Category

1. A collection of **objects**.
2. A collection or **arrows**.
3. Operations assigning to each arrow f
   (a) An object $\mathsf{dom}(f)$ called the **domain** of f, and
   (b) An object $\mathsf{cod}(f)$ called the **codomain** of f often expressed by $f : \mathsf{dom}(f) \to \mathsf{cod}(f)$
4. An associative operator $\circ$ assigning to each pair of arrows, f and g, where $\mathsf{dom}(f) = \mathsf{cod}(g)$, a **composite** arrow
$$f \circ g : \mathsf{dom}(g) \to \mathsf{cod}(f).$$
5. For each object A, an identity arrow, $id_A : A \to A$ satisfying the law that for any arrow $f : A \to B$ as
$$id_B \circ f = f = f \circ id_A.$$

HASK

In Hask, **objects are Haskell types**, so the object part of a functor, F_0, is a function from types to types. Therefore, a type constructor is suitable for use as the object function of a functor. For example

$$F_0 A = [A]$$

is suitable. In this case

$$F_1 = \mathsf{map}$$

completes the definition of a functor.

Functors

A **functor** F from category $\mathcal{C}$ to $\mathcal{D}$ is a pair of functions.

1. $F_0 : \mathcal{C}-\text{objects} \to \mathcal{D}-\text{objects}$
2. $F_1 : \mathcal{C}-\text{arrows} \to \mathcal{D}-\text{arrows}$

such that

1. If f is an arrow in $\mathcal{C}$ and $f : A \to B$ then $F_1(f) : F_0(A) \to F_0(B)$
2. $F_1(id_a) = id_{F_0(A)}$, and
3. $F_1(f \circ g) = F_1(f) \circ F_1(g)$ whenever $f \circ g$ defined.

Functors - HASK

A typical way of to define a functor in haskell is to do

Object Functor

```
data NAME type =
        Base | Constructor1 type | Consrtuctor2 type . . .
```

Function Functor

```
instance Functor Name where
        fmap f BASE = something
        fmap f (Constructor x) = Constructor (f x)
```

Small Category

Is a category where the collection of objects is a set and the collection of arrows is a set.

Cat

Is the category of small categories which has categories as objects all functors between these as arrows.

Dual

The dual of a category $\mathcal{C}$ is identical to $\mathcal{C}$ but all the arrows are reversed. This is sometimes denoted $\mathcal{C}^{op}$.