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Department of Mathematics

NUMBER THEORY, VECTOR CALCULUS AND COMPUTATIONAL METHODS (MA221TC)

UNIT 4: VECTOR DIFFERENTIATION

TUTORIAL SHEET-1

1. If $\vec{f}$ is a vector function with constant magnitude then $\vec{f}.\frac{d\vec{f}}{dt} =$ _______

Ans: 0

2. The displacement of a particle moving along a path is given by $x = (1 - t^3)$, $y = (1 + t^2)$, $z = (2t - 5)$ the magnitude of velocity vector at $t = 1$ second is _____

Ans: $\sqrt{17}$

3. For the curves whose equations are given below, find the unit tangent vectors:

   a) $x = t^2 + 1, y = 4t - 3, z = 2(t^2 - 3t)$ at $t = 0$

   b) $\vec{r} = (a\,cos\,3\,t)\,\hat{\imath} + (a\,sin\,3\,t)\,\hat{\jmath} + (4at)\,\hat{k}$ at $t = \frac{\pi}{4}$

Ans: (i) $\hat{\boldsymbol{t}} = \frac{(2\hat{\jmath} - 3\hat{k})}{\sqrt{13}}$ **(ii)** $\hat{\boldsymbol{t}} = \frac{1}{5\sqrt{2}}\left[-3\hat{\imath} - 3\hat{\jmath} + 4\sqrt{2}\hat{k}\right]$

4. A particle moves along the curve $\vec{r} = 2t^2\,\hat{\imath} + (t^2 - 4t)\,\hat{\jmath} + (3t - 5)\,\hat{k}$. Find the component of velocity and acceleration in the direction of vector$c = i - 3j + 2k$ at $t = 1$.

Ans: $\frac{16}{\sqrt{14}}$ **&** $\frac{-2}{\sqrt{14}}$

5. A person on a hang glider is spiralling upward due to rapidly rising air on a path having position vector $r(t) = 3\cos(t)\,\hat{\imath} + 3\sin(t)\,\hat{\jmath} + t^2\,\hat{k}$. Find (a) the velocity and acceleration vectors (b) the glider's speed at any time *t.*

Ans: $\boldsymbol{v} = \mathbf{3sin}\,(\boldsymbol{t})\,\hat{\boldsymbol{\imath}} + \mathbf{3\,cos}(\boldsymbol{t})\,\hat{\boldsymbol{\jmath}} + \mathbf{2}\boldsymbol{t}\hat{\boldsymbol{k}}$**;**

$\boldsymbol{a} = -\mathbf{3\,cos}\,\boldsymbol{t}\,\hat{\boldsymbol{\imath}} + -\mathbf{3\,sin}\,\boldsymbol{t}\,\hat{\boldsymbol{\jmath}} + \mathbf{2}\hat{\boldsymbol{k}}$;

$|\boldsymbol{v}| = \sqrt{\mathbf{9} + \mathbf{4}\boldsymbol{t}^{\mathbf{2}}}$

TUTORIAL SHEET-2

1. If $\phi(x, y, z) = xy^2z^3 - x^3y^2z$, then $|\nabla\phi|$ at $(1, -1, 1)$ is ____________.
Ans: $\mathbf{2\sqrt{2}}$

2. The maximum directional derivative of $\phi(x, y, z) = x^2y + yz^2 - xz^3$ at $(-1,2,1)$ is ____.
Ans: $\mathbf{\sqrt{78}}$

3. If $\phi(x, y, z) = x^2 + \sin y + z$ then $|\nabla\phi|$ at $\left(0, \frac{\pi}{2}, 1\right)$ is ____________.
Ans: $\boldsymbol{\hat{k}}$

4. Find the unit normal vector to the surface $\phi(x, y, z) = x^2y + y^2z + xz^2 - 5$ at the point $(1, -1, 2)$.
Ans: $\frac{\mathbf{1}}{\sqrt{\mathbf{38}}}(\mathbf{2}\boldsymbol{\hat{\imath}} - \mathbf{3}\boldsymbol{\hat{\jmath}} + \mathbf{5}\boldsymbol{\hat{k}})$

5. Find the constants a and b so that the surface $3x^2 - 2y^2 - 3z^2 + 8 = 0$ is orthogonal to the surface $ax^2 + y^2 = bz$ at the point $(-1, 2, 1)$.
Ans: $\boldsymbol{a} = \frac{\mathbf{4}}{\mathbf{9}}$, $\boldsymbol{b} = \frac{\mathbf{40}}{\mathbf{9}}$

7. Find the directional derivative of $\phi(x, y, z) = xyz - xy^2z^3$ at $(1, 2, -1)$ in the direction of $\hat{\imath} - \hat{\jmath} - 3\hat{k}$.
Ans: $\frac{\mathbf{29}}{\sqrt{\mathbf{11}}}$

TUTORIAL SHEET-3

1. If $\vec{f} = 3x^2\hat{\imath} + 5xy^2\hat{\jmath} + xyz^3\hat{k}$ then find $div\,\vec{f}$ at $(1,2,3)$ is ______.
Ans: 80

2. If $\vec{f} = (y^2 + z^2 - x^2)\hat{\imath} + (z^2 + x^2 - y^2)\hat{\jmath} + (x^2 + y^2 - z^2)\hat{k}$ then find $div\,\vec{f}$ and $curl\,\vec{f}$
Ans: $\boldsymbol{div\,\vec{f} = -2(x + y + z)}$ $\boldsymbol{curl\,\vec{f} = 2[(y - z)\hat{\imath} + (z - x)\hat{\jmath} + (x - y)\hat{k}]}$

3. Show that the vector field $\vec{f} = (x + 3y)\hat{\imath} + (y - 3z)\hat{\jmath} + (x - 2z)\hat{k}$ is solenoidal.

4. Determine the constant a such that the vector field $\vec{f} = 3x\hat{\imath} + (x + y)\hat{\jmath} - az\hat{k}$ is solenoidal.
Ans: 4

5. If $\vec{f} = x^2\hat{\imath} + y^2\hat{\jmath} + z^2\hat{k}$ and $\vec{g} = yz\hat{\imath} + xz\hat{\jmath} + xy\hat{k}$ then show that $\vec{f} \times \vec{g}$ is solenoidal.

6. If $\vec{f} = (2x + 3y + az)\hat{\imath} + (bx + 2y + 3z)\hat{\jmath} + (2x + cy + 3z)\hat{k}$ is irrotational vector field, then find the constants a, b, c.
Ans: $\boldsymbol{a = 2, b = 3, c = 3}$

7. If $\phi = x^2y + 2xy + z^2$ then show that $|\nabla\phi|$ is irrotational.

8. If $\phi = x^2 - y^2$ then show that ϕ satisfies the Laplacian equation.

9. If $\phi = 2x^2yz^3$ then find $\nabla^2\phi$ at $(1,1,1)$.
Ans: 1

10. If $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and $r = |\vec{r}|$ then show that $r^n\vec{r}$ is irrotational for all values of n and solenoidal for $n = -3$.

11. Show that $\vec{f} = (6xy + z^3)\hat{\imath} + (3x^2 - z)\hat{\jmath} + (3xz^2 - y)\hat{k}$ is irrotational. Find the function ϕ such that $\vec{f} = \nabla\phi$.
Ans: $\boldsymbol{\phi = 3x^2y + xz^3 - yz}$.