

Assignment 2 – Advanced Robot Navigation (RBE595 Spring 2023)

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Task 1

The state extrapolation equation

The general form of the state extrapolation equation is given as follows

$$\hat{\mathbf{x}}_{n+1,n} = \mathbf{F}\hat{\mathbf{x}}_{n,n} + \mathbf{G}\mathbf{u}_n + \mathbf{w}_n \quad (1)$$

Where:

$\hat{\mathbf{x}}_{n+1,n}$ is a predicted system state vector at time step $n + 1$

$\hat{\mathbf{x}}_{n,n}$ is an estimated system state vector at time step n

\mathbf{u}_n is a control variable or input variable - a measurable (deterministic) input to the system

\mathbf{w}_n is a process noise or disturbance - an un-measurable input that affects the state

\mathbf{F} is a state transition matrix

\mathbf{G} is a control matrix or input transition matrix (mapping control to state variables)

According to the assignment, we have information regarding the acceleration. The state vector $\hat{\mathbf{x}}_n$ that describes the estimated position and velocity of the drone in a Cartesian coordinate system (x, y, z) is given by:

The state vector $\hat{\mathbf{x}}_n$ represented as a column vector is:

$$\hat{\mathbf{x}}_n = \begin{bmatrix} \hat{x}_n \\ \hat{y}_n \\ \hat{z}_n \\ \hat{\dot{x}}_n \\ \hat{\dot{y}}_n \\ \hat{\dot{z}}_n \end{bmatrix}$$

The control vector \mathbf{u}_n that describes the measured drone acceleration in a Cartesian coordinate system (x, y, z) is given by:

$$\mathbf{u} = m\ddot{\mathbf{p}}$$

$$\mathbf{u} = m \begin{bmatrix} \ddot{x}_n \\ \ddot{y}_n \\ \ddot{z}_n \end{bmatrix}$$

The extrapolated vehicle state for time $n + 1$ can be described by the following system of equations:

$$\begin{aligned}
\hat{x}_{n+1,n} &= \hat{x}_{n,n} + \hat{\dot{x}}_{n,n}\Delta t + \frac{1}{2}\Delta t^2 u_1 \\
\hat{y}_{n+1,n} &= \hat{y}_{n,n} + \hat{\dot{y}}_{n,n}\Delta t + \frac{1}{2}\Delta t^2 u_2 \\
\hat{z}_{n+1,n} &= \hat{z}_{n,n} + \hat{\dot{z}}_{n,n}\Delta t + \frac{1}{2}\Delta t^2 u_3 \\
\hat{\dot{x}}_{n+1,n} &= \hat{\dot{x}}_{n,n} + \frac{\Delta t}{m} u_1 \\
\hat{\dot{y}}_{n+1,n} &= \hat{\dot{y}}_{n,n} + \frac{\Delta t}{m} u_2 \\
\hat{\dot{z}}_{n+1,n} &= \hat{\dot{z}}_{n,n} + \frac{\Delta t}{m} u_3
\end{aligned}$$

From the above equations, we can derive the \mathbf{F} as

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & 0 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta t & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta t \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The control matrix \mathbf{G} can be derived as follows

$$\mathbf{G} = \begin{bmatrix} 0.5\Delta t^2 & 0 & 0 \\ 0 & 0.5\Delta t^2 & 0 \\ 0 & 0 & 0.5\Delta t^2 \\ \frac{\Delta t}{m} & 0 & 0 \\ 0 & \frac{\Delta t}{m} & 0 \\ 0 & 0 & \frac{\Delta t}{m} \end{bmatrix}$$

The measurement equation

The generalized measurement equation in matrix form is given by:

$$\mathbf{z}_n = \mathbf{H}\mathbf{x}_n + \mathbf{v}_n$$

Where:

- \mathbf{z}_n is a measurement vector
- \mathbf{x}_n is a true system state (hidden state)
- \mathbf{v}_n is a random noise vector
- \mathbf{H} is an observation matrix

The measurement provides us only position and velocity of the drone. So:

$$\mathbf{z}_n = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

or

$$\mathbf{z}_n = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}$$

The dimension of \mathbf{z}_n is 3×1 and the dimension of \mathbf{x}_n is 6×1 . Therefore, the dimension of the observation matrix \mathbf{H} shall be 3×6 .

$$\mathbf{H}_{position} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

or

$$\mathbf{H}_{velocity} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Task 2

Result 3D Plots

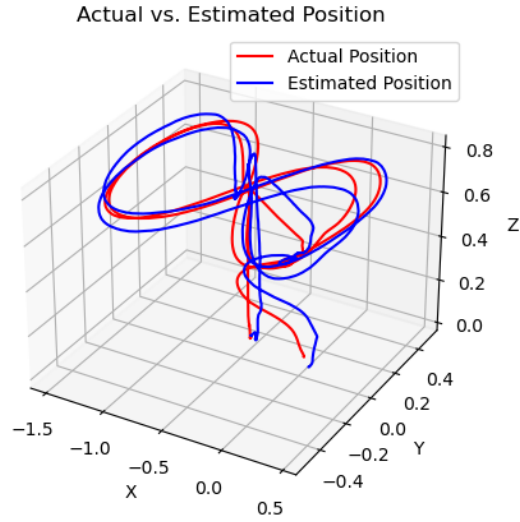


Figure 1: Tracking low-noise position data

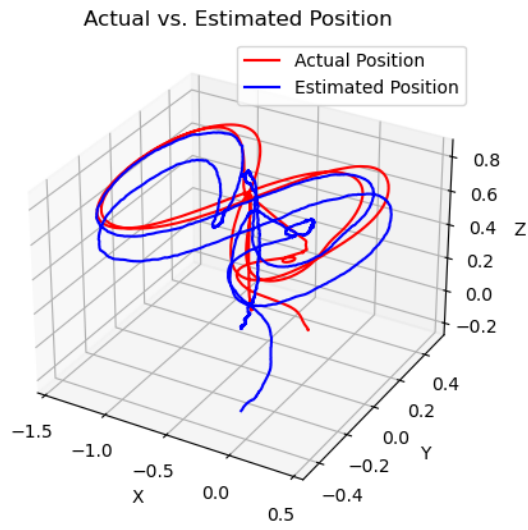


Figure 2: Tracking high-noise position data

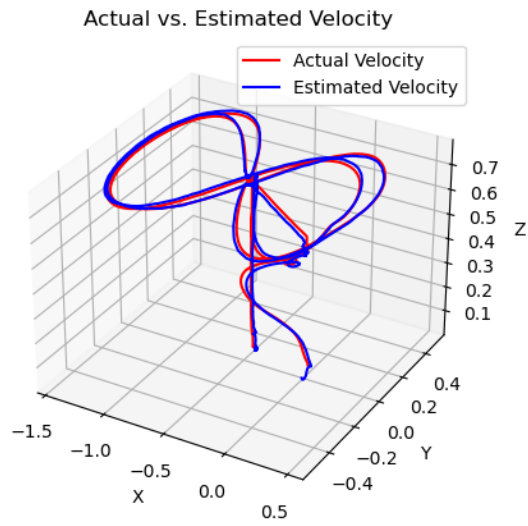


Figure 3: Tracking velocity data