P325

1d Astrophysics.

Our subjects of study are almost uniformly extraterrestrial.

So, ___where___ are they? How to specify?

Surface of earth: Longitude : Latitude
↳ 2D need 2 #'s → angles

Longitude ↓ Deg W of reference point
0° = Greenwich England

Latitude ↓ 0° N/S of equator

Sky? Right ascension + declination
2 angles also. why? "celestial sphere"

What are references?

Imagine plane of equator ~~extending~~ out forever where it intercepts celestial sphere = cel. eq.

extend earths axis → NCP, SCP

Declination ↔ latitude +/- N or S of cel. eq.

R.A. + Dec. Right ascension ↔ longitude

___but___ measured in ___hours___. why? b/c we turn inside cel sph. everyday
→ 24 hours = one cel. rot.

$24 \text{ hours} = 360^{\circ} \rightarrow 1 \text{ hr} = 15^{\circ}$

Complication: Standard to subdivide degrees into 1/60 = 1 minute of arc (1 arc-min) 1'
and 1 arcmin ~~d~~ divide 1/60 1 arc sec 1"

but, not really enough | hour of R.A. has 60 minutes and 1 min of RA has 60 sec, etc

but 1 min RA ≠ 1'
1 sec RA ≠ 1"

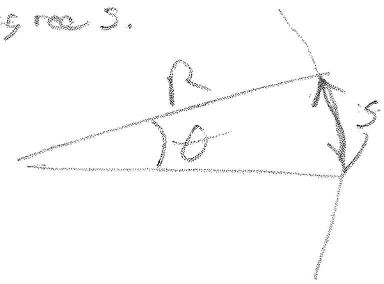
1.2. Reference for RA?
Path of sun on cel. sph?
≡ Ecliptic

we orbit sun so any time we observe sun we are looking ≡ ∥ to earth orbital plane → intersection w. cel. sph ≡ ecliptic

Because earth's axis is <u>not</u> ⊥ to orbit, cel. equator + ecliptic are different great circles on sky. Point of intersection where sun is in spring ≡ 0 h RA. Measure <u>eastward</u>

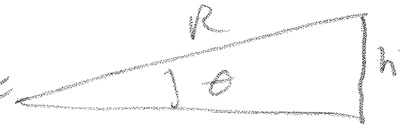
Catalogues → lots of them.

<u>Angles</u>: of course we also use <u>radians</u> to measure degrees.



$\theta \equiv \frac{s}{R}$ radians

if $\theta \ll 1$ rad then ≈



small angle approximations

<u>in radians</u> $\theta \equiv \frac{s}{R} \approx \frac{h}{R} \approx \tan\theta \approx \sin\theta$

Looking @ stars over long time, we noticed RA's + Dec's change (all together) over time

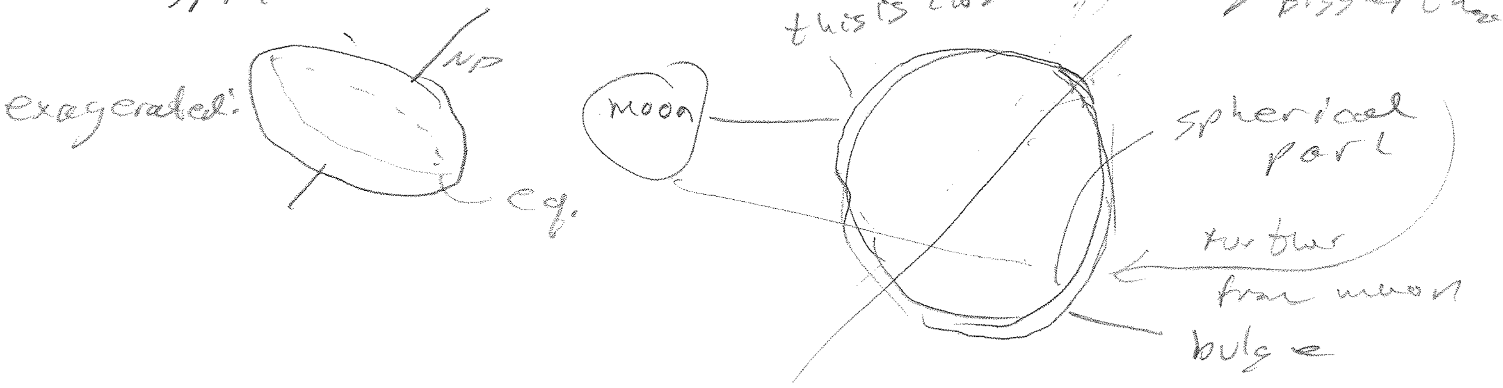
celestial coords not fixed! earth's axis of rotation is not fixed in space - has various wobbles

Main wobble is precession: NCP (+ SCP) traces out a circle ≈ 47° in diameter every ≈ 26,000 yrs

Why? If earth were isolated spinning object, would you expect axis of rotation to change? $\vec{\tau} = \frac{d\vec{L}}{dt}$ $\vec{\tau} = 0$ $\frac{d\vec{L}}{dt} = 0 \rightarrow$ <u>$\vec{L}$ = const</u>

so if $\frac{d\vec{L}}{dt} \neq 0$ $\vec{\tau} \neq 0$ what's $\vec{\tau}$ from? Sun, moon, planets

1.3 how's that work to? if earth spherical, can show $\vec{\tau}=0$ but earth is <u>oblate</u> due to spin



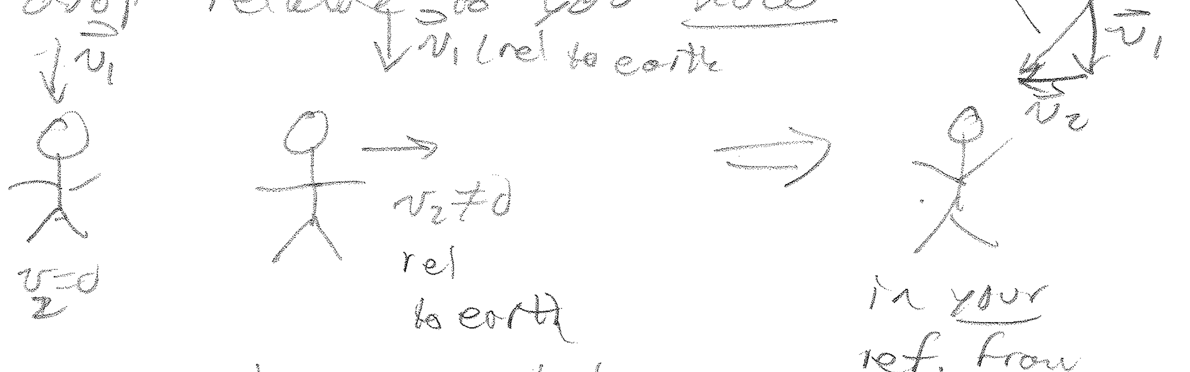
Of course earth does more than spin. it orbits sun,

Consequence direction we look @ to see stars is not where they really are,
"Aberration of light"

This is really just a relative motion issue.

Imagine standing still in rain that falls straight down. The rain hits you square on the top of your head.

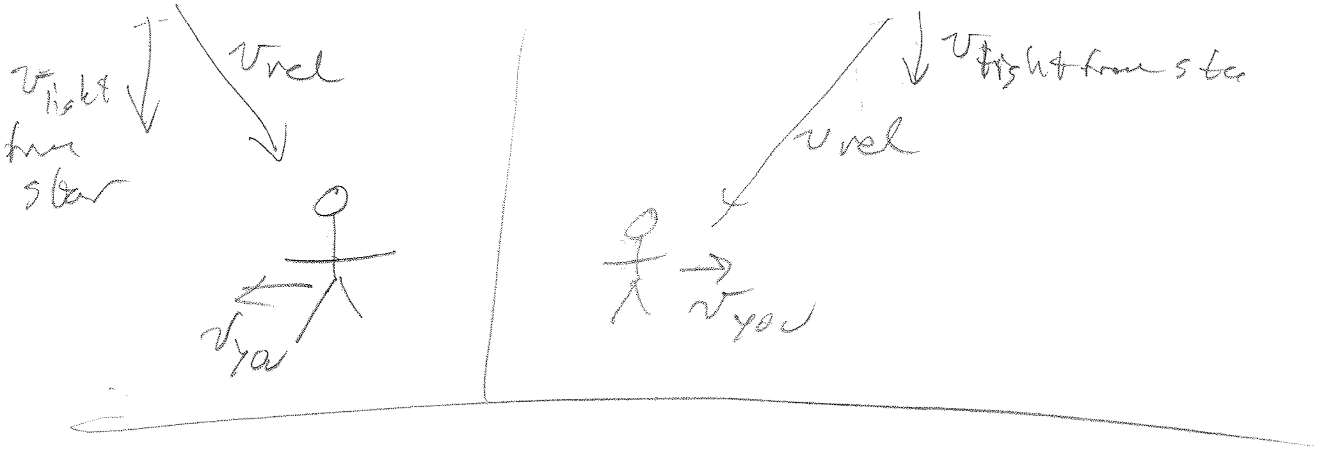
But if you are running/driving through <u>some</u> rain, what's the trajectory of a rain drop relative to you <u>now</u>



Same thing happens w. light

1.4 So you make observations of a star @ 6 month separation, what's true about relative velocity of earth at those 2 times?

So in one case rel vel looks like



Time/Clocks. A mess

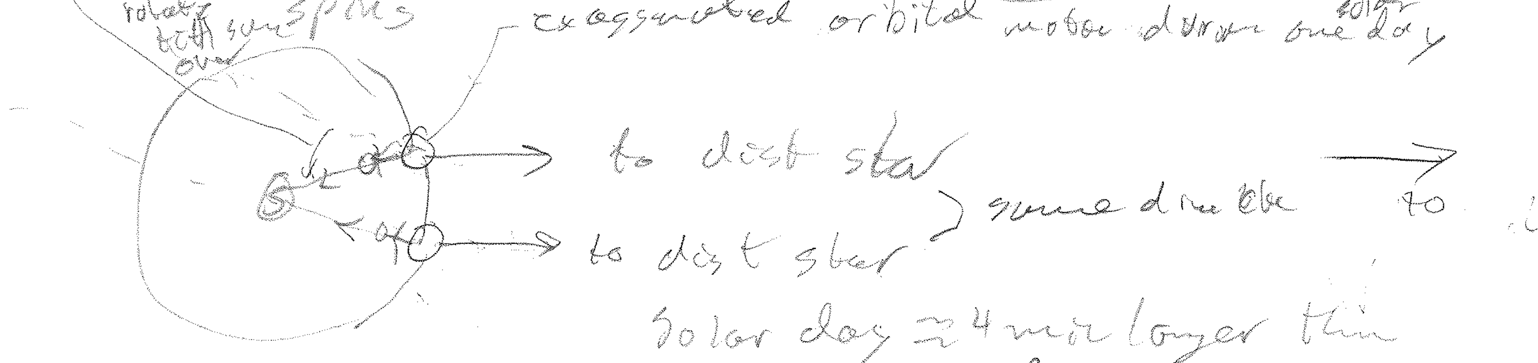
365.2422 days → leap years
leap seconds
etc.

