BASIC CALCULUS

1. Differentiation

$f(x)$	$f'(x)$	$f(x)$	$f'(x)$
a	0	a^x	$a^x \log_e a$
$x^n, n \neq -1$	nx^{n-1}	e^{ax}	ae^{ax}
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$	$\log_e x$	$\frac{1}{x}$
$\sin x$	$\cos x$	$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos x$	$-\sin x$	$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$
$\tan x$	$\sec^2 x$	$\tan^{-1} x$	$\frac{1}{1+x^2}$
$\operatorname{cosec} x$	$-\cot x \operatorname{cosec} x$	$\operatorname{cosec}^{-1} x$	$-\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$
$\sec x$	$\tan x \sec x$	$\sec^{-1} x$	$\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$
$\cot x$	$-\operatorname{cosec}^2 x$	$\cot^{-1} x$	$-\frac{1}{1+x^2}$
$\sinh x = \frac{e^x - e^{-x}}{2}$	$\cosh x$	$\sinh^{-1} x$	$\frac{1}{\sqrt{x^2+1}}$
$\cosh x = \frac{e^x+e^{-x}}{2}$	$\sinh x$	$\cosh^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$
$\tanh x$	$\operatorname{sech}^2 x$	$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\operatorname{cosech} x$	$-\coth x \operatorname{cosech} x$	$\operatorname{cosech}^{-1} x$	$-\frac{1}{\lvert x\rvert\sqrt{x^2+1}}$
$\operatorname{sech} x$	$-\tanh x \operatorname{sech} x$	$\operatorname{sech}^{-1} x$	$-\frac{1}{\lvert x\rvert\sqrt{1-x^2}}$
$\coth x$	$-\operatorname{cosech}^2 x$	$\coth^{-1} x$	$\frac{1}{1-x^2}$

DIFFERENTIAL CALCULUS

1. Transformations for polar coordinates to Cartesian coordinates: $x = r\cos\theta, y = r\sin\theta$.
2. Transformations for Cartesian coordinates to polar coordinates: $r = \sqrt{x^2+y^2}$, $\theta = \tan^{-1}\left(\frac{y}{x}\right), r \geq 0, 0 \leq \theta \leq 2\pi$.
3. The angle between the radius vector and tangent for a polar curve $r = f(\theta)$: $\tan\phi = r\frac{d\theta}{dr}$
4. The radius of curvature:
   - Cartesian curve $y = f(x)$: $\rho = \frac{[1+(y')^2]^{3/2}}{y''}$
   - Parametric curve $x = x(t), y = y(t)$: $\rho = \frac{[(x')^2+(y')^2]^{3/2}}{x'y''-x''y'}$
   - Polar curve $r = f(\theta)$: $\rho = \frac{[r^2+(r')^2]^{3/2}}{r^2+2(r')^2-rr''}$
5. Centre of curvature: $\bar{x} = x - \frac{y'[1+(y')^2]}{y''}$ and $\bar{y} = y + \frac{[1+(y')^2]}{y''}$
6. Taylor series expansion:
$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$
7. Maclaurin series expansion:
$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots$$

PARTIAL DIFFERENTIATION

1. Let $z = f(x,y)$ be a function of two variables x and y.
   - The first order partial derivative of z with respect to x, denoted by $\frac{\partial z}{\partial x}$ or $\frac{\partial f}{\partial x}$ or z_x or f_x or $\boldsymbol{p}$ is defined as $\frac{\partial z}{\partial x} = \lim_{\delta x \to 0} \frac{f(x+\delta x, y) - f(x,y)}{\delta x}$ provided the limit exists.
   - The first order partial derivative of z with respect to y, denoted by $\frac{\partial z}{\partial y}$ or $\frac{\partial f}{\partial y}$ or z_y or f_y or **q** is defined as $\frac{\partial z}{\partial y} = \lim_{\delta y \to 0} \frac{f(x, y+\delta y) - f(x,y)}{\delta y}$ provided the limit exists.
2. Notations of second order partial derivatives:
   - $\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial^2 z}{\partial x^2}$ or $\frac{\partial^2 f}{\partial x^2}$ or z_{xx} or f_{xx} or **r**
   - $\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \frac{\partial^2 z}{\partial x \partial y}$ or z_{xy} or f_{xy} or **s**
   - $\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) = \frac{\partial^2 z}{\partial y^2}$ or $\frac{\partial^2 f}{\partial y^2}$ or z_{yy} or f_{yy} or **t**
   - $\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial^2 z}{\partial y \partial x}$ or z_{yx} or f_{yx} or **s**

ORDINARY DIFFERENTIAL EQUATIONS

1. **Auxiliary/Characteristic Equation:** The equation $F(m)=0$ is known as the Auxiliary equation of $F(D)y=g(x)$.

