Homework 2

Graded

Student

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Total Points

69.16 / 80 pts

Question 1

Problem 1 — **20** / 20 pts

1.1 Screw Axis 1 — **3.3** / 3.3 pts

- ✔ **+ 3.3 pts** Correct
- **+ 1.6 pts** Correct axis. However, non-functioning Matlab Code
- **+ 0 pts** No screw axis or working code submitted

1.2 Screw Axis 2 — **3.3** / 3.3 pts

- ✔ **+ 3.3 pts** Correct
- **+ 1.6 pts** Correct screw axis. But non functioning Matlab code
- **+ 0 pts** No screw axis or working code submitted

1.3 Screw Axis 3 — **3.4** / 3.4 pts

- ✔ **+ 3.4 pts** Correct
- **+ 0 pts** Incorrect v vector
- **+ 1.6 pts** Correct Screw axis but non functioning Matlab code
- **+ 0 pts** No screw axis or working code submitted

1.4 Home Configuration — **10** / 10 pts

- ✔ **+ 10 pts** Correct
- **+ 0 pts** Incorrect home configuration due to incorrect screw axes
- **+ 5 pts** Correct Home matrix but non functioning Matlab code
- **+ 0 pts** No matrix or working code submitted

Question 2

Problem 2 **19.16** / 20 pts

2.1 **Screw Axis 1** **1.66** / 1.66 pts

- ✔ **+ 1.66 pts** Correct
- **+ 0 pts** Not submitted
- **+ 0.83 pts** S1 is correct, however p1 is not

2.2 **Screw Axis 2** **1.66** / 1.66 pts

- ✔ **+ 1.66 pts** Correct
- **+ 0 pts** Not submitted
- **+ 0.83 pts** missing v2

2.3 **Screw Axis 3** **1.67** / 1.67 pts

- ✔ **+ 1.67 pts** Correct
- **+ 0 pts** Not submitted
- **+ 0.83 pts** S3 is correct, however p3 is not
- **+ 0.5 pts** missing v3, w3 is not correct

2.4 **Screw Axis 4** **1.67** / 1.67 pts

- ✔ **+ 1.67 pts** Correct
- **+ 0.83 pts** s4 is correct, however, p4 is not.
- **+ 0 pts** Not submitted
- **+ 0.83 pts** missing v4

2.5 **Screw Axis 5** **1.67** / 1.67 pts

- ✔ **+ 1.67 pts** Correct
- **+ 0 pts** Not submitted
- **+ 0.83 pts** S5 is correct, however, p5 is not
- **+ 0.5 pts** S5 is not correct

2.6 **Screw Axis 6** 0.83 / 1.67 pts

+ 1.67 pts Correct

✔ **+ 0.83 pts** s6 is correct, however, p6 is not

+ 0 pts Not submitted

+ 0.83 pts missing v6

p6 is the intersection between the joint axis and the origin of the space frame. you do not need to translate L1+L2(0.6) in the y-axis.

2.7 **Home Configuration** 10 / 10 pts

✔ **+ 10 pts** Correct

+ 8 pts End effector frame is not correct

+ 0 pts Not submitted

+ 5 pts Missing information to define M

+ 9 pts Home configuration matrix is partially correct.

Question 3

Problem 3 **30** / 30 pts

3.1 **Screw Axis 1** **3.3** / 3.3 pts

✔ **+ 3.3 pts** Correct

+ 1.65 pts S1 is correct, however, p1 is not

+ 0 pts Not submitted

+ 1 pt missing v1, p1 is not correct

3.2 **Screw Axis 2** **3.3** / 3.3 pts

✔ **+ 3.3 pts** Correct

+ 0 pts Not submitted

+ 1.65 pts s2 is correct, however, p2 is not

+ 1 pt missing v2, p2 is not correct

3.3 **Screw Axis 3** **3.3** / 3.3 pts

✔ **+ 3.3 pts** Correct

+ 0 pts Not submitted

+ 1.65 pts s3 is correct, however, p3 is not

+ 1 pt missing v3, p3 is not correct.

