

## Unit - 2(b)

### Three - Phase A.C. Circuits

Syllabus:- Three-phase star and delta balanced circuits, voltage and current relations, measurement of three-phase power using two-wattmeter method.

#### Introduction

- \* The wire or a conductor which constitutes the coil in any electrical apparatus is called the winding. Each loop of wire is called a turn.
- \* Any electrical apparatus (generator, motor, transformer, etc.) having only one winding is called a single-phase system. If there are two windings, then they are spaced such that the voltages generated by them or the currents flowing through them have a phase difference of  $90^\circ$ . Such a system is called a two-phase system. If there are three windings, connected in such a way that the voltages generated by them or the currents flowing through them have a phase difference of  $120^\circ$ , then they are called as three-phase systems. If there are more than three windings in them, which are connected together, then they are called polyphase systems.
- \* The electrical displacement between the windings can be calculated as,

$$\text{electrical displacement} = \frac{360^\circ \text{ electrical}}{\text{number of phases}}$$

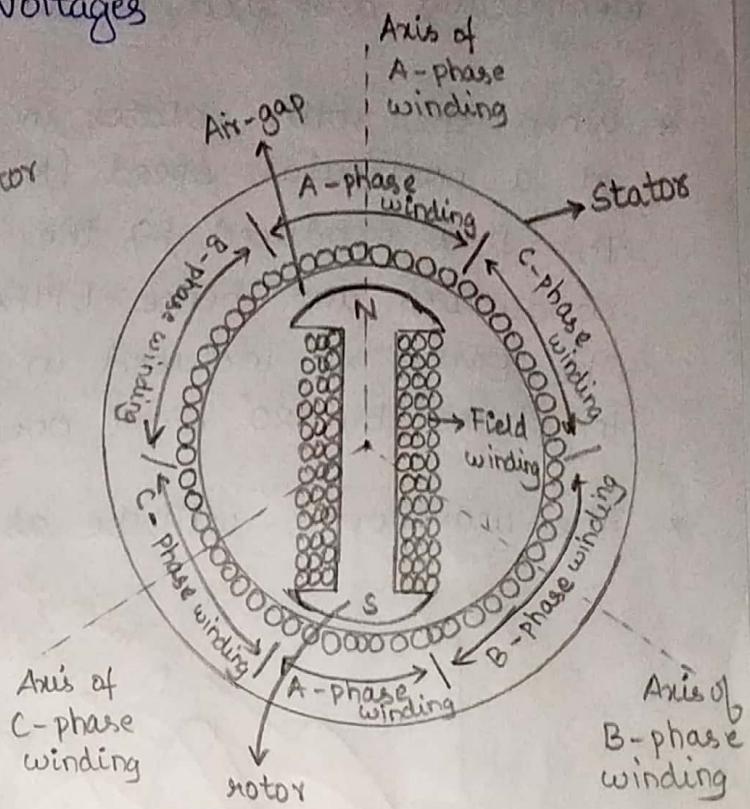
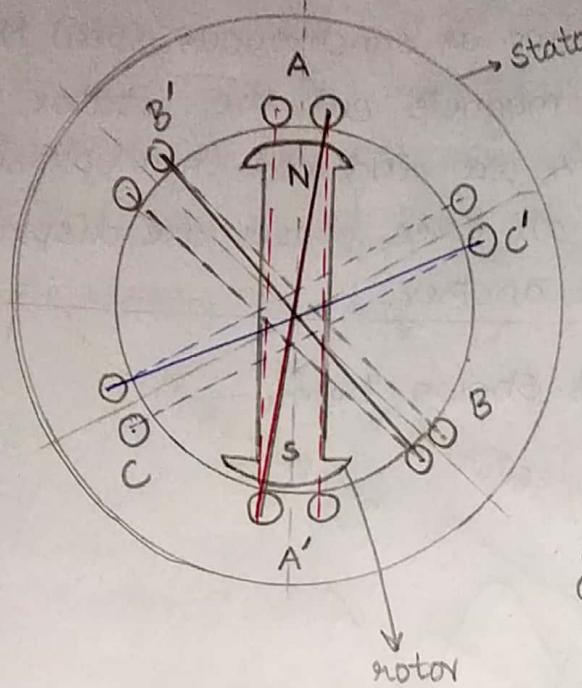
For a 3-phase system ( $3\phi$  system),

$$\text{electrical displacement} = \frac{360}{3} = 120^\circ.$$

## \* Advantages of three-phase system

1. A three-phase apparatus is more efficient than a single-phase apparatus.
2. For the same capacity, a three-phase apparatus costs less than a single-phase apparatus.
3. The size of a three-phase apparatus is smaller in size than the size of a single-phase apparatus of the same capacity, and requires less material for construction.
4. For transmitting the same amount of power, over the same distance, under the same power loss, the amount of conductor material required is less in case of a  $3\phi$  system than in the case of  $1\phi$  system. Hence, there is saving of copper.
5. In case of a  $3\phi$  system, two different voltages can be obtained, one between lines and the other between line & phase, whereas, only one voltage can be obtained in a  $1\phi$  system.
6. Three-phase motors produce uniform torque, while the torque produced by  $1\phi$  motors is pulsating.
7. Three-phase motors are self-starting whereas single-phase motors are not self-starting.
8. Three-phase generators can be driven by constant force or torque.
9. Industrial applications, such as high-power motors, welding equipment, have constant power output if they are  $3\phi$  systems.
10. The connection of  $1\phi$  generators in parallel gives rise to harmonics, whereas three-phase generators can be connected in parallel without giving rise to harmonics.

## Generation of three-phase voltages

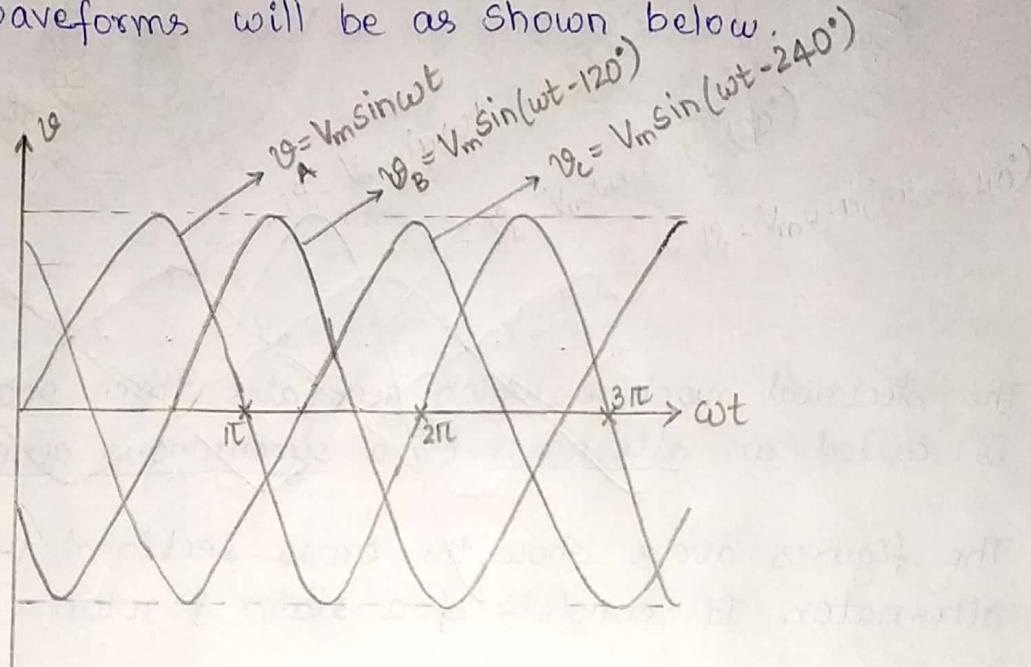


- \* The electrical machine which generates three phase voltages is called an alternator or a synchronous generator.
- \* The figures above show the cross-sectional view of an alternator. It consists of a stator & rotor.
- \* The stator is a stationary part and houses the three phase conductors in various slots. The stator is cylindrical in shape and has slots in its inner periphery for conductors. The windings (conductors) placed in these slots are connected together in such a way that the EMFs induced in them are additive, forming one winding called one phase winding. Similarly two more phase windings are formed and connected in star or delta connection.
- \* The rotor is the rotating part and usually has permanent magnets. If not, the magnetic element will have conductors known as field winding to produce flux & are called electromagnets.

