

MEEN 621 Notes

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## 1. Potential Flow

$$i = \sqrt{-1}$$

$$z = x + iy = re^{i\theta} = r(\cos \theta + i \sin \theta)$$

$$\bar{z} = x - iy = re^{-i\theta} = r(\cos \theta - i \sin \theta)$$

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$r = \sqrt{x^2 + y^2} = |z| = |z\bar{z}|^{\frac{1}{2}}$$

$$\theta = \arctan \frac{y}{x}$$

$$\text{Complex Potential } F(z) = \phi(z) + i\psi(z)$$

$$\text{Complex Velocity } w(z) = u - iv = (v_r - iv_\theta)e^{-i\theta} = \frac{dF(z)}{dz}$$

$$\bar{w}(z) = u + iv = (v_r + iv_\theta)e^{i\theta}$$

### 1.1. Cartesian $\Leftrightarrow$ Polar Velocities

$$u = v_r \cos \theta - v_\theta \sin \theta$$

$$v = v_r \sin \theta + v_\theta \cos \theta$$

$$v_r = u \cos \theta + v \sin \theta$$

$$v_\theta = -u \sin \theta + v \cos \theta$$

## 1.2. Stream Function $\Leftrightarrow$ Potential Function $\Leftrightarrow$ Velocities

Also called Cauchy Reimann Equations

$$u = \frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}$$

$$v = \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}$$

$$v_r = \frac{\partial \phi}{\partial r} = \frac{1}{r} \frac{\partial \psi}{\partial \theta}$$

$$v_\theta = \frac{1}{r} \frac{\partial \phi}{\partial \theta} = -\frac{\partial \psi}{\partial r}$$

## 1.3. Complex Potentials

	$F(z)$	$\phi$	$\psi$	$w(z)$	$u$ and $v_r$	$v$ and $v_\theta$
Uniform	$Ue^{-i\alpha}z$	$U(x \cos \alpha + y \sin \alpha)$	$U(y \cos \alpha - x \sin \alpha)$	$Ue^{-i\alpha}$	$U \cos \alpha$	$U \sin \alpha$
		$Ur \cos(\theta - \alpha)$	$Ur \sin(\theta - \alpha)$		$U \cos(\theta - \alpha)$	$-U \sin(\theta - \alpha)$
Corner	$Cz^n$			$nCz^{n-1}$		
		$Cr^n \cos n\theta$	$Cr^n \sin n\theta$		$nCr^{n-1} \cos[(n-1)\theta]$	$-nCr^{n-1} \sin[(n-1)\theta]$
Source/Sink	$\frac{m}{2\pi} \ln(z - z_0)$	$\frac{m}{4\pi} \ln[(x - x_0)^2 + (y - y_0)^2]$	$\frac{m}{2\pi} \arctan \frac{y-y_0}{x-x_0}$	$\frac{m}{2\pi(z-z_0)}$	$\frac{m}{2\pi} \left( \frac{x}{x^2+y^2} \right)$	$\frac{m}{2\pi} \left( \frac{y}{x^2+y^2} \right)$
		$\frac{m}{2\pi} \ln r$	$\frac{m}{2\pi} \theta$		$\frac{m}{2\pi r}$	0
Free Vortex	$-\frac{i\Gamma}{2\pi} \ln(z - z_0)$	$\frac{\Gamma}{2\pi} \arctan \frac{y-y_0}{x-x_0}$	$-\frac{\Gamma}{2\pi} \ln r$	$-\frac{i\Gamma}{2\pi(z-z_0)}$	$-\frac{\Gamma}{2\pi} \frac{y}{x^2+y^2}$	$\frac{\Gamma}{2\pi} \frac{x}{x^2+y^2}$
		$\frac{\Gamma}{2\pi} \theta$	$-\frac{\Gamma}{2\pi} \ln r$		0	$\frac{\Gamma}{2\pi r}$

### Legend

- Uniform
  - $U$ : Uniform velocity magnitude
  - $\alpha$ : Angle of attack - the angle at which the direction of the uniform velocity is oriented with respect to the horizontal
- Corner
  - $C$ : Indicates the direction.  $C > 0$  always
  - $n$ :  $\frac{\pi}{\text{angle of the corner}}$
- Source/Sink
  - $m$ : Volume flow rate per unit dimension normal to the page. For source,  $m > 0$ , whereas for sink,  $m < 0$