Assignment 2 – Advanced Robot Navigation (RBE595 Spring 2024)

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Task 1

The state extrapolation equation

The general form of the state extrapolation equation is given as follows

$$\hat{\mathbf{x}}_{n+1,n} = \mathbf{F}\hat{\mathbf{x}}_{n,n} + \mathbf{G}\mathbf{u}_n + \mathbf{w}_n \tag{1}$$

Where:

$\hat{\mathbf{x}}_{n+1,n}$ is a predicted system state vector at time step $n+1$
$\hat{\mathbf{x}}_{n,n}$ is an estimated system state vector at time step n
$\mathbf{u}_n$ is a control variable or input variable - a measurable (deterministic) input to the system
$\mathbf{w}_n$ is a process noise or disturbance - an un-measurable input that affects the state
$\mathbf{F}$ is a state transition matrix
$\mathbf{G}$ is a control matrix or input transition matrix (mapping control to state variables)

According to the assignment, we have information regarding the acceleration. The state vector $\hat{\mathbf{x}}_n$ that describes the estimated position and velocity of the drone in a Cartesian coordinate system (x, y, z) is given by:

The state vector $\hat{x}_n$ represented as a column vector is:

$$\hat{\mathbf{x}}_{\mathbf{n}} = \begin{bmatrix} \hat{x}_n \\ \hat{y}_n \\ \hat{z}_n \\ \hat{\dot{x}}_n \\ \hat{\dot{y}}_n \\ \hat{\dot{z}}_n \end{bmatrix}$$

The control vector $\mathbf{u}_n$ that describes the measured drone acceleration in a Cartesian coordinate system (x, y, z) is given by:

$$\mathbf{u} = m\ddot{p}$$

$$\mathbf{u} = m \begin{bmatrix} \ddot{x_n} \\ \ddot{y_n} \\ \ddot{z_n} \end{bmatrix}$$

The extrapolated vehicle state for time $n+1$ can be described by the following system of equations:

$$
\begin{aligned}
\hat{x}_{n+1,n} &= \hat{x}_{n,n} + \hat{\dot{x}}_{n,n}\Delta t + \frac{1}{2}\Delta t^2 u_1 \\
\hat{y}_{n+1,n} &= \hat{y}_{n,n} + \hat{\dot{y}}_{n,n}\Delta t + \frac{1}{2}\Delta t^2 u_2 \\
\hat{z}_{n+1,n} &= \hat{z}_{n,n} + \hat{\dot{z}}_{n,n}\Delta t + \frac{1}{2}\Delta t^2 u_3 \\
\hat{\dot{x}}_{n+1,n} &= \hat{\dot{x}}_{n,n} + \frac{\Delta t}{m} u_1 \\
\hat{\dot{y}}_{n+1} &= \hat{\dot{y}}_{n,n} + \frac{\Delta t}{m} u_2 \\
\hat{\dot{z}}_{n+1} &= \hat{\dot{z}}_{n,n} + \frac{\Delta t}{m} u_3
\end{aligned}
$$

From the above equations, we can derive the $\mathbf{F}$ as

$$
\mathbf{F} = \begin{bmatrix}
1 & 0 & 0 & \Delta t & 0 & 0 \\
0 & 1 & 0 & 0 & \Delta t & 0 \\
0 & 0 & 1 & 0 & 0 & \Delta t \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

The control matrix $\mathbf{G}$ can be derived as follows

$$
\mathbf{G} = \begin{bmatrix}
0.5\Delta t^2 & 0 & 0 \\
0 & 0.5\Delta t^2 & 0 \\
0 & 0 & 0.5\Delta t^2 \\
\frac{\Delta t}{m} & 0 & 0 \\
0 & \frac{\Delta t}{m} & 0 \\
0 & 0 & \frac{\Delta t}{m}
\end{bmatrix}
$$

The measurement equation

The generalized measurement equation in matrix form is given by:

$$
\mathbf{z}_n = \mathbf{H}\mathbf{x}_n + \mathbf{v}_n
$$

Where:

$\mathbf{z}_n$ is a measurement vector

$\mathbf{x}_n$ is a true system state (hidden state)

$\mathbf{v}_n$ is a random noise vector

$\mathbf{H}$ is an observation matrix

The measurement provides us only position and velocity of the drone. So:

$$
\mathbf{z}_n = \begin{bmatrix} x \\ y \\ z \end{bmatrix}
$$

or

$$\mathbf{z}_n = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}$$

The dimension of $\mathbf{z}_n$ is 3×1 and the dimension of $\mathbf{x}_n$ is 6×1. Therefore, the dimension of the observation matrix $\mathbf{H}$ shall be 3×6.

$$\mathbf{H}_{position} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

or

$$\mathbf{H}_{velocity} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Task 2

Result 3D Plots

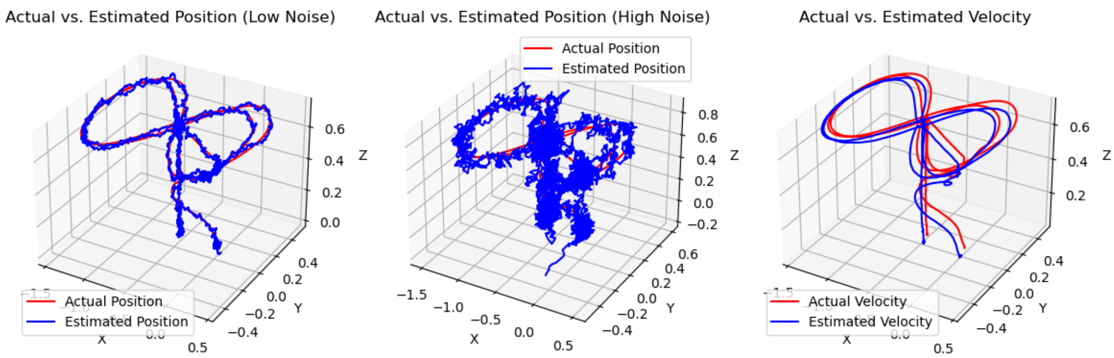

Figure 1: Resultant 3D plots