

10. Bagara Matheur - legende
 (daher unter "unlösbar")
 2. 10. Neuer Punkt Pola

Proben satz

- Kann PR X : $\mu(X)$ nur endlich in der $(\forall p \neq q \in X \quad \exists f \in \mu(X), \quad f(p) \neq f(q))$
- $\forall p \in X \quad \exists f : \kappa_f(f) = 1$) dann PR ist auch endlich.
- Charakteristisch (ganzes menge zusammen
zum max. lokale). $X = A_K$,
 $p_1, \dots, p_n \in X$, $z_i = z_{p_i} - \text{lok. roots}$.
 $r_1(z_1), \dots, r_n(z_n)$ ($r_i(z) = \sum_{j=0}^{m_i} c_j z^j$, $c_0 \neq 0$).
- T: $\exists f \in \mu(X) : A : r_i(z_i) - \text{reala.}$
Zum max. lokale $\leq p_i$ (da $v_{p_i}(f - r_i) \geq m_i$).
! Hierbei ist zulässig $p \notin \{p_1, \dots, p_n\}$

• T. P. P.: $X - Ak$ $\forall p \in P \cup X$

$$\text{diz } L(p) = \deg D + g(X) + 1 + \begin{cases} \text{diz } L^{(1)}(-D) \\ \text{diz } L(K-D) \end{cases}$$

- asymptotisch

• $X - Ak$. $M(X)$ - numerische Klass. ℓ -rach.
 $\ell \in M(X) \setminus C$. $\operatorname{tr} \deg M(X)/C = 1$,

$$[\{M(X) : C(\ell)\}] = \deg (\ell)_\infty$$

Durchschnittsmaße (zu einem) haben

$X - Ak$ $p \in X$ z_p - vektor. werte r_p

$$r(z_p) = \sum_{i=1}^n c_i z_p^i$$

OZ: Durchschnittsmaße lösen alle
geg. vektoren werte $\sum_{p \in X} r_p(z_p) \cdot p$

DVS. fakto. loken \Rightarrow sp. venn,

sons. $T(X)$ ($T - \text{fkt. } \in$) .

Even $D \in \text{Div}(X)$, $D = \sum n_p \cdot p = \sum D(p) \cdot p$

$T(D)(X) = \{ \sum r_p \cdot p : r_p \neq 0 \}$,

$$\subset T(X) \quad r_p(z_p) = \sum_{i=1}^m c_i z_p^i \quad m < -D(p)$$

Num. $D \Rightarrow T\{\text{O}\}(X) \Leftarrow r_p(z_p) = \sum_{i=1}^{-1} c_i z_p^i$
 $= \frac{c_1}{z_p} + \frac{c_2}{z_p^2} + \dots - \frac{c_n}{z_p^n}$.

Def. $t_D: T(X) \rightarrow T\{D\}(X) : \sum r_p(z_p) \cdot p \mapsto \sum p r_p(z_p) r$

$$r_p(z_p) = \sum_{n \leq i \leq m} c_i z_p^i \mapsto r_{p,D}(z_p) = \sum_{n \leq i \leq m} c_i z_p^i$$

$$i < -D(p)$$

ausrechnen $d_D: \mathcal{M}(X) \rightarrow T\{D\}(X) \quad \forall p$

$$d_D(z_p) = \sum_{n \leq i} c_i z_p^i \mapsto r_{p,D}(z_p) = \sum_{n \leq i < -D(p)} c_i z_p^i \mapsto \sum_p (r_{p,D}(z_p))$$

Dann $P_1 \leq P_2$

$\iota_{P_2}^{P_1} : T(P_1)(x) \rightarrow T(P_2)(x)$ - *Generieren*
oder.. $\ni P_1(p)$.

Only "Standard"

$\mu_1^D : T\Sigma D(X) \rightarrow T(D - (f_1))(X)$

$[r_1(z, \cdot) \cdot p] \mapsto t_{D-(f_1)}([f(z)r_1(z)] \cdot p)$

$\mu_1^D = \text{Id. } (\mu_1^D)^{-1} = \mu_{D-(f_1)}^{D-(f_1)}$

($L(P) \simeq L(D - (f_1))$)

Kern: 1) $\mu(X) \xrightarrow{\alpha_{P_1}} T(P_1)(X)$ $P_1 \leq P_2$
 $\downarrow \iota_{P_2}^{P_1}$ $T(P_2)(X)$ - *neuer Standard*.

2) $\mu_1(\alpha_P(g)) = \alpha_{P-(f_1)}(g)$

$$\Rightarrow \ker \Delta_D = L(D)$$

D \subseteq \mathbb{R}

then mean lograte: p_1, \dots, p_s
 $v_1(z_{p_1}), \dots, v_s(z_{p_s})$.

