

Additional Problems on Single - Phase Transformers.

1. A single-phase transformer with 10:1 turns ratio and rated at 25kVA, 1200/120V is used to step down the voltage of a distribution system at 50Hz. The low tension voltage is to be kept constant at 120V. Calculate the value of load impedance to be connected to the low tension winding so that the transformer is fully loaded. Find also the maximum flux if the low tension winding has 25 turns.

Solⁿ

The transformer is said to be fully loaded when it draws rated current in the secondary.

This current is also called the full-load current.

The full-load current is,

$$I_2 = \frac{\text{kVA}}{V_2} = \frac{25}{120} = 0.20833 \text{ kA}$$

$$\text{or } I_2 = 208.33 \text{ A.}$$

The impedance to be connected on the secondary so that the secondary voltage is at 120V is,

$$Z_{\text{load}} = \frac{V_2}{I_2} = \frac{120}{208.33} = 0.576$$

$$\therefore \boxed{Z_{\text{load}} = 0.576 \Omega}$$

The secondary voltage in an ideal transformer is,

$$E_2 = V_2 = 4.44 f \phi_m N_2$$

$$\therefore \phi_m = \frac{E_2}{4.44 f N_2} = \frac{120}{4.44 \times 50 \times 25} \Rightarrow \boxed{\phi_m = 0.021 \text{ Wb}}$$

2. Find the number of turns required on the HT side of a 415V/240V, 50Hz, 1 ϕ transformer, if the area of cross-section of the core is 25 cm² and the maximum flux density is 1.3 Wb/m².

Sol.ⁿ

$$A_{\text{core}} = 25 \text{ cm}^2 = 25 \times 10^{-4} \text{ m}^2$$

$$B_m = 1.3 \text{ Wb/m}^2.$$

\therefore Flux has a maximum value of,

$$\Phi_m = B_m \times A_{\text{core}} = 1.3 \times 25 \times 10^{-4} = 3.25 \text{ mWb}.$$

The HT side voltage is, $V_1 = 415 \text{ V}$.

For an ideal transformer, $E_1 = V_1$.

$$\therefore E_1 = V_1 = 4.44 f \Phi_m N_1$$

$$\Rightarrow N_1 = \frac{E_1}{4.44 f \Phi_m} = \frac{415}{4.44 \times 50 \times 3.25 \times 10^{-3}}$$

$$\Rightarrow N_1 = 580.42$$

Rounding off, we get

$$\boxed{N_1 = 580 \text{ turns}}$$

3. The required no-load ratio of a 1 ϕ , 50Hz, core-type transformer is 6000/250V. Calculate the number of turns per limb on the high voltage & the low voltage sides if the flux is to be about 0.06 Wb.

Sol.ⁿ

For the ideal transformer on no-load,

$$V_1 = E_1 = 6000 \text{ V}$$

$$V_2 = E_2 = 250 \text{ V}.$$

Using EMF equation,

$$E_1 = 4.44 f \Phi_m N_1, \text{ we get}$$

$$N_1 = \frac{E_1}{4.44 f \Phi_m} = \frac{6000}{4.44 \times 50 \times 0.06}$$

$$\boxed{N_1 = 450 \text{ turns}} \text{ (rounded off)}$$

from $E_2 = 4.44 f \Phi_m N_2$, we get

$$N_2 = \frac{E_2}{4.44 f \Phi_m} = \frac{250}{4.44 \times 50 \times 0.06}$$

$$\Rightarrow \boxed{N_2 = 19 \text{ turns}} \text{ (rounded off)}$$

In an actual core-type transformer, half of the LV windings are placed on one limb & half of the HV on the same limb around the LV winding. The remaining LV & HV windings are placed on the other limb.

$$\therefore N_1 / \text{limb} = \frac{450}{2} \Rightarrow \boxed{N_1 / \text{limb} = 225 \text{ turns}}$$

$$N_2 / \text{limb} = \frac{19}{2} \Rightarrow \boxed{N_2 / \text{limb} = 9.5 \text{ turns}}$$

4. A 125kVA transformer has a primary voltage of 2000V at 60Hz. The primary has 182 turns while the secondary has 40 turns. Neglecting losses, calculate:

- (i) primary & secondary currents on full-load
- (ii) the secondary EMF on no-load
- (iii) the maximum flux in the core.

Solⁿ

$$K = \frac{N_2}{N_1} = \frac{40}{182} = 0.22$$

$$\text{kVA rating} = 125.$$

Primary current on full-load is,

$$I_1 = \frac{VA}{V_1} = \frac{125 \times 10^3}{2000} \Rightarrow \boxed{I_1 = 62.5 \text{ A}}$$

Primary full-load current is the current that flows in the primary when the kVA of the load is equal to the transformer rating.

The secondary full-load current is,

$$I_2 = \frac{I_1}{K} \quad (\text{neglecting small no-load current})$$

$$= \frac{62.5}{0.22}$$

$$\boxed{I_2 = 284.1 \text{ A}}$$

The secondary EMF on no-load is

$$E_2 = KE_1 = KV_1 = 0.22 \times 2000$$

$$\therefore \boxed{E_2 = 440 \text{ V}}$$

The flux in the core can be calculated using the EMF equation

$$E_1 = 4.44 f \phi_m N_1$$

$$\Rightarrow \phi_m = \frac{E_1}{4.44 f N_1} = \frac{2000}{4.44 \times 60 \times 182}$$

$$\boxed{\phi_m = 0.04125 \text{ Wb}}$$

ALITER

$$E_2 = 4.44 f \phi_m N_2 \Rightarrow \phi_m = \frac{E_2}{4.44 f N_2} = \frac{440}{4.44 \times 60 \times 40}$$

$$\Rightarrow \phi_m = 0.04129 \text{ Wb.}$$

5. A 25kVA, 1 ϕ transformer has 500 turns on the primary & 40 turns on the secondary. The primary is connected to 3000V, 50Hz supply. Calculate the

- (i) primary & secondary currents on full-load
- (ii) secondary EMFs.
- (iii) maximum flux in the core.

