Homework 1

Graded

Student

Rohin Siddhartha Palaniappan Venkateswaran

Total Points

50 / 50 pts

Question 1

Frame Transformations — **20** / 20 pts

1.1 **Find the rotation matrix R required to align F0 with F1** — **5** / 5 pts

- ✔ **– 0 pts** Correct
- **– 2.5 pts** Incorrect rotation matrix.

1.2 **1.2 Given the rotation matrix R previously found in step a, calculate a corresponding set of ZYX Euler angles.** — **5** / 5 pts

- ✔ **– 0 pts** Correct
- **– 5 pts** Incorrect Euler Angle
- **– 5 pts** No submission provided

1.3 **Calculate the inverse rotation matrix** — **5** / 5 pts

- ✔ **– 0 pts** Correct
- **– 5 pts** Incorrect initial rotation matrix
- **– 5 pts** Incorrect inverse rotation

1.4 **Calculate the Homogeneous Transformation matrix T** — **5** / 5 pts

- ✔ **– 0 pts** Correct
- **– 2.5 pts** Incorrect rotation matrix
- **– 2.5 pts** Transformation matrix must be 4 by 4 matrix

Question 2

Properties of Rotation Matrices **10** / 10 pts

2.1 Calculate R **5** / 5 pts

+ 0 pts Check the math. The default settings of the trigonometric functions in Matlab is in radians.

+ 5 pts Check your K matrix

+ 5 pts Point adjustment

2.2 Calculate det(R) **5** / 5 pts

+ 0 pts You calculated the determinant of K not R

+ 0 pts Incorrect R matrix.

+ 0 pts Answer is not clear

+ 5 pts Point adjustment

Question 3

SCARA Robot **10** / 10 pts

3.1 DH Frames **5** / 5 pts

+ 0 pts The frames are not clearly established.

+ 5 pts Point adjustment

3.2 DH Table **5** / 5 pts

+ 0 pts Table is not clearly due to frames are not clearly established.

+ 0 pts Incorrect DH parameters for the second joint

+ 0 pts Incorrect DH parameters for the last joint

+ 0 pts Unnecessary addition of extra frames

+ 0 pts Incorrect DH parameters for the prismatic joint

+ 5 pts Correct. -LF

+ 5 pts Point adjustment

Question 4

RPP Robot **10** / 10 pts

4.1 **DH Frames** **5** / 5 pts

- ✔ **− 0 pts** Correct
- **− 5 pts** Incorrect Frame 2
- **− 5 pts** No DH frames
- **− 5 pts** Unnecessary DH frame added

4.2 **DH Table** **5** / 5 pts

- ✔ **− 0 pts** Correct
- **− 1 pt** Incorrect theta value
- **− 5 pts** Incorrect theta and alpha values
- **− 2.5 pts** Incorrect theta values
- **− 2.5 pts** Frame offsets incorrect
- **− 5 pts** No DH frames
- **− 2.5 pts** Incorrect alpha value
- **− 2.5 pts** Unecessary offset in theta value of first frame
- **− 2.5 pts** Unnecessary additional frame added

Worcester Polytechnic Institute

RBE/ME 501 – ROBOT DYNAMICS
HOMEWORK 1
Spring 2023 – Instructor: L. Fichera

INSTRUCTIONS

1. **This homework consists of a set of 8 problems.**
2. **Submit your solutions for Problems 1-4 as a single PDF file through Canvas > Gradescope.** The PDF should include your calculations and any supporting drawings or sketches. Use of computer software for calculations (e.g., MATLAB) is allowed.
3. **Submit your solutions for Problems 5-8 online through MATLAB grader:**
   https://grader.mathworks.com/courses/96837-rbe-501-robot-dynamics-spring-2023
   Keep an eye on your WPI inbox. You will receive an invitation to create an account on MATLAB grader. Sometimes these emails can end up in the junk folder.
4. **Soft deadline**: Tuesday 17-Jan-23 at 6:00 pm
   Hard deadline: Tuesday 31-Jan-23 at 6:00 pm

ROHIN SIDDHARTHA PALANIAPPAN VENKATESWARAN
(ID: 808068806)

The remainder of this page was intentionally left blank.

