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Tycho Brahe from Denmark made very precise measurements of the positions of the planets. Johannes Kepler worked for Tycho trying to make sense of the data. Eventually he came up with three simple relationships concerning the motion of the planets that explained all the data.

1) The shape of a planets orbit is the geometric figure called an ellipse….

(def of circle as all points R away from one point; ellipse as all point such that sum of distances from 2 points same)

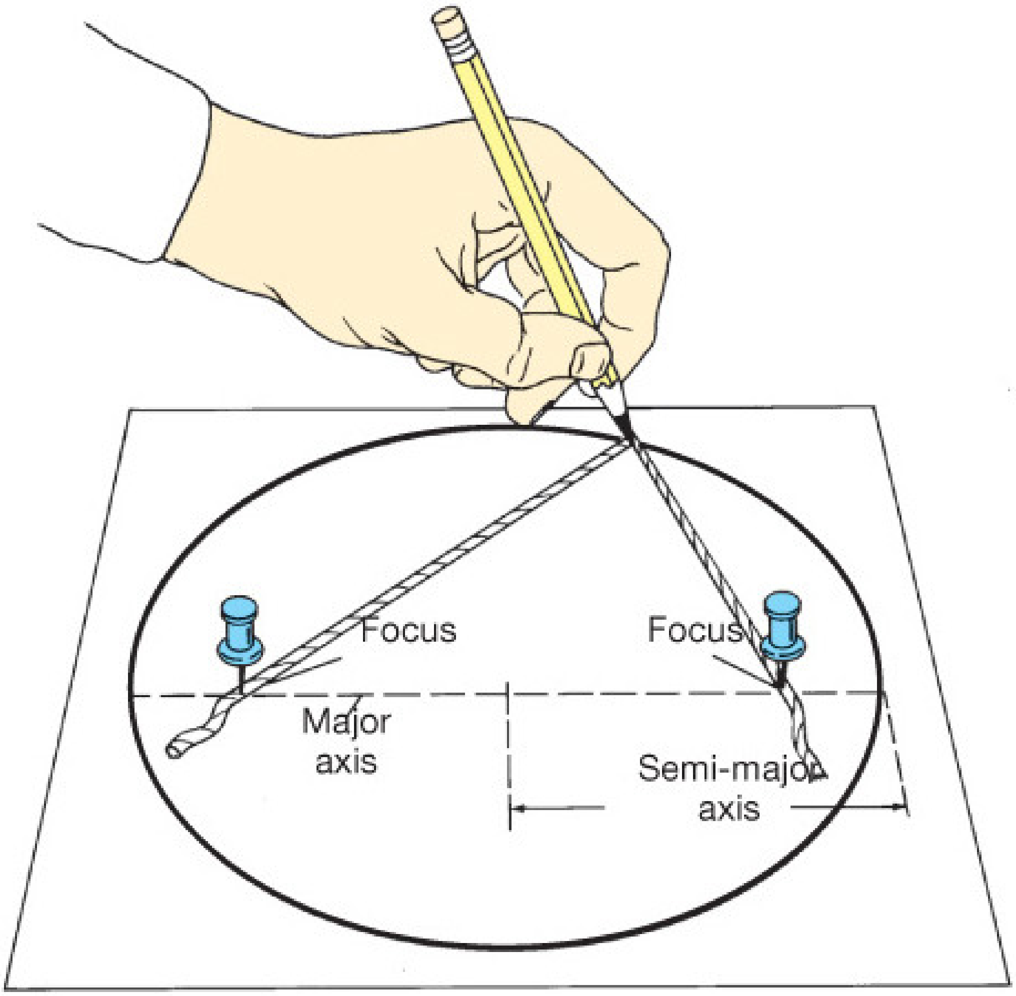


Fig 1.11 AGBU 5e

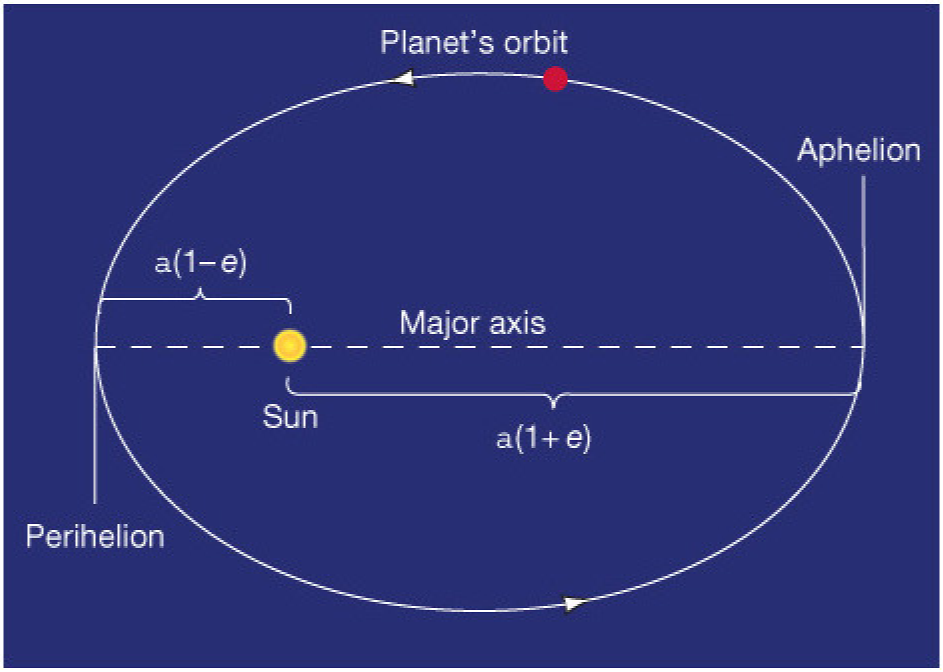
… with the sun located at one focus of the ellipse that the planet is travelling along (see below). Note: there is nothing at the other focus of the ellipse for a planets orbit. The location in its orbit where a planet is closest to the sun is called the perihelion (peri – near/around; hellion – sun). The location where a planet is farthest from the Sun is called the aphelion (ap – from).

The orbit of a planet around the sun is characterized by its size and how not-circular it is.

The entire length across the longest part of the orbit is called the major axis (2a). Half of it is called the semi-major axis (a).

Similarly, the length across the smallest part of the orbit is the minor axis (2b), and half of it is the semi-minor axis (b). Together these sort of tell us how long and skinny and orbit is compared to how circular it is. Another way to express this is by a number called the eccentricity (e). A circle has an eccentricity of zero. An extremely elongated ellipse has an eccentricity approaching 1. In general, the eccentricity is related to the semi-major and semi-minor axes of an ellipse quantitatively as $e = \sqrt{1 - \frac{b^2}{a^2}}$. It can also be related to the distances of closest and farthest approach:

