

полюс	$A - \bar{I}$	\bar{S}	z	Продолжение
	целая		алгебр.	целая

Примечание $p \leq 2$: $\mathbb{Z}[i] = \{a + bi, a, b \in \mathbb{Z}\}$
 $\mathbb{Z} = (1+i)|(1-i) \subset \mathbb{Q}(i)$

Опред. $\alpha \in \mathbb{C}$ - алгебр. элемент - если

$$f(x) \in \mathbb{Q}[x]$$

Алгебр. α над \mathbb{Q} - если алгебр., если

$$\exists g \in \mathbb{Z}[x] : g(\alpha) = 0$$

Примечание: i - простое алгебр. число $x^2 + 1 = 0$

Лемма: $r \in \mathbb{Q}$ - простое алгебр. (2) $r \in \mathbb{Z}$

□ .. □

Then $\gamma_1, \dots, \gamma_l$ — linearly indep.

$V = \left\{ \sum_{i=1}^l r_i \gamma_i \mid r_i \in \mathbb{Q} \right\}$ — basis of V over \mathbb{Q}

$$\dim_{\mathbb{Q}} V \leq l$$

Let $\gamma_1, \dots, \gamma_l$ — linearly indep. in V — then basis of V over \mathbb{Q} .
 Then for $\alpha \in \mathbb{C}$ let $\forall \gamma \in V$ $\alpha \gamma \in V$.

Let α — linearly indep.

1) Let $\alpha \gamma_1, \dots, \alpha \gamma_l$ — linearly indep. in V .

2) $\gamma_i \in V$ for $1 \leq i \leq l$ and $\alpha \gamma_i \in V$

$$\alpha \gamma_i = \sum_{j=1}^l a_{ij} \gamma_j \quad a_{ij} \in \mathbb{Q} \quad 1 \leq i \leq l$$

$$\Leftrightarrow \sum_{j=1}^l (a_{ij} - \delta_{ij} \alpha) \gamma_j = 0, \quad \delta_{ij} = \begin{cases} 1 & i=j \\ 0 & i \neq j \end{cases}$$

$$\Rightarrow \det (a_{ij} - \delta_{ij} \alpha) = 0, \quad \alpha \in \mathbb{C}$$

$$\det L = 0$$

2) d_1, d_2 - linearly independent

$$d_1^n + r_1 d_1^{n-1} + \dots + r_n = 0 \quad d_2^n + s_1 d_2^{n-1} + \dots + s_n = 0$$

V - vector space $d_1^i d_2^j$ $0 \leq i \leq n$
 $0 \leq j \leq n$

$\gamma \in V$ $d_1 \gamma, d_2 \gamma \in V$

$$\gamma = \sum b_{ij} d_1^i d_2^j \quad d_1 \gamma = \sum b_{ij} d_1^{i+1} d_2^j$$

$$d_2 \gamma = \sum b_{ij} d_1^i d_2^{j+1}$$

2) $d_1 \gamma + d_2 \gamma \in V$ $\Leftrightarrow (d_1 + d_2) \gamma \in V$

2) $d_1 + d_2$ - linearly independent

Assume $d_1, d_2 \gamma \in V$ 2) $d_1 + d_2$ - linearly independent

d_1 linearly independent $d_1^n + r_1 d_1^{n-1} + \dots + r_n = 0$ $d_2^n + s_1 d_2^{n-1} + \dots + s_n = 0$

$$d_1 + r_1 d_1^{n-1} + \dots + r_n d_1^{-1} = 0$$

2) d_1^{-1} - linearly independent, h.o. hold. \square

Q₁₂: Try R - homom (hom. es.)
 R has homom has R even:
 R - as group in eqs. group.
 Def. 1: R has Def. 2 R:

- 1) $r(x+y) = rx + ry$, $r \in R$, $x, y \in M$
- 2) $(r+s)x = rx + sx$, $r, s \in R$, $x \in M$
- 3) $(r \cdot s)x = r(sx)$
- 4) $1 \cdot x = x$, $x \in M$
 $\in R$

Q₁₃: R - homom has homom.
 Def. 1: $\exists \alpha_1, \dots, \alpha_n$, $\forall \alpha \in M$:

$$\alpha = \sum_{i=1}^n a_i \alpha_i \quad a_i \in R$$

Def. $\omega_1, \dots, \omega_n$ - small group with
 no $v = \sum_{i=1}^n r_i \omega_i$: $r_i \in \mathbb{F}$ - homom.
 before homom \mathbb{F}

