RV College of Engineering®
Mysore Road, RV Vidyaniketan Post,
Bengaluru - 560059, Karnataka, India

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Department of Mathematics
Academic Year 2023-2024 (Even Semester 2024)

Date	01/07/2024	**Time**	10:00 AM to 12 NOON
Test	Improvement Test (Quiz & Test)	**Maximum Marks**	10+50=60
Course Title	Number Theory, Vector Calculus and Computational Methods	**Course Code**	MA221TC
Semester	II	**Programs**	B.E. (AIML, BT, CD, CS, CY, IS)

Instructions: Answer all questions

PART - A

S.No.	Questions	M	BT	CO
1	If the general solution of a differential equation is $y = c_0 + c_1 e^{-x}\sin x + c_2 e^{-x}\cos x$, then the differential equation is _______.	2	L1	1
2	The Wronskian of $y_1 = \cos 2x$, $y_2 = \sin 2x$ is ________.	2	L1	1
3	Reduce the Cauchy's equation $x\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} = \frac{1}{x}$ to a differential equation with constant coefficients ____________.	2	L2	2
4	The value of $\frac{1}{\pi}\oint_C \left(3y - e^{\cos(x^2)}\right)dx + \left(7x + \sqrt{y^4+11}\right)dy$ where $C: x^2+y^2 = 9$ oriented positively is ________.	2	L2	2
5	Let S be the portion of the plane $2x + y = 4$ in the first octant bounded by $z = 1$ and $z = 4$. The surface integral $\iint_S (x\,\hat{\imath} + y\hat{\jmath} + z\hat{k})\cdot\hat{n}\,dS$, over the region R in terms of double integral with the limits is given by________ $\frac{2i+j}{\sqrt{5}}$, $dzdy$ ~~where~~ R being the projection of S taken on yz −plane. $\hat{n}\cdot\hat{k} = \frac{2}{\sqrt{5}}$	2	L3	3

PART - B

$\iint \frac{2x+y}{\sqrt{5}} \frac{dzdy}{2} = \frac{1}{2}\iint (4-y)+y\; dz\,dy$

$\iint 2\,dz\,dy$

S.No.	Questions	M	BT	CO
1a	Obtain the particular integral of the differential equation $y'' - y = x\cos x$.	4	L2	2
1b	Determine the displacement x at time $t > 0$ from the equilibrium position of the mass-spring system governed by the differential equation $t^2x'' + tx' + x = 2\cos^2(\log t)$.	6	L3	1
2	Using the method of variation of parameters solve the differential equation: $$\frac{d^2y}{dx^2} + y = \frac{1}{1+\sin x}.$$	10	L2	2
3a	Find the work done by the force $\vec{F}(x,y) = 2xy\,\hat{\imath} + 4y^2\hat{\jmath}$ acting along the piecewise curve consisting of the line segments from $(-2,2)$ to $(0,0)$ and from $(0,0)$ to $(2,3)$.	6	L3	3
3b	Evaluate the line integral $\int_C \vec{F}.\hat{T}\,ds$, where $\vec{F} = 2xy\,\hat{\imath} + x^2\hat{\jmath}$ being the conservative field, along any smooth curve C joining the point $(0,0)$ to $(1,1)$.	4	L2	4
4	Verify Green's theorem for $\oint_C (x^2 - y^2)\,dx + (2y - x)\,dy$, where C consists of the boundary of the region in the first quadrant bounded by the graphs $y = x^2$ and $y = x^3$.	10	L3	3
5	Use the surface integral in Stokes' theorem to determine the circulation of the field $\vec{F} = y\,\hat{\imath} + xz\,\hat{\jmath} + x^2\,\hat{k}$ around the boundary of the triangle cut from the plane $x + y + z = 1$ by the first octant, counterclockwise when viewed from above.	10	L4	4

BT-Blooms Taxonomy, CO-Course Outcomes, M-Marks

Marks Distribution	Particulars	CO1	CO2	CO3	CO4	L1	L2	L3	L4	L5	L6
	Max Marks	10	18	18	14	4	22	24	10	--	--

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Department of Mathematics
Academic Year 2023-2024 (Even Semester 2024)
Scheme and solution

Course: Number Theory, Vector Calculus and Computational Methods Course code: MA221TC	Improvement CIE (QUIZ & TEST) Second semester 2023-2024 Physics Cycle : AIML, BT, CD, CS, CY, IS	Maximum marks: 10+50=60

