RV Educational Institutions
RV College of Engineering

Autonomous Institution Affiliated to Visvesvaraya Technological University, Belagavi
Approved by AICTE, New Delhi

DEPARTMENT OF MATHEMATICS

Course: Fundamentals of Linear Algebra, Calculus and Statistics	**CIE-I (QUIZ & TEST)**	**Maximum marks: 10+50=60**
Course code: 22MA11C	**First semester 2022-2023** **Chemistry Cycle** **Branch: AI, BT, CD, CS, CY, IS, SPARK-C**	**Time: 9.15am to 11.15am** **Date: 17-01-2023**

Instructions to candidates:

i. ***Part A must be answered within the first two pages of the Booklet.***
ii. ***Answer all questions.***

Q.No	PART- A	M	BT	CO
1.1	The rank of the Matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{bmatrix}$ for $a = b \neq c$ is __________.	2	2	1
1.2	The value of p for which the following set of equations will have no solution is ______. $2x + 3y = 5$ $3x + py = 10$	2	2	1
1.3	If $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, then eigenvalues of A^{-1} are ______.	2	2	2
1.4	The transformation of the Lemniscate $r^2 = 2\cos 2\theta$ in cartesian system is__________.	2	1	2
1.5	If $x = at^2$ and $y = 2at$, then the radius of curvature ρ for the given curve is_______.	2	2	2

Q.No	PART -B	M	BT	CO
1	Test the consistency of the following system $3.0x_1 + 2.0x_2 + 2.0x_3 - 5.0x_4 = 8.0$ $0.6x_1 + 1.5x_2 + 1.5x_3 - 5.4x_4 = 2.7$ $1.2x_1 - 0.3x_2 - 0.3x_3 + 2.4x_4 = 2.1.$ Solve if the system is consistent.	10	3	1
2	The currents i_1, i_2, i_3 in the paths of an electrical network follow the linear equations $i_1 - i_2 + i_3 = 0, 3i_1 + 2i_2 = 7, 2i_2 + 4i_3 = 8$. Determine i_1, i_2, i_3 using Gauss-Jordan elimination method.	10	3	4
3	Employ the Rayleigh's Power method to estimate the dominant eigenvalue and its associated eigen vector for the matrix $A = \begin{bmatrix} 3 & 2 & 3 \\ 2 & 6 & 6 \\ 3 & 6 & 3 \end{bmatrix}$ by taking $[1 \quad 1 \quad 1]^T$ as initial eigenvector. Perform 6 iterations. (Consider 4 decimal places)	10	2	2
4 (a)	The temperature u_1, u_2, u_3 of a metal plate under some circumstances is given by: $u_1 - 8u_2 + 3u_3 + 4 = 0,$ $2u_1 + u_2 + 9u_3 = 12,$ $8u_1 + 2u_2 - 2u_3 = 8.$ Solve for the temperatures using Gauss- Seidel iterative method. Carry out 3 iterations. (Consider 4 decimal places)	6	3	3
4(b)	Show that the curves $r = a\theta$ and $r = \frac{a}{\theta}$ intersect orthogonally.	4	2	2
5	Find the circle of curvature of $b^2x^2 + a^2y^2 = a^2b^2$ at a point of its intersection with the y-axis.	10	2	3

BT-Blooms Taxonomy, CO-Course Outcomes, M-Marks

Marks Distribution	Particulars		CO1	CO2	CO3	CO4	L1	L2	L3	L4	L5	L6
	Test	Max Marks	14	20	16	10	2	32	26	--	-	-

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DEPARTMENT OF MATHEMATICS

Course: Fundamentals of Linear Algebra, Calculus and statistics	**CIE-II (QUIZ & TEST)**	**Maximum marks: 10+50=60**
Course code: 22MA11C	**First semester 2022-2023 Chemistry Cycle Branch: AI, BT, CD, CS, CY, IS, SPARK-C**	**Time: 1:00PM-3:00PM Date: 21-02-2023**

Instructions to students:

1. *Answer **all** questions.*
2. ***Part A** must be answered in the **first two pages** of the answer book only.*