2. **Solution of a Homogeneous ODE with constant coefficients:** For the differential equation $(a_0D^2+a_1D+a_n)y=0$, if m_1 and m_2 are the roots of auxiliary equation, then solution is given by following cases

   - If roots are real and distinct, then $y=c_1e^{m_1x}+c_2e^{m_2x}$.
   - If $m_1=m_2$ are real, then $y=c_1e^{m_1x}+c_2xe^{m_1x}$.
   - If roots are complex say $\alpha\pm i\beta$, then $y=e^{\alpha x}(c_1\cos\beta x+c_2\sin\beta x)$.

3. **Non-homogeneous Linear ODE with constant coefficients:** The general solution of $F(D)y=g(x)$ is given by $y=y_c+y_p$, where y_c is the solution of the associated homogeneous equation $F(D)y=0$ and $y_p=\frac{1}{F(D)}g(x)$ is called the particular integral.

4. **Rules for finding particular integral:**

   - If $g(x)=ke^{ax}$, then $y_p=k\frac{1}{F(D)}e^{ax}=k\frac{1}{F(a)}e^{ax}$, provided $F(a)\neq 0$.
     - If $F(a)=0$ then $y_p=k\frac{x}{[F'(D)]_{D=a}}e^{ax}$, provided $F'(a)\neq 0$.
   - If $g(x)=\sin(ax+b)$ *or* $\cos(ax+b)$, then
     - $y_p=\frac{1}{F(D^2)}\sin(ax+b)$ *or* $\frac{1}{F(D^2)}\cos(ax+b)$
       $=\frac{1}{F(-a^2)}\sin(ax+b)$ *or* $\frac{1}{F(-a^2)}\cos(ax+b)$, provided $F(-a^2)\neq 0$
     - If $F(-a^2)=0$, $y_p=\frac{x}{F'(-a^2)}\sin(ax+b)$ *or* $\frac{x}{F'(-a^2)}\cos(ax+b)$, provided $F'(-a^2)\neq 0$
   - If $g(x)=x^m$, then $y_p=\frac{1}{F(D)}x^m=[F(D)]^{-1}x^m$. Expanding the right hand side as a binomial series, the particular integral can be obtained. The following series expansions are useful:

$$(1+D)^{-1}=1-D+D^2-D^3+\cdots$$

$$(1+D)^{-2}=1-2D+3D^2-4D^3+\cdots$$

   - If $g(x)=e^{ax}V(x)$, then $y_p=\frac{1}{F(D)}e^{ax}V(x)=e^{ax}\frac{1}{F(D+a)}V(x)$

LAPLACE TRANSFORMS

1. Gamma function
   - $\Gamma(n) = \int_0^\infty e^{-x} x^{n-1} dx, (n > 0)$
   - $\Gamma(1) = 1$
   - $\Gamma(n) = 2\int_0^\infty e^{-x^2} x^{2n-1} dx$
   - $\Gamma(n+1) = n\Gamma(n)$
   - $\Gamma(n) = \frac{\Gamma(n+1)}{n}, (n < 0)$
   - $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

2. Beta Function
   - $\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1} dx$
   - $\beta(m,n) = \beta(n,m)$
   - $\beta(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta\, d\theta$
   - $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$
   - $\beta(m,n) = \int_0^\infty \frac{t^{m-1}}{(1+t)^{m+n}} dt$

3. Laplace transform of $f(t)$: $L\{f(t)\} = \int_0^\infty e^{-st} f(t) dt$
4. Transform of elementary functions:

- $L(e^{at}) = \frac{1}{s-a}, \quad s > a$
- $L(\sinh at) = L\left(\frac{e^{at}-e^{-at}}{2}\right) = \frac{a}{s^2-a^2}, s > |a|$
- $L(\sin at) = \frac{a}{s^2+a^2}, \quad s > 0$
- $L(\cosh at) = \frac{s}{s^2-a^2}, \quad s > |a|$
- $L(\cos at) = \frac{s}{s^2+a^2}, \quad s > 0$
- $L(t^n) = \frac{\Gamma(n+1)}{s^{n+1}}$
- $L[H(t-a)] = \frac{e^{-as}}{s}$, where H is Heaviside unit step function