□ Yes 

Lemma: If w_1, \dots, w_n are linearly independent
 $w = \sum_{i=1}^n r_i w_i : r_i \in \mathbb{F}$. Then $w \in L$

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2) Then we can show that L is a subspace

\square Let $w_1, w_2 \in L$, $\alpha, \beta \in \mathbb{F}$

Then $\alpha w_1 + \beta w_2 \in L$

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Lemma: Let L be a subspace of V . Then L is a subspace of V

Let $u, v \in L$, $\alpha, \beta \in \mathbb{F}$. Then $\alpha u + \beta v \in L$

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Lemma: f - belieb. mon. Abb. \mathbb{Z}
 des f zu, wo f nicht n sein
 können.

Def. \mathbb{Z} - hom. Abb. g zu.
 mon. $f \in \mathcal{O}(X)$ wo \mathbb{Z} - res
 woz.

Sei \mathbb{Z} - mon. Abb.

$$\mathcal{O}(\mathbb{Z}) = \left\{ \frac{g(\mathbb{Z})}{h(\mathbb{Z})}, g, h \in \mathcal{O}(X), h(\mathbb{Z}) \neq 0 \right\}$$

$$\mathcal{O}(\mathbb{Z}) = \{ g(\mathbb{Z}) : g \in \mathcal{O}(X) \}$$

- v.a. h hom. Abb.

Lemma: \mathbb{Z} - surj. Abb. u

$$\mathcal{O}(\mathbb{Z}) \cong \mathcal{O}(\mathbb{Z}) \quad \square \quad \mathbb{Z}$$

$\mathcal{O}(L, 1/Q)$ - further notes
Lemma 1 I - lemma lemma lemma 2
 $\mathcal{L} \in \mathcal{L} \subset \mathcal{H}$, $\bar{u} \in [Q(L, 1/Q)] \subset \mathcal{H}$

Def 1: There $F \subset \mathbb{C}$ with measure
 $\mu(F; Q) < \infty$

Lemma 2 R - the value value $\geq \forall x \in R$
 \mathcal{L} - lemma lemma $\in [F; Q]$

Def 2: F - the value value

$\mathcal{D} \subset \mathcal{D}_F \subset \{ \mathcal{L} \in F : f(\mathcal{L}) \geq 0, f \in \mathcal{F}(\mathbb{R}^+) \}$
 f - lemma

$\mathcal{D}_F \subset F$ with measure lemma lemma lemma

17: $\mathcal{D} \subset \mathcal{F}(\mathbb{R}^+)$ - lemma lemma lemma

Lemma: $\forall \beta \in \mathbb{F} \quad \exists b \in \mathbb{Z}_{\neq 0} : b\beta \in \mathcal{D}_{\mathbb{F}}$

\square \mathbb{F} - vektorraum über \mathbb{F}

$$\beta^2 + b, \beta^{n-1} + \dots + b, \dots, b, \dots \in \mathcal{D}$$

$$a_0 \beta^n + a_1 \beta^{n-1} + \dots + a_n = 0 \quad a_i \in \mathbb{F}$$

$$(a_0 \beta)^n + a_1 (a_0 \beta)^{n-1} + a_2 a_0 (a_0 \beta)^{n-2} + \dots = 0$$

$a_0 \beta$ - vektorraum $\mathcal{D} \subseteq \mathbb{F} \{x\}$ - vektorraum

Lemma: \mathbb{F} - vektorraum, $\mathcal{D} \subseteq \mathcal{D}_{\mathbb{F}}$

$\mathbb{F} \subset \mathcal{D}$ - vektorraum. $\mathbb{F} \subset \mathcal{D}_{\mathbb{F}}$

Sei $\mathbb{F} \subset \mathcal{D}$

\square $(\beta_i)_{i=1}^n$ - Sei $\beta_i \in \mathbb{F} \subset \mathcal{D}$, $\sum \beta_i \in \mathcal{D}$

$\exists b \in \mathbb{Z}_{\neq 0} : b\beta_i \in \mathcal{D}_{\mathbb{F}}, i=1, \dots, n$