Q.No	**PART- A (Quiz)**	M	BT	CO
1.1	The coefficient of x^2 in the Maclaurin series expansion of $e^{-\left(\frac{x}{3}\right)}$ is __________.	2	L1	1
1.2	If $z = x\sin(y) + y\cos(x)$ then $\frac{\partial^2 z}{\partial x \partial y} =$__________.	2	L1	1
1.3	Total differential of the function $u = x^3 e^y z^2$ is __________.	2	L2	2
1.4	The critical point of the function $f(x,y) = x^2 + 2x + 9y - 3y^2 + 5$ is _______.	2	L1	1
1.5	If $x = u\sin(v)$ and $y = u\cos(v)$ then $\frac{\partial(u,v)}{\partial(x,y)} =$________.	2	L2	2

Sl. No.	PART -B	M	BT	CO
1	Obtain the Maclaurin series expansion of $\log_e(1+e^x)$ up to the term containing x^4 and hence deduce the expansion of $\frac{1}{e^{-x}(1+e^x)}$.	10	L2	2
2. (a)	Find the value of n so that the equation $V = r^n(3\cos^2(\theta) - 1)$ satisfies the relation $\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{\sin(\theta)}\frac{\partial}{\partial \theta}\left(\sin(\theta)\frac{\partial V}{\partial \theta}\right) = 0.$	06	L2	1
2. (b)	Find $\frac{du}{dx}$ for $u = \log_e(x^2+y^2)$, where $x^3 + y^3 + 5xy = 19$.	04	L2	2
3. (a)	Using chain rule express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s if $w = x + 2y + z^2$, $x = \frac{r}{s}$, $y = r^2 + \log_e(s)$, $z = 2r$.	06	L2	2
3. (b)	The pressure P (in kilopascals), volume V (in liters), and temperature T (in kelvins) of a mole of an ideal gas are related by the equation $PV = 8.31T$. Find the rate at which the pressure is changing when the temperature is 300K and increasing at a rate of 0.1K/s and the volume is 100L and increasing at a rate of 0.2L/s.	04	L3	4
4	Find the shortest and longest distance from the point $P(1,2,-1)$ to the sphere $x^2 + y^2 + z^2 = 24$ using Lagrange's multiplier method.	10	L3	3
5	If $u = x + y + z$, $uv = y + z$ and $uvw = z$ then find $J = \frac{\partial(u,v,w)}{\partial(x,y,z)}$ and $J' = \frac{\partial(x,y,z)}{\partial(u,v,w)}$. Also show that $JJ' = 1$.	10	L3	3

BT-Blooms Taxonomy, CO-Course Outcomes, M-Marks

Marks Distribution	Particulars		CO1	CO2	CO3	CO4	L1	L2	L3	L4	L5	L6
	Test+Quiz	Max Marks	12	24	20	04	06	30	24	--	--	--

RV Educational Institutions
RV College of Engineering

Autonomous Institution Affiliated to Visvesvaraya Technological University, Belagavi

Approved by AICTE, New Delhi

Course: Fundamentals of Linear Algebra, Calculus and Statistics	DEPARTMENT OF MATHEMATICS IMPROVEMENT TEST	Maximum marks: 10+50=60
Course code: 22MA11C	First semester 2022-2023 Chemistry Cycle Branch: AI, BT, CS, CD, CY, IS, SPARK C	Time: 2pm to 4pm Date: 20-03-2023

Instructions to candidates:

i. Part A must be answered within the first two pages of the Booklet.
ii. Answer all questions.

Q.No	PART- A	M	BT	CO
1.1	If the correlation co-efficient is zero, then the two regression lines are ______ to each other.	1	L1	1
1.2	If $y = e^{at}$ is the best exponential curve for the data points $(x_i, y_i), i \in \{1,2,3,\dots n\}$, then $a =$ ______.	2	L2	2
1.3	The value of the integral $I = \int_0^2 \int_0^1 e^{x+y}\, dxdy$ is ______.	2	L2	1
1.4	The first raw moment about the point 20 is 50 then its mean is ______.	2	L1	1
1.5	Let $\sum x = 50$, $\sum y = 80$, $\sum xy = 1030$, $\sum x^2 = 750$ for a dataset (x_i, y_i), where $i \in \{1,2,3 \dots,10\}$. The best straight line fit for the given data is ______.	2	L2	2
1.6	The skewness of a normal distribution is ______.	1	L1	1

