Unit - 2a

Single - Phase A. C. Circuits

Introduction

* The path for the flow of an alternating current is called an a.c. circuit.

* In a d.c. circuit, the current (I) flowing through the circuit is given by $I = V/R$.

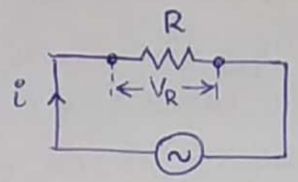
* However, in an a.c. circuit, voltage and current are changing at every instant. They give rise to magnetic (inductive) and electrostatic (capacitive) effects. So, in an a.c. circuit, inductance and capacitance must be considered in addition to resistance.

* AC circuits may be of the following basic types:-
  1. AC circuits containing pure ohmic resistance
  2. AC circuits containing pure inductance
  3. AC circuits containing pure capacitance
  4. AC circuits with <u>series</u> connection of,
     a) resistance - inductance (R-L circuit)
     b) resistance - capacitance (R-C circuit)
     c) resistance - inductance - capacitance (RLC circuit)
  5. AC circuits containing a <u>parallel</u> combination of,
     a) resistance - inductance (R-L circuit)
     b) resistance - capacitance (R-C circuit)
     c) resistance - inductance - capacitance (RLC circuit)

AC Circuit containing pure ohmic resistance

* When an alternating voltage is applied to a pure ohmic resistance, electrons flow ~~from~~ in one direction during the first half cycle and in opposite direction in the next half cycle, thus constituting an alternating current.

* In the circuit shown below, 'R' is a pure resistance to which an alternating voltage $v = V_m \sin \omega t$ is applied. Due to this, an alternating current 'i' flows through it.



$v = V_m \sin \omega t$

* The applied voltage has to supply the drop in the resistance. i.e.,

$$v = iR \longrightarrow (1)$$

But $$v = V_m \sin \omega t \longrightarrow (2)$$

$$\therefore V_m \sin \omega t = iR$$

$$\Rightarrow i = \frac{V_m}{R} \sin \omega t \longrightarrow (3)$$

The alternating current is maximum when $\sin \omega t = 1$.

i.e.,

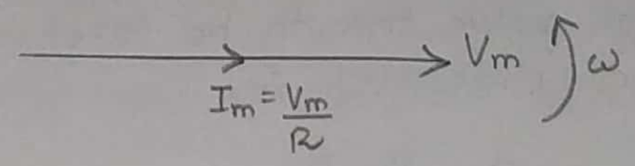
$$I_m = \frac{V_m}{R} \longrightarrow (4)$$

Thus, we get the equation for instantaneous current as,

$$\boxed{i = I_m \sin \omega t} \longrightarrow (5).$$

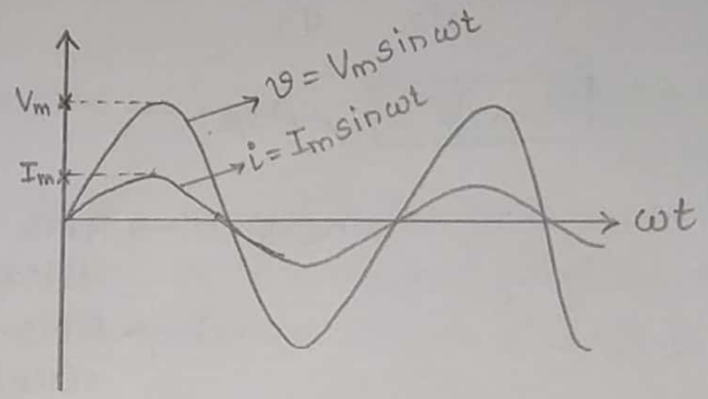
From equations (2) and (5), it is clear that the voltage & current are in phase with each other.

* The phasor diagram will be as shown below.



(Phasor Diagram)

And the waveform will be as shown below.



(Waveform)

* The instantaneous power (p) is the product of instantaneous voltage & instantaneous current, as

$$p = vi$$

Substituting from (2) and (5),

$$p = V_m \sin \omega t \; I_m \sin \omega t$$

$$= V_m I_m \sin^2 \omega t$$

$$= V_m I_m \left(\frac{1 - \cos 2\omega t}{2} \right)$$

$$\boxed{p = \frac{V_m I_m}{2} - \frac{V_m I_m \cos 2\omega t}{2}} \longrightarrow (6)$$

This consists of two parts :

(i) a constant part $(V_m I_m / 2)$.

(ii) a time-varying quantity $\frac{V_m I_m}{2} \cos 2\omega t$.

The second part has a frequency of twice the frequency of the applied voltage (and current). The average value of this part over a period of time (T) is zero. (i.e., over a complete cycle).

Since power is a scalar quantity and hence only the average value has to be taken. So, power for one complete cycle is,

$$P = \frac{V_m I_m}{2}$$

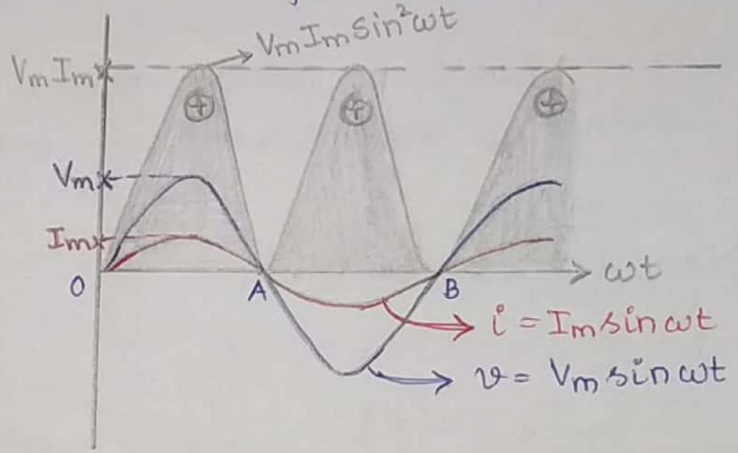
$$= \frac{V_m}{\sqrt{2}} * \frac{I_m}{\sqrt{2}}$$

$$\boxed{P = VI} \text{ watts.}$$

where $V \rightarrow$ RMS value of applied voltage

$I \rightarrow$ RMS value of the circuit current

* The power curve for such a circuit is as shown below.

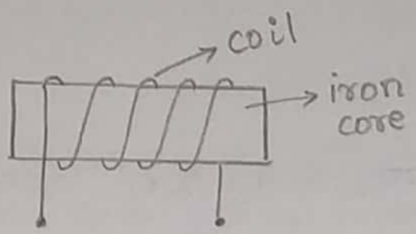


* The power in the circuit is zero at the instants O, A and B, when both voltage & current are zero.

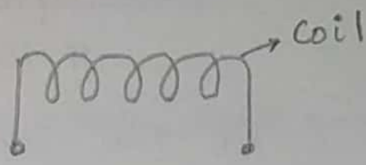
* At all other instants the power is positive. This implies that power is always dissipated in a resistive a.c. circuit as heat.

A.C. Circuit containing pure inductance

* An inductive coil is a coil of wire wound for a definite number of turns. It may or may not have an iron core. It usually has a negligible resistance.



(Iron-cored inductive coil)



(Air-cored inductive coil).

* If we use a thick copper wire and wind it on a laminated iron core, it will have negligible resistance. For understanding purposes, such a coil is called a pure inductive coil.

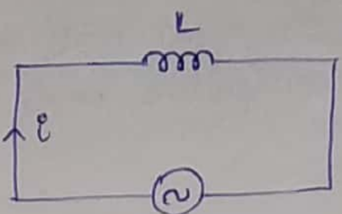
* An inductive coil or an <u>inductor</u> has a property that it opposes any sudden change in current through it. This property is called the <u>inductance</u> of the inductor.

* On application of alternating ~~current~~ voltage to a circuit containing a pure inductance, an e.m.f is induced in the inductor due to its inductance. The induced emf is such that it always opposes the rise or fall of current at every stage. Since the rise and fall of current is due to the rise and fall of supply voltage, the direction of induced emf is so as to oppose the supply voltage.

* In other words, the applied voltage has to overcome the induced emf.