$\lambda \in \mathbb{I} \quad \Rightarrow \quad b, p; \lambda \in \mathbb{I}$

$b, p; \lambda$ - bel. wesch. $\mathbb{I} \mathbb{Q}$

\Rightarrow - System (12)

Lemma: $\mathbb{I} \subset \mathbb{D}$ - asym un $\mathbb{I} \cap \mathbb{I}^{\neq}$
u asym \mathbb{I}^{\neq}

\mathbb{I} typ $\lambda \in \mathbb{I} \subset \mathbb{D} : \lambda^n + a_1 \lambda^{n-1} + \dots + a_n = 0$

- bel. know, $a_n \neq 0$

$\Rightarrow a_n = -\lambda^n - a_1 \lambda^{n-1} - \dots \in \mathbb{I}$
 $\in \mathbb{I}$

$a_n \in \mathbb{I} \cap \mathbb{I}^{\neq}$ (12).

Cases, types of vector spaces

L/K - field, $[L:K] = 2$

L_1, \dots, L_n - type L/K

$\forall L \subset L$ $L \subset L$ $[a_{ij}]$

Def: $N_{L/K}(L) = \det(a_{ij})$

$\text{Tr}_{L/K}(L) = \text{tr}(a_{ij})$

Lemma: Def is ver. or not

or not

\square ... \square

Def: L/K - separable, or $\text{Tr}_{L/K} \neq 0$

Def: $\text{Gal}(L/K) = \{ \sigma: L \rightarrow L : \sigma(x) = x \forall x \in K \}$

Definition: L/K - normal extension, then
 $[L:K] = 2^n$, and $|Gal(L/K)| = 2^n$

Def.: L/K is Galois extension
 $[L:K] = |Gal(L/K)|$

Lemma: L/K - normal extension,

$Gal(L/K) \cong \{ \sigma_1, \dots, \sigma_n \}$. Then

$$\mu_{L/K}(\alpha) = \prod_{i=1}^n \sigma_i(\alpha) \quad \tau_{L/K}(\alpha) = \sum_{i=1}^n \sigma_i(\alpha)$$

Lemma: L/K - cyclic extension, then

$\exists \alpha \in L$ such that $L = K(\alpha)$ and the
 minimal polynomial of α over K is

Lemma: L/K - cyc., then $\alpha \in L$

f - poly. poly. \mathbb{C} $f(x) = x^n + a_1 x^{n-1} + \dots + a_n$

$\text{Gal}(\mathbb{C}/\mathbb{K}) \cong \langle \sigma_1, \dots, \sigma_n \rangle$

$$f(x) = \prod_{i=1}^n (x - \sigma_i(\alpha))$$

$$\text{Ker } L_{\mathbb{K}}(\alpha) = (-1)^n a_0, \quad \text{Tr } L_{\mathbb{K}}(\alpha) = -a_{n-1}$$

3.4 : $\alpha^{(i)} = \sigma_i(\alpha) \in \mathbb{C}$ - conj. n \mathbb{C}

17 : $\mathbb{Q}(i)$ $a+bi$, $a-bi$ - conj.

$$N = a^2 + b^2 = (a+bi)(a-bi)$$

Q.42 : $\alpha_1, \dots, \alpha_n \in \mathbb{C}$

$$\Delta(\alpha_1, \dots, \alpha_n) = \det(\text{Tr } L_{\mathbb{K}}(\alpha_i \alpha_j))$$

has such property when $\alpha_1, \dots, \alpha_n$

Lemma: Given $\Delta(\alpha_1, \dots, \alpha_n) \neq 0$ then

$\alpha_1, \dots, \alpha_n$ - linearly independent in L/K .

Or: given L/K - ext. α (α_i) - linearly

ind. $\Delta(\alpha_1, \dots, \alpha_n) \neq 0$

Lemma: Given $(\alpha_i)_{i=1}^n, (\beta_i)_{i=1}^n$ - gl

linearly independent, $\alpha_i = \sum_{j=1}^n a_{ij} \beta_j$ $1 \leq i \leq n$

$$\Delta(\alpha_1, \dots, \alpha_n) = \det(a_{ij})^2 \Delta(\beta_1, \dots, \beta_n)$$

Lemma: L/K - ext.

$$\Delta(\alpha_1, \dots, \alpha_n) = (\det(\alpha_i^{(j)}))^2$$

Lemma: f - irr. mon. $\beta \in L$

L/K - free. $1, \beta, \dots, \beta^{n-1}$ - linearly indep.