? $\exists \lambda \in \mu(\lambda) : v_{p_i}(1-p_i) \geq 0, 1 \leq i \leq s$
 $\forall p \in \{p_1, \dots, p_s\} v_p(1) \geq 0$.

The mean growth $z \in T\Sigma D^1(X)$

? $z \in \bar{I} \cap \Delta_D$ ($\Delta_D: M(X) \rightarrow T\Sigma D^1(X)$)

Out.: $T\Sigma D^1(X)/\bar{I} \cap \Delta_D$ (z called Δ_D)
 $\cong H^1(D)$.

F.O. mean

$$0 \rightarrow L(D) \rightarrow M(X) \xrightarrow{\Delta_D} T\Sigma D^1(X) \rightarrow H^1(D) \rightarrow 0$$

$$0 \rightarrow M(X)/L(D) \rightarrow T\Sigma D^1(X) \rightarrow H^1(D) \rightarrow 0$$

OZ (zu winkeln): Zum ein weiteren
zyppe (..) \rightarrow zusammen
 $G_0 \xrightarrow{f_1} G_1 \xrightarrow{f_2} G_2 \rightarrow \dots \xrightarrow{f_n} G_n$
 los machen, da $\text{Im } f_i = \text{Ker } f_{i+1}$
 hoffen kann zusammen: korrektur

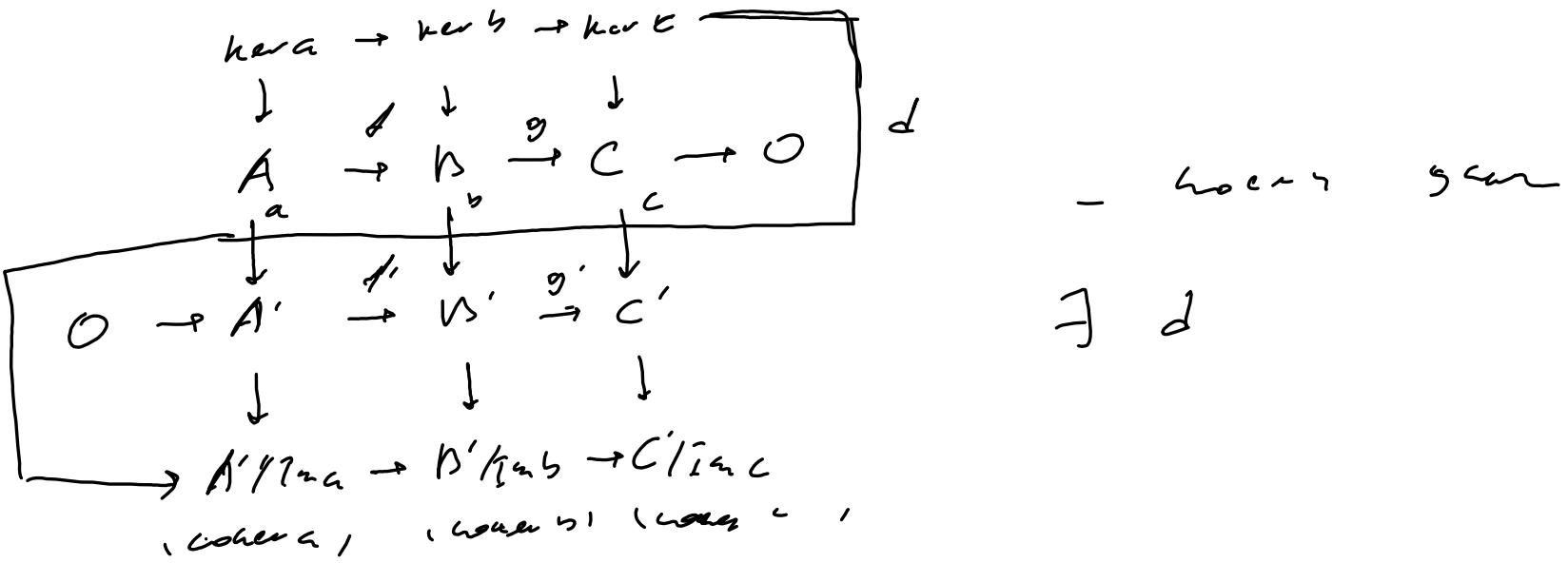
$$0 \rightarrow A \xrightarrow{f} B \xrightarrow{g} C \rightarrow 0$$

$$\text{Im } f = \text{Ker } g, \quad \text{ker } f = 0, \quad \text{Im } g = C$$

$$C \cong B / \text{Ker } g \cong B / \text{Im } f$$

$$f: A \rightarrow B, \quad C = "B/A" - \text{quotient}$$

Achtung (\rightarrow Merke):



