TRIGONOMETRY

1. Basic Functions

- $sin\ \theta = \frac{Opposite\ Side}{Hypotenuse}$
- $cos\ \theta = \frac{Adjacent\ Side}{Hypotenuse}$
- $tan\ \theta = \frac{Opposite\ Side}{Adjacent\ Side} = \frac{\sin\theta}{\cos\theta}$
- $sec\ \theta = \frac{Hypotenuse}{Adjacent\ Side} = \frac{1}{\cos\theta}$
- $cosec\ \theta = \frac{Hypotenuse}{Opposite\ Side} = \frac{1}{\sin\theta}$
- $cot\ \theta = \frac{Adjacent\ Side}{Opposite\ Side} = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$

2. Identities

- $\sin(-x) = -\sin x$
- $\cos(-x) = \cos x$
- $\tan(-x) = -\tan x$
- $\sin\left(\frac{\pi}{2} - x\right) = \cos x$
- $\cos\left(\frac{\pi}{2} - x\right) = \sin x$
- $\tan\left(\frac{\pi}{2} - x\right) = \cot x$
- $\sin\left(\frac{\pi}{2} + x\right) = \cos x$
- $\cos\left(\frac{\pi}{2} + x\right) = -\sin x$
- $\tan\left(\frac{\pi}{2} + x\right) = -\cot x$
- $\sin(\pi - x) = \sin x$
- $\cos(\pi - x) = -\cos x$
- $\tan(\pi - x) = -\tan x$
- $\sin(\pi + x) = -\sin x$
- $\cos(\pi + x) = -\cos x$
- $\tan(\pi + x) = \tan x$
- $\sin\left(\frac{3\pi}{2} - x\right) = -\cos x$
- $\cos\left(\frac{3\pi}{2} - x\right) = -\sin x$
- $\tan\left(\frac{3\pi}{2} - x\right) = \cot x$
- $\sin\left(\frac{3\pi}{2} + x\right) = -\cos x$
- $\cos\left(\frac{3\pi}{2} + x\right) = \sin x$
- $\tan\left(\frac{3\pi}{2} + x\right) = -\cot x$
- $\sin(x + y) = \sin x \cos y + \cos x \sin y$
- $\cos(x + y) = \cos x \cos y - \sin x \sin y$
- $\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$
- $\sin(2x) = 2 \sin x \cos x$
- $\cos(2x) = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x$
  $= 2\cos^2 x - 1$
- $\tan(2x) = \frac{2\tan x}{1 - \tan^2 x}$
- $\cos^2 x + \sin^2 x = 1$
- $\sec^2 x - \tan^2 x = 1$
- $\mathrm{cosec}^2\, x - \cot^2 x = 1$
- $\sin 3x = 3\sin x - 4\sin^3 x$
- $\cos 3x = 4\cos^3 x - 3\cos x$
- $\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$
- $\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$
- $\sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$
- $\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$

2. Rules of differentiation
   - $\frac{d}{dx}(fg) = gf' + fg'$
   - $\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{gf'-fg'}{g^2}$
   - $\frac{d}{dx}\left(f(t)\right) = \frac{d}{dt}\left(f(t)\right)\frac{dt}{dx}$

3. Integration

$f(x)$	$\int f(x)dx$	$f(x)$	$\int f(x)dx$
x^n	$\frac{x^{n+1}}{n+1}$	$\frac{1}{x}$	$\log_e x$
e^{ax}	$\frac{e^{ax}}{a}$	$\log_e x$	$x(\log_e x - 1)$
a^x	$\frac{a^x}{\log_e a}$	$\operatorname{cosec} x$	$\log_e(\operatorname{cosec} x - \cot x)$
$\sin x$	$-\cos x$	$\sec x$	$\log_e(\sec x + \tan x)$
$\cos x$	$\sin x$	$\cot x$	$\log_e \sin x$
$\tan x$	$\log_e \sec x$	$\sec^2 x$	$\tan x$
$\sinh x$	$\cosh x$	$\operatorname{cosec}^2 x$	$-\cot x$
$\cosh x$	$\sinh x$	$\tanh x$	$\log_e \cosh x$
$\frac{1}{\sqrt{a^2-x^2}}$	$sin^{-1}\left(\frac{x}{a}\right)$	$\frac{1}{\sqrt{a^2+x^2}}$	$sinh^{-1}\left(\frac{x}{a}\right)$
$\frac{1}{a^2+x^2}$	$\frac{1}{a}tan^{-1}\left(\frac{x}{a}\right)$	$\frac{1}{\sqrt{x^2-a^2}}$	$cosh^{-1}\left(\frac{x}{a}\right)$
$\frac{1}{a^2-x^2}$	$\frac{1}{2a}\log_e\left(\frac{a+x}{a-x}\right)$	$\frac{1}{x^2-a^2}$	$\frac{1}{2a}\log_e\left(\frac{x-a}{x+a}\right)$
$\sqrt{a^2-x^2}$	$\frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 sin^{-1}\left(\frac{x}{a}\right)\right]$	$e^{ax}\sin bx$	$\frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$
$u(x)v(x)$	$u\int v\,dx - \int\left[\frac{du}{dx}\left[\int v\,dx\right]dx\right]$	$e^{ax}\cos bx$	$\frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$

