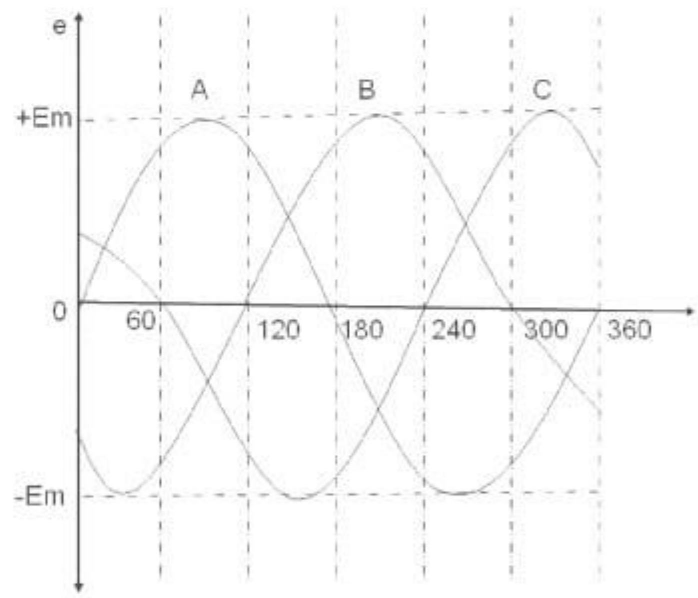
Unit-III

Three phase circuits: Generation of three-phase power, representation of balanced star and delta connected loads the relation between phase and line values of voltage and current from phasor diagrams, advantages of three-phase systems. Measurement of three-phase power by two-wattmeter method.

Transformers: Single phase transformers: Construction, principle of working, EMF equations, voltage and current ratios, losses, definition of regulation and efficiency.

THREE PHASE AC CIRCUITS

A three phase supply is a set of three alternating quantities displaced from each other by an angle of 120°. A three phase voltage is shown in the figure. It consists of three phases- phase A, phase B and phase C. Phase A waveform starts at 0°. Phase B waveform stars at 120° and phase C waveform at 240°.



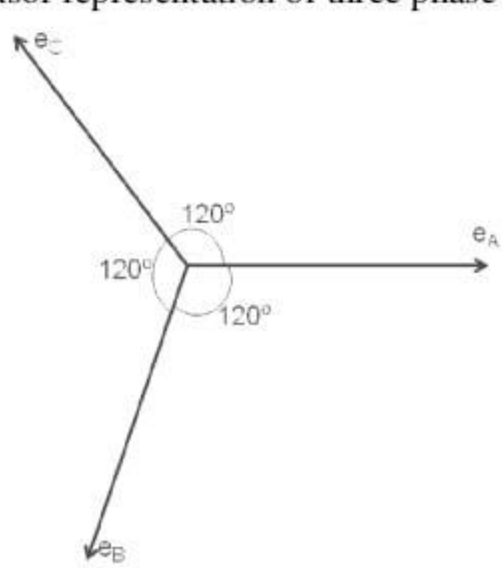
The three phase voltage can be represented by a set of three equations as shown below.

$$e_A = E_m \sin \omega t$$
$$e_B = E_m \sin(\omega t - 120^\circ)$$
$$e_C = E_m \sin(\omega t - 240^\circ) = E_m \sin(\omega t + 120^\circ)$$

The sum of the three phase voltages at any instant is equal to zero.

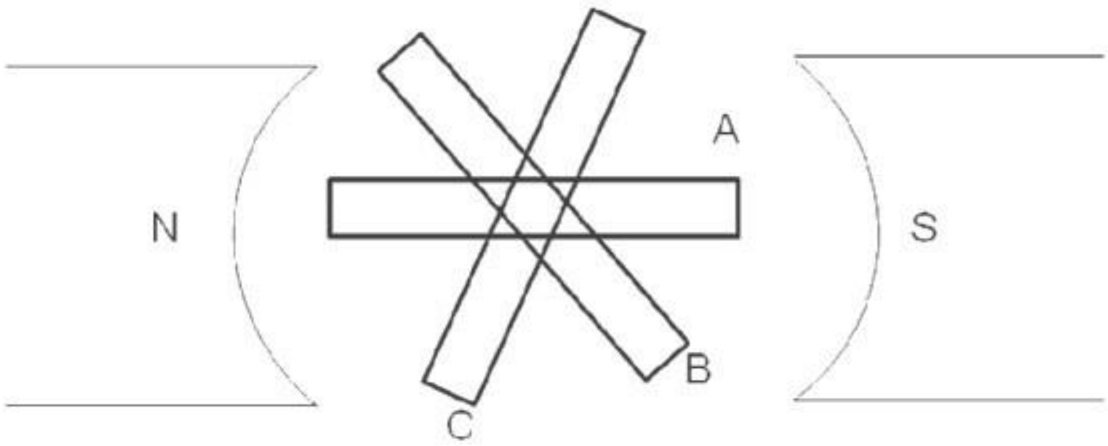
$$e_A + e_B + e_C = 0$$

The phasor representation of three phase voltages is as shown.



The phase A voltage is taken as the reference and is drawn along the x-axis. The phase B voltage lags behind the phase A voltage by 120^o. The phase C voltage lags behind the phase A voltage by 240^o and phase B voltage by 120^o.

Generation of Three Phase Voltage



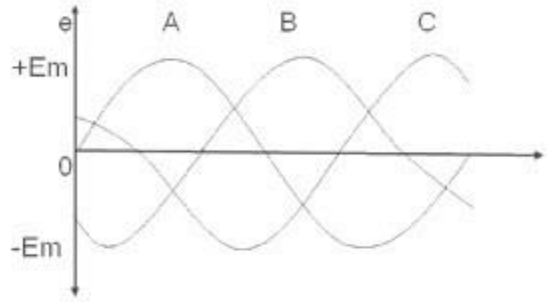
Three Phase voltage can be generated by placing three rectangular coils displaced in space by 120^o in a uniform magnetic field. When these coils rotate with a uniform angular velocity of ω rad/sec, a sinusoidal emf displaced by 120^o is induced in these coils.

Necessity and advantages of three phase systems

- 3Φ power has a constant magnitude whereas 1Φ power pulsates from zero to peak value at twice the supply frequency
- A 3Φ system can set up a rotating magnetic field in stationary windings. This is not possible with a 1Φ supply.
- For the same rating 3Φ machines are smaller, simpler in construction and have better operating characteristics than 1Φ machines
- To transmit the same amount of power over a fixed distance at a given voltage, the 3Φ system requires only $3/4^{th}$ the weight of copper that is required by the 1Φ system
- The voltage regulation of a 3Φ transmission line is better than that of 1Φ line

Phase Sequence

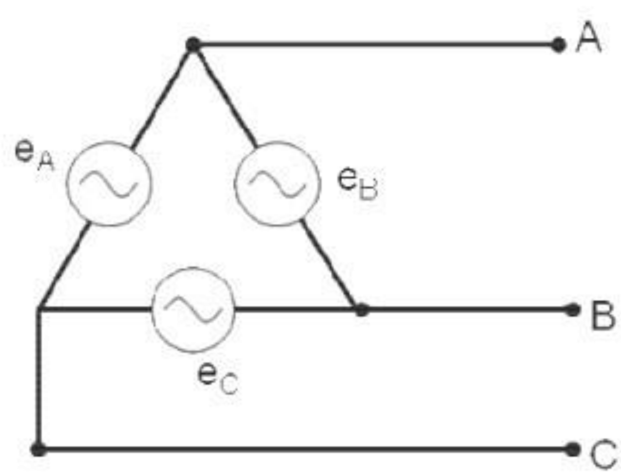
The order in which the voltages in the three phases reach their maximum value



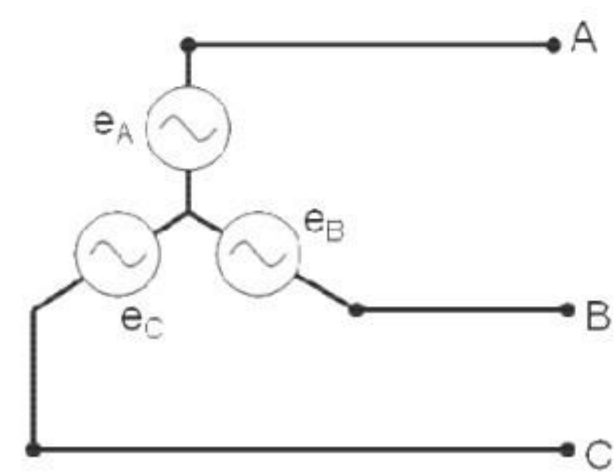
For the waveform shown in figure, phase A reaches the maximum value first, followed by phase B and then by phase C. hence the phase sequence is A-B-C.

Balanced Supply

A supply is said to be balanced if all three voltages are equal in magnitude and displaced by 120^0
A three phase supply can be connected in two ways - Either in Delta connection or in Star connection as shown in the figure.