<table>
<tr><th>Q.No</th><th>PART -B</th><th>M</th><th>BT</th><th>CO</th></tr>
<tr><td>1.a</td><td>The first four moments about the working mean 28.5 of a distribution are 0.294, 7.144, 42.409 and 454.98. Calculate the moments about the mean, co-efficient of skewness and kurtosis using moments. Also, comment upon the skewness and kurtosis of the distribution.</td><td>6</td><td>L2</td><td>1</td></tr>
<tr><td>1.b</td><td>In a partially destroyed laboratory record, only the lines of regression of y on x and x on y are available as $4x - 5y + 33 = 0$ and $20x - 9y - 107 = 0$ respectively. Calculate the mean values of x, y and the coefficient of correlation between x and y.</td><td>4</td><td>L3</td><td>3</td></tr>
<tr><td>2.a</td><td>Ten people of various heights were requested to read letters on a car at 25 yards distance. The number of letters correctly read is as given below:
<table>
<tr><td>Height (in feet)</td><td>5.1</td><td>5.3</td><td>5.6</td><td>5.7</td><td>5.8</td><td>5.9</td></tr>
<tr><td>No. of letters</td><td>11</td><td>17</td><td>19</td><td>14</td><td>8</td><td>15</td></tr>
</table>
Is there any correlation between heights and visual power?</td><td>6</td><td>L2</td><td>2</td></tr>
<tr><td>2.b</td><td>For two cities Kolkata and Mumbai, prices of commodities are given below:
<table>
<tr><td>City</td><td>Kolkata</td><td>Mumbai</td></tr>
<tr><td>Average Price</td><td>65</td><td>67</td></tr>
<tr><td>Standard Deviation</td><td>2.5</td><td>3.5</td></tr>
</table>
Correlation co-efficient between the prices of commodities in the two cities is 0.8, then find the most likely price in Mumbai corresponding to the price of ₹.70 at Kolkata.</td><td>4</td><td>L2</td><td>3</td></tr>
</table>

3 The velocity V of a liquid is known to vary with temperature according to a quadratic law $V = a + bT + cT^2$. Find the best values of a, b and c for the following observations: 10 L3 3

T	1	2	3	4	5	6	7
V	2.31	2.01	3.80	1.66	1.55	1.47	1.41

Calculate V when T = 9.

4.a The data from an experiment is given below. The variables y and x are connected by the relation $y = ax^b$, where a and b being constants. Fit this equation to the data by finding the values of a and b: 6 L2 2

x	350	400	500	600
y	61	26	7	26

4.b Evaluate the double integral 4 L2 2

$$\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{1}{1+x^2+y^2}\,dydx.$$

5 Let D be a region bounded by $x = y^2, y = x - 2$ 10 L3 4

i) Sketch the region D in the $xy -$ plane.

ii) Evaluate the double integral $\iint_D y\, dA$.

Marks Distribution	Particulars		CO1	CO2	CO3	CO4	L1	L2	L3	L4	L5	L6
	Quiz & Test	Max Marks	12	20	18	10	4	32	24	--	-	-

BT-Blooms Taxonomy, CO-Course Outcomes, M-Marks

***********ALL THE BEST***********

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RV COLLEGE OF ENGINEERING®
(An Autonomous Institution Affiliated to VTU)
I Semester B. E. Examinations May-2023
(Common to AI & ML, BT, CS, CY, CD and IS)
FUNDAMENTALS OF LINEAR ALGEBRA, CALCULUS AND STATISTICS

Time: 03 Hours

Maximum Marks: 100

Instructions to candidates:

1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2 is compulsory. Answer any one full question from 3 and 4, 5 and 6, 7 and 8, 9 and 10.
3. Use of mathematics Handbook is permitted. Do not write anything on handbook.

