HW 5

1a) $^2H + d \rightarrow n + {}^3He$

$Q = (\Delta^2H + \Delta d - \Delta n - \Delta^3He)c^2$

$= (14102 + 14102 - 8665 - 16029)\,\mu u\, c^2$

$= 3.2695\,MeV$ exothermic

$^2H + d \rightarrow p + {}^3H$

$Q = (14102 + 14102 - 7825 - 16049)\,\mu u c^2$

$= 4.0334\,MeV$

b) for 0.3 MeV:

lab	cm
1.34	1.58

c) 2.23 MeV @ -120.54° 2.29 MeV @ -112.26°

d) 1.58 MeV 1.85 MeV

e) only (d,n) makes n!

	lab	cm
E_{max} (@ 0°) =	3.22 MeV	3.65 MeV
E_{min} (@ 180°) =	1.98 MeV	1.86 MeV

$.07102 \ cm^2/g$

7a) @ 4 MeV $\mu_m = 0.07102 \frac{cm^2}{g}$ (NIST site)

$I(x) = I_0 e^{-\mu x}$

$\mu_m = \mu/\rho$

$\mu = \rho \mu_m$

$= 11.4 \frac{g}{cm^3} \cdot 0.07102 \frac{cm^2}{g} = 0.809628 / cm$

$\frac{I(x)}{I_0} = 10^{-6} = e^{-\mu x}$

$\ln 10^{-6} = -\mu x$

$x = \frac{\ln 10^{-6}}{-\mu} = \frac{-13.8155}{-0.809628/cm} = 17.064 \, cm$

b) @ 100 keV $\mu_m = 5.549 \, cm^2/g \Rightarrow \mu =$

c) 2 MeV e^- have range $\approx 1 \, g/cm^2$ from Fig 5.5.
Since water has a density $1 \, g/cm^3$,
the Range (in cm) is $\frac{1 g/cm^2}{1 g/cm^3} = 1 \, cm$.

3(a) Several approaches. 1st Note

$\frac{dE}{dx}$ in same material scales like $Z^2_{\text{projectile}}$ and some function of projectile velocity.

so $\left.\frac{dE}{dx}\right|_{\text{same material}} \propto Z^2 f(v)$ so for same velocity

protons & α's, $\frac{\left.\frac{dE}{dx}\right|_\alpha}{\left.\frac{dE}{dx}\right|_p} \propto \frac{Z_\alpha^2}{Z_p^2} = 4$. A 1MeV proton has same speed as a 4 MeV α ($E/A \approx$ same)

so

$$\left.\frac{dE}{dx}\right|_{4\text{MeV}\,\alpha} = 4 \left.\frac{dE}{dx}\right|_{1\text{MeV}\,p}$$

and since $\frac{dE}{dx} \propto \frac{1}{E^k} \approx \frac{1}{E}$, $\frac{\left.\frac{dE}{dx}\right|_{4\text{MeV}\alpha}}{\left.\frac{dE}{dx}\right|_{\text{MeV}\alpha}} = \frac{1\text{MeV}}{4\text{MeV}}$

$$\text{so } \left.\frac{dE}{dx}\right|_{4\text{MeV},\alpha} = \frac{1}{4}\left.\frac{dE}{dx}\right|_{1\text{MeV},\alpha}$$

together

$$\left.\frac{dE}{dx}\right|_{1\text{MeV},p} = \frac{1}{4}\left(\frac{1}{4}\left.\frac{dE}{dx}\right|_{1\text{MeV},\alpha}\right) = \frac{1}{16}\left.\frac{dE}{dx}\right|_{1\text{MeV},\alpha}$$

HW5

Note: $\frac{2mv^2}{I} = \frac{2m_e c^2 \beta_p^2}{I}$ + $\beta_p^2 = \frac{2E}{m_p c^2}$ $\rightarrow \frac{4E}{11 \cdot 2 \cdot 1837}$ $= \frac{198}{Z}$ (@ $E = 10^6$)

