Unit-I

DC circuits: Ohm's law and Kirchhoff's laws, analysis of series, parallel and series-parallel circuits excited by independent voltage sources. Derivation for Power and energy, Thevenin Theorem & Maximum Power Transfer Theorem applied to the series circuit and its applications.

1.0 Overview

This notes provides a general introduction to electrical theory and DC circuits. Section 2.0 provides basic information about electrical theory. It defines charge, current, voltage, and resistance. Specific information about resistors is given in Section 3.0. Section 4.0 discusses Ohm's law and power. A general overview of circuit analysis is provided in Section 5.0, while Section 6.0 gives application examples of DC circuit analysis. The network theorems of Thevenin and Maximum Power Transfer theorem are given in Section 7.0.

2.0 Fundamental Concepts

2.1 Electric Charges

Charge is a fundamental property of matter. If a neutral atom would lose an electron, it would become positively charged. A neutral atom would become negatively charged if it were to gain an extra electron. When one atom exchanges an electron with another atom, the charge gained by one will equal the charge lost by the other. Therefore, the net charge will never change for an isolated system (conservation of electric charge). Every object has electric charge, but the electric charge is hidden because of the equal amounts of positive and negative charge.

It is, however, possible to transfer electric charge. Proof of this can be seen every time you rub your shoes across a wool rug, and then you touch something (or someone) to remove that charge. This example is a static charge because charge does not move too freely, and the movement is known as static electricity.

Here are a few additional concepts of electrical charge. The SI unit for charge is the coulomb. Charges with the same electrical sign repel each other (like charges repel), and charges with opposite electrical sign attract (unlike charges attract).

2.1.1 Insulators and Conductors

Materials that do not readily allow charges to move are known as **insulators**, or **nonconductors**. Glass and rubber are common examples of insulators. If electric charges move under the influence of electric forces, the material is known as a **conductor**. Copper and aluminum are examples of conductors.

A third material class, known as **semiconductors**, have electrical properties between insulators and conductors. Silicon is a common example of a semiconductor. Materials that are perfect conductors, which allow charge to move without any interference, are called **superconductors**.

2.2 Coulomb's Law

Electrostatic force is a force (either repulsion or attraction) due to charge properties of objects. Coulomb's law is the equation that defines that force for charged particles. Charles-Augustin de Coulomb experimentally determined the relationship in 1785. Consider particle 1, which has a charge q_1, and particle 2, which has a charge q_2. If r defines the distance between the particles, the electrostatic force is given by

Coulomb's Law	$F = k\frac{\lvert q_1\rvert\lvert q_2\rvert}{r^2}$	Equation 1

The constant k is known as the Coulomb constant defined by

Coulomb Constant	$k = 8.9875 \times 10^9\ \frac{N \cdot m^2}{C^2}$	Equation 2

If you remember much about Physics, you may notice the remarkable similarity of Equation 1 to Newton's gravitational law for two masses. The gravitational force between m_1 and m_2 separated by a distance r is defined as

$$F = G\frac{m_1 m_2}{r^2}$$

where G is a gravitational constant.

Two charges, each with a magnitude of 5 μC, are 8 cm apart. What is the force of repulsion between the two charges?

Solution:

From Equation 1

$$F = k\frac{|q_1||q_2|}{r^2}$$

$$F = 8.9875 \times 10^9 \frac{N \cdot m^2}{C^2} \cdot \frac{|5 \times 10^{-6}||5 \times 10^{-6}|}{(0.08\ m)^2}$$

$$F = 35.1\ N$$

2.3 Current, Voltage, and Resistance

2.3.1 Current

Current exists when electric charge moves. In other words, current results from charges in motion. Current is the rate at which charge flows through a cross-sectional surface. Let ΔQ represent the amount of charge passing through an area, and let Δt be the time interval. The current (I) is then defined by

$$I = \frac{\Delta Q}{\Delta t} \qquad \text{Equation 3}$$

The SI unit for current is ampere (A), which is defined by the current equivalent to 1 C of charge passing through a cross-section in the time interval of 1 second.

Example 2

A charge of 8 x 10^{-3} C passes through a wire in 2 seconds. What is the current in the wire?

Solution:

From Equation 3

$$I = \frac{\Delta Q}{\Delta t} = \frac{8 \times 10^{-3}\ C}{2\ s} = 4 \times 10^{-3} A$$

$$I = 4\ mA$$

Conventional current is defined as by the direction where positive charge carriers flow through a circuit. Therefore, electric current flows from the power supply's positive terminal through the circuit and returns to the negative terminal.

If the magnitude of current changes with time, it is called **alternating current**. If the magnitude of current does not change with time, it is called **direct current**. Figure 1 illustrates both types of currents.

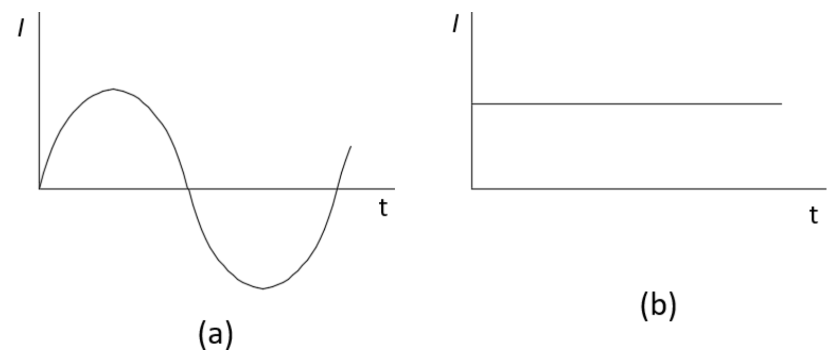


Figure 1 (a) Alternating current and (b) Direct current

2.3.2 Voltage

A voltage source is any device that converts one form of energy into electrical potential energy. Voltage source categories include chemical sources, solar and photovoltaic cells, and electrical conversion (such as a dc power supply that converts an alternating signal into a fixed magnitude signal).

2.3.3 Resistance

The ease of flow of electrons in a material will depend on the number of free electrons. If a material has few electrons per unit volume, it will have a large opposition to the flow of current. The resistance to current flow will depend on the type of material, the length, the cross-sectional area, and temperature. The resistance of a conductor is directly proportional to length, and inversely proportional to its cross-sectional area. Consider a conductor with a length L and a cross-sectional area A. The resistance is given by

Resistance $$R = \rho \frac{L}{A}$$ Equation 4

where ρ is resistivity. Resistivity is an intrinsic property of the material, which depends on the electronic structure of the material. Good conductors will have a very low resistivity, while good insulators will have a high resistivity.

Example 3

Calculate the resistance of a piece of aluminum that is 15 cm long and has a cross-sectional area of 10^{-3} m^2. Use a resistivity of aluminum equal to $2.82 \times 10^{-8} \Omega \cdot m$.

Solution:

Using Equation 4 gives

$$R = \rho \frac{L}{A} = 2.82 \times 10^{-8} \Omega \cdot m \left(\frac{0.15\ m}{10^{-3}\ m^2} \right)$$

$$R = 4.23 \times 10^{-6}\ \Omega$$

Resistivity also depends on temperature. As temperature increases, the atoms vibrate with larger amplitude. This causes, for most materials, the resistivity to increase when temperature increases. Over a limited temperature range, resistivity changes by the following.

$$\rho = \rho_0[1 + \alpha(T - T_0)] \qquad \text{Equation 5}$$

In Equation 5, ρ is the resistivity at temperature T (in °C). The value of ρ_0 is the resistivity at the reference temperature T_0, which is typically 20°C. The coefficient α is the temperature coefficient of resistivity. Because the resistance is proportional to the resistivity, a similar equation can be developed to directly calculate resistance based on temperature change.

$$R = R_0[1 + \alpha(T - T_0)] \qquad \text{Equation 6}$$

Resistivity at 20°C, along with values for the temperature coefficient of resistivity, for a few common materials are given in Table 1.