More relevant issue for us is sidereal (star) vs. solar (sun) time

solar day = sun at highest point until next

sidereal day = time until distant star at highest point

why not same? earth orbits as well as spins

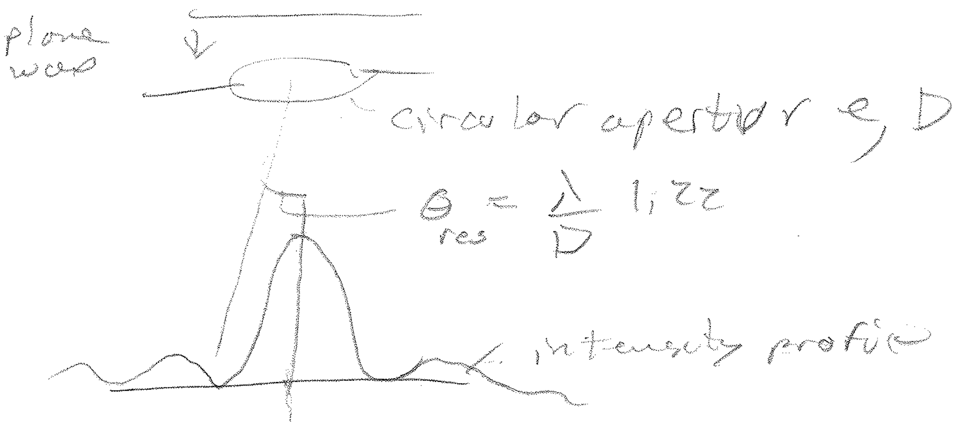


Solar day ≈ 4 min longer than sidereal

1.5 Telescopes

What's wrong w. Fig 1.6 ?

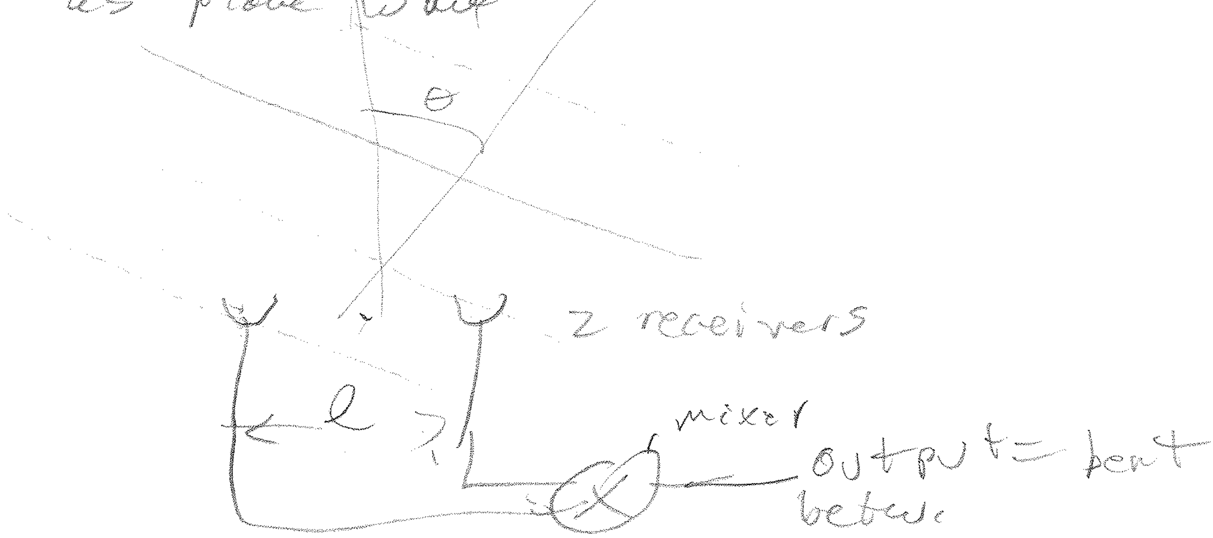
Recall : anytime em wave goes through an aperture / past an edge, there is diffraction
→ interference effect due to wave nature of em radiation



larger D → smaller angles of diffraction → finer details in image
better resolution

large D → more energy collected → brighter image
$\propto D^2$

Interferometers : apart from atmospheric distortions em waves from distant objects arrive here as plane wave



θ = 0 get constructive
$l \cos\theta = m\lambda$ " again] can use to get θ_{res} as if telescope of diameter l !

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2,1 EM Radiation

OK, he over simplifies some things

Anyone have any questions/comments on reading?

Maxwell's Equations do have sinusoidal solutions, but there are other solutions as well.

$\vec{\nabla}$ is operator $\vec{\nabla} = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}$

$\nabla \cdot \vec{E}$ divergence of $\vec{E}$ dot product

$\vec{\nabla} \times \vec{B}$ = curl

cross product $\qquad \frac{\partial}{\partial x}E_x + \frac{\partial}{\partial y}E_y + \frac{\partial}{\partial z}E_z$

$\left(\frac{\partial}{\partial x}B_y - \frac{\partial}{\partial y}B_x\right)\hat{z}$ etc

$\vec{E}$ + $\vec{B}$ are vectors so properly

$$\vec{E}(\vec{r},t) = \vec{E}_0 \cos(\omega t - \vec{k}\cdot\vec{r})$$

if $\vec{E}_0$ = const ≡ linearly polarized plane wave

$\omega = 2\pi\nu \qquad |k| = \frac{2\pi}{\lambda} \qquad \vec{k}$ = vector in direction wave travels

ω: ang freq rad/sec

ν: freq (Hz)

EM waves carry energy (+ momentum)

energy flux parallel to k

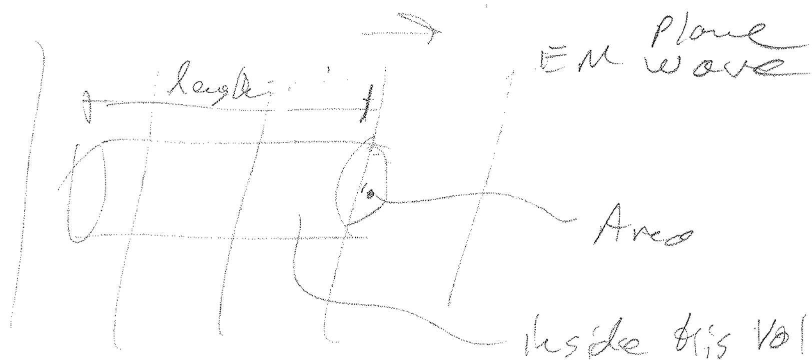
time averaged over 1 period

c.g.s. $\left\{ S = \frac{c}{8\pi}\sqrt{\frac{\varepsilon}{\mu}}\, E_0^2 \right.$ $\qquad \varepsilon$ = permittivity, μ = permeability of medium

Units? NO! It's wrong! no "c"

MKS $\left\{ S = \frac{E_0^2}{2\mu_0 c} \qquad U = \frac{E_0^2}{2\mu_0} \right.$

2.2 Useful concept for transport



inside this volume total energy = u

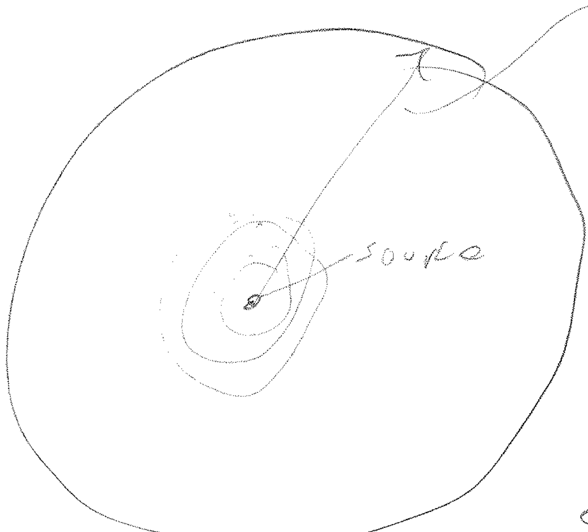
in a time $= \frac{\text{length}}{c}$ everything

in there moves out through A

so $\frac{\text{energy}}{\text{area-time}} = \text{Flux} = \frac{u \cdot \text{Vol}}{\text{area-time}}$

$$\text{Flux} = \frac{u \cdot \cancel{A} c \cancel{t}}{\cancel{A} \cancel{t}} = \boxed{uc = S}$$

Spherical wave?



some arbitrary r

If isotropic + steady

(All energy radiated evenly distributed)

then at any r,

energy/time through any spherical surf.

must be same = power radiated by source

$$S = \frac{P_{source}}{4\pi r^2} = \frac{P_{source}}{4\pi r^2}$$

NRG in wave $\propto E^2$ so $E \propto \frac{1}{r}$

2.3 Focus 2.2 Observations of Io period change? Not if dist to Io ≈ same over one period.

Predict time of eclipses when we + Jupiter in one location when we are on opposite side of sun → times are off by 22 min.

EM spectrum → you should be familiar with by now.

Michelson Morley ; Didn't need anything for EM waves to propagate in (ether)

What's wrong w/ Focus 2.3?

"toward them and away from it ½ year later"

Relativity Review:

$\underbrace{\text{sticks + clocks}}_{\equiv \text{ reference frame}}$ record events - monitors @ all necessary locations

→ something happens - we localize ≈ at one instant.
- emit a photon
- detect a photon
- lightning strikes a point

etc.

