MEEN 621 Summary

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Contents

1. Potential Flow

Complex Potential $F(z) = \phi(z) + i\psi(z)$

Complex Velocity $w(z) = u - iv = \dfrac{dF}{dz}$

1.1. Cartesian ⇔ Polar Velocities

$$u = v_r \cos\theta - v_\theta \sin\theta$$
$$v = v_r \sin\theta + v_\theta \cos\theta$$

$$v_r = u\cos\theta + v\sin\theta$$
$$v_\theta = -u\sin\theta + v\cos\theta$$

1.2. Stream Function ⇔ Potential Function ⇔ Velocities

Also called Cauchy Reimann Equations

$$u = \frac{\partial\phi}{\partial x} = \frac{\partial\psi}{\partial y}$$
$$v = \frac{\partial\phi}{\partial y} = -\frac{\partial\psi}{\partial x}$$

1.3. Complex Potentials in Cartesian Coordinates

	ϕ	ψ	u	v
Uniform	$U(x\cos\alpha + y\sin\alpha)$	$U(y\cos\alpha - x\sin\alpha)$	$U\cos\alpha$	$U\sin\alpha$
Corner				
Source	$\frac{m}{4\pi}\ln[(x-x_0)^2 + (y-y_0)^2]$	$\frac{m}{2\pi}\arctan\frac{y-y_0}{x-x_0}$	$\frac{m}{2\pi}(\frac{x}{x^2+y^2})$	$\frac{m}{2\pi}(\frac{y}{x^2+y^2})$

1.4. Complex Potentials in Polar Coordinates

	ϕ	ψ	v_r	v_θ
Uniform	$Ur\cos(\theta-\alpha)$	$Ur\sin(\theta-\alpha)$	$U\cos(\theta-\alpha)$	$-U\sin(\theta-\alpha)$
Corner	$Cr^n\cos n\theta$	$Cr^n\sin n\theta$	$nCr^{n-1}\cos[(n-1)\theta]$	$-nCr^{n-1}\sin[(n-1)\theta]$
Source	$\frac{m}{2\pi}\ln r$	$\frac{m}{2\pi}\theta$	$\frac{m}{2\pi r}$	0