Homework 3

● Graded

Student

Rohin Siddhartha Palaniappan Venkateswaran

Total Points

80 / 80 pts

Question 1

Problem 1

20 / 20 pts

✔ **+ 20 pts** Correct

– 20 pts Unable to run code

Question 2

Problem 2

20 / 20 pts

✔ **+ 20 pts** Correct

+ 10 pts Robot does not follow the desired path

+ 5 pts code does not run

+ 10 pts Code partially runs

Question 3

Problem 3

30 / 30 pts

✔ **+ 30 pts** Correct

+ 10 pts Robot does not follow the desired path

+ 5 pts code does not run

Question 4

Extra Credit Question

10 / 10 pts

✔ **+ 10 pts** Correct

+ 0 pts Click here to replace this description.

– 10 pts No submission

– 10 pts Incorrect

– 5 pts Incorrect

No questions assigned to the following page.

Worcester Polytechnic Institute

RBE/ME 501 – ROBOT DYNAMICS
HOMEWORK 3
Spring 2023 – Instructor: L. Fichera

INSTRUCTIONS

- **This homework includes 3 preliminary programming questions and 3 problems (*)**
- **To solve the 3 problems, you will have to have Peter Corke's MATLAB Robotics toolbox installed on your system** (ver 10.4 or newer). See https://petercorke.com/toolboxes/robotics-toolbox/.
    - **Submit your solutions to the programming questions on MATLAB Grader:** https://grader.mathworks.com/courses/96837-rbe-501-robot-dynamics-spring-2023/assignments/274745-homework-3
    - **Submit your solutions to the other problems on Gradescope.** **You will find separate entries to submit:**
        - A zip file containing your MATLAB code
            - All source files must be professionally commented; ideally, anyone not familiar with the code should be able to understand what it does just from reading the comments;
            - All functions **must** include a help section;
            - Use I/O and/or figures to illustrate what your code does. Code that runs silently, that is, without generating any visible output, will not be graded;
        - A PDF file detailing your solution; the PDF **must** contain:
            - All necessary schematics/diagrams used to derive the solution;
            - Commentary to explain your thought process (for instance: "*To solve the inverse kinematics, I apply method X*"); if unsure how much level of detail is enough, err on the side of verbosity; no points will be awarded for just reporting the right solution;
            - All calculations (even though they are numerically performed in MATLAB);
    - Check your code before submitting it! **Code that runs with an error will not be graded.**
    - The PDF must be clearly legible. Plots and figures must be of professional quality. Figure DPI >= 300. Remember to add axis labels and units.
- **Soft deadline**: Friday 24-Feb-23 at 6:00 pm
- **Hard deadline**: Tuesday 28-Feb-23 at 6:00 pm

(*) There is also an extra-credit question at the end.

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No questions assigned to the following page.

Preliminary Programming Questions (30 points total – 10 points for each correct solution)

This section must be completed online at:
https://grader.mathworks.com/courses/96837-rbe-501-robot-dynamics-spring-2023/assignments/274745-homework-3

1. Adjoint Transformations

2. Jacobian in the Space Frame

3. Analytic Jacobian

No questions assigned to the following page.

Problem 1 (20 points total):

The 3-DoF manipulator in Figure 1 consists of a three degrees-of-freedom manipulator, with link lengths as follows: L1: 0.3 m, L2: 0.3 m, L3: 0.3 m. A model of this manipulator was pre-created for your convenience: open the zip archive with the starting MATLAB code provided on Canvas, then run the `hw3problem1.m` file.