$\gamma = \frac{1}{\sqrt{1-\beta^2}}$ $\beta = \frac{v}{c}$ $0 \le \beta \le 1$

$\gamma \ge 1$

Watch some one else's clock tick
measure ticks w/ your clock

$\Delta t' = \gamma \Delta t$

$\Delta t'$: time interval you measure be two ticks of his clock

Δt: time interval betw ticks of his clock ≡ proper time — time when events @ same place (rest frame)

$\gamma > 1$ so $\Delta t'$ longer than Δt

7.4 Rulers moving relative to you are measured by you as being shorter parallel to direction of travel

$$\Delta x' = \frac{\Delta x}{\gamma}$$

There is no relativistic mass increase.

total energy of object

$$E = \gamma mc^2 = K + mc^2$$

Space time interval:
2 events (toss ball, catch ball) observed from each of in 2 frames in relative motion. time intervals & lengths between these events are different in each frame. But some thing is same: space-time interval,

$$ds^2 = dx^2 + dy^2 + dz^2 - c^2dt^2 = dx'^2 + dy'^2 + dz'^2 - c^2dt'^2$$

~~206~~
3.1

Total energy density radiated by B.B.: Add up over all freq

$$u = \int_0^\infty u_\nu \, d\nu = aT^4 \qquad \frac{J}{m^3}$$

$$a = \frac{8\pi^5 k^4}{15c^3h^3}$$

how much does each area radiat

$$f = \sigma T^4 \qquad \text{Stefan-Boltzmann Law}$$

$\hookrightarrow \frac{J}{s\text{-}m^2}$

σ = S-B const

$= 5.6704 \times 10^{-8}$ bits

If you ~~add~~ up over all surface = total Power radiated. If all radiates at same rate;

$$L = \text{Area} \cdot f = \underbrace{4\pi R^2}_{\text{if spherical}} \sigma T^4$$

Luminosity

Big things radiate more
Hot things radiate more

Nothing is a true B.B. but some things are close;

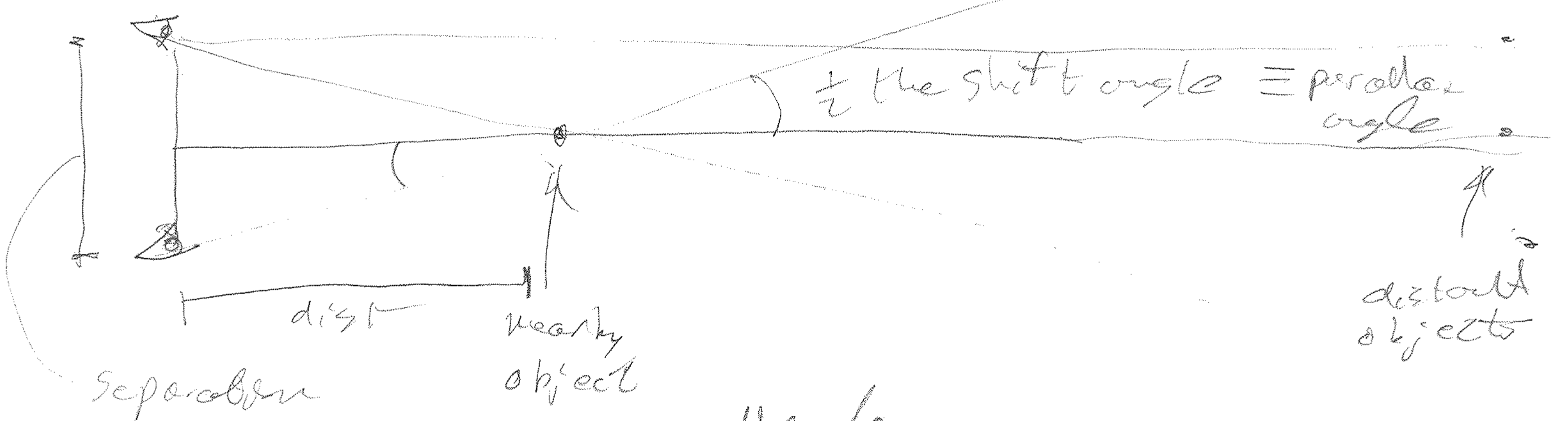
can define effective temp as temp object of same size would have to have to emit same total power as Black Body.

If object radiates @ constant ~~rate~~ uniformly in all directions (e.g. uniform sphere with same properties in all directions), Power through any spherical surface we imagine is same

since area ↑, flux decreases w/ r

$$f = \frac{L}{4\pi r^2} = \frac{\sigma R^2 T_{eff}^4}{r^2}$$

3,2 How would you tell observationally how far the sun is + how big? or anything else?
General tool: Parallax



$$\theta_p \underset{\text{small angle}}{\approx} \frac{\text{sep'n}}{\text{dist}}$$

If things are stationary or relatively slowly (a mountain, say) moving, we can move from point to point to make our observations.

What if they are moving too fast for that?
coordinated simultaneous observations

When observations are made from opposite sides of earth's orbit, and the parallax is 1",
how far away is it?

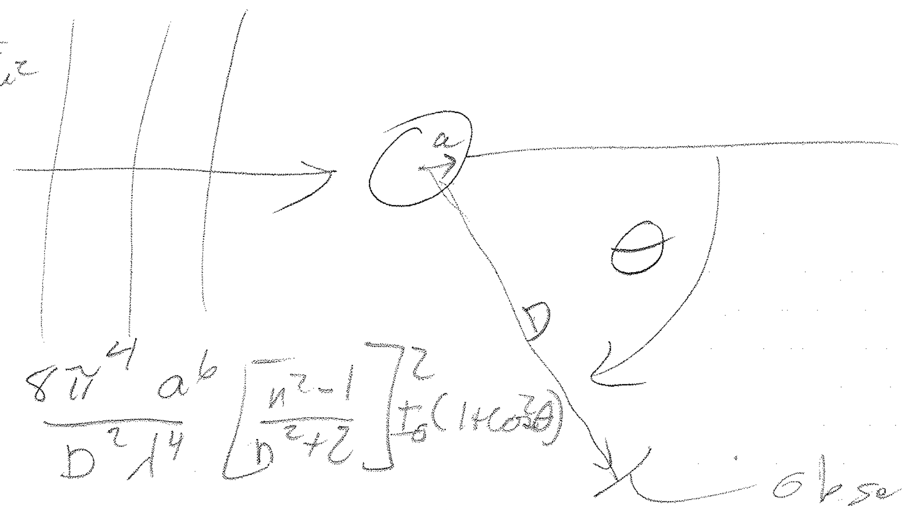
4.13 Scattering..

We observe things mostly by the light they emit. However, that light doesn't move through absolutely empty space → there's stuff in between. Also some things we see by light from another object that hits the thing & light is redirected to our eye ("reflection nebula"). All are examples of scattering and can change some aspects of the light that reaches us. That can be good or bad.

How em radiation scatters depends somewhat on what is scattering it and alot on the relative size of object + λ.

If size of object $<< \lambda$ we call it Rayleigh scattering after Lord Rayleigh described it mathematically. for spherical, dielectric (insulating) particles:

Incident intensity W/m² I_0



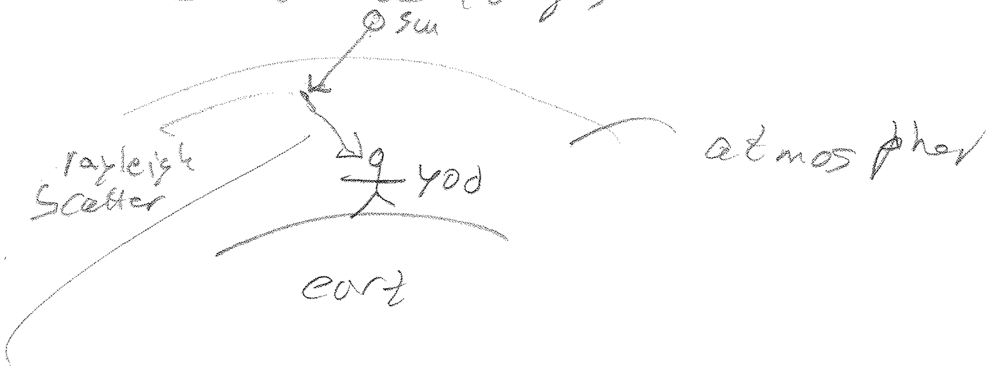
$$I \approx \frac{8\pi^4 a^6}{D^2 \lambda^4}\left[\frac{n^2-1}{n^2+2}\right]^2 I_0(1+\cos^2\theta)$$

Observation

n = relative index of refraction

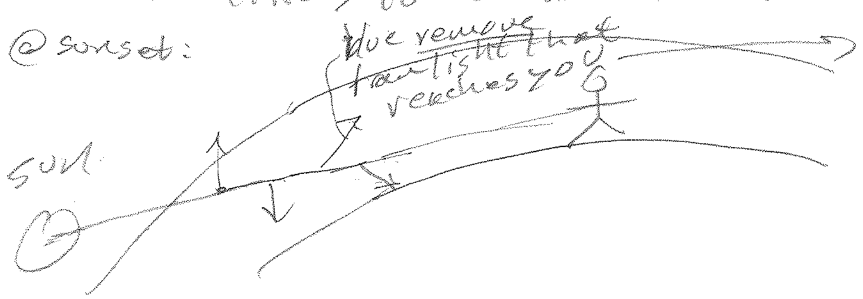
4.2 Rayleigh ctd.

$I_{scatt} \propto 1/\lambda^4$ so <u>short</u> λ scatters much more than long λ



blue light much more likely to scatter → sky = blue

@ sunset:



→ sun @ sunset = red.

For particles w. size ≈ same as λ it is called Mie scattering and $\propto \frac{1}{\lambda}$ (not as strong λ dependence.

Mathematical description more complicated

4.3 free electrons can scatter em rad.

If $E_\gamma = h\nu \ll m_e c^2$

$\rightarrow$ Thompson scattering

Basically classical radiation

Passing EM wave accelerates (wiggles) electron, accelerated electron radiates new em wave

Power scattered $P = \sigma_T \, c \, U$

σ_T: Thompson cross section

U $\rightarrow$ energy density (J/m^3) of incident radiation

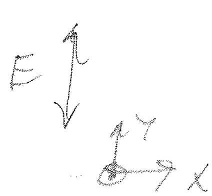
Cross section: effective area for interaction of some sort.

Related to probability of interaction happening. Large cross section = more probable

$$\sigma_T = \frac{8\pi}{3} r_e^2 = 6.6525 \times 10^{-29} \, m^2$$

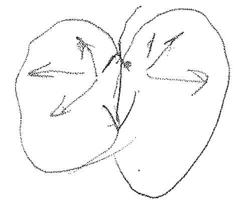
$r_e \equiv$ classical radius of electron
$= 2.818 \times 10^{-15} \, m$

Thompson scattered em radiation is due to oscillating dipole. Radiation pattern of osc. dipole



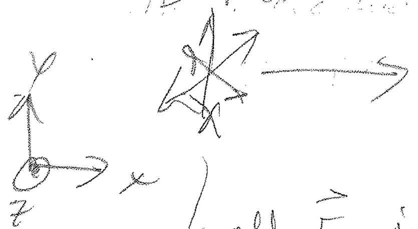
Since light is transverse em wave ($\vec{E}, \vec{B} \perp \vec{k}$) can there be radiated wave $\parallel y$?

4.4 Radiation pattern of osc. dipole:



$$\frac{dP}{d\Omega} = \frac{3}{8\pi}\sigma_T U \sin^2\theta$$

angle relative to dipole

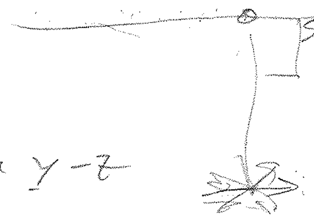


$\vec{E}$ incident

unpolarize incident light

scatterer

all $\vec{E}$s in y-z plane



must be transverse so to propagate this way we have these possible $\vec{E}$s → all in x z plane.

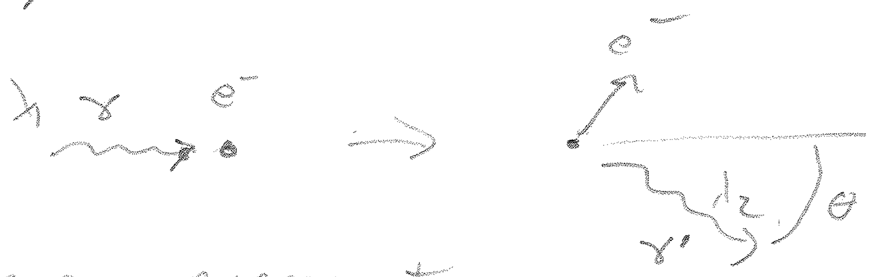
only component of incident light that is in common w. scattered is $\parallel z$

so scattered light (@90°) is linearly polarized $\perp$ to plane containing inc. + scattered $\vec{k}$.

