

10 Задача Матрица - логора
(абсолютно непрерывная)
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Пример 1

• Пусть $P \subset X$: $\mu(X)$ неограничен и неограничен.
($\forall p \neq q \in X$: $f \in \mu(X)$, $f(p) \neq f(q)$)
 $\forall p \in X$: $f : \mu_p(f) = 1$) Тогда P
не имеет предела.

• Случай абсолютной (неограниченной) непрерывности
тогда $P \subset X$: $X = A \cup B$,

$p_1, \dots, p_m \in X$, $z_1, \dots, z_{p_i} = \text{loc. roots.}$
 $r_1(z_1), \dots, r_m(z_m)$ ($r_i(z) = \sum_{j=1}^{n_i} c_j z^j$, $c_j \in \mathbb{C}$) .

I : $\exists f \in \mu(X)$: $\forall i$: $r_i(z_i) = \text{value.}$

тогда $P \subset X$: P_i ($\forall p_i (f - r_i) \geq m_i$)

! Пусть не выполнено $P \not\subset P_1, \dots, P_m$

• T.P.P. : $X - AK \quad \forall p \in p.v. X$

die $L(p) = \deg D + g(X) + 1 + \frac{\text{die } L''(1-D)}{\text{die } L(K-D)}$

- ~~apropos~~ ~~an~~

• $X - AK \quad M(X) = \text{maximaler } \mathbb{C}\text{-rank}$

$f \in M(X) \setminus \mathbb{C} \quad \text{tr des } M(X)/\mathbb{C} = 1$

$[M(X) : \mathbb{C}(f)] = \deg(f)$

Die lokale Dimension (z.B. maximal) liefert

$X - AK \quad p \in X \quad z_p = \text{loc. wgs } p$

$r(z_p) = \sum_{i=1}^n c_i z_p^i$

Def. Die lokale Dimension liefert alle

gesamten lokalen wgs $\sum_{p \in X} r_p(z_p) \cdot p$

Def. \mathcal{K} is a local system, \mathcal{O}_X is a sheaf. $T(X)$ (T-tick).

Let $D \in \text{Div}(X)$, $D = \sum r_p \cdot P = \sum D(P) \cdot P$

$T[D](X) = \{ \sum r_p \cdot P : r_p \neq 0, \sum_{i=1}^m c_i z_p^i, m < -D(P) \}$
 $\subset T(X)$

Prop. $D \geq 0$ $T\{0\}(X) \subset T[D](X)$ $r_p(z_p) = \sum_{i=1}^{-1} c_i z_p^i$
 $= \frac{c_{-1}}{z_p} + \frac{c_{-2}}{z_p^2} + \dots + \frac{c_n}{z_p^{-n}}$

Def. $t_D: T(X) \rightarrow T[D](X): \sum r_p(z_p) \cdot P \mapsto \sum r_{p,D}(z_p) \cdot P$

$r_p(z_p) = \sum_{n \leq i \leq n} c_i z_p^i \mapsto r_{p,D}(z_p) = \sum_{\substack{n \leq i \leq n \\ i < -D(P)}} c_i z_p^i$

Prop. $\mathcal{K}_p: \mathcal{K}(X) \rightarrow T[D](X) \quad \forall P$

$f(z_p) = \sum_{n \leq i} c_i z_p^i \mapsto r_{p,D}(z_p) = \sum_{n \leq i < -D(P)} c_i z_p^i \mapsto \sum_P (r_{p,D} \cdot P)$

Def $P_1 \leq P_2$

$$\iota_{P_2}^{P_1} : T(P_1)(X) \rightarrow T(P_2)(X) \quad - \text{geschieben}$$

