RV Educational Institutions ®
RV College of Engineering ®

Autonomous Institution Affiliated to Visvesvaraya Technological University, Belagavi

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DEPARTMENT OF MATHEMATICS

Course: Fundamentals of Linear Algebra, Calculus and Statistics	CIE-II	Maximum marks: 50
Course code: MAT211CT	First semester 2023-2024 Chemistry Cycle Branch: AI, BT, CS, CD, CY, IS, SPARK C	Time: 2:00PM-3:30PM Date: 27-12-2023

Sl. No.	Solutions and Scheme	Marks
1	$\frac{dx}{d\theta} = a(1-\cos\theta)$ and $\frac{dy}{d\theta} = a\sin\theta$	1
	$\frac{dy}{dx} = \cot\left(\frac{\theta}{2}\right) \Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\pi} = 0$	2
	$\frac{d^2y}{dx^2} = -\frac{1}{4a} cosec^4\left(\frac{\theta}{2}\right) \Rightarrow \left(\frac{d^2y}{dx^2}\right)_{\theta=\pi} = -\frac{1}{4a}$	2
	$\rho = -4a$	2
	$\bar{x} = a\pi$ and $\bar{y} = -2a$	2
	$(x-a\pi)^2 + (y+2a)^2 = 16a^2$	1
2	$y = e^{\tan^{-1}x} \Longrightarrow y(0) = 1$	
	$(1+x^2)y_1 = y \Longrightarrow y_1(0) = 1$	1
	$(1+x^2)y_2 + 2xy_1 = y_1 \Longrightarrow y_2(0) = 1$	2
	$(1+x^2)y_3 + 4xy_2 + 2y_1 = y_2 \Longrightarrow y_3(0) = -1$	2
	$(1+x^2)y_4 + 6xy_3 + 6y_2 = y_3 \Longrightarrow y_4(0) = -7$	2
	$(1+x^2)y_5 + 8xy_4 + 12y_3 = y_4 \Longrightarrow y_5(0) = 5$	2
	$e^{\tan^{-1}x} = 1 + x + \frac{x^2}{2} - \frac{x^3}{6} - \frac{7x^4}{24} + \frac{x^5}{24} + \cdots$	1
3. (a)	$u_x = \frac{1}{\sqrt{y^2-x^2}} - \frac{y}{x^2+y^2} \Longrightarrow xu_x = \frac{x}{\sqrt{y^2-x^2}} - \frac{xy}{x^2+y^2}$	2
	$u_y = \frac{1}{\sqrt{y^2-x^2}}\left(-\frac{x}{y}\right) + \frac{x}{x^2+y^2} \Longrightarrow yu_y = -\frac{x}{\sqrt{y^2-x^2}} + \frac{xy}{x^2+y^2}$	2
	$xu_x + yu_y = 0$	1
3. (b)	$\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial r} = \frac{\partial z}{\partial x}\cos\theta + \frac{\partial z}{\partial y}\sin\theta$	1
	$\frac{\partial z}{\partial \theta} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial \theta} = \frac{\partial z}{\partial x}(-r\sin\theta) + \frac{\partial z}{\partial y}(r\cos\theta)$	1
	$\frac{1}{r}\frac{\partial z}{\partial \theta} = \frac{\partial z}{\partial x}(-\sin\theta) + \frac{\partial z}{\partial y}(\cos\theta)$	1

	$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$	2
4. (a)	$f_x = 2y\sin x\,, f_y = 2y - 2\cos x\,, f_{xx} = 2y\cos x\,, f_{xy} = 2\sin x\,, f_{yy} = 2$ $f_x = 0$ and $f_y = 0$ gives the stationary points $(0,1)$ and $\left(\frac{\pi}{2},0\right)$ At $\left(\frac{\pi}{2},0\right)$, $f_{xx}f_{yy} - f_{xy}^2 = -4 < 0$ (Saddle point) At $(0,1)$, $f_{xx}f_{yy} - f_{xy}^2 = 4 > 0$ (Minimum point) and minimum value is -1.	2 1 1 1
4. (b)	$A = \frac{1}{2}xy$ $dA = \frac{\partial A}{\partial x}dx + \frac{\partial A}{\partial y}dy = \frac{1}{2}(ydx + xdy) = \frac{1}{2}[12(0.002) + 5(0.002)] = 0.017$ $z = \sqrt{x^2 + y^2}$ $dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy = \frac{x}{\sqrt{x^2+y^2}}dx + \frac{y}{\sqrt{x^2+y^2}}dy = \frac{5}{13}(0.002) + \frac{12}{13}(0.002)$ $dz = 0.0026$	2 1 2
5. (a)	$F = 8x^2 + 4yz - 16z + 600 + \lambda(4x^2 + y^2 + 4z^2 - 16)$ $\frac{\partial F}{\partial x} = 0 \Longrightarrow 16x + 8\lambda x = 0 \Longrightarrow x = 0 \; or \; \lambda = -2$ $\frac{\partial F}{\partial y} = 0 \Longrightarrow 4z + 2\lambda y = 0 \Longrightarrow 4z - 4y = 0 \Longrightarrow z = y$ $\frac{\partial F}{\partial z} = 0 \Longrightarrow 4y - 16 + 8\lambda z = 0 \Longrightarrow -16 - 12y = 0 \Longrightarrow y = -\frac{4}{3} = z$ $4x^2 + y^2 + 4z^2 = 16 \Longrightarrow x = \pm\frac{4}{3}$ Hottest point $\left(\pm\frac{4}{3}, -\frac{4}{3}, -\frac{4}{3}\right)$	1 Derivatives 2 y and z 2 1
5. (b)	$J = \begin{vmatrix} 3 & 2 & -1 \\ 1 & -2 & 1 \\ 2x + 2y - z & 2x & -x \end{vmatrix}$ $J = 3(2x - 2x) - 2(-3x - 2y + z) - 1(6x + 4y - 2z) = 0$	2 2