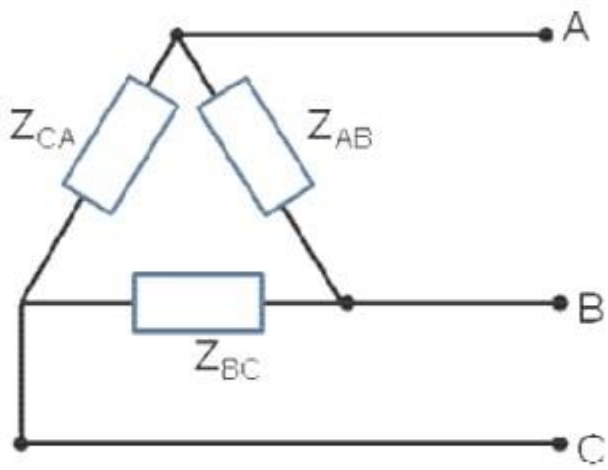
Delta Connection



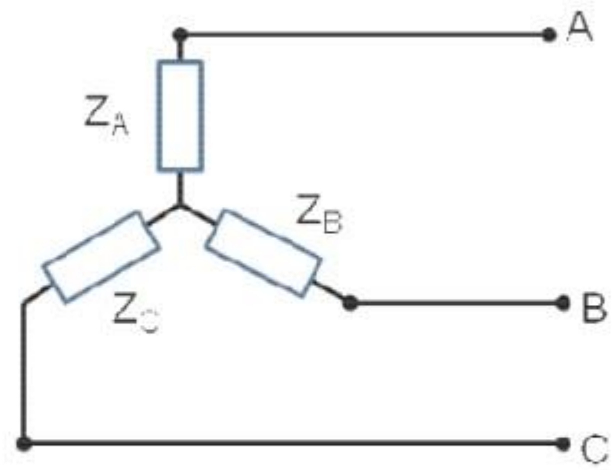
Star Connection

Balanced Load

A load is said to be balanced if the impedances in all three phases are equal in magnitude and phase
A three phase load can be connected in two ways - Either in Delta connection or in Star connection as shown in the figure.



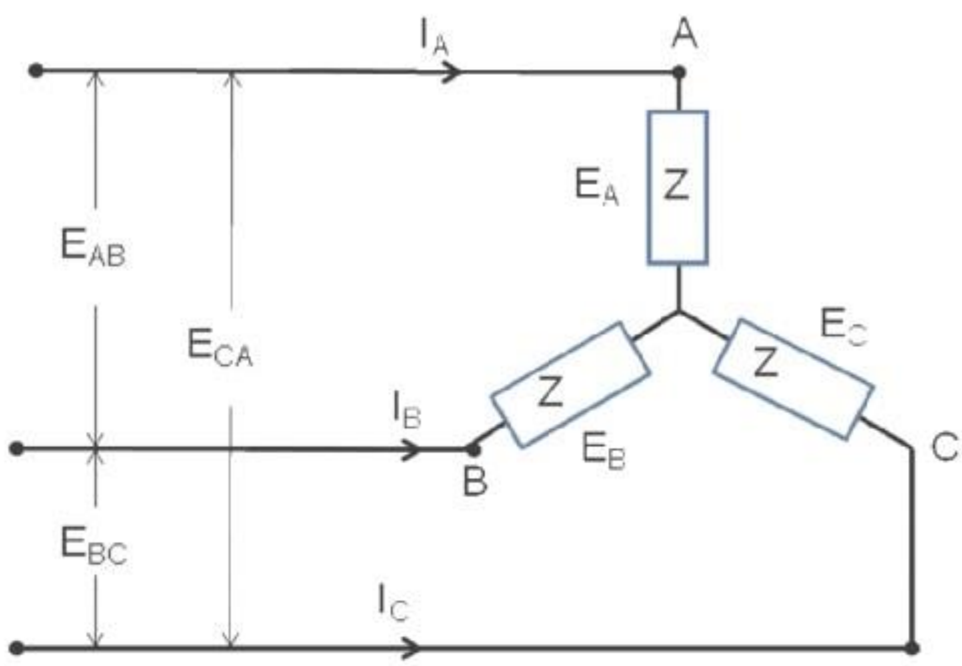
Delta Connection



Star Connection

Balanced Star Connected Load

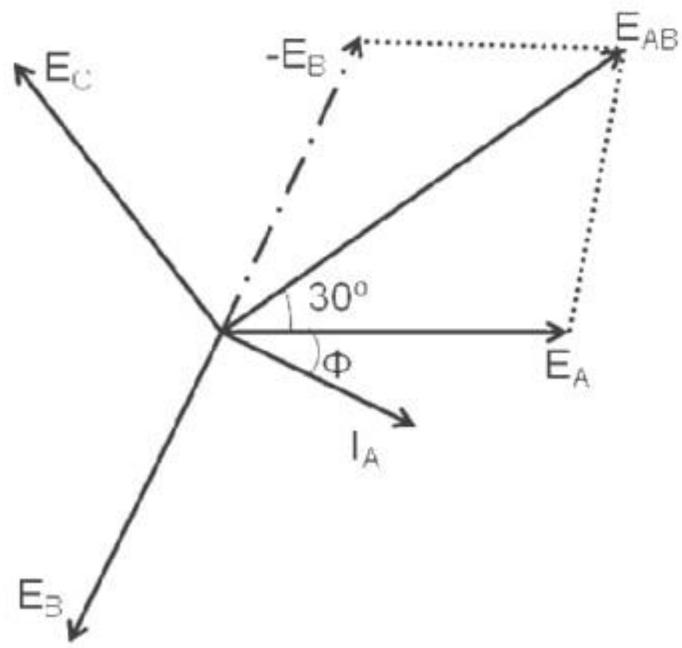
A balanced star connected load is shown in the figure. A phase voltage is defined as voltage across any phase of the three phase load. The phase voltages shown in figure are E_A, E_B and E_C. A line voltage is defined as the voltage between any two lines. The line voltages shown in the figure are E_{AB}, E_{BC} and E_{CA}. The line currents are I_A, I_B and I_C. For a star connected load, the phase currents are same as the line currents.



Using Kirchoff's voltage law, the line voltages can be written in terms of the phase voltages as shown below.

$$E_{AB} = E_A - E_B$$
$$E_{BC} = E_B - E_C$$
$$E_{CA} = E_C - E_A$$

The phasor diagram shows the three phase voltages and the line voltage E_{AB} drawn from E_A and $-E_B$ phasors. The phasor for current I_A is also shown. It is assumed that the load is inductive.



From the phasor diagram we see that the line voltage E_{AB} leads the phase voltage E_A by 30^0. The magnitude of the two voltages can be related as follows.

$$E_{AB} = 2E_A \cos 30^\circ = \sqrt{3}E_A$$

Hence for a balanced star connected load we can make the following conclusions.

$$E_l = \sqrt{3}E_{ph}$$
$$I_l = I_{ph}$$

Line voltage leads phase voltage by 30^0

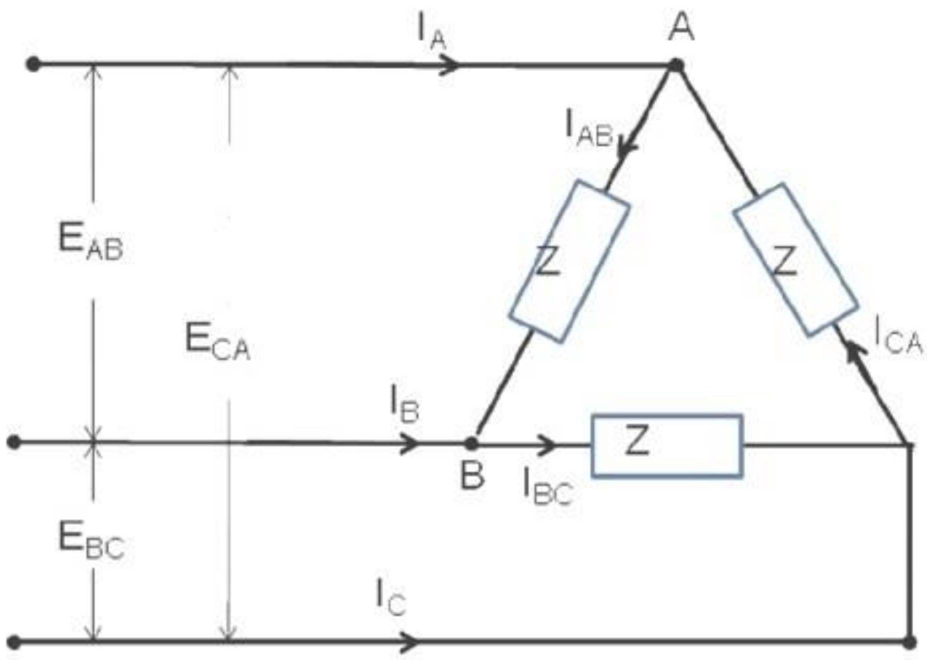
Three phase Power

In a single phase circuit, the power is given by VIcosΦ. It can also be written as $V_{ph}I_{ph}\cos\Phi$. The power in a three circuit will be three times the power in a single phase circuit.

$$P = 3E_{ph}I_{ph}\cos\Phi$$
$$P = \sqrt{3}E_l I_l \cos\Phi$$

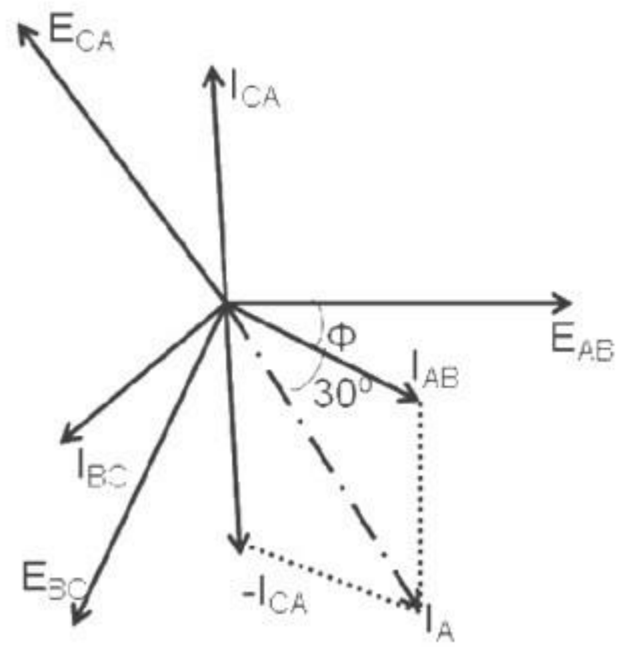
Balanced Delta Connected Load

A balanced delta connected load is shown in the figure. The phase currents are I_{AB}, I_{BC} and I_{CA}. The line currents are I_A, I_B and I_C. For a delta connected load, the phase voltages are same as the line voltages given by E_{AB}, E_{BC} and E_{CA}.