3.4 **Screw Axis 4** **3.3** / 3.3 pts

✔ **+ 3.3 pts** Correct

+ 1.65 pts S4 is correct, however p4 is not

+ 0 pts Not submitted

+ 1.65 pts v4 is not correct

+ 1 pt missing v4

3.5 **Screw Axis 5** **3.4** / 3.4 pts

✔ **+ 3.4 pts** Correct

+ 1.65 pts s5 is correct, however, p5 is not

+ 0 pts Not submitted

+ 1.65 pts missing v5

3.6 **Screw Axis 6** **3.4** / 3.4 pts

✔ **+ 3.4 pts** Correct

+ 1.65 pts s6 is correct, however, p6 is not

+ 0 pts Not submitted

+ 1.65 pts v6 is not correct

3.7 **Home Configuration** **10** / 10 pts

✔ **+ 10 pts** Correct

+ 5 pts Space frame and end the end effector frame are already set in the schematic.

+ 0 pts Not submitted

+ 8 pts p is not correct

Question 4

Extra Credit **0** / 10 pts

+ 10 pts Correct

+ 0 pts Not submitted

✔ **+ 0 pts** Incorrect

+ 0 pts Incomplete

Those two exponentials don't look like they return the same rotation matrix - try and perform the calculations.

RBE/ME 501 – ROBOT DYNAMICS
HOMEWORK 2
Spring 2023 – Instructor: L. Fichera

INSTRUCTIONS

- **This homework includes 3 preliminary programming questions and 3 problems (*)**
- **To solve the 3 problems, you will have to have Peter Corke's MATLAB Robotics toolbox installed on your system** (ver 10.4 or newer). See https://petercorke.com/toolboxes/robotics-toolbox/.
  - **Submit your solutions to the programming questions on MATLAB Grader: https://grader.mathworks.com/courses/96837-rbe-501-robot-dynamics-spring-2023/assignments/270265-homework-2**
  - **Submit your solutions to the other problems on Gradescope.**
    You will find separate entries to submit:
    - A zip file containing your MATLAB code
      - All source files must be professionally commented; ideally, anyone not familiar with the code should be able to understand what it does just from reading the comments;
      - All functions **must** include a help section;
      - Use I/O and/or figures to illustrate what your code does. Code that runs silently, that is, without generating any visible output, will not be graded;
    - A PDF file detailing your solution; the PDF **must** contain:
      - All necessary schematics/diagrams used to derive the solution;
      - Commentary to explain your thought process (for instance: "*To solve the inverse kinematics, I apply method X*"); if unsure how much level of detail is enough, err on the side of verbosity; no points will be awarded for just reporting the right solution;
      - All calculations (even though they are numerically performed in MATLAB);
  - Check your code before submitting it! **Code that runs with an error will not be graded.**
  - The PDF must be clearly legible. Plots and figures must be of professional quality. Figure DPI >= 300. Remember to add axis labels and units.
- **Soft deadline**: Tuesday 7-Feb-23 at 6:00 pm
- **Hard deadline**: Tuesday 14-Feb-23 at 6:00 pm

(*) There is also an extra-credit question at the end.

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Preliminary Programming Questions (30 points total – 10 points for each correct solution)

This section must be completed online at:

https://grader.mathworks.com/courses/96837-rbe-501-robot-dynamics-spring-2023/assignments/270265-homework-2

1. Converting exponential coordinates of rotation to rotation matrices

2. Converting twists to homogeneous transformation matrices

4. Product of Exponentials Formula

Problem 1 (20 points total):

The 3-DoF manipulator in Figure 1 consists of a three degrees-of-freedom manipulator, with link lengths as follows: L1: 0.3 m, L2: 0.3 m, L3: 0.3 m. A model of this manipulator was pre-created for your convenience: open the zip archive with the starting MATLAB code provided on Canvas, then run the `hw2problem1.m` file.