PART A – Quiz

Q.No		Marks
1.1	$m(m^2-(-1+i-1-i)m+(-1+i)(-1-i))=0$ $D(D^2+2D+2)y=0$	1+1
1.2	$W(y_1,y_2)=2$	2
1.3	$(D^3-2D^2+D)y=e^t$	2
1.4	$I=\frac{1}{\pi}(7-3)9\pi=36$	2
1.5	$\int_1^4\int_0^4 2\,dy\,dz$	2

PART B

Q.No		Marks
1a	$x\cos x = Re(xe^{ix})$ $P.I.=e^{ix}\frac{1}{(D+i)^2-1}x=e^{ix}\frac{1}{D^2+2iD-2}x$ $P.I.=-\frac{e^{ix}}{2}\left(1-\frac{D^2+2iD}{2}\right)^{-1}x=-\frac{e^{ix}}{2}\left[1+\frac{D^2+2iD}{2}+..\right]x$ $P.I=-\frac{1}{2}e^{ix}(x+i)=-\frac{1}{2}(\cos x+i\sin x)(x+i)$ $\therefore P.I.=-\frac{1}{2}(x\cos x-\sin x)$	1 1 1 1
1b	Let $t=e^z \Rightarrow z=\log t$ The given equation becomes $(D^2+1)x=2\cos^2 z$ $\therefore m=\pm i$ $x_c=c_1\cos z+c_2\sin z=c_1\cos(\log t)+c_2\sin(\log t)$ $x_p=\frac{2\cos^2 z}{D^2+1}=\frac{1+\cos 2z}{D^2+1}$ $x_p=\frac{1}{D^2+1}+\frac{\cos 2z}{D^2+1}=1-\frac{\cos 2z}{3}=1-\frac{\cos 2(\log t)}{3}$ $\therefore x=x_c+x_p$	1 2 1 2
2	$A.E.: m^2+1=0 \Rightarrow m=\pm i$ $y_c=c_1\cos x+c_2\sin x$ $W(\cos x,\sin x)=1$ $A(x)=-\int\frac{\sin x}{1+\sin x}dx=-\int\frac{\sin x-\sin^2 x}{1-\sin^2 x}dx=-\int\frac{\sin x}{\cos^2 x}dx+\int\tan^2 x\,dx=-\sec x+\tan x-x$ $B(x)=\int\frac{\cos x}{1+\sin x}dx=\log(1+\sin x)$ $y_p=(-\sec x+\tan x-x)\cos x+\log(1+\sin x)\sin x$ Therefore, $y=y_c+y_p$	3 1 2 2 1 1

Department of Mathematics

Academic Year 2023-2024 (Even Semester 2023)

Date	18/06/2024	**Time**	10:00 to 11:30 AM
Test	Test-II	**Maximum Marks**	50
Course Title	Number Theory, Vector Calculus and Computational Methods	**Course Code**	MA221TC
Semester	II	**Programs**	B.E. (AIML, BT, CD, CS, CY, IS)

Instructions: Answer all questions.

Sl. No.	Questions	M	BT	CO
1(a)	By using the Euclidean algorithm, find the greatest common divisor d of 1166 and 256, and find integers x and y to satisfy $1166x + 256y = d$.	6	L2	2
1(b)	Find the remainder when 16^{53} is divided by 7.	4	L1	1
2	Given the public key $(e, n) = (7,51)$, encrypt plain text LIV, where the alphabets $A, B, C, \ldots X, Y, Z$ are assigned the numbers $3,4,5,\cdots,26,27,28$. Give the cipher text and find the private key d.	10	L3	4
3(a)	Find all solutions of linear congruence $18x \equiv 30(mod\ 42)$.	5	L2	2
3(b)	A particle moves along the curve whose parametric equation is given by $x = (t^3 + 1)$, $y = t^2$, $z = (2t + 5)$, where 't' is time. Find the component of its acceleration at time $t = 2$ in the direction of the vector $\hat{\imath} + \hat{\jmath} + \hat{k}$.	5	L1	2
4(a)	Find the values of a and b such that the surfaces $5x^2 - 2yz = 9z$ and $ax^2 + by^3 = 4$ cut orthogonally at $(1, -1, 2)$.	5	L2	3
4(b)	Let $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and $r = \lvert\vec{r}\rvert$. Evaluate $\nabla^2 r^n$. Reduce the final answer in terms of r.	5	L2	1
5	Find the curl of the vector field $$\vec{F} = (y \sin z + y\cos(xy))\hat{\imath} + (x \sin z + x\cos(xy))\hat{\jmath} + xy\cos z\,\hat{k}.$$ If $\vec{F}$ is irrotational then find the scalar potential ϕ of $\vec{F}$ such that $\nabla\phi = \vec{F}$.	10	L3	3

