

Assignment 1

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Assignment 2

Question 1.2.4

This can be reduced in two ways:

1. `=add(succ(pred(Zero),Zero)`
 `=succ(add(pred(Zero),Zero))`
 `=succ(pred(add(Zero,Zero)))`
 `=succ(pred(Zero))`
 `=Zero`
2. `=add(succ(pred(Zero)),Zero)`
 `=add(Zero,Zero)_`
 `=Zero`

Question 1.3.2

Suppose f and g are strict then we have that $f\perp = \perp$ and $g\perp = \perp$,

$$\begin{aligned} &\Rightarrow h\perp = f(g\perp) = f\perp = \perp \\ &\Rightarrow h\perp = \perp \\ &\Rightarrow h \text{ is strict} \end{aligned}$$

Question 1.4.3

```
mylog :: Float -> Float -> Float
mylog x b = log(x)/log(b)
```

Question 1.4.7

```
uncurry :: (a->b->t) -> ((a,b)->t)
uncurry g (x,y) = g x y
```

Question 1.6.2

Defining $\text{swap}(x,y) = (y,x)$ we verify:

```
flip(curry f) x y = (curry f) y x = f(y,x)
```

```
curry(f.swap) x y = (f.swap)(x,y) = f(swap(x,y))=f(y,x)
```

Question 2.4.1

```
=cross(f,g).cross(h,k)
=cross(f,g).pair(h.fst,k.snd)      (by defn. of cross)
=pair(f.h.fst,g.k.snd)            (by property 3)
=cross(f.h,g.k)                   (by defn. of pair)
```

Question 2.4.3

```
age (d1,m1,y1) (d2,m2,y2)
  | m1>m2                = y1-y2
  | (d1>=d2) && (m1==m2) = y1-y2
  | otherwise            = (y1-y2)-1
```

Question 2.5.1

```
case2 (Left x) = 1
case2 (Right x) = 2
```

Question 2.5.2

```
=case(f,g).plus(h,k)
=case(f,g).case(Left.h,Right.k) (by defn.)
=case(case(f,g).Left.h,case(f,g).Right.l) (by prop. 3)
=case(f.h,g.k) (by prop 1. and prop 2)
```

Question 3.2.4

Let $\Pi(p)$ be the proposition $\Pi(p) \Leftrightarrow (m + n) + p = m + (n + p) \forall m, n \in \mathbb{N}$ where $p \in \mathbb{N}$.

Base: $\Pi(0)$

Utilizing the definition $x + 0 = x$ for $p = 0$ we have

$$\text{LHS} = (m + n) + 0 = (m + n) = m + n$$

$$\text{RHS} = m + (n + 0) = m + (n) = m + n$$

So $\Pi(0)$ is true.

Assumption: We will assume $\Pi(p)$, that is, $(m + n) + p = m + (n + p) \forall m, n \in \mathbb{N}$.

Induction step: (Show $\Pi(P) \Rightarrow \Pi(\text{suc}(p))$)

$$\begin{aligned} &= (m + n) + \text{suc}(p) \\ &= \text{suc}((m + n) + p) \text{ by defn. of } + \\ &= \text{suc}(m + (n + p)) \text{ by assumption} \\ &= m + \text{suc}(n + p) \text{ by defn. of } + \\ &= m + (n + \text{succ}(p)) \end{aligned}$$

So we have that $\Pi(p) \Rightarrow (m + n) + \text{suc}(p) = m + (n + \text{succ}(p)) \Rightarrow \Pi(\text{suc}(p))$. So by induction we have that $\Pi(p) \forall p \in \mathbb{N}$ as desired.