PART-A

1.1	The reduced system of set of linear equations is $\begin{bmatrix}1 & 2 & 1\\ 0 & 1 & -1\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}2\\ 3\\ 0\end{bmatrix}$. Then the solution for the systems is __________.	01
1.2	If two characteristic roots of a singular matrix A of order 3 are 4, 5 then the third characteristic root is __________.	01
1.3	The circle $x^2+y^2-2ax=0$ in polar form is __________.	01
1.4	The coefficient of $\left(x-\frac{\pi}{4}\right)$ in the Taylor's series expansion of $\sin x$ is __________.	01
1.5	The curvature of the curve $y=e^x$ at the point where it crosses the y-axis is __________.	02
1.6	The matrices taken for the computation are $A=\begin{bmatrix}2 & 2\\ 3 & 0\\ -4 & 1\end{bmatrix}$ and $B=\begin{bmatrix}1 & 1\\ 2 & 3\\ 1 & 6\end{bmatrix}$, then the rank of the matrix $A-B$.	02
1.7	If the temperature of a thin wire of finite length is $u=e^{-c^2p^2t}[a\cos(px)+b\,sin(px)]$, where a, b, p and c are constants, then $u_{xx}=$ __________.	02
1.8	For the implicit function $e^x-e^y=2xy, \frac{dy}{dx}$ using partial differentiation is __________.	02
1.9	Evaluate the integral $\int_0^{\frac{\pi}{2}}\int_0^2 r^2 sin\theta\, dr d\theta$.	02
1.10	Sketch the domain of integral $\int_0^1\int_x^{2-x}\frac{x}{y}dydx$	02
1.11	If the first three moments of a distribution about the value 2 of the variable are 3, 16 and -20, then mean and variance of the distribution is __________.	02
1.12	In a partially destroyed laboratory record of an analysis of a correlation data, the following results were noted: variance of $x=9$, equations of lines of regression of y on x is $4x-5y+33=0$ and x on y is $20x-9y=107$. For the given data the value of correlation coefficient is __________.	02

PART-B

2	a	Show that the equations $-2x + y + z = l$ $x - 2y + z = m$ $x + y - 2z = n$ have a solution only if $l + m + n = 0$. Find all possible solutions when $l = 1, m = 1, n = -2$.	05
	b	Apply Gauss-Seidel iterative method, to solve the system of equations: $x + y + 54z = 110$ $27x + 6y - z = 85$ $6x + 15y + 2z = 72$ Carry out three iterations using initial solution as $(0, 0, 0)$.	05
	c	Find the dominant eigenvalue and the corresponding eigenvector of the matrix $\begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$ by Rayleigh power method taking the initial vector as $[1\ 0\ 0]^T$. Perform four iterations.	06
3	a	Find the angle of intersection of the curves $r = a\cos\theta$ and $2r = a$.	08
	b	If ρ_1 and ρ_2 be the radii of curvature at the extremities of two conjugate semi-diameters of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, prove that $(\rho_1^{\frac{2}{3}} + \rho_2^{\frac{2}{3}})(ab)^{2/3} = a^2 + b^2$.	08
		OR	
4	a	Find the curvature and the circle of curvature of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at the point $\left(\frac{a}{4}, \frac{a}{4}\right)$.	08
	b	Obtain the Maclaurin's series expansion of $\log_e(1 + e^x)$ up to the term containing x^4 and hence deduce the series expansion of $\frac{e^x}{1+e^x}$.	08
5	a	i) If $u = x^2 + y^2$, where $x = at^2, y = 2at$, show that $\frac{du}{dt} = 4a^2t(t^2 + 2)$ using partial derivatives. ii) If $u = f\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$, find the value of $x^2u_x + y^2u_y + z^2u_z$.	08
	b	Find the volume of the greatest rectangular parallelepiped that can be inscribed in the ellipsoid, $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, using Lagrange's method of undetermined multipliers.	08
		OR	
6	a	If u *and* v are the real and imaginary parts of a complex function $f(z) = u + iv$; where $u = e^{r\cos\theta}\cos(r\sin\theta), v = e^{r\cos\theta}\sin(r\sin\theta)$ then prove that u and v satisfy the Cauchy-Reimann equations $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$ and $\frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$.	0
	b	Prove that the functions $u = x + y + z,\ u = x^2 + y^2 + z^2, w = xy + yz + zx$ are functionally dependent using the concept of Jacobians and hence find the relation between them.	0
7	a	Evaluate $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy\,dy\,dx$ by changing the order of integration. Represent the region of integration graphically.	
	b	Compute the volume of the tetrahedron formed by the planes $x = 0, y = 0, z = 0$ and $6x + 4y + 3z = 12$ using triple integration.	

OR

a A plate is in the form of a positive quadrant of the circle $x^2 + y^2 = 1$, the thickness ρ at any point is constant. Find the co-ordinates of the centre of gravity of the plate. 08

b Find the area bounded by the cardioid $r = a(1 + cos\theta)$ above the initial line using double integration. Represent the area graphically. 08

a The growth of bacteria (y) in a community after x –hours is given by the following table.