* Consider a purely inductive coil supplied by an ac voltage source represented as,

$$v = V_m \sin \omega t. \rightarrow (1)$$



$v = V_m \sin \omega t$

* Let 'L' be the inductance of the inductor. Then the induced emf is given by,

$$e = -L \frac{di}{dt} \rightarrow (2)$$

* Since the applied voltage at every instant is equal and opposite to the induced emf, we can write

$$v = -e_L \rightarrow (3)$$

$$\Rightarrow V_m \sin \omega t = -\left(-L \frac{di}{dt}\right)$$

$$\Rightarrow \frac{V_m}{L} \sin \omega t \, dt = di$$

Integrating w.r.t 't', we get

$$\int di = \int \frac{V_m}{L} \sin \omega t \, dt$$

$$\Rightarrow i = \frac{V_m}{L} \int \sin \omega t \, dt$$

$$i = \frac{V_m}{L} \left(\frac{-\cos \omega t}{\omega}\right) + A \rightarrow (4)$$

where 'A' is the constant of integration. The value of 'A' is found to be zero from initial conditions.

Therefore,

$$i = -\frac{V_m}{\omega L}\cos\omega t$$

$$= -\frac{V_m}{\omega L}\sin(\pi/2 - \omega t)$$

$$\boxed{i = \frac{V_m}{\omega L}\sin(\omega t - \pi/2)} \longrightarrow (5).$$

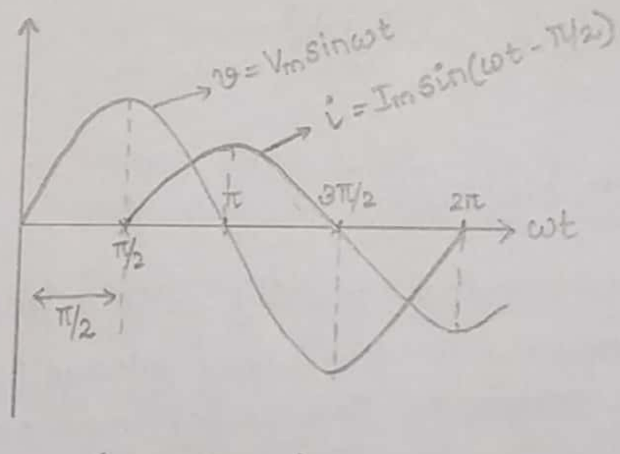
This current will be maximum when $\sin(\omega t - \pi/2) = 1$. At this value, the maximum current is

$$I_m = \frac{V_m}{\omega L} \longrightarrow (6)$$

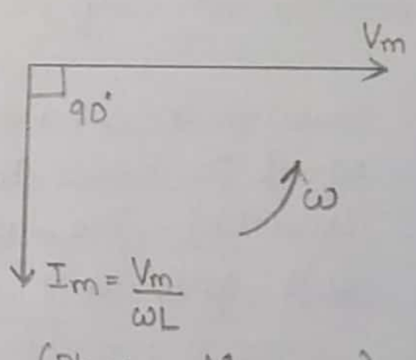
Substituting (6) in (5), we get

$$\boxed{i = I_m \sin(\omega t - \pi/2)} \longrightarrow (6).$$

Comparing equations (1) and (6), we can say that the current lags behind the voltage by $\pi/2$ radians or $90°$. The phasor diagram and waveform are shown below.



(Waveform)



(Phasor diagram)

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* Consider the expression for peak value of current,

$$I_m = \frac{V_m}{\omega L}$$

The denominator term ωL is called <u>inductive reactance</u> and is represented as X_L.

$$\therefore \boxed{X_L = \omega L}$$

since $\omega = 2\pi f$,

$$\boxed{X_L = 2\pi f L}$$

If inductance (L) is in henry (H) and 'ω' in rad/sec, then the <u>unit of X_L is ohms</u>.

* The instantaneous power is given by,

$$p = v i$$
$$= V_m \sin\omega t \; I_m \sin(\omega t - \pi/2)$$
$$p = V_m I_m \sin\omega t \sin(\omega t - \pi/2)$$

Since $\sin(\omega t - \pi/2) = -\cos\omega t$,

and $\sin\omega t \cos\omega t = \frac{\sin 2\omega t}{2}$

we get

$$p = -\frac{V_m I_m}{2} \sin 2\omega t$$

This is a sinusoidally varying quantity with a frequency equal to twice the frequency of the supply voltage (and current). The average value of this power over one full cycle is,

$$\boxed{P_{av} = 0}$$

<u>Hence, a pure inductor does not consume any power.</u>

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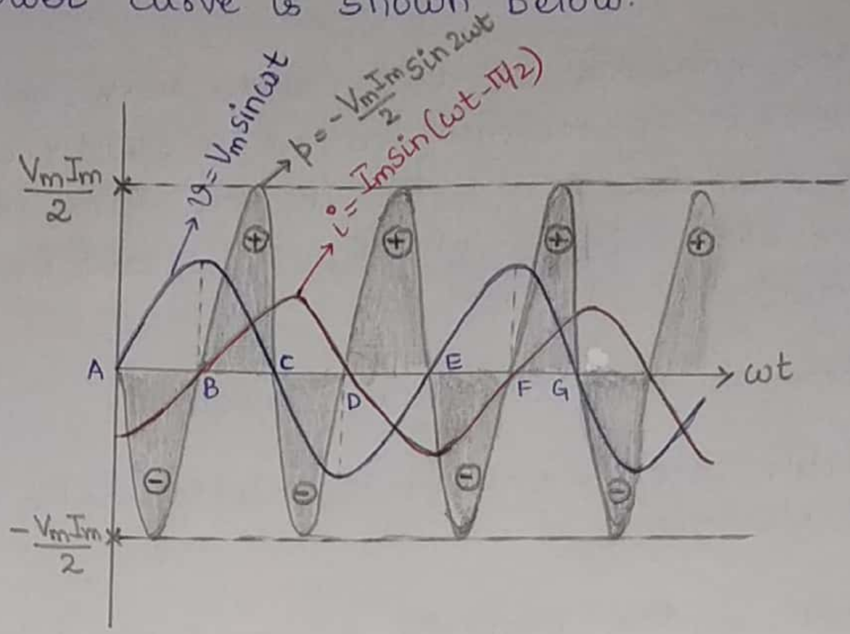
* The power curve is shown below.



→ The power equation has twice the frequency as that of the supply voltage. Hence in one cycle of supply voltage, there will be two cycles of power.

→ Between points A and B, the voltage is positive while the current is negative. Hence, the instantaneous power ($p = vi$) will be negative

→ Between points B and C, both instantaneous voltage and instantaneous currents are positive. Hence, the instantaneous power is also positive.

→ Hence the power curve is drawn with the positive and negative cycles as shown.

→ Positive lobes (half cycles) of power indicate that the power flows from source to the inductive coil. Negative lobes (half cycles) of power indicate that the coil returns the stored energy back to the source. Hence, the resultant power over one cycle is zero.

Note :- An inductor stores energy in its magnetic field.

A.C. Circuit containing pure capacitance

* A capacitor consists of two conductors, separated by an insulating medium called dielectric. These conducting surfaces could be in the form of rectangular or circular plates, or could be spherical or cylindrical in shape.

* A capacitor is also called a condenser.

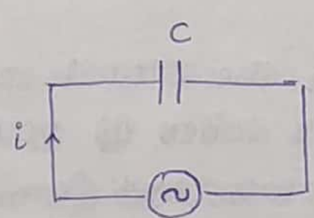
* The capacitor stores energy by electrostatic stress in the dielectric.

* Capacitance is the property of a capacitor to store electric energy. The capacitance of a capacitor is defined as the amount of charge required to create a unit potential difference between its plates.

* When a charge of 'q' coulombs is given to one of the two plates of a capacitor, and if a potential difference of 'V' volts is applied between the two, then its capacitance is

$$C = \frac{q}{V}$$

* Consider a pure capacitor (i.e., its resistance is assumed to be zero) being supplied by an alternating voltage



$v = V_m \sin \omega t$

* Let the voltage source be represented as,

$$v = V_m \sin \omega t.$$

* The instantaneous charge is,

$$q = C v$$

$$q = C V_m \sin \omega t \longrightarrow (2)$$

* Capacitor current is equal to the time rate of flow of charges.

$$\therefore \; i = \frac{dq}{dt}$$

$$= \frac{d}{dt}\left[C V_m \sin \omega t\right]$$

$$= C V_m \frac{d}{dt}\left[\sin \omega t\right]$$

$$= \omega C V_m \cos \omega t$$

$$= \frac{V_m}{(1/\omega C)} \cos \omega t$$

$$i = \frac{V_m}{(1/\omega C)} \sin(\omega t + \pi/2) \longrightarrow (3)$$

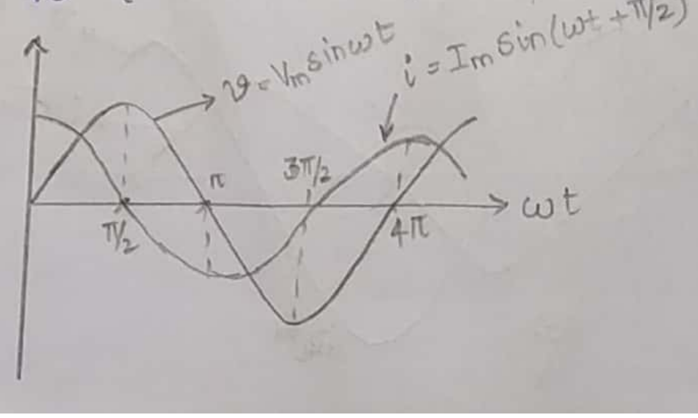
This current is maximum when $\sin(\omega t + \pi/2) = 1$.

