CT Reconstruction Algorithms - ART, SART, Back Projection and Filtered Back Projection

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In this work, I have implemented different Computed Tomography (CT) reconstruction algorithms. Majorly CT reconstructions involve 2 methods, Algebraic Reconstruction Algorithms and Back Projection Algorithms. I have implemented 2 variants of the first one [ART and SART] and 3 variants of the second one [simple BP and Filtered BP, Noise Filtered BP].

1 Algebraic Reconstruction Algorithms : ART & SART

In this section I will give details about the AR algorithms which basically solve a system of linear equations to find the reconstructed image. ART[2] is the simplest of all the AR algorithms. The general idea of AR algorithts is to solve a system of linear equations

$$Ax = b \tag{1}$$

where A is the weight matrix, x are the pixel values of the reconstructed image and b is the observed summation of the attenuation coefficients.
A is of the size $M \times N$ where M is the total number of rays, N is the size of the image.
x is of the size $N \times 1$ which is $h \times w$, where h is the height of the image and w is the width of the image. Finally b is of the size $M \times 1$.
Update equation of the ART method is"

$$x^{k+1} = x^k + \lambda \frac{1}{a_i^T a_i} \left(b_i - a_i^T x^k \right) a_i \tag{2}$$

For SART (Simeltaneous ART)[1], the update equation is"

$$x^{k+1} = x^k + \lambda V^{-1} A^T W \left(b - Ax^k \right) \tag{3}$$

where $V_{jj} = \sum_{i=1}^{M} |A_{ij}|$ and $W_{ii} = \frac{1}{\sum_{j=1}^{N} |A_{ij}|}$

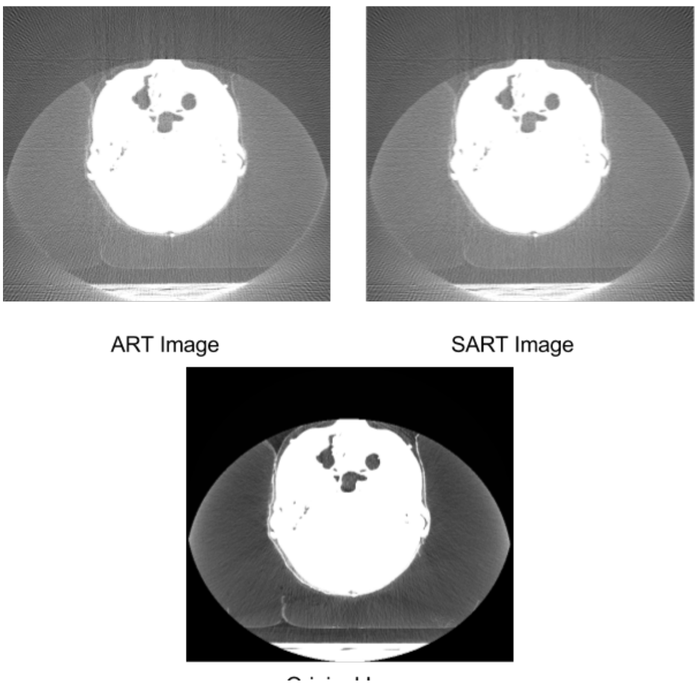

Figure 1: On the left is the ART reconstructed image and on the right is the SART reconstructed images, there is very slight difference between them since they are just different methods of finding the solution to the same system of equations but with different speeds. SART takes less time to get the result.

2 Back Projection Algorithms

Back Projection algorithms work on the basis given a single sinogram thread (reading for one angle), we simply smeare it in the opposite direction so as to get the actual image. The proof of this can be given by the fourier slice theorem. I have implemented simple BP, Filtered BP and Noise Filtered BP algorithms[3]. The simple BP gives you a blurred reconstructed image.

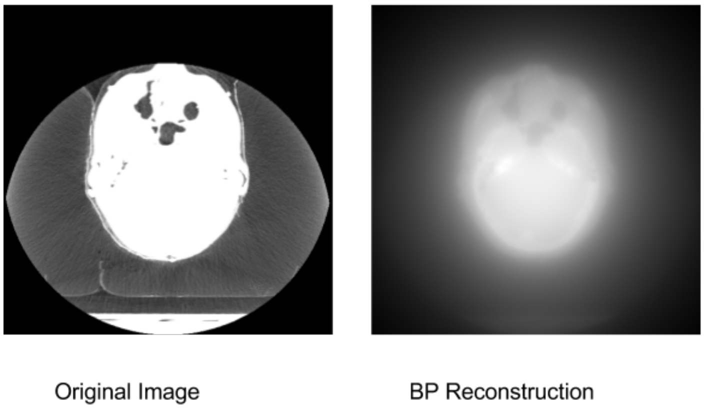

Figure 2: As you can see that BP reconstructed image is quite blurred although you can see to some extent the parts of the brain.