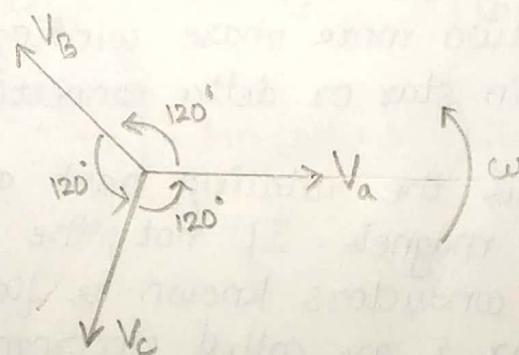
\* AA', BB' and CC' are three independent coils which are electrically displaced by  $120^\circ$  w.r.t one another.

\* When the rotor rotates in anticlockwise direction at a particular speed (known as synchronous speed  $N_s$ ), the flux produced by the magnets cut the stator conductors and hence EMFs are induced in all phases. The EMFs so induced in all three phases are displaced in phase by  $120^\circ$  w.r.t one another.

\* The waveforms will be as shown below.



\* Since the number of conductors in each phase winding is the same, the maximum value of EMFs induced in each of the winding and hence their RMS values are the same. They can be represented as shown below as phasors.



\* The equations for the induced voltages in the three windings are:

$$V_A = V_m \sin(\omega t)$$

$$V_B = V_m \sin(\omega t - 120^\circ)$$

$$V_C = V_m \sin(\omega t - 240^\circ)$$

\* Since the phasors are assumed to rotate in anticlockwise direction, we say

$$V_B \text{ lags } V_A \text{ by } 120^\circ$$

$$V_C \text{ lags } V_B \text{ by } 120^\circ \text{ (or } V_C \text{ lags } V_A \text{ by } 240^\circ\text{)}$$

\* Mathematically,

$$V_A + V_B + V_C = V_m \sin(\omega t) + V_m \sin(\omega t - 120^\circ) + V_m \sin(\omega t - 240^\circ)$$

$$= V_m \left[ 2 \sin\left(\frac{\omega t + \omega t - 120}{2}\right) \cos\left(\frac{\omega t - \omega t + 120}{2}\right) + V_m \sin(\omega t - 240^\circ) \right]$$

$$= V_m \left[ 2 \sin(\omega t - 60^\circ) \cos(60^\circ) \right] + [V_m \sin(\omega t - 240^\circ)]$$

$$= V_m \left[ 2 \sin(\omega t - 60^\circ) \times \frac{1}{2} \right] + V_m \sin(\omega t - 240^\circ)$$

$$= V_m \sin(\omega t - 60^\circ) + V_m \sin(\omega t - 240^\circ)$$

$$= V_m \left[ 2 \sin\left(\frac{\omega t - 60 + \omega t - 240}{2}\right) \cos\left(\frac{\omega t - 60 - \omega t + 240}{2}\right) \right]$$

$$= V_m \left[ 2 \sin\left(\frac{2\omega t - 300}{2}\right) \cos\left(\frac{180}{2}\right) \right]$$

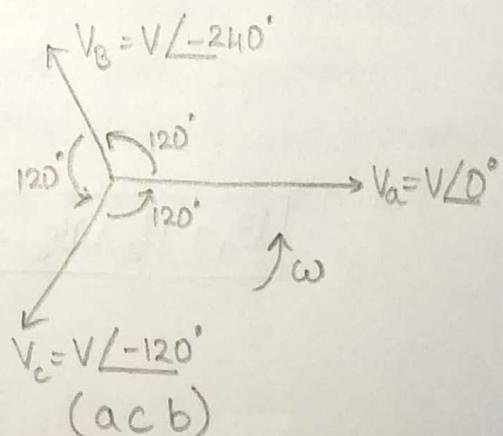
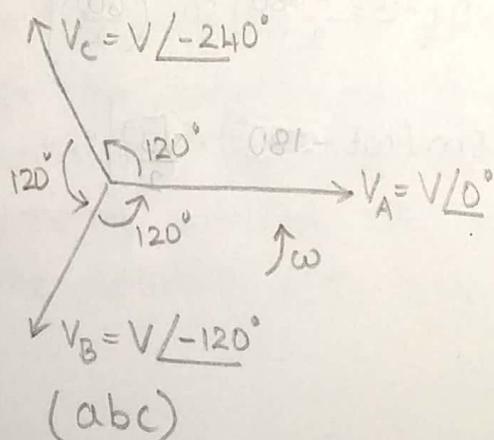
$$= V_m \left[ 2 \sin(\omega t - 150^\circ) \cdot \cos 90^\circ \right]$$

$$\boxed{V_A + V_B + V_C = 0}$$

- \* The same can be observed from the waveforms also.
- \* In phasor representation of three-phase voltages,
  - the length of each phasor represents the maximum or peak value of each voltage
  - the three voltage phasors are displaced electrically by  $120^\circ$ .

### Concept of phase sequence

- \* The order in which the voltages in the phases reach their maximum positive value is called the phase sequence.
- \* In the figure considered for generation of 3φ voltages, the three coils AA', BB' and CC' are rotating in anticlockwise direction.
- \* Then we see that phase-A (represented by coil AA') attains maximum value in positive direction first, followed by phase-B (coil BB') and then phase-C (coil CC'). Hence, the phase sequence is abc.
- \* If abc is taken as positive phase sequence then acb will be the negative phase sequence. These are shown below.



## \* Significance of phase sequence.

1. When a three phase supply of a particular phase sequence is given to a static three-phase load, certain currents flow through the lines & phases of the load. If the phase sequence is reversed, both the magnitudes & phase angles of the currents flowing in the lines and the phases of the load will change.
2. If a three phase supply is given to an induction motor, when the phase sequence of the supply is changed, not only the magnitudes & phase angles of the line & phase current change, but also the direction of rotation of motor changes.

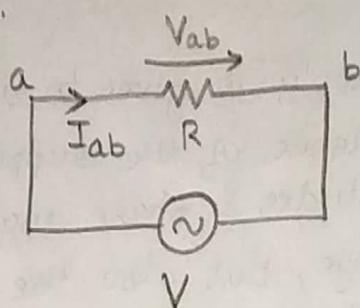
## \* Naming the phases

- The three phases may be numbered (1, 2, 3) or lettered (a, b, c) or specified colours (R, Y, B).
- By normal convention, sequence RYB is positive and RBY negative.

## \* Double subscript notation

- It is necessary to employ a systematic notation for the solution of a.c circuits and systems containing a number of EMFs acting and currents flowing so that the process of simplification & the solution are less prone to errors.

- It is normally preferred to employ double subscript notation while dealing with a.c electrical circuits.
- In this system, the order in which the subscripts are written indicates the direction in which the EMF acts or current flows.
- Consider one phase of a simple a.c circuit shown below.