***Note**: running this script will first display the robot, then it will result in an error. The error is expected, and it occurs because the forward kinematics of this robot is not implemented yet – we will work on this next.*

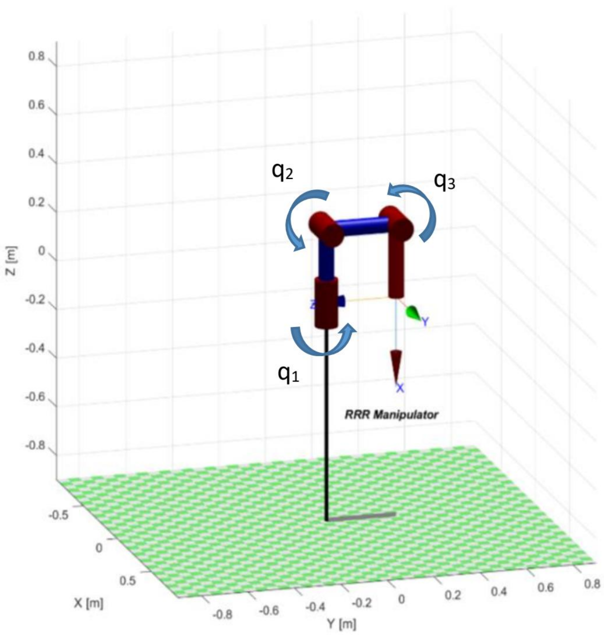
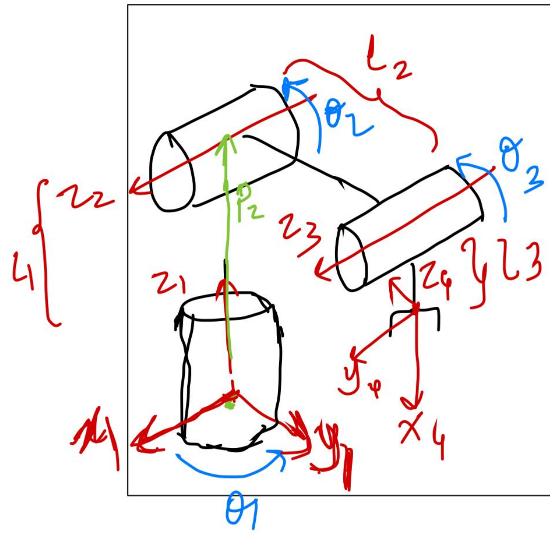


Figure 1: Three DoF manipulator.

a. **Calculate each of the screw axes $\xi_i = (\omega_i, v_i)$ with respect to the space frame (10 points).**
Show your calculations in the text box below (continues on the next page):



Let us calulate the screw axis;

$S_1 = \begin{bmatrix} w_1 \\ v_1 \end{bmatrix}$ $w_1 = [0\ 0\ 1]^T$

$v_1 = [0\ 0\ 0]^T$

$S_2 = \begin{bmatrix} w_2 \\ v_2 \end{bmatrix}$ $w_2 = [1\ 0\ 0]^T$

$v_2 = -[w_2] P_2$

$P_2 = [0\ 0\ L_1]^T$

$$V_2 = -\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ L_1 \end{bmatrix} = [0 \; L_1 \; 0]^T$$

$$S_3 = \begin{bmatrix} W_3 \\ V_3 \end{bmatrix} \quad W_3 = [1 \; 0 \; 0]^T \quad P_3 = [0 \; L_2 \; L_1]$$

$$V_3 = -[W_3]P_3$$

$$V_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ L_2 \\ L_1 \end{bmatrix}$$

$$V_3 = [0 \; L_1 \; -L_2]^T$$

$\xi_1 = [0 \; 0 \; 1 \; 0 \; 0 \; 0]^T$

$\xi_2 = [1 \; 0 \; 0 \; 0 \; L_1 \; 0]^T$

$\xi_3 = [1 \; 0 \; 0 \; 0 \; L_1 \; -L_2]^T$

Once you have calculated the screw axes, complete line 38 of `hw2problem1.m` to hardcode the screw axes into the script.