Table 1 Resistivity (at 20°C) and temperature coefficients of resistivity for common materials

Material	Resistivity ($\Omega\cdot$m)	Temperature coefficient of resistivity (°C)$^{-1}$
Copper	1.7×10^{-8}	3.9×10^{-3}
Aluminum	2.82×10^{-8}	3.9×10^{-3}
Iron	10×10^{-8}	5×10^{-3}
Silicon	640	-75×10^{-3}

Conductance is defined as the reciprocal of resistance. Therefore, conductance is a measure of the ease with which current will flow. The SI unit for conductance is siemens (S) and the symbol is G.

Conductance $$G = \frac{1}{R}$$ Equation 7

For example, if a conductor has a resistance of 250 Ω, the conductance would be equal to the inverse of 250 which is 0.004 S (or 4 mS). Similarly, the inverse of resistivity would give conductivity.

Conductivity $$\sigma = \frac{1}{\rho}$$ Equation 8

3.0 Resistors

Resistors conduct electricity but also dissipate heat. There are two basic types of resistors: composition resistors and wire-wound resistors.

Figure 2 Symbol for fixed resistor

There are also variable resistors, which come in two basic types. The two types are the rheostat and the potentiometer.

Figure 3 Potentiometer

Near one end of the resistor there will be color bands indicating resistance. Table 2 shows the color code for carbon composition resistors. The first three colors indicate magnitude of resistance. An added fourth band of gold or silver gives tolerance of ± 5% or ± 10% respectively (no fourth band indicates ± 20%). For some resistors there will be an additional fifth band indicating reliability. The reliability factor is the percentage of failure per 1000 hours of use.

Table 2 Color code for carbon composition resistors

	First Band	Second Band	Third Band	Fourth Band	Fifth Band
Color	First significant digit	Second significant digit	Multiplier	Tolerance (%)	Reliability (%)
Black	-	0	10^0	-	-
Brown	1	1	10^1	-	1
Red	2	2	10^2	-	0.1
Orange	3	3	10^3	-	0.01
Yellow	4	4	10^4	-	0.001
Green	5	5	10^5	-	-
Blue	6	6	10^6	-	-
Violet	7	7	10^7	-	-
Gray	8	8	10^8	-	-
White	9	9	10^9	-	-
Gold	-	-	10^{-1}	± 5	-
Silver	-	-	10^{-2}	± 10	-
None	-	-	-	± 20	-

As an example, consider a resistor with band 1 red, band 2 violet, band 3 orange, and band 4 is gold. The rating is determined as follows:

Red	=	2 (first significant digit)
Violet	=	7 (second significant digit)
Orange	=	10^3 (multiplier)
Gold	=	± 5% (tolerance)
Resistance	=	$27 \times 10^3\ \Omega \pm 5\%$

4.0 Ohm's Law and Power

4.1 Ohm's Law

It can be determined, for many conductors, that current is proportional to potential difference across the conductor.

$$I \propto V$$

Also, the current in a wire is inversely proportional to the wire's resistance.

$$I \propto \frac{1}{R}$$

George Simon Ohm (1787 – 1854) noted that the current (*I*) in a conductor is directly proportional to the applied voltage (*V*). The proportionality constant was later determined to be the resistance (*R*), which gives the following equation known as Ohm's law.

Ohm's Law $$V = IR$$ Equation 9

It is important to note that Ohm's law is based on experimental observation, and it is not a fundamental law of nature. Therefore, not all materials will obey Ohm's law. Materials that obey Ohm's law are called ohmic and those that do not are called non-ohmic.

Example 4

An object carries a current of 6 A when connected to a 120 V source. What is the resistance of the object?

Solution:

Using Ohm's law from Equation 9 gives

$$V = IR$$

$$R = \frac{V}{I} = \frac{120\ V}{6\ A}$$

$$R = 20\ \Omega$$

4.2 Power

The amount of work that can be accomplished in a specified amount of time is called power. Work has units of joules (J), and time has units of seconds (s). Therefore, power has units of joules per second. For electrical work, the unit is defined as a watt (W). One watt is equal to 1 joule of work that is done in 1 second. Without derivation, equations for electrical power are given below.

Power

$$P = IV$$

$$P = I^2R$$

$$P = \frac{V^2}{R}$$

Equation 10

Example 5

What power is delivered to a DC motor when the input current is 10 A and the supply voltage is 120 V?

Solution:

From Equation 10

$$P = IV = 10\ (120\ V)$$

$$P = 1{,}200\ W$$

Example 6

How much does it cost to burn a 60 W lightbulb for 8 hours if electricity costs 12 cents per kilowatt-hour?

Solution:

The energy consumed will equal the power times time.

$$E = (0.06\ kW)(8\ hr) = 0.48\ kWh$$

At 12 cents per kilowatt-hour, the cost is

$$Cost = 0.48\ kWh(\$0.12/kWh)$$

$$Cost = \$0.06$$

5.0 Single and Multi-Loop Circuits

5.1 Introduction

This section will provide foundational information dealing with circuit analysis. Content is divided into two categories: single-loop circuits, and multi-loop circuits. Single-loop circuits have all resistors arranged in series, while multi-loop circuits contain all resistors arranged in parallel. Circuits containing a combination of series and parallel arrangements will be discussed in Section 6.4.

5.2 Single-Loop Circuits (Series Circuits)

5.2.1 Resistors in Series

Figure 4 (a) shows a series circuit with two resistors. For a series circuit, there is only one pathway for the current.

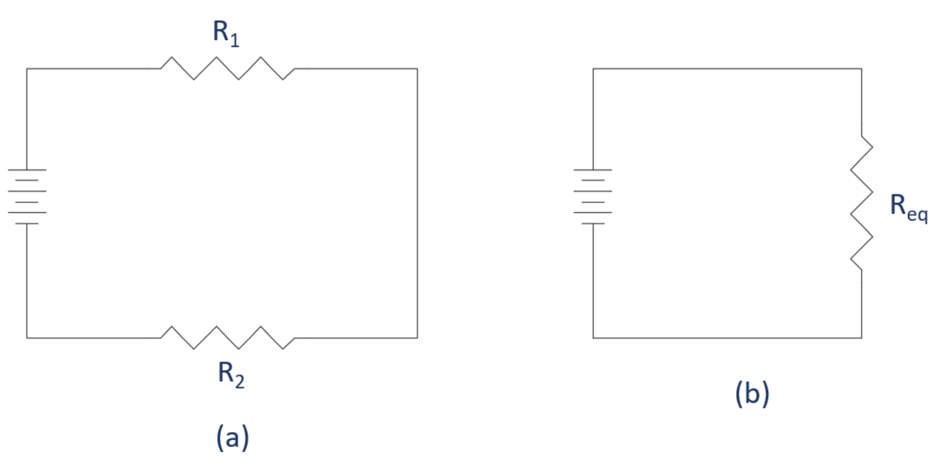


Figure 4 (a) Two resistors in series (b) Circuit with equivalent resistance

Therefore, current is the same through each resistor in a series circuit. The potential drop through the first resistor is equal to IR_1, and the potential drop through the second resistor is equal to IR_2. Therefore, the potential drop across both will be

$$V = IR_1 + IR_2 \qquad \text{Equation 11}$$

Figure 4 (b) shows the circuit with a single equivalent resistor with a resistance R_{eq}. Applying Ohm's law to the equivalent resistor gives