Hours (x)	0	1	2	3	4	5
Number of bacteria (y)	32	47	65	92	132	190

Find the best value of a and b in the formula $y = ab^x$ to fit this data and estimate the number of bacteria y at $x = 6$ hours by the method of least squares. 08

b Physiological tests of intelligence and of engineering ability were applied to 10 students. Here is a record of ungrouped data showing intelligence ratio ($I.R$) and engineering ability ($E.R$). Calculate the coefficient of correlation between intelligence ratio ($I.R$) and engineering ability ($E.R$). Also find the regression line of intelligence ratio ($I.R$) on engineering ability ($E.R$) and engineering ability ($E.R$) on intelligence ratio ($I.R$).

Student	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*
I.R	105	104	102	101	100	99	98	96	93	92
E.R	101	103	100	98	95	96	104	92	97	94

08

OR

a If the velocity V (km/hr) and Resistance R (kg/tonne) are related by a relation of the form $R = a + bV^2$, find a and b by the method of least squares with the use of the following data.

V	10	20	30	40	50
R	8	10	15	21	30

Compute the value of R when $V = 35$. 08

b The following table gives the distribution of marks in Mathematics of 50 students in an examination. Compute $\mu_1, \mu_2, \mu_3, \mu_4$ for the following distribution. Also find β_1 and β_2.

Marks	0-10	10-20	20-30	30-40	40-50	50-60
Number of students	1	6	10	15	11	7

08

22MA11C

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RV COLLEGE OF ENGINEERING®
(An Autonomous Institution Affiliated to VTU)
I Semester B. E. Supplementary Examinations Oct-2023
(Common to AI & ML, BT, CS, CY, CD and IS)
FUNDAMENTALS OF LINEAR ALGEBRA, CALCULUS AND STATISTICS

Time: 03 Hours *Maximum Marks: 100*

Instructions to candidates:

1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2 is compulsory. Answer any one full question from 3 and 4, 5 and 6, 7 and 8, 9 and 10.
3. Use of mathematics Handbook is permitted. Do not write anything on handbook.

PART-A

1	1.1	The Gauss elimination method reduces the system $[A:B]=\begin{bmatrix}3 & -1: & 4\\ 2 & 5: & -3\end{bmatrix}$ to _____________.	01
	1.2	If two latent roots of a singular matrix A of order 3 are $2, 5$ then the third latent root is _____________.	01
	1.3	The point $P=\left(3,\frac{3\pi}{2}\right)$ is located on a polar curve $r=f(\theta)$, the corresponding cartesian coordinates (x,y) is _____________.	01
	1.4	For the curve $\frac{dx}{d\theta}=-3\sin 3\theta$, $\frac{dy}{d\theta}=3\cos 3\theta$, $\frac{d^2x}{d\theta^2}=-9\cos 3\theta$ and $\frac{d^2y}{d\theta^2}=-9\sin 3\theta$, the radius of curvature is _____________.	01
	1.5	For a period of time a plane is flying along the curve $3x-2y=5$, the curvature of the curve is _____________.	01
	1.6	The coefficient of $\left(x-\frac{\pi}{3}\right)$ in the Taylor's series expansion of $\cos x$ is _______.	01
	1.7	The matrices taken for the computation are $A=\begin{bmatrix}1 & 2\\ -1 & 0\\ -4 & 1\end{bmatrix}$ and $B=\begin{bmatrix}0 & 1\\ 2 & 3\\ 1 & -10\end{bmatrix}$, then the rank of the matrix $A+B$ is _____________.	02
	1.8	If the temperature of the circular plate is $\theta=t^n e^{-\left(\frac{r^2}{4t}\right)}$, then $\frac{\partial\theta}{\partial t}=$ _____________.	02
	1.9	For the function $u(x,y)=y\cos x-(x^2+y^2)=0$, $\frac{dy}{dx}$ using partial differentiation is _____________.	02
	1.10	The value of $\int_0^{\pi/2}\int_0^1 r\sin\theta\, dr\, d\theta$ is _______.	02
	1.11	Sketch the domain of integral $\int_0^1\int_{\sqrt{y}}^{2-y} x\, dx\, dy$.	02
	1.12	If the first four moments of a distribution about the value 3 of the variable are $2, 5, 23$ and 159, then the fourth moment about the mean is _____________.	02
	1.13	In a partially destroyed laboratory record of an analysis of a correlation data, the following results were noted: variance of $x=9$, equations of lines of regression of y on x is $4x-5y+33=0$ and x on y is $20x-9y=107$. For the given data the value of correlation coefficient is _____________ and $\bar{x}=$ _____________.	02