$$e = \frac{r_{aphelion} - r_{perihelion}}{r_{aphelion} + r_{perihelion}}$$



2) If you imagine a line connecting the planet to the sun and follow that line over some time interval, it will trace out a roughly slice-of-pie shaped bit of the ellipse. At any point in the orbit, if you let it trace out just one hour's worth of slice, the area of the slice is the same as the area of any other slice made in the same way in a 1 hour interval. What does this mean? Among other things, when the planet is closer to the sun, it must be moving faster to trace out the same area in the same time (making a wider slice). (Kepler 2 applet—class website)
3) If you measure the planet's period in years and its semi-major axis in astronomical units, the two are related: $P^2=a^3$ (this only works for things orbiting our sun!-- of course Kepler wasn't thinking of anything else and didn't have data on anything else)

<u>Newton's Laws of Motion</u>
We mentioned how Kepler's laws were empirical-- that is, they fit his data, but he did not know why they worked, he had no underlying cause for the planets to behave that way.

Galileo and then Newton worked out a general understanding of the rules of motion of objects—any objects. We will see that this modifies Kepler's laws a little bit-- making them more general than just applying to planets in our solar system.

<u>Newton's first law of motion</u>: an object moves in a straight line at a constant speed unless an unbalanced force acts on it. (straight line at a constant speed ==constant velocity)
Force=a push or pull
Velocity ==speed and direction: a quantity of the general type we called a vector that has both size and direction

Balanced vs. unbalanced: tied tug of war—equal size forces in opposite directions =balanced. 3 equal forces at

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120 degrees=balanced, etc.

