UNIT 5

1 Tests of hypothesis

1.0 Decision theory in Statistics
1.1 One and two-tailed tests
1.2 Significance Levels
1.3 Step by Step procedure
1.4 Sample Differences
1.5 Coefficient of contingency

1.0 Decision theory in Statistics

Tests of Hypothesis

- In the application of statistics to the solution of many problems, we make a statistical hypothesis or decision about the population, and then proceed to test the decision from a sample study.
- If the sample study shows that the observed results differ from what would be expected on our hypothesis to a considerably greater extent than would be expected by mere chance, then we say the difference is significant.
- *Procedures which enable to be made are called tests of hypotheses or tests of significance.*

The Null Hypothesis

- One of the most useful procedures in decision theory is to make an hypothesis which we can later reject or nullify. We can then accept the alternative to the null hypothesis.

1.0 Decision theory in Statistics – continued

Level of Significance

- It is customary to use a level of significance of 5% or 1%.
- With a level of 5%, for example, we accept an hypothesis if our test shows a 95% chance of being correct.
- In this case, the test will show that the chance that the null hypothesis is correct is 5% or less.
- Similarly, with a 1% level of significance, the percentages are 99% and 1%.

Type I and Type II errors

- *Type I error: Reject an hypothesis, when it should be accepted.*
- *Type II Error: Accept an hypothesis, when it should be rejected.*
- Sometimes one type of error is more serious than the other, and where necessary, tests must be designed to reflect this.

1.1 One and Two tailed tests

One and Two tailed tests

- We are usually concerned with testing the significance of the departure of an observed value from the indicated by our hypothesis.
- This involved finding the area under the normal curve which represents a departure either above or below the assumed figure. In other words, we are concerned with measuring two tails of the normal curve.
- Two-tailed tests known as non directional hypothesis is the standard test of significance to determine if there is a relationship between variables in either direction.
- In some cases, we are interested in a difference in one direction only; here we use one-tailed test called as directional hypothesis.
- Directional differences indicate that you know one of the set of scores will be higher or lower than the other.

1.3 Significance Levels

Significance Levels

- These are determined by the areas under the normal curve.

	Critical Value (z)	
Significance level	Two-tailed test	One-tailed test
1%	2.58	2.33
5%	1.96	1.64
10%	1.64	1.28

Confidence Level	Confidence Coefficient	Confidence Level	Confidence Coefficient	Confidence Level	Confidence Coefficient
99.73%	3.00	96%	2.05	80%	1.28
99%	2.58	95.45%	2.00	75%	1.15
98%	2.33	95%	1.96	68.27%	1.00
97%	2.17	90%	1.64	50%	0.67

Normal Curve

- Note that the value of z for x % of significance under a two-tailed test is equal to that for (100 – x) % confidence in the above table, since both are calculated from the normal curve.

- The value of z for x% significance under a one-tailed test is equal to that for 2x% significance under a two-tailed test.

- From small samples, the values of z must be adjusted.

1.4 Step by step procedure

Step-by-Step procedure for testing hypothesis

1. Specify the null hypothesis
2. Specify the significance level
3. Compute the probability value (p- value)
4. Make a subjective conclusion based on the probability value and the alpha-level

Problem

- *Suppose that a doctor claims that 17 year olds have an average body temperature that is higher than the commonly accepted average human temperature of 98.6ºF.*
- *A simple random statistical sample of 25 people, each of age 17 is selected. The average temperature of the 17 year olds is found to be 98.6ºF, with a standard deviation of 0.6 degrees.*

1.4 Step by step procedure – continued

Solution

1. *The claim being investigated is that the average body temperature of 17 year olds is greater than 98.6 degrees.*

   Null hypothesis: H_0 ***x = 98.6***

   Alternative hypothesis: H1: x > 98.6

- *The statement of our problem will determine which kind of test one-tailed or two-tailed test to use. If the alternative hypothesis contains a "not equals to" sign, then we use a two-tailed test. In our case, we use one tailed test.*

2. *We choose the level of α, our significance level; typically it is 0.05 or 0.01. Here, we use a 5% level and α is 0.05.*

3. *We choose the test statistic and distribution. The sample is from a population that is normally distributed as the bell curve, so we can use the standard normal distribution. A table of z-scores will be necessary.*

1.4 Step by step procedure – continued

Solution

The test statistic is found by the formula for the mean of the sample and we use the standard error of the mean.

$n = 25; \ \sqrt{25} = 5$

$SE = 0.6 / 5 = 0.12.$

Our test statistic is $z = (98.9 - 98.6) / 0.12 = 2.5$

At 5% significance level, the critical value for a one tailed test is found from the table of z-scores to be 1.64. Since the test statistic does not fall within the critical region, we reject the null hypothesis.

We observe that a z-score of 2.5 has a p-value of 0.0062.

4. *Since, this is less than the significance level of 0.05, we reject the null hypothesis.*

1.5 Sample Differences

Sample differences

If we have two samples, we can test whether they come from the same population by using the null hypothesis that the populations are the same. Similar tests can be used for proportions.

Chi-Square Test

This test is used to determine the significance of the differences between observed and expected frequencies, where the expected statistics are available.

chi-squared statistic

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

where

χ^2 = Pearson's cumulative test statistic, which asymptotically approaches a χ^2 distribution.
O_i = an observed frequency;
E_i = an expected (theoretical) frequency, asserted by the null hypothesis;
n = the number of cells in the table.