Solⁿ

$$k = \frac{N_2}{N_1} = \frac{40}{500} = 0.08$$

Primary current on full-load is,

$$I_1 = \frac{VA}{V_1} = \frac{25 \times 10^3}{3000} \Rightarrow \boxed{I_1 = 8.33A}$$

The secondary full-load current is,

$$I_2 = \frac{I_1}{k} = \frac{8.33}{0.08} \Rightarrow \boxed{I_2 = 104.125A}$$

$$\left| \begin{aligned} \frac{I_1}{I_2} &= \frac{N_2}{N_1} \\ \Rightarrow I_2 &= \frac{N_1}{N_2} I_1 \\ I_2 &= \left(\frac{1}{k}\right) I_1 \end{aligned} \right.$$

The secondary EMF of an ideal transformer is,

$$E_2 = k E_1 = k V_1 \\ = 0.08 \times 3000$$

$$\boxed{E_2 = 240V}$$

$$\left| \begin{aligned} \frac{E_2}{E_1} &= \frac{N_2}{N_1} = \frac{V_2}{V_1} \\ \Rightarrow E_2 &= \frac{N_2}{N_1} E_1 \\ E_2 &= k E_1 = k V_1 \end{aligned} \right.$$

Given $E_1 = V_1 = 3000V$ (ideal transformer)

$$\therefore E_1 = 4.44 f \Phi_m N_1$$

$$\Rightarrow \Phi_m = \frac{E_1}{4.44 f N_1} = \frac{3000}{4.44 \times 50 \times 500}$$

$$\Rightarrow \boxed{\Phi_m = 0.02727Wb}$$

6. A single-phase, 20kVA transformer has 1000 primary turns and 2500 secondary turns. The net area of cross-section of the core is 100cm^2 . When the primary winding is connected to a 500V, 50Hz supply, calculate

- i) the maximum value of flux density in core
- ii) the voltage induced in the secondary
- iii) the primary & secondary full-load currents.

Solⁿ

$$A_{\text{core}} = 100\text{cm}^2 = 100 \times 10^{-4} \text{m}^2$$

$$\Phi_m = B_m A_{\text{core}}$$

$$k = \frac{N_2}{N_1} = \frac{2500}{1000} = 2.5$$

$$E_1 = 500\text{V} = V_1 \quad (\text{ideal transformer}).$$

$$\text{Also, } E_1 = 4.44 f \Phi_m N_1$$

$$E_1 = 4.44 f B_m A_{\text{core}} N_1$$

$$\Rightarrow B_m = \frac{E_1}{4.44 f A_{\text{core}} N_1}$$

$$= \frac{500}{4.44 \times 50 \times 100 \times 10^{-4} \times 1000}$$

$$\boxed{B_m = 0.2525 \text{ Wb/m}^2}$$

EMF in secondary is,

$$\begin{aligned} E_2 &= k E_1 \\ &= 2.5 \times 500 \end{aligned}$$

$$\boxed{E_2 = 1250\text{V}}$$

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

$$\Rightarrow E_2 = E_1 \frac{N_2}{N_1}$$

$$E_2 = k E_1$$

Primary full-load current is,

$$I_1 = \frac{VA}{V_1} = \frac{20 \times 10^3}{500}$$

$$\Rightarrow \boxed{I_1 = 40A}$$

$$I_2 = \frac{I_1}{k} = \frac{40}{2.5} = 16A \quad (\text{or}) \quad I_2 = \frac{VA}{V_2} = \frac{20 \times 10^3}{1250}$$

$$\Rightarrow \boxed{I_2 = 16A}$$

7. The primary winding of a 25kVA transformer has 200 turns and is connected to a 230V, 50Hz supply. The secondary turns are 50. Calculate

- i) no-load secondary EMF
- ii) full-load primary & secondary currents
- iii) the flux density in the core, if the cross-section of the core is 60cm^2 .

Solⁿ

$$N_1 = 200, \quad N_2 = 50 \quad \therefore k = \frac{N_2}{N_1} = \frac{50}{200} = \frac{1}{4}$$

For an ideal transformer, $E_1 = V_1$.

$$\therefore E_1 = 230V$$

$$\begin{aligned} \text{Now, } E_2 &= k E_1 \\ &= \frac{1}{4}(230) \end{aligned}$$

$$\boxed{E_2 = 57.5V}$$

Full-load primary current is,

$$I_1 = \frac{VA}{V_1} = \frac{25 \times 10^3}{230} \Rightarrow \boxed{I_1 = 108.695A}$$

Full-load secondary current is,

$$I_2 = \frac{I_1}{k} = \frac{108.695}{\frac{1}{4}}$$

$$\Rightarrow \boxed{I_2 = 434.78 \text{ A}}$$

$$E_1 = 4.44 f \Phi_m N_1 = 4.44 f (B_m A_{\text{core}}) N_1$$

$$\Rightarrow B_m = \frac{E_1}{4.44 f A_{\text{core}} N_1}$$
$$= \frac{230}{4.44 \times 50 \times 60 \times 10^{-4} \times 200}$$

$$\boxed{B_m = 0.8633 \text{ Wb/m}^2}$$

8. Find the number of turns on the primary & secondary windings of a 440/230V, 50Hz, 1 ϕ transformer, if the net core cross-sectional area is 30 cm² and the maximum flux density in the core is 1 Wb/m².