b. Calculate the manipulator kinematics using the product of exponentials formula (10 points)

To complete this step, you will have to first calculate the homogeneous transformation matrix $\boldsymbol{M}$ for the home configuration. Calculate $\boldsymbol{M}$ and enter it in the text box below:

$$\boldsymbol{M} = \quad M = \begin{bmatrix} R & P \\ 0 & 1 \end{bmatrix} \quad R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix} \quad P = \begin{bmatrix} 0 \\ L_2 \\ L_1 - L_3 \end{bmatrix}$$

Once you have calculated the home configuration, type it into the Matlab script (line 44), then use this information to calculate the forward kinematics (line 62). You should be able to use the `fkine` function developed in the preliminary questions for this homework.

Problem 2 (20 points total):

Repeat the same steps of problem 1 for the *elbow manipulator* shown below. Assume the same link lengths as in problem 1. A model of this manipulator was pre-created for your convenience: open the zip archive with the starting MATLAB code provided on Canvas, then run the `hw2problem2.m` file.

Note: *unlike the prior problem, the starting code provided in `hw2problem2.m` is minimal. You are welcome to structure this script the way you want.* *Remember to comment your code adequately and to use I/O and/or figures to illustrate what your code does.* *Code that runs silently, that is, without generating any visible output, will not be graded.*

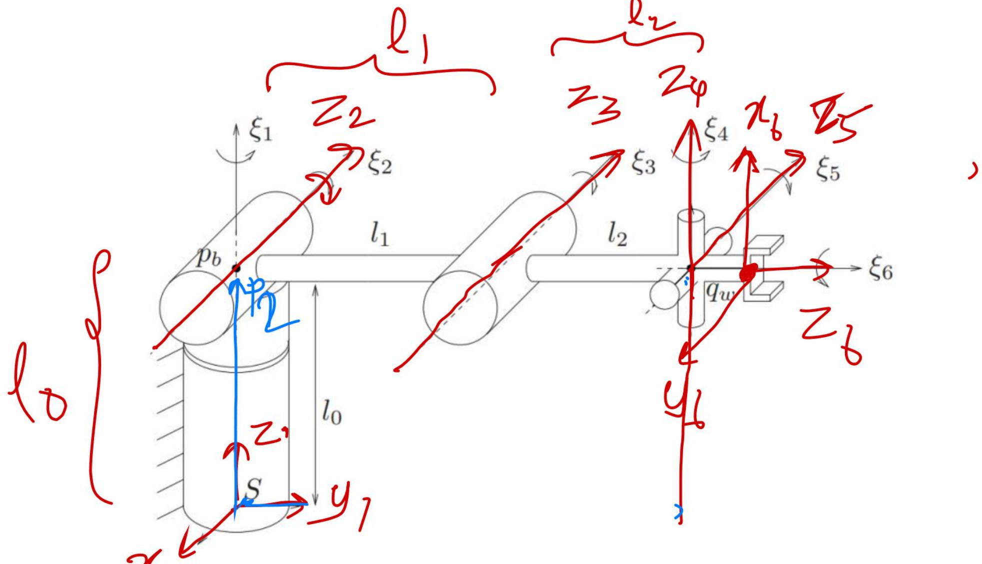


Figure 2: Elbow Manipulator

a. Calculate each of the screw axes $\xi_i = (\omega_i, v_i)$ with respect to the space frame (10 points):

