```
#Additional Question 7
>
> with(Groebner):
> with(PolynomialIdeals):
>
> I:=<x^2-y, y^4-y*z^2, x*y^3-x*z^2 >;
                          2       4      2     3      2
                 I := <x  - y, y  - y z , x y  - x z >

> IsRadical(I);
                                    true

>
> G:=Basis(I, plex(x,y,z) );
                          4      2     3      2   2
                 G := [y  - y z , x y  - x z , x  - y]

> factor(G);
                          3    2         3    2    2
                 [-y (-y  + z ), -x (-y  + z ), x  - y]

>
> P1 := Quotient(I,<x>);
                                  2       3    2
                          P1 := <x  - y, y  - z >

> P2 := Quotient(I,<y^3-z^2>);
                                P2 := <x, y>

>
> IsPrime(P1); IsPrime(P2);
                                   false

                                    true

>
#P1 gives x^2=y which subbing into y^3=z^2 gives
> f:=x^6-z^2;
                                      2    6
                               f := -z  + x

> factor(f);
                                 3       3
                              (x  - z) (x  + z)

>
> P3 := Quotient(P1,<x^3-z>);
                          2       3    2         2
                 P3 := <x  - y, y  - z , z x + y , z + x y>

> P4 := Quotient(P1,<x^3+z>);
                          2       3    2                   2
                 P4 := <x  - y, y  - z , x y - z, z x - y >

>
> IsPrime(P3); IsPrime(P4);
                                    true

                                    true

>
#So the prime decomposition should be P2 \int P3 \int P4. Check:
> P2,P3,P4;
          2       3    2         2
<x, y>, <x  - y, y  - z , z x + y , z + x y>,

      2       3    2                   2
    <x  - y, y  - z , x y - z, z x - y >
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```
> Simplify(PrimeDecomposition(I));
                 2            2             2             2
       <x, y>, <x  - y, z x + y , z + x y>, <x  - y, -z x + y , x y - z>

>
#Now we'll check irredundancy.
> IC:=IdealContainment:
>
> IC(P2,P3); IC(P3,P2);
                                     false

                                     false

> IC(P2,P4); IC(P4,P2);
                                     false

                                     false

> IC(P3,P4); IC(P4,P3);
                                     false

                                     false

>
>
#So the decompositon is irredundant.
```