3. Total differential: Let $z = f(x, y)$ be a differentiable function of two variables, x and y then total differential (or exact differential) is defined by $dz = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$.
4. Total derivative: Further, if $z = f(x, y)$, where $x = x(t), y = y(t)$, then total derivative of z is given by $\frac{dz}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$.
5. Differentiation of implicit functions: For $f(x, y) = 0$, $\frac{dy}{dx} = -\frac{\left(\frac{\partial f}{\partial x}\right)}{\frac{\partial f}{\partial y}}$.
6. Differentiation of composite functions (chain rule):
   Let z be function of x and y and that $x = \varphi(u, v)$ and $y = \phi(u, v)$ are functions of u and v then, $\frac{\partial z}{\partial u} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial u}$ and $\frac{\partial z}{\partial v} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial v}$.
7. Jacobian: If u and v are functions of variables x and y, then the determinant $\begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix}$ is called the Jacobian of u, v with respect to x, y and denoted by $\frac{\partial(u,v)}{\partial(x,y)}$.
8. If u, v are functions of r, s and r, s are functions of x, y, then $\frac{\partial(u,v)}{\partial(x,y)} = \frac{\partial(u,v)}{\partial(r,s)}\frac{\partial(r,s)}{\partial(x,y)}$.

MULTIPLE INTEGRAL

1. Area of a region $R = \iint_R dA$
2. Volume of a Solid $S = \iiint_S dx\, dy\, dz$
3. Change of variables: From Cartesian xy plane to
   - uv-plane $\iint_{R_{xy}} f(x, y)\, dx\, dy = \iint_{R_{uv}} f\big(\phi(u, v), \psi(u, v)\big)\, |J|\, du\, dv$
   - polar coordinates $\iint_{R_{xy}} f(x, y)\, dx\, dy = \iint_{R_{r\theta}} f(rcos\theta, rsin\theta)\, r\, dr\, d\theta$
4. Mass of two-dimenssional object with surface density $f(x, y)$: $M = \iint_R f(x, y)dxdy$
5. The center of gravity: $\overline{x} = \frac{1}{M}\iint_R xf(x, y)dxdy$ and $\overline{y} = \frac{1}{M}\iint_R yf(x, y)dxdy$
6. Mass of a solid S, with density $f(x, y, z)$: $M = \iiint_S f(x, y, z)dxdydz$
7. The center of gravity: $\overline{x} = \frac{1}{M}\iiint_S xf(x, y, z)dxdydz$, $\overline{y} = \frac{1}{M}\iiint_S yf(x, y, z)dxdydz$ and $\overline{z} = \frac{1}{M}\iiint_S zf(x, y, z)dxdydz$

5. **Cauchy-Euler equation:** The linear ODE of the form $(a_0x^nD^n + a_1x^{n-1}D^{n-1} + a_2x^{n-2}D^{n-2} + \cdots + a_{n-1}xD + a_n)y = g(x)$, where $a_0, a_1, \cdots a_n$are constants, is known as 'Cauchy-Euler' or equidimensional equation.

   This equation can be reduced to ODE with constant coefficients by changing the independent variable as follows –

   Take $x = e^z$, then $xDy = D_1y$,

   $$x^2D^2y = D_1(D_1 - 1)y\ ,$$

   $$x^3D^3y = D_1(D_1 - 1)(D_1 - 2)y$$

   where $D_1 = \frac{d}{dz}$

6. **Wronskian:** For two functions $y_1(x)$ and $y_2(x)$, the Wronkian is defined by $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$

7. **Method of Variation of Parameters:**

   For the second order ODE of the form $y'' + P(x)y' + Q(x)y = g(x)$. Let $y = c_1y_1 + c_2y_2$ be solution of the equation with $g(x) = 0$, the general solution is given by

