# Chapter 1

# Chow Rings and Bergmann Fans

# 1.1 Chow Rings

TO DO: Introduce the Chow Ring of a matroid. From lattice of flats to quotient ring

TO DO: Use our example matroid and construct its Chow Ring

#### 1.1.1 Properties of Chow Rings

TO DO: Use words like homogeneous polynomial, graded ring, etc...

#### 1.1.2 The Degree Map

TO DO: How do I explain this? I guess I can at least say it's linear and sends terms of full degree to 1. Maybe I'll understand it this time around

## 1.2 Bergmann Fans

TO DO: Quick definition of a fan?

### 1.2.1 How to make a Bergmann Fan

TO DO: Show definition from Chow Ring to Bergmann Fan

TO DO: Work our small example into a fan

### 1.2.2 Properties of Bergmann Fans

TO DO: Figure out their important properties. They're unimodal, so we'll include that. Oh, they're balanced as well (and so tropical). What am I missing?

TO DO: Define star of a fan. Reference A-H-K; star of a Bergmann fan is again a Bergmann fan? Or something like that

# 1.3 Relationship with the Characteristic Polynomial

TO DO: Come up with a nice way of relating the reduced characteristic polynomial with our ring (and therefore fan)

TO DO: Define  $\alpha$  and  $\beta$ . Here or in a subsection? Or should it be up when we introduce the ring itself?

### 1.3.1 How to show Log-Concavity

TO DO: Offer a self-contained proof that log-concavity is equivalent to showing that one particular relationship between  $\alpha$  and  $\beta$  found in A-H-K. They have the proof, but it requires digging through citations. Should be able to consolidate it.