October 27, 2015

Abstract

We compare solutions of partial differential equations of advection-diffusion-type

$$\frac{\partial \rho(x,t)}{\partial t} + \frac{\partial (a(x)\rho(x,t))}{\partial x} = D\frac{\partial^2 \rho(x,t)}{\partial x^2} \tag{1}$$

generated in two ways:

- By explicitly solving eq. (1) using the discretization scheme[1]

$$\rho_i^{n+1} = \rho_i^n + \Delta t \left(\frac{D}{\Delta x^2} \left(\rho_{i+1}^n - 2\rho_i^n + \rho_{i-1}^n \right) - \frac{a(x)}{\Delta x} (\rho_i^n - \rho_{i-1}^n) \right) \tag{2}$$

  for the value of the probabilty density ρ at position $i\Delta x$ and time $(n+1)\Delta t$.

- By simulating an ensemble of N particles evolving according to the corresponding stochastic differential equation

$$\mathrm{d}X = a(X)\mathrm{d}t + \sqrt{2D} \cdot \mathrm{d}W. \tag{3}$$

  The position X^{n+1} at time $(n+1)\Delta t$ of each particle is simulated using the Euler-Maruyama scheme

$$X^{n+1} = X^n + a(X^n)\Delta t + \sqrt{2D\Delta t} \cdot \xi^n \tag{4}$$

  with $\xi^n \sim \mathcal{N}(0,1)$

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[1] First-order upwind scheme for the advection part combined with the Forward-Time Central-Space-method for the difussion part