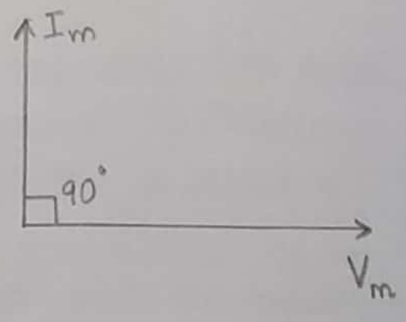
$$\therefore \; I_m = \frac{V_m}{(1/\omega C)} \longrightarrow (4)$$

Thus, equation (3) can be written as,

$$\boxed{i = I_m \sin(\omega t + \pi/2)} \longrightarrow (5).$$

Comparing equations (1) & (5), we say that the instantaneous current leads the voltage by 90° (i.e., one quarter cycle).





* In the expression for I_m, the term $1/\omega C$ is called the capacitive reactance, represented as X_C.

$$\therefore \quad \boxed{X_C = \frac{1}{\omega C}}$$

Since $\omega = 2\pi f$,

$$\boxed{X_C = \frac{1}{2\pi f C}}$$

If 'C' is in farads, 'ω' in radians/sec, then the unit of X_C is ohms.

* The instantaneous power is given by

$$p = vi$$

$$= V_m \sin\omega t \; I_m \sin(\omega t + \pi/2)$$

$$p = V_m I_m \sin\omega t \sin(\omega t + \pi/2) \longrightarrow (6)$$

Using the formula

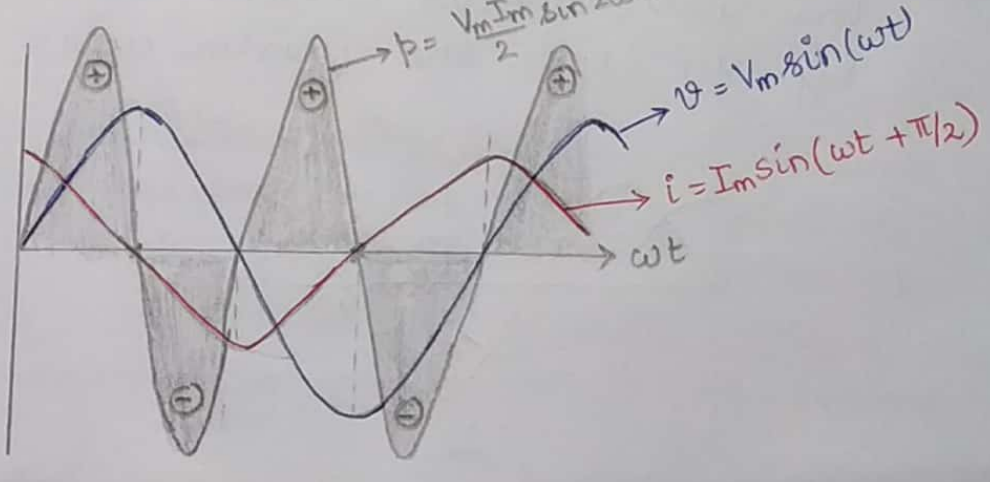
$$\sin(\omega t + \pi/2) = \cos\omega t$$

and $\quad 2\sin\omega t \cos\omega t = \sin 2\omega t$,

we get

$$\boxed{p = \frac{V_m I_m}{2} \sin(2\omega t)} \longrightarrow (7)$$

This is also a sinusoidal function with a frequency which is double the supply frequency. Over one full cycle, the average power is zero. In other words, a pure capacitor does not consume any power.



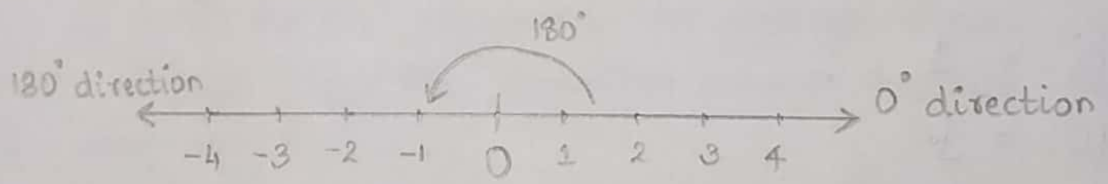
Complex numbers for A.C. circuits.

* Current, voltage and resistance in a D.C. circuit are scalar quantities. They do not change with time and they have only one dimension - the amplitude.

* In alternating circuits, the current, voltage & the opposition to charge flow are all dynamic (alternating in direction and amplitude). Even when we consider an A.C. circuit involving only one frequency, there are one more dimension to consider - the phase shift, in addition to the amplitude.

* In order to successfully analyze A.C. circuits, it is required to have one quantity that can represent these two dimensions of amplitude & phase at once. A complex number is used for this purpose.

* If we consider a number line, then any number can be represented as having a magnitude and phase.



For example, a number +3 represents a number with an amplitude of 3 and a phase of 0°. On the other hand, the number -3 represents a number with an amplitude of 3 and a phase of 180° as shown in the figure.

* This can also be represented using an operator. The angle of rotation is the operator for the number. To indicate 3 with zero phase angle, 3 is multiplied by a factor +1 while a -3 can be represented as 3 multiplied by a factor -1. The operator for -1 is 180° and the operator for +1 is 0°.

* The operator for a number can be any angle between 0° and 360°. In pure inductive and capacitive circuits, the lead/lag angle is 90°. The factor 'j' indicates 90°.

* Thus,

$3 \rightarrow$ Number 3 at 0°

$j3 \rightarrow$ number 3 at 90°

$-3 \rightarrow$ number 3 at 180°

The angle of 180° corresponds to the j operation of 90° repeated twice; this angular rotation is indicated by the factor j^2. Thus,

$$-3 = j^2 3$$

In the same way, we can say that 270° corresponds to j^3. At the same time, 270° corresponds to -90°, and this corresponds to operator $-j$.

* In summary, we can write the following:

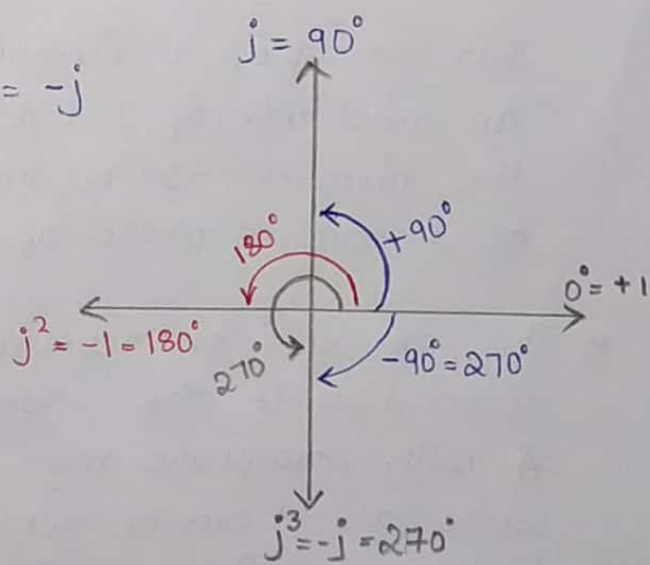
$0^\circ = 1$

$90^\circ = j$

$180^\circ = j^2 = -1$

$270^\circ = j^3 = j^2(j) = (-1)(j) = -j$

360° = same as 0°.



* An angle of 0° or a real number without any j operator is used for resistance. For example, $R = 2\Omega$ is expressed as just 2Ω.

* An angle of 90° or $+j$ is used for inductive reactance (X_L). This rule is always applicable, whether the X_L is in series or parallel in a circuit.