$$V = IR_{eq} \qquad \text{Equation 12}$$

Equating Equation 11 and Equation 12 gives

$$IR_{eq} = IR_1 + IR_2$$

$$R_{eq} = R_1 + R_2 \qquad \text{Equation 13}$$

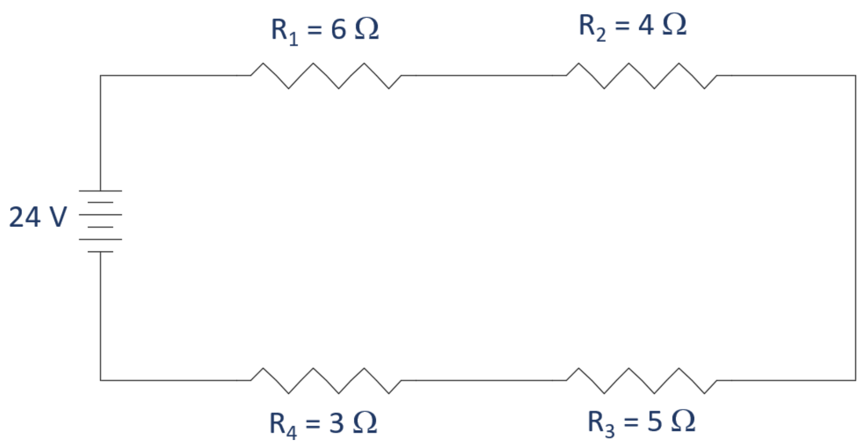
Therefore, resistors in series can be replaced by an equivalent resistor with a resistance equal to the summation of all resistors. For N resistors in series the equivalent resistance is calculated using

$$R_{eq} = R_1 + R_2 + \cdots + R_N$$

$$R_{eq} = \sum_{i=1}^{N} R_i \qquad \text{Equation 14}$$

Example 7

A circuit contains 4 resistors in series. What is the current in the circuit?



Solution:

The equivalent resistance is calculated using Equation 14.

$$R_{eq} = \sum_{i=1}^{N} R_i = 6\,\Omega + 4\,\Omega + 5\,\Omega + 3\,\Omega = 18\,\Omega$$

Ohm's law, from Equation 12, can now be used to calculate current.

$$V = IR_{eq}$$

$$I = \frac{V}{R_{eq}} = \frac{24\,V}{18\,\Omega}$$

$$I = 1\frac{1}{3}\,A$$

5.2.2 Kirchoff's Voltage Law

In single-loop circuits (series circuits), the algebraic sum of the changes in potential encountered in the loop must equal zero. This law, which is referred to as Kirchoff's loop rule or Kirchoff's voltage law (named after German physicist Gustav Robert Kirchoff), is illustrated graphically in Figure 5.

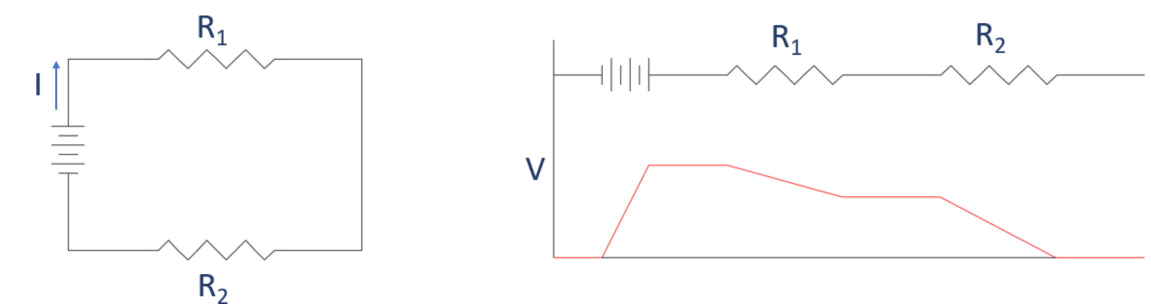


Figure 5 Voltage change through the circuit

Another way to word the voltage law is that the sum of all voltage drops in a circuit must equal the applied voltage. For a series of *N* resistors in a circuit, this can be expresses mathematically as

$$V_{supply} = V_{R1} + V_{R2} + V_{R3} + \cdots + V_{RN} \qquad \text{Equation 15}$$

5.3 Multi-Loop Circuits (Parallel Circuits)

5.3.1 Resistors in Parallel

Consider two resistors connected in parallel, as shown in Figure 6 (a). The potential difference must be the same across each resistor when arranged in parallel. The current, in general, will not be the same in each resistor (it will only be the same if the resistors have equal resistance).

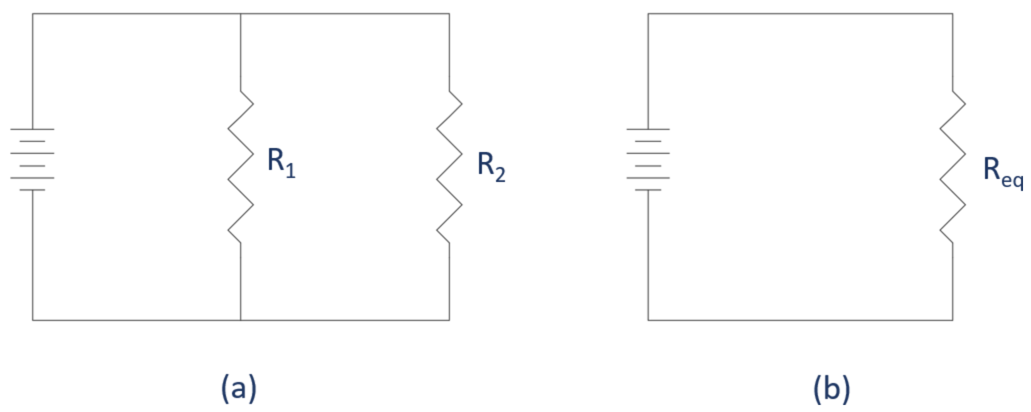


Figure 6 (a) Two resistors in parallel (b) Circuit with equivalent resistance

5.3.2 Kirchoff's Current Law

Kirchoff's current law states that the sum of the currents entering any junction must be equal to the sum of the currents leaving that junction. The concept is illustrated in Figure 7.

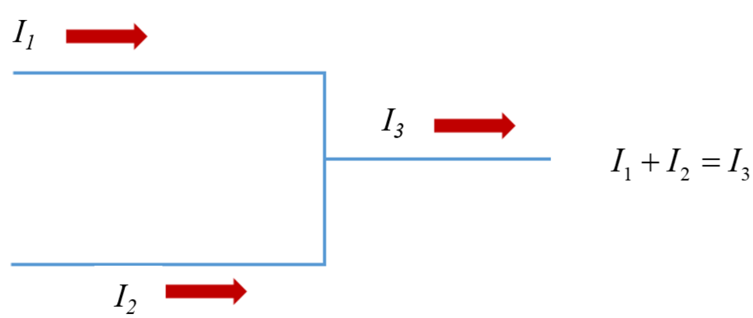


Figure 7 Kirchoff's current law

The resistors in parallel can be replaced by an equivalent resistor as shown in Figure 6 (b). Applying Ohm's law to the equivalent resistor give

$$I = \frac{V}{R_{eq}} \qquad \text{Equation 16}$$

Equation 16 can be substituted into Kirchoff's current law, which is shown in Figure 7, to give

$$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \qquad \text{Equation 17}$$

Therefore, the equivalent resistance for N resistors arranged in parallel equals

$$\frac{1}{R_{eq}} = \sum_{i=1}^{N} \frac{1}{R_i} \qquad \text{Equation 18}$$

From Equation 18 it can be determined that the equivalent resistance of two or more resistors connected in parallel will always be less than the smallest resistor in the group.

