

HW2 Physics 321 Fall 2019 Due next Thurs.  
Show all work.

1. Consider a signal with background and noise, such as we discussed in class. To be most efficient you would like to measure in the intervals  $T_1$  (signal+background+noise) and  $T_2$  (background+noise) so as to minimize the relative error ( $\sigma_{\text{noise}}/V_{\text{signal}}$ ) if possible. This is equivalent to maximizing  $V_{\text{signal}}/\sigma_{\text{noise}}$ . Is there an optimal ratio of  $T_1/T_2$  (given a total time  $T_1+T_2$ ) to achieve this? If so find it. Show why or why not in any case.

2. Show that the second of the 3 orthogonality relations for sin and cos is true. Use the trig addition identities for sin and cos to do the integrals by hand—no Mathematica etc. here.

3. In class we worked out how to use Fourier series to determine the effect of a lo-pass filter on a square wave signal. Plot the original function and also plot on the same axes  $V_{\text{out}}$  for the first 5 non-zero terms of the series.

4. For a full wave rectifier we can approximate the voltage as a function of time as

$$V(t) = \sin(\omega t) \quad 0 < \omega t < \pi.$$

$$V(t) = -\sin(\omega t) \quad -\pi < \omega t < 0$$

a) Derive the Fourier coefficients for this waveform. (Note: Symmetry arguments will again halve your work.) You can use Mathematica (or other source) for the integrals needed. If you don't do them by hand, specify the source (or include a Mathematica notebook).

b) Plot (NOT by hand) the original function and the results of the Fourier expansion for the cases of keeping the first 1, 3, 5 terms in the expansion. (total of 4 separate functions to be plotted) spread them across multiple plots if all 4 on one renders the result illegible, but always include the original function for comparison).

c) Also separately plot (NOT by hand) the coefficients of the series vs  $\omega_n \equiv n\omega$ . Include terms up to  $n=19$ . What does such a plot tell you physically?