  E.g., An inductor with 14Ω reactance is represented as $+j14\Omega$.

  This is because V_L represents voltage across the inductor, which always leads the current through the inductance by 90°. The $+j$ is also used for V_L.

* An angle of -90° or $-j$ is used for capacitive reactance (X_C). This rule is always applicable to X_C, whether it is in series or parallel.

  E.g., a capacitor with 14Ω reactance is represented as $X_C = -j14\Omega$.

  This is because V_C represents the voltage across a capacitor, which always lags the capacitor current by 90°. The $-j$ is also used for V_C.

Note:- Considering the above convention, $-j$ is used for I_L and $+j$ for I_C, where I_L represents the current through the inductor and I_C the current through the capacitor.

* An actual (or practical) inductor & capacitor always have an associated resistance. Hence, both inductive and capacitive reactances are always associated with some resistance.

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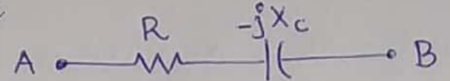
* A practical inductance with a reactance of X_L is usually represented with a series resistance as shown below. This is called a series R-L element.

* Similarly, a practical capacitor is represented with a reactance of X_C with its resistance. i.e., a series R-C element



Electrical Impedance

* Electrical impedance (Z) is the measure of the opposition that a circuit presents to a current when a voltage is applied.

* Quantitatively, the impedance of a two-terminal circuit element is the ratio of the complex representation of a sinusoidal voltage between its terminals to the complex representation of the current through it.

* The impedance can be represented in two forms:-

  a) the polar form

  b) the rectangular (cartesian) form.

* The polar form representation of impedances is suited for multiplication & division of impedances. Here it is represented to have a magnitude ($|Z|$) and a phase angle $[\theta = \arg(Z)]$. Thus, in polar form

$$\bar{Z} = |Z| \angle \theta$$

* In cartesian form, impedance is defined as

$$Z = R + jX$$

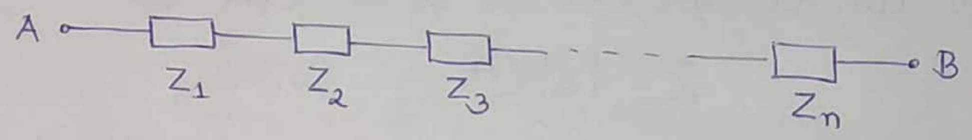
where the real part of impedance is the resistance and the imaginary part is the reactance.

* Thus,

$$\boxed{\begin{array}{ll} \text{for RL element}, & Z = R + jX_L \\ \text{for RC element}, & Z = R - jX_C \end{array}}$$

* Combining Impedances

→ For components connected in series, the current through each impedance is the same; the total impedance is the sum of component impedances.

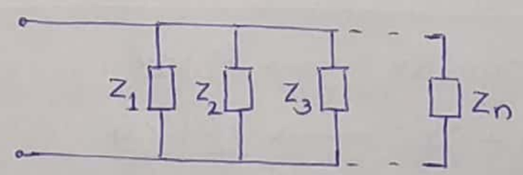


$$Z_{eq} = Z_1 + Z_2 + Z_3 + \cdots + Z_n$$

$$= (R_1 \pm jX_1) + (R_2 \pm jX_2) + (R_3 \pm jX_3) + \cdots + (R_n \pm jX_n)$$

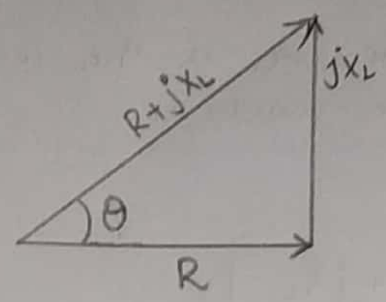
$$Z_{eq} = (R_1 + R_2 + R_3 + \cdots + R_n) + j(\pm X_1 \pm X_2 \pm X_3 \pm \cdots \pm X_n)$$

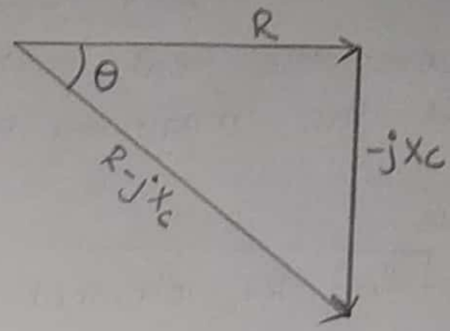
→ For parallel combination,



$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \cdots + \frac{1}{Z_n}$$

* Since $+j$ represents a 90° rotation and $-j$ indicates -90° rotation, impedances can be expressed as phasors as shown below:





This is called the impedance triangle.

* Rectangular to polar conversion

→ Consider an impedance represented in rectangular form as,

$$Z = R + jX$$

→ It can be written in polar form as

$$Z = |Z| \angle \theta$$

using the impedance triangle & applying trigonometric rules.

→ Thus,

$$|Z| = \sqrt{R^2 + X^2}$$

and $\theta = \tan^{-1}(X/R)$.

In other words,

$$\boxed{\begin{array}{l} \text{Rectangular} \longrightarrow \text{Polar} \\ R + jX \longrightarrow |Z| \angle \theta \\ R + jX \longrightarrow \sqrt{R^2+X^2} \angle \tan^{-1}(X/R) \end{array}}$$

* Polar to rectangular conversion

→ Consider an impedance represented in polar form as,

$$Z = |Z| \angle \theta.$$

→ It can be converted to rectangular form represented as

$$Z = R + jX$$

using the power triangle as,

$$R = |Z| \cos\theta$$
$$X = |Z| \sin\theta$$

In other words,

Polar	→	Rectangular
$\lvert Z\rvert \angle \theta$	→	$R + jX$
$\lvert Z\rvert \angle \theta$	→	$R = Z\cos\theta$ $X = Z\sin\theta$

* Example :- a) Convert $14.14 \angle 45^\circ$ to rectangular form.

$$Z \angle \theta = 14.14 \angle 45^\circ$$

$$R = Z\cos\theta = 14.14 \cos 45^\circ = 10$$
$$X = Z\sin\theta = 14.14 \sin 45 = 10$$

$$\therefore 14.14 \angle 45^\circ = 10 + j10$$

This is of the form $R + jX$, the reactance is inductive.

b) Convert $100 \angle -30^\circ$ to rectangular form

$$R = Z\cos\theta = 100 \cos(-30^\circ) = 86.6$$
$$X = Z\sin\theta = 100 \sin(-30^\circ) = -50.$$

$$\therefore 100 \angle -30^\circ = 86.6 - j50$$

This is of the form $R - jX$, the reactance is capacitive.

c) Convert $8+j6$ to polar form.

$$R+jX = 8+j6$$

$$\therefore |Z| = \sqrt{R^2+X^2} = \sqrt{8^2+6^2} = 10$$

$$\angle\theta = \tan^{-1}(X/R) = \tan^{-1}(6/8) = 37^\circ.$$

$$\therefore 8+j6 = 10\angle 37^\circ.$$

d) convert $4-j4$ to polar form

$$R-jX = 4-j4$$

$$|Z| = \sqrt{R^2+X^2} = \sqrt{4^2+(-4)^2} = 5.66$$

$$\angle\theta = \tan^{-1}(X/R) = \tan^{-1}(-4/4) = -45^\circ.$$

$$\therefore 4-j4 = 5.66\angle -45^\circ.$$

Note:- Inductive reactance gives a positive angle when represented in polar form, while capacitive reactance gives a negative angle.

Addition & Multiplication of complex numbers

* Complex numbers are added after converting them to rectangular form. If $Z_1 = 3+j4$ and $Z_2 = 4-j2$, then

$$Z_1+Z_2 = (3+j4)+(4-j2) = (3+4)+j(4-2) = (7+j2)$$

In general,

$$Z_1+Z_2+\dots+Z_n = (R_1+R_2+\dots+R_n)+j(\pm X_1 \pm X_2 \pm \dots \pm X_n)$$

* Complex numbers are multiplied after representing them in polar form.

$$Z_1 * Z_2 = (|Z_1|\angle\pm\theta_1) * (|Z_2|\angle\pm\theta_2)| = Z_1 Z_2 \angle \pm\theta_1 \pm\theta_2$$

Thus, magnitudes are multiplied, while angles are added algebraically.

Subtraction & division of complex numbers

* Complex numbers are subtracted in rectangular form.

$$Z_1 - Z_2 = (R_1 \pm jX_1) - (R_2 \pm jX_2) = (R_1 - R_2) - j(\pm X_1 \pm X_2).$$

* Complex numbers are divided in polar form.

