Physics 216 Laboratory 13, Spring 2018
Special Relativity

Background
Since our budget doesn't allow for a fleet of relativistic spaceships, we'll use a series of computer simulations to investigate some situations in which the concepts of special relativity are important. All the simulations that you will be examining today, along with a number of other simulations that you are encouraged to play with, are found at http://www.compadre.org/PQP/special-relativity/.

Once you have selected the appropriate simulation, you will see a set of control buttons under the animation that will allow you to control the animation (play, pause, step backward/forward one frame). If there are several animations on the page, a ***Restart*** link on the page will restore the entire page to its initial condition.

Investigation 1: Time dilation and length contraction
Open Section 2.4 from the menu on the left. You should see two animations, with a description under each animation. Read the description under the top animation, and then set $|\beta|=|u/c|$ to 0.5 and press the *play* button.

In 5 ticks of the clock in the stationary frame (red clock), how many ticks (estimate fractional ticks) does the green clock make?

Show that these measurements agree with the time dilation formula.

What would you see from the green reference frame? Why?

Now scroll down the page to the other animation (rotated light clocks). Once you've read the text under the animation, set $|\beta|=|u/c|$ to 0.5 and press the *play* button.

Pause the animation at a point where the green clock is still visible. Using the small crosshairs in the lower left region of the animation, measure the length of the red and green clock in the red reference frame by recording the position of each end of the clock. Be sure to include units.

Length of green clock in red reference frame:

Length of red clock in red reference frame:

Show that these measurements agree with the length contraction formula.

What would you see from the green reference frame? Why?

√ Checkpoint 1

Investigation 2: Worldline matching

Open Section 2.6 from the frame on the left of the page. In this simulation, you will explore worldlines, which are used to depict the trajectory of a particle in spacetime.

Your goal is to match a series of worldlines that can be loaded by clicking one of the links below the control buttons. The position of the red ball can be controlled by using the mouse to move the red ball in the box below the spacetime diagram.

For each of the worldlines, sketch the spacetime diagram and describe in words how you had to move the red ball to match the worldlines.

Worldline 1

Worldline 2

Worldline 3

Worldline 4

Worldline 5 (along the line of v=c)

Did you find any bad physics in the animations? If so, what was it?

√ Checkpoint 2

Investigation 3: Simultaneity

Open Section 2.3 from the frame on the left of the page. In this simulation, you will explore the nature of simultaneity. There are two different “experiments.” In each, an event occurs and this event is then observed by four observers at different locations in two different coordinate systems.

Run Experiment 1: From the frame of the ground.
Describe the order in which the light flash reaches each of the four observers, according to an observer in the reference frame of the ground.

Are there any simultaneous events in this reference frame? If so, which events?

Run Experiment 2: From the frame of the railcar.
Describe the order in which the light flash reaches each of the four observers, according to an observer in the reference frame of the railcar.

Are there any simultaneous events in this reference frame? Are they the same events as in the other reference frame?

Are the results of these two experiments consistent? Explain.

Investigation 4: Simultaneity using relativistic measuring sticks

Open Problem 2.7 from the frame on the left of the page. In this simulation, you will explore the nature of simultaneity from a slightly different perspective. In this simulation, two measuring sticks pass each other with a given relative speed. A light flash occurs when the right ends of the sticks coincide (event 1) and again when the left ends coincide (event 2).

Observe the events at several different relative speeds and from both reference frames. Record what happens and when, according to each reference frame, in the table below. Try to find a relative speed such that the events are simultaneous in one or both reference frames.

v/c	Event 1 (right ends) in Green frame	Event 2 (left ends) in Green frame	Event 1(right ends) in Red frame	Event 2(left ends) in Red frame

Based on your observations, answer the following.

Does one of the sticks have a greater proper length, and if so, which one? How can you tell?

Does event 1 always occur before event 2? Does the answer depend on the reference frame?

Is there any speed for which the events are simultaneous in <u>both</u> reference frames?

Is there any speed for which the events are simultaneous in <u>one</u> reference frame? What is special at that speed?

At a given speed, will the red and green observers always agree on which event occurred first? Explain.

√ **Checkpoint 3**

Investigation 5: Relativistic pole vaulting

Open Section 2.7. In this simulation, you will explore the famous pole and barn paradox. A pole vaulter carries a long pole towards a barn as shown in the animation (and the spacetime diagram). You can place the cursor on a point in the animation and click to show the x and y coordinates in the selected frame.

Start by watching the animation from the frame of the barn.

How long is the barn in this frame? How long is the pole in this frame? Which of these is a proper length?

How fast is the pole moving relative to the barn (v/c) (show how you got this)?

What do events A, B, A', and B' refer to? Which, if any, of these events are simultaneous in this reference frame?

Initially consider the back door of the barn to be closed. It can be opened instantaneously when the front of the pole reaches it. Similarly, the front door will be closed instantaneously when the back end of the pole reaches it. Given that, are there any times when the pole is completely inside the barn so that both doors will be closed at the same time (even if just for an instant)? Explain.

Next, look at the animation from the frame of the pole vaulter.
How fast is the barn moving relative to the pole (v/c) (show how you got this)? Does this agree with what you measured from the barn frame? Should it?

How long is the pole in this reference frame? Why is it this long?

How long is the barn in this reference frame? Why is it this long?

What do events A, B, A', and B' refer to? Which if any of these events are simultaneous in this reference frame?

From this frame, are there any times when the pole is completely inside the barn so that both doors will be closed at the same time?

Given that from the barn frame both doors can be closed at the same time (even if only for an instant), describe the sequence of events (in terms of where/when the front/back of pole is relative to the front/back door of barn and if/when front/back door are opened/closed).

√ **Checkpoint 4**

Investigation 6: Twin paradox

Open Section 2.8. In this simulation, you will explore the famous twin paradox. One twin (red) remains on Earth, while a second twin (green) begins a trip to a distant location at t = 0 years, and then returns, reaching Earth again at t = 10 years.

First run the simulation from the view of the Earth-bound (red) twin.

How fast is the green twin traveling relative to the red twin (v/c)?

The two twins have agreed to send each other a light pulse once a year on the anniversary of the green twin's departure. Select Pulses from Earth-Bound Twin and play.

What is the frequency of the red twin's light pulse in her own reference frame?

By how many years has the red twin aged during her twin's trip?

How many light pulses reach the green twin during his outbound trip?

During his inbound trip?

Select Pulses from Traveling Twin and play.
By how many years has the green twin aged during his trip?

The red twin argues that from her point of view, the green twin is moving and therefore the green twin should age more slowly. On the other hand, the green twin argues that from his point of view, the red twin is moving and therefore the red twin should age more slowly. But when they both get back together, the green twin has actually aged less. What's going on here?

Select Pulses from Traveling Twin: ST. Now we have added the traveling twin's clock and numbers on the spacetime diagram to mark the arrival of the traveling twin's light pulse.

One of the most important concepts in special relativity is the idea of the spacetime interval, $(\Delta s)^2 = c^2(\Delta t)^2 - (\Delta x)^2$, which is the same in any two inertial frames. This can be thought of as the Pythagorean Theorem of spacetime.

Calculate the spacetime interval for the <u>red</u> twin during the <u>outbound</u> trip.

Repeat for the <u>green</u> twin during the <u>outbound</u> trip.

Calculate the spacetime interval for the <u>red</u> twin during the <u>inbound</u> trip.

Repeat for the <u>green</u> twin during the <u>inbound</u> trip.

Why is there a difference in the spacetime intervals for the twins for the entire trip?

Is there really a paradox in the fact that it's the green twin who ages more slowly? Explain.

√ **Checkpoint 5**

Investigation 7: Relativistic Doppler shift and the twin paradox (optional)

In this simulation, you will explore the famous twin paradox from a slightly different perspective. To open the simulation, select ***Special Relativity*** from the frame on the left of the page. Open ***Chapter 3*** and then open Section 3.6 from the frame on the left of the page.

In this simulation, one twin (Pink) remains on earth, while a second twin (Green) leaves in a rocket ship to travel to a distant planet at t = 0 years. When Green reaches the planet, they turn around and return to Earth 20 years later (as measured by Pink who remained on Earth). Pink has aged 20 years, but Green has not aged as much. To measure this effect quantitatively, each twin sends out a flash of light on their birthday. In each twin's own reference frame, then, these flashes occur with a frequency of one. What does it look like from the other reference frame?

First look at it from Pink's view on Earth. The time clock in the upper left corner shows the years as measured on Earth, while the counter counts the flashes that Pink receives from Green.

What is the frequency of the pulses that she receives from Green that were emitted on his outbound trip?

What is the frequency of the pulses that she receives from Green that were emitted on his inbound trip?

Are your answers consistent with the equation for the relativistic Doppler shift? Justify your answer with a short calculation.

How many signals does Pink receive from Green's outbound trip? From his inbound trip? Total?

Now, from Green's point of view, the situation is a bit different. In order to make this trip, he has to switch reference frames after turning around so in the animation, we show the view of Pink (and the Earth) from the two frames (White and Black) that Green jumps between. The white pulse counter counts pulses only when Green is in the white reference frame and a black counter will count pulses only when Green is in the black frame.

Verify that the total number of pulses Green receives is equal to Pink's change in age when Green returns to Earth. Notice that Green's clock (in the upper left hand corner) shows him to be younger.

What is the frequency of the pulses that Green receives while in the White reference frame? While in the Black reference frame?

Show that these equations, too, are consistent with the Doppler shift equation.

How many signals does Green receive on his outbound trip? On his inbound trip? Total?

Summarize: How many years elapse for Pink during the time of the trip? How many years elapse for Green during the time of his trip? How do you know?

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Worldline 4

Worldline 5 (along the line of v=c)

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Investigation 3: Simultaneity

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Is there any speed for which the events are simultaneous in <u>both</u> reference frames?

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√ Checkpoint 3

Investigation 5: Relativistic pole vaulting

Open Section 2.7. In this simulation, you will explore the famous pole and barn paradox. A pole vaulter carries a long pole towards a barn as shown in the animation (and the spacetime diagram). You can place the cursor on a point in the animation and click to show the x and y coordinates in the selected frame.

Start by watching the animation from the frame of the barn.

How long is the barn in this frame? How long is the pole in this frame? Which of these is a proper length?

How fast is the pole moving relative to the barn (v/c) (show how you got this)?

What do events A, B, A', and B' refer to? Which, if any, of these events are simultaneous in this reference frame?

Initially consider the back door of the barn to be closed. It can be opened instantaneously when the front of the pole reaches it. Similarly, the front door will be closed instantaneously when the back end of the pole reaches it. Given that, are there any times when the pole is completely inside the barn so that both doors will be closed at the same time (even if just for an instant)? Explain.

Next, look at the animation from the frame of the pole vaulter.
How fast is the barn moving relative to the pole (v/c) (show how you got this)? Does this agree with what you measured from the barn frame? Should it?

How long is the pole in this reference frame? Why is it this long?

How long is the barn in this reference frame? Why is it this long?

What do events A, B, A', and B' refer to? Which if any of these events are simultaneous in this reference frame?

From this frame, are there any times when the pole is completely inside the barn so that both doors will be closed at the same time?

Given that from the barn frame both doors can be closed at the same time (even if only for an instant), describe the sequence of events (in terms of where/when the front/back of pole is relative to the front/back door of barn and if/when front/back door are opened/closed).