Three resistors, which resistances of 3 Ω, 6 Ω, and 9 Ω, are connected in parallel. What is the equivalent resistance?

Solution:

From Equation 18

$$\frac{1}{R_{eq}} = \sum_{i=1}^{N} \frac{1}{R_i} = \frac{1}{3\ \Omega} + \frac{1}{6\ \Omega} + \frac{1}{9\ \Omega}$$

$$R_{eq} = 1.64\ \Omega$$

Further examples of parallel circuits will be provided in Section 6.3.

6.0 DC Circuit Analysis

6.1 Introduction

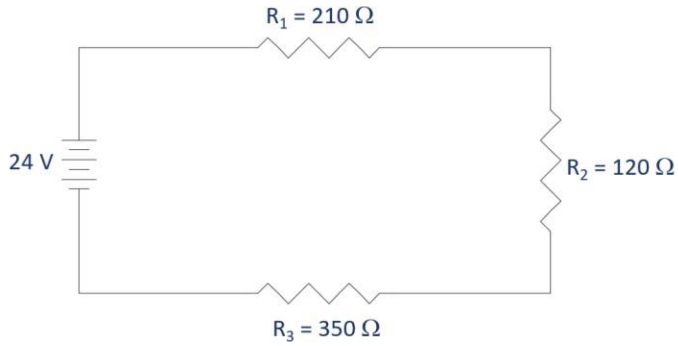
This section will provide application examples of series and parallel DC circuits. The concepts covered in previous sections will be used to analyze the circuits. In addition, some new concepts will be introduced to aid in the analysis of the circuits. Application examples will be separated into categories of series circuits and parallel circuits. The last section will consider problems that are a combination of series and parallel arrangements.

6.2 Series Circuits

We will begin with examples involving only series circuits.

Example 9

Determine the amount of current that will flow for the circuit shown. What is voltage drop across resistor R_2?



Solution:

The total equivalent resistance can be calculated using Equation 14.

$$R_{eq} = R_1 + R_2 + R_3$$

$$R_{eq} = 210\ \Omega + 120\ \Omega + 350\ \Omega = 680\ \Omega$$

Rearranging Equation 12 will give the current.

$$V = IR_{eq}$$

$$I = \frac{V}{R_{eq}} = \frac{24\ V}{680\ \Omega}$$

$$I = 35.29\ mA$$

The voltage drop across resistor R_2 can be determined by multiplying the current and the resistance.

$$V_{R2} = IR_2 = 0.03529\ (120\ \Omega)$$

$$V_{R2} = 4.24\ V$$

6.2.1 Voltage Divider Rule

The second part of the previous example asked for the voltage drop across a resistor. The method used required the current. However, another method exists for calculating voltage drops across a given resistor in a series circuit. The **voltage divider rule** states that the ratio between any two voltage drops in a series circuit is equal to the ratio of the two resistances. Mathematically, the voltage divider rule states

$$V_{Rx} = V_T \frac{R_x}{R_T} \qquad \text{Equation 19}$$

Example 10

Use the voltage divider rule to determine the voltage drop across each resistor in the previous example.

Solution:

Using Equation 19 for each resistor gives

$$V_{R1} = V_T \frac{R_1}{R_T} = 24\,V \frac{210\,\Omega}{680\,\Omega}$$

$$V_{R1} = 7.41V$$

$$V_{R2} = V_T \frac{R_2}{R_T} = 24\,V \frac{120\,\Omega}{680\,\Omega}$$

$$V_{R2} = 4.24\,V$$

$$V_{R3} = V_T \frac{R_3}{R_T} = 24\,V \frac{350\,\Omega}{680\,\Omega}$$

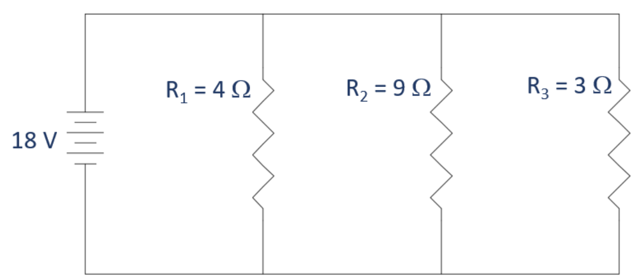
$$V_{R3} = 12.35\,V$$

6.3 Parallel Circuits

Now we can consider examples only using parallel arrangements.

Example 11

Three resistors are connected in parallel as shown. Determine the current in each resistor.



Solution:

For a parallel circuit, the voltage will be the same through each resistor. Using Ohm's law gives

$$I_1 = \frac{V}{R_1} = \frac{18\,V}{4\,\Omega}$$

$$I_1 = 4.5\,A$$

$$I_2 = \frac{V}{R_2} = \frac{18\,V}{9\,\Omega}$$

$$I_2 = 2\,A$$

$$I_3 = \frac{V}{R_3} = \frac{18\,V}{3\,\Omega}$$

$$I_3 = 6\,A$$

The concept of parallel circuits can be treated the same as fluid flow in a branch. Fluid will follow the path of least resistance. Notice that the flow (current) followed the path of least resistance because the largest current flow was through the lowest resistor.

Next, we will consider a problem using Kirchoff's current law discussed in Section 5.3.2. The current law was briefly introduced in Section 5.3.2, but it will be expanded here. It is important to understand the sign convention. The four rules below apply, with each assuming we are moving from point *a* to point *b*.

1 If a resistor is traversed in the direction of the current:

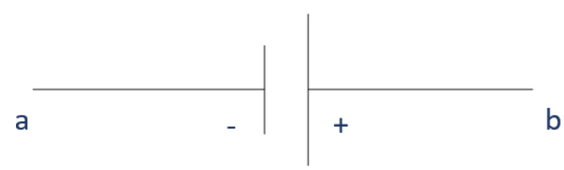
$$\Delta V = V_b - V_a = -IR$$

2 If a resistor is traversed in the direction opposite the current:

$$\Delta V = V_b - V_a = +IR$$

3 If a voltage source is traversed from – to +:

$$\Delta V = V_b - V_a = +V$$



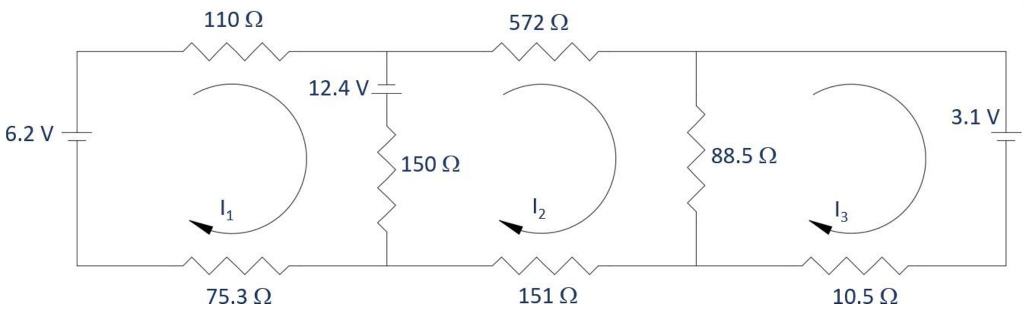
4 If a voltage source is traversed from + to -:

$$\Delta V = V_b - V_a = -V$$

The rules, and the process of using Kirchoff's current law, are best illustrated in an example problem. For each loop in the following example use the rules above to determine the proper sign.

Example 12

Use Kirchoff's law to determine the value of the three currents.



