

MEEN 621 Notes

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1. Potential Flow

$$i = \sqrt{-1}$$

$$z = x + iy = re^{i\theta} = r(\cos \theta + i \sin \theta)$$

$$\bar{z} = x - iy = re^{-i\theta} = r(\cos \theta - i \sin \theta)$$

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$r = \sqrt{x^2 + y^2} = |z| = |z\bar{z}|^{\frac{1}{2}}$$

$$\theta = \arctan \frac{y}{x}$$

$$\text{Complex Potential } F(z) = \phi(z) + i\psi(z)$$

$$\text{Complex Velocity } w(z) = u - iv = (v_r - iv_\theta)e^{-i\theta} = \frac{dF(z)}{dz}$$

$$\bar{w}(z) = u + iv = (v_r + iv_\theta)e^{i\theta}$$

1.1. Cartesian \Leftrightarrow Polar Velocities

$$u = v_r \cos \theta - v_\theta \sin \theta$$

$$v = v_r \sin \theta + v_\theta \cos \theta$$

$$v_r = u \cos \theta + v \sin \theta$$

$$v_\theta = -u \sin \theta + v \cos \theta$$

1.2. Stream Function \Leftrightarrow Potential Function \Leftrightarrow Velocities

Also called Cauchy Reimann Equations

$$u = \frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}$$

$$v = \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}$$

$$v_r = \frac{\partial \phi}{\partial r} = \frac{1}{r} \frac{\partial \psi}{\partial \theta}$$

$$v_\theta = \frac{1}{r} \frac{\partial \phi}{\partial \theta} = -\frac{\partial \psi}{\partial r}$$

1.3. Complex Potentials in Cartesian Coordinates

	ϕ	ψ	u	v
Uniform	$U(x \cos \alpha + y \sin \alpha)$	$U(y \cos \alpha - x \sin \alpha)$	$U \cos \alpha$	$U \sin \alpha$
Corner				
Source	$\frac{m}{4\pi} \ln[(x - x_0)^2 + (y - y_0)^2]$	$\frac{m}{2\pi} \arctan \frac{y-y_0}{x-x_0}$	$\frac{m}{2\pi} \left(\frac{x}{x^2+y^2} \right)$	$\frac{m}{2\pi} \left(\frac{y}{x^2+y^2} \right)$
Free Vortex	$\frac{\Gamma}{2\pi} \arctan \frac{y-y_0}{x-x_0}$	$-\frac{\Gamma}{2\pi} \ln r$	0	$\frac{\Gamma}{2\pi r}$

1.4. Complex Potentials in Polar Coordinates

	ϕ	ψ	v_r	v_θ
Uniform	$Ur \cos(\theta - \alpha)$	$Ur \sin(\theta - \alpha)$	$U \cos(\theta - \alpha)$	$-U \sin(\theta - \alpha)$
Corner	$Cr^n \cos n\theta$	$Cr^n \sin n\theta$	$nCr^{n-1} \cos[(n-1)\theta]$	$-nCr^{n-1} \sin[(n-1)\theta]$
Source	$\frac{m}{2\pi} \ln r$	$\frac{m}{2\pi} \theta$	$\frac{m}{2\pi r}$	0
Free Vortex	$\frac{\Gamma}{2\pi} \theta$	$-\frac{\Gamma}{2\pi} \ln r$	0	$\frac{\Gamma}{2\pi r}$

Legend

- Uniform
 - U : Uniform velocity magnitude
 - α : Angle of attack - the angle at which the direction of the uniform velocity is oriented with respect to the horizontal
- Corner
 - C : Indicates the direction. $C > 0$ always
 - n : $\frac{\pi}{\text{angle of the corner}}$
- Source/Sink
 - m : Volume flow rate per unit dimension normal to the page. For source, $m > 0$, whereas for sink, $m < 0$