\square $\Sigma \cong \mathbb{Z}$

$$\text{Sau } P_i \leq P_{i-1} \quad (L(P_i) \leq L(P_{i-1}))$$

$$\begin{array}{ccccccc}
 0 & \rightarrow & H(X)/L(P_i) & \xrightarrow{\alpha_{P_i}} & T\Sigma P_i(X) & \rightarrow & H'(P_i) \rightarrow 0 \\
 & & \downarrow & & \downarrow t_{P_i}^* & & \downarrow \\
 0 & \rightarrow & H(X)/L(P_{i-1}) & \xrightarrow{\alpha_{P_{i-1}}} & T\Sigma P_{i-1}(X) & \rightarrow & H'(P_{i-1}) \rightarrow 0
 \end{array}$$

$$\text{Dann } \text{Ker}(H'(P_i) \rightarrow H'(P_{i-1})) = H'(P_i/P_{i-1})$$

Lemma: $P_1 \leq P_2$ $H'(P_1, P_2)$ - hoch o. lch
 \Leftrightarrow $\dim H'(P_1/P_2) = (\dim P_2 - \dim L(P_2)) -$
 $- (\dim P_1 - \dim L(P_1))$

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$$0 \rightarrow H(X)/I(P_+) \rightarrow T\{P_+\}(X) \rightarrow H'(P_+) \rightarrow 0$$

\downarrow^a \downarrow^b \downarrow^c

$$0 \rightarrow H(X)/I(P_-) \rightarrow T\{P_-\}(X) \rightarrow H'(P_-) \rightarrow 0$$

$$0 \rightarrow \ker a \xrightarrow{A} \ker b \xrightarrow{B} \ker c \rightarrow 0$$

$L(P_1)/L(P_2)$ $\ker t \frac{P_1}{P_2}$ $L'(P_1/P_2)$

$$\ker t_{P_0}^P : \mathbb{Z} = [r_0(z_0) \cdot p \in T(P_0)](X)$$

$$r_p(z_p) = \int_{-P_2(p)}^{-P_1(p)} c_k z_p^k$$

$$\text{es en } P_-(P) - P_+(P) \quad \text{wenn } r_r < r$$

$$\dim \ker t_{P_-}^{P_+} = \bigcap_P (P_-(P) - P_+(P)) = \deg P_- - \deg P_+$$

$$\dim L(P_-) / L(P_+) = \dim L(P_-) - \dim L(P_+).$$

$$0 \rightarrow A \rightarrow B \rightarrow C \rightarrow 0 \quad - \text{holomorphen} \quad - \text{nach}$$

h.a. $C = B/A$

$$\Rightarrow \dim C = \dim B - \dim A$$

$$\begin{aligned} \Rightarrow \dim L'(P_+/P_-) &= (\deg P_- - \deg P_+) - \\ &- (\dim L(P_-) - \dim L(P_+)) \end{aligned}$$

Klausur: $f \in \mu(x) \setminus \mathbb{C}$, $D = (f)_\infty$.

\Rightarrow nur $n \geq n_0$ haben $\dim L'(O/KD)$

Ansatz: ($\forall n \geq n_0 \quad \dim L'(O/(Kd)D) = \dim L'(O/KD)$)

$$\begin{aligned} \text{D} \quad \dim H^1(\mathcal{O}, \kappa D) &= (\deg \kappa D - \dim L(\kappa D)) - \\ &- (\deg \mathcal{O} - \dim L(\mathcal{O})) = \\ &\quad " \mathcal{O} \quad " \quad \text{, (X-meth.)} \end{aligned}$$

$$z \in \deg D = \dim L(4D) + 1$$

D = (1) ω

पर तारे के $\{ \mu(X_i : L) \} = \text{des } D$

$$\dim L(uh) \geq (k - k_0 + 1) \deg h$$

$$\dim \mu'(D \cap D) \leq \deg D - (\kappa \cdot \kappa, +) \deg D + 1 =$$

$$z(k, -1) \text{ des } D + 1$$

Mr. C. Cochran & Son.

