2D BWR Control Blade Drop LRA Benchmark Problem Specification

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The LRA benchmark problem, as presented here, is modeled by the set of 2-dimensional, 2-group neutron diffusion equations with spatially discontinuous properties, a specified axial buckling, 2 delayed neutron precursor families, and an adiabatic thermal feedback. The equations are summarized in section 1. The properties, together with the initiating perturbation of the transient, are summarized in section 2.

1 Multiphysics Equations Summary

The multidimensional 2-group neutron diffusion equations with several simplifying assumptions are given by:

$$\begin{aligned}\frac{\partial}{\partial t} n_1(t,\vec{\mathbf{x}}) &= \nabla\cdot D_1 \nabla \phi_1(t,\vec{\mathbf{x}}) - \left(\Sigma_{a1}(T) + \Sigma_{s21} + B_z^2 D_1\right)\phi_1(t,\vec{\mathbf{x}}) + \\ &\quad + \frac{\nu(1-\beta)}{k^0_{\mathit{eff}}}\sum_{g'=1}^{2}\Sigma_{fg'}\phi_{g'}(t,\vec{\mathbf{x}}) + \sum_{m=1}^{2}\lambda_m c_m(t,\vec{\mathbf{x}}),\end{aligned} \tag{1a}$$

$$\frac{\partial}{\partial t} n_2(t,\vec{\mathbf{x}}) = \nabla\cdot D_2 \nabla \phi_2(t,\vec{\mathbf{x}}) - \left(\Sigma_{a2}(t) + B_z^2 D_2\right)\phi_2(t,\vec{\mathbf{x}}) + \Sigma_{s21}\phi_1(t,\vec{\mathbf{x}}), \tag{1b}$$

in which $n_g(t,\vec{\mathbf{x}})$ is the group g neutron density, $\phi_g(t,\vec{\mathbf{x}})$ is the group g scalar flux, D_g is the group g neutron diffusion coefficient, $\vec{\mathbf{x}}$ is the (x,y) location in geometric space, subscript $g \in (1,2)$ refers to the energy group index, subscript $m \in (1,2)$ refers to the precursor family index, $T(t,\vec{\mathbf{x}})$ is temperature, ν is the average total number of neutrons born per fission, Σ_{ag} and Σ_{fg} are the group g absorption and fission cross sections, respectively, Σ_{s21} is the group 1 to 2 scattering cross section, B_z^2 is the axial buckling, λ_m is the precursor family m decay constant, $c_m(t,\vec{\mathbf{x}})$ is the precursor family m spatial density, β is the total delayed neutron fraction and k^0_{eff} is the effective multiplication constant at which the problem starts (typically,

*The original problem, along with a 3-dimensional extension and a reference solution, is specified in ANL Code Center: Benchmark Problem Book, supplement 2, benchmark problem 14, Tech. Report ANL-7416, Argonne National Laboratory, 1977.

to make the reactor critical). The multigroup parameters are all assumed to be given. The simplifying assumptions are as follows: *a*) there is no upscattering; *b*) all neutrons are born fast; *c*) the reactor starts as a critical system with a known eigenvalue; *d*) the average number of neutrons born per fission ν is independent of energy; *e*) all properties except $\Sigma_{a1}(T)$ are homogeneous over the nodes ($T(t, \vec{\mathbf{x}})$ may be heterogeneous), and are discontinuous at node boundaries; and *f*) $\Sigma_{a2}(t)$ may be time-dependent to model the variation in thermal absorption cross section due to control blade movement.

The group fluxes relate to group neutron densities through:

$$\phi_g(t, \vec{\mathbf{x}}) = V_{ng} n_g(t, \vec{\mathbf{x}}), \tag{2}$$

in which V_{ng} is the group g neutron velocity, assumed given.

Equations (1) apply within nodes, and not at property discontinuities. Equation (3) enforces the current continuity at the property discontinuities $\vec{\mathbf{x}} = \vec{\mathbf{x}}_d \in \partial S$ with ∂S as the set of all points on the interface:

$$J^-_{\vec{\mathbf{x}}_d} = \left[\vec{\mathbf{n}} \cdot D_g \nabla \phi_g\right]^-_{\vec{\mathbf{x}}_d} = J^+_{\vec{\mathbf{x}}_d} = \left[\vec{\mathbf{n}} \cdot D_g \nabla \phi_g\right]^+_{\vec{\mathbf{x}}_d}, \tag{3}$$

in which J is net neutron current density, the $\mp$ superscripts indicate evaluations immediately at the two different sides of the discontiunity interface and $\vec{\mathbf{n}}$ is the directional unit vector normal to the interface at location $\vec{\mathbf{x}}_d$.