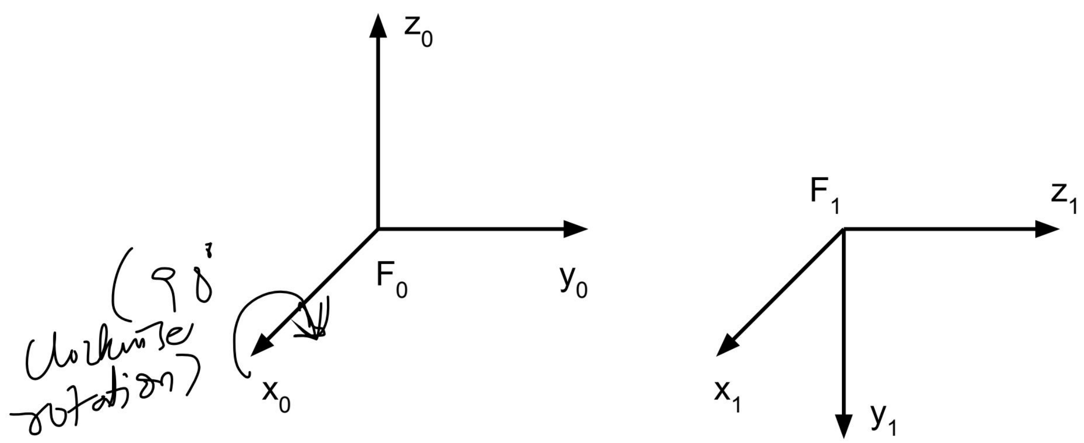

Figure 1: Two Cartesian reference frames.

Problem 1: Given the reference frames F_0 and F_1 as shown in Fig. 1:

1.1 Calculate the rotation matrix **R** required to align F_0 with F_1.

$$R_x(\gamma) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$\boxed{R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}}$$

1.2 Given the rotation matrix **R** previously found in step a, calculate a corresponding set of ZYX Euler angles $\boldsymbol{\phi} = [\varphi, \vartheta, \psi]$.

$$T = \begin{bmatrix} \cos\phi\cos\vartheta & \cos\phi\sin\vartheta\sin\psi - \sin\phi\cos\psi & \cos\phi\sin\vartheta\cos\psi + \sin\phi\sin\psi \\ \sin\phi\cos\vartheta & \sin\phi\sin\vartheta\sin\psi + \cos\phi\cos\vartheta & \sin\phi\sin\vartheta\cos\psi - \cos\phi\sin\psi \\ -\sin\vartheta & \cos\vartheta\sin\psi & \cos\vartheta\cos\psi \end{bmatrix}$$

Comparing the entries, we get

$-\sin\vartheta = 0$

$\boxed{\vartheta = 0, \pi}$

$\cos\vartheta\cos\psi = 0$

for $\vartheta = 0$:

$\cos\psi = 0$

$\boxed{\psi = \pi/2, 3\pi/2}$

$\boldsymbol{\phi} = [\ 0 \quad 0 \quad \pi/2\ ]$

$\phi = [\ \pi \quad \pi \quad 3\pi/2\ ]$

$\cos\phi\cos\vartheta = 1$

for $\vartheta = 0$, $\cos\phi = 1$, $\boxed{\phi = 0, \pi}$

1.3 Calculate the inverse rotation $\mathbf{R}^{-1}$.

$$R^{-1} = R^{T} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$R^{-1} =$

1.4 Calculate the homogeneous transformation matrix **T** between F_0 and F_1. Assume that the translational component of the transformation is given by $\boldsymbol{p} = [0 \quad 10 \quad 0]^{\mathrm{T}}$.

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

T =

Problem 2: Given the *skew-symmetric* matrix $\boldsymbol{K} = \begin{bmatrix} 0 & -0.0875 & 0.5670 \\ 0.0875 & 0 & -0.8190 \\ -0.5670 & 0.8190 & 0 \end{bmatrix}$ and angle value $\theta = 20°$:

2.1 Calculate the 3x3 matrix given by $\boldsymbol{R} = \boldsymbol{I} + \sin(\theta)\,\boldsymbol{K} + [1 - \cos(\theta)]\boldsymbol{K}^2$.

$$R = \begin{bmatrix} 0.9802 & -0.0019 & 0.1982 \\ 0.0579 & 0.9591 & -0.2771 \\ -0.1896 & 0.2837 & 0.9402 \end{bmatrix}$$

R =

2.2 Calculate the determinant of $\boldsymbol{R}$.

det(R) = 1

<u>Problem 3</u>: Given the <u>SCARA</u> manipulator shown below:

3.1 Assign reference frames to each of the joint axes following the Denavit-Hartenberg convention[1]. The frame at the first joint {1} and the end effector frame {5} were pre-assigned for your convenience (red ink).

