# DEPARTMENT OF MATHEMATICS

| Course: Fundamentals of Linear Algebra, Calculus and Statistics | Improvement CIE | Maximum marks: 50 |
|---|---|---|
| Course code: MAT211CT | First semester 2023-2024<br>Chemistry Cycle<br>Branch: AI, BT, CS, CD, CY, IS, SPARK | Time: 2:00PM-3:30PM<br>Date: 22-01-2024 |

*Instructions to candidates: Answer all questions.*

| Q.No | QUESTIONS | M | BT | CO |
|---|---|---|---|---|
| 1(a) | Test for consistency and solve the system:<br>$x+y+z=4\,,2x+y-z=1\,,x-y+2z=2.$ | 5 | L2 | 1 |
| 1(b) | Apply Rayleigh's power method to find the largest eigenvalue of the matrix<br>$A=\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$. Take the initial vector as $X_0=[1\ 1\ 1\ ]^T$ . Perform 4 iterations. | 5 | L2 | 2 |
| 2(a) | Apply Gauss-Seidel iteration method to solve the following system of equations:<br>$20x+y-2z=17\,,3x+20y-z=-18\,,2x-3y+20z=25.$<br>Carry out 4 iterations. | 5 | L2 | 2 |
| 2(b) | Evaluate $\int_0^1\int_x^1 e^{\frac{x}{y}}\,dy\,dx$ by changing the order of integration. | 5 | L2 | 2 |
| 3(a) | Determine the area enclosed by the parabola $y=x^2$ and the line $y=x+2$. | 5 | L3 | 3 |
| 3(b) | Transform to polar coordinates and hence evaluate $\int_0^2\int_0^{\sqrt{2x-x^2}}\frac{x}{\sqrt{x^2+y^2}}\,dy\,dx$ . | 5 | L2 | 2 |
| 4 | Evaluate $\int_0^{\log_e 2}\int_0^x\int_0^{x+y} e^{x+y+z}\,dz\,dy\,dx$ . | 10 | L2 | 3 |
| 5 | Obtain the center of gravity of a triangular lamina with vertices $(0,0),(0,3)$ and $(3,0)$ if the density function is $\rho(x,y)=xy$. | 10 | L3 | 4 |

BT-Blooms Taxonomy, CO-Course Outcomes, M-Marks

| Marks Distribution | Particulars | | CO1 | CO2 | CO3 | CO4 | L1 | L2 | L3 | L4 | L5 | L6 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Test | Max Marks | 5 | 20 | 15 | 10 | -- | 35 | 15 | -- | - | - |

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**DEPARTMENT OF MATHEMATICS**
**Scheme and Solution – Chemistry cycle**

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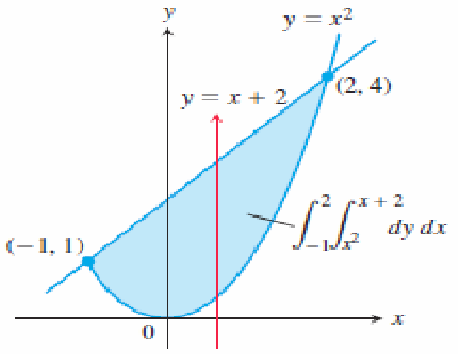
| **Course: Fundamentals of Linear Algebra, Calculus and Statistics** | **Improvement CIE** | **Maximum marks: 50** |
| **Course code: MAT211CT** | **First semester 2023-2024**<br>**Chemistry Cycle**<br>**Branch: AI, BT, CS, CD, CY, IS, SPARK** | |

| Q.No | | Marks |
|---|---|---|
| 1a) | $[A:B]=\begin{bmatrix}1&1&1&:&4\\2&1&-1&:&1\\1&-1&2&:&2\end{bmatrix}\sim\begin{bmatrix}1&1&1&:&4\\0&-1&-3&:&-7\\0&0&7&:&-12\end{bmatrix}$<br>Rank (A) =Rank ([A: B]) =No of variables=3. The system has unique solution.<br>$x=\frac{3}{7}, y=\frac{13}{7} z=\frac{12}{7}$ | 3<br>1<br>1 |
| 1b) | $\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix}=\begin{bmatrix}6\\0\\4\end{bmatrix}=6\begin{bmatrix}1\\0\\0.667\end{bmatrix}$<br>$\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}\begin{bmatrix}1\\0\\0.667\end{bmatrix}=\begin{bmatrix}7.334\\-2.667\\4.001\end{bmatrix}=7.334\begin{bmatrix}1\\-0.364\\0.546\end{bmatrix}$<br>$\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}\begin{bmatrix}1\\-0.364\\0.546\end{bmatrix}=\begin{bmatrix}7.820\\-3.638\\4.002\end{bmatrix}=7.820\begin{bmatrix}1\\-0.465\\0.512\end{bmatrix}$<br>$\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}\begin{bmatrix}1\\-0.465\\0.512\end{bmatrix}=\begin{bmatrix}7.954\\-3.907\\4.001\end{bmatrix}=7.954\begin{bmatrix}1\\-0.4912\\0.5029\end{bmatrix}$ | 2<br>1<br>1<br>1 |
| 2a) | $x=\frac{17-y+2z}{20}, y=\frac{-18-3x+z}{20}, z=\frac{25-2x+3y}{20}$ and initial values (0,0,0)<br>Iteration-1: x= 0.85,y= -1.0275, z= 1.0109. Iteration-2: x= 1.0025,y= -0.9998, z= 1.0000.<br>Iteration-3: x= 0.9999,y= -0.9999, z= 1.0000. Iteration-4: x= 0.9999,y= -0.9999, z= 1.0000. | 1<br>1+1<br>1+1 |
| 2b) | $\int_0^1\int_x^1 e^{\frac{x}{y}}\,dy\,dx=\int_0^1\int_0^y e^{\frac{x}{y}}\,dx\,dy$<br>$\int_0^1\int_x^1 e^{\frac{x}{y}}\,dy\,dx=\int_0^1\left[ye^{\frac{x}{y}}\right]_0^y dy=\int_0^1[y.e-y]dy=\int_0^1 y(e-1)dy=\left[\frac{(e-1)y^2}{2}\right]_0^1$<br>$\int_0^1\int_x^1 e^{\frac{x}{y}}\,dy\,dx=\frac{e-1}{2}$ | 2<br>2<br>1 |
| 3(a) |  | 2 |

