1 Time evolution of coarse states on static interaction networks with different topologies

1.1 Fully connected network

Figure 1 shows the ensemble average over 1000 realizations of the average state in a fully connected network as a function of time. Simulations are performed for two different coupling constants $\mathcal{N}(\bar{\nu} = 0.5)$ and $\mathcal{N}(\bar{\nu} = 0.05)$ and two different initial conditions $\mu_{0n} \sim \mathcal{B}(\mu; 0.5)$ respectively $\mu_{0n} \sim \mathcal{B}(\mu; 0.9)$ for $n = 1, ..., 4000$. The values of the microscopic parameters are chosen in correspondence with these in the AHS-lattice-model [1]. Each agent is interacting with all other agents. For small values of the average coupling constant $\bar{\nu}$, the mixed state is the unique macroscopic stable equilibrium. If the average value of the coupling constant $\bar{\nu}$ is increased, two new macroscopic states arise, suggesting the presence of a pitchfork bifurcation.

1.2 Erdős-Rényi network

Above described experiments are repeated with the same simulation parameters and system size, but with all agents randomly connected with on average 100 other agents. The connections don't change in time and the agents only interact with agents they are connected with. Figure 2 shows that it took at least 20 iterations to achieve the locked-in states, which is longer than the time needed to reach equilibrium in a fully connected network. For the mixed state however, the relaxation time does not differ significantly between both models.

2 Variance of asymptotic equilibria

The expectation value $\mathbb{E}(\rho_N)$ and the variance $\mathtt{var}(\rho_N(t))$ are monitored as a function of time for a collection of different means of the initial states. In Figure **??** the several orbits on the $(\mathbb{E}(\rho_N), \mathtt{var}(\rho_N(t)))$-plane are shown for low, intermediate and high values of the average interaction strength.

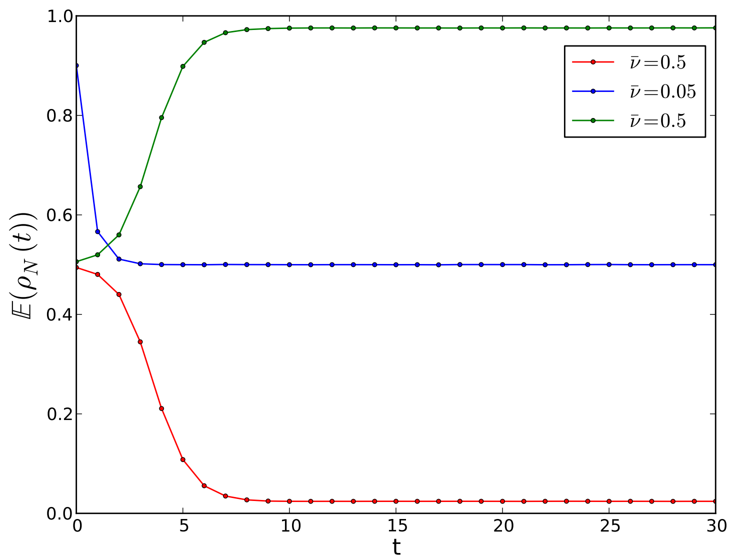


Figure 1: Average time evolution of coarse states in a fully-connected network with 4000 nodes.