5.1 Compton scattering.

Thompson $h\nu \ll m_e c^2$

Compton $h\nu \gtrsim m_e c^2$

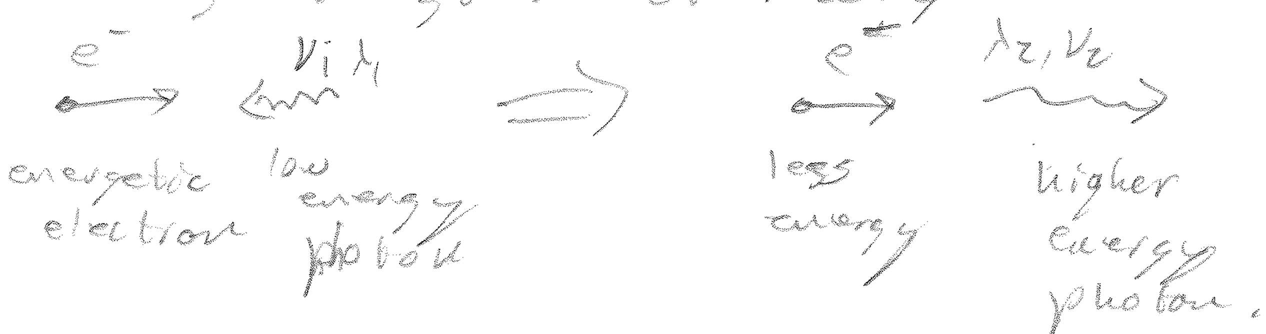


conserve energy + momentum relativistically

$$\Rightarrow \lambda_2 - \lambda_1 = \underbrace{\frac{h}{m_e c}}_{\equiv \lambda_C}(1-\cos\theta)$$

as shown electron gains energy so $E_2 < E_1$

but things can go the other way



"inverse compton scattering"

if $E_e \gg m_e c^2$ (extreme relativistic limit)
and $\gamma h\nu_1 \ll m_e c^2$

$\nu_2 \approx \gamma^2 \nu_1$

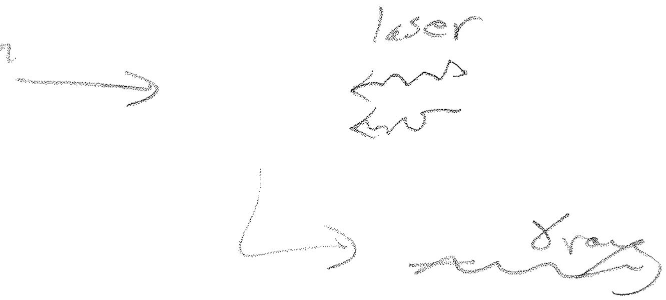
and scattering cross section $= \gamma^2 \sigma_T$
(Thompson)

if $\gamma h\nu_1 \gg m_e c^2$

$\nu_2 = \gamma m_e c^2 / h$

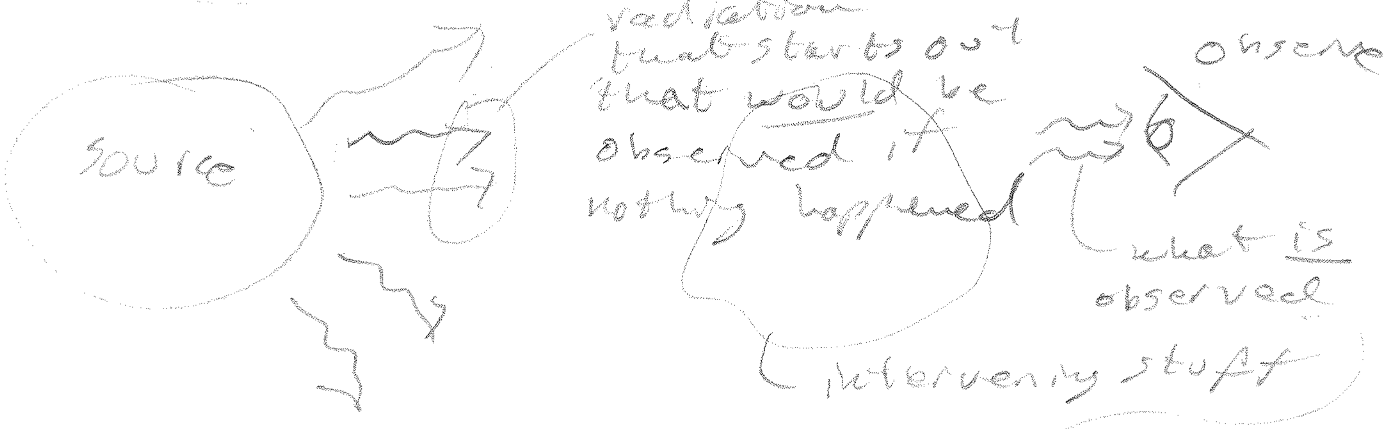
5.2 Inverse compton used to make high energy photons for nuclear physics research

H.E. electron beam



Radiation transport:

what gets from A to B?
depends on situation, largely on what's in between + what energy we have.



= original
− removed
+ added

α_ν = absorbtion coeff per length

$\alpha_\nu dx$ fraction absorbed in thin layer

attenuation if α_ν = const.

$$I_{\nu\,abs}(x) = I_\nu(0)\, e^{-\alpha_\nu x}$$

If α_ν varies w. position

$$\tau_\nu = \underbrace{\int_0^L \alpha_\nu dx}_{\text{add up over line of sight through absorber}} \equiv \text{optical thickness}$$

$$I_{\nu\,abs}(L) = I_\nu(0)\, e^{-\tau_\nu}$$

5.3 <u>But</u> cloud or whatever could <u>emit</u> as well

If an object is in thermal equilibrium
(steady state, temp same everywhere + const)
then : absorption = emission
+ can write $\varepsilon_\nu = \alpha_\nu \underbrace{B_\nu(T)}$

emission coeff B.B. rad. $\varepsilon_\nu = \frac{W}{m^3\text{-Hz-sr}}$

and then what reaches observer after L of material

$$I_\nu(L) = I_\nu(0)e^{-\tau_\nu} + \frac{\varepsilon_\nu}{\alpha_\nu}[1 - e^{-\tau_\nu}]$$

Note: optical depth τ_ν is how many factors of e by which incident light (radiation) is attenuated $\frac{1}{e} \approx \frac{1}{3}$ $\frac{1}{e^2} \approx \frac{1}{10}$ etc.
so optical depths of $\gg 1$ are <u>opaque</u>

useful approxs:

$$1 - e^{-x} \approx x \text{ if } x \ll 1$$

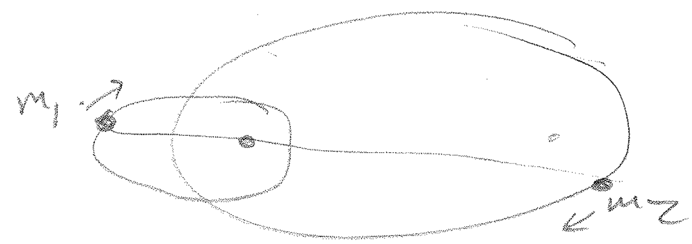
$$\approx 1 \text{ if } x \gg 1$$

If only emitting Brightness $= \frac{\varepsilon_\nu}{\alpha_\nu}[1 - e^{-\tau_\nu}]$

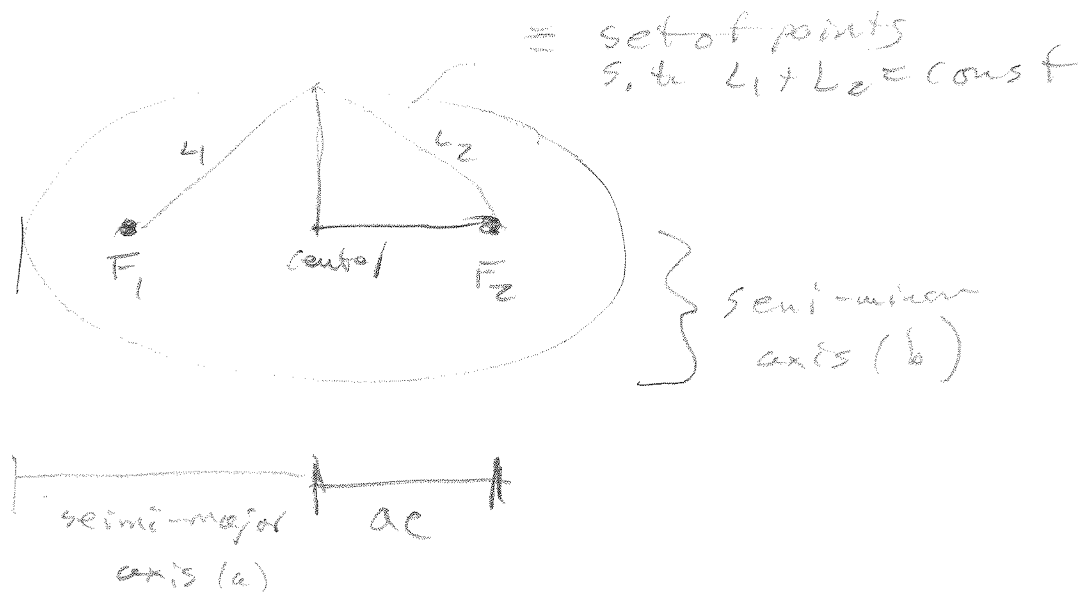
$$= B_\nu(T)[1 - e^{-\tau_\nu}]$$

5.4 Keplers Laws:

KI 2 objects orbiting each other follow elliptical paths with one focus of ellipse @ c.m.



Ellipse properties



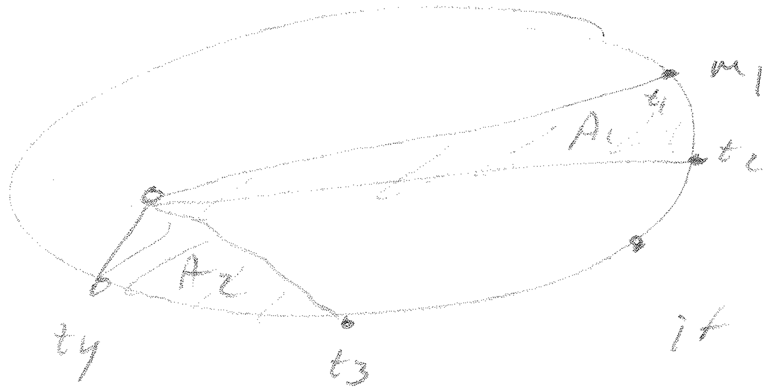
e ≡ eccentricity

e = 0 → circle

e = 1 → line

5.5

KII As object orbits in its ellipse, a line from obj to the Focus @ c.m. sweeps out equal area in equal times

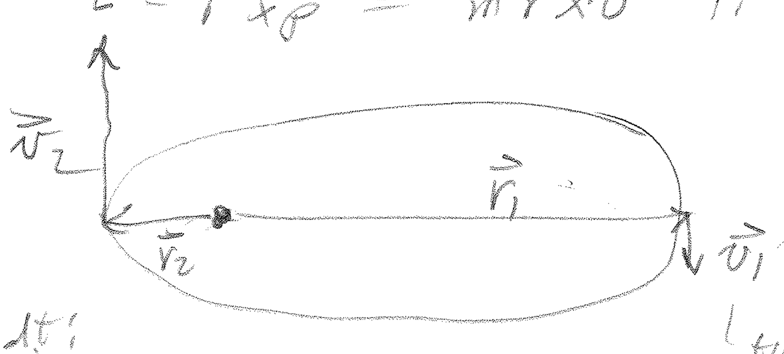


if $t_2 - t_1 = t_4 - t_3$
then $A_1 = A_2$

This is really a statement of conservation of angular momentum. For 2 objects under gravity, can they exert torques on each other around c.m.? So $\vec{L}$ around c.m. must be const. (for each as well as both).

