3 & 3 & -3 & : & -1 \\ 2 & 2 & -1 & 1 & : & 10 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -1 & 1 & 2 & : & 6 \\ 0 & -3 & 0 & 5 & : & 6 \\ 0 & -7 & 1 & 11 & : & 25 \\ 0 & -4 & 3 & 3 & : & 2 \end{bmatrix}$$

$$\begin{array}{ccccc} 0 & -21 & 0 & 35 & 42 \\ 0 & -21 & 3 & 33 & : 75 \\ \hline \end{array}$$

$$\sim \begin{bmatrix} 1 & -1 & 1 & 2 & : & 6 \\ 0 & -3 & 0 & 5 & : & 6 \\ 0 & 0 & 3 & -2 & : & 33 \\ 0 & 0 & 9 & -11 & : & -18 \end{bmatrix}$$

$$\begin{array}{ccccc} 0 & -12 & 0 & 20 & 24 \\ 0 & -12 & 9 & 9 & 6 \end{array}$$

$$\sim \begin{bmatrix} 1 & -1 & 1 & 2 & : & 6 \\ 0 & -3 & 6 & 5 & : & 6 \\ 0 & 0 & 3 & -2 & : & 33 \\ 0 & 0 & 0 & 5 & : & 117 \end{bmatrix}$$

$$\boxed{x_4 = 117/5} = 23.4$$

$$x_3 = \dfrac{33 + 2\left(\frac{117}{5}\right)}{3}$$

$$\boxed{x_3 = 133/5} = 26.6$$

$$\boxed{x_2 = 37}$$

$$x_2 = \dfrac{6 - 5\left(117/5\right)}{-3}$$

$$x_1 = x_2 - x_3 + 2x_4 + 6$$

$$x_1 = -\frac{152}{5} = -30.4 \checkmark$$

3 (a)

$$\begin{bmatrix} 1 & 4 & 5 & : & 25 \\ 2 & -3 & 1 & : & -1 \\ 3 & -4 & 1 & : & 2 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 4 & 5 & : & 25 \\ 0 & 11 & 9 & : & 51 \\ 0 & 16 & 14 & : & 73 \end{bmatrix}$$

$$\sim \begin{bmatrix} 11 & 0 & 19 & : & 71 \\ 0 & 11 & 9 & : & 51 \\ 0 & 0 & +10 & : & -13 \end{bmatrix}$$

$$\begin{array}{cccc} 11 & 44 & 55 & 275 \\ 0 & 44 & 36 & 204 \\ 0 & 176 & 144 & 816 \\ 0 & 176 & 154 & 803 \end{array}$$

$$\sim \begin{bmatrix} 110 & 0 & 6 & 957 \\ 0 & 110 & 0 & : 627 \\ 0 & 0 & 10 & : -13 \end{bmatrix}$$

$$\begin{array}{cccc} 0 & 110 & 90 & : 510 \\ 0 & 0 & 90 & : -117 \end{array}$$

$$x = 8.7, \quad y = 5.7, \quad z = -1.3$$

$$\begin{array}{cccc} 110 & 0 & 190 & 710 \\ 0 & 0 & 190 & : -247 \end{array}$$

3(b)

$$\begin{bmatrix} 1 & -2 \\ 3 & -4 \end{bmatrix} \Rightarrow d^2 + 3d + 2 = 0$$

$$d = -1, -2$$

$$-4 + 6$$

For $d = -2$,

$$\begin{bmatrix} 1+2 & -2 \\ 3 & -4+2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$3x - 2y = 0$$

$$x = \frac{2}{3}y, \quad y = k$$

For $d = -1$,

$$\begin{bmatrix} 2 & -2 \\ 3 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$2x - 2y = 0$$

$$\boxed{x = y}, \quad y = k$$

4(a)
$$\begin{bmatrix} 4 & 3 & 0 \\ 0 & 3 & 0 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \\ 4 \end{bmatrix} = 7 \begin{bmatrix} 1 \\ 0.4285 \\ 0.5714 \end{bmatrix}$$

$$\bar{A} x^{(1)} = \begin{bmatrix} 5.2857 \\ 1.2857 \\ 2.5714 \end{bmatrix} = 5.2857 \begin{bmatrix} 1 \\ 0.2432 \\ 0.4864 \end{bmatrix}$$

$$A x^{(2)} = \begin{bmatrix} 4.7297 \\ 0.7297 \\ 2.2162 \end{bmatrix} = 4.7297 \begin{bmatrix} 1 \\ 1.1542 \\ 0.4685 \end{bmatrix}$$

$$A x^{(3)} = \begin{bmatrix} 4.4628 \\ 0.4628 \\ 2.0914 \end{bmatrix} = 4.4628 \begin{bmatrix} 1 \\ 0.1037 \\ 0.4686 \end{bmatrix}$$

4(b) $y = x e^{-x}$

$y' = -x e^{-x} + e^{-x}$ $\Rightarrow$ $(-x+1)e^{-x} = 0$

$\Rightarrow \quad x = 1$

$y'' = + x e^{-x} - e^{-x} - e^{-x}$

$(y'')_{x=1} = e^{-1} - 2e^{-1} = -e^{-1}$

$(y')_{x=1} = -e^{-1} + e^{-1} = 0$

$\therefore \quad S = \dfrac{[1+(y')^2]^{3/2}}{y''} = e$

5(a) $r = a(1+\cos\theta)$ $\qquad r^2 = a^2 \cos 2\theta$

$\dfrac{dr}{d\theta} = -a\sin\theta$ $\qquad 2r\dfrac{dr}{d\theta} = -2a^2 \sin 2\theta$

$\phi_1 = \dfrac{\pi}{2} + \dfrac{2\theta}{2}$ $\qquad \phi_2 = \dfrac{\pi}{2} + \dfrac{1}{2} 2\theta$

$|\phi_1 - \phi_2| = \dfrac{3\theta}{2}$

$\cos^2\theta - 2\cos\theta - 2 = 0$

$\Rightarrow \quad \cos\theta = 1 \pm \sqrt{3}$

$1 - 2\sin^2(\theta/2) = 1 - \sqrt{3}$

$\sin(\theta/2) = \dfrac{3^{1/4}}{\sqrt{2}}$

$\dfrac{\theta}{2} = \sin^{-1}(3/4)^{1/4}$