$\tilde{\nu}$ usually in cm

$\int \frac{dE}{E} = -\tau \int dt$ dipole

$\frac{m_p}{m_p + m_e}$ + binomial exp.

Important features of Hydrogenic Wave functions

Ortho-normality

$\underbrace{\text{normality}}_{\text{normalized}}$

$\underbrace{\text{Ortho}}$ orthogonal ?!?

$$\int \psi^*_{n\ell m} \psi_{n'\ell' m'} r^2 \sin\theta \, dr \, d\theta \, d\varphi = \delta_{nn'} \delta_{\ell\ell'} \delta_{mm'}$$

$$\delta_{ij} = \text{Kroenecker delta} = \begin{cases} 1 & i = j \\ 0 & \text{else} \end{cases}$$

Becaus they are product solutions, normalize parts independently

$$\int_0^\infty R^*_{n\ell}(r) R_{n\ell}(r) \, r^2 dr = 1$$

$$\int_{d\Omega} Y^*_{\ell m}(\theta, \varphi) \, Y_{\ell m}(\theta, \varphi) \sin\theta \, d\theta \, d\varphi = 1$$

$Y_{\ell m}$'s separately orthonormal $\int_{d\Omega} Y^*_{\ell m} Y_{\ell' m'} d\Omega = \delta_{\ell\ell'} \delta_{mm'}$

and $\int_0^\infty R^*_{n\ell} R_{n'\ell'} r^2 dr = \delta_{nn'}$

E-Dipole operator as transition

Perturbation $H_{pert} = H' = \underbrace{e\vec{r}}_{\text{E dipole operator}} \cdot \underbrace{\vec{E}(t)}_{|\vec{E}_0| Re(e^{-i\omega t} \hat{e}_{rad})}$

electric field direction (polarization vector)

Rate $\propto |e\vec{E}_0|^2 \; \underbrace{|\langle 2 | \vec{r} \cdot \hat{e}_{rad} | 1 \rangle|^2}$

from 1 to 2

properties:

$\langle 2 | \vec{r} \cdot \hat{e}_{rad} | 1 \rangle = \underbrace{D_{12} I_{ang}}$

since $\psi_{nlm} = R_{nl} Y_{lm}$

$D_{12} = \int_0^\infty R_{n_2 l_2} \, r \, R_{n_1 l_1} r^2 dr$ } not usually zero, but can be small

$I_{ang} = \int_{d\Omega} Y_{l_2 m_2} (\hat{r} \cdot \hat{e}_{rad}) Y_{l_1 m_1} \sin\theta \, d\theta \, d\phi$

→ leads to selection rules (e.g. it is identically zero unless particular conditions apply

If z defined (external field) or light polarized

π-transitions ($\vec{E} \parallel z$) $I^{\pi}_{ang} = 0$ or $m_1 = m_2$

equiv $\Delta m_l = 0$

σ-transitions ($\vec{E} \perp z$) $I^{\sigma} \neq 0$ $\sigma^+ \to m_{l1} - m_{l2} = -1$ } $\Delta m_l = \pm 1$

$\sigma^- \to m_{l1} - m_{l2} = +1$

If not z or pol. $\Delta m_l = 0, \pm 1$

Parity what is it?
behavior under $\vec{r} \to -\vec{r}$ (sort of like even-odd f(x))

$\hat{P}\,\psi = p\,\psi$ state of definite parity
$p = \pm 1$ $\left(\begin{smallmatrix}\text{even}\\ \text{odd}\end{smallmatrix}\right)$

$P\,Y_{lm} = (-1)^l\,Y_{lm}$

I_{ang} $(-1)^{l_2+l_1+1}\,I_{ang}$ E dipole $\Delta l \neq 0$