How can we write $\vec{L}$ for orbiting obj?

$\vec{L} = \vec{r} \times \vec{p} = m\vec{r} \times \vec{v}$ if m = const



$\vec{L}_1 = \vec{L}_0$

$\cancel{m} r_1 v_1 = \cancel{m} r_2 v_2$ (since I chose $\vec{r}_1 \perp \vec{v}_1$)

$r_1 v_1 = r_2 v_2$

In same dt:

$\frac{dA_1}{dt} = \frac{1}{2} r_1 \frac{dr_1}{dt} = \frac{1}{2} r_1 v_1$

$\frac{1}{2} r_1 v_1 = \frac{1}{2} r_2 v_2$

$\frac{dA_2}{dt} = \frac{1}{2} r_2 \frac{dr_2}{dt} = \frac{1}{2} r_2 v_2$

harder for general case, but true also

$\frac{dA_1}{dt} = \frac{dA_2}{dt}$

5.6 K III relates ~~size~~ of orbit to time to go around

K found for planets in our SS

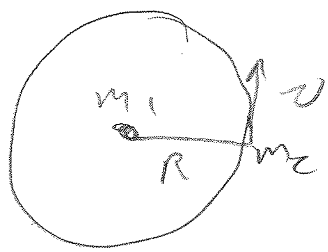
P^2 in years $= a^3$ in AU

earth $(1\,yr)^2 = (1\,AU)^3$ semi-major axis

That's a specific case of more general rule.

If $m_1 \gg m_2$ and $e=0$

m_2 moves in circle centered on m_1. speed? cons



$$\frac{m_2 v^2}{R} = \frac{G m_1 m_2}{R^2}$$

$$v = \frac{2\pi R}{T}$$

$$\frac{\cancel{m_2}}{\cancel{R}}\left(\frac{2\pi R}{T}\right)^2 = \frac{G m_1 \cancel{m_2}}{R^{\cancel{2}}}$$

$$4\pi^2 R^3 = G M_1 T^2$$

$$T^2 = \frac{4\pi^2}{G M_1} R^3$$

$$T^2 = \frac{4\pi^2}{G M_1} a^3$$

more complete:

$$T^2 = \frac{4\pi^2}{G(m_1+m_2)} a^3$$

times?

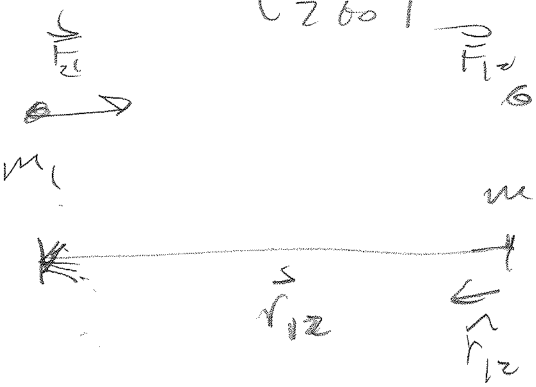
4.1 Newtonian Gravity

$$\vec{F}_{12} = \frac{G m_1 m_2}{r_{12}^2}\,\hat{r}_{12}$$

$\vec{F}_{12}$: 1 on 2 $\qquad$ r_{12}^2: 2 on 1 $\qquad$ $\hat{r}_{12}$: 2 to 1

$$G = 6.67\times10^{-11}\ \frac{N m^2}{kg^2}$$

$\vec{F}_{12} = -\vec{F}_{21}$



shown for point masses. for uniform spheres it is center to center. If non uniform/non spherical harder.

Grav potential energy:

$$U = -\frac{G m_1 m_2}{r_{12}}$$

scalar; Joules

$U \to 0$ as $r_{12} \to 0$

(recall we can choose zero of potential energy anywhere we want)

If I launch something from earth's surface, straight up for simplicity, + neglect air resistance, it will fall back down unless speed ≥ some min value → escape speed

To just not come back final $KE \to 0$ + $U \to 0$

so $r_{12} \to$? (∞)

Start at $r_{12} = R_{earth}$ $\qquad$ $\Delta U + \Delta KE = 0$ cons. of mech. E

$$0 - \left(-\frac{G M_e m}{R_e}\right) + 0 - \frac{1}{2} m v_{esc}^2 \rightarrow v_{esc} = \left(\frac{2 G M_e}{R_e}\right)^{\frac{1}{2}}$$

6.2 $v_{esc} = \left(\frac{2GM}{R}\right)^{1/2}$ general: starting R far from M.

You may recall in 218 the electrostatic P.E due to assembling a uniform sphere of charge. Similarly, an extended massive object represents Grav. P.E. ⇔ work done in bringing the bits together w/o acceleration ($U < 0$). If spherically symmetric, build w/ shells. At some point in process

$dm = 4\pi r^2 dr \rho$

mass assembled so far $m(r) = \frac{4}{3}\pi r^3 \rho$ if ρ = uniform

$$dU = -\frac{G\,m(r)\,dm}{r}$$

Nota Bene: Its not hardly ever! why?

$$U = \int_0^R -\frac{G\,m(r)\,dm}{r} = -G\frac{16\pi^2}{3}\rho^2 \int_0^R \frac{r^3 r^2}{r} dr$$

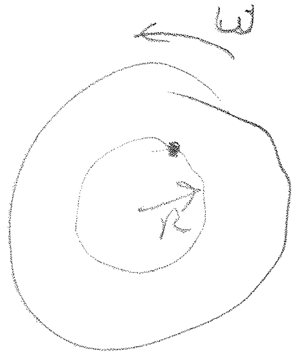
$$= -G\frac{16\pi^2}{3}\rho^2\frac{R^5}{5}$$ but if ρ = uniform $= \frac{M}{\frac{4}{3}\pi R^3}$

$$= -G\frac{16\pi^2}{3}\frac{R^5}{5}\left(\frac{9M^2}{16\pi^2 R^6}\right) = \frac{-3GM^2}{5R}$$

7.1 Tides;

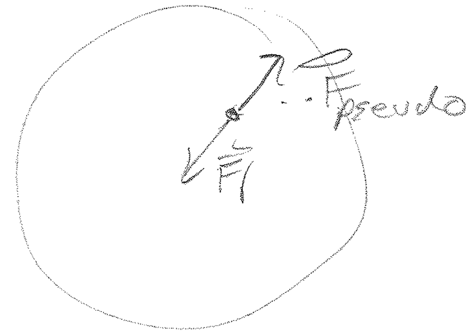
Easiest sense is from non-inertial frame:

merry-go-round



or

transfer to coords fixed on merry go round



radial equations of motion (coords fixed on ground)

$F_r = m a_r$

e.g. friction

$\frac{mv^2}{R}$

$= m\omega^2 R$

w.r.t. m.g.r., $a_r = 0$ so

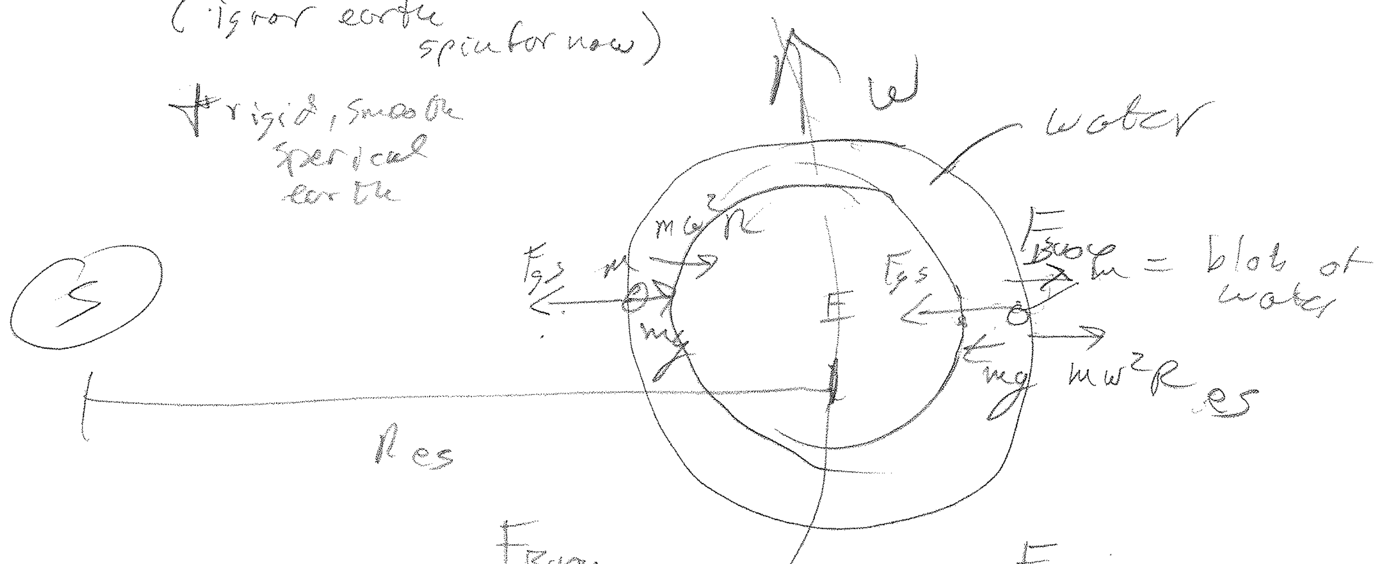
$F_r - F_{pseudo} = 0$

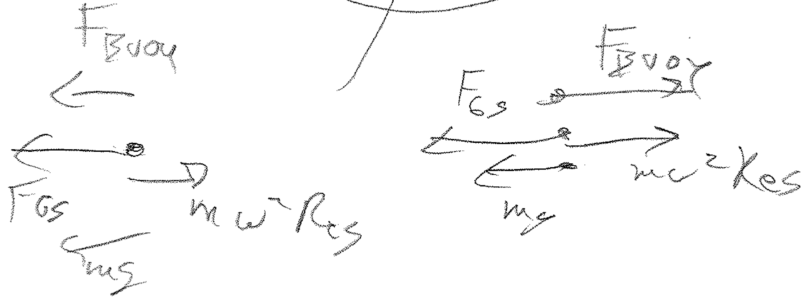
$m\omega^2 R$

---

earth orbits sun (ignore earth spin for now)

+ rigid, smooth spherical earth





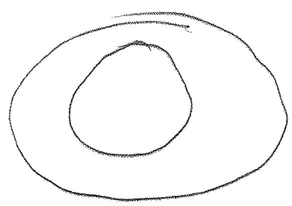
7.2 So in addition to the forces we "expect" ($mg + \vec{F}_{Buoy}$) there's a tidal force

$$F_{tidtot} = \underbrace{F_{gs}}_{\frac{G M_s m}{r_{ms}^2}} - \underbrace{m\omega^2 r_{es}}_{\text{orbital accel of earth}} = F_{gs} - \frac{G M_s m}{r_{es}^2}$$

r_{ms} ∓ r_{ms} from ~~R_e~~ R_{es} to $R_e + R_{es}$ ~~water~~ in closest/farthest (nearside/farside) ~~an~~ $= R_{es}$ in between.

