

# CARBONIC ANHYDRASE: KINETICS OF REMOVAL OF Zn(II) By 2,6-PYRIDINECARBOXYLATE

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## Abstract

In the past decade, supervised activity recognition methods have been studied by many researchers, however these methods still face many challenges in real world settings. Supervised activity recognition methods assume that we are provided with labeled training examples from a set of predefined activities. Annotating and hand labeling data is a very time consuming and laborious task. Also, the assumption of consistent pre-defined activities might not hold in reality. More importantly, these algorithms do not take into account the streaming nature of data, or the possibility that the patterns might change over time. In this chapter, we will provide an overview of the state of the art *unsupervised* methods for activity recognition. In particular, we will describe a scalable activity discovery and recognition method for complex large real world datasets, based on sequential data mining and stream data mining methods.

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# Introduction

Carbonic anhydrase (CA) catalyzes the interconversion of carbon dioxide and carbonic acid/bicarbonate as follows:



As shown in figure 1, the CA active site contains a  $\text{Zn}^{2+}$  cofactor (denoted as  $\text{CA}\cdot\text{Zn}$ ), which it relies upon for its catalytic activity. The zinc ion can be stripped from the enzyme using a Lewis base ligand, which donates electrons to the ion to form a covalent bond. The ligand being studied in this experiment is 2,6-pyridinecarboxylate, commonly called dipicolinate (or dipic). Figure 2 shows the structure of dipic. In this experiment, the rate of zinc removal by dipic will be measured.

## Mechanism

When  $[\text{dipic}] \gg [\text{CA}]$ , that is, when  $\frac{[\text{dipic}]}{[\text{CA}]} \geq 25$ , then the removal of zinc is pseudo-first-order with respect to  $\text{CA}\cdot\text{Zn}$  because the concentration of dipic, denoted as L, does not change appreciably. Thus the formation of the inactive enzyme, apoCA, can be modeled using the following rate equation:

$$\frac{d[\text{apoCA}]}{dt} = k_{obs}[\text{CA}\cdot\text{Zn}] \quad (2)$$

The pseudo-first-order rate constant,  $k_{obs}$ , increases as  $[\text{L}]$  increases, but levels off at sufficiently high concentrations of L. Biochemists will recognize behavior similar to Michaelis-Menten enzyme kinetics in which the enzyme,  $\text{CA}\cdot\text{Zn}$ , and the substrate, L, reversibly form a  $\text{CA}\cdot\text{Zn}\cdot\text{L}$  complex with association constant  $K_{EML}$  (EML stands for Enzyme-Metal-Ligand):





Figure 1:  $\text{Zn}^{2+}$  cofactor in the active site of human carbonic anhydrase II<sup>1</sup>

This can be modeled as follows:

$$K_{EML} = \frac{[\text{CA} \cdot \text{Zn} \cdot \text{L}]}{[\text{CA} \cdot \text{Zn}][\text{L}]} \quad (4)$$

$\text{CA} \cdot \text{Zn} \cdot \text{L}$  can either revert back to the original species or irreversibly convert to the inactive form of the enzyme, apoCA, and the covalently bound zinc-dipic molecule, ZnL:





Figure 2: Structure of 2,6-pyridinedicarboxylate (dipic)<sup>2</sup>

This yields the following differential rate law:

$$\frac{d[\text{apoCA}]}{dt} = k_d[\text{CA}\cdot\text{Zn}\cdot\text{L}] \quad (6)$$

Note the difference in assumptions between this reaction and the Michaelis-Menten model; M-M kinetics assumes the enzyme is reformed and that only the substrate is modified, while (5) shows that both the substrate and the enzyme are permanently modified. Recall that  $[\text{L}] \gg [\text{CA}\cdot\text{Zn}]$ , so  $[\text{L}]$  can be assumed to stay constant at  $[\text{L}]_0$ , which is substituted into a rearranged form of equation (4),

$$[\text{CA}\cdot\text{Zn}\cdot\text{L}] = K_{EM\text{L}}[\text{CA}\cdot\text{Zn}][\text{L}]_0 \quad (7)$$