√ **Checkpoint 4**

Investigation 6: Twin paradox

Open Section 2.8. In this simulation, you will explore the famous twin paradox. One twin (red) remains on Earth, while a second twin (green) begins a trip to a distant location at t = 0 years, and then returns, reaching Earth again at t = 10 years.

First run the simulation from the view of the Earth-bound (red) twin.

How fast is the green twin traveling relative to the red twin (v/c)?

The two twins have agreed to send each other a light pulse once a year on the anniversary of the green twin's departure. Select Pulses from Earth-Bound Twin and play.

What is the frequency of the red twin's light pulse in her own reference frame?

By how many years has the red twin aged during her twin's trip?

How many light pulses reach the green twin during his outbound trip?

During his inbound trip?

Select Pulses from Traveling Twin and play.
By how many years has the green twin aged during his trip?

The red twin argues that from her point of view, the green twin is moving and therefore the green twin should age more slowly. On the other hand, the green twin argues that from his point of view, the red twin is moving and therefore the red twin should age more slowly. But when they both get back together, the green twin has actually aged less. What's going on here?

Select Pulses from Traveling Twin: ST. Now we have added the traveling twin's clock and numbers on the spacetime diagram to mark the arrival of the traveling twin's light pulse.

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Calculate the spacetime interval for the <u>red</u> twin during the <u>outbound</u> trip.

Repeat for the <u>green</u> twin during the <u>outbound</u> trip.

Calculate the spacetime interval for the <u>red</u> twin during the <u>inbound</u> trip.

Repeat for the <u>green</u> twin during the <u>inbound</u> trip.

Why is there a difference in the spacetime intervals for the twins for the entire trip?

Is there really a paradox in the fact that it's the green twin who ages more slowly? Explain.

√ **Checkpoint 5**

Investigation 7: Relativistic Doppler shift and the twin paradox (optional)

In this simulation, you will explore the famous twin paradox from a slightly different perspective. To open the simulation, select ***Special Relativity*** from the frame on the left of the page. Open ***Chapter 3*** and then open Section 3.6 from the frame on the left of the page.

In this simulation, one twin (Pink) remains on earth, while a second twin (Green) leaves in a rocket ship to travel to a distant planet at t = 0 years. When Green reaches the planet, they turn around and return to Earth 20 years later (as measured by Pink who remained on Earth). Pink has aged 20 years, but Green has not aged as much. To measure this effect quantitatively, each twin sends out a flash of light on their birthday. In each twin's own reference frame, then, these flashes occur with a frequency of one. What does it look like from the other reference frame?

First look at it from Pink's view on Earth. The time clock in the upper left corner shows the years as measured on Earth, while the counter counts the flashes that Pink receives from Green.

What is the frequency of the pulses that she receives from Green that were emitted on his outbound trip?

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Are your answers consistent with the equation for the relativistic Doppler shift? Justify your answer with a short calculation.

How many signals does Pink receive from Green's outbound trip? From his inbound trip? Total?

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Worldline 2

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Worldline 5 (along the line of v=c)

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√ Checkpoint 3

Investigation 5: Relativistic pole vaulting

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How long is the barn in this frame? How long is the pole in this frame? Which of these is a proper length?

How fast is the pole moving relative to the barn (v/c) (show how you got this)?

What do events A, B, A', and B' refer to? Which, if any, of these events are simultaneous in this reference frame?

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How fast is the barn moving relative to the pole (v/c) (show how you got this)? Does this agree with what you measured from the barn frame? Should it?

How long is the pole in this reference frame? Why is it this long?

How long is the barn in this reference frame? Why is it this long?

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√ **Checkpoint 4**

Investigation 6: Twin paradox

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How many light pulses reach the green twin during his outbound trip?

During his inbound trip?

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By how many years has the green twin aged during his trip?

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In this simulation, you will explore the famous twin paradox from a slightly different perspective. To open the simulation, select ***Special Relativity*** from the frame on the left of the page. Open ***Chapter 3*** and then open Section 3.6 from the frame on the left of the page.

In this simulation, one twin (Pink) remains on earth, while a second twin (Green) leaves in a rocket ship to travel to a distant planet at t = 0 years. When Green reaches the planet, they turn around and return to Earth 20 years later (as measured by Pink who remained on Earth). Pink has aged 20 years, but Green has not aged as much. To measure this effect quantitatively, each twin sends out a flash of light on their birthday. In each twin's own reference frame, then, these flashes occur with a frequency of one. What does it look like from the other reference frame?

First look at it from Pink's view on Earth. The time clock in the upper left corner shows the years as measured on Earth, while the counter counts the flashes that Pink receives from Green.

What is the frequency of the pulses that she receives from Green that were emitted on his outbound trip?

What is the frequency of the pulses that she receives from Green that were emitted on his inbound trip?

Are your answers consistent with the equation for the relativistic Doppler shift? Justify your answer with a short calculation.

How many signals does Pink receive from Green's outbound trip? From his inbound trip? Total?

Now, from Green's point of view, the situation is a bit different. In order to make this trip, he has to switch reference frames after turning around so in the animation, we show the view of Pink (and the Earth) from the two frames (White and Black) that Green jumps between. The white pulse counter counts pulses only when Green is in the white reference frame and a black counter will count pulses only when Green is in the black frame.

Verify that the total number of pulses Green receives is equal to Pink's change in age when Green returns to Earth. Notice that Green's clock (in the upper left hand corner) shows him to be younger.

What is the frequency of the pulses that Green receives while in the White reference frame? While in the Black reference frame?

Show that these equations, too, are consistent with the Doppler shift equation.

How many signals does Green receive on his outbound trip? On his inbound trip? Total?

Summarize: How many years elapse for Pink during the time of the trip? How many years elapse for Green during the time of his trip? How do you know?

√ **Checkpoint 6**

Physics 216 Laboratory 13, Spring 2018
Special Relativity

Background

Since our budget doesn't allow for a fleet of relativistic spaceships, we'll use a series of computer simulations to investigate some situations in which the concepts of special relativity are important. All the simulations that you will be examining today, along with a number of other simulations that you are encouraged to play with, are found at http://www.compadre.org/PQP/special-relativity/.

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Investigation 1: Time dilation and length contraction

Open Section 2.4 from the menu on the left. You should see two animations, with a description under each animation. Read the description under the top animation, and then set $|\beta|=|u/c|$ to 0.5 and press the *play* button.

In 5 ticks of the clock in the stationary frame (red clock), how many ticks (estimate fractional ticks) does the green clock make?

Show that these measurements agree with the time dilation formula.

What would you see from the green reference frame? Why?

Now scroll down the page to the other animation (rotated light clocks). Once you've read the text under the animation, set $|\beta|=|u/c|$ to 0.5 and press the *play* button.

Pause the animation at a point where the green clock is still visible. Using the small crosshairs in the lower left region of the animation, measure the length of the red and green clock in the red reference frame by recording the position of each end of the clock. Be sure to include units.

Length of green clock in red reference frame:

Length of red clock in red reference frame:

Show that these measurements agree with the length contraction formula.

What would you see from the green reference frame? Why?

√ Checkpoint 1

Investigation 2: Worldline matching

Open Section 2.6 from the frame on the left of the page. In this simulation, you will explore worldlines, which are used to depict the trajectory of a particle in spacetime.

Your goal is to match a series of worldlines that can be loaded by clicking one of the links below the control buttons. The position of the red ball can be controlled by using the mouse to move the red ball in the box below the spacetime diagram.

For each of the worldlines, sketch the spacetime diagram and describe in words how you had to move the red ball to match the worldlines.

Worldline 1

Worldline 2

Worldline 3

Worldline 4

Worldline 5 (along the line of v=c)

Did you find any bad physics in the animations? If so, what was it?

√ Checkpoint 2

Investigation 3: Simultaneity

Open Section 2.3 from the frame on the left of the page. In this simulation, you will explore the nature of simultaneity. There are two different "experiments." In each, an event occurs and this event is then observed by four observers at different locations in two different coordinate systems.

Run Experiment 1: From the frame of the ground.
Describe the order in which the light flash reaches each of the four observers, according to an observer in the reference frame of the ground.

Are there any simultaneous events in this reference frame? If so, which events?

Run Experiment 2: From the frame of the railcar.
Describe the order in which the light flash reaches each of the four observers, according to an observer in the reference frame of the railcar.

Are there any simultaneous events in this reference frame? Are they the same events as in the other reference frame?

Are the results of these two experiments consistent? Explain.

Investigation 4: Simultaneity using relativistic measuring sticks

Open Problem 2.7 from the frame on the left of the page. In this simulation, you will explore the nature of simultaneity from a slightly different perspective. In this simulation, two measuring sticks pass each other with a given relative speed. A light flash occurs when the right ends of the sticks coincide (event 1) and again when the left ends coincide (event 2).

Observe the events at several different relative speeds and from both reference frames. Record what happens and when, according to each reference frame, in the table below. Try to find a relative speed such that the events are simultaneous in one or both reference frames.

v/c	Event 1 (right ends) in Green frame	Event 2 (left ends) in Green frame	Event 1(right ends) in Red frame	Event 2(left ends) in Red frame

Based on your observations, answer the following.

Does one of the sticks have a greater proper length, and if so, which one? How can you tell?

Does event 1 always occur before event 2? Does the answer depend on the reference frame?

Is there any speed for which the events are simultaneous in <u>both</u> reference frames?

Is there any speed for which the events are simultaneous in <u>one</u> reference frame? What is special at that speed?

At a given speed, will the red and green observers always agree on which event occurred first? Explain.

√ Checkpoint 3

Investigation 5: Relativistic pole vaulting

Open Section 2.7. In this simulation, you will explore the famous pole and barn paradox. A pole vaulter carries a long pole towards a barn as shown in the animation (and the spacetime diagram). You can place the cursor on a point in the animation and click to show the x and y coordinates in the selected frame.

Start by watching the animation from the frame of the barn.

How long is the barn in this frame? How long is the pole in this frame? Which of these is a proper length?

How fast is the pole moving relative to the barn (v/c) (show how you got this)?

What do events A, B, A', and B' refer to? Which, if any, of these events are simultaneous in this reference frame?

Initially consider the back door of the barn to be closed. It can be opened instantaneously when the front of the pole reaches it. Similarly, the front door will be closed instantaneously when the back end of the pole reaches it. Given that, are there any times when the pole is completely inside the barn so that both doors will be closed at the same time (even if just for an instant)? Explain.

Next, look at the animation from the frame of the pole vaulter.
How fast is the barn moving relative to the pole (v/c) (show how you got this)? Does this agree with what you measured from the barn frame? Should it?

How long is the pole in this reference frame? Why is it this long?

How long is the barn in this reference frame? Why is it this long?

What do events A, B, A', and B' refer to? Which if any of these events are simultaneous in this reference frame?

From this frame, are there any times when the pole is completely inside the barn so that both doors will be closed at the same time?

Given that from the barn frame both doors can be closed at the same time (even if only for an instant), describe the sequence of events (in terms of where/when the front/back of pole is relative to the front/back door of barn and if/when front/back door are opened/closed).

√ **Checkpoint 4**

Investigation 6: Twin paradox

Open Section 2.8. In this simulation, you will explore the famous twin paradox. One twin (red) remains on Earth, while a second twin (green) begins a trip to a distant location at t = 0 years, and then returns, reaching Earth again at t = 10 years.

First run the simulation from the view of the Earth-bound (red) twin.

How fast is the green twin traveling relative to the red twin (v/c)?

The two twins have agreed to send each other a light pulse once a year on the anniversary of the green twin's departure. Select Pulses from Earth-Bound Twin and play.