3 b) For 1 MeV protons (same v + z_{proj})

$$\frac{\frac{1}{\rho_{Pb}}\frac{dE}{dx}\Big|_{Pb}}{\frac{1}{\rho_{Al}}\frac{dE}{dx}\Big|_{Al}} = \frac{Z_{Pb}}{Z_{Al}} \cdot \frac{A_{Al}}{A_{Pb}} \cdot \frac{\ln\left(\frac{2mv_p^2}{I_{Pb}}\right)}{\ln\left(\frac{2mv_p^2}{I_{Al}}\right)}$$

$$\frac{\frac{dE}{\rho dx}\Big|_{Pb}}{\frac{dE}{\rho dx}\Big|_{Al}} = \left(\frac{82}{13}\right)\left(\frac{27}{207}\right)\frac{\ln\left(\frac{198}{82}\right)}{\ln\left(\frac{198}{13}\right)} = 0.8227 \frac{0.8815}{2.72} = 0.266$$

$$\frac{dE}{\rho dx}\Big|_{Pb} = 0.2667 \frac{dE}{\rho dx}\Big|_{Al} = 0.2667\left(200 \frac{MeV}{g/cm^2}\right) = 53.26$$

$$\frac{dE}{dx}\Big|_{Pb} = \rho_{Pb}\frac{dE}{\rho dx}\Big|_{Pb} = 607.15 \frac{MeV}{cm}$$

c) $$\frac{dE}{dx}\Big|_{\alpha, Pb} = \frac{dE}{dx}\Big|_{p, Pb} \cdot 4 = 607.16 \cdot 4 = 2428 \ MeV/cm$$

4. a) From NNDC chart of nuclides ^{16}N endpoint is 10.4 MeV (Highest energy branch)

b) ESTAR says in Si Range of 10 MeV e^- is 5.642 g/cm^2

12.5 MeV e^- is 6.849 g/cm^2

linear interp gives Range of x 5.835 g/cm^2

$\rho_{Si} = 2.33 \ g/cm^3$

so Range $= \frac{5.835 \ g/cm^2}{2.33 \ g/cm^3} = 2.50$ cm

c) CSDA is over the whole path length travel (all the steps in the zigs + zags) so it's would be an ~~≈~~ upper limit.

5. a) Lilley 5.8

Eq. 5.13 @ $\theta = 180°$ ($\cos\theta = -1$) is

$$E_\gamma' = \frac{E_\gamma}{1 + \frac{2E_\gamma}{mc^2}}$$ only E_γ can vary so where is E_γ' a max

$$f = E_\gamma'/mc^2 = \frac{E_\gamma/mc^2}{1 + 2\,E_\gamma/mc^2} = \frac{x}{1+2x}$$

$$\frac{\partial f}{\partial x} = \frac{1}{1+2x} - \frac{x\,2}{(1+2x)^2} = 0$$

$$1 - \frac{2x}{1+2x} = 0 \rightarrow 1 = \frac{2x}{1+2x} \quad 1+2x = 2x \Big\}\text{only at } \infty$$

$$@ \quad E_\gamma \rightarrow \infty \qquad E_\gamma' = \frac{mc^2}{2} = \frac{0.511}{2} = 0.2555 \text{ MeV}$$ ✓

b) all remaining energy $E_\gamma - E_\gamma' = T_e$
so as $E_\gamma \uparrow$, E_γ' approaches a fixed 0.2555 MeV less than that.

6a) Estimate 4" to middle of brain (≈10 cm) Treat as water.

pstar gives: 70 MeV → 4.075 g/cm² (= cm for H_2O)
65 MeV → 3.567 g/cm²
60 MeV → 3.089 g/cm²

so ≈ 70 MeV

b) To travel 1 cm less would take 60 MeV
implies ≈ 10 MeV in last cm → 10 MeV/cm

c.f. $\frac{70 \text{ MeV}}{10 \text{ cm}} = 7 \text{ MeV/cm}$ } (less)