Let us calulate the screw axis;

$$S_1 = \begin{bmatrix} W_1 \\ V_1 \end{bmatrix} \quad W_1 = [0\ 0\ 1]^T$$

$$V_1 = [0\ 0\ 0]^T \text{ since } P_1 = [0\ 0\ 0]^T$$

$$S_2 = \begin{bmatrix} W_2 \\ V_2 \end{bmatrix} \quad W_2 = [-1\ 0\ 0]^T \quad P_2 = [0\ l_1\ l_0]^T$$

$$V_2 = -[W_2]P_2$$

$$V_2 = -\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ L_1 \\ L_0 \end{bmatrix} = [0\ -L_0\ L_1]$$

$$S_2 = \begin{bmatrix} \omega_2 \\ V_2 \end{bmatrix} \quad \omega_2 = [-1 \ 0 \ 0]^T \quad p_2 = [0 \ 0 \ L_0]^T$$

$$V_2 = -\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ L_0 \end{bmatrix} = [0 \ -L_0 \ 0]^T$$

$$S_4 = \begin{bmatrix} \omega_4 \\ V_4 \end{bmatrix} \quad \omega_4 = [0 \ 0 \ 1]^T \quad p_4 = [0 \ L_1 + L_2 \ 0]^T$$

$$V_4 = -\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ L_1 + L_2 \\ 0 \end{bmatrix} = [L_1 + L_2 \ 0 \ 0]^T$$

$$S_5 = \begin{bmatrix} \omega_5 \\ V_5 \end{bmatrix} \quad \omega_5 = [-1 \ 0 \ 0]^T \quad p_5 = [0 \ L_1 + L_2 \ L_0]^T$$

$$V_5 = -\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ L_1 + L_2 \\ L_0 \end{bmatrix} = [0 \ -L_0 \ L_1 + L_2]^T$$

$$S_6 = \begin{bmatrix} \omega_6 \\ V_6 \end{bmatrix} \quad \omega_6 = [0 \ 1 \ 0]^T \quad p_6 = [0 \ L_1 + L_2 \ L_0]^T$$

$$V_6 = -\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ L_1 + L_2 \\ L_0 \end{bmatrix} = [-L_0 \ 0 \ 0]^T$$

$\xi_1 = [0 \ 0 \ 1 \ 0 \ 0 \ 0]^T$

$\xi_2 = [-1 \ 0 \ 0 \ 0 \ -L_0 \ 0]^T$

$\xi_3 = [-1 \ 0 \ 0 \ 0 \ -L_0 \ L_1]^T$

$\xi_4 = [0 \ 0 \ 1 \ L_1 + L_2 \ 0 \ 0]^T$

$\xi_5 = [-1 \ 0 \ 0 \ 0 \ -L_0 \ L_1 + L_2]^T$

$\xi_6 = [0 \ 1 \ 0 \ -L_0 \ 0 \ 0]^T$

b. Calculate the home configuration (10 points):

$$M = R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad p = \begin{bmatrix} 0 \\ L_1 + L_2 \\ L_0 \end{bmatrix}$$

Once you have calculated the screw axes and home configuration, don't forget to fill out and run the Matlab script for this problem.

$$M = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_1 + L_2 \\ 1 & 0 & 0 & L_0 \end{bmatrix}$$

Problem 3 (30 points total):

Repeat the same steps of problem 1 for the Staubli TX-40 *robot.* The robot schematic and dimensions are shown in Figure 3. Run `hw2problem3.m` to create a model of the robot.