5. Properties of Laplace transform:
   - $L[a f(t) + b g(t)] = a L[f(t)] + b L[g(t)]$.
   - If $L[f(t)] = F(s)$, then $L[f(at)] = \frac{1}{a} F\left(\frac{s}{a}\right)$, where a is a positive constant.
   - Let a be any real constant then $L[e^{at} f(t)] = F(s-a)$
   - If $L[f(t)] = F(s)$, then $L[t^n f(t)] = (-1)^n \frac{d^n}{ds^n} F(s), n = 1, 2, 3, \ldots$
   - If $L[f(t)] = F(s)$, then $L\left(\frac{f(t)}{t}\right) = \int_s^\infty F(s) ds.$
   - If $L[f(t)] = F(s)$, then $L\{f^n(t)\} = s^n L\{f(t)\} - s^{n-1} f(0) - s^{n-2} f'(0) - \ldots\ldots - f^{n-1}(0)$
   - If $L[f(t)] = F(s)$, then $L \int_0^t f(t) dt = \frac{1}{s} F(s)$

- The r^{th} moment about any point A: $\mu_r' = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - A)^r,\ r = 1, 2, 3 \cdots$

3. Relation between raw (Moments about origin or any point) and Central Moments:
   - $\mu_r = \mu_r' - {}^rC_1\, \mu_{r-1}'\, \mu_1' + {}^rC_2\, \mu_{r-2}'\, {\mu'}^2_1 - \ldots + (-1)^r {\mu_1'}^r$, r = 1, 2, 3 …
   - $\mu_r' = \mu_r + {}^rC_1 \mu_{r-1}\, \mu_1' + {}^rC_2 \mu_{r-2} {\mu_1'}^2 - \ldots + {\mu_1'}^r$

4. Measures of Kurtosis: $\beta_2 = \frac{\mu_4}{\mu_2^2}$

5. Measures of Skewness: Karl Pearson's coefficient of Skewness: $S_k = \frac{\sqrt{\beta_1}\,(\beta_2+3)}{2(5\beta_2 - 6\beta_1 - 9)}$, where $\beta_1 = \frac{\mu_3^2}{\mu_2^3}$

6. Fitting of a straight line: $y = a + bx$ for the data $(x_1, y_1), (x_2, y_2), \cdots (x_n, y_n)$

   The normal equations for estimating the values of a and b are

   $\sum y = na + b\sum x,$

   $\sum xy = a\sum x + b\sum x^2.$

7. Fitting of a second-degree equation (quadratic): $y = a + bx + cx^2$

   The normal equations for estimating the values of a , b , c are

   $\sum y = na + b\sum x + c\sum x^2,$

   $\sum xy = a\sum x + b\sum x^2 + c\sum x^3,$

   $\sum x^2 y = a\sum x^2 + b\sum x^3 + c\sum x^4.$

8. Correlation Coefficient (Karl Pearson correlation coefficient):
   - $r = \frac{\sum(x-\bar{x})(y-\bar{y})}{n\sigma_x\sigma_y}$, where $\sigma_x^2 = \frac{\sum(x-\bar{x})^2}{n}$ variance of the x series, $\sigma_y^2 = \frac{\sum(y-\bar{y})^2}{n}$ variance of the y series,
   - $\bar{x} = \frac{\sum x}{n} \rightarrow$ Mean of the x series $\bar{y} = \frac{\sum y}{n} \rightarrow$ mean of the y series.
   - $r = \frac{n\sum xy - (\sum x)(\sum y)}{\sqrt{\{n\sum x^2 - (\sum x)^2\}\{n\sum y^2 - (\sum y)^2\}}}$

9. Regression line of y on x: $y - \bar{y} = b_{yx}(y - \bar{y})$, where $b_{yx} = r\frac{\sigma_y}{\sigma_x} = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2} = \frac{n\sum xy - \sum x\sum y}{n\sum x^2 - (\sum x)^2}$

10. Regression line of x on y: $x - \bar{x} = b_{xy}(y - \bar{y})$ where $b_{xy} = r\frac{\sigma_x}{\sigma_y} = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(y-\bar{y})^2} = \frac{n\sum xy - \sum x\sum y}{n\sum y^2 - (\sum y)^2}$