BT-Blooms Taxonomy, CO-Course Outcomes, M-Marks

Marks Distribution	Particulars	CO1	CO2	CO3	CO4	L1	L2	L3	L4	L5	L6
	Max Marks	9	16	15	10	9	21	20	--	--	--

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Department of Mathematics
Academic year 2023-2024 (Even Semester 2023)

Date	18/06/2024	**Time**	10:00 to 11:30 PM
Test	Test-II	**Maximum Marks**	50
Course Title	Number Theory, Vector Calculus and Computational Methods	**Course Code**	MA221TC
Semester	II	**Programs**	B.E. (AIML, BT, CD, CS, CY, IS)

SCHEME AND SOLUTION

Sl. No.	Questions	M
1(a)	$1166 = 4 \times 256 + 142$; $256 = 1 \times 142 + 114$ $142 = 1 \times 114 + 28$; $114 = 4 \times 28 + 2$; $28 = 14 \times 2 + 0$ $\gcd(1166,256) = 2$ $2 = 5 \times 114 - 4 \times 142 = 41 \times 256 - 9 \times 1166$	1+1 1 1 1+1
1(b)	$16 \equiv 2\ (mod\ 7)$, $2^3 \equiv 1\ (mod\ 7)$, $2^{53} = 2^{37\times3+2} = (2^3)^{37} \times 2^2 \equiv 1 \times 2^2\ (mod\ 7)$ $16^{53} \equiv 2^{53}\ (mod\ 7) \equiv 4\ (mod\ 7)$	1 2 1
2	$\phi(51) = 32$, Plain text $L = 14, I = 11, V = 24$ Cipher Text: L: $14^7 \equiv 23\ (mod\ 51)$ $L \longrightarrow U$ I: $11^7 \equiv 20\ (mod\ 51)$ $I \longrightarrow R$ V: $24^7 \equiv 12\ (mod\ 51)$ $V \longrightarrow J$ Cipher text is URJ Private key $(d, 32)$ $7d \equiv 1\ (mod\ 32) \Longrightarrow d = 23$	1 2 2 2 1 2
3(a)	$\gcd(18,42) = 6$, Therefore, the linear congruence has 6 incongruent solutions. Solutions are $x \equiv 4\ (mod\ 42)$, $x \equiv 11\ (mod\ 42)$, $x \equiv 18\ (mod\ 42)$, $x \equiv 25\ (mod\ 42)$, $x \equiv 32 (mod\ 42)$, $x \equiv 39 (mod\ 42)$	1 2 1+1
3(b)	$\vec{r}(t) = (t^3 + 1)i + t^2 j + (2t + 5)k$ $\vec{v}(t) = \frac{d\vec{r}}{dt} = 3t^2 \boldsymbol{i} + 2tj + 2k$, $\vec{a}(t) = 6t\boldsymbol{i} + 2j$ $\vec{a}(2) = 12\boldsymbol{i} + 2j$ $\vec{u} = \boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$, $\hat{u} = \frac{1}{\sqrt{3}}(\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k})$ Component of acceleration= $\vec{a}(2) \cdot \hat{u} = \frac{14}{\sqrt{3}}$	1 1 1 1 1
4a	Let $\phi_1 = 5x^2 - 2yz - 9z$ and $\phi_2 = ax^2 + by^3$ $\nabla\phi_1 = 10x\boldsymbol{i} - 2zj - (2y + 9)k$, $\nabla\phi_2 = 2axi + 3by^2 j$ At $(1, -1, 2)$ $\vec{n}_1 = \nabla\phi_1 = 10\boldsymbol{i} - 4j - 7k$, $\vec{n}_2 = \nabla\phi_2 = 2a\boldsymbol{i} + 3bj$ $\vec{n}_1 \cdot \vec{n}_2 = 0 \Longrightarrow 20a - 12b = 0 \Longrightarrow 5a = 3b$ As $(1, -1, 2)$ lies on $ax^2 + by^3 = 4$, $a - b = 4 \Longrightarrow 3a - 3b = 12 \Longrightarrow a = -6, b = -10$ or The point $(1, -1, 2)$ is not on the surface ϕ_1	1 1 1 1 1 5
4b	$r = \sqrt{x^2 + y^2 + z^2}$, $\nabla^2 r^n = \frac{\partial^2}{\partial x^2}(r^n) + \frac{\partial^2}{\partial y^2}(r^n) + \frac{\partial^2}{\partial z^2}(r^n)$. $\frac{\partial}{\partial x}(r^n) = nr^{n-1}\frac{\partial r}{\partial x} = nxr^{n-2}$ $\frac{\partial^2}{\partial x^2}(r^n) = \frac{\partial}{\partial x}(nxr^{n-2}) = nr^{n-2} + nx(n-2)r^{n-3}\frac{\partial r}{\partial x} = nr^{n-2} + n(n-2)x^2 r^{n-4}$ Similarly	1 2