***Note**: This is the same robot whose kinematics you modeled in Problem 1 of Homework 2.*

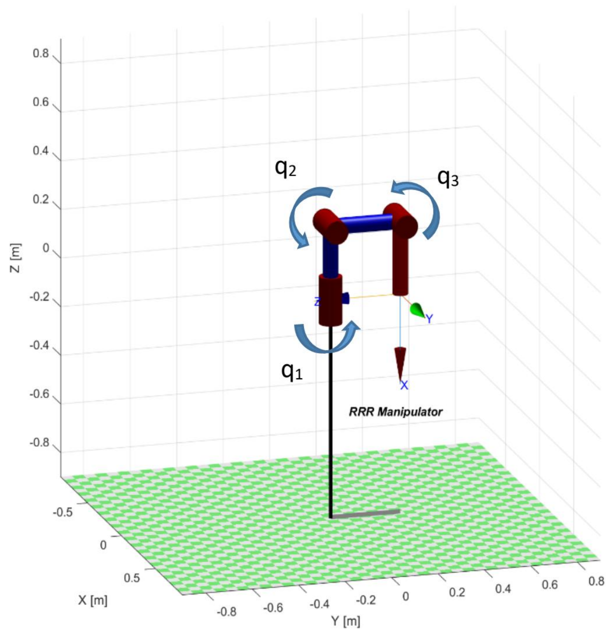

Figure 1: Three DoF manipulator.

Calculate the inverse kinematics for 20 different configurations using a numerical approach

Walking through the starting code and get familiar with how target configurations are being generated (lines 64-84). To solve this problem, you will have to fill out line 101, i.e., you will have to generate a vector `deltaQ` to be added to the vector of joint variables (line 103) to move the robot closer to the target configuration. In the box below, comment on what numerical method you chose to implement; take note of the configurations for which the inverse kinematics algorithm fails (if any), investigate the cause of the failure (ill-conditioned Jacobian? Unreachable configuration?), and attempt to find a different solution for these configurations.

The numerical method which was used for this problem is the Newton Raphson method by finding the Jacobian in the space frame. All test cases have passed and there are no failed configurations.

No questions assigned to the following page.

Problem 2 (20 points total):

Solve the inverse kinematics for the *elbow manipulator* shown below. Assume the same link lengths as in problem 1. A model of this manipulator was pre-created for your convenience: open the zip archive with the starting MATLAB code provided on Canvas, then run the `hw3problem2.m` file.

Note 1: *This is the same robot whose kinematics you modeled in Problem 2 of Homework 2.*

Note 2: *Unlike problem 1, where we had to solve the inverse kinematics for both position and orientation, this problem only gives as input the desired end effector position – see lines 66-78 of the starting code. You can solve this problem using the analytic Jacobian we have derived in class, and you can leave the orientation of the end effector free to vary.*

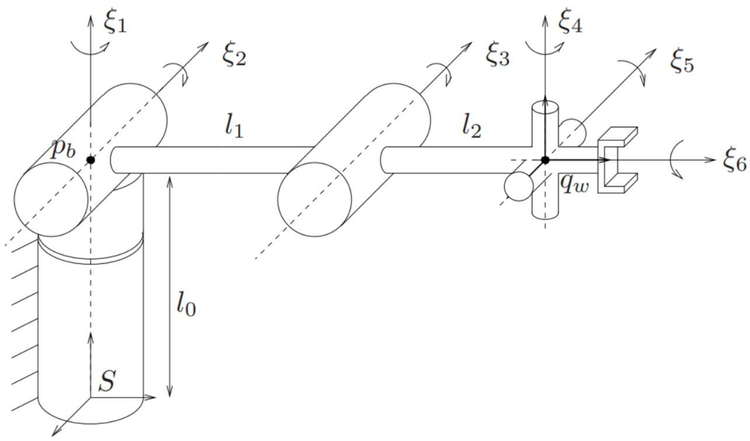

Figure 2: Elbow Manipulator

In the box below, comment on what numerical method you chose to implement; take note of the configurations for which the inverse kinematics algorithm fails (if any), investigate the cause of the failure (ill-conditioned Jacobian? Unreachable configuration?), and attempt to find a different solution for these configurations.

The analytic Jacobian has been used to solve this problem - Three methods for Inverse Kinematics have been tested: pseudoinverse, transpose and Least squares method. The robot does not run into any singularconfiguration while tracing the path and the Inverse Kinematic Algorithm does not fail.

No questions assigned to the following page.

