```
#4.3 Question 8
>
> f:=x^4+x^3*y+x^3*z^2-x^2*y^2+x^2*y*z^2-x*y^3-x*y^2*z^2-y^3*z^2:
> g:=x^4+2*x^3*z^2-x^2*y^2+x^2*z^4-2*x*y^2*z^2-y^2*z^4:
> factor(f); factor(g);
                                2                 2
                          (x + z ) (x - y) (y + x)

                                2 2
                          (x + z )  (x - y) (y + x)

>
#(a)
#Using Thm 11
> GB:=Groebner[Basis]( [ t*f,(1-t)*g], plex(t,x,y,z) ):
> map(indets,GB);
                    [{x, y, z}, {x, y, z, t}, {x, y, z, t}]

#So <f> \int <g> = < GB[1] > where
> factor(GB[1]);
                                2 2                2
                          (x + z )  (x - y) (y + x)

>
#Let's check using 4.3 Prop 13 where we have that if <f> \int <g> = <h> then h=LCM(f,g)
> h:=factor(lcm(f,g));
                                  2 2                2
                         h := (x + z )  (x - y) (y + x)

>
# Rad( <f><g> ) = Rad( <fg> ) and by 4.2 Prop 12 we have that Rad( <fg> ) = < h > where
> h:=convert(f*g,sqrfree);
                                 2   2           2      2 3
                    h := (x - y)  (x  + y x + x z  + y z )

>
#(b)
>
> G:=gcd(f,g);
                              2      3    2  2    2  2
                         G := -y  x + x  - y  z  + x  z

> factor(G);
                                  2
                             (x + z ) (x - y) (y + x)

>
#(c)
>
> p:=x^2+x*y+x*z+y*z;
                                  2
                          p := x  + y x + x z + y z

> q:=x^2-x*y-x*z+y*z;
                                  2
                          q := x  - y x - x z + y z

>
> GB:=Groebner[Basis]([t*f,t*g,(t-1)*p,(t-1)*q],plex(t,x,y,z));
       3      3  2    2    2      3      2  2    3  2    2  4    2  4
GB := [x  y - x  z  + x  y z  - x y  + x y  z  - y  z  - x  z  + y  z ,

     4      3  2    2  2    2  4        2  2    2  4
    x  + 2 x  z  - x  y  + x  z  - 2 x y  z  - y  z ,

       2        2      2        2
    t y  z + t z  y - y  z - y z , t y x + t x z - y x - x z,

              2          2
    t y z + t x  - y z - x ]
```

```
> map(indets,GB);
         [{x, y, z}, {x, y, z}, {y, z, t}, {x, y, z, t}, {x, y, z, t}]

# <f,g> \intersect <p,q> = <GB[1],GB[2]> where
> GB[1];GB[2];
          3      3  2    2    2      3      2  2    3  2    2  4    2  4
        x  y - x  z  + x  y z  - x y  + x y  z  - y  z  - x  z  + y  z

                 4      3  2    2  2    2  4        2  2    2  4
                x  + 2 x  z  - x  y  + x  z  - 2 x y  z  - y  z
```