$$\Delta(1, \lambda, -\lambda^{n-1}) = (-1)^{\frac{n(n-1)}{2}} \mu(\lambda(1)).$$

Proposition K field \Rightarrow then \Rightarrow

wh hold reason

$$K/\mathbb{Q} \quad \Rightarrow \quad K:\mathbb{Q} = n$$

$$\text{Recall: } \alpha \in D_n \quad \Rightarrow \quad \mu_{K/\mathbb{Q}}(\alpha) \cdot \text{Tr}_{K/\mathbb{Q}}(\alpha) \in \mathbb{Z}$$

$$\alpha_1, \dots, \alpha_n \in D_n \quad \Rightarrow \quad \text{span } K/\mathbb{Q} \quad \Rightarrow$$

$$\Delta(\alpha_1, \dots, \alpha_n) \in \mathbb{Z}$$

$$\text{Recall: } I \subset D_n \quad \Rightarrow \quad \text{span}$$

$$\alpha_1, \dots, \alpha_n \in I \quad \Rightarrow \quad \text{span} \quad \text{max} \quad \text{then}$$

$$|\Delta(\alpha_1, \dots, \alpha_n)| = \text{det}.$$

$$I \subseteq \mathbb{Z} \alpha_1 + \dots + \mathbb{Z} \alpha_n$$

04. The last sum (4) has
 user only.

Let $(\alpha_i), (\beta_i)$ — given

system $\alpha_i \in [\alpha_i, \beta_i]$

(α_i) — given, $\det \neq 0$

$$D(\alpha_1, \dots, \alpha_n) = D(\beta_1, \dots, \beta_n)$$

we given have n of α_i δ_k .

On the other hand permutation

$\mathbb{Z}[i]$ $\mathbb{Z}[\omega]$ — given here.

Other view: therefore $\mathbb{Z}[\sqrt{-5}]$

$$\mathbb{Z}[\sqrt{-5}] \cong (1 + 2\sqrt{-5})(1 - 2\sqrt{-5})$$

up, the other (24.)

K - maximal ideal $D \subsetneq D_K$ - maximal
ideal $(\mathbb{Z}[i], \mathbb{Z}[\sqrt{5}])$

$P \subset D$ - maximal ideal \Rightarrow
 $\exists p \in P \Rightarrow \exists p \in P \vee p \in P$

Lemma: $I \subset D$ - maximal ideal

D/I - local ring with maximal ideal
ideal $I \subset (a)$, $a \in \mathbb{Z}_{>0}$. $|D/(a)| = a^k$

1) $I \subset D$, $I \cap D \neq \emptyset \neq \emptyset$

$\exists a \in I \cap \mathbb{Z}$ $a \neq 0$, $a \in \mathbb{Z}$

$(a) \subset I \Rightarrow D/I \subset D/(a)$

$\exists (a_1, a_2) \subset I$ - maximal ideal K ,

$$\mathcal{D} \approx \mathbb{Z} \omega_1 + \dots + \mathbb{Z} \omega_n$$

$$\text{Theorem} \quad \mathcal{S} = \{ \lfloor c_i \omega_i : 0 \leq c_i < a, c_i \in \mathbb{Z} \}$$

$$\forall \omega \in \mathcal{D} \quad \omega = \sum_{i=1}^n m_i \omega_i \quad m_i \in \mathbb{Z}$$

$$\forall i \quad m_i = q_i a + c_i \quad 0 \leq c_i < a$$

$$\omega \equiv \lfloor c_i \omega_i \pmod{a}$$

$$\text{So} \quad \lfloor c_i \omega_i \equiv \lfloor c_i' \omega_i \pmod{a} \quad 0 \leq c_i, c_i' < a$$

$$\lfloor (c_i - c_i') \omega_i \equiv 0 \pmod{a}$$

$$\Rightarrow \gamma a, \quad \gamma \in \mathcal{D}$$

$$\gamma = \lfloor b_i \omega_i$$

$$\gamma a = \lfloor b_i a \omega_i \quad \Rightarrow \quad c_i - c_i' = b_i a$$

$$c_i \equiv c_i' \pmod{a} \Rightarrow c_i = c_i'$$

$$D(a) \cong S, \quad |D(a)| = a^{\aleph_1} \quad \square$$

Def: R heißt hämmernd wenn

$$\forall I_1 \subset I_2 \subset \dots \subset I_n \subset \dots$$

oder $\exists \mu : \forall n \geq \mu$

$$I_n \supseteq I_{n+1}$$

Lemma: D - vervollständigt.