What is the frequency of the red twin's light pulse in her own reference frame?

By how many years has the red twin aged during her twin's trip?

How many light pulses reach the green twin during his outbound trip?

During his inbound trip?

Select Pulses from Traveling Twin and play.
By how many years has the green twin aged during his trip?

The red twin argues that from her point of view, the green twin is moving and therefore the green twin should age more slowly. On the other hand, the green twin argues that from his point of view, the red twin is moving and therefore the red twin should age more slowly. But when they both get back together, the green twin has actually aged less. What's going on here?

Select Pulses from Traveling Twin: ST. Now we have added the traveling twin's clock and numbers on the spacetime diagram to mark the arrival of the traveling twin's light pulse.

One of the most important concepts in special relativity is the idea of the spacetime interval, $(\Delta s)^2 = c^2(\Delta t)^2 - (\Delta x)^2$, which is the same in any two inertial frames. This can be thought of as the Pythagorean Theorem of spacetime.

Calculate the spacetime interval for the <u>red</u> twin during the <u>outbound</u> trip.

Repeat for the <u>green</u> twin during the <u>outbound</u> trip.

Calculate the spacetime interval for the <u>red</u> twin during the <u>inbound</u> trip.

Repeat for the <u>green</u> twin during the <u>inbound</u> trip.

Why is there a difference in the spacetime intervals for the twins for the entire trip?

Is there really a paradox in the fact that it's the green twin who ages more slowly? Explain.

√ **Checkpoint 5**

Investigation 7: Relativistic Doppler shift and the twin paradox (optional)

In this simulation, you will explore the famous twin paradox from a slightly different perspective. To open the simulation, select ***Special Relativity*** from the frame on the left of the page. Open ***Chapter 3*** and then open Section 3.6 from the frame on the left of the page.

In this simulation, one twin (Pink) remains on earth, while a second twin (Green) leaves in a rocket ship to travel to a distant planet at t = 0 years. When Green reaches the planet, they turn around and return to Earth 20 years later (as measured by Pink who remained on Earth). Pink has aged 20 years, but Green has not aged as much. To measure this effect quantitatively, each twin sends out a flash of light on their birthday. In each twin's own reference frame, then, these flashes occur with a frequency of one. What does it look like from the other reference frame?

First look at it from Pink's view on Earth. The time clock in the upper left corner shows the years as measured on Earth, while the counter counts the flashes that Pink receives from Green.

What is the frequency of the pulses that she receives from Green that were emitted on his outbound trip?

What is the frequency of the pulses that she receives from Green that were emitted on his inbound trip?

Are your answers consistent with the equation for the relativistic Doppler shift? Justify your answer with a short calculation.

How many signals does Pink receive from Green's outbound trip? From his inbound trip? Total?

Now, from Green's point of view, the situation is a bit different. In order to make this trip, he has to switch reference frames after turning around so in the animation, we show the view of Pink (and the Earth) from the two frames (White and Black) that Green jumps between. The white pulse counter counts pulses only when Green is in the white reference frame and a black counter will count pulses only when Green is in the black frame.

Verify that the total number of pulses Green receives is equal to Pink's change in age when Green returns to Earth. Notice that Green's clock (in the upper left hand corner) shows him to be younger.

What is the frequency of the pulses that Green receives while in the White reference frame? While in the Black reference frame?

Show that these equations, too, are consistent with the Doppler shift equation.

How many signals does Green receive on his outbound trip? On his inbound trip? Total?

Summarize: How many years elapse for Pink during the time of the trip? How many years elapse for Green during the time of his trip? How do you know?

√ **Checkpoint 6**

Physics 216 Laboratory 13, Spring 2018
Special Relativity

Background
Since our budget doesn't allow for a fleet of relativistic spaceships, we'll use a series of computer simulations to investigate some situations in which the concepts of special relativity are important. All the simulations that you will be examining today, along with a number of other simulations that you are encouraged to play with, are found at http://www.compadre.org/PQP/special-relativity/.

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Investigation 1: Time dilation and length contraction
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In 5 ticks of the clock in the stationary frame (red clock), how many ticks (estimate fractional ticks) does the green clock make?

Show that these measurements agree with the time dilation formula.

What would you see from the green reference frame? Why?

Now scroll down the page to the other animation (rotated light clocks). Once you've read the text under the animation, set $|\beta|=|u/c|$ to 0.5 and press the *play* button.

Pause the animation at a point where the green clock is still visible. Using the small crosshairs in the lower left region of the animation, measure the length of the red and green clock in the red reference frame by recording the position of each end of the clock. Be sure to include units.

Length of green clock in red reference frame:

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What would you see from the green reference frame? Why?

√ Checkpoint 1

Investigation 2: Worldline matching

Open Section 2.6 from the frame on the left of the page. In this simulation, you will explore worldlines, which are used to depict the trajectory of a particle in spacetime.

Your goal is to match a series of worldlines that can be loaded by clicking one of the links below the control buttons. The position of the red ball can be controlled by using the mouse to move the red ball in the box below the spacetime diagram.

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Worldline 1

Worldline 2

Worldline 3

Worldline 4

Worldline 5 (along the line of v=c)

Did you find any bad physics in the animations? If so, what was it?

√ Checkpoint 2

Investigation 3: Simultaneity

Open Section 2.3 from the frame on the left of the page. In this simulation, you will explore the nature of simultaneity. There are two different "experiments." In each, an event occurs and this event is then observed by four observers at different locations in two different coordinate systems.

Run Experiment 1: From the frame of the ground.
Describe the order in which the light flash reaches each of the four observers, according to an observer in the reference frame of the ground.

Are there any simultaneous events in this reference frame? If so, which events?

Run Experiment 2: From the frame of the railcar.
Describe the order in which the light flash reaches each of the four observers, according to an observer in the reference frame of the railcar.

Are there any simultaneous events in this reference frame? Are they the same events as in the other reference frame?

Are the results of these two experiments consistent? Explain.

Investigation 4: Simultaneity using relativistic measuring sticks

Open Problem 2.7 from the frame on the left of the page. In this simulation, you will explore the nature of simultaneity from a slightly different perspective. In this simulation, two measuring sticks pass each other with a given relative speed. A light flash occurs when the right ends of the sticks coincide (event 1) and again when the left ends coincide (event 2).

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Based on your observations, answer the following.

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Does event 1 always occur before event 2? Does the answer depend on the reference frame?

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At a given speed, will the red and green observers always agree on which event occurred first? Explain.

√ **Checkpoint 3**

Investigation 5: Relativistic pole vaulting

Open Section 2.7. In this simulation, you will explore the famous pole and barn paradox. A pole vaulter carries a long pole towards a barn as shown in the animation (and the spacetime diagram). You can place the cursor on a point in the animation and click to show the x and y coordinates in the selected frame.

Start by watching the animation from the frame of the barn.

How long is the barn in this frame? How long is the pole in this frame? Which of these is a proper length?

How fast is the pole moving relative to the barn (v/c) (show how you got this)?

What do events A, B, A', and B' refer to? Which, if any, of these events are simultaneous in this reference frame?

Initially consider the back door of the barn to be closed. It can be opened instantaneously when the front of the pole reaches it. Similarly, the front door will be closed instantaneously when the back end of the pole reaches it. Given that, are there any times when the pole is completely inside the barn so that both doors will be closed at the same time (even if just for an instant)? Explain.

Next, look at the animation from the frame of the pole vaulter.
How fast is the barn moving relative to the pole (v/c) (show how you got this)? Does this agree with what you measured from the barn frame? Should it?

How long is the pole in this reference frame? Why is it this long?

How long is the barn in this reference frame? Why is it this long?

What do events A, B, A', and B' refer to? Which if any of these events are simultaneous in this reference frame?

From this frame, are there any times when the pole is completely inside the barn so that both doors will be closed at the same time?

Given that from the barn frame both doors can be closed at the same time (even if only for an instant), describe the sequence of events (in terms of where/when the front/back of pole is relative to the front/back door of barn and if/when front/back door are opened/closed).

√ **Checkpoint 4**

Investigation 6: Twin paradox

Open Section 2.8. In this simulation, you will explore the famous twin paradox. One twin (red) remains on Earth, while a second twin (green) begins a trip to a distant location at t = 0 years, and then returns, reaching Earth again at t = 10 years.

First run the simulation from the view of the Earth-bound (red) twin.

How fast is the green twin traveling relative to the red twin (v/c)?

The two twins have agreed to send each other a light pulse once a year on the anniversary of the green twin's departure. Select Pulses from Earth-Bound Twin and play.

What is the frequency of the red twin's light pulse in her own reference frame?

By how many years has the red twin aged during her twin's trip?

How many light pulses reach the green twin during his outbound trip?

During his inbound trip?

Select Pulses from Traveling Twin and play.
By how many years has the green twin aged during his trip?

The red twin argues that from her point of view, the green twin is moving and therefore the green twin should age more slowly. On the other hand, the green twin argues that from his point of view, the red twin is moving and therefore the red twin should age more slowly. But when they both get back together, the green twin has actually aged less. What's going on here?

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Calculate the spacetime interval for the <u>red</u> twin during the <u>inbound</u> trip.

Repeat for the <u>green</u> twin during the <u>inbound</u> trip.

Why is there a difference in the spacetime intervals for the twins for the entire trip?

Is there really a paradox in the fact that it's the green twin who ages more slowly? Explain.

√ **Checkpoint 5**

Investigation 7: Relativistic Doppler shift and the twin paradox (optional)

In this simulation, you will explore the famous twin paradox from a slightly different perspective. To open the simulation, select ***Special Relativity*** from the frame on the left of the page. Open ***Chapter 3*** and then open Section 3.6 from the frame on the left of the page.

In this simulation, one twin (Pink) remains on earth, while a second twin (Green) leaves in a rocket ship to travel to a distant planet at t = 0 years. When Green reaches the planet, they turn around and return to Earth 20 years later (as measured by Pink who remained on Earth). Pink has aged 20 years, but Green has not aged as much. To measure this effect quantitatively, each twin sends out a flash of light on their birthday. In each twin's own reference frame, then, these flashes occur with a frequency of one. What does it look like from the other reference frame?

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Summarize: How many years elapse for Pink during the time of the trip? How many years elapse for Green during the time of his trip? How do you know?

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Physics 216 Laboratory 13, Spring 2018
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√ Checkpoint 1

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Worldline 1

Worldline 2

Worldline 3

Worldline 4

Worldline 5 (along the line of v=c)

Did you find any bad physics in the animations? If so, what was it?

√ Checkpoint 2

Investigation 3: Simultaneity

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Are there any simultaneous events in this reference frame? If so, which events?

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Describe the order in which the light flash reaches each of the four observers, according to an observer in the reference frame of the railcar.

Are there any simultaneous events in this reference frame? Are they the same events as in the other reference frame?

Are the results of these two experiments consistent? Explain.

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Based on your observations, answer the following.

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Does event 1 always occur before event 2? Does the answer depend on the reference frame?

Is there any speed for which the events are simultaneous in <u>both</u> reference frames?

Is there any speed for which the events are simultaneous in <u>one</u> reference frame? What is special at that speed?

At a given speed, will the red and green observers always agree on which event occurred first? Explain.

√ **Checkpoint 3**

Investigation 5: Relativistic pole vaulting

Open Section 2.7. In this simulation, you will explore the famous pole and barn paradox. A pole vaulter carries a long pole towards a barn as shown in the animation (and the spacetime diagram). You can place the cursor on a point in the animation and click to show the x and y coordinates in the selected frame.