Using Kirchoff's current law, the line currents can be written in terms of the phase currents as shown below.

$$I_A = I_{AB} - I_{CA}$$
$$I_B = I_{BC} - I_{AB}$$
$$I_C = I_{CA} - I_{BC}$$



The phasor diagram shows the three voltages E_{AB}, E_{BC} and E_{CA} and the three phase currents I_{AB}, I_{BC} and I_{CA} lagging behind the respective phase voltages by an angle Φ. This is drawn by assuming that the load is inductive. From the phase currents I_{AB} and $-I_{CA}$, the line current I_A is drawn as shown in the figure.

From the phasor diagram we see that the line current I_A lags behind the phase phase current I_{AB} by 30^o. The magnitude of the two currents can be related as follows.

$$I_A = 2I_{AB}\cos 30^\circ = \sqrt{3}I_{AB}$$

Hence for a balanced delta connected load we can make the following conclusions.

$$I_l = \sqrt{3}I_{ph}$$

$$E_l = E_{ph}$$

Line current lags behind phase current by 30^0

Three phase Power

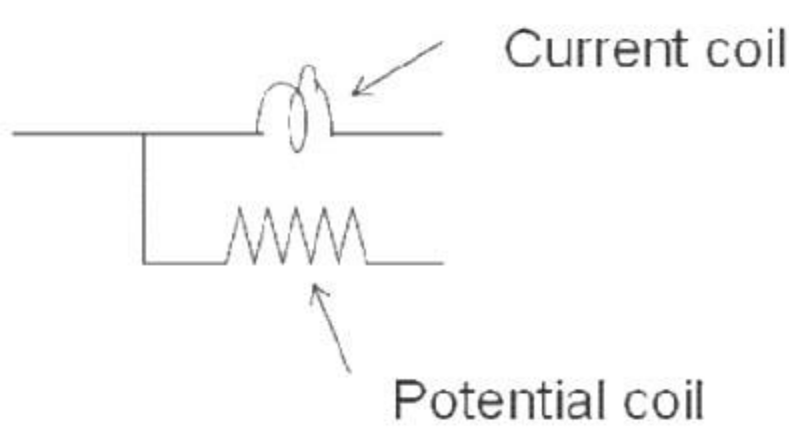
The three phase power for a delta connected load can be derived in the same way as that for a star connected load.

$$P = 3E_{ph}I_{ph}\cos\Phi$$

$$P = \sqrt{3}E_l I_l \cos\Phi$$

Measurement of power and power factor by two wattmeter method

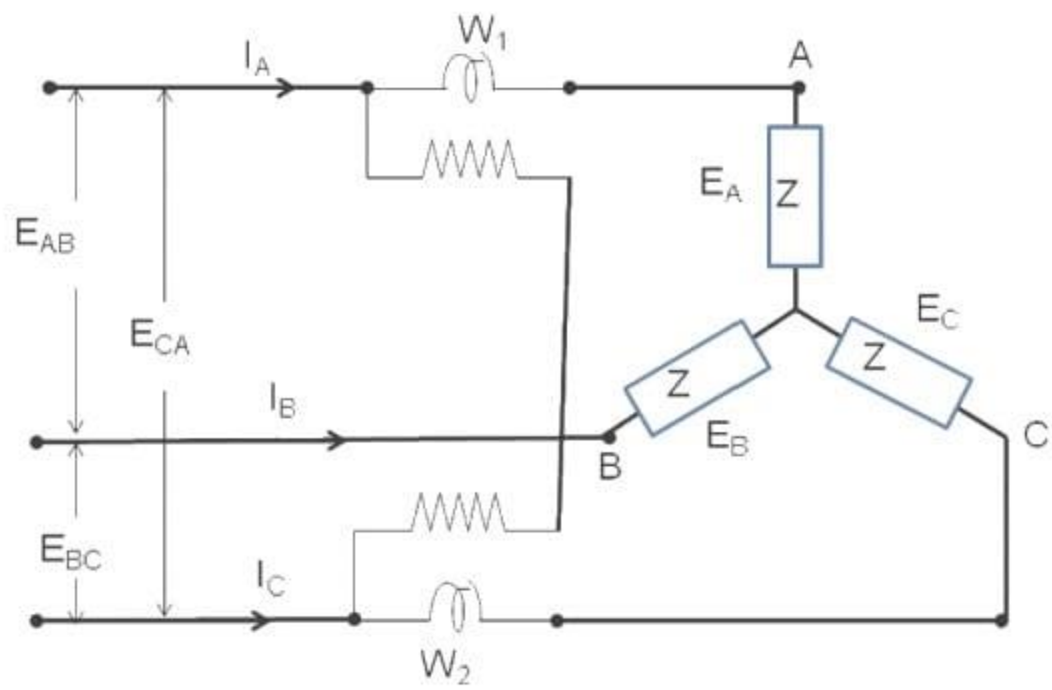
The power in a three phase circuit can be measured by connecting two wattmeters in any of the two phases of the three phase circuit. A wattmeter consists of a current coil and a potential coil as shown in the figure.



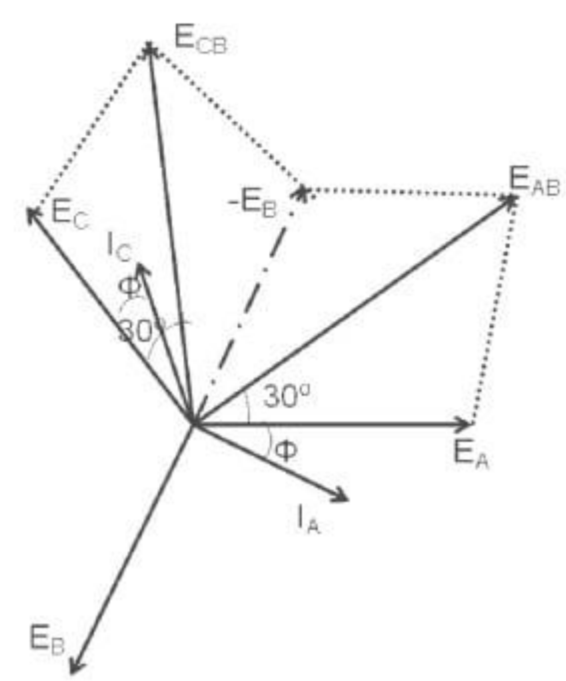
The wattmeter is connected in the circuit in such a way that the current coil is in series and carries the load current and the potential coil is connected in parallel across the load voltage. The wattmeter reading will then be equal to the product of the current carried by the current coil, the voltage across the potential coil and the cosine of the angle between the voltage and current.

The measurement of power is first given for a balanced star connected load and then for a balanced delta connected load.

(i) Balanced star connected load



The circuit shows a balanced star connected load for which the power is to be measured. Two wattmeter W_1 and W_2 are connected in phase A and phase C as shown in the figure.



The current coil of wattmeter W_1 carries the current I_A and its potential coil is connected across the voltage E_{AB}. A phasor diagram is drawn to determine the angle between I_A and E_{AB} as shown.
From the phasor diagram we determine that the angle between the phasors I_A and E_{AB} is (30+Φ). Hence the wattmeter reading W_1 is given by

$W_1=E_{AB}I_A\cos(30+\Phi)$

The current coil of wattmeter W_2 carries the current I_C and its potential coil is connected across the voltage E_{CB}. From the phasor diagram we determine that the angle between the phasors I_C and E_{CB} is (30-Φ). Hence the wattmeter reading W_2 is given by