Solⁿ

$$k = \frac{V_2}{V_1} = \frac{230}{440} = 0.5227$$

$$A_{\text{core}} = 30 \text{ cm}^2 = 30 \times 10^{-4} \text{ m}^2$$

$$B_m = 1 \text{ Wb/m}^2.$$

$$\therefore \Phi_m = B_m A_{\text{core}} = 30 \times 10^{-4} \times 1 = 30 \times 10^{-4} \text{ Wb}$$

No. of turns of primary is,

$$N_1 = \frac{E_1}{4.44 f \Phi_m} = \frac{440}{4.44 \times 50 \times 30 \times 10^{-4}} \Rightarrow \boxed{N_1 = 661}$$

No. of turns of secondary is

$$N_2 = N_1 \frac{E_2}{E_1} = 661 \times \frac{230}{440} \Rightarrow \boxed{N_2 = 346}$$

9. A 1ϕ , core-type transformer has square core of 20cm side. The permissible flux density is 1 Wb/m^2 . Calculate the number of turns per limb of high & low voltage winding for a 3000/220V ratio.

Solⁿ

$$k = \frac{V_2}{V_1} = \frac{220}{3000} = 0.0733, \quad A_{\text{core}} = 20\text{cm} \times 20\text{cm} \\ = 400\text{cm}^2 \\ = 400 \times 10^{-4} \text{ m}^2$$

Turns of primary winding is,

$$N_1 = \frac{E_1}{4.44 f B_m A_{\text{core}}} \\ = \frac{3000}{4.44 \times 50 \times 1 \times 400 \times 10^{-4}}$$

$$\boxed{N_1 = 338}$$

No. of turns of secondary winding is,

$$N_2 = N_1 \frac{V_2}{V_1} = 338 \times \frac{220}{3000}$$

$$\Rightarrow \boxed{N_2 = 25}$$

Usually, in a transformer half of the primary turns and half the secondary turns are wound on each limb.

$$\therefore N_1/\text{limb} = \frac{338}{2} = \underline{\underline{169}}$$

$$N_2/\text{limb} = \frac{25}{2} = \underline{\underline{12.5}}$$

- 10> The design requirements of a 6000/450V, 50Hz core-type transformer are:-

EMF/turn = 15V.

Max. flux density = 1 Wb/m^2

Calculate the suitable number of primary & secondary turns & the net cross-sectional area of the core.

Solⁿ.

For a transformer, we know that

EMF/turn of primary = EMF/turn of secondary

$$\therefore \frac{E_1}{N_1} = \frac{E_2}{N_2} = 15 \text{ V (given).}$$

Given $E_1 = 6000 \text{ V}$

$E_2 = 450 \text{ V}$

$$\therefore N_1 = \frac{E_1}{15} = \frac{6000}{15} \Rightarrow \boxed{N_1 = 400}$$

and $N_2 = \frac{E_2}{15} = \frac{450}{15} \Rightarrow \boxed{N_2 = 30}$

Also,

EMF/turn is

$$\frac{E_1}{N_1} = 15 = 4.44 f \Phi_m \quad (\because E_1 = 4.44 f \Phi_m N_1)$$

$$\therefore \Phi_m = \frac{15}{4.44 f}$$

$$= \frac{15}{4.44 \times 50}$$

$$= 0.0675 \text{ Wb.}$$

However, the flux can be represented in terms of flux density as,

$$\Phi_m = B_m \times A_{\text{core}}$$

$$\Rightarrow A_{\text{core}} = \frac{\Phi_m}{B_m} = \frac{0.0675}{1}$$

$$\therefore \boxed{A_{\text{core}} = 0.0675 \text{ m}^2}$$

11. A 4000V/400V, 1 ϕ transformer draws a no-load current of 0.8A and consumes 600W. Calculate the components of the no-load current.

Solⁿ

$$P_0 = 600W$$

$$I_0 = 0.8A.$$

When on no-load, the no-load current has two components: the magnetizing component (I_μ) and the core-loss component (I_w).

When on no-load, the transformer only consumes iron losses (and a very small amount of copper loss which can be neglected). This can be represented as,

$$P_0 = V_0 I_w$$

$$\Rightarrow I_w = \frac{P_0}{V_1} = \frac{600}{4000}$$

$$\Rightarrow \boxed{I_w = 0.15A}$$

The no-load current is given by

$$I_0^2 = I_\mu^2 + I_w^2$$

$$\begin{aligned}\therefore I_\mu &= \sqrt{I_0^2 - I_w^2} \\ &= \sqrt{(0.8)^2 - (0.15)^2}\end{aligned}$$

$$\boxed{I_\mu = 0.7858A}$$

- 12> A 4400/400V, 1 ϕ transformer draws a no-load current of 1A at a power factor of 0.25. Find the components of the no-load current.

Solⁿ

$$I_0 = 1A$$

$$\cos \phi_0 = 0.25 \Rightarrow \phi_0 = 75.52^\circ$$

$$\Rightarrow \sin \phi_0 = 0.968$$

\therefore The core-loss component is,

$$I_w = I_0 \cos \phi_0$$

$$= 1 \times 0.25$$

$$\boxed{I_w = 0.25A}$$

The magnetizing component is,

$$I_\mu = I_0 \sin \phi_0$$

$$= 1 \times 0.968$$

$$\boxed{I_\mu = 0.968A}$$

$$(OR) \quad I_\mu = \sqrt{I_0^2 - I_w^2}$$

$$= \sqrt{1^2 - 0.25^2}$$

$$I_\mu = 0.968A$$

13. A 1ϕ transformer, 440/110V takes a no-load current of 4A at 0.25 pf lagging. If the secondary takes a current of 100A at 0.9 pf lag, calculate the current drawn by primary. Draw the phasor diagram.

Solⁿ

$$I_0 = 4A$$

$$\cos \phi_0 = 0.25 \Rightarrow \phi_0 = 75.52^\circ$$

ϕ_0 is the angle between V_1 and I_0 .