Start by watching the animation from the frame of the barn.

How long is the barn in this frame? How long is the pole in this frame? Which of these is a proper length?

How fast is the pole moving relative to the barn (v/c) (show how you got this)?

What do events A, B, A', and B' refer to? Which, if any, of these events are simultaneous in this reference frame?

Initially consider the back door of the barn to be closed. It can be opened instantaneously when the front of the pole reaches it. Similarly, the front door will be closed instantaneously when the back end of the pole reaches it. Given that, are there any times when the pole is completely inside the barn so that both doors will be closed at the same time (even if just for an instant)? Explain.

Next, look at the animation from the frame of the pole vaulter.
How fast is the barn moving relative to the pole (v/c) (show how you got this)? Does this agree with what you measured from the barn frame? Should it?

How long is the pole in this reference frame? Why is it this long?

How long is the barn in this reference frame? Why is it this long?

What do events A, B, A', and B' refer to? Which if any of these events are simultaneous in this reference frame?

From this frame, are there any times when the pole is completely inside the barn so that both doors will be closed at the same time?

Given that from the barn frame both doors can be closed at the same time (even if only for an instant), describe the sequence of events (in terms of where/when the front/back of pole is relative to the front/back door of barn and if/when front/back door are opened/closed).

√ **Checkpoint 4**

Investigation 6: Twin paradox

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First run the simulation from the view of the Earth-bound (red) twin.

How fast is the green twin traveling relative to the red twin (v/c)?

The two twins have agreed to send each other a light pulse once a year on the anniversary of the green twin's departure. Select Pulses from Earth-Bound Twin and play.

What is the frequency of the red twin's light pulse in her own reference frame?

By how many years has the red twin aged during her twin's trip?

How many light pulses reach the green twin during his outbound trip?

During his inbound trip?

Select Pulses from Traveling Twin and play.
By how many years has the green twin aged during his trip?

The red twin argues that from her point of view, the green twin is moving and therefore the green twin should age more slowly. On the other hand, the green twin argues that from his point of view, the red twin is moving and therefore the red twin should age more slowly. But when they both get back together, the green twin has actually aged less. What's going on here?

Select Pulses from Traveling Twin: ST. Now we have added the traveling twin's clock and numbers on the spacetime diagram to mark the arrival of the traveling twin's light pulse.

One of the most important concepts in special relativity is the idea of the spacetime interval, $(\Delta s)^2 = c^2(\Delta t)^2 - (\Delta x)^2$, which is the same in any two inertial frames. This can be thought of as the Pythagorean Theorem of spacetime.

Calculate the spacetime interval for the red twin during the outbound trip.

Repeat for the green twin during the outbound trip.

Calculate the spacetime interval for the red twin during the inbound trip.

Repeat for the green twin during the inbound trip.

Why is there a difference in the spacetime intervals for the twins for the entire trip?

Is there really a paradox in the fact that it's the green twin who ages more slowly? Explain.

√ **Checkpoint 5**

Investigation 7: Relativistic Doppler shift and the twin paradox (optional)

In this simulation, you will explore the famous twin paradox from a slightly different perspective. To open the simulation, select ***Special Relativity*** from the frame on the left of the page. Open ***Chapter 3*** and then open Section 3.6 from the frame on the left of the page.

In this simulation, one twin (Pink) remains on earth, while a second twin (Green) leaves in a rocket ship to travel to a distant planet at t = 0 years. When Green reaches the planet, they turn around and return to Earth 20 years later (as measured by Pink who remained on Earth). Pink has aged 20 years, but Green has not aged as much. To measure this effect quantitatively, each twin sends out a flash of light on their birthday. In each twin's own reference frame, then, these flashes occur with a frequency of one. What does it look like from the other reference frame?

First look at it from Pink's view on Earth. The time clock in the upper left corner shows the years as measured on Earth, while the counter counts the flashes that Pink receives from Green.

What is the frequency of the pulses that she receives from Green that were emitted on his outbound trip?

What is the frequency of the pulses that she receives from Green that were emitted on his inbound trip?

Are your answers consistent with the equation for the relativistic Doppler shift? Justify your answer with a short calculation.

How many signals does Pink receive from Green's outbound trip? From his inbound trip? Total?

Now, from Green's point of view, the situation is a bit different. In order to make this trip, he has to switch reference frames after turning around so in the animation, we show the view of Pink (and the Earth) from the two frames (White and Black) that Green jumps between. The white pulse counter counts pulses only when Green is in the white reference frame and a black counter will count pulses only when Green is in the black frame.

Verify that the total number of pulses Green receives is equal to Pink's change in age when Green returns to Earth. Notice that Green's clock (in the upper left hand corner) shows him to be younger.

What is the frequency of the pulses that Green receives while in the White reference frame? While in the Black reference frame?

Show that these equations, too, are consistent with the Doppler shift equation.

How many signals does Green receive on his outbound trip? On his inbound trip? Total?

Summarize: How many years elapse for Pink during the time of the trip? How many years elapse for Green during the time of his trip? How do you know?

√ **Checkpoint 6**

Physics 216 Laboratory 13, Spring 2018
Special Relativity

Background

Since our budget doesn't allow for a fleet of relativistic spaceships, we'll use a series of computer simulations to investigate some situations in which the concepts of special relativity are important. All the simulations that you will be examining today, along with a number of other simulations that you are encouraged to play with, are found at http://www.compadre.org/PQP/special-relativity/.

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Investigation 1: Time dilation and length contraction

Open Section 2.4 from the menu on the left. You should see two animations, with a description under each animation. Read the description under the top animation, and then set $|\beta|=|u/c|$ to 0.5 and press the *play* button.

In 5 ticks of the clock in the stationary frame (red clock), how many ticks (estimate fractional ticks) does the green clock make?

Show that these measurements agree with the time dilation formula.

What would you see from the green reference frame? Why?

Now scroll down the page to the other animation (rotated light clocks). Once you've read the text under the animation, set $|\beta|=|u/c|$ to 0.5 and press the *play* button.

Pause the animation at a point where the green clock is still visible. Using the small crosshairs in the lower left region of the animation, measure the length of the red and green clock in the red reference frame by recording the position of each end of the clock. Be sure to include units.

Length of green clock in red reference frame:

Length of red clock in red reference frame:

Show that these measurements agree with the length contraction formula.

What would you see from the green reference frame? Why?

√ Checkpoint 1

Investigation 2: Worldline matching

Open Section 2.6 from the frame on the left of the page. In this simulation, you will explore worldlines, which are used to depict the trajectory of a particle in spacetime.

Your goal is to match a series of worldlines that can be loaded by clicking one of the links below the control buttons. The position of the red ball can be controlled by using the mouse to move the red ball in the box below the spacetime diagram.

For each of the worldlines, sketch the spacetime diagram and describe in words how you had to move the red ball to match the worldlines.

Worldline 1

Worldline 2

Worldline 3

Worldline 4

Worldline 5 (along the line of v=c)

Did you find any bad physics in the animations? If so, what was it?

√ Checkpoint 2

Investigation 3: Simultaneity

Open Section 2.3 from the frame on the left of the page. In this simulation, you will explore the nature of simultaneity. There are two different "experiments." In each, an event occurs and this event is then observed by four observers at different locations in two different coordinate systems.

Run Experiment 1: From the frame of the ground.
Describe the order in which the light flash reaches each of the four observers, according to an observer in the reference frame of the ground.

Are there any simultaneous events in this reference frame? If so, which events?

Run Experiment 2: From the frame of the railcar.
Describe the order in which the light flash reaches each of the four observers, according to an observer in the reference frame of the railcar.

Are there any simultaneous events in this reference frame? Are they the same events as in the other reference frame?

Are the results of these two experiments consistent? Explain.

Investigation 4: Simultaneity using relativistic measuring sticks

Open Problem 2.7 from the frame on the left of the page. In this simulation, you will explore the nature of simultaneity from a slightly different perspective. In this simulation, two measuring sticks pass each other with a given relative speed. A light flash occurs when the right ends of the sticks coincide (event 1) and again when the left ends coincide (event 2).

Observe the events at several different relative speeds and from both reference frames. Record what happens and when, according to each reference frame, in the table below. Try to find a relative speed such that the events are simultaneous in one or both reference frames.

v/c	Event 1 (right ends) in Green frame	Event 2 (left ends) in Green frame	Event 1(right ends) in Red frame	Event 2(left ends) in Red frame

Based on your observations, answer the following.

Does one of the sticks have a greater proper length, and if so, which one? How can you tell?

Does event 1 always occur before event 2? Does the answer depend on the reference frame?

Is there any speed for which the events are simultaneous in <u>both</u> reference frames?

Is there any speed for which the events are simultaneous in <u>one</u> reference frame? What is special at that speed?

At a given speed, will the red and green observers always agree on which event occurred first? Explain.

√ **Checkpoint 3**

Investigation 5: Relativistic pole vaulting

Open Section 2.7. In this simulation, you will explore the famous pole and barn paradox. A pole vaulter carries a long pole towards a barn as shown in the animation (and the spacetime diagram). You can place the cursor on a point in the animation and click to show the x and y coordinates in the selected frame.

Start by watching the animation from the frame of the barn.

How long is the barn in this frame? How long is the pole in this frame? Which of these is a proper length?

How fast is the pole moving relative to the barn (v/c) (show how you got this)?

What do events A, B, A', and B' refer to? Which, if any, of these events are simultaneous in this reference frame?

Initially consider the back door of the barn to be closed. It can be opened instantaneously when the front of the pole reaches it. Similarly, the front door will be closed instantaneously when the back end of the pole reaches it. Given that, are there any times when the pole is completely inside the barn so that both doors will be closed at the same time (even if just for an instant)? Explain.

Next, look at the animation from the frame of the pole vaulter.
How fast is the barn moving relative to the pole (v/c) (show how you got this)? Does this agree with what you measured from the barn frame? Should it?

How long is the pole in this reference frame? Why is it this long?

How long is the barn in this reference frame? Why is it this long?

What do events A, B, A', and B' refer to? Which if any of these events are simultaneous in this reference frame?

From this frame, are there any times when the pole is completely inside the barn so that both doors will be closed at the same time?

Given that from the barn frame both doors can be closed at the same time (even if only for an instant), describe the sequence of events (in terms of where/when the front/back of pole is relative to the front/back door of barn and if/when front/back door are opened/closed).

√ **Checkpoint 4**

Investigation 6: Twin paradox

Open Section 2.8. In this simulation, you will explore the famous twin paradox. One twin (red) remains on Earth, while a second twin (green) begins a trip to a distant location at t = 0 years, and then returns, reaching Earth again at t = 10 years.

First run the simulation from the view of the Earth-bound (red) twin.

How fast is the green twin traveling relative to the red twin (v/c)?

The two twins have agreed to send each other a light pulse once a year on the anniversary of the green twin's departure. Select Pulses from Earth-Bound Twin and play.

What is the frequency of the red twin's light pulse in her own reference frame?

By how many years has the red twin aged during her twin's trip?

How many light pulses reach the green twin during his outbound trip?

During his inbound trip?

Select Pulses from Traveling Twin and play.
By how many years has the green twin aged during his trip?

The red twin argues that from her point of view, the green twin is moving and therefore the green twin should age more slowly. On the other hand, the green twin argues that from his point of view, the red twin is moving and therefore the red twin should age more slowly. But when they both get back together, the green twin has actually aged less. What's going on here?