$W_2=E_{CB}I_C\cos(30-\Phi)$

Line voltages $E_{AB}=E_{CB}=E_L$
And line currents $I_A=I_C=I_L$

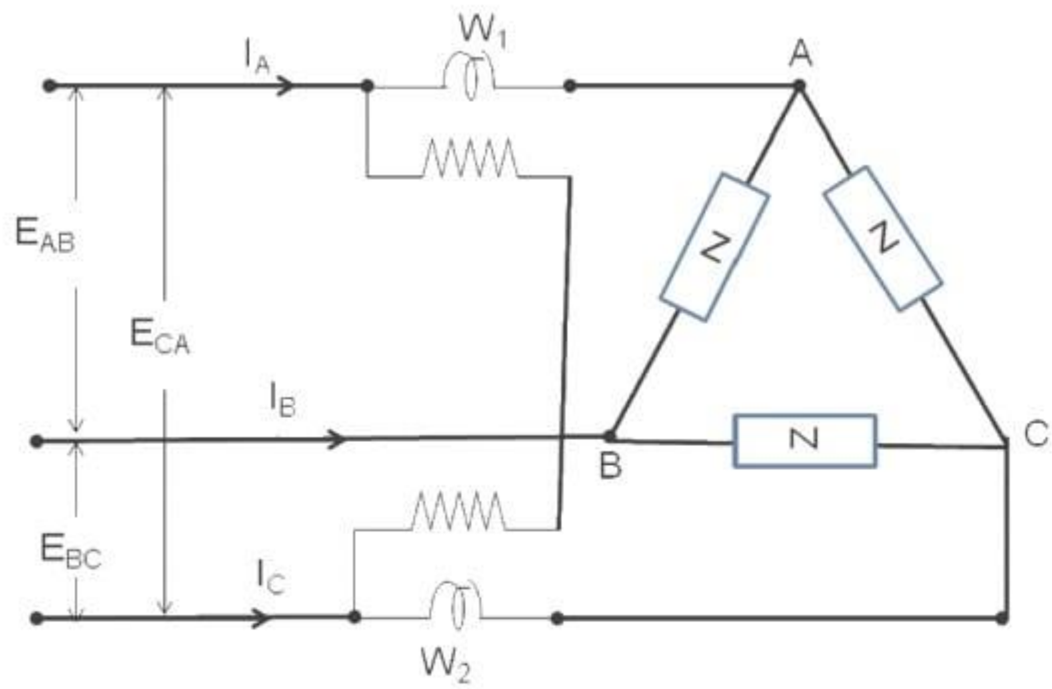
Hence

$$W_1 = E_L I_L \cos(30+\Phi)$$
$$W_2 = E_L I_L \cos(30-\Phi)$$
$$W_1 + W_2 = E_L I_L \cos(30+\Phi) + E_L I_L \cos(30-\Phi)$$
$$W_1 + W_2 = E_L I_L (2\cos 30^\circ \cos\Phi)$$
$$W_1 + W_2 = \sqrt{3} E_L I_L \cos\Phi$$

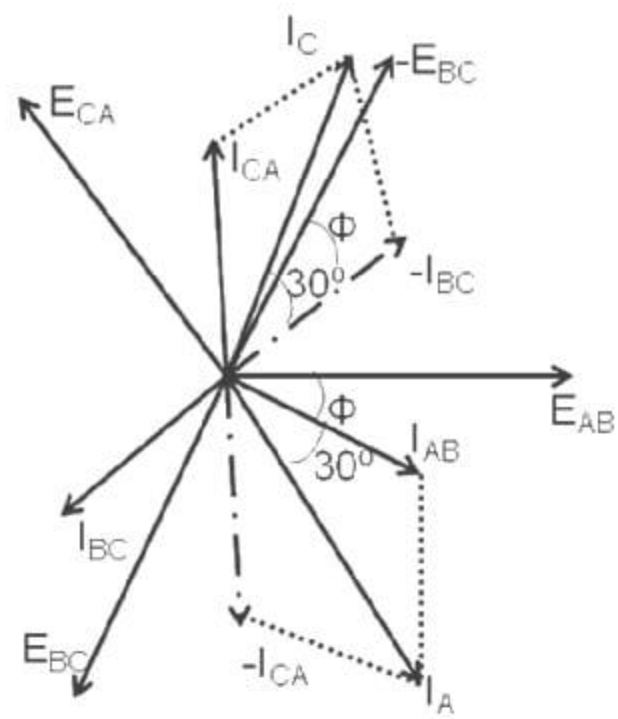
From the above equations we observe that the sum of the two wattmeter reading gives the three phase power.

(ii) Balanced delta connected load

The circuit shows a balanced delta connected load for which the power is to be measured. Two wattmeter W_1 and W_2 are connected in phase A and phase C as shown in the figure.



The current coil of wattmeter W_1 carries the current I_A and its potential coil is connected across the voltage E_{AB}. A phasor diagram is drawn to determine the angle between I_A and E_{AB} as shown.



From the phasor diagram we determine that the angle between the phasors I_A and E_{AB} is $(30+\Phi)$. Hence the wattmeter reading W_1 is given by

$W_1 = E_{AB} I_A \cos(30+\Phi)$

The current coil of wattmeter W_2 carries the current I_C and its potential coil is connected across the voltage E_{CB}. From the phasor diagram we determine that the angle between the phasors I_C and E_{CB} is $(30-\Phi)$. Hence the wattmeter reading W_2 is given by

$W_2=E_{CB}I_C\cos(30-\Phi)$

Line voltages $E_{AB}=E_{CB}=E_L$

And line currents $I_A=I_C=I_L$

Hence

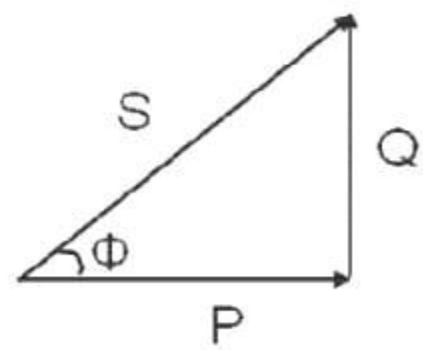
$$W_1 = E_L I_L \cos(30+\Phi)$$
$$W_2 = E_L I_L \cos(30-\Phi)$$
$$W_1 + W_2 = E_L I_L \cos(30+\Phi) + E_L I_L \cos(30-\Phi)$$
$$W_1 + W_2 = E_L I_L (2\cos 30^\circ \cos\Phi)$$
$$W_1 + W_2 = \sqrt{3} E_L I_L \cos\Phi$$

From the above equations we observe that the sum of the two wattmeter reading gives the three phase power.

Determination of Real power, Reactive power and Power factor

$$W_1 = E_L I_L \cos(30+\Phi)$$
$$W_2 = E_L I_L \cos(30-\Phi)$$
$$W_1 + W_2 = \sqrt{3} E_L I_L \cos\Phi$$
$$W_2 - W_1 = E_L I_L \sin\Phi$$
$$\tan\Phi = \sqrt{3}\left(\frac{W_2 - W_1}{W_1 + W_2}\right)$$
$$\Phi = \tan^{-1}\left[\sqrt{3}\left(\frac{W_2 - W_1}{W_1 + W_2}\right)\right]$$
$$P = W_1 + W_2$$
$$Q = \sqrt{3}(W_2 - W_1)$$
$$pf = \cos\Phi = \cos\left[\tan^{-1}\left[\sqrt{3}\left(\frac{W_2 - W_1}{W_1 + W_2}\right)\right]\right]$$

The power factor can also be determined from the power triangle



From the power triangle,

$$P = W_1 + W_2$$
$$Q = \sqrt{3}(W_2 - W_1)$$
$$S = \sqrt{(W_1 + W_2)^2 + 3(W_2 - W_1)^2}$$
$$pf = \cos\Phi = \frac{P}{S} = \frac{W_1 + W_2}{\sqrt{(W_1 + W_2)^2 + 3(W_2 - W_1)^2}}$$

Wattmeter readings at different Power Factors

$$(i) upf$$
$$\Phi = 0^\circ$$
$$W_1 = E_L I_L \cos(30 + \Phi) = E_L I_L \cos(30) = \frac{\sqrt{3}}{2} E_L I_L$$
$$W_2 = E_L I_L \cos(30 - \Phi) = E_L I_L \cos(30) = \frac{\sqrt{3}}{2} E_L I_L$$
$$W_1 = W_2$$
$$(ii) pf = 0.866$$
$$\Phi = 30^\circ$$
$$W_1 = E_L I_L \cos(30 + \Phi) = E_L I_L \cos(30 + 30) = \frac{E_L I_L}{2}$$
$$W_2 = E_L I_L \cos(30 - \Phi) = E_L I_L \cos(30 - 30) = E_L I_L$$
$$W_2 = 2W_1$$

$$(iii)\, pf = 0.5$$
$$\Phi = 60^\circ$$
$$W_1 = E_L I_L \cos(30+\Phi) = E_L I_L \cos(30+60) = 0$$
$$W_2 = E_L I_L \cos(30-\Phi) = E_L I_L \cos(30-60) = \frac{\sqrt{3}}{2} E_L I_L$$

$$(iv)\, pf < 0.5$$
$$\Phi > 60^\circ$$
$$W_1 = E_L I_L \cos(30+\Phi) < 0$$
$$W_2 = E_L I_L \cos(30-\Phi) > 0$$

$$(v)\, pf = 0$$
$$\Phi = 90^\circ$$
$$W_1 = E_L I_L \cos(30+\Phi) = E_L I_L \cos(30+90) = \frac{E_L I_L}{2}$$
$$W_2 = E_L I_L \cos(30-\Phi) = E_L I_L \cos(30-90) = -\frac{E_L I_L}{2}$$
$$W_1 = -W_2$$

Problem 1

A balanced 3Φ delta connected load has per phase impedance of (25+j40)Ω. If 400V, 3Φ supply is connected to this load, find (i) phase current (ii) line current (iii) power supplied to the load.

$$Z_{ph} = \sqrt{25^2 + 40^2} = 47.17\Omega$$
$$\Phi = \tan^{-1}\left(\frac{40}{25}\right) = 60^\circ$$
$$Z_{ph} = 47.17\angle 60^\circ\,\Omega$$
$$E_L = 400V = E_{ph}$$
$$(i) I_{ph} = \frac{E_{ph}}{Z_{ph}} = \frac{400}{47.17\angle 60^\circ} = 8.48\angle -60^\circ\, A$$
$$(ii) I_L = \sqrt{3} I_{ph} = \sqrt{3}\times 8.48 = 14.7\angle -90^\circ\, A$$
$$(iii) P = \sqrt{3} E_L I_L \cos\Phi = \sqrt{3}\times 400 \times 14.7 \times \cos 60^\circ$$
$$P = 5397.76W$$