Solution:

Loop 1

$$6.2 - 110I_1 + 12.4 - 150I_1 + 150I_2 - 75.3I_1 = 0$$

$$-335I_1 + 150I_2 = -18.6 \quad [1]$$

Loop 2

$$-150I_2 + 150I_1 - 12.4 - 572I_2 - 88.5I_2 + 88.5I_3 - 151I_2 = 0$$

$$150I_1 - 962I_2 + 88.5I_3 = 12.4 \quad [2]$$

Loop 3

$$-88.5I_3 + 88.5I_2 - 3.1 - 10.5I_3 = 0$$

$$88.5I_2 - 99I_3 = 3.1 \quad [3]$$

Putting equations [1], [2], and [3] in matrix form gives

$$\begin{bmatrix} -335 & 150 & 0 \\ 150 & -962 & 88.5 \\ 0 & 88.5 & -99 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -18.9 \\ 12.4 \\ 3.1 \end{bmatrix}$$

Solving the system of equations for the current values gives

$$I_1 = 52.7\ mA$$
$$I_2 = -8.22\ mA$$
$$I_3 = -38.7\ mA$$

6.3.1 Current Divider Rule

The current divider rule states that amount of current in one of two parallel resistances is equal to the product of total current and other resistance divided by the sum of resistance.

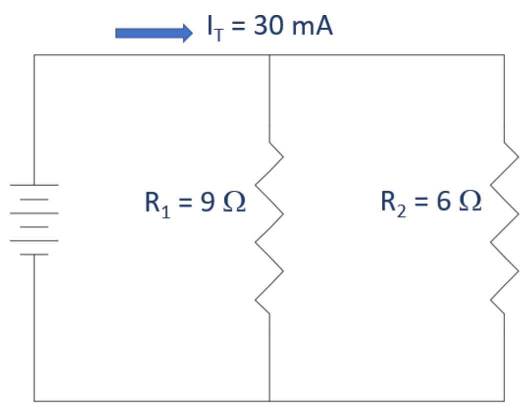
Current Divider Rule

$$I_1 = \frac{I_T R_2}{R_1 + R_2}$$
$$I_2 = \frac{I_T R_1}{R_1 + R_2}$$

Equation 20

Example 13

Use the current divider rule to determine the currents flowing through R_1 and R_2.



Solution:

Using the current divider rule in Equation 20

$$I_1 = \frac{I_T R_2}{R_1 + R_2} = \frac{30\ (6\ \Omega)}{(9\ \Omega) + (6\ \Omega)}$$

$$I_1 = 12\ mA$$

$$I_2 = \frac{I_T R_1}{R_1 + R_2} = \frac{30\ (9\ \Omega)}{(9\ \Omega) + (6\ \Omega)}$$

$$I_2 = 18\ mA$$

6.4 Series-Parallel Circuits

Circuits that contain a combination of series and parallel circuits are known as series-parallel circuits. The equations for equivalent resistance for series and parallel circuits can be applied to systems with a combination of series and parallel arrangements. Consider the arrangement shown in Figure 8. The 5 Ω and 9 Ω resistors are arranged in series, and they can be simplified to the 14 Ω equivalent resistor using Equation 14. The 6 Ω and 3 Ω resistors are arranged in parallel, and they can be simplified to the equivalent resistance of 2 Ω using Equation 18. The new arrangement has two resistors (14 Ω and 2 Ω) in series, which can be simplified again to the final equivalent resistance of 16 Ω.

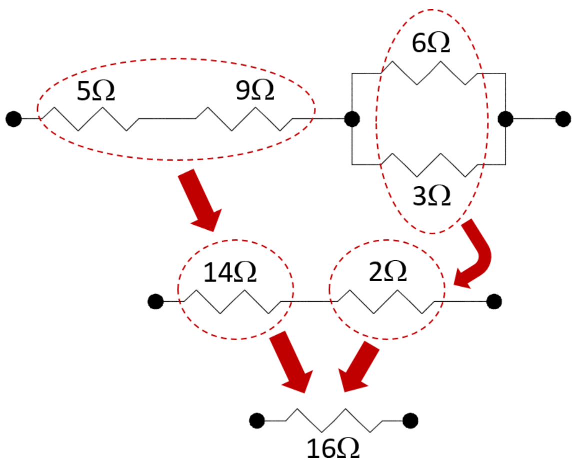
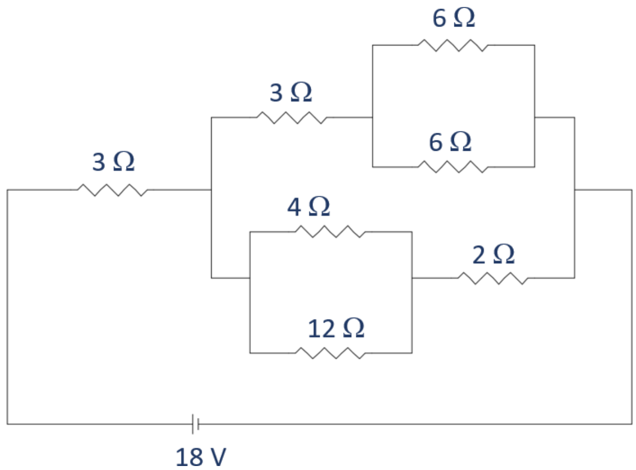


Figure 8 Equivalent resistance concept for series-parallel circuits

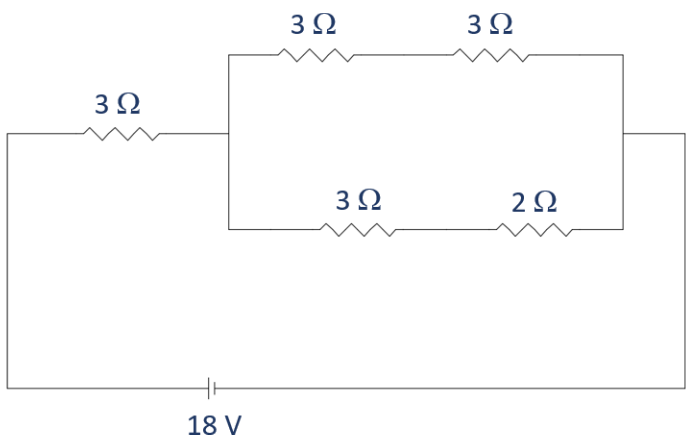
Example 14

Determine the current in the 12 Ω resistor for the circuit shown.

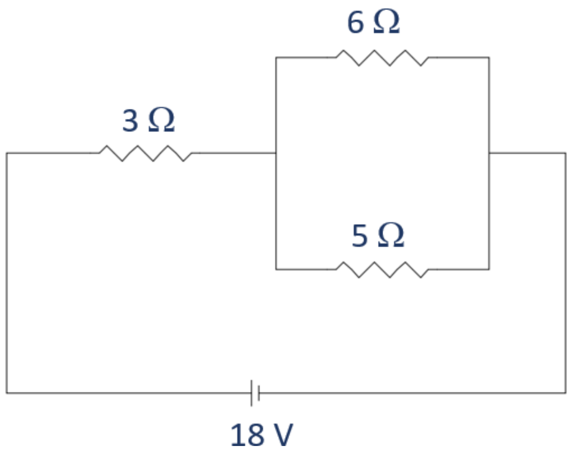


Solution:

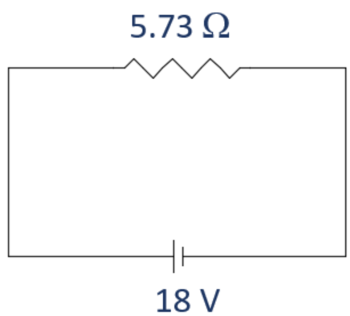
The first step is to determine the equivalent resistance for all resistors. The two 6 Ω resistors and the 4 Ω and 12 Ω resistors in parallel can be reduced using Equation 18.



