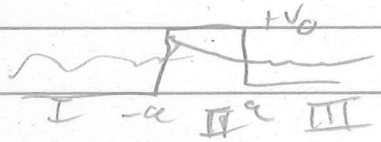
2.33



$$\psi = \begin{cases} Ae^{ikx} + Be^{-ikx} & \text{I} \\ Ce^{lx} + De^{-lx} & \text{II} \\ Fe^{ikx} & \text{III} \end{cases} \qquad k = \frac{\sqrt{2mE}}{\hbar} \qquad l = \frac{\sqrt{2m(V_0-E)}}{\hbar}$$

Continuity:

I $\quad x=-a \quad Ae^{-ika} + Be^{ika} = Ce^{-la} + De^{la}$

II $\quad x=+a \quad Ce^{la} + De^{-la} = Fe^{ika}$

derivative

III $\quad x=-a \quad Aike^{-ika} - Bike^{ika} = Cle^{-la} - Dle^{la}$

$$Ae^{-ika} - Be^{ika} = \frac{l}{ik}\left(Ce^{-la} - De^{la}\right)$$

IV $\quad x=+a \quad Cle^{la} - Dle^{-la} = Fike^{ika}$

$$Ce^{la} - De^{-la} = F\frac{ik}{l}e^{ika}$$

II + IV $\quad Ce^{la} = \frac{Fe^{ika}}{2}\left(1 + \frac{ik}{l}\right) \qquad C = \frac{Fe^{ika}e^{-la}}{2}\left(1 + \frac{ik}{l}\right)$

II − IV $\quad De^{-la} = \frac{Fe^{ika}}{2}\left(1 - \frac{ik}{l}\right) \qquad D = \frac{Fe^{ika}e^{la}}{2}\left(1 - \frac{ik}{l}\right)$

I + III $\quad 2Ae^{-ika} = Ce^{-la} + De^{la} + \frac{l}{ik}\left(Ce^{-la} - De^{la}\right)$

$$2Ae^{-ika} = Ce^{-la}\left(1 + \frac{l}{ik}\right) + De^{la}\left(1 - \frac{l}{ik}\right)$$

$$2Ae^{-ika} = \frac{Fe^{ika}}{2}e^{-2la}\left(1 + \frac{l}{ik}\right)\left(1 + \frac{ik}{l}\right) + \frac{Fe^{ika}}{2}e^{2la}\left(1 - \frac{ik}{l}\right)\left(1 - \frac{l}{ik}\right)$$

Goal: Collect factors + move F + e^{ika} to LHS:

4 $\frac{A}{F} e^{-2ika} = e^{-2la}\left(1+\frac{l}{ik}+\frac{ik}{l}+1\right) + e^{2la}\left(1-\frac{ik}{l}-\frac{l}{ik}+1\right)$

$$= e^{-2la}\left(2+\frac{l^2-k^2}{ikl}\right) + e^{2la}\left(2+\frac{k^2-l^2}{lik}\right)$$

$$= e^{-2la}\left(\frac{2ikl+l^2-k^2}{ikl}\right) + e^{2la}\left(\frac{2ikl+k^2-l^2}{ikl}\right)$$

$$= e^{-2la}\left(\frac{(l+ik)^2}{ikl}\right) + e^{2la}\left(\frac{(k+il)^2}{ikl}\right)$$

$\frac{A}{F} 4ikl e^{-2ika} = e^{-2la}(l+ik)^2 + e^{2la}(k+il)^2$

Goal: Remove i's:

$\left|\frac{A}{F}\right|^2 16k^2l^2 = \left[e^{-2la}(l+ik)^2 + e^{2la}(k+il)^2\right]\left[e^{-2la}(l-ik)^2 + e^{2la}(k-il)^2\right]$

$$= e^{-4la}(l+ik)^2(l-ik)^2 + e^{4la}(k+il)^2(k-il)^2$$
$$+ (l+ik)^2(k-il)^2 + (k+il)^2(l-ik)^2$$

$$= e^{-4la}\left[(l+ik)(l-ik)\right]^2 + e^{4la}\left[(k+il)(k-il)\right]^2$$

$$+ \left[lk - il^2 + ik^2 + kl\right]^2 + \left[kl - ik^2 + il^2 + lk\right]^2$$

$$= e^{-4la}\left[l^2+k^2\right]^2 + e^{4la}\left[k^2+l^2\right]^2$$
$$\left[2lk + i(k^2-l^2)\right]^2 + \left[2lk - i(k^2-l^2)\right]^2$$

$$= \left[l^2+k^2\right]^2\left(e^{-4la}+e^{4la}\right) + 4l^2k^2 - (k^2-l^2)^2 + \cancel{4lik(k^2-l^2)}$$
$$+ 4l^2k^2 - (k^2-l^2) - \cancel{4lik(k^2-l^2)}$$

$\left|\frac{A}{F}\right|^2 16k^2l^2 = \left[l^2+k^2\right]^2\left(e^{-4la}+e^{4la}\right) + 8l^2k^2 - 2(k^2-l^2)^2$

$$\left|\frac{A}{F}\right|^2 = \frac{(l^2+k^2)^2}{8k^2l^2}\cosh 4la + \frac{1}{2} - \frac{(k^2-l^2)^2}{8k^2l^2}$$

see if l + k terms simplify:

$$l^2+k^2 = \cancel{\frac{2mE}{\hbar^2}} + \frac{2mV_0}{\hbar^2} - \cancel{\frac{2mE}{\hbar^2}}$$

$$k^2 - l^2 = \frac{2mE}{\hbar^2} - \frac{2mV_0}{\hbar^2} + \frac{2mE}{\hbar^2}$$

$$= \frac{4mE}{\hbar^2} - \frac{2mV_0}{\hbar^2}$$

$$k^2 l^2 = \frac{2mE}{\hbar^2}\frac{2m(V_0-E)}{\hbar^2} = \frac{4m^2E(V_0-E)}{\hbar^4}$$

$$\left|\frac{A}{F}\right|^2 = \left(\frac{2mV_0}{\hbar^2}\right)^2 \frac{\hbar^4}{8\,4m^2E(V_0-E)}\cosh 4la + \frac{1}{2} - \left(\frac{4mE}{\hbar^2} - \frac{2mV_0}{\hbar^2}\right)^2 \frac{\hbar^4}{8\,4m^2E(V_0-E)}$$

$$\left|\frac{A}{F}\right|^2 = \frac{V_0^2}{8E(V_0-E)}\cosh 4la + \frac{1}{2} - \frac{(2E-V_0)^2}{8E(V_0-E)}$$

$$\left|\frac{A}{F}\right|^2 = \frac{V_0^2\cosh 4la - (2E-V_0)^2 + 4E(V_0-E)}{8E(V_0-E)}$$

$$= \frac{V_0^2\cosh 4la - 4E^2 - V_0^2 + 4EV_0 + 4EV_0 - 4E^2}{8E(V_0-E)}$$

$$\left|\frac{A}{F}\right|^2 = \frac{V_0^2\cosh 4la - 8E^2 + 8EV_0 - V_0^2}{8E(V_0-E)}$$

$$\left|\frac{A}{F}\right|^2 = \frac{V_0^2(\cosh 4la - 1) - 8E(E-V_0)}{8E(V_0-E)}$$

$$= \frac{V_0^2(\cosh 4la - 1) + 8E(V_0-E)}{8E(V_0-E)}$$

$$T^{-1} = \left|\frac{A}{F}\right|^2 = \frac{V_0^2}{8E(V_0-E)}(\cosh 4la - 1) + 1$$

Probably there's a $\frac{1}{2}$ angle formula to get into some form.