$$\text{By } \varphi_{\alpha} \quad \alpha < \kappa_1 < \kappa_2 \quad 0 < \kappa_1 D < \kappa_2 D \\ \kappa_2 = \varphi_{\alpha}'(0/\kappa_1 D)$$

$$H'(0) \longrightarrow H'(\eta, D)$$

↓ ← main site

$\ker \varphi$

$\mu'(m, D)$

$$\Rightarrow h'(0/a, D) \leq h'(0/a_-, D)$$

$$z_1 \in \text{d}^{\circ} \mathcal{U}'(0/\mathfrak{m}, D) \leq \text{d}^{\circ} \mathcal{U}'(0/\mathfrak{m}_2, D)$$

=> } wo wi. < word. Mar- 15

$$D = (\mathcal{L})_n \quad \exists \mu \quad \deg_{\infty D} - \dim L(\infty D) \leq \mu$$

Aussage. $\forall A \in \text{Div } X \quad \exists \mu :$

$$\deg A - \dim L(A) \leq \mu$$

$$\square f \in \mu(X), \quad D = (\mathcal{L})_n, \quad \exists \mu :$$

$$\deg \infty D - \dim L(\infty D) \leq \mu$$

μ auswählen so dass $\forall A \in \text{Div } X \quad \exists g \in \mu(X)$.

$$\exists \alpha \quad B = A - (g) \leq \infty D$$

$$\deg B = \deg A, \quad L(B) \cong L(A),$$

$$\deg A - \dim L(A) \geq \deg B - \dim L(B) =$$

$$\{\deg B \leq \infty D\}$$

$$\geq (\deg \infty D - \dim L(\infty D)) - \dim L(\infty D) \leq$$

$$\leq \deg \infty D - \dim L(\infty D) \leq \mu \quad \text{□}$$

Kern - : $\sum A_\alpha$ - reines Sch
 $\deg A_\alpha = \dim L(A_\alpha)$ erster, , mo

$$H'(A_\alpha) = 0$$

\square \exists z s.t. $H'(A_\alpha) \neq 0$, m.a. $\exists z \in T\mathcal{E} A_\alpha \setminus \{x\}$
 $z \neq \alpha_{A_\alpha}(t) \quad \forall t \in \mathcal{M}(X)$

Thm B - sch : $A_\alpha \leq B$ \wedge $t(z) = 0$
 $\wedge T\mathcal{E} B \setminus \{x\}$

$$\Rightarrow \{t(z)\} = 0 \quad \wedge \quad H'(B)$$

$$\Rightarrow [z]_{H'(A_\alpha)} \in H'(A_\alpha \setminus B)$$

$$\Rightarrow H'(A_\alpha \setminus B) \neq 0$$

$$1 \leq \dim H'(A_\alpha \setminus B) = (\deg B - \dim L(\partial)) - \\ - (\deg A_\alpha - \dim L(A_\alpha))$$

$$\deg A_\alpha - \dim L(A_\alpha) \leq \deg B - \dim L(B) - 1$$

$$\rightarrow \subset \text{ker } A_\alpha$$

$$\Rightarrow H'(A_\alpha) \neq 0$$

Zoone, $\forall D \in \text{Div}(X)$ d.h. $H^*(D) < \infty$

$D = D_0 - \text{vanne}$, $H^*(A_0) = 0$

$$D - A_0 = P - N \quad P, N \geq 0$$

$A_0 \leq A_0 + P = H^*(A_0) \rightarrow H^*(A_0 + P)$
- omsa vek.

z, $H^*(A_0 + P) = 0$

$$H^*(P) \geq H^*(A_0 + P - N) \simeq H^*(A_0 + P - N / A_0 + P)$$
$$\text{dim} < \infty$$

$H^*(P)$ - homolog vek. \square
 (P, P)

Eckenen: $X = \mathbb{A}^n$, $D \in \text{Div } X$

$$\text{dim } L(D) - \text{dim } H^*(D) = \deg D + 1 - \text{dim } H^*(O)$$

\square

$$D_+ \leq D_-$$

$$H'(P_+), H'(P_-), H'(P_+ | P_-) - \dim < \infty$$

$$\dim H'(P_+ | P_-) = \dim H'(P_+) - \dim H'(P_-)$$

$$(\deg P_+ - \dim L(P_-)) - (\deg P_- - \dim L(P_+))$$

$$\geq \dim L(P_+) - \deg P_+ - \dim H'(P_+) = \\ = \dim L(P_+) - \deg P_- - \dim H'(P_-)$$

$$\overline{L}_1 < \dim L(P) - \deg P - \dim H'(P) \leq \omega_1 + 2$$

$$\text{In } P=0 : 1-0-H'(0) \quad \square$$

$$H'(P) = L^{(1-P)}, \text{ - wahr?}$$

$$\text{Für } D \subset \text{Div } X \quad \omega \in L^{(1-D)} \quad ((\omega) \geq D) \quad \forall p \quad v_p(\omega) \geq D(p)$$