So F_{tidal} is maximum away from center of earth on both near + far side + 0 in between!

Pulls water:



Same for moon only larger effect.

Reality: - continents → messy

- earth spins

↳ 1) 2 high + 2 low tides/day

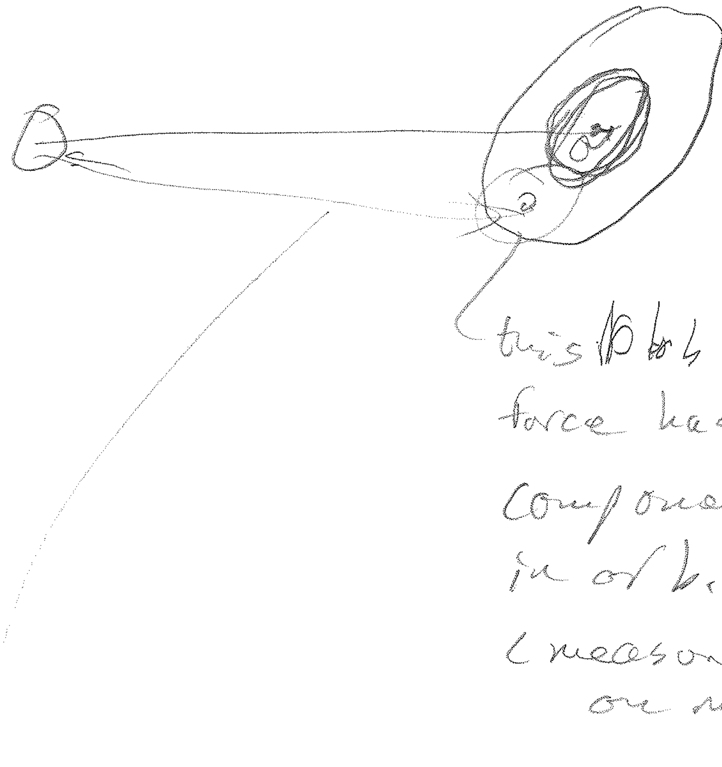
→ 2)

earth rot. + friction drags tidal bulge ahead of e-moon line

- earth not perfectly rigid

→ land tides

7.3 Consequences:



nonspherical part of land + water

this blob pulls on moon + force has some tangential component → speeds moon up in orbit → R_{cm} ↑ by 3.82 cm/yr (measure by retroreflectors on moon ...) Focus 3.4

→ corresponding force on blob (land + water) torque the earth → spin slows. Day lengthens by ≈ 2 ms / century. See Focus 3.2

Land masses + rotation funnels water in odd ways so you can get tides @ shores considerably larger than the 1-2 ft. you expect on average (Bay of Fundy ≈ 10's of feet)

7.4. OK, so what's the long term prognosis
earth slows
but earth rot. responsible for ~~tides~~ tides leading
moon which is why earth is slowing.
??
when will it stop?

moon already tidally locked spins once per orbit (same face towards us)

tidal locking

spins @ same rate moon goes around
there's no torque any more!
moon is now fixed over one longitude
(may still be inclined? → another complication...)

↓ many other moons tidally locked as well.

does it stay that way? no!
we forgot the sun. If earth spinning faster
than once a year, there will be the
(weaker) drag from solar tides. This
~~Al~~so slows the earth even more
now bulge lags moon + earth ~~slows~~
slows moon which brings moon in
closer. Details are complicated
but eventually the moon will get
to close to hold together

→ Roche Limit!

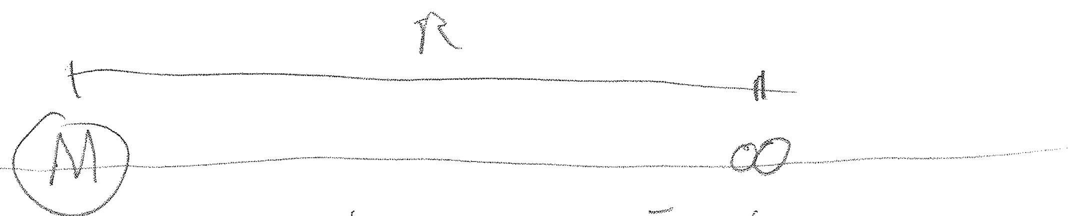
7.5 Roche Limit

Consider something held together by mutual gravity:

$m_1 = m_2$

maybe an asteroid or comet or small moon.

Consider it approaching a more massive object along a radial direction



since one is closer to M, F is larger we can ask when will the difference in forces $F_{Mm_1} - F_{Mm_2}$ be greater than mutual attraction? at that point the objects will accelerate at different rates + separate as they fall → breakup

$$\Delta F = \frac{GMm_1}{(R-r)^2} - \frac{GMm_2}{(R+r)^2} = GMm_1\left(\frac{1}{(R-r)^2} - \frac{1}{(R+r)^2}\right)$$

if $r \ll R$

$$\left(1-\frac{r}{R}\right)^{-2} \simeq 1+\frac{2r}{R}$$

$$\left(1+\frac{r}{R}\right)^{-2} \simeq 1-\frac{2r}{R}$$

$$= \frac{GMm_1}{R^2}\left(\frac{1}{\left(1-\frac{r}{R}\right)^2} - \frac{1}{\left(1+\frac{r}{R}\right)^2}\right)$$

$$= \frac{GMm_1}{R^2}\left(1+\frac{2r}{R} - 1 + \frac{2r}{R}\right)$$

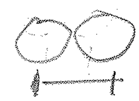
$$= \frac{GMm_1\, 4r}{R^3}$$ (give or take your model)

7.6 so @ limit

$$\Delta F = 4\frac{\cancel{G} M m_1 r}{R_{lim}^3} = \frac{\cancel{G} m_1^2}{4r^2}$$

$$R_{lim}^3 = 16\frac{M}{m_1} r^3 \Rightarrow R_{lim} = r\left(\frac{16M}{m_1}\right)^{1/3}$$

Book's model is a little different



$r_{book} = 2r$

$$r = \frac{r_{book}}{2} \text{ so } R_{lim} = \frac{r_{book}}{2}\left(\frac{16M}{m_1}\right)^{1/3} = r_{book}\left(\frac{16}{8}\frac{M}{m_1}\right)^{1/3} = r_{book}\left(\frac{2M}{m_1}\right)^{1/3}$$

so "rock piles" will pull apart at this limit ≡ Roche limit

objects with some cohesive forces can get closer, but generally not much

these breakups are thought to produce material for planetary rings.

8.1 Causes of gravity

Newtons theory of Gravi. $\left(\vec{F}_{12} = G \frac{m_1 m_2}{r_{12}^2} \hat{r}_{12} \right)$

is really a law at best.

He had no idea about <u>how</u> these masses exerted their forces, He also assumed, as was done with all classical forces-at-a-distance forces (electro statics, magnetics) that forces were felt instantaneously.

Einstein did better. w. General rel. but first...

What does the special in special rel. refer to?

The special case of inertial frames (+ flat space-time)

What was the principle?

→ same laws of physics regardless inertial frame + all frames get same speed for 'c'.

Light clock can be used to derive all main effects in S.R.

General rel. considers accelerated motion + curved space time.

Same idea: laws of physics same in any frame.

One principle is "equivalence" principle.

Consider you are in a room w. no windows you have a scale + you have your normal earth weight.

How can you tell if your room is sitting still on earth, or instead is accelerating through space @ 1g?

* needs to be uniform field really

8.2 Equivalence principle says if you are far from large masses, there is no way to tell + in such cases, S.R. applies locally.

From this you conclude that light is deflected by gravity.

↳ = Newtonian value.

But, there's more to G.R. Einstein's description of gravity is tied up with the geometry of space + time. W.d.t.i.m?

Recall set up for S.R. → regular lattice of x, y, z coords w. clocks

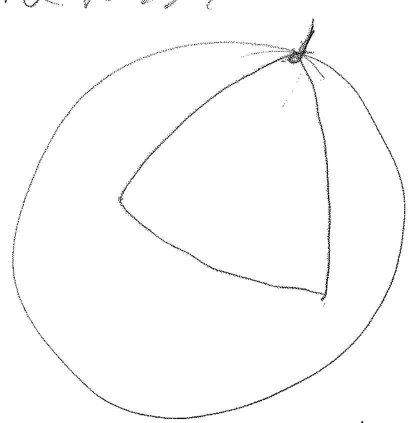
→ euclidean geometry shortest path ↔ straight line, shortest distance betw 2 points $= \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$

What is there besides euclidean? Tons.

Simplify: 2-D x-y euclidean = quad ruled graph paper, normal geometry

non-euclidean: surface of a sphere?

Weirdness? Take a walk straight for $\frac{1}{4}$ circumf



turn left 90°
repeat 2x more
what have you done?
where are you @ 3rd turn?

Is that a triangle?

Σ interior angles = ?

What if triangle smaller? → close to euclidean
if sides << R → locally flat

8.3 Einsteins model for GR is that space-time is a single 4-dimensional space and that the space is curved by the presence of energy/mass. Objects move through curved space time along "geodesics" which are the curved spacetime generalization of "a free particle moves in a straight line".

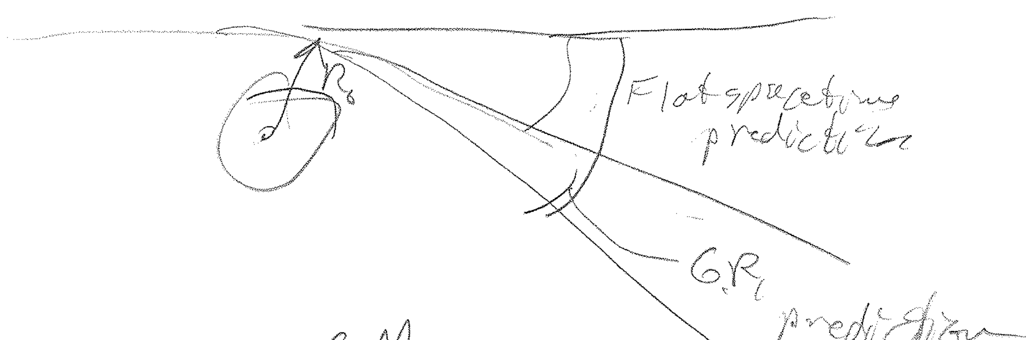
So G.R. is odd it doesn't explain the force of gravity, it replaces it with "objects moving freely in curved space time follow geodesics".