   $y = A(x)y_1 + B(x)y_2$, where $A(x) = -\int \frac{y_2 g(x)}{W} dx + c_1$ and $B(x) = \int \frac{y_1 g(x)}{W} dx + c_2$, *and* $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \neq 0$

PARTIAL DIFFERENTIAL EQUATIONS

1. **Lagrange's linear equation:** The first order linear partial differential equation of the form $P\frac{\partial u}{\partial x} + Q\frac{\partial u}{\partial y} = R$, where P, Q and R are functions of x, y, z is known as Lagrange's Linear equation.
2. **Subsidiary/Auxiliary Equation:** The equation $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$ is known as the subsidiary/ auxiliary equation of as Lagrange's Linear equation $P\frac{\partial u}{\partial x} + Q\frac{\partial u}{\partial y} = R$.
3. **One-Dimensional Wave Equation:** $\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$, where $c^2 = \frac{T}{\rho}$ the phase speed, T is the tension, and ρ density of the string.
4. **One-Dimensional Heat Equation:** $\frac{\partial u}{\partial t} = c^2\frac{\partial^2 u}{\partial x^2}$, where $c^2 = \frac{\kappa}{s\rho}$ the thermal diffusivity, κ thermal conductivity, s specific heat and ρ density of the material of the body.
5. **Two-Dimensional Laplace equation:** $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

- Let $f(t)$ be a periodic function of period T then $L\{f(t)\} = \frac{1}{1-e^{-ST}}\int_0^T e^{-st} f(t)dt.$
- If $L\{f(t)\} = F(s)$, then $L\,[f(t-a)\,H(t-a)] = e^{-as}\,F(s)$
- $f(t)$ be a continuous function at $t = a$, then $\int_0^\infty f(t)\delta(t-a)dt = f(a)$, where $\delta(t-a)$ is unit impulse function.

6. Inverse Laplace transform of $F(s)$ using Convolution theorem: If $L^{-1}[F(s)] = f(t)$ and $L^{-1}[G(s)] = g(t)$, then $L^{-1}[F(s)G(s)] = \int_0^t f(u)g(t-u)du = f(t) * g(t)$.

NUMBER THEORY

1. The number of all positive divisors of a, denoted by $T(a)$, where $a = p_1^{a_1} p_2^{a_2} \cdots p_n^{a_n}$
$$T(a) = (1+a_1)(1+a_2)\cdots(1+a_n)$$
2. The sum of all positive divisors of a, denoted by $S(a)$,
$$S(a) = \left(\frac{p_1^{a_1+1}-1}{p_1-1}\right)\left(\frac{p_2^{a_2+1}-1}{p_2-1}\right)\cdots\left(\frac{p_n^{a_n+1}-1}{p_n-1}\right)$$
3. Euler's theorem: if $(a, m) = 1$, then $a^{\phi(m)} \equiv 1\ (mod\ m)$.
4. If p is a prime number, then $\phi(p) = p - 1$
5. If p is a prime number and $k > 0$, then $\phi(p^k) = p^k - p^{k-1}$
6. If the integer $n > 1$ has the prime factorization, $n = p_1^{k_1} \times p_2^{k_2} \times \cdots \times p_r^{k_r}$, then
$$\phi(n) = n\left(1-\frac{1}{p_1}\right)\left(1-\frac{1}{p_2}\right)\cdots\left(1-\frac{1}{p_r}\right)$$
7. Cipher text: $c = m^e (mod\ n)$, where m is the message.
8. Decryption: $m = c^d (mod\ n)$, where d is the private key.

STATISTICS

1. Moments for ungrouped data:
   - The r^{th} moment about origin: $\mu_r' = \frac{1}{n}\sum_{i=1}^n x_i^r$, where $r = 1, 2, 3 \ldots$, $x_1, x_2 \cdots x_n$ are n observations
   - The r^{th} central moment: $\mu_r = \frac{1}{n}\sum_{i=1}^n (x_i - \bar{x})^r$, where $r = 1, 2, 3 \cdots$, $\bar{x}$ is mean
2. Moments for grouped data:
   - The r^{th} moment about origin: $\mu_r' = \frac{1}{N}\sum_{i=1}^n f_i x_i^r$, $r = 1, 2, 3 \cdots$, where observations $x_1, x_2, \ldots, x_n$ are the mid points of the class-intervals and $f_1, f_2, \ldots, f_n$ are their corresponding frequencies and $N = \sum_{i=1}^n f_i$
   - The r^{th} central moment: $\mu_r = \frac{1}{N}\sum_{i=1}^n f_i (x_i - \bar{x})^r$, $r = 1, 2, 3 \cdots$