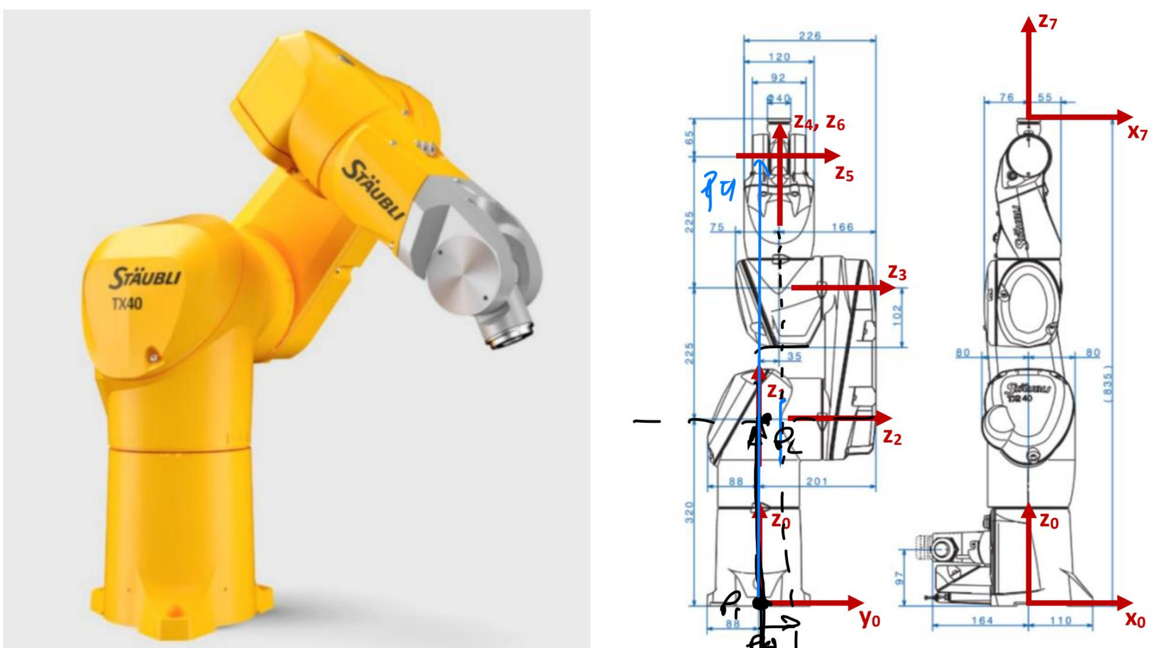


Figure 3: (a) The Staubli TX-40 robotic arm; (b) Joint axes and dimensions. All dimensions are in millimeters. Joints are numbered consecutively from 1 to 6. Frame {0} is the space frame. Frame {7} is the end-effector frame. ***Note****: the schematic does* *not* *show joint frames – only the joint axes (z_1, z_2, …, z_6). Recall that joint frames are not needed to calculate the kinematics with the Product of Exponentials Formula.*

a. **Calculate each of the screw axes $\xi_i = (\omega_i, v_i)$ with respect to the space frame (20 points):**

$$S_1 = \begin{bmatrix} w_1 \\ v_1 \end{bmatrix} \quad w_1 = [0 \ 0 \ 1]^T \quad v_1 = [0 \ 0 \ 0]^T \quad P_1 = [0 \ 0 \ 0]^T$$

$$S_2 = \begin{bmatrix} w_2 \\ v_2 \end{bmatrix} \quad w_2 = [0 \ 1 \ 0]^T \quad v_2 = -[w_2] P_2$$

$$P_2 = [0 \ 0 \ 320]^T$$

$$v_2 = -\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 320 \end{bmatrix}$$

$$v_2 = [320 \ 0 \ 0]^T$$

$$S_3=\begin{bmatrix} w_3 \\ v_3 \end{bmatrix} \quad w_3=\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T \quad v_3=-[w_3]p_3$$

$$p_3=\begin{bmatrix} 0 & 0 & 545 \end{bmatrix}^T$$

$$v_3=-\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 545 \end{bmatrix}=\begin{bmatrix} 545 & 0 & 0 \end{bmatrix}^T$$

$$S_4=\begin{bmatrix} w_4 \\ v_4 \end{bmatrix} \quad w_4=\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T \quad v_4=-[w_4]p_4$$

$$p_4=\begin{bmatrix} 0 & 35 & 0 \end{bmatrix}^T$$

$$v_4=-\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 35 \\ 0 \end{bmatrix}=\begin{bmatrix} 35 & 0 & 0 \end{bmatrix}^T$$