P.T.O.

Department of Mathematics

Academic year 2023-2024 (Even Semester 2023)

	$\frac{\partial^2}{\partial y^2}(r^n) = nr^{n-2} + n(n-2)y^2r^{n-4}$ and $\frac{\partial^2}{\partial z^2}(r^n) = nr^{n-2} + n(n-2)z^2r^{n-4}$ $\nabla^2 r^n = n(n+1)r^{n-2}$
5	$curl\ \vec{F} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y\sin z + y\cos(xy) & x\sin z + x\cos(xy) & xy\cos z \end{vmatrix}$ $= [x\cos z - x\cos z]\boldsymbol{i} - [y\cos z - y\cos z]\boldsymbol{j} + [\sin z + \cos(xy) - xy\sin(xy) - \sin z - \cos(xy) + xy\sin(xy)]\boldsymbol{k}$ $= 0$ Hence $\vec{F}$ is irrotational. Let ϕ be the potential of $\vec{F}$. $\frac{\partial\phi}{\partial x} = y\sin z + y\cos(xy), \frac{\partial\phi}{\partial y} = x\sin z + x\cos(xy), \frac{\partial\phi}{\partial z} = xy\cos z$ $d\phi = [y\sin z + y\cos(xy)]dx + [x\sin z + x\cos(xy)]dy + xy\cos z\, dz$ $= d(xy\sin z) + d(\sin(xy))$ $\phi = xy\sin z + \sin(xy) + c$

Date	13/05/2024	Time	10:00 to 11:30 AM	
TEST	Test-I	Maximum Marks	50	
Course Title	Number Theory, Vector calculus and Computational methods		Course Code	MA221TC
Semester	II	Programs	B. E. (AIML,BT,CD,CS,CY,IS)	

Instructions: Answer all questions.

Sl. No.	Questions	M	BT	CO
1(a)	From the following table estimate the number of students who obtained marks between 50 and 75 using appropriate interpolation formula Marks: 30-40, 40-50, 50-60, 60 -70, 70 - 80 Number of students: 31, 73, 124, 159, 190	6	L2	3
1(b)	Using Lagrange Interpolation Formula fit a polynomial for the given data: x: 0, 1, 3, 5 $y(x)$: 1, 5, 49, 231	4	L2	2
2	The following table gives the values of pressure P and specific volume V of saturated steam: V: 40, 50, 60, 70, 80 P: 304.5, 270.4, 250.2, 204.6, 184.9 Find the value of pressure P at volume V= 45. Also find the rate of change of pressure with respect to volume at V= 50 and $\frac{d^2P}{dV^2}$ at V=70.	10	L3	3
3(a).	If $F(D) = (D^4 - \omega^4)$, where D is the linear differential operator, with $D = \frac{d}{dz}$. Obtain the general solution of $F(D)v = 0$.	4	L1	1
3(b)	Find the particular solution of the initial value problem $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = 18e^{-t}$, given $y(0) = 0$ and $y'(0) = 4$.	6	L2	2
4	Solve $\frac{d^2y}{dx^2} - y = e^{-2x}\cos(e^{-x})$ by method of variation of parameters.	10	L2	2
5	Obtain the radial displacement x in a rotating disc at a distance s from the axis is given by the differential equation $2s^2\frac{d^2x}{ds^2} + 3s\frac{dx}{ds} - x = \frac{\cos(\log s)}{s} - 2s$.	10	L3	4

BT-Blooms Taxonomy, CO-Course Outcomes, M-Marks

Marks Distribution	Particulars	CO1	CO2	CO3	CO4	L1	L2	L3	L4	L5	L6
	Max Marks	4	20	16	10	4	26	20	--	--	--

....All the best....