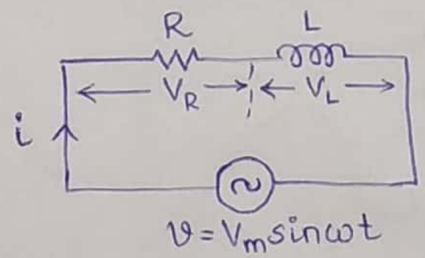
$$\frac{Z_1}{Z_2} = \frac{|Z_1| \angle \pm\theta_1}{|Z_2| \angle \pm\theta_2} = \frac{|Z_1|}{|Z_2|} \angle \pm\theta_1 - (\pm\theta_2).$$

Thus, magnitudes are divided, while the angles are subtracted algebraically.

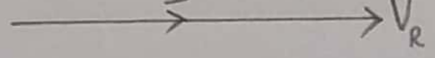
Series R - L Circuit

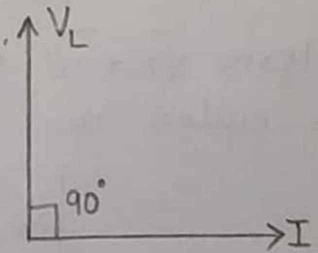
* Let us consider an A.C. circuit containing a pure resistance and a pure inductance connected in series.

* This circuit can also be envisioned as a practical inductor with inductance L and its resistance being R.



* Let V_R be the voltage drop across the resistance and V_L be the voltage across the inductor. The phasors of V_L and V_R w.r.t I are shown below.





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* Let,

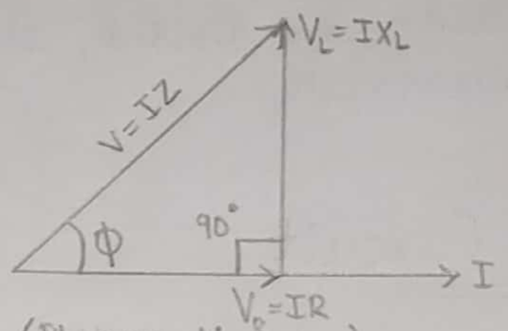
V = RMS value of applied voltage

I = RMS value of current.

Voltage drop across resistance, $V_R = IR$ --- in phase with 'I'

Voltage across the inductor, $V_L = IX_L$ --- leads 'I' by 90°

* The combined phasor diagram is shown below. The applied voltage is the phasor sum of V_R and V_L. The current I is in phase with V_R, and lags V_L by 90°.



(Phasor diagram)

* By comparing the phasors of V and I, we can say that the applied voltage and current have a phase difference of $\phi < 90°$. In other words, current through the R-L circuit lags voltage by an angle $\phi < 90°$.

* From the phasor diagram,

$$V^2 = V_R^2 + V_L^2$$

$$V^2 = (IR)^2 + (IX_L)^2$$

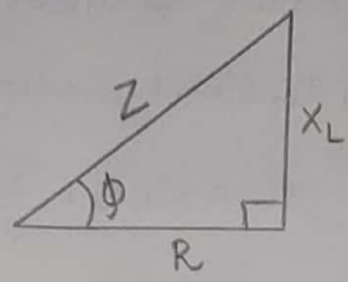
$$\Rightarrow V = I\sqrt{R^2 + X_L^2}$$

or $$\boxed{I = \frac{V}{\sqrt{R^2 + X_L^2}}}$$

The term $\sqrt{R^2 + X_L^2}$ offers opposition to current flow and is called the impedance (Z) of the circuit. Thus,

$$\boxed{I = \frac{V}{Z}}$$

* Using the phasor diagram, we can write the impedance triangle as shown below.



(Impedance triangle)

Thus,

$$R = Z\cos\phi$$

$$X = Z\sin\phi$$

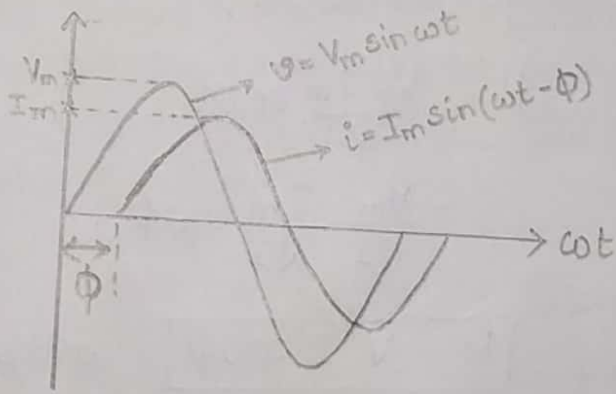
$$\therefore \sin\phi = X/Z$$

$$\cos\phi = R/Z$$

$$\Rightarrow \tan\phi = \frac{\sin\phi}{\cos\phi} = \frac{X_L/Z}{R/Z} = \frac{X_L}{R}$$

$$\Rightarrow \boxed{\phi = \tan^{-1}\left(\frac{X_L}{R}\right)}$$

* The waveform is shown below.



* Therefore, if the applied voltage is

$$v = V_m \sin\omega t$$

then the current is

$$\boxed{i = I_m \sin(\omega t - \phi)}$$

* The instantaneous power will be,

$$p = vi$$
$$= V_m \sin\omega t \; I_m \sin(\omega t - \phi)$$

$$p = V_m I_m \sin\omega t \; \sin(\omega t - \phi)$$

Using the formula $\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$, we can write

$$p = V_m I_m \times \frac{1}{2}\left[\cos(\omega t - \omega t + \phi) - \cos(\omega t + \omega t - \phi)\right]$$

$$= \frac{V_m I_m}{2}\left[\cos\phi - \cos(2\omega t - \phi)\right]$$

$$\boxed{p = \frac{V_m I_m}{2}\cos\phi - \frac{V_m I_m}{2}\cos(2\omega t - \phi)}$$

→ The second term is a periodically varying quantity, whose frequency is two times the frequency of applied voltage, has a zero average value.

→ The first term varies cosinusoidally, and its average value is,

$$P = \frac{V_m I_m}{2}\cos\phi$$

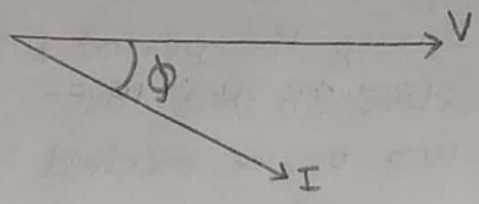
$$= \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\phi$$

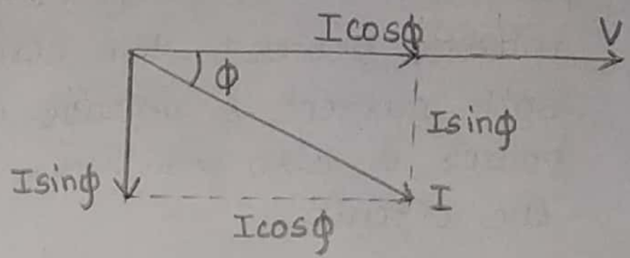
$$\boxed{P = V I \cos\phi} \text{ watts.}$$

→ In other words, the average or mean power consumed by the circuit is the product of the rms voltage, rms current and the cosine of the angle between voltage & current.

* The term 'cos ϕ' is known as the power factor of the circuit.

* Considering voltage as the reference phasor, the phasor diagram will be as shown below.