Carbonic anhydrase can exist in one of three forms: the metalloenzyme  $\text{CA}\cdot\text{Zn}$ , the enzyme-metal-ligand complex  $\text{CA}\cdot\text{Zn}\cdot\text{L}$ , or the inactivated enzyme apoCA. Initially, all CA is tied up in the metalloenzyme, and none exists as  $\text{CA}\cdot\text{Zn}\cdot\text{L}$  or apoCA. As the activated form of the enzyme gets bound to L and then inactivated,

$$[\text{CA}\cdot\text{Zn}] = [\text{CA}\cdot\text{Zn}]_0 - [\text{apoCA}] - [\text{CA}\cdot\text{Zn}\cdot\text{L}], \quad (8)$$

which can be combined with equation (7) to yield

$$[\text{CA}\cdot\text{Zn}] = [\text{CA}\cdot\text{Zn}]_0 - [\text{apoCA}] - K_{EML}[\text{CA}\cdot\text{Zn}][\text{L}]_0 \quad (9)$$

and can be rearranged as follows:

$$[\text{CA}\cdot\text{Zn}] = \frac{[\text{CA}\cdot\text{Zn}]_0 - [\text{apoCA}]}{1 + K_{EML}[\text{L}]_0} \quad (10)$$

Equations (6) and (7) can be combined to give

$$\frac{d[\text{apoCA}]}{dt} = k_d K_{EML}[\text{CA}\cdot\text{Zn}][\text{L}]_0 \quad (11)$$

Therefore, the rate of apoCA formation is first-order with respect to CA·Zn. Combining equations (11) and (10) yields

$$\frac{d[\text{apoCA}]}{dt} = k_d K_{EML}[\text{L}]_0 \frac{[\text{CA}\cdot\text{Zn}]_0 - [\text{apoCA}]}{1 + K_{EML}[\text{L}]_0} \quad (12)$$

Rearranging and integrating,

$$\begin{aligned} \int_{[\text{apoCA}]_0}^{[\text{apoCA}]_t} \frac{d[\text{apoCA}]}{[\text{CA}\cdot\text{Zn}]_0 - [\text{apoCA}]_t} &= \int_0^t \frac{k_d K_{EML}[\text{L}]_0}{1 + K_{EML}[\text{L}]_0} dt \\ &= \frac{k_d K_{EML}[\text{L}]_0}{1 + K_{EML}[\text{L}]_0} t \end{aligned} \quad (13)$$

The left side must be integrated using u-substitution:

$$u = [\text{CA}\cdot\text{Zn}]_0 - [\text{apoCA}]_t$$

$$du = -d[\text{apoCA}]$$

To change the integral boundaries,

$$u(t=0) = [\text{CA}\cdot\text{Zn}]_0, \text{ since no inactivated enzyme has been formed}$$

$$u(t=t) = [\text{CA}\cdot\text{Zn}]_0 - [\text{apoCA}]_t$$

Therefore,

$$\begin{aligned} \int_{[\text{apoCA}]_0}^{[\text{apoCA}]_t} \frac{d[\text{apoCA}]}{[\text{CA}\cdot\text{Zn}]_0 - [\text{apoCA}]_t} &= - \int_{[\text{CA}\cdot\text{Zn}]_0}^{[\text{CA}\cdot\text{Zn}]_0 - [\text{apoCA}]_t} \frac{du}{u} \\ &= -\ln(u) \Big|_{[\text{CA}\cdot\text{Zn}]_0}^{[\text{CA}\cdot\text{Zn}]_0 - [\text{apoCA}]_t} \\ &= -\ln \left( \frac{[\text{CA}\cdot\text{Zn}]_0 - [\text{apoCA}]_t}{[\text{CA}\cdot\text{Zn}]_0} \right) \end{aligned} \quad (14)$$

Combining the evaluated integrals from equations (14) and (13) yields

$$\ln \left( \frac{[\text{CA}\cdot\text{Zn}]_0 - [\text{apoCA}]_t}{[\text{CA}\cdot\text{Zn}]_0} \right) = - \frac{k_d K_{EML} [\text{L}]_0}{1 + K_{EML} [\text{L}]_0} t \quad (15)$$