Problem 3 (30 points total):

Solve the inverse kinematics for the Staubli TX-40 *robot.* The robot schematic and dimensions are shown in Figure 3. Run `hw3problem3.m` to create a model of the robot.

Note: *This is the same robot whose kinematics you modeled in Problem 3 of Homework 2.*

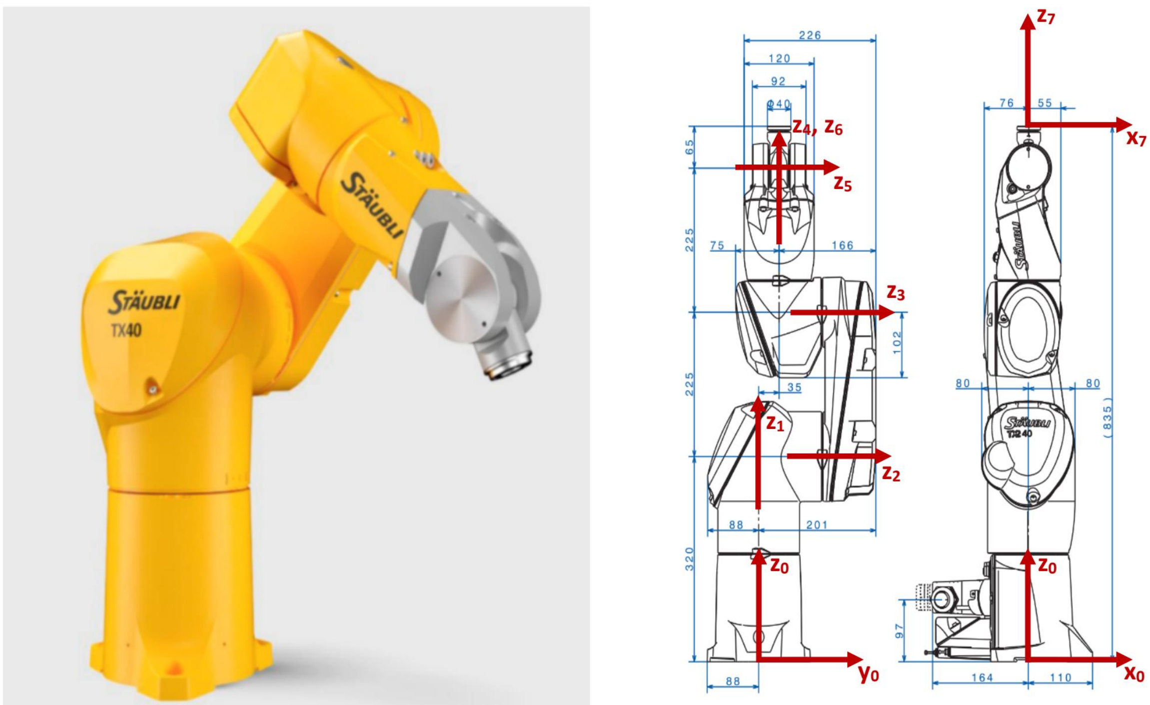

Figure 3: (a) The Staubli TX-40 robotic arm; (b) Joint axes and dimensions. All dimensions are in millimeters. Joints are numbered consecutively from 1 to 6. Frame {0} is the space frame. Frame {7} is the end-effector frame. ***Note****: the schematic does* *not* *show joint frames – only the joint axes (z_1, z_2, …, z_6). Recall that joint frames are not needed to calculate the kinematics with the Product of Exponentials Formula.*

In the box below, comment on what numerical method you chose to implement; take note of the configurations for which the inverse kinematics algorithm fails (if any), investigate the cause of the failure (ill-conditioned Jacobian? Unreachable configuration?), and attempt to find a different solution for these configurations.

The numerical method which was used for this problem is the Newton Raphson method by finding the Jacobian in the space frame. All test cases have passed and there are no failed configurations. While tracing the trajectory, the robot does not run into any singular configurations.

No questions assigned to the following page.