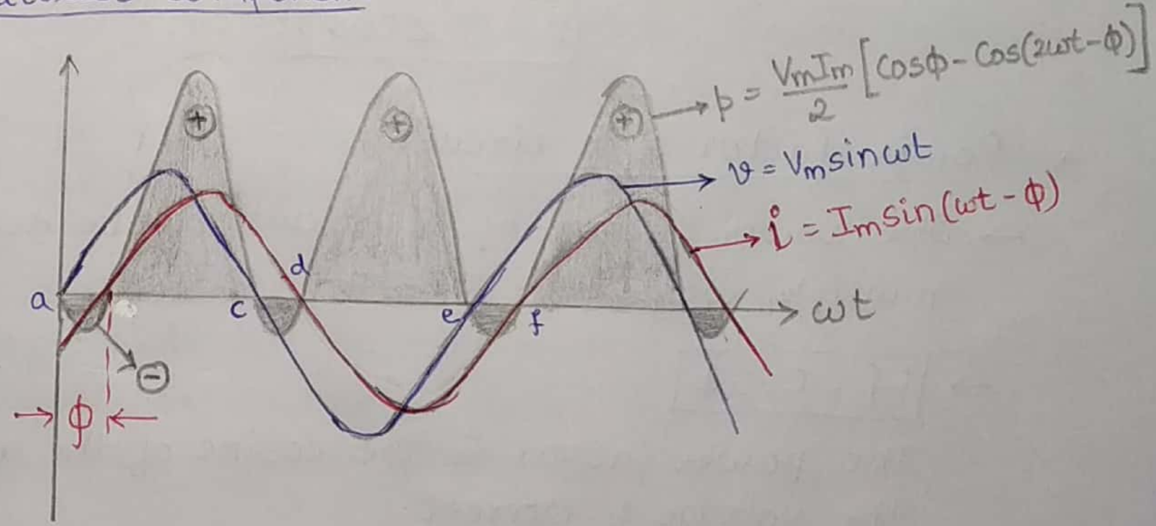




The current phasor can be resolved into two components:

i) I cos ϕ, which is in phase with the applied voltage. Hence it is known as "in-phase" component. Only this component contributes to the power consumed by the circuit. Hence, it is known as real component or active component or wattful component.

ii) I sin ϕ, which is in quadrature (at 90°) with the applied voltage. Hence it is known as "quadrature component". This component does not contribute anything for the power consumed by the circuit. Hence it is called the reactive component or as wattless component.



* The power curve ~~under~~ for R-L circuit (series) indicates that the greater part is positive and the smaller part is negative, so that the net power over the cycle is positive.

* During the time interval a to b, the applied voltage is positive while the current is negative. Hence the power ($p = vi$) is negative. During this time, the inductance returns power to the circuit. During the period b to c, both current & voltage are positive so that the power is also positive, indicating power received by the circuit.

* The power consumed is due to ohmic resistance only, as a pure inductance does not consume any power. The power so consumed is given by

$$\boxed{P = VI\cos\phi}$$

Now, $\cos\phi = \dfrac{R}{Z}$.

$$\therefore P = VI\left(\frac{R}{Z}\right)$$

$$= \frac{V}{Z}(IR)$$

$$\boxed{P = I^2R}$$

Since $R = Z\cos\phi$,

$$\boxed{P = I^2 Z\cos\phi}$$

* Power factor of a circuit

→ The power factor of a circuit can be defined in multiple ways.

→ $\boxed{Pf = \cos\phi}$

The power factor is the cosine of the angle between the voltage & current.

→ From the impedance triangle,

$$\boxed{pf = \frac{R}{Z}}$$

The power factor is the ratio of the resistance of the circuit to the impedance of the circuit.

→ Since power is given by

$$P = VI \cos\phi,$$

$$\boxed{\cos\phi = \frac{P}{VI}}$$

Power factor is the ratio of average power to the product of the rms value of voltage & rms value of current.

* Importance (Significance) of power factor

→ The power in an A.C. circuit is given by,

$$P = VI \cos\phi$$

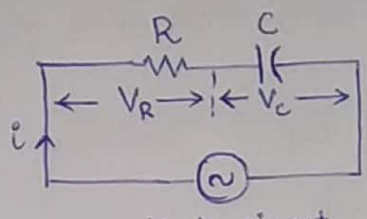
If the power factor of the load is small, the active power generated by the generator, and the active power received & transmitted is reduced. To generate the same capacity of power, the capacity of the generator has to be increased which involves additional cost.

→ At constant voltage if the power factor decreases then the load draws higher current to satisfy its power requirement. This leads to higher losses (I^2R) and reduces transmission efficiency.

→ In addition, higher current drawn due to low pf will necessiate ~~the~~ an increase in current carrying capacity of the conductor. Hence, large conductor sizes will be required, which leads to higher cost.

Series R - C Circuit

* Consider an a.c. circuit containing a pure resistance and a pure capacitance connected in series and supplied by a source $v = V_m \sin \omega t$.

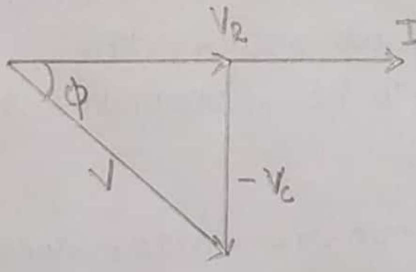


* Let V = RMS value of supply voltage
  I = RMS value of supply current.

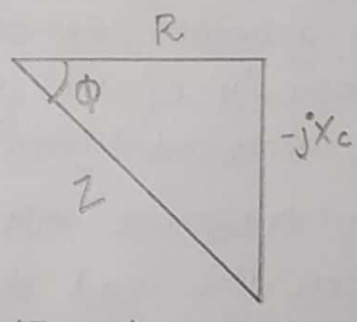
∴ Voltage drop across the resistance, $V_R = IR$ ... in phase with 'I'.

Voltage across the capacitance, $V_C = I X_C$ ... Lagging 'I' by 90°.

* The voltage triangle & the impedance triangle are as shown below.



(Voltage triangle)



(Impedance triangle)

* From the voltage triangle,

$$V^2 = V_R^2 + (-V_C)^2 \Rightarrow V = \sqrt{V_R^2 + V_C^2}$$

$$\Rightarrow V = \sqrt{(IR)^2 + (-IX_C)^2}$$

$$V = I\sqrt{R^2 + X_C^2}$$

$$\therefore \frac{V}{I} = \sqrt{R^2 + X_C^2}$$

$$\Rightarrow \boxed{Z = \sqrt{R^2 + X_C^2}}$$

where 'Z' is the impedance of the R-C circuit.

From the impedance triangle,

$$\boxed{R = Z\cos\phi}$$

$$\boxed{X_C = Z\sin\phi}$$

$$\therefore \frac{Z\sin\phi}{Z\cos\phi} = \frac{X_C}{R}$$

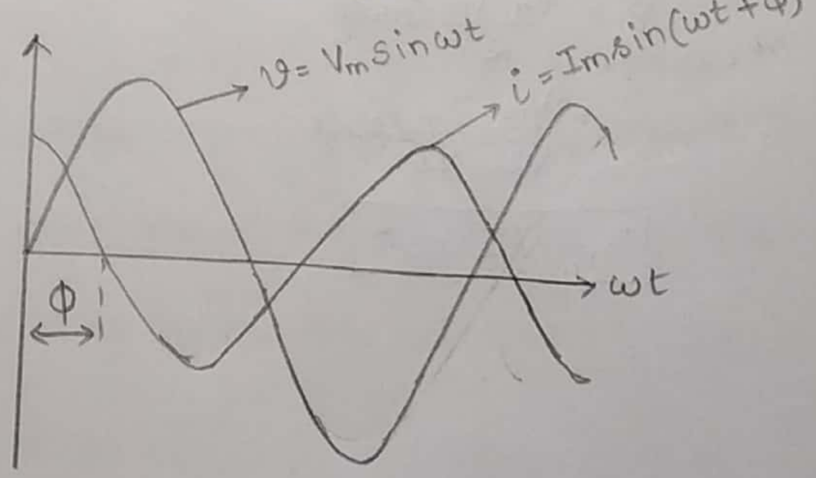
$$\Rightarrow \tan\phi = \frac{X_C}{R}$$

$$\Rightarrow \boxed{\phi = \tan^{-1}\left(\frac{X_C}{R}\right)}$$

* The power factor is given by

$$\boxed{\cos\phi = \frac{R}{Z}}$$

* The voltage triangle shows that the current leads the voltage by an angle $\phi < 90^\circ$. The corresponding waveform is shown below.



* Therefore, the current is given by,

$$\boxed{i = I_m \sin(\omega t + \phi)}$$

* The power is,

$$p = vi$$

$$= V_m \sin\omega t * I_m \sin(\omega t + \phi)$$

$$p = V_m I_m \sin\omega t \sin(\omega t + \phi)$$

using $\sin A \sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$, we get

$$p = \frac{V_m I_m}{2}\left[\cos(-\phi) - \cos(2\omega t + \phi)\right]$$

since $\cos(-\theta) = \cos\theta$,

$$p = \frac{V_m I_m}{2}\left[\cos\phi - \cos(2\omega t + \phi)\right]$$

$$\boxed{p = \frac{V_m I_m}{2}\cos\phi - \frac{V_m I_m}{2}\cos(2\omega t + \phi)}$$

→ The second term is a periodically varying quantity, whose frequency is twice the frequency of applied voltage, has an average value over one cycle zero. Hence, it does not contribute to the average value of power consumed by the circuit.