The resulting parallel circuits now have resistors in series, which can be simplified using Equation 14.



The two resistors in parallel can be reduced to one 2.73 Ω, which leaves two resistors in series. Summing the two in series gives the final equivalent resistance.



Ohm's law can now be used to determine the total current.

$$I_T = \frac{V}{R_{eq}} = \frac{18\ V}{5.73\ \Omega} = 3.14\ A$$

Move now to the previous figure. The full 3.14 A will pass through the 3 Ω resistor and will be split between the two in parallel. The current divider rule can be used to determine the current passing through the 5 Ω resistor.

$$I_{5\Omega} = \frac{I_T R_1}{R_1 + R_2} = \frac{3.14\ (6\ \Omega)}{6\ \Omega + 5\ \Omega} = 1.71\ A$$

Now reference the original figure. The 1.71 A will be split between the 4 Ω and 12 Ω resistors. Using the current divider rule to determine the current through the 12 Ω resistor gives

$$I_{12\Omega} = \frac{I_T R_1}{R_1 + R_2} = \frac{1.71\ (4\Omega)}{4\ \Omega + 12\ \Omega}$$

$$I_{12\Omega} = 0.429\ A$$

7.0 Network Theorems

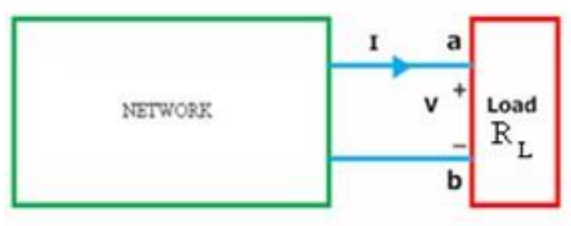
Any complicated network i.e. several sources, multiple resistors are present if the single element response is desired then use the network theorems. Network theorems are also can be termed as network reduction techniques. Each and every theorem got its importance of solving network. Let us see some important theorems with DC and AC excitation with detailed procedures.

7.1 Thevenin's Theorem and Norton's theorem:

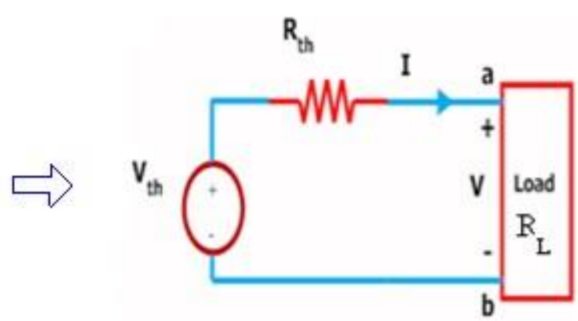
Thevenin's Theorem and Norton's theorem are two important theorems in solving Network problems having many active and passive elements. Using these theorems the networks can be reduced to simple equivalent circuits with one active source and one element. In circuit analysis many a times the current through a branch is required to be found when it's value is changed with all other element values remaining same. In such cases finding out every time the branch current using the conventional mesh and node analysis methods is quite awkward and time consuming. But with the simple equivalent circuits (with one active source and one element) obtained using these two theorems the calculations become very simple. Thevenin's and Norton's theorems are dual theorems.

Thevenin's Theorem Statement:

Any linear, bilateral two terminal network consisting of sources and resistors(Impedance),can be replaced by an equivalent circuit consisting of a voltage source in series with a resistance (Impedance).The equivalent voltage source $\mathbf{V_{Th}}$ is the open circuit voltage looking into the terminals(with concerned branch element removed) and the equivalent resistance $\mathbf{R_{Th}}$ while all sources are replaced by their internal resistors at ideal condition i.e. voltage source is short circuit and current source is open circuit.



(a)



(b)

Figure (a) shows a simple block representation of a network with several active / passive elements with the load resistance $\mathbf{R_L}$ connected across the terminals **'a & b'** and figure (b) shows the **Thevenin equivalent circuit** with $\mathbf{V_{Th}}$ connected across $\mathbf{R_{Th}}$ & $\mathbf{R_L}$ **.**

Main steps to find out $\mathbf{V_{Th}}$ and $\mathbf{R_{Th}}$:

1. The terminals of the branch/element through which the current is to be found out are marked as say **a & b** after removing the concerned branch/element.

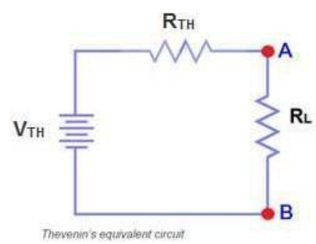
2. Open circuit voltage V_{OC} across these two terminals is found out using the conventional network mesh/node analysis methods and this would be V_{Th} **.**

3. **Thevenin resistance R_{Th}** is found out by the method depending upon whether the network contains dependent sources or not.

   a. With dependent sources: $R_{Th} = V_{oc} / I_{sc}$

   b. Without dependent sources : R_{Th} = **Equivalent resistance looking into the concerned terminals** with all voltage & current sources replaced by their internal impedances (i.e. ideal voltage sources short circuited and ideal current sources open circuited)

4. Replace the network with V_{Th} in series with R_{Th} and the concerned branch resistance **(or)** load resistance across the load terminals(A&B) as shown in below fig.



Example: Find V_{TH}, R_{TH} and the load current and load voltage flowing through R_L resistor as shown in fig. by using Thevenin's Theorem?

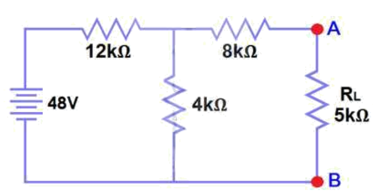


Fig.(a)

Solution:

The resistance R_L is removed and the terminals of the resistance R_L are marked as **A & B** as shown in the fig. (1)

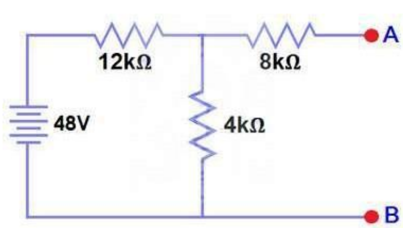
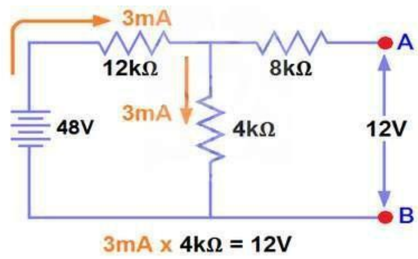


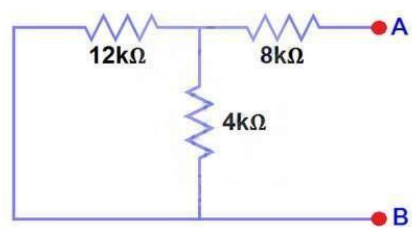
Fig.(1)

Calculate / measure the Open Circuit Voltage. This is the Thevenin Voltage (V_{TH}). We have already removed the load resistor from fig.(a), so the circuit became an open circuit as shown in fig (1). Now we have to calculate the Thevenin's Voltage. Since 3mA Current flows in both 12kΩ and 4kΩ resistors as this is a series circuit because current will not flow in the 8kΩ resistor as it is open. So 12V (3mA x 4kΩ) will appear across the 4kΩ resistor. We also know that current is not flowing through the 8kΩ resistor as it is open circuit, but the 8kΩ resistor is in parallel with 4k resistor. So the same voltage (i.e. 12V) will appear across the 8kΩ resistor as 4kΩ resistor. Therefore 12V will appear across the AB terminals. So, $\mathbf{V_{TH} = 12V}$



Fig(2)