\forall I_n hämmernd \Rightarrow P stetig beschränkt.

\square gilt. \square

Lemma: I, J - unter D . Then

$$1) \text{ für } I \supseteq J, \text{ wo } J \supseteq D$$

$$2) \text{ für } \exists \omega \in D, (\omega) I \supseteq J I, \text{ wo } (\omega) \supseteq J$$

$$\square \quad \underline{1}, \underline{2}, \dots, \underline{n} \in I \quad - \quad \text{Knoten } D$$

$$I \subseteq I' \quad \forall : \quad \underline{x}_i \in \bigcup_{j \in I} \beta_{ij} \underline{x}_j, \quad \beta_{ij} \in \mathbb{R}$$

2, det $(A_{ij} - \delta_{ij}) = 0$

$$\parallel 1 \neq [(\sim, \beta) : j = 0 \quad \& \quad 1 \in J = 2) \quad j = 2]$$

$$2/ \quad (\omega) \quad \bar{I} \quad 2 \quad \bar{I} \quad \bar{I}$$

$$\beta \in \mathbb{J} \quad \beta \alpha_1 = \omega \alpha_1 \quad \left(\frac{\beta}{\omega} \right) \cdot \mathbb{I} \subset \mathbb{I}$$

$$w_1, \dots, w_k \quad - \quad \sqrt{k} \quad D \quad | \quad \in D \quad \sim$$

$$\sim \mathbb{E} w_1 + \dots + \mathbb{E} w_k$$

L' - 205 m. 1 A

$$\frac{\Delta}{\omega} \cdot \gamma \in \mathbb{I} \quad \forall \quad \gamma \in \mathbb{I} \quad \text{z,} \quad \frac{\Delta}{\omega} \in \mathbb{D}$$

$$\mathbb{I} \subset \omega, \quad \frac{1}{\omega} \mathbb{I} \subset \mathbb{D} - \text{wenn}$$

$$(\omega) \mathbb{I} \approx \mathbb{I}$$

$$\mathbb{I} \approx (\omega^{-1}) \mathbb{I}$$

$$\stackrel{1)}{=} \frac{1}{\omega} \mathbb{I} \approx \mathbb{D}$$

$$\Leftrightarrow \mathbb{I} \approx (\omega)$$

Q4: Given $\mathbb{I}, \mathbb{J} \in \mathcal{D}$ let

such, can $\exists \alpha, \beta \in \mathcal{D}$:

$$(\alpha) \mathbb{I} \approx (\beta) \mathbb{J} \quad \left(\left(\frac{\alpha}{\beta} \right) \mathbb{I} \approx \mathbb{J} \right)$$

Answer: given. standard. $\mathbb{I} \sim \mathbb{I}$.

$\mathbb{D} \dots$

Q4: Given $|\mathcal{D}| \sim 1$ how many
of them $n_k \approx |\mathcal{D}| \sim 1$ (for $n_k \approx \infty$)

Answer: $n_k \approx 1 \Leftrightarrow \mathcal{D} = K \mathbb{I} K$.

Answer: K - unit vector, $\mathcal{D} \approx \mathcal{D}_u$

$\exists \mu \in \mathbb{Z}_{>0} (\mu \geq \mu_n) : \forall \alpha, \beta \in \mathcal{D},$
 $\rho \neq 0 \quad \exists t, 1 \leq t \leq \mu, \exists \omega \in \mathcal{D}$

$$|\mu(\alpha - \omega \beta)| < |\mu(\beta)| \quad \mu \geq \mu_{n, Q}$$

$$\square \quad \gamma \in \frac{\mathbb{Z}}{\rho} \in K \quad \text{Yes.} \quad \text{then}$$

$$\exists \mu \quad \exists t \quad \exists \omega \in \mathcal{D} \quad |\mu(t\gamma - \omega)| < 1$$