The precursor densities are modeled by Eq. (4):

$$\frac{\partial}{\partial t} c_m(t, \vec{\mathbf{x}}) = \frac{\nu \beta_m}{k^0_{\mathit{eff}}} \sum_{g'=1}^{2} \Sigma_{fg'} \phi_{g'}(t, \vec{\mathbf{x}}) - \lambda_m c_m(t, \vec{\mathbf{x}}), \tag{4}$$

in which β_m is the precursor family m delayed neutron fraction.

Lastly, the adiabatic heatup and power equations are:

$$\frac{\partial}{\partial t} T(t, \vec{\mathbf{x}}) = \alpha \sum_{g'=1}^{2} \Sigma_{fg'} \phi_{g'}(t, \vec{\mathbf{x}}), \tag{5a}$$

$$P(t, \vec{\mathbf{x}}) = \kappa \sum_{g'=1}^{2} \Sigma_{fg'} \phi_{g'}(t, \vec{\mathbf{x}}), \tag{5b}$$

in which α is the temperature conversion factor, $P(t, \vec{\mathbf{x}})$ is the power density in space and κ is thermal energy per fission. Both equations only apply in the fuel region, because there is no fissile material (and therefore zero power and constant temperature) in the reflector region. To rewrite Eq. (5a) in terms of thermal energy density, we first express the volumetric heat capacity ρc_p:

$$\rho c_p = \frac{\kappa}{\alpha}, \tag{6}$$

and then multiply Eq. (5a) by it:

$$\frac{\partial}{\partial t} u_v(t, \vec{\mathbf{x}}) = P(t, \vec{\mathbf{x}}) = \kappa \sum_{g'=1}^{2} \Sigma_{fg'} \phi_{g'}(t, \vec{\mathbf{x}}), \tag{7}$$

in which $u_v(t, \vec{\mathbf{x}})$ is the thermal energy density in space.

The average core power density and temperature are given by:

$$\overline{P}(t) = \frac{1}{A_{core}} \iint_{A_{core}} dA\, P(t, \vec{\mathbf{x}}), \tag{8a}$$

$$\overline{T}(t) = \frac{1}{A_{core}} \iint_{A_{core}} dA\, T(t, \vec{\mathbf{x}}), \tag{8b}$$

in which A_{core} is the area of the fueled part of the core. The problem is 2-dimensional, so the integrals are over the fueled core area, and not the fueled core volume.

Two types of boundary conditions (BCs) are present in the problem: the zero flux BC and the zero current BC. They are represented by Eqs. (9a) and (9b), respectively:

$$\phi_g(t, \vec{\mathbf{x}}_{BC}) = 0, \tag{9a}$$

$$J_g^{BC}(t, \vec{\mathbf{x}}_{BC}) = 0, \tag{9b}$$

in which $\vec{\mathbf{x}}_{BC} \in \partial S_{BC}$ refers to the set of points on the boundary ∂S_{BC} at which the relevant BC applies and J_g^{BC} refers to the group g net current density across the boundary at point $\vec{\mathbf{x}}_{BC}$.

2 Properties Summary

Figure 1 details the reactor layout and boundary conditions.

The initial region properties are given in Table 1.

Equation (10) models the Doppler feedback of the fast absorption cross-section:

$$\Sigma_{a1}(T) = \Sigma_{a1}^{0}\left[1 + \gamma\left(\sqrt{T} - \sqrt{T^0}\right)\right], \tag{10}$$

in which in which Σ_{a1}^{0} is the fast macroscopic absorption cross section at initial temperature T^0, obtained from Table 1, and γ is the Doppler feedback constant, given in Table 2.

Additional parameters for all regions are given in Table 2.