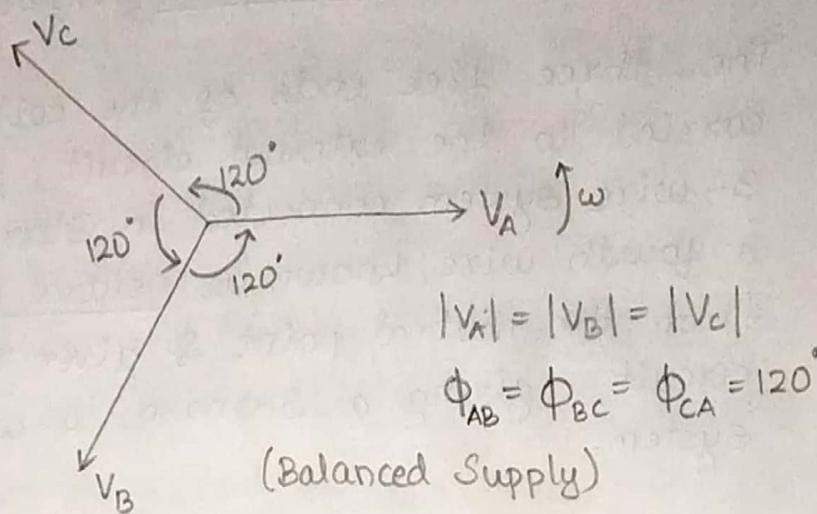


- \* If the EMF is expressed as  $V_{ab}$ , it indicates that the EMF acts from  $a$  to  $b$ ; if it is expressed as  $V_{ba}$ , then the EMF acts from  $b$  to  $a$ .
- \* From this, we can say
$$V_{ba} = -V_{ab}$$
- \* Similarly,  $I_{ab}$  indicates that current flows in the direction from  $a$  to  $b$  but  $I_{ba}$  indicates that current flows in the direction  $b$  to  $a$ .
- \* For this also, we can write

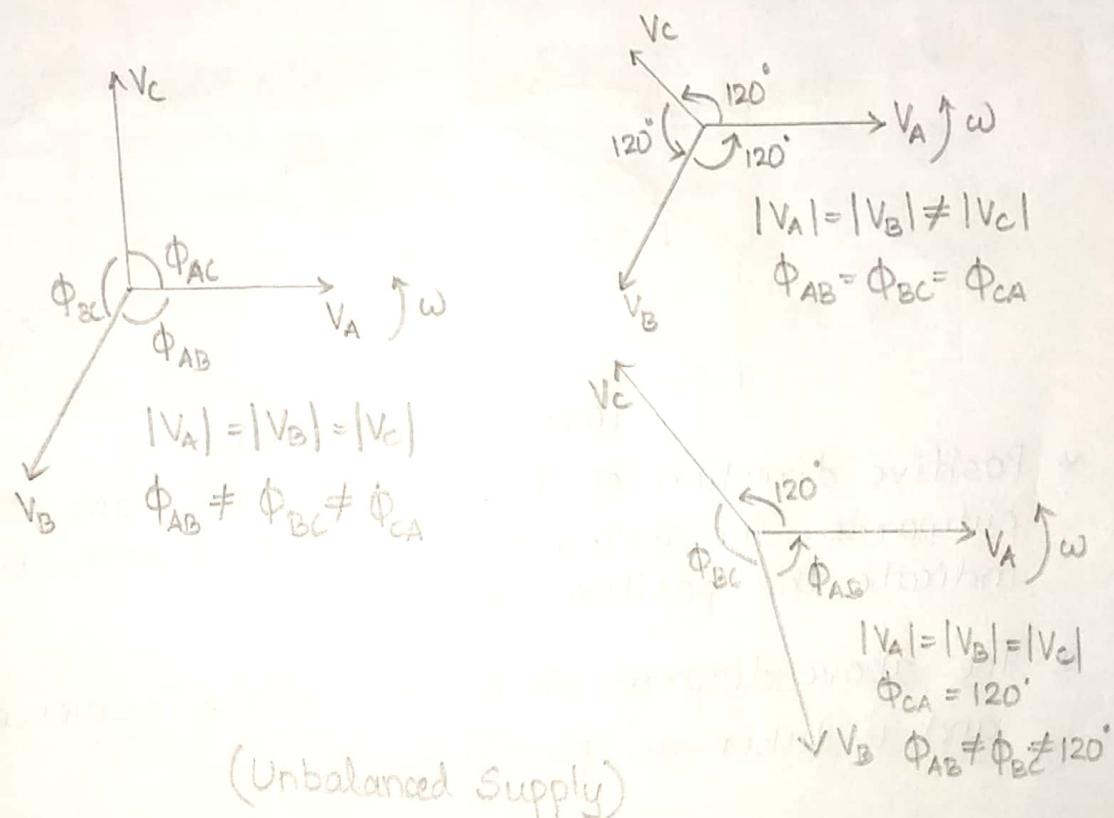
$$I_{ba} = -I_{ab}$$

## Balanced 3 $\phi$ Supply

- \* A three phase supply is said to be balanced, when all the three voltages have the same magnitude and differ in phase by  $120^\circ$  with respect to one another.

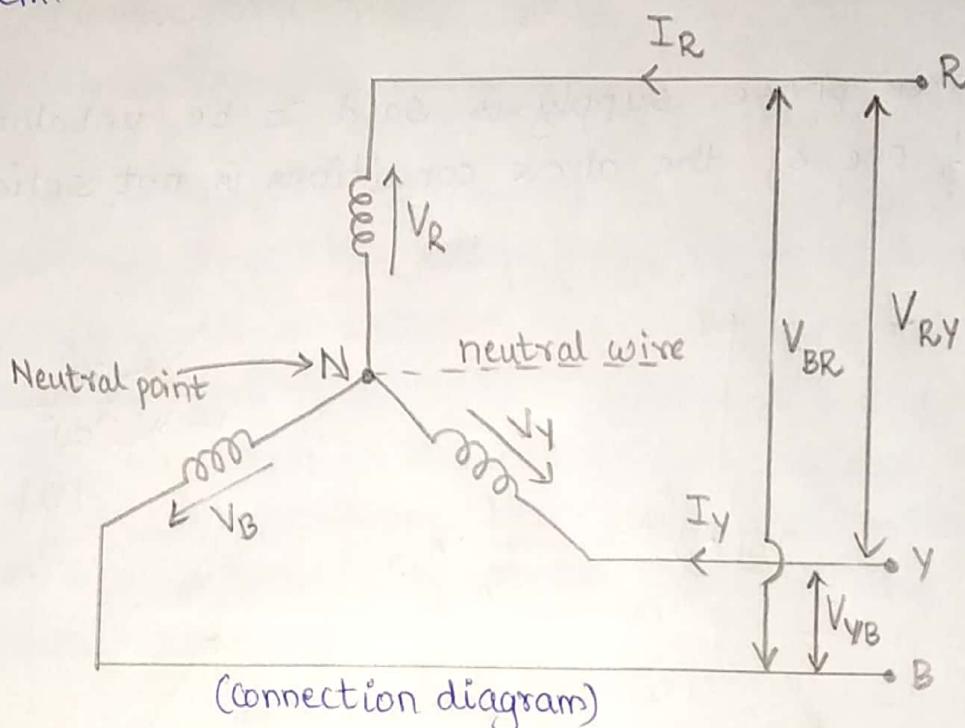


- \* Three phase supply is said to be unbalanced, even if one of the above conditions is not satisfied.



## Star or Wye (Y) Connection

- \* A star connection is obtained when one end of the three coils are connected to a common point and the other three ends are free.
- \* The common point at which the coils are connected is called the neutral point (N), or the star point.
- \* The three free ends of the coil are usually carried to the external circuit, giving a 3-phase, 3-wire system connected in Star. However, sometimes a fourth wire, known as neutral wire, is taken from the neutral point & given to the external circuit, giving a 3-phase, 4-wire star connected system.