Given $I_2 = 100A$

$$\cos \phi_2 = 0.9 \Rightarrow \phi_2 = 25.84^\circ$$

ϕ_2 is the angle between V_2 and I_2 .

The turns ratio is,

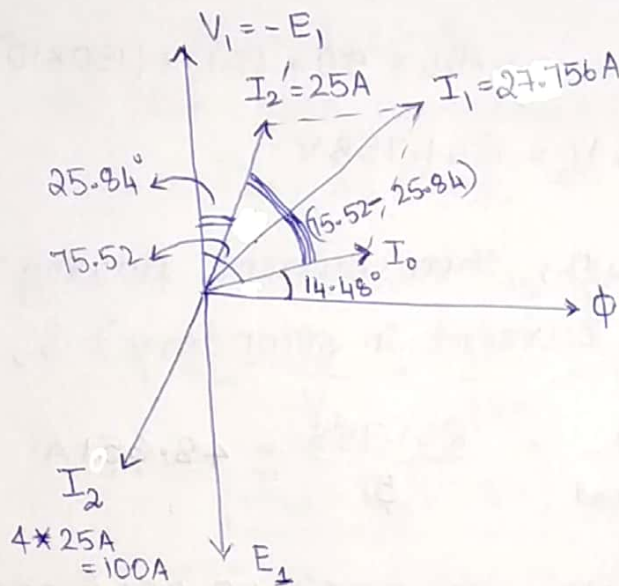
$$\frac{V_2}{V_1} = \frac{110}{440} = 0.25$$

If I_2 is the secondary current, then neglecting the very small no-load current, the primary current is,

$$I_2' = k I_2$$

$$= 0.25 \times 100$$

$$I_2' = 25 \text{ A}$$



Angle between I_2' and I_0

$$= 75.52 - 25.84$$

$$= 49.68^\circ$$

(Phasor considering no-load current)

If we add the no-load current vectorially, then the primary current is

$$I_1 = I_0 + I_2'$$

$$= \sqrt{I_0^2 + (I_2')^2 + 2 I_0 I_2' \cos 49.68^\circ}$$

$$I_1 = 27.756 \text{ A}$$

14. A 1ϕ transformer has a core cross-sectional area of 150cm^2 . It operates at a peak flux density of 1.1 Wb/m^2 from a 50Hz supply. If the secondary winding has 66 turns, calculate the output in kVA when connected to a load of 4Ω impedance. Neglect the voltage drops in the transformer.

Solⁿ

$$\text{Output kVA} = (V_2 I_2 / 1000)$$

Since we are neglecting the voltage drops,

$$\begin{aligned} E_2 = V_2 &= 4.44 f \Phi_m N_2 \\ &= 4.44 f B_m A_{\text{core}} N_2 \\ &= 4.44 \times 50 \times (1.1) \times (150 \times 10^{-4}) \times 66 \\ \Rightarrow V_2 &= 241.758\text{V} \end{aligned}$$

If $Z_{\text{load}} = 4\Omega$, then current flowing through the load (i.e., current in secondary) is,

$$I_2 = \frac{V_2}{Z_{\text{load}}} = \frac{241.758}{4} = 60.44\text{ A}$$

$$\therefore \text{Output kVA} = 241.758 \times 60.44 / 1000$$

$$\Rightarrow \boxed{\text{Output} = 14.612\text{ kVA}}$$

15). A 1ϕ , $230/110\text{V}$, 50Hz transformer takes an input of 350 volt-ampere at no-load while working at rated voltage. The core loss is 110W . Calculate

- the loss component of no-load current
- magnetizing component of no-load current
- no-load power factor

Solⁿ

No load power = Iron-loss.
(P_0).

$$P_0 = \left(\text{kVA of transformer} \right) \times \left(\text{no-load power factor} \right)$$

$$110 = 350 \times \cos \phi_0$$

$$\Rightarrow \cos \phi_0 = \frac{110}{350}$$

$$\therefore \boxed{\cos \phi_0 = 0.314} \Rightarrow \sin \phi_0 = 0.949$$

The no-load components are :-

i) magnetizing component (I_m)

$$I_m = I_0 \sin \phi_0$$

ii) core-loss component (I_w)

$$I_w = I_0 \cos \phi_0$$

I_w can be calculated using core-losses as,

$$I_w = \frac{P_0}{V_1} = \frac{110}{230} = 0.478 \text{ A.}$$

$$\therefore \boxed{I_w = 0.478 \text{ A}}$$

$$\therefore I_0 \cos \phi_0 = 0.478$$

$$I_0 (0.314) = 0.478$$

$$\Rightarrow I_0 = 1.522 \text{ A}$$

Hence,

$$I_m = I_0 \sin \phi_0$$

$$= 1.522 \times 0.949$$

$$\boxed{I_m = 1.44 \text{ A}}$$

16. A 1Φ , 230/110V transformer has iron loss of 100W at 60Hz, and 60W at 40Hz. Calculate the hysteresis & eddy current loss at 50Hz.

Sol:

Let the flux density be constant at Φ_m .

$$W_e \propto f^2 \Rightarrow W_e = Af^2 \quad \text{and} \quad W_i = W_e + W_h$$
$$W_h \propto f \Rightarrow W_h = Bf$$

At 40Hz,

$$W_i = Af^2 + Bf$$

$$60 = A(40)^2 + B(40)$$

$$\therefore 1600A + 40B = 60 \longrightarrow (1).$$

At 60Hz,

$$W_i = Af^2 + Bf$$

$$100 = A(60)^2 + B(60)$$

$$3600A + 60B = 100 \longrightarrow (2).$$

Solving (1) & (2), we get

$$A = 0.00833$$

$$B = 1.167$$

At 50Hz,

$$W_i = Af^2 + Bf$$

$$= [(0.00833)(50)^2] + [(1.167)(50)]$$

$$\boxed{W_i = 79.175 \text{ W}}$$

$$W_h = Bf = 1.167 \times 50 \Rightarrow \boxed{W_h = 58.35 \text{ W}}$$

$$W_e = Af^2 = 0.00833 \times 50^2 \Rightarrow \boxed{W_e = 20.825 \text{ W}}$$

17. A supply of frequency 50Hz, produces a flux density varying sinusoidally, having a maximum value of 1 Wb/m^2 and an eddy current loss of 60W. Find the eddy current loss when the supply frequency is raised to 60Hz & the maximum value of flux reduced to 0.5 Wb/m^2 .