\bar{z}_p en z_p - roen $\omega \geq 0$. 1 p

$$\omega = \left(\sum_{n \geq D(p)} c_n z_p^n \right) dz_p$$

$$O_{\Omega} : \int_{\Gamma} r_p(z_p) \cdot p \mapsto \int_{\Gamma} \text{Res}_{\omega}(r_p \omega)$$

$$\text{Res}_{\omega} : T\{\Omega\}(X) \rightarrow \mathbb{C}$$

$$\underline{\text{Kern}} : \forall f \in \mathcal{M}(X) \quad \text{Res}_{\omega}(\omega_p(f)) = 0$$

$$(\text{u.e. } \omega_p(f) \in \text{ker Res}_{\omega})$$

$$\square \quad \omega_p(f) = \int_{\Gamma} r_p(z_p) \cdot p$$

$$f(z_p) = \sum_m a_m z_p^m \quad r_p(z_p) = \sum_{m < -D(p)} a_m z_p^m$$

$$\text{Res}_{\omega} \omega_p(f) = \int_{\Gamma} \text{Res}_p(r_p \omega)$$

$$r_p \omega = \left(\sum_{m < -D(p)} a_m z_p^m \cdot \sum_{n \geq D(p)} c_n z_p^n \right) dz_p$$

$$\text{Res}_p(r_p \omega) = \int_{\Gamma} c_1 a_{-1-n} = \text{Res}_p(f \omega)$$

$$f \omega = \sum_m a_m z_p^m \sum_{n \geq D(p)} c_n z_p^n,$$

$$\text{Kern } \rightarrow \text{u.e. } \frac{1}{z_p} \text{ ist der neue } \omega$$

$$\int_{\Gamma} \text{Res}_{\omega}(r_p \omega) = \int_{\Gamma} \text{Res}_p(\omega) = 0 \quad \text{d.h. } T$$

$$\therefore \text{Res}_{\omega} \omega_p(f) = 0 \quad \square$$

\tilde{z}_p in $\Omega_p \subset \text{ker Res}_{\omega}$

$\text{Res}_{\omega} : O_{\Omega} / \text{Ker Res}_{\omega}$

\hookrightarrow $O_{\Omega} / \text{Ker Res}_{\omega} \cong H^*(P)$

Res_{ω} - nah. operation in $H^*(P)$

u.e. $\text{Res}_{\omega} \in H^*(P)^*$

$H^*(P)^* = \{ \text{nah. op. in } T\{\Omega\}(X) : \sum \omega \in \text{Ker } \omega \}$

$\forall \omega \in L^{**}(-P) \quad \omega \mapsto \text{Res}_{\omega} \in H^*(P)^*$

m.e. $\text{Res} : L^{**}(-P) \rightarrow H^*(P)^* \cong H^*(P)$

Reziproz (yoursame Cech):

$\text{Res} : L^{**}(-P) \rightarrow H^*(P)^* - \text{nah.}$

$\dim H^*(P)^* = \dim L^{**}(-P)$

$\dim H^*(P) = \dim L^{**}(k-P)$

\square Sehr schön, mir) \square

Kenn, die $h'(0) = g(x)$

$\square h = \text{const}$, der $h = 2^g - 2$

die $h'(k) = \text{die } L(h-k) = \text{die } L(0) = 1$

die $h'(0) = \text{die } L(k)$

p. p. \rightarrow zu zeigen

$\text{die } L(k) - \text{die } h'(k) = \deg k + 1 = \text{die } h'(0)$

$\text{die } L(k) + \text{die } h'(0) = \deg n + 1 + \text{die } h'(k)$

$2 \cdot \text{die } h'(0)$

rechnen $\underline{g = g(x)} = \frac{\text{die } h'(0)}{2 \cdot \text{die } L''(0)} = \text{die } L(k) =$
 $= \text{die } S'(x)$

die $h'(0)$ neu erhalten haben

Frage (P, P):

$$\dim L(P) = \deg P + 1 - g + \begin{matrix} \dim L(K-P) \\ " \\ \dim L''(-P) \end{matrix}$$

Clausurklausur

$$\dim L(D) \geq \deg D + 1 - g$$

Lemma, $\deg D \geq 2g - 1$

$$\dim L(D) = \deg D + 1 - g, \text{ um } h'(D) = 0$$

$$\therefore \deg D \geq 2g - 1 \quad \forall z \in T(D)(X)$$

$$\exists z \in \mu(X), \quad \lambda_b(z) = z.$$