- \* Positive direction of EMFs are taken from star point outwards. The arrow head on EMFs and currents indicate the positive direction.
- \* The above diagram shows the phase sequence RYB and is taken as positive.

- \* The phase winding is the winding represented between the line (which goes to the external circuit) and the neutral point.
- \* The voltage between any line and the neutral point i.e., the voltage across the phase winding, is called the line-to-neutral voltage or phase voltage. It is represented in general as  $V_{ph}$ . Here,  $V_R$ ,  $V_Y$  and  $V_B$  are the phase voltages.
- \* The voltage between any two lines is called as the line-to-line voltage, or simply as line voltage. It is represented in general as  $V_L$ . Here,  $V_{BR}$ ,  $V_{YB}$  and  $V_{RY}$  are the line voltages.
- \* In a star connection, there are two windings between each pair of lines. For example, winding of R-phase and Y-phase are in between the R-Y line pair. Since their one end is connected together & voltages across them as shown in the connection diagram, the voltages are in opposition. Hence, the line voltage is the phasor difference of phase EMFs of the two phases concerned.

$$\therefore V_{RY} = V_R - V_Y$$

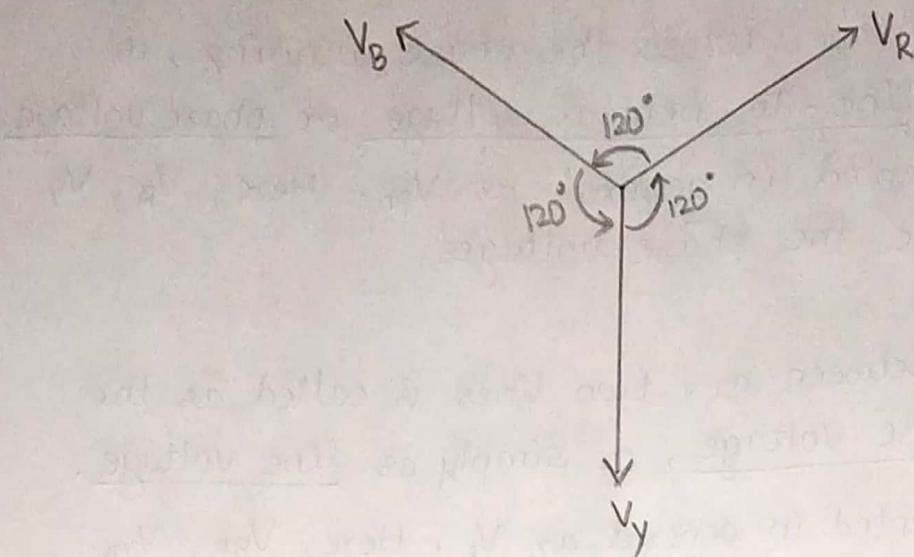
$$V_{YB} = V_Y - V_B$$

$$V_{BR} = V_B - V_R$$

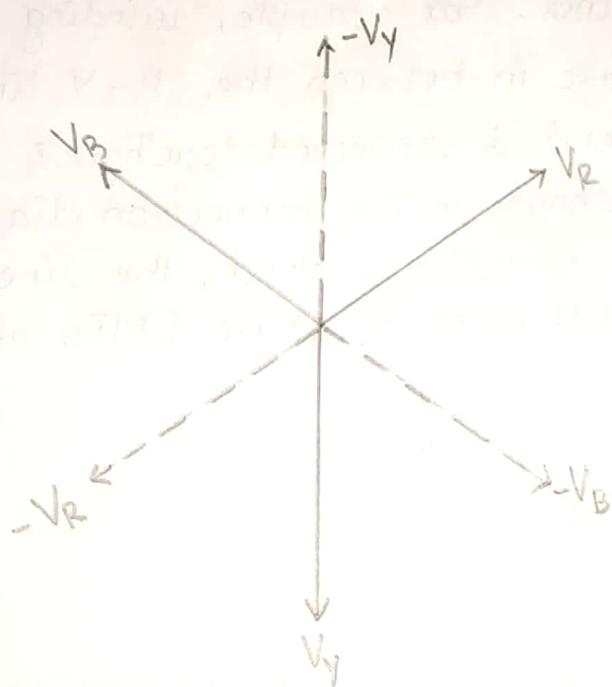
- \* This can also be expressed using phasor diagram to show the various voltages and their phase differences.

\* Method of drawing phasor diagram

Step-1 :- Draw the phase voltages ( $V_R$ ,  $V_Y$  and  $V_B$ ) with equal magnitude, having  $120^\circ$  phase difference.



Step-2 :- Extend the phasors in the opposite direction.  
Mark them with negative sign.



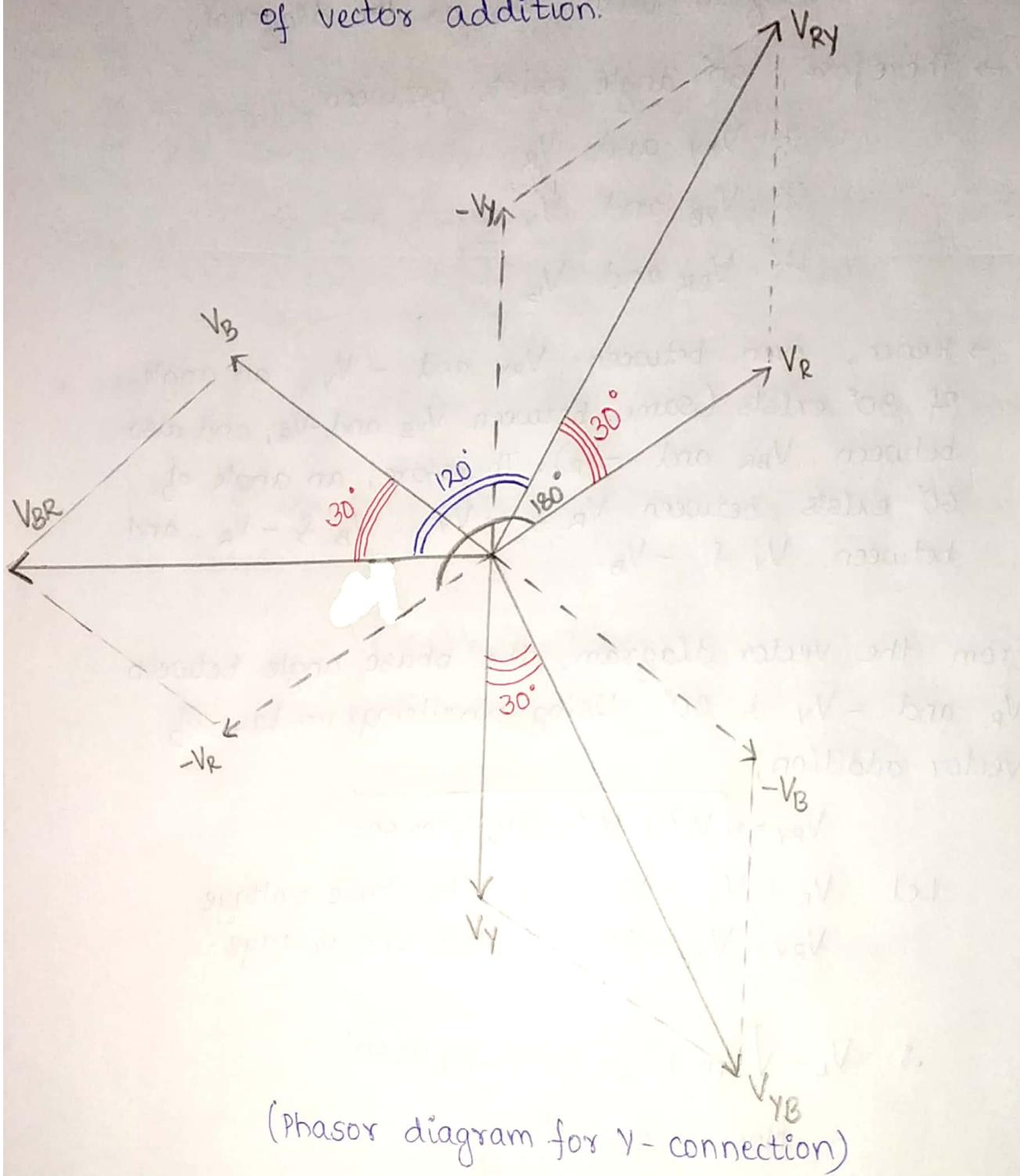
Step-3 :- Using the previously stated relationship,

$$V_{RY} = V_R - V_Y$$

$$V_{YB} = V_Y - V_B$$

$$V_{BR} = V_B - V_R$$

This is obtained using parallelogram law of vector addition.