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DEPARTMENT OF MATHEMATICS

Course: Number theory, Vector calculus and Computational methods	**CIE-I SCHEME AND SOLUTION**	**Date: 13-05-2024**
Course code: MA221TC	**II Semester(AIML, BT, CD, CS, CY, IS)**	

Answer all questions

Q.No 1(a)

Less than (x)	No of students (y)	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
40	31				
		73			
50	104		51		
		124		-16	
60	228		35		12
		159		-4	
70	387		31		
		190			
80	577				

M: 2

$$x_n = 80, x = 75$$

$$p = \frac{75-80}{10} = -0.5$$ 1

$$y_{75} = y_{n+} \, p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \dots\dots\dots \frac{p(p+1)\dots\dots(p+n-1)}{n!}\nabla^n y_n$$

$$y_{75} = 577 - 0.5(190) - \frac{0.5(0.5)(31)}{2!} - \frac{0.5(0.5)(1.5)(-4)}{3!} - \frac{0.5(0.5)(1.5)(2.5)(12)}{4!}$$ 1

= 477.9063 ~ 478 students 1

∴ 50-75 marks obtained by 478-104 =374 1

1(b) Lagrange Interpolation

$$f(x) = \frac{(x-x_1)(x-x_2)\dots\dots\dots(x-x_n)}{(x_0-x_1)(x_0-x_2)\dots\dots\dots(x_0-x_n)}y_0 + \frac{(x-x_0)(x-x_2)\dots\dots\dots\dots(x-x_n)}{(x_1-x_0)(x_1-x_2)\dots\dots\dots(x_1-x_n)}y_1 + \dots$$

$$+ \frac{(x-x_0)(x-x_1)\dots\dots\dots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\dots\dots\dots(x_n-x_{n-1})}y_n$$

3(a)	$(D^4-\omega^4)v=0$ $m^4-\omega^4=(m^2-\omega^2)(m^2+\omega^2)=0$ $m=\pm\omega,\pm i\omega$ $v=C_1e^{\omega z}+C_2e^{-\omega z}+C_3cos\omega z+C_4sin\omega z$	2 2
3(b)	$(D^2+4D+4)y=18e^{-t}$ $m^2+4m+4=0$ $m=-2,-2$ $y_c=(C_1+C_2t)\,e^{-2t}$ $y_p=\dfrac{18e^{-t}}{(D^2+4D+4)}$	1 1
Replace D by -1	$y_p=\frac{18e^{-t}}{((-1)^2-4+4)}$ y_p=$18e^{-t}$ $y=y_c+y_p$ $y=(C_1+C_2t)e^{-2t}+18e^{-t}$ $y'=-2(C_1+C_2t)e^{-2t}+C_2e^{-2t}-18e^{-t}$ $y(0)=0=C_1+18$ $C_1=-18$ $y'(0)=4=-2(-18)+C_2-18$ $C_2=-14$ $y=-(18+14t)e^{-2t}+18e^{-t}$	1 1 1 1
4	A E $\quad m^2-1=0, m=\pm 1$ $y_c=C_1e^x+C_2e^{-x}$ $y_p=uy_1+vy_2$ $y_1=\ e^x, y_2=e^{-x}$ $w=\begin{vmatrix} e^x & e^{-x} \\ e^x & -e^{-x} \end{vmatrix}=\ -2$ $u=-\int\frac{y_2X}{w}dx=-\int\frac{e^{-x}e^{-2x}\cos(e^{-x})}{-2}dx$ $=-\frac{1}{2}\int e^{-2x}\cos(e^{-x})(-e^{-x})dx$	1 1 2 1
Put $z=e^{-x}, dz=-e^{-x}dx$,	$u=-\frac{1}{2}\int z^2\cos z\,dz$ $=-\frac{1}{2}[z^2\sin z-2z(-\cos z)+2(-\sin z)]$ $=-\frac{1}{2}e^{-2x}\sin e^{-x}-e^{-x}(\cos e^{-x})+\sin e^{-x}$	1