All voltage & current sources replaced by their internal impedances (i.e. ideal voltage sources short circuited and ideal current sources open circuited) as shown in fig.(3)



Fig(3)

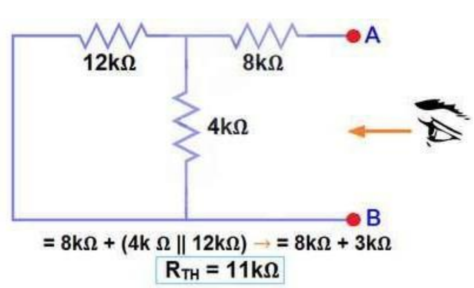
Calculate /measure the Open Circuit Resistance. This is the Thevenin Resistance (R_{TH})We have Reduced the 48V DC source to zero is equivalent to replace it with a short circuit as shown in figure (3) We can see that 8kΩ resistor is in series with a parallel connection of 4kΩ resistor and 12k Ω resistor. i.e.:

8kΩ + (4k Ω || 12kΩ) ….. (|| = in parallel with)

R_{TH} = 8kΩ + [(4kΩ x 12kΩ) / (4kΩ + 12kΩ)]

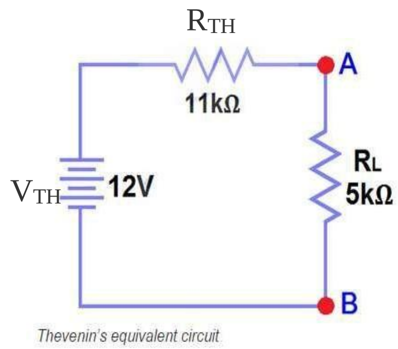
R_{TH} = 8kΩ + 3kΩ

R_{TH} = 11kΩ



Fig(4)

Connect the R_{TH} in series with Voltage Source V_{TH} and re-connect the load resistor across the load terminals(A&B) as shown in fig (5) i.e. Thevenin circuit with load resistor. This is the Thevenin's equivalent circuit



Fig(5)

Now apply Ohm's law and calculate the total load current from fig 5.

$I_L = V_{TH} / (R_{TH} + R_L) = 12V / (11k\Omega + 5k\Omega) = 12/16k\Omega$

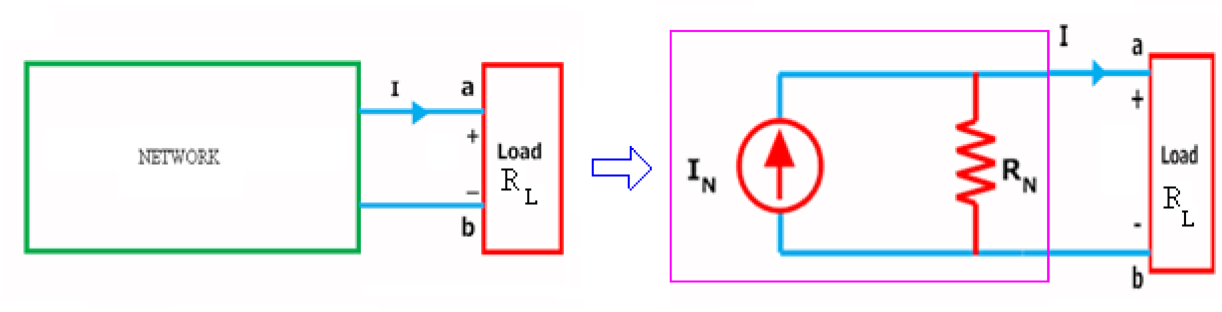
$\mathbf{I_L = 0.75mA}$

And $V_L = I_L x\ R_L = 0.75mA \times 5k\Omega$

$\mathbf{V_L = 3.75V}$

Norton's Theorem Statement :

Any linear, bilateral two terminal network consisting of sources and resistors(Impedance),can be replaced by an equivalent circuit consisting of a current source in parallel with a resistance (Impedance),the current source being the short circuited current across the load terminals and the resistance being the internal resistance of the source network looking through the open circuited load terminals.



(a) (b)

Figure (a) shows a simple block representation of a network with several active / passive elements with the load resistance $\mathbf{R_L}$ connected across the terminals **'a & b'** and figure (b) shows the **Norton equivalent circuit** with $\mathbf{I_N}$ connected across $\mathbf{R_N}$ & $\mathbf{R_L}$ **.**

Main steps to find out $\mathbf{I_N}$ and $\mathbf{R_N}$:

1. The terminals of the branch/element through which the current is to be found out are marked as say **a & b** after removing the concerned branch/element.

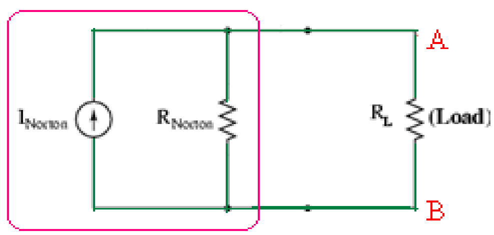
2. Open circuit voltage V_{OC} across these two terminals and I_{SC} through these two terminals are found out using the conventional network mesh/node analysis methods and they are same as what we obtained in Thevenin's equivalent circuit**.**

3. Next **Norton resistance R_N** is found out depending upon whether the network contains dependent sources or not.

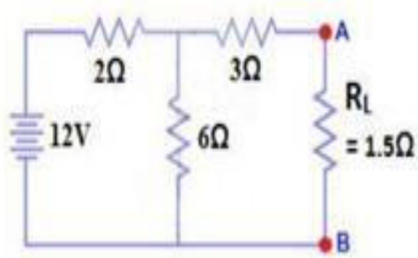
   a) With dependent sources: $\mathbf{R_N = V_{oc} / I_{sc}}$

   b) Without dependent sources : $\mathbf{R_N}$ = Equivalent resistance looking into the concerned terminals with all voltage & current sources replaced by their internal impedances (i.e. ideal voltage sources short circuited and ideal current sources open circuited)

4. Replace the network with $\mathbf{I_N}$ in parallel with $\mathbf{R_N}$ and the concerned branch resistance across the load terminals(A&B) as shown in below fig

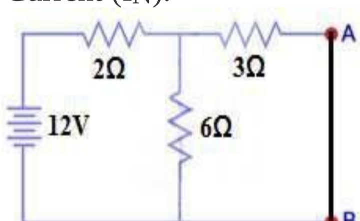


Example: Find the current through the resistance R_L (1.5 Ω) of the circuit shown in the figure (a) below using Norton's equivalent circuit.?



Fig(a)

Solution: To find out the Norton's equivalent ckt we have to find out $\mathbf{I_N = I_{sc}}$ **,**$\mathbf{R_N = V_{oc} / I_{sc}}$**.** Short the 1.5Ω load resistor as shown in (Fig 2), and Calculate / measure the Short Circuit Current. This is the Norton Current (I_N).



Fig(2)

We have shorted the AB terminals to determine the Norton current, I_N. The 6Ω and 3Ω are then in parallel and this parallel combination of 6Ω and 3Ω are then in series with 2Ω.So the Total Resistance of the circuit to the Source is:-

2Ω + (6Ω || 3Ω) ….. (|| = in parallel with)

R_T = 2Ω + [(3Ω x 6Ω) / (3Ω + 6Ω)]

R_T = 2Ω + 2Ω

R_T = 4Ω

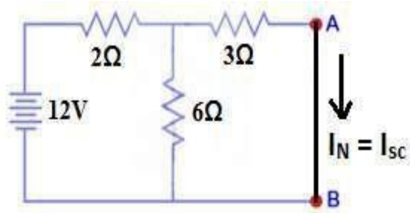
I_T = V / R_T

I_T = 12V / 4Ω= 3A..