\* Why the angles  $30^\circ$  and  $60^\circ$ ?

- The angle between  $V_R$  and  $-V_R$  is  $180^\circ$ . The angle between  $V_{RY}$  and  $V_{BR}$  is  $120^\circ$ . The difference angle is  $180 - 120 = 60^\circ$ .
- The phasors  $V_R$  and  $-V_Y$  form a parallelogram with  $V_{RY}$  as the diagonal. Hence, the  $60^\circ$  angle divides equally as even  $V_B$  and  $-V_R$  form a parallelogram with  $V_{BR}$  as the diagonal.
- Therefore,  $30^\circ$  angle exists between
  - ∴  $V_{RY}$  and  $V_R$
  - ∴  $V_{YB}$  and  $V_Y$
  - ∴  $V_{BR}$  and  $V_B$
- Hence, even between  $V_{RY}$  and  $-V_Y$ , an angle of  $30^\circ$  exists (same between  $V_{YB}$  and  $-V_B$ , and also between  $V_{BR}$  and  $-V_R$ ). Therefore, an angle of  $60^\circ$  exists between  $V_R$  &  $-V_Y$ ,  $V_B$  &  $-V_R$ , and between  $V_Y$  &  $-V_B$ .

\* From the vector diagram, the phase angle between  $V_R$  and  $-V_Y$  is  $60^\circ$ . Using parallelogram law of vector addition,

$$V_{RY} = \sqrt{V_R^2 + V_Y^2 + 2V_R V_Y \cos 60^\circ}$$

Let  $V_R = V_Y = V_B = V_{ph}$ , the phase voltage  
 $V_{RY} = V_{YB} = V_{BR} = V_L$ , the line voltage.

$$\therefore V_L = \sqrt{V_{ph}^2 + V_{ph}^2 + 2V_{ph} V_{ph} \cos 60^\circ}$$

Now,  $\cos 60^\circ = \frac{1}{2}$ .

$$\therefore V_L = \sqrt{V_{ph}^2 + V_{ph}^2 + (2V_{ph}^2 \times \frac{1}{2})}$$

$$V_L = \sqrt{3} V_{ph}^2$$

$$\Rightarrow \boxed{V_L = \sqrt{3} \cdot V_{ph}}$$

Thus, in a star connected system, the line voltage is equal to  $\sqrt{3}$  times the phase voltage.

- \* From the connection diagram, each line is connected to a separate phase, so that the current flowing in the lines & phases are the same.

$$\therefore \text{Line current} = \text{Phase Current}$$

$$\boxed{I_L = I_{ph}}$$

- \* Let  $\phi$  be the angle between phase voltage & phase current. Then,

$$\text{Power output per phase} = V_{ph} I_{ph} \cos \phi$$

The total power of a  $3\phi$  circuit is,

$$\boxed{P = 3 V_{ph} I_{ph} \cos \phi}$$

But for a star connection,

$$I_{ph} = I_e$$

$$V_{ph} = \frac{V_L}{\sqrt{3}}$$

$$\therefore P = 3 \frac{V_L}{\sqrt{3}} I_e \cos \phi = (\sqrt{3})(\sqrt{3}) \left( \frac{V_L}{\sqrt{3}} \right) I_e \cos \phi$$

$$\therefore \boxed{P = \sqrt{3} V_L I_e \cos \phi}$$

Thus, for a star connection,

$$V_L = \sqrt{3} V_{ph}$$

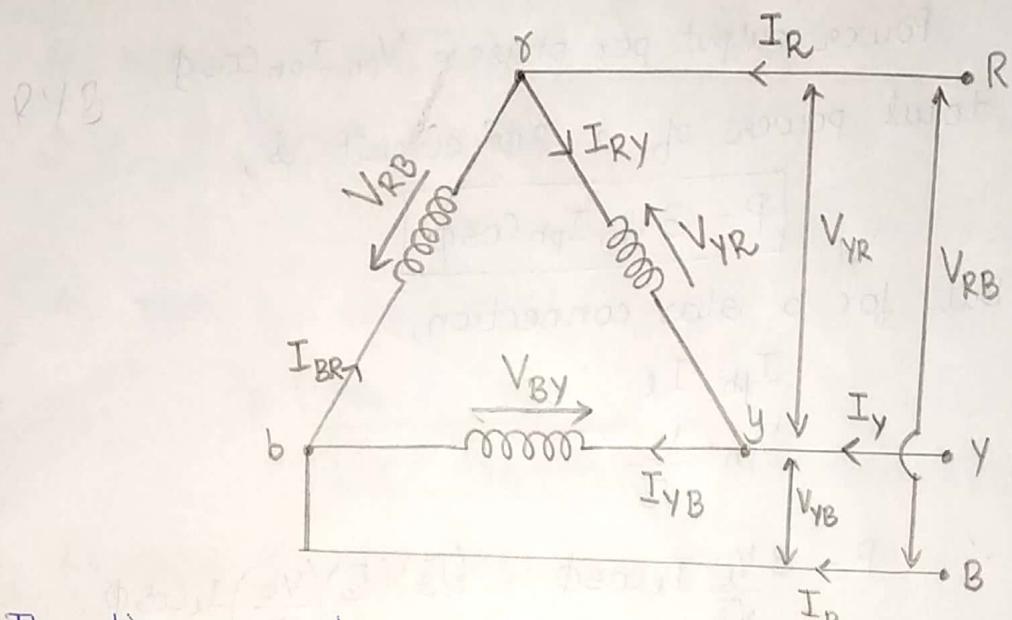
$$I_L = I_{ph}$$

$$P = 3V_{ph} I_{ph} \cos\phi = \sqrt{3} V_L I_L \cos\phi$$

$\phi \rightarrow$  Angle between  $V_{ph}$  &  $I_{ph}$   
(not  $V_L$  and  $I_L$ )

### Delta ( $\Delta$ ) or Mesh Connection.

- \* When three coils are connected end-to-end as shown below, a delta or mesh connection is obtained.
- \* The direction of EMFs is also shown below.