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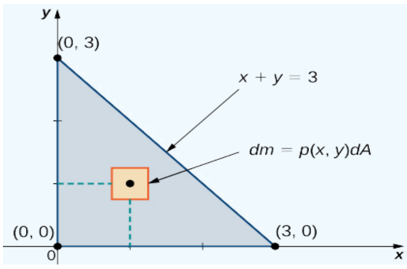
|---|---|---|
| | $$A=\int_{-1}^{2}\int_{x^2}^{x+2} dy\,dx=\int_{-1}^{2}(x+2-x^2)\,dx$$ $$A=\left[\frac{x^2}{2}+2x-\frac{x^3}{3}\right]_{-1}^{2}=\frac{9}{2}\,sq.units$$ | 2<br>1 |
| 3b) | $$I=\int_0^{\frac{\pi}{2}}\int_0^{2\cos\theta}\frac{r\cos\theta\,.r}{r}dr\,d\theta$$ $$I=\int_0^{\frac{\pi}{2}}\int_0^{2\cos\theta} r\cos\theta\,dr\,d\theta=\int_0^{\frac{\pi}{2}}\cos\theta\left[\frac{r^2}{2}\right]_0^{2cos\theta}d\theta=\int_0^{\frac{\pi}{2}}2\cos^3\theta\,d\theta=2\int_0^{\frac{\pi}{2}}\frac{1}{4}[\cos 3\theta+3\cos\theta]\,d\theta$$ $$I=\frac{1}{2}\left[\frac{\sin 3\theta}{3}+3\sin\theta\right]_0^{\frac{\pi}{2}}=\frac{1}{2}\left[-\frac{1}{3}+3\right]=\frac{4}{3}$$ | 2<br>2<br>1 |
| 4 | $$I=\int_0^{\log 2}\int_0^{x}e^{x+y}\,[e^z]_0^{(x+y)}dy\,dx$$ $$I=\int_0^{\log 2}\int_0^{x}e^{x+y}\,(e^{x+y}-1)dy\,dx=\int_0^{\log 2}\int_0^{x}(e^{2(x+y)}-e^{x+y})\,dy\,dx$$ $$I=\int_0^{\log 2}\left[e^{2x}.\frac{e^{2y}}{2}-e^x.e^y\right]_0^x dx=\int_0^{\log 2}\left(\frac{e^{4x}}{2}-e^{2x}-\frac{e^{2x}}{2}+e^x\right)dx$$ $$I=\left[\frac{e^{4x}}{8}-\frac{e^{2x}}{2}-\frac{e^{2x}}{4}+e^x\right]_0^{log2}=\left[\frac{e^{4\log 2}}{8}-\frac{e^{2\log 2}}{2}-\frac{e^{2\log 2}}{4}+e^{\log 2}\right]-\left(\frac{1}{8}-\frac{1}{2}-\frac{1}{4}+1\right)$$ $$I=\left(\frac{e^{\log 16}}{8}-\frac{e^{\log 4}}{2}-\frac{e^{\log 4}}{4}+e^{\log 2}\right)-\frac{3}{8}$$ $$I=\left(\frac{16}{8}-\frac{4}{2}-\frac{4}{4}+2\right)-\frac{3}{8}=1-\frac{3}{8}=\frac{5}{8}$$ | 1<br>2<br>2<br>2<br>2<br>1 |
| 5 |  $$m=\int_0^3\int_0^{3-x}\rho(x,y)\,dy\,dx$$ $$m=\int_0^3\int_0^{3-x}xy\,dy\,dx=\frac{27}{8}$$ $$\bar{x}=\frac{8}{27}\int_0^3\int_0^{3-x}x^2y\,dy\,dx=\frac{8}{27}\times\frac{81}{20}=\frac{6}{5}\,,\bar{y}=\frac{8}{27}\int_0^3\int_0^{3-x}xy^2\,dy\,dx=\frac{8}{27}\times\frac{81}{20}=\frac{6}{5}$$ $$\therefore(\bar{x},\bar{y})=\left(\frac{6}{5},\frac{6}{5}\right)$$ | 2<br>2<br>2<br>2+2 |