Wikipedia: "path of orbiting planet is projection of a 4D geodesic onto 3 spatial coords."

Equiv. principle requires grav mass = inertial mass

All kinds of fun

Consequence of curved space is extra deflection of light (2x) if passing a massive object.

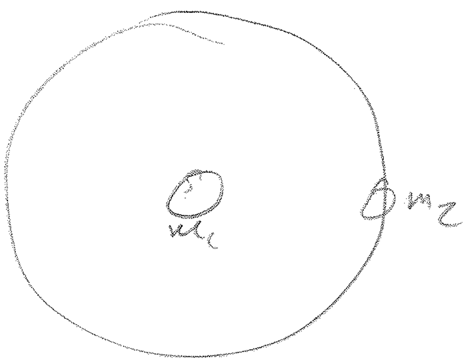


$$\Theta_{GR} = 2(1+\gamma)\frac{GM}{R_0 c^2} \text{ radians}$$

γ ↳ parameter of spacetime = 1 in Einsteins GR
= 0 in Newton.

8.4 "Gravitational lensing"

9.1 Orbital speed: circular
Reminder



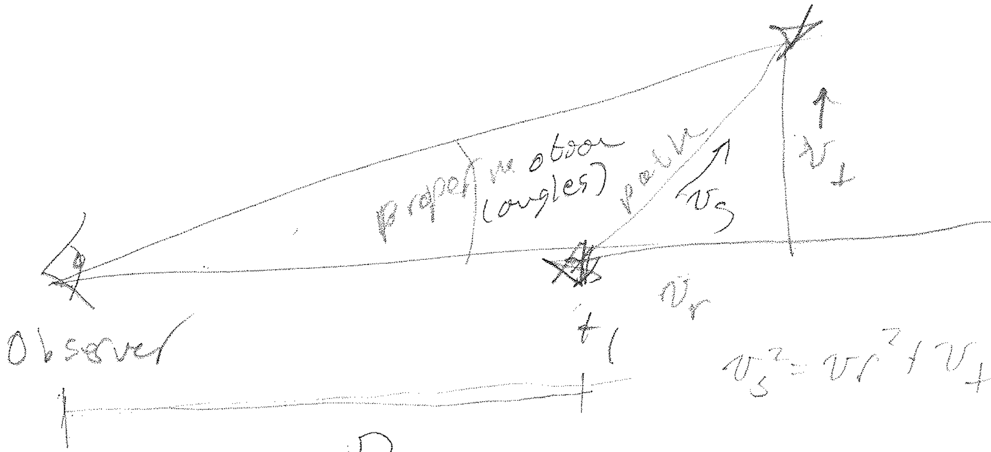
grav force provides
centripetal accel.

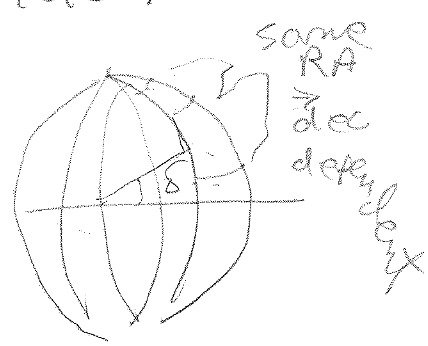
$$\frac{G m_1 \cancel{m_2}}{r^{\cancel{2}}} = \frac{\cancel{m_2} v^2}{\cancel{r}}$$

$$v = \sqrt{\frac{G m_1}{r}}$$

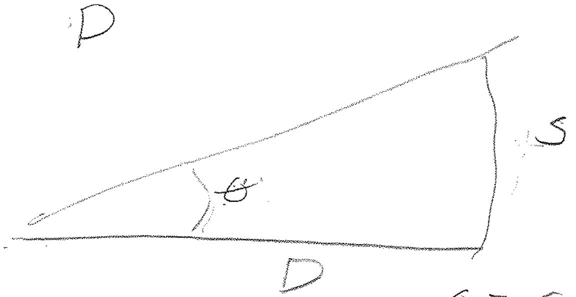
Stellar motion:

$t_2 = 1$ yr later.

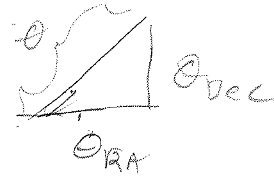




$$v_s^2 = v_r^2 + v_\perp^2$$



on the sky.
θ will have same RA + dec



$\cancel{\theta = \frac{s}{D}}$ $\quad \frac{\Delta\theta}{\Delta t} = \frac{1}{D}\frac{\Delta s}{\Delta t}$

$$= \frac{v_\perp}{D}$$

so if we measure angular motion + know D, we get $v_\perp$

large angular motion is ≈ 1000 mas/year (milli arc sec)
(Table 9.2)

9a2 Radial motion;

→ Doppler shift.

we know there are characteristic spectral lines in stellar spectra.

If emitter is moving toward/away, it will suffer doppler shift

$$f' = \sqrt{\frac{1 \pm \beta}{1 \mp \beta}}\, f_0 \qquad \beta = v/c$$

$$c = f_0 \lambda_0 \qquad \lambda_0 = \sqrt{\frac{1 \pm \beta}{1 \mp \beta}}\, \lambda'$$

or if $v \ll c$

$$\frac{\lambda_{observed} - \lambda_{emitted}}{\lambda_{emitted}} \equiv z = \frac{v_r}{c}$$

If motion is away $\lambda_{observed}$ = longer
"red shift"

If motion is toward $\lambda_{observed}$ = shorter
"blue shift"

9.3 What if we have a lot of objects orbiting each other? Globular star cluster.

Virial theorem: For a bound grav. system in $\approx$ equilibrium there's a relation between KE + PE $\langle KE \rangle = \frac{1}{2} PE$

$$\frac{1}{2} M \langle v \rangle^2 = \frac{G M_c M_s}{2 R_c}$$

N_s = # stars
$M_c = N_s M_s$
R_c = typ / avg

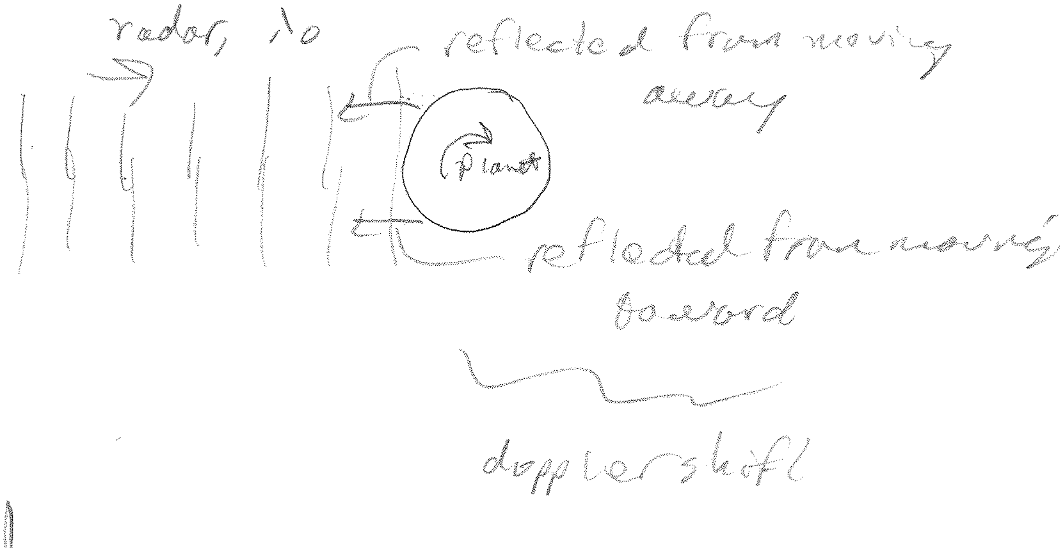
avg over time

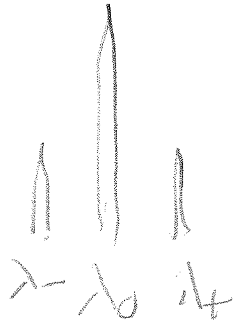
Rotation

Solar system: In some cases we can make visual observation of rotation rates, mars, jupiter saturn, etc

careful not solid!

Venus - cloud covered, mercury too close to sun.
→ Radar from Arecibo

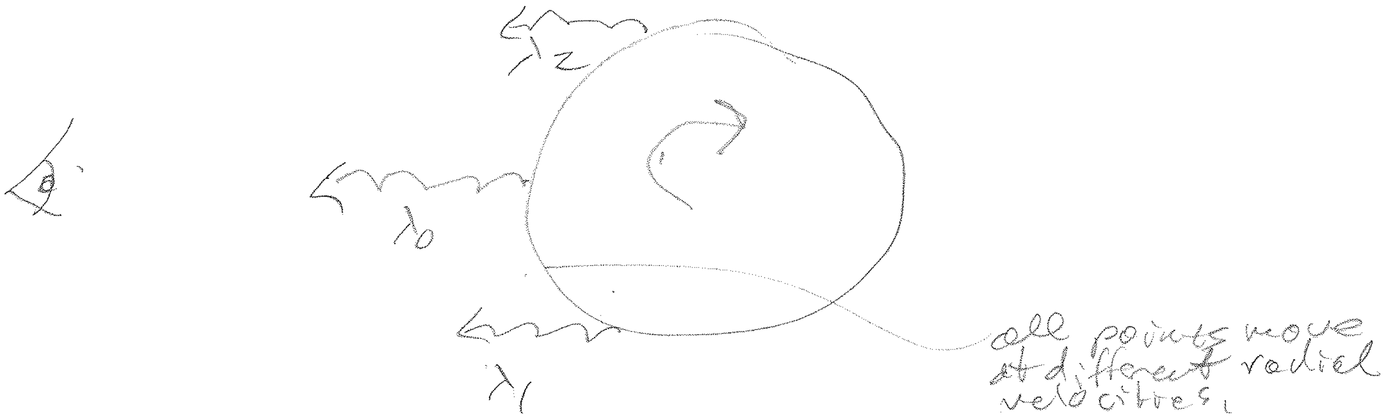


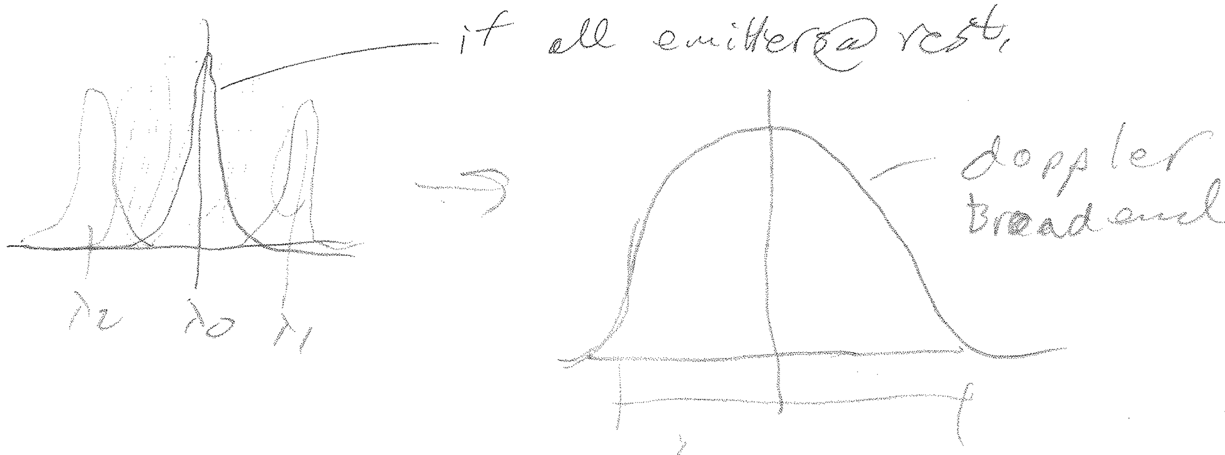


9.4

What if we can't bounce radar? (= dist star)

Stars emit their own light





from width can get estimate of velocity

with estimate of size we can get rotation rate.