Problem 2

Two wattmeter method is used to measure the power absorbed by a 3Φ induction motor. The wattmeter readings are 12.5kW and -4.8kW. Find (i) the power absorbed by the machine (ii) load power factor (iii) reactive power taken by the load.

$$W_1 = 12.5kW$$
$$W_2 = -4.8kW$$
$$(i) P = W_1 + W_2 = 12.5 - 4.8 = 7.7kW$$
$$(ii) \tan\Phi = \sqrt{3}\left(\frac{W_2 - W_1}{W_1 + W_2}\right) = \sqrt{3}\left(\frac{-4.8 - 12.5}{12.5 - 4.8}\right) = -3.89$$
$$\Phi = \tan^{-1}[-3.89] = -75.6^\circ$$
$$pf = \cos\Phi = \cos(-75.6^\circ) = 0.2487$$
$$(iii) Q = \sqrt{3}(W_2 - W_1) = \sqrt{3}(-4.8 - 12.5) = 29.96kVAR$$

Problem 3

Calculate the active and reactive components of each phase of a star connected 10kV, 3Φ alternator supplying 5MW at 0.8 pf.

$$E_L = 10kV$$
$$P = 5MW$$
$$pf = \cos\Phi = 0.8$$
$$\Phi = 36.87^\circ$$
$$P = \sqrt{3}E_L I_L \cos\Phi$$
$$I_L = \frac{P}{\sqrt{3}E_L \cos\Phi} = \frac{5\times10^6}{\sqrt{3}\times10\times10^3\times0.8} = 360.84A$$
$$P_{ph} = \frac{5\times10^6}{3} = 166.7MW$$
$$Q_{ph} = E_{ph} I_{ph} \sin\Phi = \frac{10\times10}{\sqrt{3}} \times 360.8 \times \sin 36.87^\circ = 1.25MVAR$$

Problem 4

A 3Φ load of three equal impedances connected in delta across a balanced 400V supply takes a line current of 10A at a power factor of 0.7 lagging. calculate (i) the phase current (ii) the total power (iii) the total reactive kVAR

$$E_L = 400V = E_{ph}$$
$$I_L = 10A$$
$$pf = \cos\Phi = 0.7lag$$
$$\Phi = 45.57^\circ$$
$$(i) I_{ph} = \frac{I_L}{\sqrt{3}} = \frac{10}{\sqrt{3}} = 5.8A$$
$$(ii) P = \sqrt{3} E_L I_L \cos\Phi = \sqrt{3} \times 400 \times 10 \times 0.7 = 4.84kW$$
$$(iii) Q = \sqrt{3} E_L I_L \sin\Phi = \sqrt{3} \times 400 \times 10 \times \sin 45.57^\circ = 4.94kVAR$$

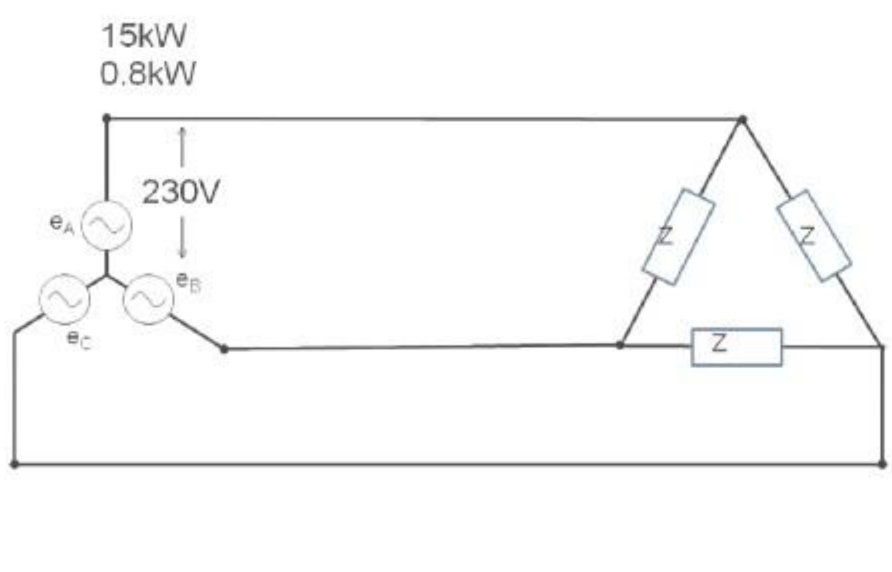
Problem 5

The power flowing in a 3Φ, 3 wire balanced load system is measured by two wattmeter method. The reading in wattmeter A is 750W and wattmeter B is 1500W. What is the power factor of the system?

$$W_1 = 750W$$
$$W_2 = 1500W$$
$$\Phi = \tan^{-1}\left[\sqrt{3}\left(\frac{W_2 - W_1}{W_1 + W_2}\right)\right] = \tan^{-1}\left[\sqrt{3}\left(\frac{1500 - 750}{750 + 1500}\right)\right]$$
$$\Phi = 30^\circ$$
$$pf = \cos\Phi = \cos 30^\circ = 0.866$$

Problem 6

A 3Φ star connected supply with a phase voltage of 230V is supplying a balanced delta connected load. The load draws 15kW at 0.8pf lagging. Find the line currents and the current in each phase of the load. What is the load impedance per phase.



$Alternator$

$$E_{ph} = 230V$$

$$E_L = \sqrt{3} \times 230V = 398.37V$$

$$P = 15kW$$

$$pf = \cos\Phi = 0.8 lagging$$

$$I_L = \frac{P}{\sqrt{3} E_L \cos\Phi} = 27.17A$$

$Load$

$$E_{ph} = E_L = 398.37V$$

$$I_L = 27.17A$$

$$I_{ph} = \frac{I_L}{\sqrt{3}} = 15.68A$$

$$Z_{ph} = \frac{E_{ph}}{I_{ph}} = 25.4\Omega$$

Single Phase
Transformers

What is a Transformer?

A ***transformer*** is a static piece of equipment used either for raising or lowering the voltage of an AC supply with a corresponding decrease or increase in current.

The use of transformers in transmission system is shown in the Figure below.

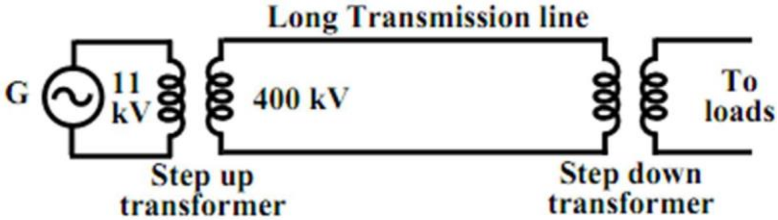


Fig.3-1 **A simple single phase power system.**

Principle of Operation

A transformer in its simplest form will consist of a rectangular laminated magnetic structure on which two coils of different number of turns are wound as shown in Figure 3.2a.

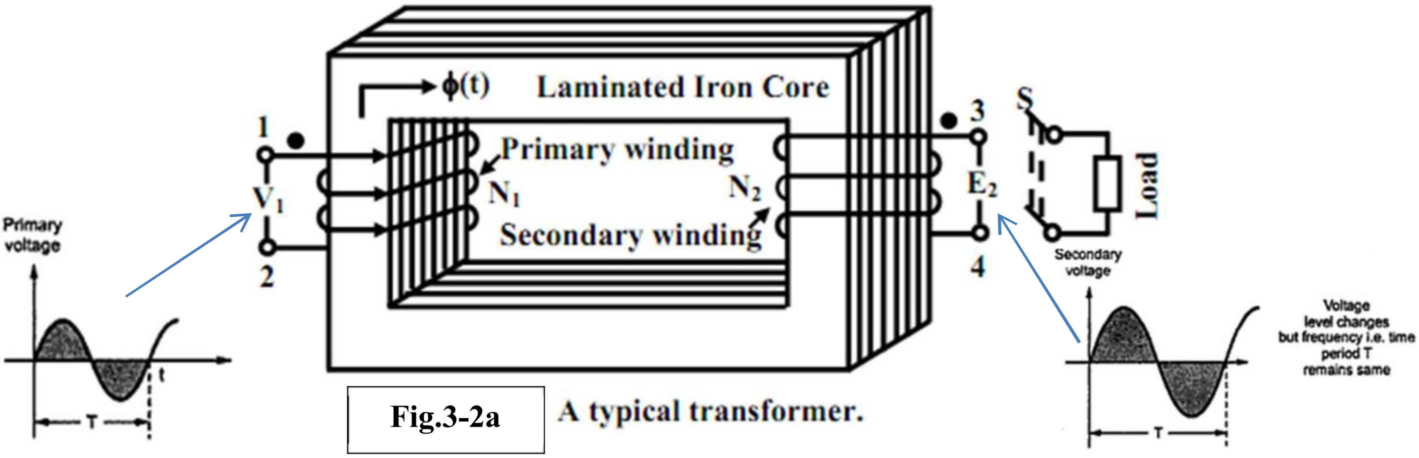


Fig.3-2a **A typical transformer.**

The winding to which AC voltage is impressed is called ***the primary*** of the transformer and the winding across which the load is connected is called ***the secondary*** of the transformer.