$$S_5=\begin{bmatrix} w_5 \\ v_5 \end{bmatrix} \quad w_5=\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T \quad v_5=-[w_5]p_5$$

$$p_5=\begin{bmatrix} 0 & 0 & 770 \end{bmatrix}^T$$

$$v_5=-\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 770 \end{bmatrix}=\begin{bmatrix} -770 & 0 & 0 \end{bmatrix}^T$$

$$S_6=\begin{bmatrix} w_6 \\ v_6 \end{bmatrix} \quad w_6=\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$$

$$v_6=-[w_6]p_6$$

$$p_6=\begin{bmatrix} 0 & 35 & 0 \end{bmatrix}^T$$

$$v_6=\begin{bmatrix} 35 & 0 & 0 \end{bmatrix}^T$$

$$\xi_1=[0\ 0\ 1\ 0\ 0\ 0]^T$$

$$\xi_2=[0\ 1\ 0\ -H_1\ 0\ 0]^T$$

$$\xi_3=[0\ 1\ 0\ -(H_1+H_2)\ 0\ 0]^T$$

$$\xi_4=[0\ 0\ 1\ W\ 0\ 0]^T$$

$$\xi_5=[0\ 1\ 0\ -(H_1+H_2+H_3)\ 0\ 0]^T$$

$$\xi_6=[0\ 0\ 1\ W\ 0\ 0]^T$$

a. Calculate the home configuration (10 points):

$$M = \quad R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad p = \begin{bmatrix} 0 & W & H_1 + H_2 + H_3 + H_4 \end{bmatrix}^T$$

Once you have calculated the screw axes and home configuration, don't forget to fill out and run the Matlab script for this problem.

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & W \\ 0 & 0 & 1 & H_1 + H_2 + H_3 + H_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Extra Credit (10 points):

Prove the following Proposition: **The exponential map exp : $\mathfrak{so}(3)$ → SO(3) is not injective**, i.e., $\exists\ \omega_1, \omega_2 \in \mathbb{R}^3$ such that $e^{[\omega_1]} = e^{[\omega_2]}$ and $\omega_1 \neq \omega_2$.

Proof:

The exponential map so(3) → SO(3) maps the elements of Lie Algebra so(3) [set of all skew symmetric matries in $\mathbb{R}^3$] to the special orthogonal group SO(3) [set of all 3 X 3 orthogonal matrices with determinant = 1].

To prove that the exponential map is not injective, we have to show that there exists $w_1, w_2 \in \mathbb{R}^3$ such that

$$e^{[w_1]} = e^{[w_2]} \text{ for } w_1 \neq w_2.$$

Consider the following skew symmetric matrices in so(3).

$w_1 = [0, -\gamma, \gamma]$

$w_2 = [0, \gamma, \gamma]$ [γ is a non zero real number]

These are 2 different matrices, but their exponential map is the same.

(contd. next page)

<u>Extra Credit (10 points):</u>

Prove the following Proposition: **The exponential map exp : $\mathfrak{so}$(3) → SO(3) is <u>not</u> injective**, i.e., $\exists\ \omega_1, \omega_2 \in \mathbb{R}^3$ such that $e^{[\omega_1]} = e^{[\omega_2]}$ and $\omega_1 \neq \omega_2$.

Proof:

$$e^{[w_1]} = e^{[0,-\gamma,\gamma]} = [[\cos\gamma, -\sin\gamma, 0], [\sin\gamma, \cos\gamma, 0], [0, 0, 1]]$$

$$e^{[w_2]} = e^{[0,\gamma,\gamma]} = [[\cos\gamma, \sin\gamma, 0], [-\sin\gamma, \cos\gamma, 0], [0, 0, 1]]$$

which basically denote the rotation matrices of angle γ about the y axis.

Hence, $e^{[w_1]} = e^{[w_2]}$ where $w_1 \neq w_2$; therefore it is proved that the exponential map so(3) → SO(3) is not injective.