Extra Credit (10 points):

Prove the following Proposition: **Given a twist $\mathcal{V} \in \mathbb{R}^6$ and a homogeneous transformation matrix $T = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix} \in SE(3)$, the adjoint transformation associated with T can be formulated as $\mathcal{V}' = Ad_T \mathcal{V}$, where $Ad_T = \begin{bmatrix} R & 0 \\ [p]R & R \end{bmatrix} \in \mathbb{R}^{6\times 6}$.**

Proof:

We know that $T = \begin{bmatrix} R & P \\ 0 & 1 \end{bmatrix} \in SE(3)$.

Given T, the Adjoint Trasformation Ad_T can be defined as a linear transformation on the Lie Algebra of $SE(3)$ given as

$$Ad_T = T X T^{-1} \quad \text{where } X \in SE(3).$$

We know that $\mathcal{V} \in \mathbb{R}^6$ and

$$\mathcal{V} = [w \quad v]^T$$

$$w \in \mathbb{R}^3, \quad v \in \mathbb{R}^3.$$

Now $Ad_T(\mathcal{V}) = T \mathcal{V} T^{-1}$

Note:-

$$T^{-1} = \begin{bmatrix} R^T & -R^T P \\ 0 & 1 \end{bmatrix}$$

No questions assigned to the following page.

Extra Credit (10 points):

Prove the following Proposition: **Given a twist $\mathcal{V} \in \mathbb{R}^6$ and a homogeneous transformation matrix $T = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix} \in SE(3)$, the adjoint transformation associated with T can be formulated as $\mathcal{V}' = Ad_T\,\mathcal{V}$, where $Ad_T = \begin{bmatrix} R & 0 \\ [p]R & R \end{bmatrix} \in \mathbb{R}^{6\times6}$.**

Proof:

Substituting the value of T and expanding, we get

$$T\mathcal{V}T^{-1} = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix}\begin{bmatrix} w \\ v \end{bmatrix}\begin{bmatrix} R^T & -R^Tp \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} Rw \\ Rv + p\times Rw \end{bmatrix}\begin{bmatrix} R^T & -R^Tp \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} Rw \\ Rv + [p]Rw \end{bmatrix}\begin{bmatrix} R^T & -R^Tp \\ 0 & 1 \end{bmatrix}$$

We modify $Rv + [p]Rw \rightarrow [p]Rw + Rv$.

Similarly $\begin{bmatrix} R & 0 \\ [p]R & R \end{bmatrix} \Rightarrow \begin{bmatrix} R & 0 \\ [p]R & R \end{bmatrix}\begin{bmatrix} w \\ v \end{bmatrix} = \begin{bmatrix} Rw \\ [p]Rw + Rv \end{bmatrix}$

Hence; $Ad_T(\mathcal{V}) = \begin{bmatrix} R & 0 \\ [p]R & R \end{bmatrix}\begin{bmatrix} w \\ v \end{bmatrix}$

No questions assigned to the following page.

Extra Credit (10 points):

Prove the following Proposition: **Given a twist $\boldsymbol{\mathcal{V}} \in \mathbb{R}^6$ and a homogeneous transformation matrix $\boldsymbol{T} = \begin{bmatrix} \boldsymbol{R} & \boldsymbol{p} \\ \boldsymbol{0} & \boldsymbol{1} \end{bmatrix} \in SE(3)$, the adjoint transformation associated with $\boldsymbol{T}$ can be formulated as $\boldsymbol{\mathcal{V}}' = \boldsymbol{Ad_T}\,\boldsymbol{\mathcal{V}}$, where $\boldsymbol{Ad_T} = \begin{bmatrix} \boldsymbol{R} & \boldsymbol{0} \\ [\boldsymbol{p}]\boldsymbol{R} & \boldsymbol{R} \end{bmatrix} \in \mathbb{R}^{6\times 6}$.**

Proof:

$$= \begin{bmatrix} R\omega \\ [p]R\omega + Rv \end{bmatrix}$$

Hence proved.