Uebung... $\cong \iota_{P_2}(P)$

Oben "geschieben"

$$\mu_1^D : T(D)(X) \rightarrow T(D - (1))(X)$$

$$[r_i(z_i) \cdot p] \mapsto \iota_{p-(1)}([(1(z) r_i(z)) \cdot p])$$

$$\mu_1^D = \text{Uebung} \quad (\mu_1^D)^{-1} = \mu_{1/f}^{D-(1)}$$

$$(L(P) \cong L(P - (1)))$$

$$\text{Lemma: } 1) \mu(X) \xrightarrow{\alpha_{P_1}} T(P_1)(X) \xrightarrow{\downarrow \iota_{P_2}^{P_1}} T(P_2)(X)$$

$P_1 \leq P_2$
- weiter
gehen.

$$2) \mu_1(\alpha_P(g)) = \alpha_{p-(1)}(1g)$$

Def (in \mathcal{A}) : Two are called
 exact () if

$$G_0 \xrightarrow{d_1} G_1 \xrightarrow{d_2} G_2 \rightarrow \dots \rightarrow G_n$$

is exact, if $\text{Im } d_i \subseteq \text{Ker } d_{i+1}$

if $\text{Im } d_i = \text{Ker } d_{i+1}$: exact

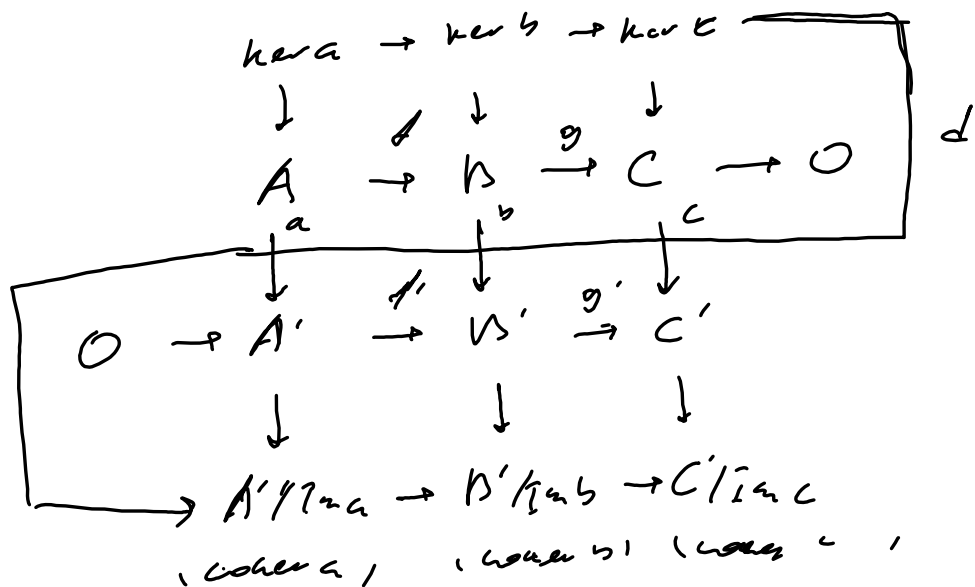
$$0 \rightarrow A \xrightarrow{f} B \xrightarrow{g} C \rightarrow 0$$

$\text{Im } f \subseteq \text{Ker } g$, $\text{Ker } f = 0$, $\text{Im } g = C$

$$C \cong B / \text{Ker } g \cong B / \text{Im } f$$

$f: A \rightarrow B$, $C \cong B / A$ - quotient

Lemma () :



- coher a, g coher

$\exists d$

1) Sur

Let $P_1 \leq P_2$ ($L(P_1) \leq L(P_2)$)

$$\begin{array}{ccccccc}
 0 \rightarrow H(X)/L(P_1) & \xrightarrow{\alpha_{P_1}} & T(P_1)(X) & \rightarrow & H'(P_1) & \rightarrow & 0 \\
 \downarrow & & \downarrow \epsilon_{P_1}^{P_1} & & \downarrow & & \\
 0 \rightarrow H(X)/L(P_2) & \xrightarrow{\alpha_{P_2}} & T(P_2)(X) & \rightarrow & H'(P_2) & \rightarrow & 0
 \end{array}$$