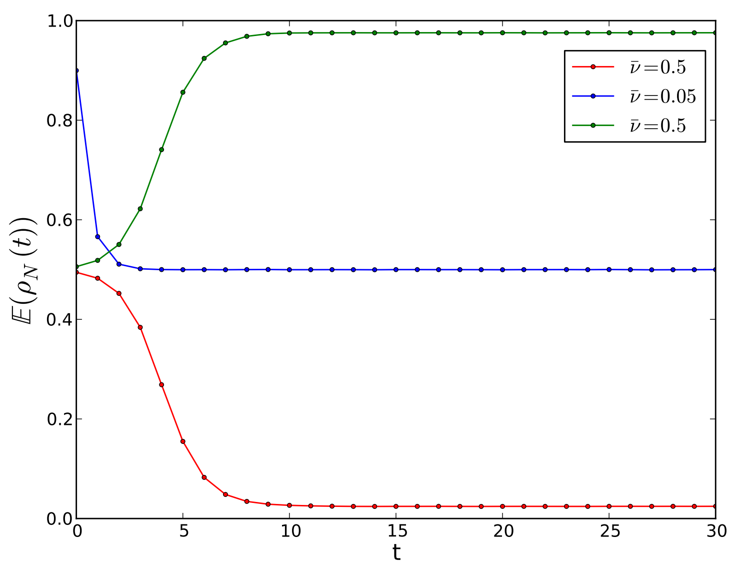
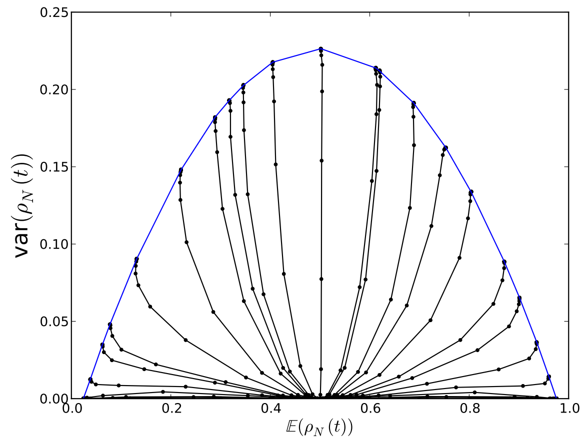
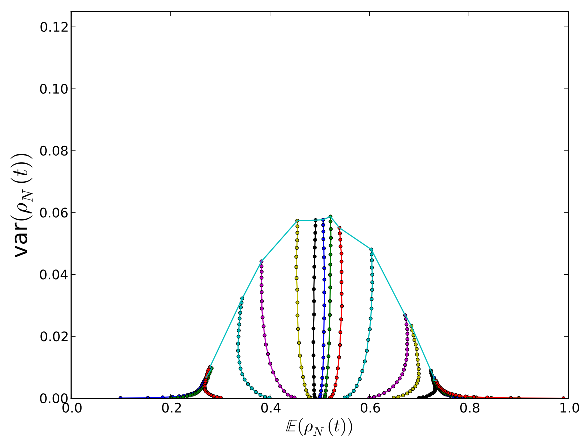


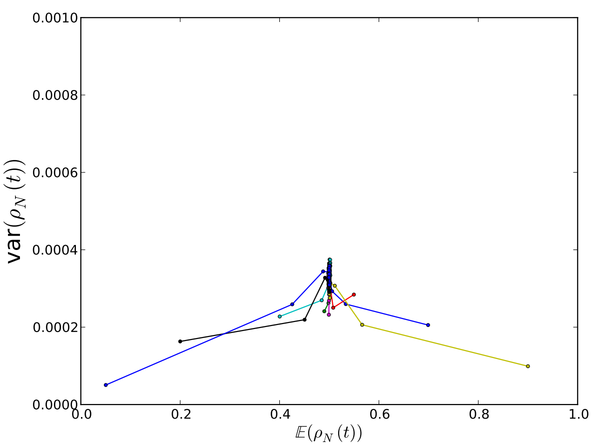
Figure 2: Average time evolution of coarse states in an Erdős-Rényi network of 4000 nodes with average degree 100.



(a) High coupling strength: $\bar{\nu} = 0.5$, $\bar{\zeta} = 0.236$.



(b) Intermediate coupling strength: $\bar{\nu} = 0.263$, $\bar{\zeta} = 0.0879$.



(c) Low coupling strength: $\bar{\nu} = 0.05$, $\bar{\zeta} = 0.0167$.

