

HOMEWORK

1. EXERCICE 1

Consider the asymmetric random walk on \mathbb{Z} given by

$$X_n = \sum_{k=1}^n \varepsilon_k,$$

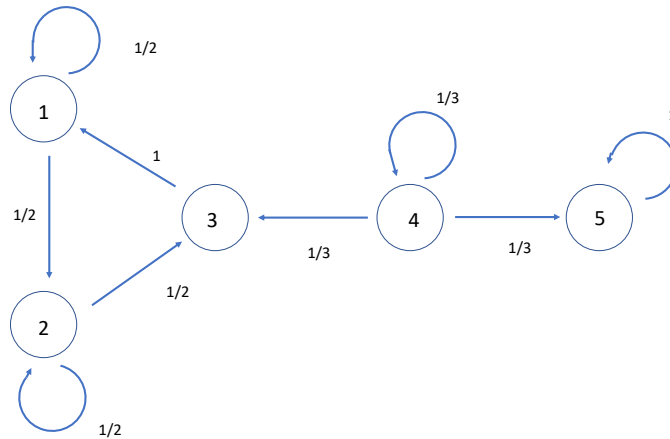
where we sum i.i.d variables that take values in $\{-1, 1\}$ with probabilities

$$\mathbb{P}(\varepsilon_1 = 1) = 1 - \mathbb{P}(\varepsilon_1 = -1) = p \in]\frac{1}{2}, 1[.$$

- 1) Find out the distribution of X_n
- 2) Is this irreducible Markov chain recurrent ? transient ?

2. EXERCICE 2

Consider the Markov chain X_n defined on $S = \{1, 2, 3, 4, 5\}$ whose transition diagram is



- 1) Write the transition matrix of this Markov chain.
- 2) What are the transient and recurrent communication classes of this Markov chain ?
- 3) For any j in S compute $\mathbb{P}_j(X_n = 5)$.
- 4) Let T be defined as $\inf\{n \geq 1; X_n \in \{3, 5\}\}$. Compute $\mathbb{E}[T|X_0 = 4]$.
- 5) Describe the long-run dynamics of X_n , starting from $X_0 = 4$.