Thus $\ker(H'(P_1) \rightarrow H'(P_2)) = H'(P_1/P_2)$

Lemma: $P_1 \leq P_2$ $H'(P_1, P_2)$ - hochgradig
 v.1. d.h. $H'(P_1/P_2) = (\deg P_2 - \deg L(P_2)) -$
 $-(\deg P_1 - \deg L(P_1))$

□

$$\begin{array}{ccccccc} 0 & \rightarrow & K(X)/L(P_1) & \rightarrow & T\{P_1\}(X) & \rightarrow & H'(P_1) \rightarrow 0 \\ & & \downarrow a & & \downarrow b & & \downarrow c \\ 0 & \rightarrow & K(X)/L(P_2) & \rightarrow & T\{P_2\}(X) & \rightarrow & H'(P_2) \rightarrow 0 \end{array}$$

Es reicht \circ genü-

$$\begin{array}{ccccccc} & A & & B & & C & \\ 0 & \rightarrow & \ker a & \rightarrow & \ker b & \rightarrow & \ker c \rightarrow 0 \\ & \downarrow & & \downarrow & & \downarrow & \\ & L(P_2)/L(P_1) & & \ker t_{P_2}^{P_1} & & H'(P_1/P_2) & \end{array}$$

$$\ker t_{P_2}^{P_1} : Z \approx \left[r_p(z_p) \cdot P \in T\{P_1\}(X) \right]$$

$$r_p(z_p) = \sum_{-P_2(P) \leq k < -P_1(P)} c_k z_p^k$$

es $P_-(P) = P_+(P)$ wenn $r_P(2P)$


die $\ker \tau_{P_-}^{P_+} = \bigcup_P (P_-(P) - P_+(P)) = \deg P_- - \deg P_+$

die $L(P_-) / L(P_+) = \text{die } L(P_-) - \text{die } L(P_+)$.

$0 \rightarrow A \rightarrow B \rightarrow C \rightarrow 0$ — haben noch.
noch

h. e. $C = B/A$

2, die $C = \text{die } B - \text{die } A$

2, die $H^1(P_+/P_-) = (\deg P_- - \deg P_+) -$
 $-(\text{die } L(P_-) - \text{die } L(P_+))$ 

Lemma: $f \in M(X) \setminus \mathbb{C}$, $D = (f)_\infty$.

\exists no gen $n \geq 1$ haben die $H^1(O(nD))$

und s. (A) zeigen die $H^1(O(nD)) = \text{die } H^1(O(nD))$

$$\begin{aligned} \square \quad \dim H'(0/kD) &= (\deg kD - \dim L(kD)) - \\ &= (\deg 0 - \dim L(0)) = \\ &= 1 \quad (\text{X-nach.}) \end{aligned}$$

$$= k \deg D - \dim L(kD) + 1$$

$$D = (1) \sim$$

$$p \text{ genau } k \text{ mal} \quad \{ \mu(X) : p(L) \} = \deg D$$

$$\exists k_0 \quad \forall k \geq k_0 \quad \dim L(kD) \geq (k - k_0 + 1) \deg D$$

$$\dim H'(0/kD) \leq k \deg D - (k - k_0 + 1) \deg D + 1 =$$

$$= (k_0 - 1) \deg D + 1$$

$$k_0 \leq k \text{ hochster } k \text{ ord.}$$

$$\text{Folgerung} \quad 0 < k_1 < k_2$$

$$0 < k_1 D < k_2 D$$

$$k_0 = H'(0/k, D)$$

$$H'(0) \longrightarrow H'(k_1 D)$$

$$\downarrow$$

$$\leftarrow \text{ord } k_1 D$$

$$\begin{array}{l} \nearrow \\ k_0 = \\ H'(0/k_1, D) \end{array}$$

$$H'(k_2 D)$$

$$\Rightarrow H'(0/k_1, D) \subseteq H'(0/k_2, D)$$

$$\Rightarrow \dim H'(0/k_1, D) \leq \dim H'(0/k_2, D)$$

$$\Rightarrow \exists k_0 \text{ ver. } \leq \text{ nach. } k_0 \quad \square$$

$D = (f)_\infty \quad \exists M \quad \text{deg } u_D - \dim L(u_D) \leq M$

Lemma. $\forall A \in \text{Div } X \quad \exists M$:

$\text{deg } A - \dim L(A) \leq M$

$\square \quad f \in K(X), \quad D = (f)_\infty, \quad \exists M$:

$\text{deg } u_D - \dim L(u_D) \leq M$

\forall Lemma $\forall A \in \text{Div } X \quad \exists g \in K(X)$.

$\exists \alpha \quad B = A - (g) \leq u_D$

$\text{deg } B = \text{deg } A, \quad L(B) \simeq L(A)$,

$\text{deg } A - \dim L(A) = \text{deg } B - \dim L(B) =$

$[B \leq u_D]$

$= (\text{deg } u_D - \dim L(u_D)) - \dim L(B, u_D) \leq$

$\leq \text{deg } u_D - \dim L(u_D) \leq M \quad \square$

Beh. - : Es in A_0 - neues gel
des A_n - die $L(A_0)$ schen, wo

$$\mu'(A_0) = 0$$

\square $\exists y \in A_1$ $L'(A_2) \neq \emptyset$, $\text{w. a. } \exists z \in T \Sigma A_2 B(X)$

$$Z \neq \Delta_0(H) \quad \forall I \in \mu(X)$$

\mathcal{H}_{gen} B - ~~set~~ : $A_0 \subseteq \mathbb{N}$ \wedge $t(\mathbb{Z}) = 0$
 $\wedge \quad \tau[\mathbb{N}] (X)$

$$2, \quad \{t(z)\} = 0 \quad \text{and} \quad L'(1)$$

$$2) \quad [Z]_{H'(A_1)} \in H'(A_1/B)$$

$$2, \quad L'(A_0/B) \neq 0$$

$$1 \leq \dim H^1(A, \mathcal{O}) = (\deg A - \dim L(A)) - (\deg A_0 - \dim L(A_0))$$

$$\deg P_0 = \dim L(P_0) \leq \deg P - \dim L(N) - 1$$

$$- \gamma < \subset \text{ then } A,$$

2) $\mu'(A_0) = 0$ 

Lemma: $\forall P \in \text{Div}(X)$ d.h. $h'(P) < \infty$

□ Für A_0 - nat. $h'(A_0) \geq 0$

$$D - A_0 = P - N \quad P, N \geq 0$$

$$A_0 \leq A_0 + P \Rightarrow h'(A_0) \leq h'(A_0 + P)$$

- offensichtlich

$$\geq h'(A_0 + P) \geq 0$$

$$h'(P) \geq h'(A_0 + P - N) \geq h'(A_0 + P - N / A_0 + P)$$

d.h. $< \infty$

$$h'(P) = \lim_{(P, P)} \text{hohes hohes} \quad \square$$

Lemma: $X = AK$, $P \in \text{Div } X$

$$\dim L(P) = \dim h'(P) \geq \deg D + 1 = \dim h'(O)$$

□

H.v. es z_p - kein $h_0 > 4$. 1 p

$$\omega = \left(\sum_{h \geq D(P)} c_h z_p^h \right) dz_p$$

$$\text{Oz: } \sum_P r_p(z_p) \cdot p \mapsto \sum_P \text{Res}_p(r_p \omega)$$

$$\text{Res}_\omega : T\{D\}(X) \rightarrow \mathbb{C}$$

$$\text{Lemma: } \forall f \in \mathcal{H}(X) \quad \text{Res}_\omega(L_P(f)) = 0$$