Figure 3: Expectation and variance of average states, starting from different initial states u_0. For each trajectory, we initialise $M = 500$ independent simulations for $N = 1000$ agents and iterate the model until $t = 20$.

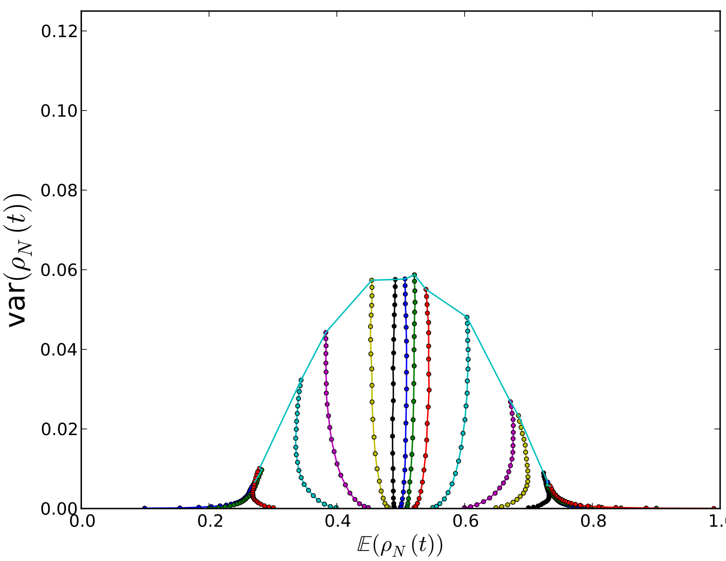


Figure 4: Expectation and variance of average states for low average coupling: $\bar{\nu} = 0.05$, $\bar{\zeta} = 0.0167$). For each trajectory, we initialise $M = 500$ independent simulations for $N = 1000$ agents and iterate the model until $t = 20$.

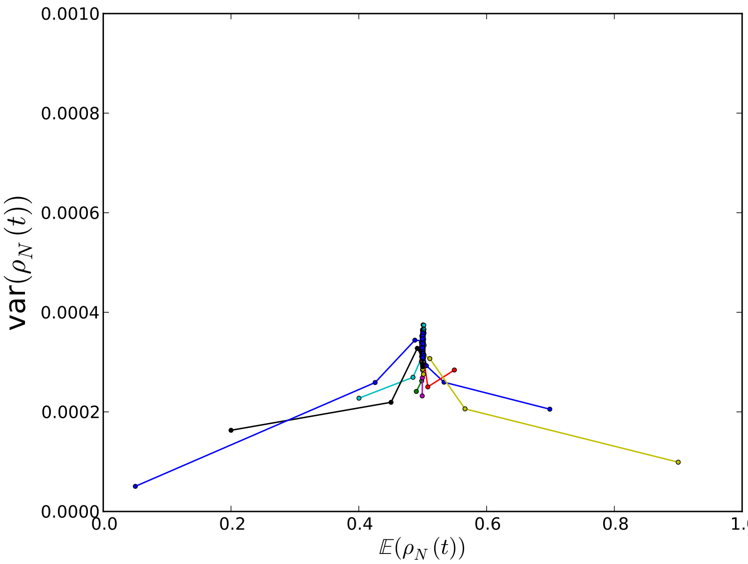


Figure 5: Expectation and variance of average states for low average coupling: $\bar{\nu} = 0.05$, $\bar{\zeta} = 0.0167$). For each trajectory, we initialise $M = 500$ independent simulations for $N = 1000$ agents and iterate the model until $t = 20$.

References

[1] Daniele Avitabile, Rebecca Hoyle, and Giovanni Samaey. Noise reduction in coarse bifurcation analysis of stochastic agent-based models: an example of consumer lock-in. *SIAM Journal on Applied Dynamical Systems*, 13(4):1583–1619, 2014.

[2] Duncan J Watts and Steven H Strogatz. Collective dynamics of 'small-world' networks. *Nature*, 393:440–442, 1998.