Assuming steady-state conditions for  $\text{CA}\cdot\text{Zn}\cdot\text{L}$ , that is,  $[\text{CA}\cdot\text{Zn}\cdot\text{L}]$  is always negligible, then  $\frac{[\text{CA}\cdot\text{Zn}]_0 - [\text{apoCA}]_t}{[\text{CA}\cdot\text{Zn}]_0}$  is simply the fraction of  $\text{CA}\cdot\text{Zn}$  remaining after reaction time  $t$  and can be referred to as  $F_{\text{CA}\cdot\text{Zn}}$ :

$$\ln(F_{\text{CA}\cdot\text{Zn}}) = - \frac{k_d K_{EML} [\text{L}]_0}{1 + K_{EML} [\text{L}]_0} t \quad (16)$$

Since  $-\frac{k_d K_{EML} [\text{L}]_0}{1 + K_{EML} [\text{L}]_0}$  is a constant for a given concentration of dipic, equation (16) exhibits linear behavior while the solution contains active enzyme. Therefore, a linear least squares regression procedure can be performed for measurements of  $\ln(F_{\text{CA}\cdot\text{Zn}})$  over time (until those measurements level off, which indicates that all the enzyme is used up), and the slope, denoted  $-k_{obs}$ , will be

$$-k_{obs} = - \frac{k_d K_{EML} [\text{L}]_0}{1 + K_{EML} [\text{L}]_0} \quad (17)$$

Taking the reciprocal of equation (17) yields

$$\begin{aligned}
\frac{1}{k_{obs}} &= \frac{1 + K_{EML}[L]_0}{k_d K_{EML}[L]_0} \\
&= \frac{1}{k_d K_{EML}[L]_0} + \frac{K_{EML}[L]_0}{k_d K_{EML}[L]_0} \\
&= \frac{1}{k_d K_{EML}} \times \frac{1}{[L]_0} + \frac{1}{k_d}
\end{aligned} \tag{18}$$

Again, a linear pattern is observed; measuring  $k_{obs}$  at several different dipic concentrations enables one to perform a least squares regression procedure on  $\frac{1}{k_{obs}}$  versus  $\frac{1}{[L]_0}$  to determine the slope,  $m = \frac{1}{k_d K_{EML}}$ , and intercept,  $b = \frac{1}{k_d}$ .  $k_d$  and  $K_{EML}$  are thus calculated as follows:

$$k_d = \frac{1}{b} \tag{19}$$

$$K_{EML} = \frac{b}{m} \tag{20}$$

## Rate Measurements

While equations (16) and (17) seem to imply that the determination of  $k_{obs}$  requires knowing  $[CA \cdot Zn]$  or  $[apoCA]$  at any point in time, in reality a plot of  $f(t)$  vs  $t$ , where  $f(t)$  is proportional to  $F_{CA \cdot Zn}$ , will still exhibit the same slope  $-k_{obs}$ . The Michaelis-Menten kinetics model states that, for sufficiently large substrate concentrations, the enzyme is fully saturated and the reaction velocity asymptotically approaches a number,  $V_{max} = k_{cat}[CA \cdot Zn]$ , where  $k_{cat}$  (the “turnover number”) is amount of substrate that can be converted to product by a single saturated enzyme molecule in a given unit of time.<sup>3</sup> Thus for sufficiently large concentrations of substrate, the enzyme catalytic activity fulfills the requirement of proportionality to  $F_{CA \cdot Zn}$ . Ideally, the determination of the enzymatic activity from  $V_{max}$ , that is, an enzyme assay, can be done using a substrate which is cheap, readily available, and easily measurable. As Figure 3 shows, carbonic anhydrase happens to hydrolyze para-nitrophenyl acetate (pNPA) to form para-nitrophenol and acetic acid:

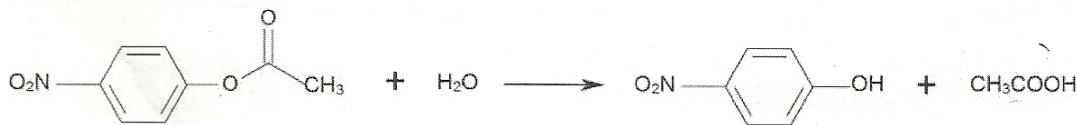


Figure 3: Carbonic anhydrase-catalyzed hydrolysis of para-nitrophenyl acetate (pNPA) to para-nitrophenol and acetic acid<sup>2</sup>

Para-nitrophenol absorbs strongly at 348nm. Recall that Beer's law states

$$A = \epsilon bc \quad (21)$$

where  $A$  is the measured absorbance at a particular wavelength,  $\epsilon$  is the molar absorptivity,  $b$  is the measurement path length, and  $c$  is the concentration.<sup>4</sup> Therefore,  $\left(\frac{dA}{dt}\right)_{cat}$  is proportional to the change in para-nitrophenol (that is, the reaction velocity). Because the reaction occurs to a nontrivial extent without enzyme catalysis, a baseline correction is needed,  $\left(\frac{dA}{dt}\right)_{uncat}$ , which can be obtained either from a solution containing only pNPA or a mixture of pNPA and apoCA (that is, after sufficient time has passed to inactivated all carbonic anhydrase before taking the assay). Thus, equation (16) can be implemented using

$$\ln \left( \frac{dA}{dt} \right)_{cat} = \ln \left( \frac{\left( \frac{dA}{dt} \right)_t}{\left( \frac{dA}{dt} \right)_{uncat}} \right) \quad (22)$$

(note: this also fulfills the logarithm function's unitless requirement.)



## Sample Calculations

As Equation (22) demonstrates, a baseline correction is needed to convert the raw absorbance velocity,  $\left(\frac{dA}{dt}\right)_t$ , to a proportion which can take the place of  $F_{CA \cdot Zn}$  in Equation (16).  $\left(\frac{dA}{dt}\right)_{uncat}$  was found to be  $3.18 \times 10^{-5} \text{ sec}^{-1}$ . One data point that was measured for the 0.10 M dipic concentration experiment was  $\left(\frac{dA}{dt}\right)_{t=1.08 \text{ min}} = 0.00295 \text{ sec}^{-1}$ . Therefore, **(note: consider changing t in prior equation to be less ambiguous)**

$$\begin{aligned} \ln \left( \frac{dA}{dt} \right)_{cat} &= \ln \left( \frac{\left(\frac{dA}{dt}\right)_t}{\left(\frac{dA}{dt}\right)_{uncat}} \right) \\ &= \ln \left( \frac{0.00295 \text{ sec}^{-1}}{3.18 \times 10^{-5} \text{ sec}^{-1}} \right) \\ &= 4.530 \end{aligned}$$

Recall from Equation (16) that  $-k_{obs}$  is the slope obtained from a least-squares regression of multiple assay  $\ln \left(\frac{dA}{dt}\right)_{cat}$  readings versus CA/dipic reaction time. For -10 M dipic, this was found to be  $-k_{obs} = -0.090 \text{ min}^{-1}$  (so  $k_{obs} = 0.090 \text{ min}^{-1}$ ).

Equation (18) implies that a least-squares regression of  $\frac{1}{k_{obs}}$  versus  $\frac{1}{[L]_0}$  gives a slope,  $m = \frac{1}{k_d K_{EML}}$ , and intercept,  $b = \frac{1}{k_d}$ , which can be used to calculate  $k_d$  and  $K_{EML}$  for the dipic/CA reaction. Empirically,  $m$  was found to have a value of  $0.5372 \text{ M} \cdot \text{mins}$ , and  $b$  was found to be  $8.513 \text{ mins}$ . Thus,

$$\begin{aligned} K_{EML} &= \left( \frac{b}{m} \right) \\ &= \frac{(9.0)}{(0.5)} \\ &= 20 \end{aligned}$$

$$\begin{aligned}
 k_d &= \left(\frac{1}{b}\right) \text{ mins}^{-1} \\
 &= \frac{1}{(9.0)} \\
 &= 0.1 \text{ mins}^{-1}
 \end{aligned}$$

## References

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