Now we have to find I_{SC} = I_N… Apply CDR… (Current Divider Rule)…

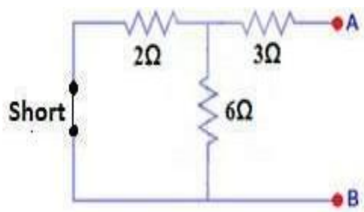
I_{SC} = I_N = 3A x [(6Ω / (3Ω + 6Ω)] = 2A.

I_{SC}= I_N = 2A.



Fig(3)

All voltage & current sources replaced by their internal impedances (i.e. ideal voltage sources short circuited and ideal current sources open circuited) and Open Load Resistor. as shown in fig.(4)



Fig(4)

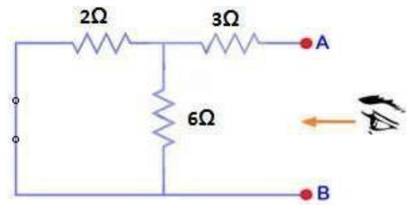
Calculate /measure the Open Circuit Resistance. This is the Norton Resistance (R_N) We have Reduced the 12V DC source to zero is equivalent to replace it with a short circuit as shown in fig(4), We can see that 3Ω resistor is in series with a parallel combination of 6Ω resistor and 2Ω resistor. i.e.:

3Ω + (6Ω || 2Ω) ….. (|| = in parallel with)

R_N = 3Ω + [(6Ω x 2Ω) / (6Ω + 2Ω)]

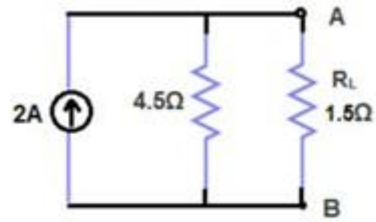
R_N = 3Ω + 1.5Ω

R_N = 4.5Ω



Fig(5)

Connect the R_N in Parallel with Current Source I_N and re-connect the load resistor. This is shown in fig (6) i.e. Norton Equivalent circuit with load resistor.



Fig(6)

Now apply the Ohm's Law and calculate the load current through Load resistance across the terminals A&B. Load Current through Load Resistor is

$I_L = I_N$ x $[R_N / (R_N + R_L)]$

$I_L =$ 2A x (4.5Ω /4.5Ω +1.5kΩ)

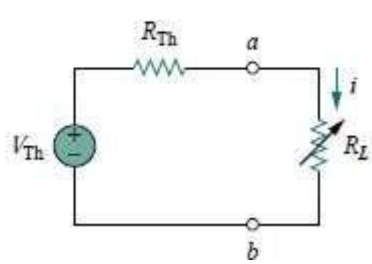
I_L = 1.5A **I_L = 1. 5A**

7.2 Maximum Power Transfer Theorem:

In many practical situations, a circuit is designed to provide power to a load. While for electric utilities, minimizing power losses in the process of transmission and distribution is critical for Efficiency and economic reasons, there are other applications in areas such as communications where it is desirable to maximize the power delivered to a load. electrical applications with electrical loads such as Loud speakers, antennas, motors etc. it would be required to find out the condition under which maximum power would be transferred from the circuit to the load.

Maximum Power Transfer Theorem Statement:

Any linear, bilateral two terminal network consisting of a resistance load, being connected to a dc network, receives maximum power when the load resistance is equal to the internal resistance (Thevenin's equivalent resistance) of the source network as seen from the load terminals.



According to Maximum Power Transfer Theorem, for maximum power transfer from the network to the load resistance , **R_L** must be equal to the source resistance i.e. Network's Thevenin equivalent resistance **R_{Th} . i.e. $R_L = R_{Th}$**

The load current **I** in the circuit shown above is given by,

$$I = \frac{V_{TH}}{R_{TH}+R_L}$$

The power delivered by the circuit to the load:

$$P = I^2 R = \frac{V_{TH}^2}{(R_{TH}+R_L)^2} R_L$$

The condition for maximum power transfer can be obtained by differentiating the above expression for power delivered with respect to the load resistance (Since we want to find out the value of $\mathbf{R_L}$ for maximum power transfer) and equating it to zero as :

$$\frac{\partial P}{\partial R_L} = 0 = \frac{V_{TH}^2}{(R_{TH}+R_L)^2} - \frac{2V_{TH}^2}{(R_{TH}+R_L)^3} R_L = 0$$

Simplifying the above equation, we get:

$$(R_{TH} + R_L) - 2R_L = 0 \Longrightarrow R_L = R_{TH}$$

Under the condition of maximum power transfer, the power delivered to the load is given by :

$$P_{MAX} = \frac{V_{TH}^2}{(R_L+R_L)^2} \times R_L = \frac{V_{TH}^2}{4R_L}$$

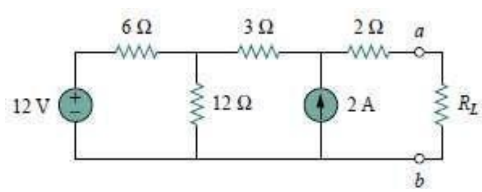
Under the condition of maximum power transfer, the efficiency $\boldsymbol{\eta}$ of the network is then given by:

$$P_{LOSS} = \frac{V_{TH}^2}{(R_L+R_L)^2} \times R_{TH} = \frac{V_{TH}^2}{4R_L}$$

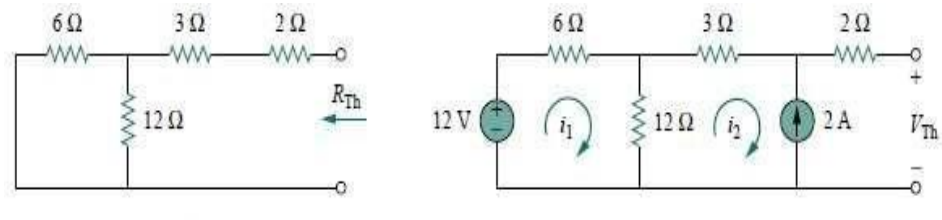
$$\boldsymbol{\eta} = \frac{\text{output}}{\text{input}} = \frac{\frac{V_{TH}^2}{4R_L}}{\left(\frac{V^2}{4R_L} + \frac{V_{TH}^2}{4R_L}\right)} = 0.50$$

For maximum power transfer the load resistance should be equal to the Thevenin equivalent resistance (or Norton equivalent resistance) of the network to which it is connected . Under the condition of maximum power transfer the efficiency of the system is 50 %.

Example: Find the value of $\mathbf{R_L}$ for maximum power transfer in the circuit of Fig. Find the maximum power.?



Solution: We need to find the Thevenin resistance R_{Th} and the Thevenin voltage V_{Th} across the terminals *a-b*. To get R_{Th}, we use the circuit in Fig. (a)



(a) (b)

$R_{Th} = 2 + 3 + (6 \,//\, 12) = 5 + \left(\frac{6\times12}{6+12}\right) = 5+4 = 9\Omega$

To get V_{Th}, we consider the circuit in Fig.(b). Applying mesh analysis,

$-12 + 18i_1 - 12i_2 = 0,$

$i_2 = -2$ A,

Solving for i1, we get $i_1 = -2/3$.

Applying KVL around the outer loop to get V_{Th} across terminals *a-b*, we obtain,

$-12 + 6i_1 + 3i_2 + 2(0) + V_{Th} = 0$

$V_{Th} = 22$ V

For maximum power transfer, $R_L = R_{Th} = 9\Omega$ and the maximum power is,

$$P_{MAX} = \frac{V_{TH}^2}{4R_L} = \frac{22\times22}{4\times9} = 13.44\text{W}$$