Select Pulses from Traveling Twin: ST. Now we have added the traveling twin's clock and numbers on the spacetime diagram to mark the arrival of the traveling twin's light pulse.

One of the most important concepts in special relativity is the idea of the spacetime interval, $(\Delta s)^2 = c^2(\Delta t)^2 - (\Delta x)^2$, which is the same in any two inertial frames. This can be thought of as the Pythagorean Theorem of spacetime.

Calculate the spacetime interval for the <u>red</u> twin during the <u>outbound</u> trip.

Repeat for the <u>green</u> twin during the <u>outbound</u> trip.

Calculate the spacetime interval for the <u>red</u> twin during the <u>inbound</u> trip.

Repeat for the <u>green</u> twin during the <u>inbound</u> trip.

Why is there a difference in the spacetime intervals for the twins for the entire trip?

Is there really a paradox in the fact that it's the green twin who ages more slowly? Explain.

√ **Checkpoint 5**

Investigation 7: Relativistic Doppler shift and the twin paradox (optional)

In this simulation, you will explore the famous twin paradox from a slightly different perspective. To open the simulation, select ***Special Relativity*** from the frame on the left of the page. Open ***Chapter 3*** and then open Section 3.6 from the frame on the left of the page.

In this simulation, one twin (Pink) remains on earth, while a second twin (Green) leaves in a rocket ship to travel to a distant planet at t = 0 years. When Green reaches the planet, they turn around and return to Earth 20 years later (as measured by Pink who remained on Earth). Pink has aged 20 years, but Green has not aged as much. To measure this effect quantitatively, each twin sends out a flash of light on their birthday. In each twin's own reference frame, then, these flashes occur with a frequency of one. What does it look like from the other reference frame?

First look at it from Pink's view on Earth. The time clock in the upper left corner shows the years as measured on Earth, while the counter counts the flashes that Pink receives from Green.

What is the frequency of the pulses that she receives from Green that were emitted on his outbound trip?

What is the frequency of the pulses that she receives from Green that were emitted on his inbound trip?

Are your answers consistent with the equation for the relativistic Doppler shift? Justify your answer with a short calculation.

How many signals does Pink receive from Green's outbound trip? From his inbound trip? Total?

Now, from Green's point of view, the situation is a bit different. In order to make this trip, he has to switch reference frames after turning around so in the animation, we show the view of Pink (and the Earth) from the two frames (White and Black) that Green jumps between. The white pulse counter counts pulses only when Green is in the white reference frame and a black counter will count pulses only when Green is in the black frame.

Verify that the total number of pulses Green receives is equal to Pink's change in age when Green returns to Earth. Notice that Green's clock (in the upper left hand corner) shows him to be younger.

What is the frequency of the pulses that Green receives while in the White reference frame? While in the Black reference frame?

Show that these equations, too, are consistent with the Doppler shift equation.

How many signals does Green receive on his outbound trip? On his inbound trip? Total?

Summarize: How many years elapse for Pink during the time of the trip? How many years elapse for Green during the time of his trip? How do you know?

√ **Checkpoint 6**

Physics 216 Laboratory 13, Spring 2018
Special Relativity

Background

Since our budget doesn't allow for a fleet of relativistic spaceships, we'll use a series of computer simulations to investigate some situations in which the concepts of special relativity are important. All the simulations that you will be examining today, along with a number of other simulations that you are encouraged to play with, are found at http://www.compadre.org/PQP/special-relativity/.

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In 5 ticks of the clock in the stationary frame (red clock), how many ticks (estimate fractional ticks) does the green clock make?

Show that these measurements agree with the time dilation formula.

What would you see from the green reference frame? Why?

Now scroll down the page to the other animation (rotated light clocks). Once you've read the text under the animation, set $|\beta|=|u/c|$ to 0.5 and press the *play* button.

Pause the animation at a point where the green clock is still visible. Using the small crosshairs in the lower left region of the animation, measure the length of the red and green clock in the red reference frame by recording the position of each end of the clock. Be sure to include units.

Length of green clock in red reference frame:

Length of red clock in red reference frame:

Show that these measurements agree with the length contraction formula.

What would you see from the green reference frame? Why?

√ Checkpoint 1

Investigation 2: Worldline matching

Open Section 2.6 from the frame on the left of the page. In this simulation, you will explore worldlines, which are used to depict the trajectory of a particle in spacetime.

Your goal is to match a series of worldlines that can be loaded by clicking one of the links below the control buttons. The position of the red ball can be controlled by using the mouse to move the red ball in the box below the spacetime diagram.

For each of the worldlines, sketch the spacetime diagram and describe in words how you had to move the red ball to match the worldlines.

Worldline 1

Worldline 2

Worldline 3

Worldline 4

Worldline 5 (along the line of v=c)

Did you find any bad physics in the animations? If so, what was it?

√ Checkpoint 2

Investigation 3: Simultaneity

Open Section 2.3 from the frame on the left of the page. In this simulation, you will explore the nature of simultaneity. There are two different "experiments." In each, an event occurs and this event is then observed by four observers at different locations in two different coordinate systems.

Run Experiment 1: From the frame of the ground.
Describe the order in which the light flash reaches each of the four observers, according to an observer in the reference frame of the ground.

Are there any simultaneous events in this reference frame? If so, which events?

Run Experiment 2: From the frame of the railcar.
Describe the order in which the light flash reaches each of the four observers, according to an observer in the reference frame of the railcar.

Are there any simultaneous events in this reference frame? Are they the same events as in the other reference frame?

Are the results of these two experiments consistent? Explain.

Investigation 4: Simultaneity using relativistic measuring sticks

Open Problem 2.7 from the frame on the left of the page. In this simulation, you will explore the nature of simultaneity from a slightly different perspective. In this simulation, two measuring sticks pass each other with a given relative speed. A light flash occurs when the right ends of the sticks coincide (event 1) and again when the left ends coincide (event 2).

Observe the events at several different relative speeds and from both reference frames. Record what happens and when, according to each reference frame, in the table below. Try to find a relative speed such that the events are simultaneous in one or both reference frames.

v/c	Event 1 (right ends) in Green frame	Event 2 (left ends) in Green frame	Event 1(right ends) in Red frame	Event 2(left ends) in Red frame

Based on your observations, answer the following.

Does one of the sticks have a greater proper length, and if so, which one? How can you tell?

Does event 1 always occur before event 2? Does the answer depend on the reference frame?

Is there any speed for which the events are simultaneous in <u>both</u> reference frames?

Is there any speed for which the events are simultaneous in <u>one</u> reference frame? What is special at that speed?

At a given speed, will the red and green observers always agree on which event occurred first? Explain.

√ Checkpoint 3

Investigation 5: Relativistic pole vaulting

Open Section 2.7. In this simulation, you will explore the famous pole and barn paradox. A pole vaulter carries a long pole towards a barn as shown in the animation (and the spacetime diagram). You can place the cursor on a point in the animation and click to show the x and y coordinates in the selected frame.

Start by watching the animation from the frame of the barn.

How long is the barn in this frame? How long is the pole in this frame? Which of these is a proper length?

How fast is the pole moving relative to the barn (v/c) (show how you got this)?

What do events A, B, A', and B' refer to? Which, if any, of these events are simultaneous in this reference frame?

Initially consider the back door of the barn to be closed. It can be opened instantaneously when the front of the pole reaches it. Similarly, the front door will be closed instantaneously when the back end of the pole reaches it. Given that, are there any times when the pole is completely inside the barn so that both doors will be closed at the same time (even if just for an instant)? Explain.

Next, look at the animation from the frame of the pole vaulter.
How fast is the barn moving relative to the pole (v/c) (show how you got this)? Does this agree with what you measured from the barn frame? Should it?

How long is the pole in this reference frame? Why is it this long?

How long is the barn in this reference frame? Why is it this long?

What do events A, B, A', and B' refer to? Which if any of these events are simultaneous in this reference frame?

From this frame, are there any times when the pole is completely inside the barn so that both doors will be closed at the same time?

Given that from the barn frame both doors can be closed at the same time (even if only for an instant), describe the sequence of events (in terms of where/when the front/back of pole is relative to the front/back door of barn and if/when front/back door are opened/closed).

√ **Checkpoint 4**

Investigation 6: Twin paradox

Open Section 2.8. In this simulation, you will explore the famous twin paradox. One twin (red) remains on Earth, while a second twin (green) begins a trip to a distant location at t = 0 years, and then returns, reaching Earth again at t = 10 years.

First run the simulation from the view of the Earth-bound (red) twin.

How fast is the green twin traveling relative to the red twin (v/c)?

The two twins have agreed to send each other a light pulse once a year on the anniversary of the green twin's departure. Select Pulses from Earth-Bound Twin and play.

What is the frequency of the red twin's light pulse in her own reference frame?

By how many years has the red twin aged during her twin’s trip?

How many light pulses reach the green twin during his outbound trip?

During his inbound trip?

Select Pulses from Traveling Twin and play.
By how many years has the green twin aged during his trip?

The red twin argues that from her point of view, the green twin is moving and therefore the green twin should age more slowly. On the other hand, the green twin argues that from his point of view, the red twin is moving and therefore the red twin should age more slowly. But when they both get back together, the green twin has actually aged less. What’s going on here?

Select Pulses from Traveling Twin: ST. Now we have added the traveling twin's clock and numbers on the spacetime diagram to mark the arrival of the traveling twin's light pulse.

One of the most important concepts in special relativity is the idea of the spacetime interval, $(\Delta s)^2 = c^2(\Delta t)^2 - (\Delta x)^2$, which is the same in any two inertial frames. This can be thought of as the Pythagorean Theorem of spacetime.

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Repeat for the <u>green</u> twin during the <u>outbound</u> trip.

Calculate the spacetime interval for the <u>red</u> twin during the <u>inbound</u> trip.

Repeat for the <u>green</u> twin during the <u>inbound</u> trip.

Why is there a difference in the spacetime intervals for the twins for the entire trip?

Is there really a paradox in the fact that it's the green twin who ages more slowly? Explain.

√ **Checkpoint 5**

Investigation 7: Relativistic Doppler shift and the twin paradox (optional)

In this simulation, you will explore the famous twin paradox from a slightly different perspective. To open the simulation, select ***Special Relativity*** from the frame on the left of the page. Open ***Chapter 3*** and then open Section 3.6 from the frame on the left of the page.

In this simulation, one twin (Pink) remains on earth, while a second twin (Green) leaves in a rocket ship to travel to a distant planet at t = 0 years. When Green reaches the planet, they turn around and return to Earth 20 years later (as measured by Pink who remained on Earth). Pink has aged 20 years, but Green has not aged as much. To measure this effect quantitatively, each twin sends out a flash of light on their birthday. In each twin's own reference frame, then, these flashes occur with a frequency of one. What does it look like from the other reference frame?

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Summarize: How many years elapse for Pink during the time of the trip? How many years elapse for Green during the time of his trip? How do you know?

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Physics 216 Laboratory 13, Spring 2018
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√ Checkpoint 1

Investigation 2: Worldline matching

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Worldline 1

Worldline 2

Worldline 3

Worldline 4

Worldline 5 (along the line of v=c)

Did you find any bad physics in the animations? If so, what was it?

√ Checkpoint 2

Investigation 3: Simultaneity

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Are there any simultaneous events in this reference frame? If so, which events?

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v/c	Event 1 (right ends) in Green frame	Event 2 (left ends) in Green frame	Event 1(right ends) in Red frame	Event 2(left ends) in Red frame

Based on your observations, answer the following.

