1 Differentiation

$\frac{d}{dx}\sin x = \cos x$

$\frac{d}{dx}\cos x = -\sin x$

$\frac{d}{dx}\tan x = \sec^2 x$

$\frac{d}{dx}\sec x = \sec x \tan x$

$\frac{d}{dx}\csc x = -\csc x \cot x$

$\frac{d}{dx}\cot x = -\csc^2 x$

2 Formulas

$\sin 2x =$

$\cos 2x =$

$\sin^2 x + \cos^2 x = 1$

Pythagorean Identity (tan): $\tan^2 x + 1 = \sec^2 x$

Pythagorean Identity (cot): $\sec^2 x - 1 = \tan^2 x$

3 Theorem

Fundamental Theorem of Calculus (Part 1):
If f is continuous at [a,b], then $\int_a^x f(t)dt$ is continuous on [a,b] and differentiable at (a,b), and the derivative is
$F'(x) = \frac{d}{dx}\int_a^x f(t)dt = f(x)$

Mean Value Theorem:
If f is continous at [a,b], differentiable at (a,b), then there must be a $c \in (a,b)$ which

$f'(c) = \frac{f(b)-f(a)}{b-a}$

Intermediate Value Theorem:
If f is proved continuous as [a,b], then let M be a number which lies between f(a) and f(b), there will be a number c $\in$ [a,b] such that $f(c) = M$.

4 Laplace Transform

$\frac{1}{(s-a)(s-b)} = \frac{A}{s-a} + \frac{B}{s-b}$

$\frac{1}{(s-a)^3} = \frac{A}{s-a} + \frac{B}{(s-a)^2} + \frac{C}{(s-a)^3}$

$\frac{1}{(s-a)^2+b^2} = \frac{A(s-a)}{(s-a)^2+b^2} + \frac{B(b)}{(s-a)^2+b^2}$

Convolution: $f * g = \int_0^t f(u)g(t-u)du$

First shift:

Second shift:

5 Areas

Polar Equation: $r = \sin\theta$

Volume of Solid:

Area of Polar Equation: $A = \int_a^b \frac{1}{2} r^2 d\theta$

6 ODE

1. Solve the following ODE using D-operator.

$$y'' + 2y' + y = e^{-x}$$

Solution:
Let $y = e^{rx}$,

$$\begin{aligned} r^2 + 2r + 1 &= 0 \\ (r+1)^2 &= 0 \end{aligned}$$

Therefore,

$$y = C_1 e^{-x} + C_2 x e^{-x}$$

Write $D = \frac{d}{dx}$:

$$\begin{aligned} D^2 y + 2Dy + y &= e^{-x} \\ (D^2 + 2D + 1)y &= e^{-x} \end{aligned}$$

Let $f = e^{-x}$.

$$\begin{aligned} Df &= -e^{-x} \\ D^2 f &= e^{-x} \\ D^2 f - f &= 0 \\ (D^2 - 1)f &= 0 \end{aligned}$$

Back to the equation,

$$\begin{aligned} (D^2 + 2D + 1)(D^2 - 1)y &= 0 \\ (D+1)^2(D+1)(D-1)y &= 0 \end{aligned}$$

2. Solve the following Cauchy-Euler equation:

$$x^2 y'' + 3xy' + 6y = 0$$

3. Find the Laplace Transform of

$$t \sin bt$$