Solⁿ.

$$B_{m1} = 1 \text{ Wb/m}^2$$

$$f_1 = 50 \text{ Hz}$$

$$W_{e1} = 60 \text{ W}$$

$$B_{m2} = 0.5 \text{ Wb/m}^2$$

$$f_2 = 60 \text{ Hz}$$

$$W_{e2} = ?$$

$$W_e = k_e B_m^2 f^2 t^2 \nu \text{ watts}$$

$$\Rightarrow W_e \propto B_m^2 f^2$$

$$\therefore \frac{W_{e2}}{W_{e1}} = \frac{B_{m2}^2 f_2^2}{B_{m1}^2 f_1^2}$$

$$\therefore W_{e2} = W_{e1} \frac{B_{m2}^2 f_2^2}{B_{m1}^2 f_1^2}$$

$$= 60 \times \frac{(0.5)^2 \times (60)^2}{(1)^2 \times (50)^2}$$

$$\boxed{W_{e2} = 21.6 \text{ W}}$$

- 18> A 20kVA, 5000/500V, 1ϕ transformer has primary and secondary winding resistances of 10Ω and 0.1Ω , and leakage reactances of 20Ω and 0.2Ω respectively. The magnetization reactance is 500Ω and the core-loss resistance is 10000Ω . Calculate the primary current when

a) secondary is open

b) secondary load current is 20A at 0.8 pf lag.

Solⁿ

The core-loss component of no-load current is,

$$I_w = \frac{V_1}{R_0} = \frac{5000}{10000} = 0.5A.$$

The magnetization component of no-load current is,

$$I_u = \frac{V_1}{X_m} = \frac{5000}{5000} = 1A.$$

∴ The no-load current is,

$$I_0 = \sqrt{I_w^2 + I_u^2} = \sqrt{0.5^2 + 1^2} = 1.118A.$$

a) When secondary is open

$$I_2 = 0$$

$$\Rightarrow I_2' = kI_2 = 0.$$

$$\therefore \overline{I_1} = \overline{I_0} + \overline{I_2'} = \overline{I_0} \quad (\text{phasors}).$$

∴ Primary current when secondary is open is

$$I_1 = 1.118A.$$

The angle of lag can be found using

$$\tan \phi = \frac{X_m}{R_0} = \frac{5000}{10000} = 0.5$$

$$\Rightarrow \phi = \tan^{-1}(0.5)$$

$$\Rightarrow \phi = 26.56^\circ.$$

$$\therefore \boxed{I_1 = 1.118A / 26.56^\circ}$$

b) When secondary is on load

$$\text{Given } I_2 = 20A$$

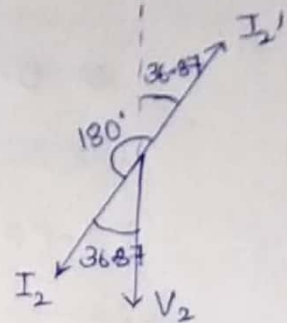
$$\phi_2 = \cos^{-1}(0.8) = 36.87^\circ.$$

$$\therefore I_2 = 20A / \underline{36.87^\circ} \quad (-ve \text{ angle shows lag}).$$

Thus, the load component of the primary current is,

$$\begin{aligned} I_2' &= k I_2 \\ &= \left(\frac{V_2}{V_1} \right) I_2 \\ &= \frac{500}{5000} \times 20 / \underline{-36.87^\circ} \end{aligned}$$

$$I_2' = 2A / \underline{-36.87^\circ}$$



Thus, the primary current is,

$$\begin{aligned} \bar{I}_1 &= \bar{I}_0 + \bar{I}_2' \\ &= 1.118 / \underline{26.56^\circ} + 2 / \underline{-36.87^\circ} \\ &= 1 + j0.5 + 1.6 - j1.2 \\ &= 2.6 + j1.7 \end{aligned}$$

$$\boxed{\bar{I}_1 = 2.692A / \underline{-15.06^\circ}}$$

19. A 10kVA, 400/200V, 50 Hz, 1 ϕ transformer, has a full-load copper loss of 200W and has a full-load efficiency of 96% at 0.8 pf lagging. Calculate the iron loss. What would be the efficiency at half full load & unity power factor?

Solⁿ

Rating = 10kVA, 400/200V

Full-load ($x=1$)

$$W_{cuFL} = 200W$$

$$\eta = 0.96$$

$$\cos\phi_2 = 0.8$$

Half full load ($x=1/2$)

$$\eta = ?$$

$$\cos\phi_2 = 1.$$

a) The efficiency at full-load, 0.8 pf lag is

$$\eta = \frac{x \times VA \times \cos\phi_2}{x \times VA \times \cos\phi_2 + W_i + x^2 W_{cu}}$$

$$\Rightarrow 0.96 = \frac{1 \times 10 \times 1000 \times 0.8}{(1 \times 10 \times 1000 \times 0.8) + W_i + 200}$$

$$\Rightarrow 0.96(8000 + W_i + 200) = 8000$$

$$\Rightarrow 0.96 W_i = 128$$

$$\Rightarrow \boxed{W_i = 133.33 \text{ W}}$$

Since iron loss remains constant, we can say that $W_i = 133.33 \text{ W}$ at all loads.

b) For half full-load, unity power factor.

$$\eta = \frac{(\frac{1}{2})(10 \times 1000) \times 1}{[(\frac{1}{2})(10 \times 1000) \times 1] + 133.33 + (\frac{1}{2})^2(200)} = 0.9646$$

$$\boxed{\eta = 96.46\%}$$

20. A 1ϕ transformer working at 0.8 pf has an efficiency of 94% at both $3/4^{\text{th}}$ full-load and full load of 600 kW. Calculate the efficiency at half-full-load, unity power factor.