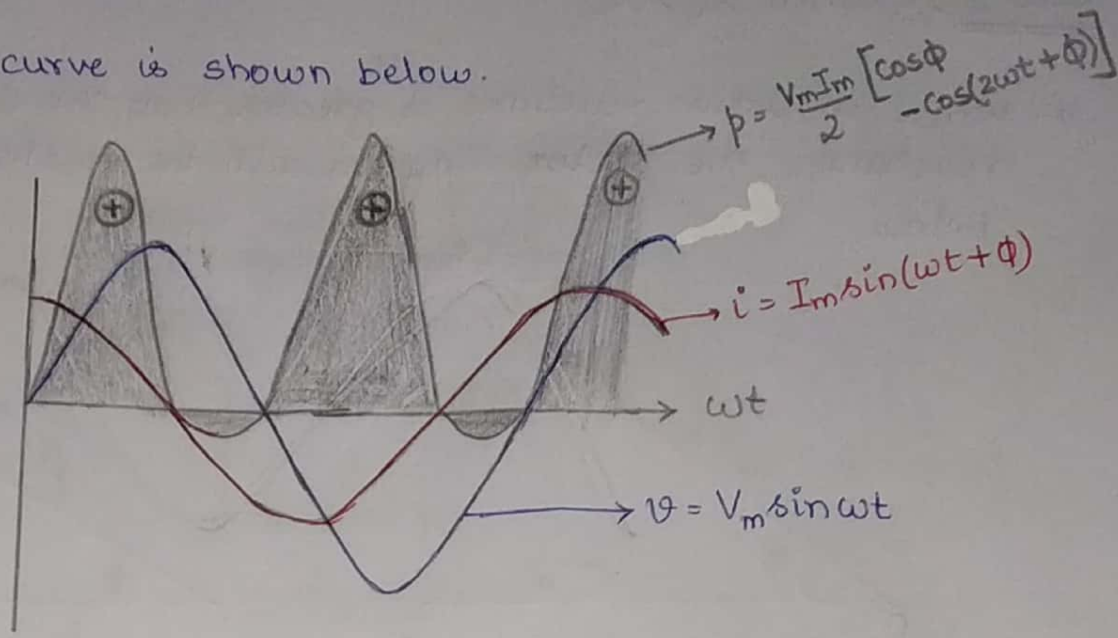
→ The average power consumed is given by,

$$P = \frac{V_m I_m}{2}\cos\phi$$

$$= \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}}\cos\phi$$

$$\boxed{P = VI\cos\phi}$$

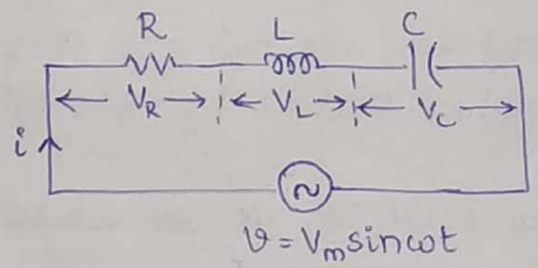
* The power curve is shown below.



Again, the net power is positive since the greater part of the power curve is positive and smaller part is negative.

Series R-L-C Circuit.

* Consider an RLC series circuit as shown below, being supplied by an A.C source $v = V_m \sin\omega t$.



* Let

  V = RMS value of the supply voltage

  I = RMS value of the supply current.

Voltage drop across the resistance, $V_R = IR$ --- in phase with I

Voltage drop across the inductor, $V_L = IX_L$ --- leads 'I' by 90°

Voltage drop across the capacitor, $V_C = IX_C$ --- lags 'I' by 90°.

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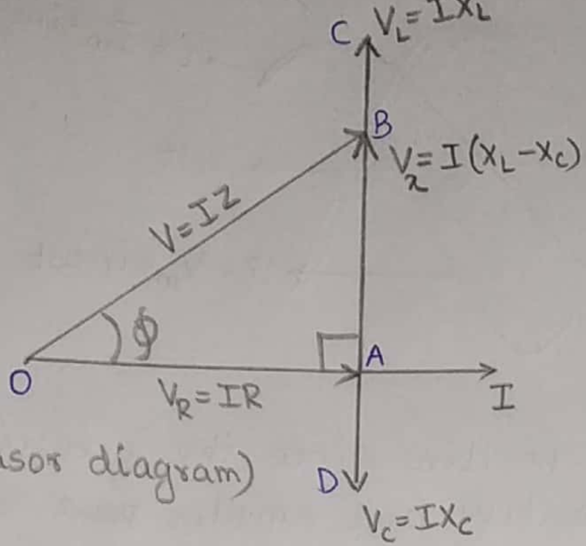
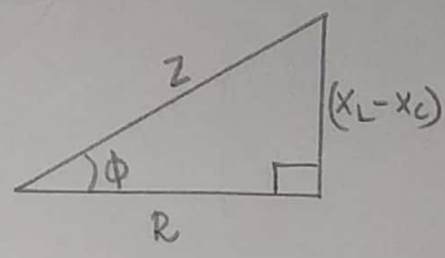
Case-1 :- When $X_L > X_C$

* When inductive reactance is greater than the capacitive reactance, the phasor diagram will be as shown below.



(Phasor diagram)



(Impedance triangle)

* In this phasor,

$$OA = V_R = IR$$
$$AC = V_L = IX_L$$
$$AD = V_C = IX_C$$
$$AB = I(X_L - X_C) \qquad (\because X_L > X_C)$$

* From the phasor diagram, we can see that the current lags the applied voltage by $\phi < 90^\circ$.

* From the impedance triangle, we can write

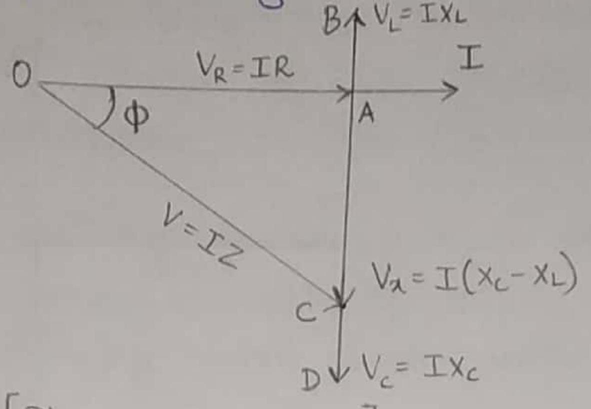
$$\boxed{Z = \sqrt{R^2 + (X_L - X_C)^2}}$$

* This is similar to R-L circuit (I lags V by $\phi < 90^\circ$). Therefore,

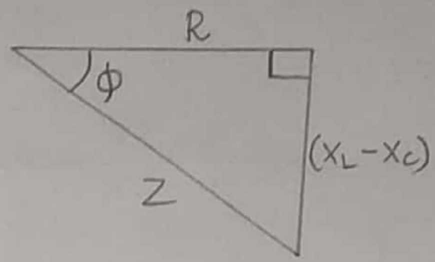
$$\boxed{\begin{aligned} v &= V_m \sin \omega t \\ i &= I_m \sin(\omega t - \phi) \\ P &= VI \cos \phi \end{aligned}}$$

Case - 2 :- When $X_L < X_C$.

* When inductive reactance is less than capacitive reactance, the phasor diagram will be as shown below.



[Phasor Diagram]



[Impedance triangle]

* Here,

$$OA = V_R = IR$$
$$AB = V_L = IX_L$$
$$AD = V_C = IX_C$$
$$AC = I(X_C - X_L) \qquad (\because X_C > X_L)$$

* From the phasor diagram, we can see that the current leads voltage by an angle $\phi < 90^\circ$.