The initiating perturbation, the control blade drop of region R, is given by Eq. (11):

$$\Sigma_{a2,R}(t) = \begin{cases} \Sigma_{a2,3}^{0} \cdot \left(1 - \left[0.060\,618\,4\,\mathrm{s}^{-1} \cdot t\right]\right) & \text{if } t \leq 2\,\mathrm{s}, \\ \Sigma_{a2,3}^{0} \cdot \left(1 - \left[0.060\,618\,4\,\mathrm{s}^{-1} \cdot 2\,\mathrm{s}\right]\right) & \text{if } t > 2\,\mathrm{s}, \end{cases} \tag{11}$$

in which $\Sigma_{a2,R}(t)$ is the thermal absorption cross-section in region R, and $\Sigma_{a2,3}^{0}$ is the initial thermal absorption cross-section in regions 3 and R, obtained from Table 1.

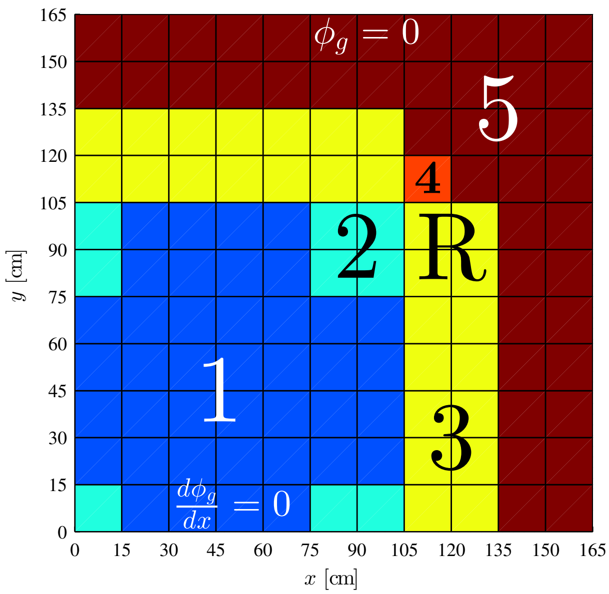

Figure 1: Problem Layout and BCs

Table 1: Benchmark Problem Initial Region Properties

Region	**Material**	g	D_g[cm]	$\Sigma_{ag}[\mathrm{cm}^{-1}]$[a]	$\nu\Sigma_{fg}[\mathrm{cm}^{-1}]$	$\Sigma_{s21}[\mathrm{cm}^{-1}]$
1	Fuel 1,	1	1.255	0.008 252	0.004 602	0.025 33
	blade in	2	0.211	0.1003	0.1091	
2	Fuel 1,	1	1.268	0.007 181	0.004 609	0.027 67
	blade out	2	0.1902	0.070 47	0.086 75	
3, R[b]	Fuel 2,	1	1.259	0.008 002	0.004 663	0.026 17
	blade in	2	0.2091	0.083 44	0.1021	
4	Fuel 2,	1	1.259	0.008 002	0.004 663	0.026 17
	blade out	2	0.2091	0.073 324	0.1021	
5	Reflector	1	1.257	0.000 603 4	0	0.047 54
		2	0.1592	0.019 11	0	

[a] All Σ_{a1} are given at $T = T^0 = 300\,\mathrm{K}$.
[b] Region R is the subregion of region 3, from which the blade is dropped during the initiating perturbation. They are initially identical; Σ_{a2} in region R is then modified according to Eq. (11).

The initial average core power density $\overline{P}^0$ is:

$$\overline{P}^0 = 1.0 \times 10^{-6}\,\mathrm{W/cm^3}. \tag{12}$$

Table 2: Additional Reactor-Wide Parameters

Parameter	g or m	**Value**
V_{ng}[cm/s]	1	3.0×10^7
	2	3.0×10^5
β_m	1	0.0054
	2	0.001 087
$\lambda_m[\mathrm{s}^{-1}]$	1	0.006 54
	2	1.35
$B_z^2[\mathrm{cm}^{-2}]$		1.0×10^{-4}
ν[n/fission]		2.43
$\alpha[\mathrm{K\,cm}^3]$		3.83×10^{-11}
$\gamma[\mathrm{K}^{-1/2}]$		2.034×10^{-3}
κ[J/fission]		3.204×10^{-11}

The initial core temperature T^0 is uniform:

$$T^0 = 300\,\mathrm{K}. \tag{13}$$

k^0_{eff} is chosen such that the reactor is initially critical and at steady state. The delayed neutron precursors are in equilibrium with the initial critical flux shape. k^0_{eff} remains in Eqs. (1) once the perturbation starts.