1.5 Sample Differences – continued

- Values of χ^2 are tabulated in statistical tables for various degrees of freedom. Some typical values are given below:

DF	0.20	0.10	0.05	0.01
1	1.6	2.7	3.8	6.6
2	3.2	4.6	6.0	9.2
3	4.6	6.3	7.8	11.3
4	6.0	7.8	9.5	13.3
5	7.3	9.2	11.1	15.1
10	13.4	16.0	18.3	23.2
20	25.0	28.4	31.4	37.6

- *where an h X k table is involved, the number of degrees of freedom is v = (h – 1) (k –1) – m; m is the number of population parameters which have to be estimated for the sample statistics.*
- *For discrete data, a correction known as Yates' correction should be used which changes the formula to*

$X^2 = \Sigma (| o_i - e_i | - 0.5)^2 \ / \ e_i$; *Here* $| o_i - e_i |$ *is the positive value of* $o_i - e_i$

1.5 Sample Differences – continued

Example

Assume that 125 school children are shown three television commercials, A,B,C and are asked to pick which they liked the best.

The results are shown below:

Gender	A	B	C	Total
Boys	30	29	16	75
Girls	12	33	5	50
Total	42	62	21	125

We would like to know, if the choice of favorite commercial was related to the gender of the child.

1.5 Sample Differences – continued

Solution

H_0: Gender and favorite commercial channel are independent.

We calculate the expected count for each cell.

Multiply the row total by the column total and divide by n.

Gender	A	B	C
Boys	(75 X 42) / 125 = **25.2**	(75 X 62) / 125 = **37.2**	(75 X 21) / 125 = **12.6**
Girls	(50 X 42) / 125 = **16.8**	(50 X 62) / 125 = **24.8**	(50 X 21) / 125 = **8.4**

We compute $\chi^2 = (30 - 25.2)^2 / 25.2 + (29 - 37.2)^2 / 37.2 + (16 - 12.6)^2 / 12.6 + (12 - 16.8)^2 / 16.8 + (33 - 24.8)^2 / 24.8 + (5 - 8.4)^2 / 8.4 = 9.098$

Degrees of freedom = (nr –1) X (nc – 1) = (2–1) X (3 –1) = 2

$X^2{}_{df} = 2$ & $\alpha = 5\% = 6$ and $X^2{}_{df} = 2$ & $\alpha = 1\% = 9.2$

At $\alpha = 1\%$ *level, we accept t* H_0*: since* X^2 *cal* $= 9.098 < X^2{}_{df} = 2$ & $\alpha = 1\% = 9.2$

1.5 Sample Differences – continued

Computing Chi-square in a 2 X 2 contingency table

- The observed frequencies in a study which involves two classifications of each of two variables can be set out in a contingency table as follows:

	I	II	Total
A	a1	a2	N_A
B	b1	b2	N_B
Total	N_1	N_2	N

- If there is no correlation between the two variables, we shall expect the N1 values in I to be distributed between A and B in the same proportion as the total. That is, we expect

  $a1 = N_A / N \times N_1$ and similarly for the other values.

  So, $\chi^2 = N (a1b2 - a2b1)^2 / N_1N_2N_AN_B$

- *With Yates' correction this becomes*

  $\chi^2 = N (|a1b2 - a2b1| - 0.5N)^2 / N_1N_2N_AN_B$

1.5 Sample Differences – continued

Example

A company using a door-to-door sales procedure is :testing a new sales approach, and has the following results on a comparative test under otherwise identical conditions:

Sales Approach	Sales	No Sales	Total Visits
Old	84	116	200
New	98	102	200
Total	182	218	400

Use χ^2 *test to determine the significance of the observed difference.*

Solution:

H_0: New sales approach has not improved the sales.

Expected frequencies from the above data are a1 = b1 = 182 / 2 = 91; a2 = b2 = 218 /2 = 109

$\chi^2 = N\,(a1b2 - a2b1)^2 \,/\, N_1 N_2 N_A N_B$

$X^2 = \{ 400\,[(84 \text{ X } 102) - (98 \text{ X } 116)]^2 \} \,/\, 182 \text{ X } 218 \text{ X } 200 \text{ X } 200 = 1.98$

Degrees of freedom= (h –1) X (v – 1) = (2 –1) X (2 – 1) = 1

$X^2_{\text{calculated}}$ *(1.98)* $< X^2_{\text{tab (1, } \alpha = 10\%)}$ *(2.7) ,* ***we accept*** H_0

1.6 Coefficient of contingency

Coefficient of Contingency & correlation of attributes

Coefficient of Contingency, $C = \sqrt{\chi^2 / (\chi^2 + N)}$

Larger the value of C, greater the association; C is always < 1

Correlation of attributes, $r = \sqrt{\chi^2 / \{ N \times (k-1)\}}$ *; Here k is the number of rows / columns in the contingency table*

Example

$\chi^2 = 1.98; N = 400$

Coefficient of contingency, $C = \sqrt{1.98 / 401.98} = 0.07$

Correlation of attributes, $r = \sqrt{1.98 / [(400 - (2-1)]} = 0.07$