10.1 Ch. 5

Everyone's had modern, so we know electrons "orbit" (QM) nuclei made of protons + neutrons

Relative mass of e + p? p + n?

$M_p \approx m_n = 1836\, M_e$

relative size of atom/nucleus?

$1\mathring{A} = 10^{-10}$ m / 10^{-15} m

fm

femtometer

aka 1 fermi

"isolated atoms mostly empty space"

$M_e = 9.1 \times 10^{-31}$ kg $= 0.511$ MeV/c^2

$m_p = 1.673 \times 10^{-27}$ kg $= 938.27$ MeV/c^2

$m_n = 1.675$ kg $= 939.57$ MeV/c^2

Temperature is a measure of average kinetic energy of particles of system

- KE relative to c.m. (bulk motion + internal

- only meaningful if particles exchanging energy freely → equilibrium.

(classical)

Particles in a dilute gas in thermal equilibrium have characteristic distribution of speeds

→ Boltzman distribution.

$$N(v)\,dv = N_{tot}\, f(v)\, dv$$

$$f(v) = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2} v^2 \cdot e^{-\left(mv^2/2kT\right)}$$

T = absolute temp.

k = Boltzmann Const $= 1.38 \times 10^{-23}$ J/K

10.4

Gas Const
8.31 J/mol Kelvin

most probable → how to find?

$f_{max} \rightarrow \frac{df}{dV} = 0$ solve for $V = \sqrt{\frac{2kT}{m}} = \sqrt{\frac{2RT}{M}}$

M molar mass in kg

mean (avg) speed? $\langle V \rangle = \dfrac{\int f(v)\, v\, dV}{\int f(v)\, dV \equiv 1}$

$f(v)$ = normalize probability dist

$= \sqrt{\frac{8kT}{\pi m}} = \sqrt{\frac{8RT}{\pi M}}$

$v_{rms} = \sqrt{\langle v^2 \rangle} = \sqrt{\frac{3kT}{m}}$

since $\langle KE \rangle = \frac{1}{2} m \langle v^2 \rangle = \frac{1}{2} m \frac{3kT}{m} = \frac{3}{2} kT$

How do planets keep atmospheres?

Boltzmann dist $v \rightarrow \infty > v_{escape}$

Matter of degree: how <u>many</u> molecules are above v_{esc}? How do I determine that?

10.3

$PV = N_{tot} kT$ $\qquad NkT$

$P = n kT$

n: # / volume

$n = \dfrac{\rho}{\bar{m}} = \left(\dfrac{\text{mass}}{\text{volume}}\right) \dfrac{1}{\left(\text{avg mass of stuff in that volume}\right)}$

$= \dfrac{\rho}{\mu m_H}$

μ → mean molecular mass rel. to. H.

general $\bar{m} = \dfrac{N_1 m_1 + N_2 m_2 + N_3 m_3 \ldots}{N_1 + N_2 + \ldots}$

m_i = masses of constituent i
(could include electrons if ionized).

what if ionized?

H: $\bar{m}_H = \dfrac{m_p + m_e}{2} \simeq \dfrac{m_H}{2}$

He $\bar{m}_{He} = \dfrac{2m_e + 4m_H}{3} \simeq \dfrac{4}{3} m_H$

A $\bar{m}_A = \dfrac{Zm_e + Am_H}{Z+1} \simeq \dfrac{2Zm_H}{Z+1} \simeq 2m_H$

$A \simeq 2Z$

10.4 For this atmosphere (or general

$$P_{ressur}(r) = P_{ressur}(R)\, e^{-\frac{(r-R)}{H}}$$

$H =$ scale height $= \frac{kT}{\bar{m} g}$

if $r-R \ll R_p$ + $R = R_p$

$$P(z) = P_0\, e^{-z/H}$$

~~What~~ if object is all gas

10.0 Sound is a compressional wave (pressure/density) in matter - we usually think of it in gas but liquids + solids also support sound waves.

we're used to

$$P(x) = P(0)\sin(kx-\omega t)$$

more general (Euler's relation)

$$P(x) = P(0)\, e^{i(kx-\omega t)}$$

$$\omega^2 = \left(\frac{2\pi}{\lambda}\right)^2 \left(\frac{\partial P}{\partial \rho}\right)$$

for ideal gas we can figure the $\frac{\partial P}{\partial \rho}$

$$c_s = \left(\frac{\gamma k T_0}{\bar{m}}\right)^{1/2}$$

speed of sound

$$\gamma = \frac{c_p}{c_v}$$

$\gamma = 5/3$ monoatomic

$= 7/5$ for diatomic

T_0 = pressure of undisturbed gas

11.0 Sound waves (pressure/density)

in matter - solids/liquids/gases

we're most familiar w. gases

travels in +x dir

$$P(x) = P_0 \sin(kx - \omega t)$$

more general

$$P(x) = P_0 e^{i(kx-\omega t)}$$

$$\omega^2 = \left(\frac{2\pi}{\lambda}\right)^2 \left(\frac{\partial P}{\partial \rho}\right)$$

for ideal gas we know $\frac{\partial P}{\partial \rho}$

so

$$c_s = \left(\frac{\gamma k T_0}{\bar{m}}\right)^{1/2} \qquad \gamma = \frac{c_p}{c_v} = 5/3 \text{ monoatomic}$$

$$= 7/5 \text{ diatomic}$$

$\vdots$

c_s speed of sound

T_0 = undisturbed gas temp.

11.1 Plasma

Thermodynamic processes in general depend on the ratio of a relevant energy to the thermal energy.

Thermal energy $\propto kT$

$$k = 1.38 \times 10^{-23}\ \text{J/K}$$

$$k = 8.61 \times 10^{-5}\ \text{eV/K}$$

or $$\frac{1}{k} = 11{,}600\ \text{K/eV}$$

So thermal energy scale is on the order of 1 eV @ ≈ 11,600 K.

Typical energy to remove outermost electron of a typical atom is on the order of eV

So where temperatures are in this range, thermal collisions stand a good chance of removing an electron.

→ Plasma is what we call this material where thermal equilibrium keeps it ionized

Degree of ionization ≠ 100% typically

→ inner electrons more tightly bound

11.1 For gas we've seen $\langle KE \rangle = \frac{3}{2} kT$

In general in thermal equilibrium, the energy scale that is relevant is kT

$k = 1.38 \times 10^{-23}$ J/K but for atomic processes, energy scale is eV

$k = 8.61 \times 10^{-5}$ eV/K

or $\frac{1}{k} = \frac{1}{11,600}$ K/eV

Ionization.

ple

11.2 a system of + & − charges such as a plasma can have new dynamics we haven't seen before.

For example there's a collective mode of motion of + against minus: displace all electrons one way there's an electrical restoring force & the system oscillates at the "plasma frequency"

$$\nu_p = \left[\frac{e^2 N_e}{4\pi^2 \varepsilon_0 m_e}\right]^{1/2} \doteq 8.98\, N_e^{1/2}\ \mathrm{Hz}$$

density of electrons in m^{-3}

11.2 Plasma

Plasma freq.

$$\nu_p = \left[\frac{e^2 N_e}{4\pi^2 \varepsilon_0 m_e}\right]^{1/2} = 8.98\, N_e^{1/2} \text{ Hz}$$

\# density of e

11.3 Because plasmas have free charges they interact strongly with magnetic fields. Coupled ^propagating^ disturbance of B & particles in plasma give new waves → Alfven waves

11/4 Identifying distant materials from their light

- Spectroscopy

Emission — p 220

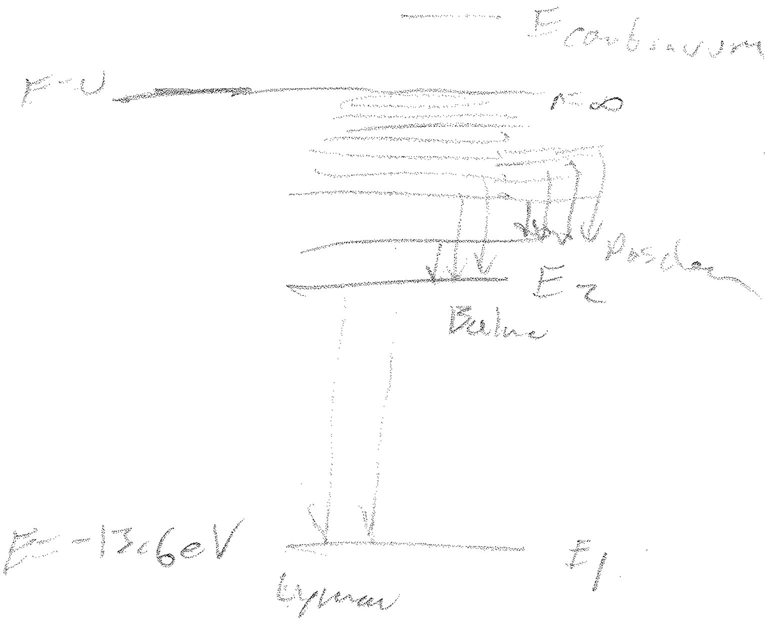
Absorbtion?

how's that work? show solar spectrum in text

Solar abundances

Sun: 92.1% H	by #	71.54% H	by mass
7.8% He		27.03% He	
0.1% else		1.42% else	

Bohr model of hydrogen spectrum



$$E_n = -\frac{m e^4}{8 \varepsilon_0^2 h^2 n^2} \qquad n = 1, 2, \ldots$$

$$h\nu = \Delta E$$

$$\frac{1}{\lambda_{mn}} \approx R_\infty \left(\frac{1}{m^2} - \frac{1}{n^2} \right)$$

$$R_\infty = 1.097 \times 10^7 / m.$$