We've all seen things that don't continue in a straight line at a constant speed. For example, on a highway I get the car up to 65 miles an hour and take my foot off the gas. What happens? So does that violate Newton's first law? What forces?

Newton's second law of motion: if there is an unbalanced force, velocity changes in a specific way. Here we have to note that a change in velocity (which could be a change in the speed, or in the direction, or both) is defined as acceleration.

The average acceleration == (how much the velocity changes)/(how long the change takes to happen)
acceleration is also a vector quantity. The direction of acceleration is the same as the direction of the unbalanced force.
As long as an unbalanced force acts, velocity will be changing:
speeding up/slowing down
turning
-twirling mass on string. Force on mass? More force or less if faster?
-car around corner

Size of accel=(size of unbalanced force)/(mass of object force acts on)

Note: mass is not the same as weight!

Newton's third law of motion: Forces never occur alone. If one object exerts a force on a second object, the second object exerts a force on the first. → a better way to think about forces is that they really are the result of mutual interactions between objects. Furthermore Newton found that these paired forces are the same size and act in opposite directions, as well as acting on different objects.
--carts on track with plunger.

So what does this have to do with Kepler?
-- since planets move around and curves (ellipses in general, circles in special cases) even if they move at constant speed they are continually changing direction which means they are accelerating. This means there must be some force acting on them just like the string pulls the ball around in a circular or other path. Newton realized this force was the gravitational attraction between masses.
-Fgrav = (some constant)* mass1*mass2/(separation)^2
-always attractive (tries to pull masses toward each other--sketch)
-this form technically for point masses, works perfectly for spheres and well for near-spherical objects and for size<<separation

Specifically:

$$F_{grav} = \frac{Gm_1m_2}{r^2}$$

r=distance between centers

G=6.67x10^{-11} N*m^2/kg^2 (Metric force=Newton; 1 N approx weight of an apple on earth (.25lb))

How does this work with orbiting planets etc.? You can think about it this way. If you drop a cannonball it falls to the ground. What is responsible for its falling to the ground? Now imagine throwing it horizontally what

happens? What happens if you throw it even faster? What happens if you throw it so fast that it travels a large distance compared to the size of the Earth?
-Newton cannon applet. https://physics.weber.edu/schroeder/software/NewtonsCannon.html
Keeping this up eventually we get to the point where the cannonball is "falling" just as fast as the surface of the Earth is curving away from it (run applet to full circle)

The circular path around the earth occurs at a speed when the gravitational force is just what's needed to make them cannonball travel around in circular motion at constant speed.

For a small mass (cannonball) orbiting a much larger one (earth):

$$v_{circ} = \sqrt{\frac{GM_{earth}}{r}}$$

Where r is the distance from the earth's center

This is an approximation, but is accurate for any small mass orbiting any big mass, just substitute the big mass for Mearth

What if we keep increasing the speed? (show on applet)
-orbit gets elliptical
-slows down on way out, speeds up on way in (=Kepler II)
if launched fast enough, Fgrav can't turn it around→escape speed

$$v_{escape} = \sqrt{\frac{2GM_{earth}}{R}}$$

R= starting separation center to center (again, for escaping objects other than earth put in that mass for Mearth).

We can easily get a modified form of Kepler's third law from the circular velocity: there's an extra bit that comes in:

$$P^2 = \frac{4\pi^2 a^3}{G(m_1+m_2)}$$

for two objects m1 and m2 *orbiting each other*
Note: this is in SI/MKS units (not a.u.'s, not years, not solar masses). This is a purely metric relationship P=sec, M's=kg, a=meters

This also shows us that it's the total mass of the system that matters. For Kepler, he was dealing with the sun which is so much more massive than any of the planets that Msun+Mplanet is approximately Msun. But what if the two masses are nearly equal?
-air pucks tied with string
-binary star applet http://www.astro.ucla.edu/undergrad/astro3/orbits.html
-you can see that they are orbiting on ellipses with their common center of mass at the focus. This is like two figure skaters spinning around each other (explore effects of changing mass ratios in applet)

We will see that this form of Kepler's 3rd law will help us determine the masses of stars other than the sun since we can often measure the period of binary star orbits and using our angular measure techniques can figure out what the semi-major axis is.