CS 2209b - Quiz 3 - Solutions

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Question 1

(a) James blames no one besides himself.

$$\sim (\exists x)\,((\sim x = j) \bullet Bjx)$$
$$\forall x\,(Bjx \supset x = j)$$

(b) The smallest one is either red or blue.

$$(\exists x)\,(y)\,(Sxy) \supset (Rx \lor Bx)$$

(c) Someone besides Tom and Jack is cruel.

$$(\exists x)\,((\sim x = t) \bullet (\sim x = j) \bullet Cx)$$

(d) Only mike is tall.

$$\forall x\,(Tx \supset (x = m))$$
$$\sim (\exists x)\,((\sim x = m) \bullet Tx)$$

Question 2

Prove or refute the arguments using (extended) S/I rules.

(a) Prove $\sim (x)\,(\sim (\sim Qx \bullet Tx)) \lor Px$

Proof.

1	$\sim (\exists x) \sim Tx$	
2	$(\exists x) \sim (Px \lor Qx)$	
3	assume: $(x)\,(\sim (\sim Qx \bullet Tx)) \lor Px$	
4	$\therefore \quad (x)\,Tx$	Reverse: 1
5	$\therefore \quad \sim (Pa \lor Qa)$	Specify: 2
6	$\therefore \quad \sim Pa$	NOR: 5
7	$\therefore \quad \sim Qa$	NOR: 5
8	$\therefore \quad \sim (\sim Qa \bullet Ta) \lor Pa$	Specify: 3
9	$\therefore \quad \sim (\sim Qa \bullet Ta)$	DS: 8, 7
10	$\therefore \quad Ta$	Specify: 4
11	$\therefore \quad Qa$	DS: 9, 10
12	$\therefore \quad ((A \bullet C) \bullet \sim E)$	Contradiction of 3: 7, 11

$\square$

(b) Prove $((\exists x)\, Sx \bullet \sim (x) \sim (Ex \; \vee \; Fx))$

Proof.

1	$\sim (x)\,(Ex \supset\, \sim Sx)$	
2	$\therefore \quad (\exists x) \sim (Ex \supset\, \sim Sx)$	
3	assume: $\sim ((\exists x)\, Sx \bullet \sim (x) \sim (Ex \; \vee \; Fx))$	
4	assume: $\sim (\exists x)\, Sx$	
5	$\therefore \quad (x) \sim Sx$	Reverse: 4
6	$\therefore \quad \sim (Ea \supset\, \sim Sa)$	Specify: 2
7	$\therefore \quad Sa$	NIF: 6
8	$\therefore \quad \sim Sa$	Specify: 5
9	$\therefore \quad (\exists x)\, Sx$	Contradiction of 4: 7, 8
10	$\therefore \quad (x) \sim (Ex \; \vee \; Fx)$	DS: 3, 9
11	$\therefore \quad \sim (Ea \supset Sa)$	Specify: 2
12	$\therefore \quad Ea$	NIF : 11
13	$\therefore \quad \sim (Ea \; \vee \; Fa)$	Specify: 10
14	$\therefore \quad \sim Ea$	NOR: 11
15	$\therefore \quad ((\exists x)\, Sx \bullet \sim (x) \sim (Ex \; \vee \; Fx))$	Contradiction of 3: 12, 14

□

(c) Prove $(Me \lor {\sim} (\exists x)\, Mx)$

Proof.

1	$\sim (\exists x)\,(Mx \bullet Px)$		
2	$(x)\,(\sim e = x \supset Px)$		
3	$\therefore \quad (x) \sim (Mx \bullet Px)$	Reverse: 1
4	assume: $\sim (Me \lor {\sim} (\exists x)\, Mx)$	
5	$\therefore \quad \sim Me$	NOR: 4
6	$\therefore \quad (\exists x)\, Mx$	NOR: 4
7	$\therefore \quad Ma$	Specify: 7
8	$\therefore \quad \sim (Ma \bullet Pa)$	Specify: 3
9	$\therefore \quad \sim Pa$	CS: 8
10	$\therefore \quad (\sim e = a \supset Pa)$	Specify: 2
11	$\therefore \quad e = a$	MT: 10
12	$\therefore \quad Me$	Eq: 7, 11
13	$\therefore \quad (Me \lor {\sim} (\exists x)\, Mx)$	Contradiction of 4: 5, 12

$\square$

(d)

1	$(x)\,(\sim Ex \supset Us)$	
2	$\sim (Ut \supset {\sim} h = t)$	
3	$\therefore \quad Eh$	

Is *invalid.* Let the domain be $\{h, t\}$ and set $\sim Eh, Et, Ut, Uh$.

Question 3

(a) Someone is a student. Someone is not a student. $\therefore$ There is more than one being. (Use Sx for x is a student).

Proof.

1	$(\exists x)\,Sx$	
2	$(\exists x) \sim Sx$	
3	$\therefore \quad Sa$	Specify: 1
4	$\therefore \quad \sim Sb$	Specify: 2
5	assume: $\sim (\exists x)\,(\exists y) \sim x = y$	
6	$\therefore \quad \forall x \sim (\exists y) \sim x = y$	Reverse: 5
7	$\therefore \quad (x)\,(y)\,x = y$	Reverse: 6
8	$\therefore \quad a = b$	Specify: 7
9	$\therefore \quad Sb$	Sub. Equals: 8
10	$\therefore \quad (\exists x)\,(\exists y) \sim x = y$	Contradiction of 5: 4, 9

□

(b) If someone confesses, then someone goes to jail. I confess. I don't go to jail. ∴ someone besides me goes to jail. (Use Cx for x confesses, Jx for x goes to jail, i for I).

Proof.

1	$(\exists x)\, Cx \supset (\exists x)\, Jx$	
2	Ci	
3	$\sim Ji$	
4	assume: $\sim (\exists x)\,((\sim x = i) \bullet Jx)$	
5	$(x) \sim ((\sim x = i) \bullet Jx)$	Reverse: 4
6	assume: $(\exists x)\, Cx$	
7	$\therefore$ $(\exists x)\, Jx$	MP: 1
8	$\therefore$ Ja	Specify: 7
9	$\therefore$ $\sim ((\sim a = i) \bullet Ja)$	Specify: 5
10	$\therefore$ $a = i$	DS: 9
11	$\therefore$ Ji	Eq: 8, 10
12	$\therefore$ $\sim (\exists x)\, Cx$	Contradiction of 6: 3, 11
13	$\therefore$ $(x) \sim Cx$	Reverse: 12
14	$\therefore$ $\sim Ci$	Specify: 13
15	$\therefore$ $(\exists x)\,((\sim x = i) \bullet Jx)$	Contradiction of 4: 2, 14

□