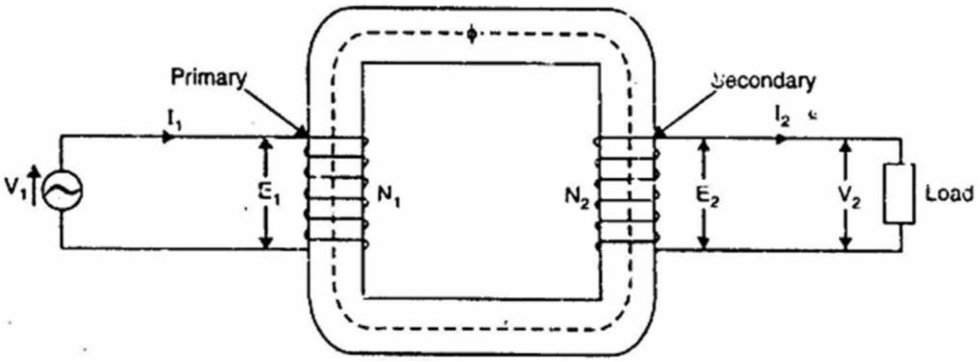


Fig.3-2b

Depending upon the number of turns of the primary (N_1) and secondary (N_2), an alternating emf (E_2) is induced in the secondary. This induced emf (E2) in the secondary causes a secondary current I_2. Consequently, terminal voltage V_2 will appear across the load. If $V_2 > V_1$, it is called a ***step up-transformer***. On the other hand, if $V_2 < V_1$, it is called a ***step-down transformer***.

When an alternating voltage V_1 is applied to the primary, an alternating flux Φ is set up in the core. This alternating flux links both the windings and induces emfs E1 and E2 in them according to ***Faraday's laws of electromagnetic induction***. The emf E1 is termed as primary emf and emf E2 is termed as Secondary emf.

Clearly, $$E_1 = -N_1 \frac{d\phi}{dt}$$

and $$E_2 = -N_2 \frac{d\phi}{dt}$$

$$\therefore \quad \frac{E_2}{E_1} = \frac{N_2}{N_1}$$

Note that magnitudes of E_2 and E_1 depend upon the number of turns on the secondary and primary respectively. If $N_2 > N_1$, then $E_2 > E_1$ (or $V_2 > V_1$) and we get a step-up transformer. On the other hand, if $N_2 < N_1$, then $E_2 < E_1$ (or $V_2 < V_1$) and we get a step-down transformer. If load is connected across the secondary winding, the secondary e.m.f. E_2 will cause a current I_2 to flow through the load. Thus, a transformer enables us to transfer a.c. power from one circuit to another with a change in voltage level.

The following points may be noted carefully:

(i) The transformer action is based on the laws of ***electromagnetic induction***.

(ii) There is no electrical connection between the primary and secondary.

(iii) There is no change in frequency i.e., output power has the same frequency as the input power.

Can DC Supply be used for Transformers?

The DC supply cannot be used for the transformers. This is because the transformer works on the principle of ***mutual induction***, for which current in one coil must change uniformly. If DC supply is given, the current will not change due to constant supply and transformer will not work

There can be saturation of the core due to which transformer draws very large current from the supply when connected to DC.

Thus DC supply should not be connected to the transformers.

Construction

We usually design a power transformer so that it approaches the characteristics of an ideal transformer. To achieve this, following design features are incorporated:

(i) The core is made of silicon steel which has low hysteresis loss and high permeability. Further, core is laminated in order to reduce eddy current loss. These features considerably reduce the iron losses and the no-load current.

(ii) Instead of placing primary on one limb and secondary on the other, it is a usual practice to wind one-half of each winding on one limb. This ensures tight coupling between the two windings. Consequently, leakage flux is considerably reduced.

(iii) The winding resistances are minimized to reduce Copper loss and resulting rise in temperature and to ensure high efficiency.

Transformers are of two types: (i) core-type transformer (see Fig.3-3) and (ii) shell-type transformer (see Fig.3-4).

Core-Type Transformer: In a core-type transformer, half of the primary winding and half of the secondary winding are placed round each limb to reduce the leakage flux.

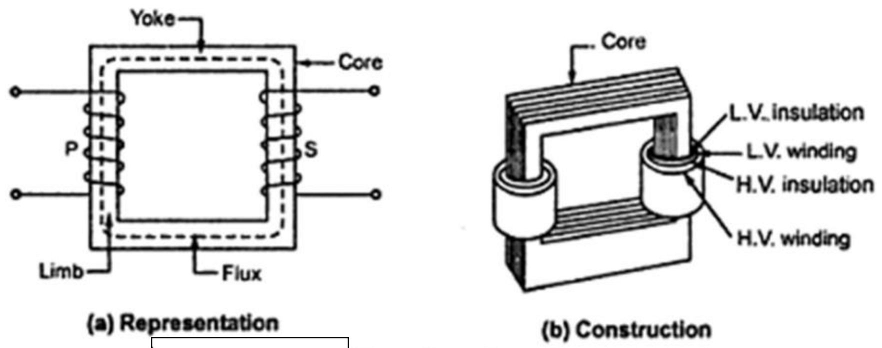


Fig.3-3 **Core type transformer**

Shell-Type Transformer: This method of construction involves the use of a double magnetic circuit. Both the windings are placed round the central limb to ensure a low-reluctance flux path.

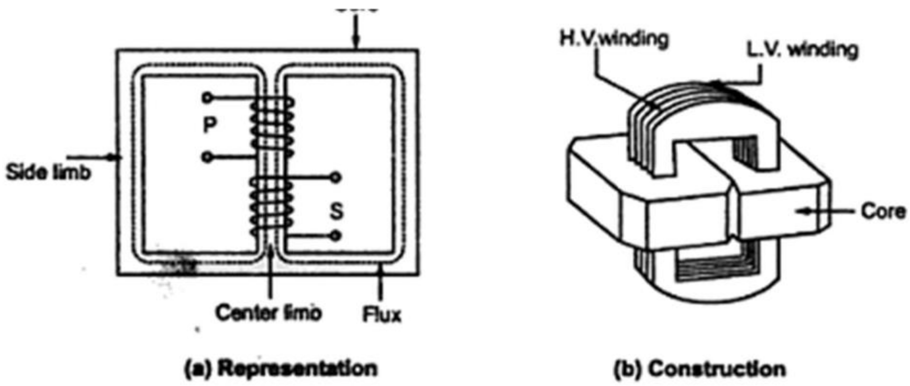


Fig.3-4 **Shell type transformer**

Comparison of Core and Shell Type Transforms

Core Type	Shell Type
The winding encircles the core.	The core encircles most part of the winding
It has single magnetic circuit	It has double magnetic circuit
The core has two limbs	The core has three limbs
The cylindrical coils are used.	The multilayer disc or sandwich type coils are used.
The winding are uniformly distributed on two limbs hence natural cooling is effective	The natural cooling does not exist as the windings are surrounded by the core.
Preferred for low voltage transformers.	Preferred for high voltage transformers.

EMF Equation of a Transformer

Consider that an alternating voltage V_1 of frequency f is applied to the primary as shown in Fig. 3-2b. The sinusoidal flux Φ produced by the primary can be represented as:

$$\phi = \phi_m \sin\omega t$$

The instantaneous e.m.f. e_1 induced in the primary is

$$e_1 = -N_1 \frac{d\phi}{dt} = -N_1 \frac{d}{dt}(\phi_m \sin\omega t)$$

$$= -\omega N_1 \phi_m \cos\omega t = -2\pi f N_1 \phi_m \cos\omega t$$

$$= 2\pi f N_1 \phi_m \sin(\omega t - 90°) \quad \text{(i)}$$

It is clear from the above equation that maximum value of induced e.m.f. in the primary is

$$E_{m1} = 2\pi f N_1 \phi_m$$

The r.m.s. value E^ of the primary e.m.f. is

$$E_1 = \frac{E_{m1}}{\sqrt{2}} = \frac{2\pi f N_1 \phi_m}{\sqrt{2}}$$

or $$E_1 = 4.44 f N_1 \phi_m$$

Similarly $$E_2 = 4.44 f N_2 \phi_m$$

In an ideal transformer, $E_1 = V_1$ and $E_2 = V_2$.

Note. It is clear from exp. (i) above that e.m.f. E_1 induced in the primary lags behind the flux ϕ by 90°. Likewise, e.m.f. E_2 induced in the secondary lags behind flux ϕ by 90°.

Voltage Transformation Ratio (K)

From the above equations of induced emf, we have,

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = K$$

The constant K is called voltage transformation ratio. Thus if K = 5 (i.e.

N2/N1 = 5), then E2 = 5 E1.

Concept of Ideal Transformer

A transformer is said to be ideal if it satisfies following properties:

i) It has no losses.

ii) Its windings have zero resistance.

iii) Leakage flux is zero i.e. 100 % flux produced by primary links with the secondary.

iv) Permeability of core is so high that negligible current is required to establish the flux in it.

NOTE:

For an ideal transformer, the primary applied voltage V1 is same as the primary induced emf E1 as there are no voltage drops.

For ideal transformer:

(i) $E_1 = V_1$ and $E_2 = V_2$ as there is no voltage drop in the windings.

$$\therefore \quad \frac{E_2}{E_1} = \frac{V_2}{V_1} = \frac{N_2}{N_1} = K$$

(ii) there are no losses. Therefore, volt-amperes input to the primary are equal to the output volt-amperes i.e.

$$V_1 I_1 = V_2 I_2$$

$$\text{or} \qquad \frac{I_2}{I_1} = \frac{V_1}{V_2} = \frac{1}{K}$$

Hence, currents are in the inverse ratio of voltage transformation ratio. This simply means that if we raise the voltage, there is a corresponding decrease of current.