Solⁿ

Full-load

$$\eta = 94\%$$

$$\cos\phi_2 = 0.8$$

$$x = 1$$

$3/4^{\text{th}}$ full load

$$\eta = 94\%$$

$$\cos\phi_2 = 0.8$$

$$x = 3/4$$

Full load = 600 kW.

$$\therefore \text{Transformer kVA} = \frac{600}{\cos\phi_2} = \frac{600}{0.8} = 750 \text{ kVA}$$

At full-load condition

$$\eta = \frac{x \times VA \times \cos \phi_2}{(x \times VA \times \cos \phi_2) + W_i + x^2 W_{cuFL}}$$

$$0.94 = \frac{1 \times 750 \times 1000 \times 0.8}{(1 \times 750 \times 1000 \times 0.8) + W_i + W_{cuFL}}$$

$$\Rightarrow 0.94 [600000 + W_i + W_{cuFL}] = 600000$$

$$\Rightarrow W_i + W_{cuFL} = 38297.87$$

—————→ (1).

At $\frac{3}{4}$ th full-load condition

$$\eta = \frac{x \times VA \times \cos \phi_2}{(x \times VA \times \cos \phi_2) + (W_i) + x^2 W_{cuFL}}$$

$$0.94 = \frac{(3/4 \times 750 \times 1000 \times 0.8)}{(3/4 \times 750 \times 1000 \times 0.8) + W_i + (3/4)^2 W_{cuFL}}$$

$$\Rightarrow 0.94 (450000 + W_i + 9/16 W_{cuFL}) = 450000$$

$$\Rightarrow W_i + \frac{9}{16} W_{cuFL} = 28723.40$$

—————→ (2)

$$(1) - (2) \Rightarrow$$

$$W_{cuFL} - \frac{9}{16} W_{cuFL} = 9574.47$$

$$\Rightarrow W_{cuFL} = \underline{\underline{21884.50 \text{ kW}}}$$

Substituting in (1),

$$W_i = 38297.87 - W_{cuFL}$$

$$= 38297.87 - 21884.50$$

$$W_i = \underline{\underline{16.41 \text{ kW}}}$$

At half-load, unity pf

$$x = \frac{1}{2}$$

$$\cos \phi_2 = 1.$$

$$\therefore \eta = \frac{(\frac{1}{2} \times 750 \times 1000 \times 1)}{(\frac{1}{2} \times 750 \times 1000 \times 1) + (16413.36) + (\frac{1}{4} \times 21884.5)}$$

$$\eta = 0.932$$

$$\text{or } \boxed{\eta = 93.2\%}$$

21) A 10kVA, 400/200V, 1ϕ transformer has a maximum efficiency of 98% at 90% full-load at 0.8 pf lag. Calculate its efficiency at full load & 0.6 pf.

Solⁿ

$$\eta = \left[\frac{x \times VA \times \cos \phi_2}{(x \times VA \times \cos \phi_2) + W_i + x^2 W_{cu}} \right]$$

For 90% full load

$$x = 0.9 \quad \cos \phi_2 = 0.8$$

$$\eta = 0.98.$$

$$\therefore 0.98 = \frac{0.9 \times 10 \times 1000 \times 0.8}{(0.9 \times 10 \times 1000 \times 0.8) + W_i + (0.9)^2 W_{cu}}$$

$$\Rightarrow 0.98 [7200 + W_i + 0.81 W_{cu}] = 7200$$

$$\therefore W_i + 0.81 W_{cu} = 146.94$$

If at this load the efficiency is maximum, then

$$0.81 W_{cu} = W_i$$

$$\Rightarrow 2W_i = 146.94$$

$$\therefore W_i = 73.46 \text{ W}$$

$$\text{Now, } 0.81 W_{cu} = 73.46$$

$$\Rightarrow W_{cu} = 90.69 \text{ W}$$

At full-load condition

$$x = 1 \quad \cos \phi_2 = 0.6$$

$$W_i = 73.46 \text{ W}$$

$$\therefore \eta = \frac{1 \times 10 \times 1000 \times 0.6}{(1 \times 10 \times 1000 \times 0.6) + 73.46 + [1^2 \times 90.69]} = 0.9733$$

$$\Rightarrow \boxed{\eta = 97.33\%}$$

22> The maximum efficiency at full-load & unity power factor of a 1ϕ , 25kVA, 500/1000V, 50Hz transformer is 98%. Determine its efficiency at

i) 75% load, 0.9 pf

ii) 50% load, 0.8 pf.