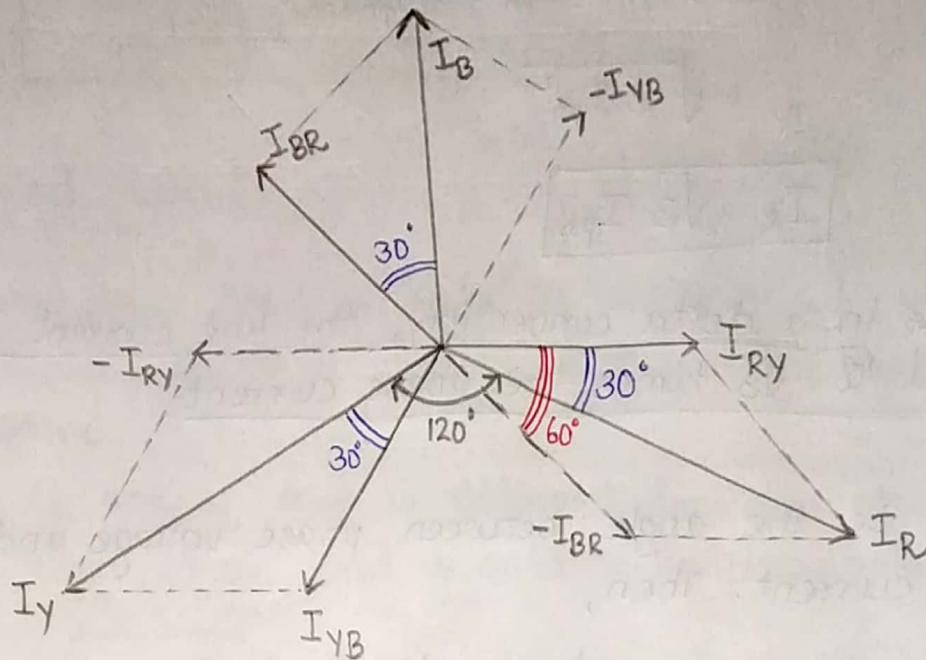
- \* The line currents are  $I_R$ ,  $I_Y$  and  $I_B$ , while the phase currents are  $I_{RY}$ ,  $I_{YB}$  and  $I_{BR}$ .
- \* Let  $V_{YR}$ ,  $V_{BY}$  and  $V_{RB}$  be the voltage across the phases. This is same as the voltage between

two lines. Hence in a delta connection,

Line Voltage = Phase Voltage

$$V_L = V_{ph}$$

- \* The phasor diagram for a delta connection (following steps similar to that of star connection) can be written as shown below.



At node  $y$ , apply KCL

$$I_R + I_{BR} = I_{RY}$$

$$\therefore [I_R = I_{RY} - I_{BR}] \text{ --- to draw } I_R \text{ phasor}$$

At node  $y$ , apply KCL.

$$I_y + I_{RY} = I_{YB}$$

$$\therefore [I_y = I_{YB} - I_{RY}] \text{ --- to draw } I_y \text{ phasor}$$

At node 'b', apply KCL

$$I_B + I_{YB} = I_{BR}$$

$$\therefore [I_B = I_{BR} - I_{YB}] \text{ --- to draw } I_B \text{ phasor.}$$

The phase angle between currents  $I_{RY}$  and  $-I_{BR}$  is  $60^\circ$ .

$$\therefore I_R = \sqrt{I_{RY}^2 + I_{BR}^2 + 2I_{RY}I_{BR}\cos 60^\circ}$$

Let  $I_{RY} = I_{BR} = I_{ph}$ , the phase current  
 $I_R = I_y = I_B = I_e$ , the line current.

$$\therefore I_e = \sqrt{I_{ph}^2 + I_{ph}^2 + 2I_{ph}I_{ph}\frac{1}{2}}$$
$$= \sqrt{3} I_{ph}$$

$$I_e = \sqrt{3} I_{ph}$$

Thus in a delta connection, the line current is equal to  $\sqrt{3}$  times the phase current.

\* Let  $\phi$  be the angle between phase voltage and phase current. Then,

$$\text{power per phase} = V_{ph} I_{ph} \cos \phi$$

Total power is,

$$P = 3 V_{ph} I_{ph} \cos \phi$$

$$= 3 V_e \left( \frac{I_e}{\sqrt{3}} \right) \cos \phi$$

$$= (\sqrt{3})(\sqrt{3})(V_e) \left( \frac{I_e}{\sqrt{3}} \right) \cos \phi$$

$$P = \sqrt{3} V_e I_e \cos \phi$$

Thus, the power equation for both star & delta connections are the same.

\* Thus, for a delta connection,

$$V_e = V_{ph}$$

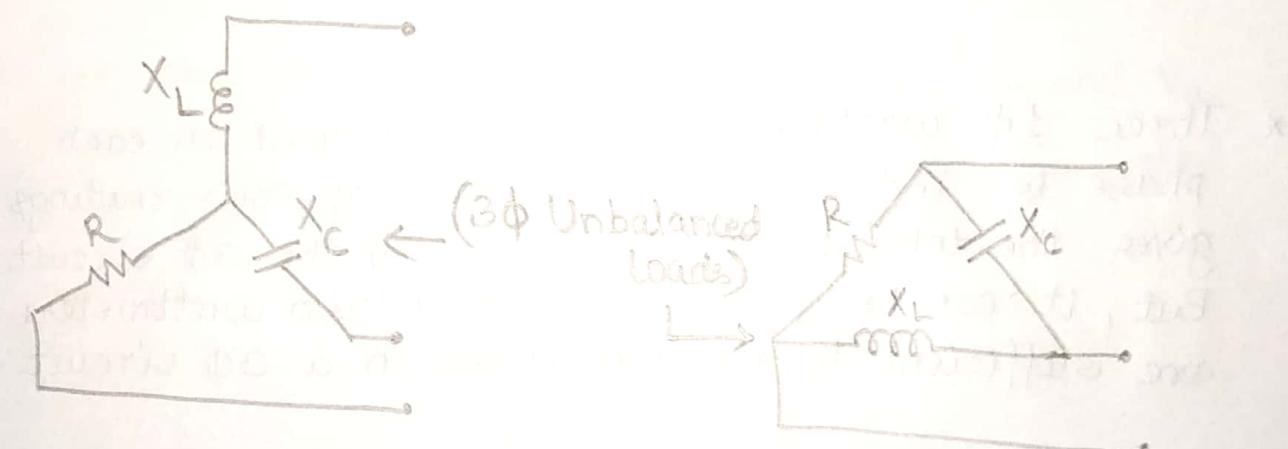
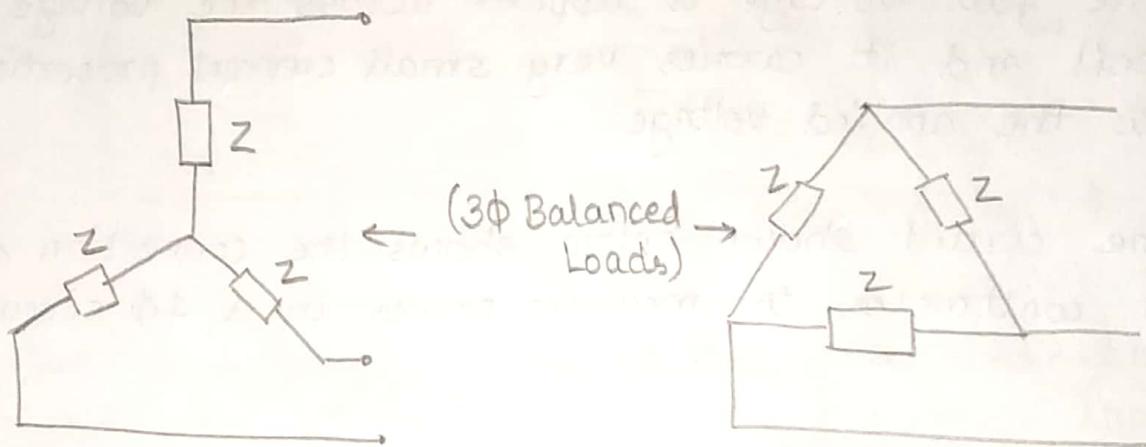
$$I_e = \sqrt{3} I_{ph}$$

$$P = 3V_{ph} I_{ph} \cos\phi = \sqrt{3} V_e I_e \cos\phi$$

$\phi \rightarrow$  Angle between  $V_{ph}$  and  $I_{ph}$   
(not  $V_e$  and  $I_e$ ).