* From the impedance triangle, we can write

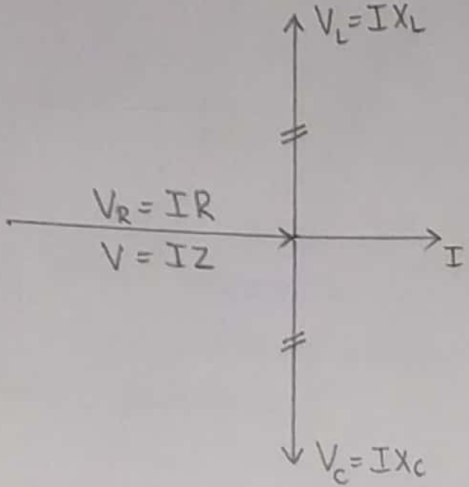
$$\boxed{Z = \sqrt{R^2 + (X_C - X_L)^2}}$$

* Since current leads voltage by an angle ϕ, this circuit is similar to an RC circuit. Therefore,

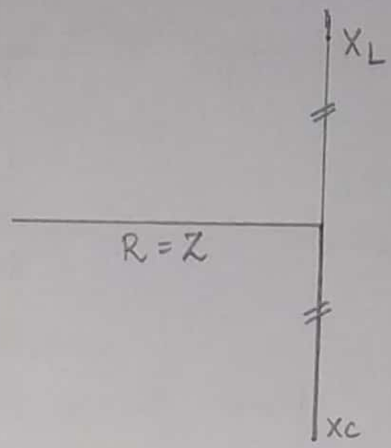
$$\boxed{\begin{aligned} v &= V_m \sin \omega t \\ i &= I_m \sin(\omega t + \phi) \\ P &= VI \cos\phi \end{aligned}}$$

Case-3 :- When $X_L = X_C$

* When the inductive & capacitive reactances are equal, the phasor diagram is shown below.



[Phasor Diagram]



[Impedance]

* V_L and V_C cancel each other. Therefore,

$$V = V_R$$

$$\Rightarrow IZ = IR$$

$$\Rightarrow \boxed{Z = R}$$

Since $V_L = V_C$, $IX_L = IX_C \Rightarrow X_L = X_C$. Hence,

$Z = \sqrt{R^2 + (X_L - X_C)^2} \Rightarrow Z = R$

* Thus, this circuit behaves like a purely resistive circuit. Hence,

$$\boxed{\begin{array}{l} v = V_m \sin \omega t \\ i = I_m \sin \omega t \end{array}}$$

$$\cos\phi = \frac{R}{Z} = \frac{R}{R} = 1 \quad \phi$$

Thus,
$$P = VI\cos\phi = VI * 1$$

$$\therefore \boxed{P = VI}$$

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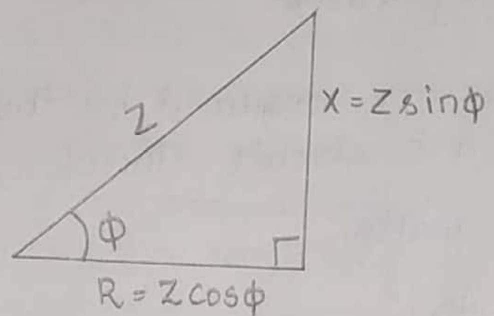
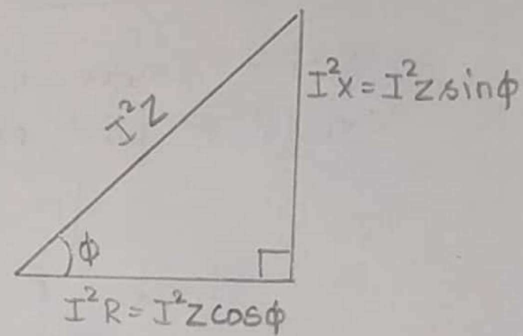
Power Triangle

* Consider the impedance triangle of an a.c circuit shown below. In this, the 'R' remains constant, while the overall shape of the triangle changes based on 'X', the reactance, as the frequency changes.

* This can be converted to a power triangle representing the three elements of power in an A.C circuit. By multiplying every phasor with I^2, we get the power triangle.



[Impedance Triangle]



[Power triangle]

For an A.C circuit,

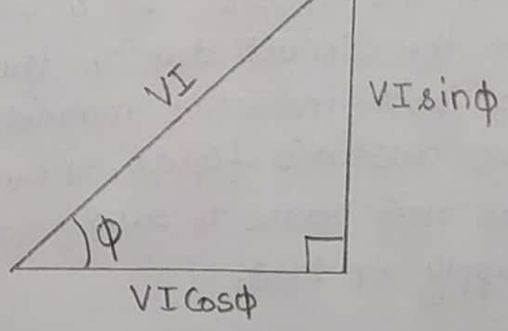
$$V = IZ.$$

$$\therefore \quad I^2 Z \cos\phi = I(IZ)\cos\phi = VI\cos\phi$$

$$I^2 Z \sin\phi = I(IZ)\sin\phi = VI\sin\phi$$

$$I^2 Z = I(IZ) = VI.$$

* Then the power triangle can be written as,



* There are three different powers in an A.C. circuit.

a) Active power ($VI\cos\phi$)

b) Reactive power ($VI\sin\phi$)

c) Apparent power (VI).

* Active power (P)

→ This power is also known as real power or true power and actually performs work in an electric circuit.

→ The unit of active power is watts. Thus,

$$\boxed{P = VI\cos\phi} \text{ watts.}$$

→ Active power is the power consumed by the resistive part of the a.c circuit. Therefore,

$$\boxed{P = I^2R} \text{ watts.}$$

Since $R = Z\cos\phi$,

$$\boxed{P = I^2 Z\cos\phi} \text{ watts.}$$

→ This is also the average power of the circuit over one cycle.

* Reactive Power (Q)

→ Reactive power or wattless power exists in A.C circuits when the voltage & current are not in phase.

→ Unlike real power, the reactive power does not do any real "work" in the circuit. It actually takes power away from the circuit due to the creation and reduction of both inductive magnetic fields and capacitive electrostatic fields, thereby making it harder for the real power to supply power directly to a supply or load.

→ If reactive power does not do any useful work, then why is it required? The answer for this lies in the reactive components of the a.c. circuit. Whenever current flows in an A.C. circuit, electrostatic and electromagnetic fields are created. These fields need to be sustained for proper working of the reactive components. Reactive power is the portion of power that helps in establishing & sustaining the electric & magnetic fields required by the alternating current equipment.

→ The energy stored by an inductor in its magnetic field tries to control the current, while the energy stored in a capacitor electrostatic fields tries to control the voltage. There is storage of energy during one alternation, while the entire stored energy is completely returned during the next alternation. Therefore, both inductor & capacitor do not consume or dissipate any power.

→ Reactive power has the unit volt-ampere reactive (VAr) and is given by

$$\boxed{Q = VI\sin\phi} \text{ VAr.}$$

It can also be written as,

$$\boxed{Q = I^2 X} \text{ VAr}$$

Since $X = Z\sin\phi$,

$$\boxed{Q = I^2 Z\sin\phi} \text{ VAr}$$

* Apparent power (s)

→ Most of our loads are reactive. This means that they all need reactive power along with active power. This is very important because although the current associated with reactive power does not do any useful work at the load, it still must be supplied to the load.

→ Conductors, transformers and generators must be sized to carry the total current, not just the current that does useful work.

→ Thus, if $V = V_m/\sqrt{2}$ is the effective value of voltage across the load and $I = I_m/\sqrt{2}$ is the rms value of the current through the load, then apparently, it seems that the power going to the load should be the product of V and I. Hence, this power is called the apparent power(s).

$$\therefore \boxed{S = VI} \text{ volt-amperes}$$

→ Apparent power has the unit volt-amperes (VA).

* Using the concept of apparent power, we can write

Real power = Apparent power * $\cos\phi$
Reactive power = Apparent power * $\sin\phi$.

In other words,

$$\boxed{\begin{aligned} P &= S\cos\phi \\ Q &= S\sin\phi \end{aligned}}$$

Important notes

* In an A.C. circuit if the active and apparent powers are equal, then

$$VI\cos\phi = VI$$

$$\Rightarrow \cos\phi = 1.$$

Thus, the system will behave like a purely resistive circuit. Only dissipation will occur.

* On the other hand if the reactive and apparent powers are equal, then

$$VI\sin\phi = VI$$

$$\Rightarrow \sin\phi = 1$$

$$\Rightarrow \cos\phi = 0$$

$$\Rightarrow P = VI\cos\phi = 0.$$

Thus, the system will behave like a purely reactive circuit & does not consume any power.

* The power factor of a circuit also signifies the nature of the circuit. i.e., whether the equivalent circuit is resistive, inductive or capacitive. The power factor is expressed as a numerical value with the term 'leading' or 'lagging'. For example, pf = 0.7 lag or pf = 0.8 lead. The lagging or leading refers to the phase of current vector w.r.t voltage. Therefore,

→ a lagging power factor indicates an inductive (R-L) circuit (current lags voltage)

→ a leading power factor implies a capacitive (R-C) circuit (current leads voltage)

→ a unity power factor signifies a purely resistive circuit.

* The vector sum of active and reactive powers is called the complex power. The apparent power is the magnitude of the complex power.