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**Department of Mathematics**

**NUMBER THEORY, VECTOR CALCULUS AND COMPUTATIONAL METHODS (MA221TC)**

**UNIT 4: LINEAR ORDINARY DIFFERENTIAL EQUATIONS OF HIGHER ORDER**

**TUTORIAL SHEET – 1**

1. Solve the following differential equations.
   a. $2y'' - 5y' - 3y = 0$
   b. $(D^2 - 10D + 25)y = 0$
   c. $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 7y = 0$
   d. $y''' + 3y'' - 4y = 0$
   e. $\frac{d^4y}{dx^4} + 2\frac{d^2y}{dx^2} + y = 0$
   f. $3y''' + 5y'' + 10y' - 4y = 0$
   g. $\frac{d^3u}{dt^3} + \frac{d^2u}{dt^2} - 2u = 0$
   h. $y'' + 16y = 0, y(0) = 2, y'(0) = -2$
   i. $y'' - 2y' + y = 0,\ y(0) = 0,\ y(1) = 3$
   j. $y'' + y = 0,\ y'(0) = 0,\ y'\left(\frac{\pi}{2}\right) = 0$
   k. $y'' + 2ky' + (k^2 + w^2)y = 0,\ y(0) = 1,\ y'(0) = -k$
   l. $y''' - 4y' = 0,\ y'(0) = y(0) = y''(0) = 1$

2. The roots of the cubic auxiliary equation are $m_1 = 4,\ m_2 = m_3 = -5$, what is the corresponding homogeneous linear differential equation?

3. Show that the solution of the differential equation

   $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 4y = 0$ is $y = e^{-2x}[c_1 \cosh\sqrt{2}\,x + c_2 \sinh\sqrt{2}\,x]$.

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# TUTORIAL SHEET - 2

1. Find the solution of the following differential equations:
   (i) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \cosh x$
   (ii) $\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = (e^{3x} + 3)^2$

2. Solve the following differential equations.
   a. $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = \cos^2 x + 2^x$
   b. $\frac{d^2x}{dt^2} + x = \sin t \sin 2t \sin 3t$
   c. $\frac{d^4y}{dx^4} - y = \cosh(x - 1) + \sin x$
   d. $\frac{d^3y}{dx^3} + y = 65\cos(2x + 1) + e^x$
   e. $(2D^2 + 2D + 3)y = x^2 + 2x - 1$
   f. $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + \frac{dy}{dx} = e^{2x} + x^3$
   g. $y'' - y = x, y(0) = 0, y(1) = 0$
   h. $\frac{d^2y}{dx^2} + 4y = \sin 2x, y(0) = 1, y\left(\frac{1}{2}\right) = 0$

3. Solve the following differential equations:
   (i) $\frac{d^2y}{dt^2} + 4y = t^2e^{3t}$
   (ii) $\frac{d^3y}{dx^3} - 7\frac{dy}{dx} + 6y = e^{2x}(1 + x)$
   (iii) $\frac{d^2y}{dx^2} + 4y = 2e^x \sin^2 x$
   (iv) $\frac{d^2x}{dt^2} - 4x = t^2 \cos t$
   (v) $\frac{d^2u}{dx^2} - 4u = x \sinh x$
   (vi) $\frac{d^2y}{dx^2} - y = xe^x \sin x$

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# TUTORIAL SHEET - 3

1. Find the solution of the following differential equations:

   (i) $2x\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - \frac{y}{x} = 5 - \frac{\sin(\log_e x)}{x}$

   (ii) $x^3\frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} + 2y = 10\left[x + \frac{1}{x}\right]$

   (iii) $(5+2x)^2\frac{d^2y}{dx^2} - 6(5+2x)\frac{dy}{dx} + 8y = 6x$

   (iv) $x^2\frac{d^2y}{dx^2} - 2y + x = 0,\ y(2) = y(3) = 0.$

2. Solve the following differential equations by the method of variation of parameters:

   (i) $y'' + y = \frac{1}{1+\sin x}$

   (ii) $y'' + 2y' + y = 4e^{-x}\log_e x$

   (iii) $\frac{d^2y}{dx^2} - y = e^{-2x}\cos(e^{-x})$

   (iv) $xy'' - y' = (3+x)x^2e^x$

3. In a simple harmonic motion, the amplitude of motion is 5 meters and the period is 4 seconds. Find the time required by the particle in passing between the points which are at distance of 4 meters and 2 meters from the center of the force and are on same side of it. Also find the velocities at these points. (Equation of SHM is $\frac{d^2x}{dt^2} = -\mu^2 x$).

4. Find the current in the RLC circuit assuming zero initial current and charge and R=160 ohm, L=20 Henry, C= 2.10-3 Farad and E = 48.1 sin(10t) volts.

   (The differential equation for the charge Q is $L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{c} = E$)

**Objective Questions:**

1. The order and degree of the differential equation $y'' - (y')^{3/2} = 2$ is ___and ____.
2. The complementary function of the differential equation is $c_1\cos 3x + c_2\sin 3x$, then Wronskian is _________.
3. $\frac{1}{D^2+1}\sin 2t =$_________.
4. Particular integral of $\frac{d^2x}{dt^2} + \frac{1}{b}(x-a) = 0$ is_________.
5. If $x = e^{2t}$ is the solution of the equation $\frac{d^2x}{dt^2} - 5\frac{dx}{dt} + kx = 0$ then value of $k$ is ________.
6. Reduce the differential equation $x^2y'' - xy' + y = \log_e x$ to a linear differential equation with constant coefficient.