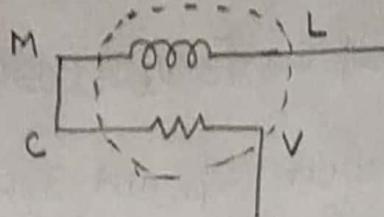
## Balanced Load

- \* A three phase load is said to be balanced, when the impedances of all the three phases are exactly the same.
- \* Even if one of them is different from the other, then the three-phase load is said to be unbalanced.



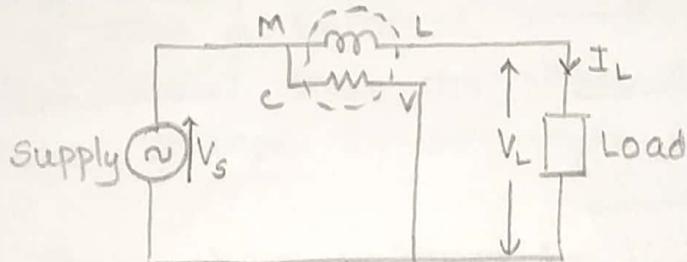
## Measurement of Power in a 3φ Circuit

- \* A Wattmeter is an instrument used to measure power in an electrical circuit.



[Wattmeter Representation]

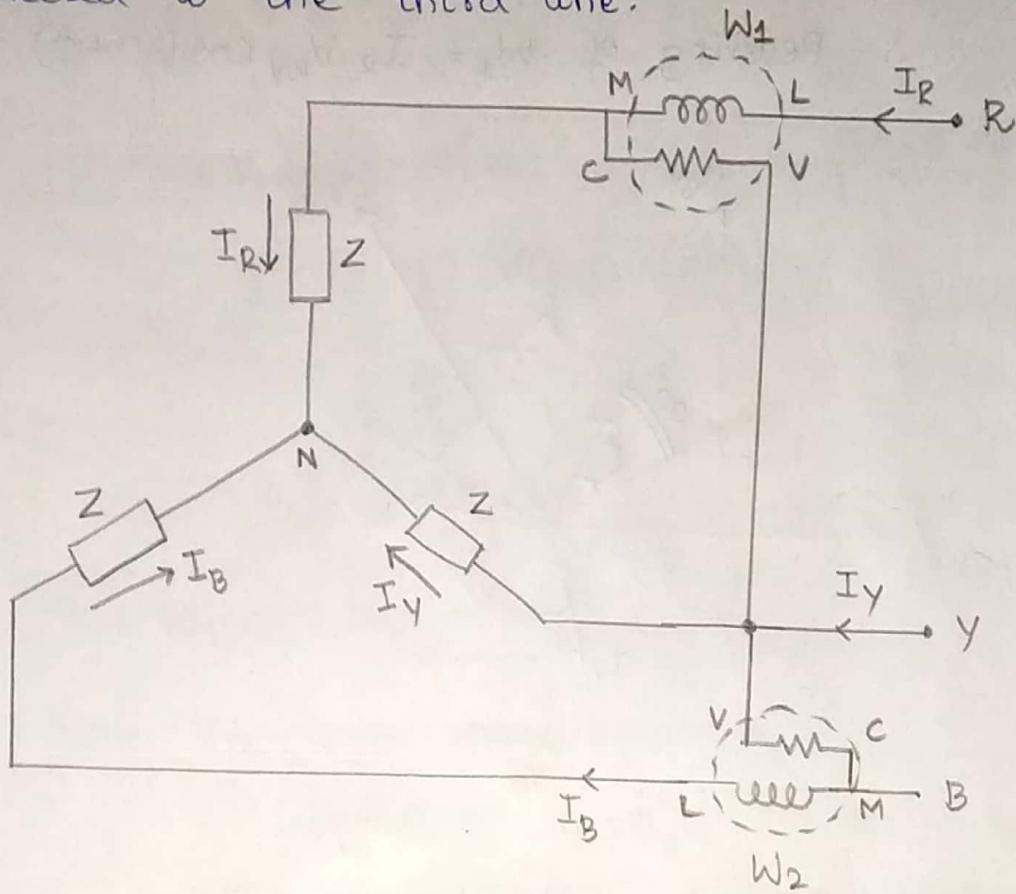
- \* It consists of
  - i) a current coil ML, which is connected in series with the circuit and through which the line current flows.
  - ii) a voltage (potential) coil CV, which is connected across the circuit.
- \* The full voltage is applied across the voltage coil and it carries very small current proportional to the applied voltage.
- \* The circuit shown below shows the connection of a wattmeter to measure power in a 1φ circuit.



- \* Three 1φ wattmeters may be connected in each phase and the algebraic sum of their readings gives the total power consumed by the 3φ circuit. But, it can be proved that only two wattmeters are sufficient to measure power in a 3φ circuit.

## Two Wattmeters Method.

- \* This method is normally used to measure power in a  $3\phi$ , 3-wire balanced (load & supply) systems.
- \* For this method, the current coils in the two wattmeters are inserted in any two lines and the voltage coil of the two wattmeters are connected to the third line.



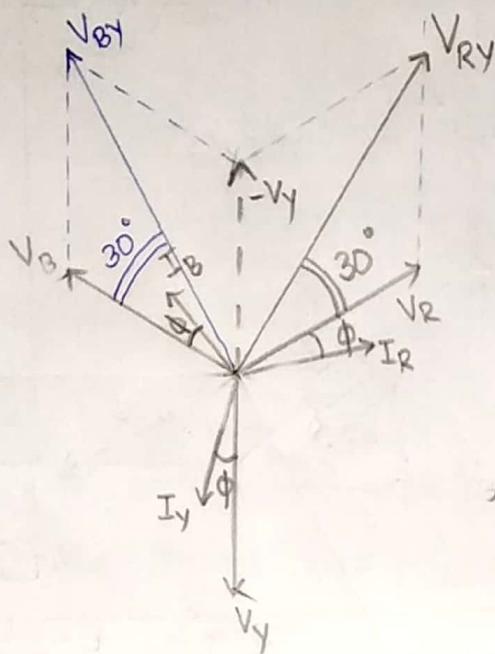
- \* Let us assume a balanced inductive star-connected load. This implies that the phase current lags the phase voltage by an angle  $\phi$ .
- \* Let the three phase voltages be  $V_R$ ,  $V_Y$  and  $V_B$  and let the currents be  $I_R$ ,  $I_Y$  and  $I_B$  (all are RMS values).

\* Current through wattmeter  $W_1 = I_R$   
 Potential difference across the voltage coil of  $W_1 \} = V_{RY} = V_R - V_Y$

The  $V_{RY}$  is the resultant of  $V_R$  and  $-V_Y$  as shown in the vector diagram

Clearly, the phase difference between  $V_{RY}$  and  $I_R$  is  $30 + \phi$

$$\therefore \text{Reading of } W_1 = I_R V_{RY} \cos(30 + \phi)$$



\* Current through wattmeter  $W_2 = I_B$   
 Potential difference between the voltage coil of  $W_2 \} = V_{BY} = V_B - V_Y$

The  $V_{BY}$  is the resultant of  $V_B$  and  $-V_Y$ .