Does one of the sticks have a greater proper length, and if so, which one? How can you tell?

Does event 1 always occur before event 2? Does the answer depend on the reference frame?

Is there any speed for which the events are simultaneous in <u>both</u> reference frames?

Is there any speed for which the events are simultaneous in <u>one</u> reference frame? What is special at that speed?

At a given speed, will the red and green observers always agree on which event occurred first? Explain.

√ Checkpoint 3

Investigation 5: Relativistic pole vaulting

Open Section 2.7. In this simulation, you will explore the famous pole and barn paradox. A pole vaulter carries a long pole towards a barn as shown in the animation (and the spacetime diagram). You can place the cursor on a point in the animation and click to show the x and y coordinates in the selected frame.

Start by watching the animation from the frame of the barn.

How long is the barn in this frame? How long is the pole in this frame? Which of these is a proper length?

How fast is the pole moving relative to the barn (v/c) (show how you got this)?

What do events A, B, A', and B' refer to? Which, if any, of these events are simultaneous in this reference frame?

Initially consider the back door of the barn to be closed. It can be opened instantaneously when the front of the pole reaches it. Similarly, the front door will be closed instantaneously when the back end of the pole reaches it. Given that, are there any times when the pole is completely inside the barn so that both doors will be closed at the same time (even if just for an instant)? Explain.

Next, look at the animation from the frame of the pole vaulter.
How fast is the barn moving relative to the pole (v/c) (show how you got this)? Does this agree with what you measured from the barn frame? Should it?

How long is the pole in this reference frame? Why is it this long?

How long is the barn in this reference frame? Why is it this long?

What do events A, B, A', and B' refer to? Which if any of these events are simultaneous in this reference frame?

From this frame, are there any times when the pole is completely inside the barn so that both doors will be closed at the same time?

Given that from the barn frame both doors can be closed at the same time (even if only for an instant), describe the sequence of events (in terms of where/when the front/back of pole is relative to the front/back door of barn and if/when front/back door are opened/closed).

√ **Checkpoint 4**

Investigation 6: Twin paradox

Open Section 2.8. In this simulation, you will explore the famous twin paradox. One twin (red) remains on Earth, while a second twin (green) begins a trip to a distant location at t = 0 years, and then returns, reaching Earth again at t = 10 years.

First run the simulation from the view of the Earth-bound (red) twin.

How fast is the green twin traveling relative to the red twin (v/c)?

The two twins have agreed to send each other a light pulse once a year on the anniversary of the green twin's departure. Select Pulses from Earth-Bound Twin and play.

What is the frequency of the red twin's light pulse in her own reference frame?

By how many years has the red twin aged during her twin's trip?

How many light pulses reach the green twin during his outbound trip?

During his inbound trip?

Select Pulses from Traveling Twin and play.
By how many years has the green twin aged during his trip?

The red twin argues that from her point of view, the green twin is moving and therefore the green twin should age more slowly. On the other hand, the green twin argues that from his point of view, the red twin is moving and therefore the red twin should age more slowly. But when they both get back together, the green twin has actually aged less. What's going on here?

Select Pulses from Traveling Twin: ST. Now we have added the traveling twin's clock and numbers on the spacetime diagram to mark the arrival of the traveling twin's light pulse.

One of the most important concepts in special relativity is the idea of the spacetime interval, $(\Delta s)^2 = c^2(\Delta t)^2 - (\Delta x)^2$, which is the same in any two inertial frames. This can be thought of as the Pythagorean Theorem of spacetime.

Calculate the spacetime interval for the red twin during the outbound trip.

Repeat for the green twin during the outbound trip.

Calculate the spacetime interval for the red twin during the inbound trip.

Repeat for the green twin during the inbound trip.

Why is there a difference in the spacetime intervals for the twins for the entire trip?

Is there really a paradox in the fact that it's the green twin who ages more slowly? Explain.

√ **Checkpoint 5**

Investigation 7: Relativistic Doppler shift and the twin paradox (optional)

In this simulation, you will explore the famous twin paradox from a slightly different perspective. To open the simulation, select ***Special Relativity*** from the frame on the left of the page. Open ***Chapter 3*** and then open Section 3.6 from the frame on the left of the page.

In this simulation, one twin (Pink) remains on earth, while a second twin (Green) leaves in a rocket ship to travel to a distant planet at t = 0 years. When Green reaches the planet, they turn around and return to Earth 20 years later (as measured by Pink who remained on Earth). Pink has aged 20 years, but Green has not aged as much. To measure this effect quantitatively, each twin sends out a flash of light on their birthday. In each twin's own reference frame, then, these flashes occur with a frequency of one. What does it look like from the other reference frame?

First look at it from Pink's view on Earth. The time clock in the upper left corner shows the years as measured on Earth, while the counter counts the flashes that Pink receives from Green.

What is the frequency of the pulses that she receives from Green that were emitted on his outbound trip?

What is the frequency of the pulses that she receives from Green that were emitted on his inbound trip?

Are your answers consistent with the equation for the relativistic Doppler shift? Justify your answer with a short calculation.

How many signals does Pink receive from Green's outbound trip? From his inbound trip? Total?

Now, from Green's point of view, the situation is a bit different. In order to make this trip, he has to switch reference frames after turning around so in the animation, we show the view of Pink (and the Earth) from the two frames (White and Black) that Green jumps between. The white pulse counter counts pulses only when Green is in the white reference frame and a black counter will count pulses only when Green is in the black frame.

Verify that the total number of pulses Green receives is equal to Pink's change in age when Green returns to Earth. Notice that Green's clock (in the upper left hand corner) shows him to be younger.

What is the frequency of the pulses that Green receives while in the White reference frame? While in the Black reference frame?

Show that these equations, too, are consistent with the Doppler shift equation.

How many signals does Green receive on his outbound trip? On his inbound trip? Total?

Summarize: How many years elapse for Pink during the time of the trip? How many years elapse for Green during the time of his trip? How do you know?

√ **Checkpoint 6**

Physics 216 Laboratory 13, Spring 2018
Special Relativity

Background

Since our budget doesn't allow for a fleet of relativistic spaceships, we'll use a series of computer simulations to investigate some situations in which the concepts of special relativity are important. All the simulations that you will be examining today, along with a number of other simulations that you are encouraged to play with, are found at http://www.compadre.org/PQP/special-relativity/.

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Investigation 1: Time dilation and length contraction

Open Section 2.4 from the menu on the left. You should see two animations, with a description under each animation. Read the description under the top animation, and then set $|\beta|=|u/c|$ to 0.5 and press the *play* button.

In 5 ticks of the clock in the stationary frame (red clock), how many ticks (estimate fractional ticks) does the green clock make?

Show that these measurements agree with the time dilation formula.

What would you see from the green reference frame? Why?

Now scroll down the page to the other animation (rotated light clocks). Once you've read the text under the animation, set $|\beta|=|u/c|$ to 0.5 and press the *play* button.

Pause the animation at a point where the green clock is still visible. Using the small crosshairs in the lower left region of the animation, measure the length of the red and green clock in the red reference frame by recording the position of each end of the clock. Be sure to include units.

Length of green clock in red reference frame:

Length of red clock in red reference frame:

Show that these measurements agree with the length contraction formula.

What would you see from the green reference frame? Why?

√ Checkpoint 1

Investigation 2: Worldline matching

Open Section 2.6 from the frame on the left of the page. In this simulation, you will explore worldlines, which are used to depict the trajectory of a particle in spacetime.

Your goal is to match a series of worldlines that can be loaded by clicking one of the links below the control buttons. The position of the red ball can be controlled by using the mouse to move the red ball in the box below the spacetime diagram.

For each of the worldlines, sketch the spacetime diagram and describe in words how you had to move the red ball to match the worldlines.

Worldline 1

Worldline 2

Worldline 3

Worldline 4

Worldline 5 (along the line of v=c)

Did you find any bad physics in the animations? If so, what was it?

√ Checkpoint 2

Investigation 3: Simultaneity

Open Section 2.3 from the frame on the left of the page. In this simulation, you will explore the nature of simultaneity. There are two different “experiments.” In each, an event occurs and this event is then observed by four observers at different locations in two different coordinate systems.

Run Experiment 1: From the frame of the ground.
Describe the order in which the light flash reaches each of the four observers, according to an observer in the reference frame of the ground.

Are there any simultaneous events in this reference frame? If so, which events?

Run Experiment 2: From the frame of the railcar.
Describe the order in which the light flash reaches each of the four observers, according to an observer in the reference frame of the railcar.

Are there any simultaneous events in this reference frame? Are they the same events as in the other reference frame?

Are the results of these two experiments consistent? Explain.

Investigation 4: Simultaneity using relativistic measuring sticks

Open Problem 2.7 from the frame on the left of the page. In this simulation, you will explore the nature of simultaneity from a slightly different perspective. In this simulation, two measuring sticks pass each other with a given relative speed. A light flash occurs when the right ends of the sticks coincide (event 1) and again when the left ends coincide (event 2).

Observe the events at several different relative speeds and from both reference frames. Record what happens and when, according to each reference frame, in the table below. Try to find a relative speed such that the events are simultaneous in one or both reference frames.

v/c	Event 1 (right ends) in Green frame	Event 2 (left ends) in Green frame	Event 1(right ends) in Red frame	Event 2(left ends) in Red frame

Based on your observations, answer the following.

Does one of the sticks have a greater proper length, and if so, which one? How can you tell?

Does event 1 always occur before event 2? Does the answer depend on the reference frame?

Is there any speed for which the events are simultaneous in <u>both</u> reference frames?

Is there any speed for which the events are simultaneous in <u>one</u> reference frame? What is special at that speed?

At a given speed, will the red and green observers always agree on which event occurred first? Explain.

√ **Checkpoint 3**

Investigation 5: Relativistic pole vaulting

Open Section 2.7. In this simulation, you will explore the famous pole and barn paradox. A pole vaulter carries a long pole towards a barn as shown in the animation (and the spacetime diagram). You can place the cursor on a point in the animation and click to show the x and y coordinates in the selected frame.

Start by watching the animation from the frame of the barn.

How long is the barn in this frame? How long is the pole in this frame? Which of these is a proper length?

How fast is the pole moving relative to the barn (v/c) (show how you got this)?

What do events A, B, A', and B' refer to? Which, if any, of these events are simultaneous in this reference frame?

Initially consider the back door of the barn to be closed. It can be opened instantaneously when the front of the pole reaches it. Similarly, the front door will be closed instantaneously when the back end of the pole reaches it. Given that, are there any times when the pole is completely inside the barn so that both doors will be closed at the same time (even if just for an instant)? Explain.

Next, look at the animation from the frame of the pole vaulter.
How fast is the barn moving relative to the pole (v/c) (show how you got this)? Does this agree with what you measured from the barn frame? Should it?

How long is the pole in this reference frame? Why is it this long?

How long is the barn in this reference frame? Why is it this long?

What do events A, B, A', and B' refer to? Which if any of these events are simultaneous in this reference frame?

From this frame, are there any times when the pole is completely inside the barn so that both doors will be closed at the same time?

Given that from the barn frame both doors can be closed at the same time (even if only for an instant), describe the sequence of events (in terms of where/when the front/back of pole is relative to the front/back door of barn and if/when front/back door are opened/closed).

√ **Checkpoint 4**

Investigation 6: Twin paradox

Open Section 2.8. In this simulation, you will explore the famous twin paradox. One twin (red) remains on Earth, while a second twin (green) begins a trip to a distant location at t = 0 years, and then returns, reaching Earth again at t = 10 years.