Volt-Ampere Rating

Transformer rating is specified as the product of voltage and current and called ***VA rating***.

$$\text{kVA rating of a transformer} = \frac{V_1 I_1}{1000} = \frac{V_2 I_2}{1000}$$

The full load primaty and secondary currents which indicate the safe maximum values of currents which transformer windings can carry can be given as:

$$I_1 \text{ full load} = \frac{\text{kVA rating} \times 1000}{V_1} \quad \text{... (1000 to convert kVA to VA)}$$

$$I_2 \text{ full load} = \frac{\text{kVA rating} \times 1000}{V_2}$$

Ideal Transformer on No Load

Consider an ideal transformer in Fig. 3-5. For no load $I_2 = 0$. I_1 is just necessary to produce flux in the core, which is called ***magnetizing*** current denoted as I_m. I_m is very small and lags V_1 by 90^0 as the winding is purely inductive.

.

According to Lenz's law, the induced e.m.f opposes the cause producing it which is supply voltage V_1. Hence E_1 and E_2 are in antiphase with V_1 but equal in magnitude and E_1 and E_2 are in phase.

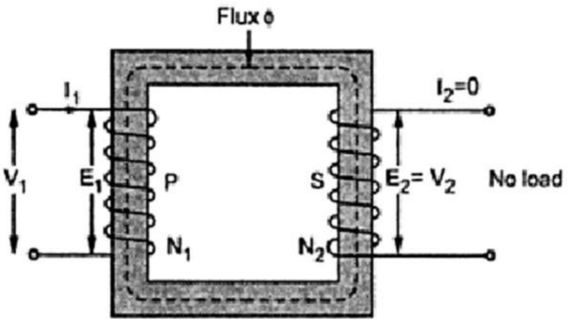
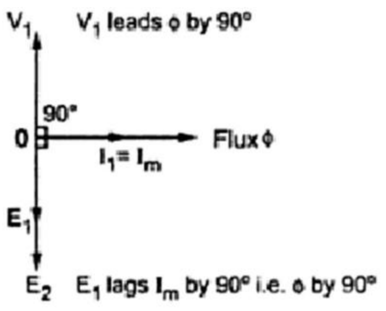


Fig.3-5

This can be illustrated in the phase diagram as shown below:



Phasor diagram for ideal transformer on no load

Losses in a Transformer

The power losses in a transformer are of two types, namely;

1. **Core or Iron losses** 2. **Copper losses**

NOTE: The above losses appear in the form of heat and produce (i) ***an increase in temperature*** and (ii) a ***drop in efficiency***.

A- Core or Iron losses (P_i)

These consist of ***hysteresis and eddy current losses*** and occur in the transformer core due to the alternating flux. These can be determined by open-circuit test (see next sections).

$$\text{Hysteresis loss, } = k_h\ f\ B_m^{1.6} \quad \text{watts}/m^3$$

$$\text{Eddy current loss, } = k_e\ f^2\ B_m^2 t^2 \quad \text{watts}/m^3$$

Both hysteresis and eddy current losses depend upon

- Maximum flux density B_m in the core and
- Supply frequency f.

NOTE: Since transformers are connected to constant-frequency, constant voltage supply, both f and B_m are constant. Hence, core or iron losses are practically the same at all loads. Hence,

$$\text{Iron or Core losses, } P_i = \text{Hysteresis loss} + \text{Eddy current loss} = \text{Constant losses}$$

NOTE: The hysteresis loss can be minimized ***by using steel of high silicon content*** whereas eddy current loss can be reduced ***by using core of thin laminations***.

B- Copper losses (P_C)

These losses occur in both the primary and secondary windings due to their ohmic resistance. These can be determined by short-circuit test

Total copper Cu losses:

$$\boxed{\begin{aligned} P_C &= I_1^2 R_1 + I_2^2 R_2 \\ &= I_1^2 R_{01} \text{ or } I_2^2 R_{02} \end{aligned}}$$

Hence, total losses in a transformer are:

$$\boxed{\begin{aligned} \text{Total losses in a transformer} &= P_1 + P_C \\ &= \text{Constant losses} + \text{Variable losses} \end{aligned}}$$

Efficiency of a Transformer

Like any other electrical machine, the efficiency of a transformer is defined as **the ratio of output power (in watts or kW) to input power (watts or kW)** i.e.

$$\text{Efficiency} = \frac{\text{Output power}}{\text{Input power}}$$

In practice, open-circuit and short-circuit tests are carried out to find the efficiency,

$$\boxed{\text{Efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{\text{Output}}{\text{Output} + \text{Losses}}}$$

NOTE: The losses can be determined by transformer tests.

Condition for Maximum Efficiency

Output power = $V_2 I_2 \cos\phi_2$

If R_{02} is the total resistance of the transformer referred to secondary, then,

$$\text{Total Cu loss, } P_C = I_2^2 R_{02}$$

$$\text{Total losses} = P_i + P_C$$

$$\text{Transformer } \eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{02}}$$

$$= \frac{V_2 \cos\phi_2}{V_2 \cos\phi_2 + P_i / I_2 + I_2 R_{02}}$$

For a load of given pf, efficiency depends upon load current I_2. Hence, the efficiency to be maximum ***the denominator should be minimum*** i.e.

$$\frac{d}{dI_2}\left(V_2 \cos\phi_2 + P_i / I_2 + I_2 R_{02}\right) = 0$$

$$\text{or} \qquad 0 - \frac{P_i}{I_2^2} + R_{02} = 0$$

$$\text{or} \qquad P_i = I_2^2 R_{02}$$

i.e., Iron losses = Copper losses

Hence efficiency of a transformer will be maximum when copper losses are equal to constant or iron losses.

From above, the load current I2 corresponding to maximum efficiency is:

$$\boxed{I_2 = \sqrt{\frac{P_i}{R_{02}}}}$$

NOTE: In a transformer, iron losses are constant whereas copper losses are variable. In order to obtain **maximum efficiency**, the load current should be such that total Cu losses become equal to iron losses.

Output kVA corresponding to Maximum Efficiency

Let P_C = Copper losses at full-load kVA

P_i = Iron losses

x = Fraction of full-load kVA at which efficiency is maximum

Total Cu losses = $x^2 \times P_C$

for maximum efficiency $x^2 \times P_C = P_i$

or

$$x = \sqrt{\frac{P_i}{P_C}} = \sqrt{\frac{\text{Iron loss}}{\text{F.L. Cu loss}}}$$

⇒Output kVA corresponding to maximum efficiency:

$$\text{Full - load kVA} \times \sqrt{\frac{\text{Iron loss}}{\text{F.L. Cu loss}}}$$

NOTE: The value of kVA at which the efficiency is maximum, is independent of pf of the load.

Efficiency from Transformer Tests

The full-load efficiency of the transformer at any pf can be obtained as:

$$\boxed{\text{F.L. efficiency, } \eta_{\text{F.L.}} = \frac{\text{Full - load VA} \times \text{p.f.}}{(\text{Full - load VA} \times \text{p.f.}) + P_i + P_C}}$$

where: P_i = Iron loss can be obtained from open-circuit test
P_c = Copper loss can be obtained from short-circuit test
F.L. = Full Load

Also the efficiency for any load,

$$\text{Corresponding total losses} = P_i + x^2 P_C$$

$$\text{Corresponding } \eta_x = \frac{(xx \text{ Full - load VA}) \times \text{p.f.}}{(xx \text{ Full - load VA} \times \text{p.f.}) + P_i + x^2 P_C}$$

where xx= Fraction of full-load

<u>NOTE</u>: Iron loss remains the same at all loads.

Voltage Regulation of a Transformer:

Definition: The voltage regulation is defined as the change in the magnitude of receiving and sending voltage of the transformer. The voltage regulation determines the ability of the transformer to provide the constant voltage for variable loads.

When the transformer is loaded with continuous supply voltage, the terminal voltage of the transformer varies. The variation of voltage depends on the load and its power factor.

Mathematically, the voltage regulation is represented as:

$$\text{Voltage Regulation} = \frac{E_2 - V_2}{E_2}$$

$$\text{\% Voltage Regulation} = \frac{E_2 - V_2}{E_2} \text{ x } 100$$

where,
E2 – secondary terminal voltage at no load
V2 – secondary terminal voltage at full load

The voltage regulation by considering the primary terminal voltage of the transformer is expressed as,

$$\text{\% Voltage Regulation} = \frac{V_1 - E_1}{V_1} \text{ x } 100$$

Problems on Transformers

Q1

A single phase, 20 kVA transformer has 1000 primary turns and 2500 secondary turns. The net cross sectional area of the core is 100 cm². When the primary winding is connected to 500V, 50 Hz supply, calculate (i) the maximum value of the flux density in the core (ii) the voltage induced in the secondary winding and (iii) the primary and secondary full load currents.

i) $E_1 = 4.44\, f\, \phi_m\, N_1$ i.e. $500 = 4.44 \times 50 \times \phi_m \times 1000$

$\therefore\ \phi_m = 2.252 \times 10^{-3}$ Wb

$$B_m = \frac{\phi_m}{a} = \frac{2.252 \times 10^{-3}}{100 \times 10^{-4}} = 0.2252 \text{ Wb/m}^2$$

ii) $\dfrac{E_2}{E_1} = \dfrac{N_2}{N_1}$ i.e. $\dfrac{E_2}{500} = \dfrac{2500}{1000}$, $\therefore\ E_2 = 1250$ V

iii) $I_1 = \dfrac{\text{kVA} \times 1000}{E_1} = \dfrac{20 \times 1000}{500} = 40$ A, $I_2 = \dfrac{20 \times 1000}{1250} = 16$ A

Q2

Find the number of turns on the primary and secondary side of a 440/230 V, 50 Hz, single phase transformer, if the net area of a cross section of the core is 30 cm² and the maximum value of the flux density is 1 Wb/ m².