The phase difference between  $V_{BY}$  and  $I_B$  is  $30 - \phi$

$$\therefore \text{Reading of } W_2 = I_B V_{BY} \cos(30 - \phi)$$

Since the load is balanced,

$V_{RY} = V_{BY} = V_L$ , the line voltage

$I_R = I_B = I_L$ , the line current.

$$\therefore W_1 = V_L I_L \cos(30 + \phi)$$

$$W_2 = V_L I_L \cos(30 - \phi).$$

$$\therefore W_1 + W_2 = V_L I_L \cos(30 + \phi) + V_L I_L \cos(30 - \phi)$$

$$= V_L I_L [\cos(30 + \phi) + \cos(30 - \phi)]$$

$$= V_L I_L [2 \cos\left(\frac{30 + \phi + 30 - \phi}{2}\right) \cos\left(\frac{30 + \phi - 30 + \phi}{2}\right)]$$

$$= V_L I_L [2 \cos\left(\frac{60}{2}\right) \cos\left(\frac{2\phi}{2}\right)]$$

$$= V_L I_L [2 \cos 30 \cos \phi]$$

$$= V_L I_L \left[ 2 \frac{\sqrt{3}}{2} \cos \phi \right]$$

$$\boxed{W_1 + W_2 = \sqrt{3} V_L I_L \cos \phi}$$

Since the three phase power is

$$P = \sqrt{3} V_L I_L \cos \phi,$$

$$\boxed{W_1 + W_2 = P}$$

Thus, we prove that only two wattmeters are sufficient to measure three-phase power.

Note:-

1. This method of measuring three-phase power is valid for unbalanced load also.

2. At the same time, the same conclusion can be drawn when the load is delta connected.

\* Measurement of power factor

The expression for power read by the two wattmeters are,

$$W_1 = V_L I_L \cos(30 - \phi)$$

$$W_2 = V_L I_L \cos(30 + \phi)$$

$$\therefore W_1 + W_2 = V_L I_L \cos(30 - \phi) + V_L I_L \cos(30 + \phi)$$

$$W_1 + W_2 = \sqrt{3} V_L I_L \cos \phi \quad \rightarrow (1)$$

and

$$W_1 - W_2 = V_L I_L \cos(30 - \phi) - V_L I_L \cos(30 + \phi)$$

$$= \left\{ 2 V_L I_L \left[ \sin \left( \frac{30 - \phi + 30 + \phi}{2} \right) \sin \left( \frac{30 - \phi - 30 - \phi}{2} \right) \right] \right\}$$

$$= -2 V_L I_L \left[ \sin \left( \frac{60}{2} \right) \sin \left( -\frac{2\phi}{2} \right) \right]$$

$$= -2 V_L I_L \left[ \sin 30 \sin(-\phi) \right]$$

$$= -(-2) V_L I_L \left( \frac{1}{2} \sin \phi \right)$$

$$W_1 - W_2 = V_L I_L \sin \phi \quad \rightarrow (2).$$

Divide equation (2) by equation (1).

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{V_L I_L \sin \phi}{\sqrt{3} V_L I_L \cos \phi} = \frac{1}{\sqrt{3}} \tan \phi$$

$$\Rightarrow \boxed{\tan \phi = \frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}}$$

$\therefore$  Power factor is,

$$\boxed{PF = \cos \left[ \tan^{-1} \left( \frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)} \right) \right]}$$

## Effect of power factor on $W_1$ and $W_2$ .

1. When power factor is unity ( $Pf = 1$ )

$$Pf = \cos \phi = 1$$

$$\therefore \phi = 0^\circ$$

$$W_1 = V_L I_L \cos(30 - \phi) = V_L I_L \cos(30 - 0) = \frac{\sqrt{3}}{2} V_L I_L$$

$$W_2 = V_L I_L \cos(30 + \phi) = V_L I_L \cos(30 + 0) = \frac{\sqrt{3}}{2} V_L I_L$$

Thus, when power factor is unity, both wattmeters read equal positive values.

Inference:- In a purely resistive 3 $\phi$  circuit,  $Pf = 1$  and hence both wattmeters will read equal positive values.

2. When power factor is 0.5

$$Pf = \cos \phi = 0.5$$

$$\Rightarrow \phi = 60^\circ$$

$$\therefore W_1 = V_L I_L \cos(30 - \phi) = V_L I_L \cos(30 - 60) = V_L I_L \cos(-30) = \frac{\sqrt{3}}{2} V_L I_L$$

$$W_2 = V_L I_L \cos(30 + \phi) = V_L I_L \cos(30 + 60) = V_L I_L \cos(90) = 0.$$

Thus, when power factor is 0.5, one of the wattmeters, reads zero.

3. When power factor is zero

$$Pf = \cos \phi = 0$$

$$\Rightarrow \phi = 90^\circ$$

$$\therefore W_1 = V_L I_L \cos(30 - \phi) = V_L I_L \cos(30 - 90) = V_L I_L \cos(-60) = \frac{V_L I_L}{2}$$

$$W_2 = V_L I_L \cos(30 + \phi) = V_L I_L \cos(30 + 90) = V_L I_L \cos(120) = -\frac{V_L I_L}{2}$$

Thus, one wattmeter reads negative value at zero power factor.

- \* However, a physical meter cannot show a negative reading. When it tries to give such downscale reading, we say that the wattmeter 'kicks back'. In such a case, the current coil connections (M and L) are to be interchanged to get a reading. However, this wattmeter reading has to be considered as negative.
- \* In summary,

Power factor (phase angle)	Wattmeter readings
between 0 & 0.5 $(\phi > 60^\circ)$	one of the wattmeters, reads negative
equal to 0.5 $(\phi = 60^\circ)$	one wattmeter reads zero
between 0.5 & 1 $(\phi < 60^\circ)$	both wattmeters read positive values.
equal to 1 $(\phi = 0^\circ)$	both wattmeters read equal positive value.

#### Note:-

When a wattmeter kicks back, we can also interchange the voltage coil connections & take the value as negative.

## Important points about 3φ system.

1. For 3φ system, unless otherwise mentioned, it is normal practice to specify the values of line currents and line voltages.

Example:- If a 3φ, 11 kV circuit carries a current of 500A, it implies

$$V_L = 11 \text{ kV}$$

$$I_L = 500 \text{ A}.$$

2. The current in any phase ( $I_{ph}$ ) can be determined by dividing the voltage across that phase ( $V_{ph}$ ) by the impedance of that phase ( $Z_{ph}$ ). Thus,

$$I_{ph} = \frac{V_{ph}}{Z_{ph}}$$

3. Relationship between power in star ( $P_{star}$ ) and in delta circuits ( $P_{delta}$ ).

→ Consider that the same 3φ load is connected first in star & then in delta. Hence,  $Z_{ph}$  is same for both the connections. If it is supplied from the same source, then  $V_L$  is the same for star & delta loads.

→ In a star load,

$$\begin{aligned} P_{star} &= 3 V_{ph} I_{ph} \cos \phi \\ &= 3 V_{ph} \left( \frac{V_{ph}}{Z_{ph}} \right) \cos \phi \end{aligned}$$

$$P_{star} = \frac{3 V_{ph}^2 \cos \phi}{Z_{ph}}$$

but  $V_{ph} = \frac{V_L}{\sqrt{3}}$  for star connection.

$$\therefore P_{\text{star}} = \frac{3(V_L/\sqrt{3})^2 \cos\phi}{Z_{\text{ph}}}$$

$$\therefore P_{\text{star}} = \frac{V_L^2 \cos\phi}{Z_{\text{ph}}} \rightarrow (1)$$

→ If the same load is connected in delta, then

$$P_{\text{delta}} = 3 V_{\text{ph}} I_{\text{ph}} \cos\phi$$

$$= 3 V_{\text{ph}} \left( \frac{V_{\text{ph}}}{Z_{\text{ph}}} \right) \cos\phi$$

$$P_{\text{delta}} = \frac{3 V_{\text{ph}}^2 \cos\phi}{Z_{\text{ph}}}$$

But  $V_{\text{ph}} = V_L$ . for delta connection

$$\therefore P_{\text{delta}} = \frac{3 V_L^2 \cos\phi}{Z_{\text{ph}}} \rightarrow (2)$$

→ Comparing (1) and (2), we can write

$P_{\text{delta}} = 3 * P_{\text{star}}$

Thus, a delta connected load consumes three times the power it would have consumed if connected in star connection.