Solⁿ

At full-load, upf

$$\eta = 0.98$$

$$x = 1$$

$$\cos \phi_2 = 1$$

$$\eta = \frac{(x \times VA \times \cos \phi_2)}{(x \times VA \times \cos \phi_2) + W_i + x^2 W_{cu}}$$

$$\therefore \eta = 0.98 = \frac{1 \times 25 \times 1000 \times 1}{(1 \times 25 \times 1000 \times 1) + W_i + W_{cu}}$$

$$\Rightarrow 0.98 [25000 + W_i + W_{cu}] = 25000$$

$$\Rightarrow W_i + W_{cu} = 510.20$$

But at η_{\max} , iron loss = copper loss.

$$\text{Since } W_i + x^2 W_{cu} = 510.24$$

$$\Rightarrow W_i + W_i = 510.24$$

$$\Rightarrow W_i = 255.10 \text{ W.}$$

$$\text{and } W_{cu} = 255.10 \text{ W.}$$

At 75% load, 0.9 pf

$$x = 0.75$$

$$\cos \phi_2 = 0.9$$

$$\eta = \left[\frac{0.75 \times 25 \times 1000 \times 0.9}{(0.75 \times 25 \times 1000 \times 0.9) + 255.10 + (0.75^2 \times 255.10)} \right] \times 100$$

$$\boxed{\eta = 97.69\%}$$

At 50% load, 0.8 pf

$$x = 0.5$$

$$\cos \phi_2 = 0.8$$

$$\therefore \eta = \left[\frac{(0.5 \times 25 \times 1000 \times 0.8)}{(0.5 \times 25 \times 1000 \times 0.8) + 255.10 + (0.5^2 \times 255.10)} \right] \times 100$$

$$\boxed{\eta = 96.9\%}$$

23> A transformer working at UPF has an efficiency of 90% at both half load & full load of 500W. Calculate its efficiency at 75% of full-load.

Solⁿ

At full-load, UPF

$$x = 1$$

$$\cos \phi_2 = 1$$

$$\eta = 0.9$$

$$\eta = \frac{x \times VA \times pf}{(x \times VA \times pf) + W_i + x^2 W_{cu}}$$

$$VA = \frac{W}{\cos \phi} = \frac{500}{1} =$$

$$\Rightarrow \text{Rating} = 500 \text{ VA}$$

$$\therefore 0.9 = \frac{(1 \times 500 \times 1)}{(1 \times 500 \times 1) + W_i + W_{cu}}$$

$$\Rightarrow 0.9 [500 + W_i + W_{cu}] = 500$$

$$\Rightarrow W_i + W_{cu} = 55.55 \longrightarrow (1).$$

At 50% load, UPF

$$\eta = 0.9$$

$$x = \frac{1}{2}$$

$$\cos \phi_2 = 1.$$

$$\therefore 0.9 = \frac{0.5 \times 500 \times 1}{(0.5 \times 500 \times 1) + W_i + 0.5^2 W_{cu}}$$

$$0.9 [250 + W_i + 0.25 W_{cu}] = 250$$

$$\Rightarrow W_i + 0.25 W_{cu} = 27.77 \longrightarrow (2)$$

$$(1) - (2) \Rightarrow$$

$$W_{cu} - 0.25 W_{cu} = 55.55 - 27.77$$

$$\Rightarrow 0.75 W_{cu} = 27.78$$

$$\Rightarrow W_{cu} = 37.04 \text{ W.}$$

Substituting in (1), we get

$$W_i = 18.51 \text{ W.}$$

At 75% full-load, UPF

$$x = 0.75$$

$$\cos \phi_2 = 1$$

$$\eta = \frac{0.75 \times 500 \times 1}{(0.75 \times 500 \times 1) + 18.51 + 0.75^2 (37.04)} = 0.9050$$

$$\therefore \boxed{\eta = 90.50\%}$$

24> In a 25kVA, 2000/200V, 1 Φ transformer, the iron & copper losses at full load are 350W and 400W respectively. Calculate the efficiency at UPF, half load. Find also the copper loss for maximum efficiency.

Solⁿ

$$\eta = \left[\frac{(x \times \text{kVA} \times \text{pf} \times 1000)}{(x \times \text{kVA} \times \text{pf} \times 1000) + W_i + x^2 W_{cu}} \right] \times 100$$

At half-load

$$\Rightarrow \eta = \left[\frac{(\frac{1}{2} \times 25 \times 1000 \times 1)}{(\frac{1}{2} \times 25 \times 1000 \times 1) + 350 + (\frac{1}{2})^2 400} \right] \times 100$$

$$\boxed{\eta = 96.525\%}$$

For maximum efficiency

$$x^2 W_{cu} = W_i$$

Iron loss is constant at 350W.

\therefore Maximum efficiency occurs when

$$\boxed{\text{Copper loss} = x^2 W_{cu} = 350W}$$

Note

It occurs at load fraction,

$$x = \sqrt{\frac{W_i}{W_{cu}}} = \sqrt{\frac{350}{400}} = 0.935$$

$$\begin{aligned} \text{i.e. kVA to get } \eta_{\max} \text{ is} &= 0.935 \times \text{kVA} \\ &= 0.935 \times 25 \end{aligned}$$

\therefore kVA at which maximum efficiency occurs is 23.375kVA.

25). A 25kVA, 2000/200V, 1 ϕ transformer has iron loss and copper loss of 350W and 400W respectively. Calculate the efficiency at full load, unity power factor.

Solⁿ

$$\eta = \frac{x \times VA \times pf}{(x \times VA \times pf) + W_i + x^2 W_{cu}}$$

At Given

$$W_i = 350W$$

$$W_{cu} = 400W$$

$$VA = 25 \times 10^3$$

For full-load, UPF

$$x = 1$$

$$pf = 1$$

$$\therefore \eta = \left[\frac{1 \times 25 \times 10^3 \times 1}{(1 \times 25 \times 10^3 \times 1) + 350 + (1^2 \times 400)} \right] 100$$

$$\Rightarrow \boxed{\eta = 97.08\%}$$

26) & 27). A 50kVA transformer has an efficiency of 98% at full-load, 0.8 pf and an efficiency of 96.9% at $\frac{1}{4}$ th full-load, UPF. Calculate the iron loss & full-load copper loss. Also find the losses when load is $\frac{1}{4}$ th at 0.8 lag, if $\eta = 0.969$