First run the simulation from the view of the Earth-bound (red) twin.

How fast is the green twin traveling relative to the red twin (v/c)?

The two twins have agreed to send each other a light pulse once a year on the anniversary of the green twin's departure. Select Pulses from Earth-Bound Twin and play.

What is the frequency of the red twin's light pulse in her own reference frame?

By how many years has the red twin aged during her twin's trip?

How many light pulses reach the green twin during his outbound trip?

During his inbound trip?

Select Pulses from Traveling Twin and play.
By how many years has the green twin aged during his trip?

The red twin argues that from her point of view, the green twin is moving and therefore the green twin should age more slowly. On the other hand, the green twin argues that from his point of view, the red twin is moving and therefore the red twin should age more slowly. But when they both get back together, the green twin has actually aged less. What's going on here?

Select Pulses from Traveling Twin: ST. Now we have added the traveling twin's clock and numbers on the spacetime diagram to mark the arrival of the traveling twin's light pulse.

One of the most important concepts in special relativity is the idea of the spacetime interval, $(\Delta s)^2 = c^2(\Delta t)^2 - (\Delta x)^2$, which is the same in any two inertial frames. This can be thought of as the Pythagorean Theorem of spacetime.

Calculate the spacetime interval for the red twin during the outbound trip.

Repeat for the green twin during the outbound trip.

Calculate the spacetime interval for the red twin during the inbound trip.

Repeat for the green twin during the inbound trip.

Why is there a difference in the spacetime intervals for the twins for the entire trip?

Is there really a paradox in the fact that it's the green twin who ages more slowly? Explain.

√ **Checkpoint 5**

Investigation 7: Relativistic Doppler shift and the twin paradox (optional)

In this simulation, you will explore the famous twin paradox from a slightly different perspective. To open the simulation, select ***Special Relativity*** from the frame on the left of the page. Open ***Chapter 3*** and then open Section 3.6 from the frame on the left of the page.

In this simulation, one twin (Pink) remains on earth, while a second twin (Green) leaves in a rocket ship to travel to a distant planet at t = 0 years. When Green reaches the planet, they turn around and return to Earth 20 years later (as measured by Pink who remained on Earth). Pink has aged 20 years, but Green has not aged as much. To measure this effect quantitatively, each twin sends out a flash of light on their birthday. In each twin's own reference frame, then, these flashes occur with a frequency of one. What does it look like from the other reference frame?

First look at it from Pink's view on Earth. The time clock in the upper left corner shows the years as measured on Earth, while the counter counts the flashes that Pink receives from Green.

What is the frequency of the pulses that she receives from Green that were emitted on his outbound trip?

What is the frequency of the pulses that she receives from Green that were emitted on his inbound trip?

Are your answers consistent with the equation for the relativistic Doppler shift? Justify your answer with a short calculation.

How many signals does Pink receive from Green's outbound trip? From his inbound trip? Total?

Now, from Green's point of view, the situation is a bit different. In order to make this trip, he has to switch reference frames after turning around so in the animation, we show the view of Pink (and the Earth) from the two frames (White and Black) that Green jumps between. The white pulse counter counts pulses only when Green is in the white reference frame and a black counter will count pulses only when Green is in the black frame.

Verify that the total number of pulses Green receives is equal to Pink's change in age when Green returns to Earth. Notice that Green's clock (in the upper left hand corner) shows him to be younger.

What is the frequency of the pulses that Green receives while in the White reference frame? While in the Black reference frame?

Show that these equations, too, are consistent with the Doppler shift equation.

How many signals does Green receive on his outbound trip? On his inbound trip? Total?

Summarize: How many years elapse for Pink during the time of the trip? How many years elapse for Green during the time of his trip? How do you know?

√ **Checkpoint 6**

Physics 216 Laboratory 13, Spring 2018
Special Relativity

Background
Since our budget doesn't allow for a fleet of relativistic spaceships, we'll use a series of computer simulations to investigate some situations in which the concepts of special relativity are important. All the simulations that you will be examining today, along with a number of other simulations that you are encouraged to play with, are found at http://www.compadre.org/PQP/special-relativity/.

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Investigation 1: Time dilation and length contraction
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In 5 ticks of the clock in the stationary frame (red clock), how many ticks (estimate fractional ticks) does the green clock make?

Show that these measurements agree with the time dilation formula.

What would you see from the green reference frame? Why?

Now scroll down the page to the other animation (rotated light clocks). Once you've read the text under the animation, set $|\beta|=|u/c|$ to 0.5 and press the *play* button.

Pause the animation at a point where the green clock is still visible. Using the small crosshairs in the lower left region of the animation, measure the length of the red and green clock in the red reference frame by recording the position of each end of the clock. Be sure to include units.

Length of green clock in red reference frame:

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Show that these measurements agree with the length contraction formula.

What would you see from the green reference frame? Why?

√ Checkpoint 1

Investigation 2: Worldline matching

Open Section 2.6 from the frame on the left of the page. In this simulation, you will explore worldlines, which are used to depict the trajectory of a particle in spacetime.

Your goal is to match a series of worldlines that can be loaded by clicking one of the links below the control buttons. The position of the red ball can be controlled by using the mouse to move the red ball in the box below the spacetime diagram.

For each of the worldlines, sketch the spacetime diagram and describe in words how you had to move the red ball to match the worldlines.

Worldline 1

Worldline 2

Worldline 3

Worldline 4

Worldline 5 (along the line of v=c)

Did you find any bad physics in the animations? If so, what was it?

√ Checkpoint 2

Investigation 3: Simultaneity

Open Section 2.3 from the frame on the left of the page. In this simulation, you will explore the nature of simultaneity. There are two different "experiments." In each, an event occurs and this event is then observed by four observers at different locations in two different coordinate systems.

Run Experiment 1: From the frame of the ground.
Describe the order in which the light flash reaches each of the four observers, according to an observer in the reference frame of the ground.

Are there any simultaneous events in this reference frame? If so, which events?

Run Experiment 2: From the frame of the railcar.
Describe the order in which the light flash reaches each of the four observers, according to an observer in the reference frame of the railcar.

Are there any simultaneous events in this reference frame? Are they the same events as in the other reference frame?

Are the results of these two experiments consistent? Explain.

Investigation 4: Simultaneity using relativistic measuring sticks

Open Problem 2.7 from the frame on the left of the page. In this simulation, you will explore the nature of simultaneity from a slightly different perspective. In this simulation, two measuring sticks pass each other with a given relative speed. A light flash occurs when the right ends of the sticks coincide (event 1) and again when the left ends coincide (event 2).

Observe the events at several different relative speeds and from both reference frames. Record what happens and when, according to each reference frame, in the table below. Try to find a relative speed such that the events are simultaneous in one or both reference frames.

v/c	Event 1 (right ends) in Green frame	Event 2 (left ends) in Green frame	Event 1(right ends) in Red frame	Event 2(left ends) in Red frame

Based on your observations, answer the following.

Does one of the sticks have a greater proper length, and if so, which one? How can you tell?

Does event 1 always occur before event 2? Does the answer depend on the reference frame?

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Is there any speed for which the events are simultaneous in <u>one</u> reference frame? What is special at that speed?

At a given speed, will the red and green observers always agree on which event occurred first? Explain.

√ **Checkpoint 3**

Investigation 5: Relativistic pole vaulting

Open Section 2.7. In this simulation, you will explore the famous pole and barn paradox. A pole vaulter carries a long pole towards a barn as shown in the animation (and the spacetime diagram). You can place the cursor on a point in the animation and click to show the x and y coordinates in the selected frame.

Start by watching the animation from the frame of the barn.

How long is the barn in this frame? How long is the pole in this frame? Which of these is a proper length?

How fast is the pole moving relative to the barn (v/c) (show how you got this)?

What do events A, B, A', and B' refer to? Which, if any, of these events are simultaneous in this reference frame?

Initially consider the back door of the barn to be closed. It can be opened instantaneously when the front of the pole reaches it. Similarly, the front door will be closed instantaneously when the back end of the pole reaches it. Given that, are there any times when the pole is completely inside the barn so that both doors will be closed at the same time (even if just for an instant)? Explain.

Next, look at the animation from the frame of the pole vaulter.
How fast is the barn moving relative to the pole (v/c) (show how you got this)? Does this agree with what you measured from the barn frame? Should it?

How long is the pole in this reference frame? Why is it this long?

How long is the barn in this reference frame? Why is it this long?

What do events A, B, A', and B' refer to? Which if any of these events are simultaneous in this reference frame?

From this frame, are there any times when the pole is completely inside the barn so that both doors will be closed at the same time?

Given that from the barn frame both doors can be closed at the same time (even if only for an instant), describe the sequence of events (in terms of where/when the front/back of pole is relative to the front/back door of barn and if/when front/back door are opened/closed).

√ **Checkpoint 4**

Investigation 6: Twin paradox

Open Section 2.8. In this simulation, you will explore the famous twin paradox. One twin (red) remains on Earth, while a second twin (green) begins a trip to a distant location at t = 0 years, and then returns, reaching Earth again at t = 10 years.

First run the simulation from the view of the Earth-bound (red) twin.

How fast is the green twin traveling relative to the red twin (v/c)?

The two twins have agreed to send each other a light pulse once a year on the anniversary of the green twin's departure. Select Pulses from Earth-Bound Twin and play.

What is the frequency of the red twin's light pulse in her own reference frame?

By how many years has the red twin aged during her twin's trip?

How many light pulses reach the green twin during his outbound trip?

During his inbound trip?

Select Pulses from Traveling Twin and play.
By how many years has the green twin aged during his trip?

The red twin argues that from her point of view, the green twin is moving and therefore the green twin should age more slowly. On the other hand, the green twin argues that from his point of view, the red twin is moving and therefore the red twin should age more slowly. But when they both get back together, the green twin has actually aged less. What's going on here?

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Repeat for the green twin during the inbound trip.

Why is there a difference in the spacetime intervals for the twins for the entire trip?

Is there really a paradox in the fact that it's the green twin who ages more slowly? Explain.

√ **Checkpoint 5**

Investigation 7: Relativistic Doppler shift and the twin paradox (optional)

In this simulation, you will explore the famous twin paradox from a slightly different perspective. To open the simulation, select ***Special Relativity*** from the frame on the left of the page. Open ***Chapter 3*** and then open Section 3.6 from the frame on the left of the page.

In this simulation, one twin (Pink) remains on earth, while a second twin (Green) leaves in a rocket ship to travel to a distant planet at t = 0 years. When Green reaches the planet, they turn around and return to Earth 20 years later (as measured by Pink who remained on Earth). Pink has aged 20 years, but Green has not aged as much. To measure this effect quantitatively, each twin sends out a flash of light on their birthday. In each twin's own reference frame, then, these flashes occur with a frequency of one. What does it look like from the other reference frame?

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Summarize: How many years elapse for Pink during the time of the trip? How many years elapse for Green during the time of his trip? How do you know?

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Worldline 1

Worldline 2

Worldline 3

Worldline 4

Worldline 5 (along the line of v=c)

Did you find any bad physics in the animations? If so, what was it?

√ Checkpoint 2

Investigation 3: Simultaneity

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Are there any simultaneous events in this reference frame? If so, which events?

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Are there any simultaneous events in this reference frame? Are they the same events as in the other reference frame?

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Open Problem 2.7 from the frame on the left of the page. In this simulation, you will explore the nature of simultaneity from a slightly different perspective. In this simulation, two measuring sticks pass each other with a given relative speed. A light flash occurs when the right ends of the sticks coincide (event 1) and again when the left ends coincide (event 2).