$$(\text{u.e. } L_P(f) \in \ker \text{Res}_\omega)$$

$$\square \quad L_P(f) = \sum_P r_p(z_p) \cdot p$$

$$f(z_p) = \sum_n a_n z_p^n \quad r_p(z_p) = \sum_{h < -D(P)} a_h z_p^h$$

$$\text{Res}_\omega L_P(f) = \sum_P \text{Res}_p(r_p \omega)$$

$$r_p \omega = \left(\sum_{h < -D(P)} a_h z_p^h \cdot \sum_{n \geq D(P)} c_n z_p^n \right) dz_p$$

$$\text{Res}_p(r_p \omega) = \sum_{h \geq D(P)} c_h a_{-1-h} = \text{Res}_p(f \omega)$$

$$(f \omega = \sum_n a_n z_p^n \cdot \sum_{h \geq D(P)} c_h z_p^h) \quad \left(\text{hier ist } \sum_{h \geq D(P)} c_h z_p^h \text{ die } \frac{1}{z_p} \text{ der } \omega \right)$$

$$\sum_P \text{Res}_p(r_p \omega) = \sum_P \text{Res}_p(f \omega) = 0 \quad \text{denn } \sum_P \text{Res}_p(f \omega) = 0$$

$$\Rightarrow \text{Res}_\omega L_P(f) = 0 \quad \square$$

$$\text{H.v. } \sum_n L_P \subset \ker \text{Res}_\omega$$

$$\text{Res}_\omega \text{ Oz } T\{D\}(X) / \ker \text{Res}_\omega$$

$$\Rightarrow \text{Oz } T\{D\}(X) / \sum_n L_P \cong H^1(P)$$

$$\text{Res}_\omega \text{ — nat. isomorphism } \text{ in } H^1(P)$$

$$\text{u.e. } \text{Res}_\omega \in H^1(P)^*$$

$$H^1(P)^* = \{ \text{nat. isom. in } T\{D\}(X) : \sum_n L_P \}$$

$$\forall \omega \in L^{(1)}(-D) \quad \omega \mapsto \text{Res}_\omega \in H^1(P)^*$$

$$\text{H.v. } \text{Res} : L^{(1)}(-D) \Rightarrow H^1(P)^* \cong H^1(P)$$

$$\text{Proposition (geometrische Version):}$$

$$\text{Res} : L^{(1)}(-D) \rightarrow H^1(P)^* \text{ — isom.}$$

$$\text{d.h. } H^1(P)^* \cong \text{d.h. } L^{(1)}(-D)$$

$$\text{d.h. } H^1(P) \cong \text{d.h. } L(K-D)$$

$$\square \text{ Ser 9.4, (Mir) } \square$$

Lemma 1 die $H'(0) = g(X)$

□ $k = (0)$, da $k = 2g - 2$

die $H'(k) = \text{die } L(k - k) = \text{die } L(0) = 1$

die $H'(0) = \text{die } L(k)$

p.p. \rightarrow ~~422~~ Lemma

die $L(k) - \text{die } H'(k) = \text{da } k + 1 \rightarrow \text{die } H'(0)$

die $L(k) + \text{die } H'(0) = \text{da } k + 1 + \text{die } H'(k)$

$$\frac{\quad}{2 \text{ die } H'(0)} \quad 2g - 2 + 1 + 1 = 2g \quad \square$$

Lemma $g = g(X) = \frac{\text{die } H'(0)}{2 \text{ die } L''(0)} = \text{die } L(k) = \frac{\text{die } L''(X)}{2}$

die $H'(0)$ neu ~~offen~~ hoch

Theorem (P. P):

$$\dim L(D) = \deg D + 1 - g + \dim L(K - D)$$
$$\stackrel{u}{\dim} L''(-D)$$

Corollary (Riemann)

$$\dim L(D) \geq \deg D + 1 - g$$

Lemma , $\deg D \geq 2g - 1$

$$\dim L(D) = \deg D + 1 - g, \quad \dim L'(D) = 0$$

m.o. da $D \geq 2g - 1 \quad \forall z \in T(D)(X)$

$$\exists z \in K(X) : L_D(z) = \mathbb{C}.$$