Solⁿ

(a) At full-load, 0.8 pf

$$\eta = 0.98$$

$$x = 1$$

$$\cos \phi_2 = 0.8$$

$$\eta = \frac{x \times kVA \times 1000 \times \cos \phi_2}{[x \times kVA \times 1000 \times \cos \phi_2 + W_i + x^2 W_{cu}]}$$

$$\therefore \eta = 0.98 = \frac{1 \times 50 \times 1000 \times 0.8}{(1 \times 50 \times 1000 \times 0.8) + W_i + W_{cu}}$$

$$\therefore W_i + W_{cu} = 816.32 \longrightarrow (1)$$

At $\frac{1}{4}$ th load, UPF

$$\alpha = \frac{1}{4}$$

$$\cos \phi_2 = 1$$

$$\eta = 0.969$$

$$\therefore \eta = 0.969 = \frac{\left[\frac{1}{4} \times 50 \times 1000 \times 1 \right]}{\left(\frac{1}{4} \times 50 \times 1000 \times 1 \right) + W_i + \frac{1}{16} W_{cu}}$$

$$\Rightarrow W_i + \frac{1}{16} W_{cu} = 387.5 \longrightarrow (2)$$

Solving (1) & (2), we get

$$W_i = 358.9 \text{ W}$$

$$W_{cu} = 457.4 \text{ W}$$

b) At $\frac{1}{4}$ th load, 0.8 pf lag

$$\alpha = \frac{1}{4}$$

$$\eta = 0.969$$

$$\cos \phi_2 = 0.8$$

$W_i = ?$ (since $\cos \phi_2$ and η are both changing, W_i changes).

$$W_{cu} = ?$$

$$\eta = 0.969 = \frac{\left(\frac{1}{4} \times 50 \times 1000 \times 0.8 \right)}{\left(\frac{1}{4} \times 50 \times 1000 \times 0.8 \right) + W_i + \frac{1}{16} W_{cu}}$$

$$\Rightarrow W_i + \frac{1}{16} W_{cu} = 319.91 \longrightarrow (3)$$

Solving (1) & (3), we get

$$W_i = 286.8 \text{ W}$$

$$W_{cu} = 529.5 \text{ W}$$

28> Determine the efficiency of a 150kVA, single-phase transformer at 50% full-load of 0.8 pf lag if the iron loss & copper loss at full load are 1400W and 1600W respectively.

Solⁿ

Given $W_i = 1400W$

$\cos\phi_2 = 0.8$

$W_{cu} = 1600W$, when $x = 1$

$x = \frac{1}{2}$

$$\eta = \frac{x \times VA \times \cos\phi}{x \times VA \times \cos\phi + W_i + x^2 W_{cu}} \times 100$$

$$= \frac{0.5 \times 150 \times 1000 \times 0.8}{(0.5 \times 150 \times 1000 \times 0.8) + 1400 + \frac{1}{4}(1600)} \times 100$$

$$\boxed{\eta = 97.08\%}$$

29> A 20kVA, 440/220V, 1 ϕ , 50Hz transformer has an iron loss of 324W. The copper loss is found to be 100W when delivering half full-load current. calculate

(i) the efficiency when delivering full load current at 0.8 pf lag

(ii) the percent full-load when the efficiency will be maximum.

Solⁿ

(i) $W_i = 324W$

$$x^2 W_{cu} = \left(\frac{1}{2}\right)^2 W_{cu} = 100W$$

$$\Rightarrow W_{cu} = 400W$$

At full-load, 0.8 pf lag,

$$\eta = \frac{1 \times 20 \times 1000 \times 0.8}{(1 \times 20 \times 1000 \times 0.8) + 324 + 400} \times 100 \Rightarrow \boxed{\eta = 95.67\%}$$

(ii) Efficiency is maximum when

$$x^2 W_{cu} = W_i$$

$$\therefore x = \sqrt{\frac{W_i}{W_{cu}}} = \sqrt{\frac{324}{400}} = 0.9.$$

\therefore At load equal to 90% full load, we get maximum efficiency.

30) For a single-phase, 50 Hz, 150 kVA transformer, the required no-load voltage ratio is 5000/250V. Calculate

- the number of turns in each winding for a maximum core flux of 0.06 Wb.
 - the efficiency at half rated kVA, and unity power factor
 - the efficiency at full-load, and 0.8 pf lag
 - kVA load for maximum efficiency
- if full-load copper losses are 1800W and the core losses are 1500W.

Solⁿ

a) The EMF equation is,

$$E_2 = 4.44 f \Phi_m N_2$$

$$\Rightarrow N_2 = \frac{E_2}{4.44 f \Phi_m} = \frac{250}{4.44 \times 50 \times 0.06} = 18.76$$

$$\Rightarrow \boxed{N_2 = 19 \text{ turns}}$$

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} \Rightarrow N_1 = N_2 \frac{E_1}{E_2} = 19 \times \frac{5000}{250}$$

$$\Rightarrow \boxed{N_1 = 380 \text{ turns}}$$

b) At half-rated kVA, the current is half full-load current.

$$\therefore x = \frac{1}{2}$$

$$\cos \phi_2 = 1 \text{ (given).}$$

$$\therefore \eta = \frac{(\frac{1}{2})(150) \times 1 \times 1000}{(\frac{1}{2} \times 150 \times 1000 \times 1) + 1500 + \frac{1}{4}(1800)} \times 100$$

$$\boxed{\eta = 97.46\%}$$

c) At full-load, 0.8 pf lag,

$$x = 1$$

$$\cos \phi_2 = 0.8$$

$$\eta = \frac{1 \times 150 \times 1000 \times 0.8}{(1 \times 150 \times 1000 \times 0.8) + 1500 + 1800} \times 100$$

$$\Rightarrow \boxed{\eta = 97.3\%}$$

d) The kVA at which efficiency is maximum is,

$$S = \text{full load kVA} \sqrt{\frac{W_i}{W_{cuFL}}}$$

$$= 150 \times \sqrt{\frac{1500}{1800}}$$

$$\boxed{S = 136.93 \text{ kVA}}$$