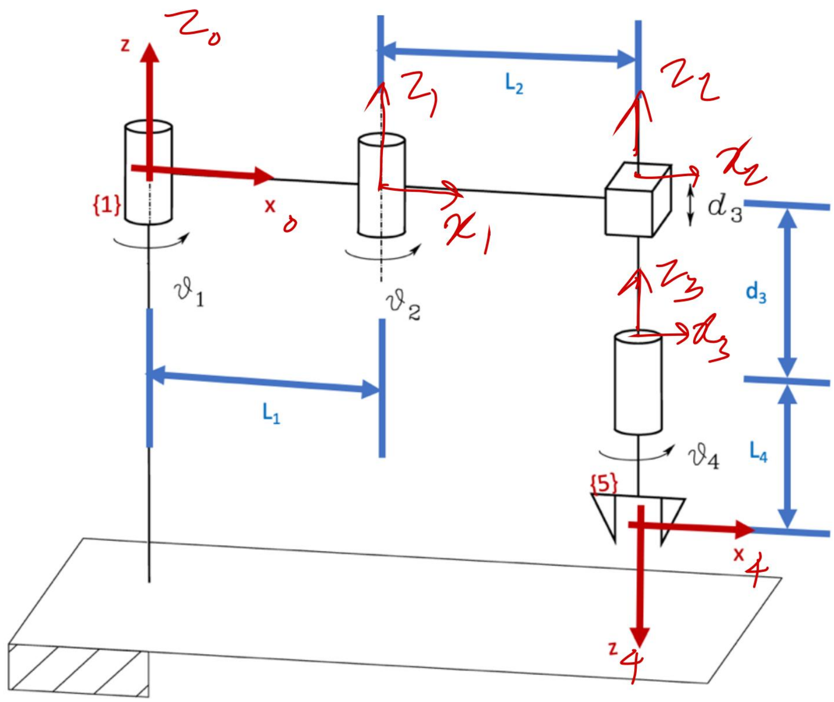

Figure 2: 4 DoF SCARA Manipulator

3.2 Create the table of D-H parameters. Use link lengths as noted in Fig. 2 (blue ink).

	θ	d	a	α
1	ϑ_1	0	L_1	0
2	ϑ_2	0	L_2	0
3	0	$-d_3$	0	0
4	ϑ_4	$-L_4$	0	180°

---

[1] Multiple variants of the Denavit-Hartenberg convention exist. For this homework, use the version described in Siciliano et al. (2010), *Robotics: modelling, planning and control* (§2.8.2). Link: https://www.springer.com/us/book/9781846286414.

Problem 4: Given the RPP (Revolute-Prismatic-Prismatic) robotic manipulator illustrated in Fig. 3:

4.1 Assign reference frames to each of the joint axes following the Denavit-Hartenberg convention. Frames at the first joint of the robot and at the end effector were pre-assigned for your convenience (red ink).

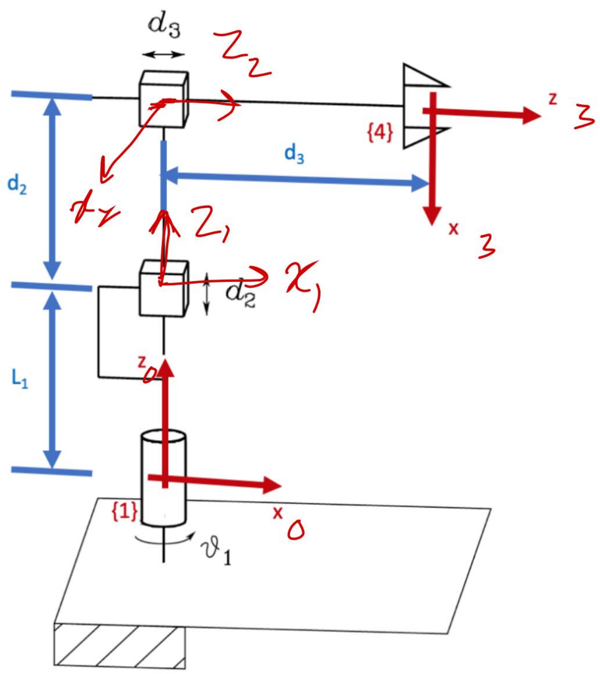

Figure 3: 3-DoF RPP (Revolute-Prismatic-Prismatic) robotic manipulator.

4.2 Create the table of D-H parameters. Use link lengths as noted in the picture (blue ink).

	θ	d	a	α
1	ϑ_1	L_1	0	0
2	-90°	d_2	0	-90°
3	90	d_3	0	0

[See the following problems online at https://grader.mathworks.com/courses/96837-rbe-501-robot-dynamics-spring-2023/assignments/264377-homework-1.]

5. Rotation matrices in MATLAB (10 points)
6. DH Parameters in MATLAB (10 points)
7. Forward kinematics of the SCARA robot (20 points)
8. Forward kinematics of the RPP robot (10 points)