DELIRES TP2: Plug & Play ADMM

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First example: Missing Pixels

Question 1 In order to test the importance of the initialization, we ran four experiences using the shepperd initialization, initialization with zeros, initialization with degraded image, and a random initialization. We found that the shepperd initialization gives a good visual results, on the contrary with other methods that give very noisy images.

Question 2 To measure the effect of the initial value of ρ on standard and adaptive PnP, we tried several values of ρ_0 that vary from 0.01 to 1. It was found that adaptive PnP ADMM is more sensitive to the initial value of ρ, because its SNR results varied from 31.6 to 20.2, while the standard PnP ADMM results varied from 31.1 to 23.6.

Add a new inverse problem: Deblurring

Question 4 We have, using the Plancherel theorem:

$$\begin{aligned} g(z) &= argmin_x F(x) + \frac{\rho}{2}||x - z||_2^2 \\ &= argmin_x \frac{1}{2\sigma^2}||A(x) - \widetilde{x}||_2^2 + \frac{\rho}{2}||x - z||_2^2 \\ &= argmin_{\hat{x}} \frac{1}{2\sigma^2}||\hat{x}.\hat{h} - \hat{\widetilde{x}}||_2^2 + \frac{\rho}{2}||\hat{x} - \hat{z}||_2^2 \\ &= argmin_{\hat{x}} \sum_i \frac{1}{2\sigma^2}|\hat{x}_i.\hat{h}_i - \hat{\widetilde{x}}_i|^2 + \frac{\rho}{2}|\hat{x}_i - \hat{z}_i|^2 \end{aligned}$$

By solving the problem, we find:

$$\hat{x}_i^* = \frac{\frac{1}{2\sigma^2}\hat{h}_i\hat{\widetilde{x}}_i + \rho\hat{z}_i}{\frac{1}{2\sigma^2}|\hat{z}_i|^2 + \rho}$$

Regarding the implementation results, I can't put the photos because I couldn't configure Matlab on my personal computer, and I left my results on the Telecom computer.

Add a new regularizer: Transform domain soft thresholding (TDT)

Question 6 We denote

$$f(v) = \frac{1}{2}(v - w)^2 + \lambda|v|$$

And we have

$$\partial f(v) = \lambda.\partial(|.|)(v) + v - w$$

Since f is sub-differential, so using the optimality conditions, we have $0 \in \partial f(prox_{\lambda||.||}(w))$, so

$$\partial(|.|)(x) = \begin{cases} -1 & \text{if } x < 0 \\ [-1, 1] & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$$

which gives us

$$\partial f(x) = \begin{cases} x - \omega - \lambda & \text{if } x < 0 \\ [x - \omega - \lambda, x - \omega + \lambda] & \text{if } x = 0 \\ x - \omega + \lambda & \text{if } x > 0 \end{cases}$$

so:

$$prox_{\lambda||.||}(w) = \begin{cases} \omega + \lambda & \text{if } \omega < -\lambda \\ 0 & -\lambda \leqslant \omega \leqslant \lambda \\ \omega - \lambda & \text{if } \omega > \lambda \text{ if } x > 0 \end{cases}$$

what had to be demonstrated!

Question 7 By denoting $w = Wx$, and using the orthogonality of W,we have:

$$\begin{aligned} ||W^{-1}w - z||^2 &= \langle\ W^{-1}w - z, W^{-1}w - z\rangle \\ &= \langle\ W^t(w - Wz), W^t(w - Wz)\rangle \\ &= \langle\ (w - Wz), WW^t(w - Wz)\rangle \\ &= \langle\ (w - Wz), (w - Wz)\rangle \\ &= ||w - Wz||^2 \end{aligned}$$

So, we have:

$$\begin{aligned} D_\sigma &= W^{-1} argmin_w \frac{1}{2\sigma}||W^{-1}w - z||^2 + \lambda||w||_1 \\ &= W^{-1} argmin_w \frac{1}{2\sigma}||w - Wz||^2 + \lambda||w||_1 \end{aligned}$$

Question 8 We have:

$$\frac{1}{2\sigma}||w - Wz||^2 + \lambda||w||_1 = \sum_i \frac{1}{2\sigma}(w_i - (Wz)_i)^2 + \lambda|w_i|$$

So using previous questions:

$$\begin{aligned} D_\sigma(z) &= W^{-1} argmin_w \frac{1}{2\sigma}||w - Wz||^2 + \lambda||w||_1 \\ &= \left(prox_{\lambda||.||}\left((Wz)_i\right)\right) \\ &:= P \end{aligned}$$

so: $D_\sigma(z) = W^{-1}P$.

Verify conditions for convergence

Question 9

Question 10

Question 11