So simple BP doesn't give a good amount of informatation, so in order to tackle this problem we do something known as filtered BP in which before smearing the sinogram, we filter it out using 1D filters. There have been many filters proposed in the past. I have done my experiments with Hann, Hamming, Shepp Logan, Ramp filters.

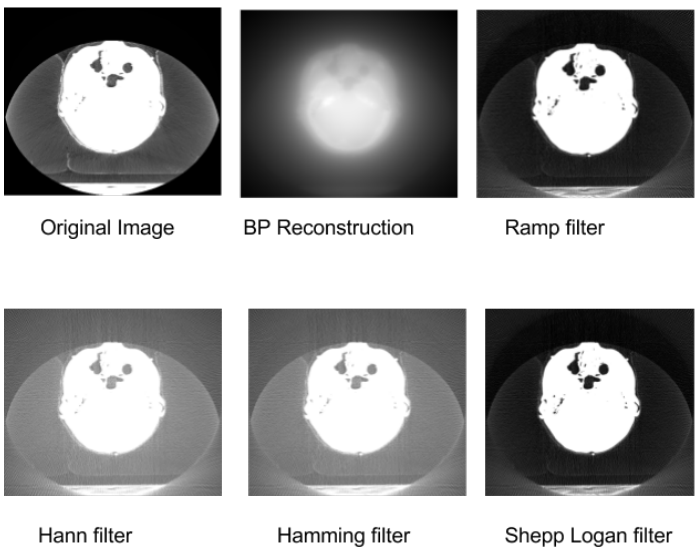

Figure 3: Comparison of outputs of different filters with the original ground truth.

2.1 Noise Weighted Filtered BP

NWFBP is given by the following:

$$F_n(w) = |w|. \left[1 - \left(1 - \alpha \frac{w_n}{|w|} \right)^k \right] \tag{4}$$

where $w_n = \frac{1}{0.1.n.p_{max}}$, $\alpha = 0.001$ and $k = 6000$

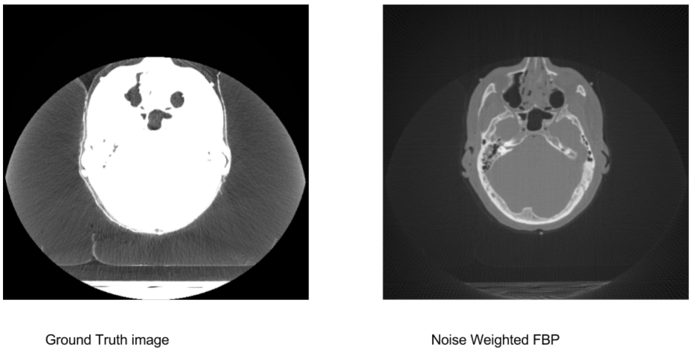

Figure 4: Comparison of outputs of Noise Weighted Filtered BP output with ground truth

Comparision of Ground Truth and Reconstructed images	
Method Name	PSNR error
Simple BP	-104.916002
Ramp Filter	-52.124499
Hann Filter	-50.997027
Hamming Filter	-51.072466
Shepp Logan Filter	-52.121333
Noise Weighted Filter	-49.315911

On an average Noise Weighted FBP perform relatively better than other filters.

[1] A. C. Kak A. H. Andersen. Simultaneous algebraic reconstruction technique (sart): A superior implementation of the art algorithm. *Ultrasonic Imageing*, 1984.

[2] Pengpen T Soleimani M. 2015 introduction: a brief overview of iterative algorithms in x-ray computed tomography. *Phil. Trans. R. Soc. A 373: 20140399.*, 2015.

[3] Gengsheng L. Zeng. Comparison of a noise-weighted filtered backprojection algorithm with the standard mlem algorithm for poisson noise. *Journal of Nuclear Medicine Technology*, 2013.