$E_1 = 4.44\, f\, \phi_m\, N_1 = 4.44\, f\, B_m\, a\, N_1$ i.e., $440 = 4.44 \times 50 \times 1 \times 30 \times 10^{-4} \times N_1$

$\therefore\ N_1 = 660.67$ turns

$\dfrac{E_2}{E_1} = \dfrac{N_2}{N_1}$ i.e. $\dfrac{230}{440} = \dfrac{N_2}{660.67}$, $\therefore\ N_2 = 345.35$ turns

Q3

The primary winding of a 25 kVA transformer has 200 turns and is connected to 230V, 50Hz supply. The secondary turns are 50. Calculate

i) No load secondary induced emf

ii) Full load primary and secondary currents

iii) The flux density in the core, if the cross section of the core is 60 cm^2.

i) $\dfrac{E_2}{E_1} = \dfrac{N_2}{N_1}$ i.e. $\dfrac{E_2}{230} = \dfrac{50}{200}$, $\therefore$ $E_2 = 57.5$ V

ii) $I_1 = \dfrac{25 \times 1000}{230} = 108.7$ A

$\dfrac{I_1}{I_2} = \dfrac{N_2}{N_1}$ i.e. $\dfrac{108.7}{I_2} = \dfrac{50}{200}$, $\therefore$ $I_2 = 434.8$ A

iii) $E_1 = 4.44\, f\, \phi_m\, N_1$ i.e. $230 = 4.33 \times 50 \times \phi_m \times 200$

$\therefore$ $\phi_m = 5.18 \times 10^{-3}$ Wb, $B_m = \dfrac{\phi_m}{a} = \dfrac{5.18 \times 10^{-3}}{60 \times 10^{-4}} = 0.863$ Wb/m^2

Q4

A 100 kVA, 50 Hz single phase transformer has a turns ratio of 1000/ 250. The primary winding is connected to 500V, 50 Hz supply. Fund the secondary open circuit voltage and the max. value of the flux in the core.

$\dfrac{E_2}{E_1} = \dfrac{N_2}{N_1}$ i.e. $\dfrac{E_2}{500} = \dfrac{250}{1000}$ $\therefore$ $E_2 = 125$V

$E_1 = 4.44\, f\, \phi_m\, N_1$ i.e. $500 = 4.44 \times 50 \times \phi_m \times 1000$

$\therefore$ $\phi_m = 2.25 \times 10^{-3}$ Wb. = 2.25 mWb

Q5

A single phase, 50 Hz core type transformer has square cores of 20 cm side. The permissible maximum flux density is 1 Wb/m^2. Calculate the number of turns per limb on the high and low voltage sides for a 3000/ 220 V ratio.

$E_1 = 4.44\, f\, \phi_m\, N_1 = 4.44\, f\, B_m\, a\, N_1$ i.e. $3000 = 4.44 \times 50 \times 1 \times (20 \times 20 \times 10^{-4})\, N_1$

$\therefore$ $N_1 = 337.82$ turns $\cong 338$ turns

$N_2 = 337.82 \times \dfrac{220}{3000} = 24.77 \cong 25$ turns.

Usually, in a transformer ½ of the primary turns and ½ of the secondary turns are wound on each limb.

$\therefore$ $N_1/\text{limb} = \dfrac{338}{2} = 169$, $N_2/\text{limb} = \dfrac{25}{2} = 12.5$

Q6	**The design requirements of a 6000/450 V, 50 Hz core type transformer are: approximate e.m.f./turn = 15 V, max. flux density = 1 Wb/m^2. Calculate suitable number of primary and secondary turns and the net cross sectional area of the core.** $N_1 = \frac{6000}{15} = 400$ and $N_2 = \frac{450}{15} = 30$ $E_1 = 4.44\, f\, B_m\, a\, N_1$ i.e. $6000 = 4.44 \times 50 \times 1 \times a \times 400$ $\therefore\ a = 0.0676\ m^2 = 676\ cm^2$
Q7	**A 10 kVA, 400/200 V, 50 Hz, single phase transformer has a full load copper loss of 200 W and has a full load efficiency of 96 % at 0.8 p.f. lagging. Determine the iron loss. What would be the efficiency at half of the full load and unity p.f.?** Total full load losses = Input – Output $= \left(\frac{10 \times 1000 \times 0.8}{0.96}\right) - (10 \times 1000 \times 0.8)$ $= 333.33\ W$ Iron loss = $333.33 - 200 = 133.33\ W$ Efficiency at half of the full load and u.p.f. is given by, $\eta_{1/2 F.L} = \frac{1/2 \times 10 \times 1000 \times 1}{1/2 \times 10 \times 1000 \times 1 + 133.33 + (1/2)^2 \times 200}$ $= \frac{5000}{5000 + 133.33 + 50}$ $= 0.9646$ or 96.46%

Q8

A 600 kVA transformer has an efficiency of 92% at full load, unity p.f. and half full load, 0.9 p.f. Determine its efficiency at 75% of full load and 0.9 p.f.

$$0.92 = \frac{600 \times 1000 \times 1}{600 \times 1000 \times 1 + W_i + W_{cu}}$$

$\therefore \; W_i + W_{cu} = 52{,}173.9 \text{ W}$ --- (1)

Also $$0.92 = \frac{1/2 \times 600 \times 1000 \times 0.9}{1/2 \times 600 \times 1000 \times 0.9 + W_i + (1/2)^2 W_{cu}}$$

$\therefore \; W_i + 0.25\, W_{cu} = 23{,}478.26 \text{ W}$ --- (2)

Solving equations (1) and (2) we get,

$W_i = 13{,}913.03$ W and $W_{cu} = 38{,}260.87$ W.

$$\therefore \; \eta_{0.75 f.l, 0.9 p.f} = \frac{0.75 \times 600 \times 1000 \times 0.9}{0.75 \times 600 \times 1000 \times 0.9 + 13{,}913.03 + (0.75)^2 \times 38{,}260.87}$$

$= 0.9196$ or 91.96%

Q9

The maximum efficiency at full load and unity p.f of a single phase 25 kVA, 500/1000V, 50Hz transformer is 98%. Determine its efficiency at (i) 75% load, 0.9 p.f. and (ii) 50% load, 0.8 p.f.

Maximum efficiency occurs when $W_i = W_{cu}$

$$\therefore \; 0.98 = \frac{1 \times 25 \times 1000 \times 1}{1 \times 25 \times 1000 \times 1 + W_i + W_{cu}}$$

$\therefore \; W_i = W_{cu} = 255.1$ W.

$$\eta_{0.75 f.l, 0.9 p.f} = \frac{0.75 \times 25 \times 1000 \times 0.9}{0.75 \times 25 \times 1000 \times 0.9 + 255.1 + (0.75)^2 \times 255.1}$$

$= 0.9769$ or $97.69\,\%$

$$\eta_{0.5 f.l, 0.8 p.f} = \frac{0.75 \times 25 \times 1000 \times 0.8}{0.5 \times 25 \times 1000 \times 0.8 + 2551. + (0.5)^2 \times 255.1}$$

$= 0.969$ or 96.9%

Q10	A 10 kVA, 400/200 V, single phase transformer has a maximum efficiency of 98 % at 90 % of the full load at 0.8 p.f. Find its efficiency at full load and 0.6 p.f.

Output at 90% full load and 0.8 p.f.

$$= 10 \times 0.9 \times 0.8 = 7.2 \text{ kW}$$

$$\text{Input at 90\% full load} = \frac{\text{Output}}{\eta} = \frac{7.2}{0.98} = 7.347 \text{ kW}$$

$$\text{Total losses} = \text{Input} - \text{Output} = 7.347 - 7.2 = 0.147 \text{ kW}$$

When the efficiency is maximum, $W_i = W_{cu}$

$$\therefore W_i = \frac{0.147}{2} = 0.0735 \text{ kW}$$

$$W_{cu} \text{ at 90\% full load} = 0.0735 \text{ kW}$$

$$W_{cu} \text{ at full load} = \left(\frac{1}{0.9}\right)^2 \times 0.0735 = 0.0907 \text{ kW}$$

$\therefore$ Efficiency at full load, 0.6 p.f. is given by,

$$\eta_{FL} = \frac{10 \times 0.6}{10 \times 0.6 + 0.0735 + 0.0907} = 0.9734 \text{ or } 97.34\%$$

At full load

$$0.9 = \frac{1 \times 500}{500 + W_i + W_{cu}}\ , \quad \therefore \quad W_i + W_{cu} = 55.56 \text{ watts} \qquad \text{--- (1)}$$

At ½ full load

$$0.9 = \frac{250}{250 + W_i + (1/2)^2 W_{cu}}$$

$$\therefore \; W_i + \frac{W_{cu}}{4} = 27.78 \text{ watts}$$

i.e. $\quad 4\,W_i + W_{cu} = 111.12$ watts --- (2)

Solving equations (1) and (2), we get

$W_i = 18.52$ watts and $W_{cu} = 37.04$ watts

$$\eta_{0.75fl} = \frac{0.75 \times 500}{0.75 \times 500 + 18.52 + (0.75)^2 \times 37.04}$$

$$= 0.9056 \quad \text{or} \quad 90.56\%$$