Observe the events at several different relative speeds and from both reference frames. Record what happens and when, according to each reference frame, in the table below. Try to find a relative speed such that the events are simultaneous in one or both reference frames.

v/c	Event 1 (right ends) in Green frame	Event 2 (left ends) in Green frame	Event 1(right ends) in Red frame	Event 2(left ends) in Red frame

Based on your observations, answer the following.

Does one of the sticks have a greater proper length, and if so, which one? How can you tell?

Does event 1 always occur before event 2? Does the answer depend on the reference frame?

Is there any speed for which the events are simultaneous in <u>both</u> reference frames?

Is there any speed for which the events are simultaneous in <u>one</u> reference frame? What is special at that speed?

At a given speed, will the red and green observers always agree on which event occurred first? Explain.

√ **Checkpoint 3**

Investigation 5: Relativistic pole vaulting

Open Section 2.7. In this simulation, you will explore the famous pole and barn paradox. A pole vaulter carries a long pole towards a barn as shown in the animation (and the spacetime diagram). You can place the cursor on a point in the animation and click to show the x and y coordinates in the selected frame.

Start by watching the animation from the frame of the barn.

How long is the barn in this frame? How long is the pole in this frame? Which of these is a proper length?

How fast is the pole moving relative to the barn (v/c) (show how you got this)?

What do events A, B, A', and B' refer to? Which, if any, of these events are simultaneous in this reference frame?

Initially consider the back door of the barn to be closed. It can be opened instantaneously when the front of the pole reaches it. Similarly, the front door will be closed instantaneously when the back end of the pole reaches it. Given that, are there any times when the pole is completely inside the barn so that both doors will be closed at the same time (even if just for an instant)? Explain.

Next, look at the animation from the frame of the pole vaulter.
How fast is the barn moving relative to the pole (v/c) (show how you got this)? Does this agree with what you measured from the barn frame? Should it?

How long is the pole in this reference frame? Why is it this long?

How long is the barn in this reference frame? Why is it this long?

What do events A, B, A', and B' refer to? Which if any of these events are simultaneous in this reference frame?

From this frame, are there any times when the pole is completely inside the barn so that both doors will be closed at the same time?

Given that from the barn frame both doors can be closed at the same time (even if only for an instant), describe the sequence of events (in terms of where/when the front/back of pole is relative to the front/back door of barn and if/when front/back door are opened/closed).

√ **Checkpoint 4**

Investigation 6: Twin paradox

Open Section 2.8. In this simulation, you will explore the famous twin paradox. One twin (red) remains on Earth, while a second twin (green) begins a trip to a distant location at t = 0 years, and then returns, reaching Earth again at t = 10 years.

First run the simulation from the view of the Earth-bound (red) twin.

How fast is the green twin traveling relative to the red twin (v/c)?

The two twins have agreed to send each other a light pulse once a year on the anniversary of the green twin's departure. Select Pulses from Earth-Bound Twin and play.

What is the frequency of the red twin's light pulse in her own reference frame?

By how many years has the red twin aged during her twin's trip?

How many light pulses reach the green twin during his outbound trip?

During his inbound trip?

Select Pulses from Traveling Twin and play.
By how many years has the green twin aged during his trip?

The red twin argues that from her point of view, the green twin is moving and therefore the green twin should age more slowly. On the other hand, the green twin argues that from his point of view, the red twin is moving and therefore the red twin should age more slowly. But when they both get back together, the green twin has actually aged less. What's going on here?

Select Pulses from Traveling Twin: ST. Now we have added the traveling twin's clock and numbers on the spacetime diagram to mark the arrival of the traveling twin's light pulse.

One of the most important concepts in special relativity is the idea of the spacetime interval, $(\Delta s)^2 = c^2(\Delta t)^2 - (\Delta x)^2$, which is the same in any two inertial frames. This can be thought of as the Pythagorean Theorem of spacetime.

Calculate the spacetime interval for the red twin during the outbound trip.

Repeat for the green twin during the outbound trip.

Calculate the spacetime interval for the red twin during the inbound trip.

Repeat for the green twin during the inbound trip.

Why is there a difference in the spacetime intervals for the twins for the entire trip?

Is there really a paradox in the fact that it's the green twin who ages more slowly? Explain.

√ **Checkpoint 5**

Investigation 7: Relativistic Doppler shift and the twin paradox (optional)

In this simulation, you will explore the famous twin paradox from a slightly different perspective. To open the simulation, select ***Special Relativity*** from the frame on the left of the page. Open ***Chapter 3*** and then open Section 3.6 from the frame on the left of the page.

In this simulation, one twin (Pink) remains on earth, while a second twin (Green) leaves in a rocket ship to travel to a distant planet at t = 0 years. When Green reaches the planet, they turn around and return to Earth 20 years later (as measured by Pink who remained on Earth). Pink has aged 20 years, but Green has not aged as much. To measure this effect quantitatively, each twin sends out a flash of light on their birthday. In each twin's own reference frame, then, these flashes occur with a frequency of one. What does it look like from the other reference frame?

First look at it from Pink's view on Earth. The time clock in the upper left corner shows the years as measured on Earth, while the counter counts the flashes that Pink receives from Green.

What is the frequency of the pulses that she receives from Green that were emitted on his outbound trip?

What is the frequency of the pulses that she receives from Green that were emitted on his inbound trip?

Are your answers consistent with the equation for the relativistic Doppler shift? Justify your answer with a short calculation.

How many signals does Pink receive from Green's outbound trip? From his inbound trip? Total?

Now, from Green's point of view, the situation is a bit different. In order to make this trip, he has to switch reference frames after turning around so in the animation, we show the view of Pink (and the Earth) from the two frames (White and Black) that Green jumps between. The white pulse counter counts pulses only when Green is in the white reference frame and a black counter will count pulses only when Green is in the black frame.

Verify that the total number of pulses Green receives is equal to Pink's change in age when Green returns to Earth. Notice that Green's clock (in the upper left hand corner) shows him to be younger.

What is the frequency of the pulses that Green receives while in the White reference frame? While in the Black reference frame?

Show that these equations, too, are consistent with the Doppler shift equation.

How many signals does Green receive on his outbound trip? On his inbound trip? Total?

Summarize: How many years elapse for Pink during the time of the trip? How many years elapse for Green during the time of his trip? How do you know?

√ **Checkpoint 6**

Physics 216 Laboratory 13, Spring 2018
Special Relativity

Background

Since our budget doesn't allow for a fleet of relativistic spaceships, we'll use a series of computer simulations to investigate some situations in which the concepts of special relativity are important. All the simulations that you will be examining today, along with a number of other simulations that you are encouraged to play with, are found at http://www.compadre.org/PQP/special-relativity/.

Once you have selected the appropriate simulation, you will see a set of control buttons under the animation that will allow you to control the animation (play, pause, step backward/forward one frame). If there are several animations on the page, a ***Restart*** link on the page will restore the entire page to its initial condition.

Investigation 1: Time dilation and length contraction

Open Section 2.4 from the menu on the left. You should see two animations, with a description under each animation. Read the description under the top animation, and then set $|\beta|=|u/c|$ to 0.5 and press the *play* button.

In 5 ticks of the clock in the stationary frame (red clock), how many ticks (estimate fractional ticks) does the green clock make?

Show that these measurements agree with the time dilation formula.

What would you see from the green reference frame? Why?

Now scroll down the page to the other animation (rotated light clocks). Once you've read the text under the animation, set $|\beta|=|u/c|$ to 0.5 and press the *play* button.

Pause the animation at a point where the green clock is still visible. Using the small crosshairs in the lower left region of the animation, measure the length of the red and green clock in the red reference frame by recording the position of each end of the clock. Be sure to include units.

Length of green clock in red reference frame:

Length of red clock in red reference frame:

Show that these measurements agree with the length contraction formula.

What would you see from the green reference frame? Why?

√ Checkpoint 1

Investigation 2: Worldline matching

Open Section 2.6 from the frame on the left of the page. In this simulation, you will explore worldlines, which are used to depict the trajectory of a particle in spacetime.

Your goal is to match a series of worldlines that can be loaded by clicking one of the links below the control buttons. The position of the red ball can be controlled by using the mouse to move the red ball in the box below the spacetime diagram.

For each of the worldlines, sketch the spacetime diagram and describe in words how you had to move the red ball to match the worldlines.

Worldline 1

Worldline 2

Worldline 3

Worldline 4

Worldline 5 (along the line of v=c)

Did you find any bad physics in the animations? If so, what was it?

√ Checkpoint 2

Investigation 3: Simultaneity

Open Section 2.3 from the frame on the left of the page. In this simulation, you will explore the nature of simultaneity. There are two different "experiments." In each, an event occurs and this event is then observed by four observers at different locations in two different coordinate systems.

Run Experiment 1: From the frame of the ground.
Describe the order in which the light flash reaches each of the four observers, according to an observer in the reference frame of the ground.

Are there any simultaneous events in this reference frame? If so, which events?

Run Experiment 2: From the frame of the railcar.
Describe the order in which the light flash reaches each of the four observers, according to an observer in the reference frame of the railcar.

Are there any simultaneous events in this reference frame? Are they the same events as in the other reference frame?

Are the results of these two experiments consistent? Explain.

Investigation 4: Simultaneity using relativistic measuring sticks

Open Problem 2.7 from the frame on the left of the page. In this simulation, you will explore the nature of simultaneity from a slightly different perspective. In this simulation, two measuring sticks pass each other with a given relative speed. A light flash occurs when the right ends of the sticks coincide (event 1) and again when the left ends coincide (event 2).

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At a given speed, will the red and green observers always agree on which event occurred first? Explain.

√ Checkpoint 3

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Start by watching the animation from the frame of the barn.

How long is the barn in this frame? How long is the pole in this frame? Which of these is a proper length?

How fast is the pole moving relative to the barn (v/c) (show how you got this)?

What do events A, B, A', and B' refer to? Which, if any, of these events are simultaneous in this reference frame?

Initially consider the back door of the barn to be closed. It can be opened instantaneously when the front of the pole reaches it. Similarly, the front door will be closed instantaneously when the back end of the pole reaches it. Given that, are there any times when the pole is completely inside the barn so that both doors will be closed at the same time (even if just for an instant)? Explain.

Next, look at the animation from the frame of the pole vaulter.
How fast is the barn moving relative to the pole (v/c) (show how you got this)? Does this agree with what you measured from the barn frame? Should it?

How long is the pole in this reference frame? Why is it this long?

How long is the barn in this reference frame? Why is it this long?

What do events A, B, A', and B' refer to? Which if any of these events are simultaneous in this reference frame?

From this frame, are there any times when the pole is completely inside the barn so that both doors will be closed at the same time?

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√ **Checkpoint 4**

Investigation 6: Twin paradox

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Repeat for the <u>green</u> twin during the <u>outbound</u> trip.

Calculate the spacetime interval for the <u>red</u> twin during the <u>inbound</u> trip.

Repeat for the <u>green</u> twin during the <u>inbound</u> trip.

Why is there a difference in the spacetime intervals for the twins for the entire trip?

Is there really a paradox in the fact that it's the green twin who ages more slowly? Explain.

√ **Checkpoint 5**

Investigation 7: Relativistic Doppler shift and the twin paradox (optional)

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Summarize: How many years elapse for Pink during the time of the trip? How many years elapse for Green during the time of his trip? How do you know?

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