Contents

Contents v

1 Introduction 1

2 Finite Markov Chains 5
2.1 A few examples 5
2.2 Transition probability matrix 10
2.3 Long-run properties 14
2.4 Classification of states 18
2.5 Periodicity of a class 24
2.6 Regular Markov chains 31
2.7 Absorption of transient states 33
2.8 Reversibility 51
2.9 Gambler's ruin problem 53
2.A Inverting Matrices 60
2.B Solving Difference Equations 62

3 Branching Processes 75
3.1 Introduction and prerequisites 75
3.2 Generations of offsprings 78
3.3 Ultimate extinction 82

4 Renewal Theory 93
4.1 Introduction and prerequisites 93
4.2 Pattern generation 94
4.3 Two patterns competing 105

5 Poisson Process 113
5.1 Basics 113
5.2 Various modifications 117

6 Birth and Death Processes I 131
6.1 Basics 131
6.2 Pure-birth process 133
6.3 Pure-death process 135
6.4 Linear-growth model 136
6.5 Linear growth with immigration 140
6.6 $M/M/\infty$ queue 144
6.7 Power–supply problem 144
6.A Solving simple PDE's 146

7 Birth and Death Processes II 157
7.1 Constructing Stationary Distribution 157
7.2 Little's formulae 162
7.3 Absorption issues 163
7.4 Probability of ultimate absorption 165
7.5 Mean time till absorption 167

8 Continuous-Time Markov Chains 173
8.1 Basics 173
8.2 Long-run properties 176
8.A Functions of matrices 182

9 Brownian Motion 193
9.1 Basics 193
9.2 Case of $d = 0$ 194
9.3 Diffusion with drift 200
9.4 First-passage time 203

10 Autoregressive Models 209
10.1 Basics 209
10.2 Yule model 213
10.3 General Autoregressive model 218
10.4 General Autoregressive model $AR(m)$ 221
10.A Normal Distribution and Partial Correlation 225

A Basic Probability Review 237
A.1 Probability 237
A.2 Common distributions 250

B Maple Programming 255

Index 265