PART-B

2	a	Determine the values of μ for which the system $u + v + w = 1$ $2u + v + 4w = \mu$ $4u + v + 10w = \mu^2$ has a solution. Solve the system for each possible cases.	06
	b	Apply Gauss-Seidel iterative method, to solve the system of equations: $8x + y + z = 8$ $x + 3y + 5z = 5$ $2x + 4y + z = 4$ Carry out three iterations using initial solution as $(0, 0, 0)$.	05
	c	Find the dominant eigenvalue and the corresponding eigenvector of the matrix $\begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$ by Rayleigh power method taking the initial vector as $[1\,0\,0]^T$. Perform four iterations.	05
3	a	Prove that the angle of intersection of the curves $r = a\log_e\theta$ and $r = a/\log_e\theta$ is given by $\tan^{-1}\left(\frac{2e}{1-e^2}\right)$.	08
	b	Find the radius of curvature of the curve $y^2 = \frac{4a^2(2a-x)}{x}$ at the point where the curve meets the x-axis.	08
		OR	
4	a	Find the curvature and the circle of curvature of the cissoid $y^2(2-x) = x^3$ at the point $(1, 1)$.	08
	b	Obtain the Maclaurin's series expansion of $\log_e(\sec x)$ up to the term containing x^6 and hence deduce the series expansion of $\tan x$.	08
5	a	i) If $u = \tan^{-1}\left(\frac{y}{x}\right)$, where $x = e^t - e^{-t}, y = e^t + e^{-t}$, show that $\frac{du}{dt} = -\frac{2}{e^{2t}+e^{-2t}}$ using partial derivatives. ii) Find $u\frac{\partial f}{\partial u} + v\frac{\partial f}{\partial v} + w\frac{\partial f}{\partial w}$ if $x = u + v + w,\ y = uv + vw + wu,\ z = uvw$ and $f = f(x, y, z)$.	08
	b	The temperature T at any point (x, y, z) in space is $T = 400xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$ using Lagrange's method of undetermined multipliers.	08
		OR	
	a	If $u = \frac{1}{r}\{\psi(r - at) + \phi(r + at)\}$, prove that $\frac{\partial^2 u}{\partial t^2} = \frac{a^2}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right)$.	08
	b	If $x = a^u\cos v$ and $y = a^u\sin v$, then find $J' = \frac{\partial(x,y)}{\partial(u,v)}$ and $J = \frac{\partial(u,v)}{\partial(x,y)}$. Also show that $JJ' = 1$.	08
	a	Change the order of integration and hence evaluate $\int_0^1\int_x^{\sqrt{2-x^2}}\frac{x}{\sqrt{x^2+y^2}}\,dy\,dx$. Represent the region of integration graphically.	08
	b	Compute the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ using triple integration.	08
		OR	
	a	A plate is in the form of a quadrant of the circle $x^2 + y^2 = 1$, but of varying thickness at any point given by $\rho = kxy$, where k is a constant. Find the co-ordinates of the centre of gravity of the plate.	08
	b	Find the area lying inside the cardioid $r = a(1 + \cos\theta)$ and outside the circle $r = a$ using double integration. Represent the region of integration graphically.	08

9 a The following pair of observations was noted in an experimental work on cosmic rays. Find by the method least squares the best values of a and b for the equation $y = ax^b$ which fits the data.

x	2	3	4	5	6
y	8.3	15.4	33.1	65.2	127.4

Construct the table of values and compute the value of y when $x = 7$. 08

b The experimental values relating centripetal force and radius for a mass travelling at constant velocity in a circle are as shown:

$Radius\ (x)$	55	30	15	12	11	9
$Force\ (y)$	5	10	15	20	25	30

Calculate the coefficient of correlation between radius and force. Determine the equations of the regression line of force on radius and regression line of radius on force. 08

OR

10 a The distance (in km) of 60 engineers from their residence to their place of work were found as follows:

$Distance\ (in\ km)$	$0-5$	$5-10$	$10-15$	$15-20$	$20-25$	$25-30$
$No. of\ engineers$	8	11	15	12	9	5

Compute the first four moments about the mean and also find the measures β_1 and β_2 for the above distribution and comment on the nature of the distribution. 08

b The velocity V of a liquid is known to vary with temperature according to a quadratic law $V = a + bt + cT^2$. Find the best values of $a, b\ and\ c$ by least squares method for the following data:

T	1	1.5	2	2.5	3	3.5	4
V	1.1	1.3	1.6	2.0	2.7	3.4	4.1

Compute the value of V when $T = 5$. 08