$$\text{then } \omega_1, \dots, \omega_n \text{ is a system } \mathcal{D}$$

$$\gamma = \sum c_i \omega_i, \quad c_i \in \mathbb{Q}$$

$$|\mu \gamma| = \left| \prod_j \gamma^{(j)} \right| = \left| \prod_j \left(\sum_i c_i \omega_i^{(j)} \right) \right| \leq$$

$$\leq C \left(\max_i |c_i| \right)^s$$

$$C = \prod_j \left(\sum_i |\omega_i^{(j)}| \right)$$

$$\text{Then } \alpha > \sqrt{c} \quad \mu \in \mathbb{K}^n$$

$$c_i \in \mathbb{Q} \quad c_i = \begin{matrix} [c_i] + \lambda [c_i] \\ \text{"} a_i \text{"} \\ \in \mathbb{A} \end{matrix} \quad b_i \quad 0 \leq b_i < 1$$

$$\gamma = [c_i, w_i] \quad [\gamma] = [a_i, w_i] \quad \gamma = [\gamma] + \lambda \gamma \\ \lambda \gamma = [b_i, w_i]$$

$$\text{Then } \varphi: \mathbb{K} \rightarrow \mathbb{R}^n \quad [c_i, w_i] \mapsto (c_1, \dots, c_n)$$

$$\varphi(\gamma) = \varphi([\gamma]) + \varphi(\lambda \gamma)$$

$$\in \mathbb{R}^n \text{ with } \mathbb{K} = \\ = \{ (x_1, \dots, x_n) : 0 \leq x_i < 1 \}$$

$$\text{Then } \mathbb{K} \text{ has dimension } n$$

$$\frac{1}{n} \quad n^{\frac{1}{n}} \quad \text{unbounded}$$

$$1 \leq j \leq n^{\epsilon} + 1$$

$$\varphi(\{j\}) \subset K$$

$$2) \quad \exists j_1, j_2$$

$$1 \leq j_2 < j_1 \leq n^{\epsilon} + 1$$

$$\{j_1, \delta\}, \{j_2, \delta\} \in \text{ограничение}$$

$$n^{\epsilon} \leq n$$

$$j_1, \delta - j_2, \delta = \underbrace{\{j_1, \delta\} - \{j_2, \delta\}}_{\omega \in D} + \underbrace{\{j_1, \delta\} - \{j_2, \delta\}}_{\delta \in K}$$

$$\underbrace{(j_1 - j_2), \delta}_{\delta \in K}$$

$$\begin{aligned} & \text{но } \omega \in K \text{ и } \omega_j \\ & < \frac{1}{n} \end{aligned}$$

$$\mu \delta \leq \frac{1}{n^{\epsilon}} < 1 \quad \square$$

$$\text{теперь: } h_n < \infty$$

$$\square \quad I \subset D - \text{где}, \quad \lambda \in [1, \infty)$$

$$|\mu(\lambda)| \in \mathbb{R}_{>0}, \quad \lambda \in [1, \infty) - \text{некоторые}$$

$$\text{выбранные}$$

$$\exists \mu \quad \forall \alpha \in I \quad \exists \beta, \gamma \in \mu \quad \exists \omega \in \mathcal{D}$$

$$|\mu(\beta - \omega)| < |\mu(\beta)|$$

$$\alpha, \beta \in I \quad \beta - \omega \in I \quad \Rightarrow \quad \beta - \omega \geq 0$$

$$\beta \geq \omega$$

$$\text{Th. 0} \quad \mu: \beta \geq \omega, \beta \quad \mu: \alpha \in (\beta)$$

$$\Rightarrow \quad \mu: I \subset (\beta)$$

$$J = \bigcup_{\beta} \mu: I \quad : \quad \mu: I \subset (\beta) \quad \Rightarrow$$

$$\forall \alpha \in \mu: I \quad \exists \gamma \in \mathcal{D} \quad \alpha \geq \beta \gamma$$

$$J \subset \mathcal{D} - \text{where}$$

$$\mu: I \subset (\beta, J)$$

$$\beta \in I \quad \mu: \beta \in \mu: I \quad \mu: \beta \in (\beta, J) \\ \Rightarrow \mu: \in J \quad (\mu: \beta \geq \omega \gamma)$$

$$a \in \mathcal{A} \quad a \in \mathcal{I} \quad 2, (a) \in \mathcal{I} \quad 2, \mathcal{D} \cap \mathcal{I} \subset \mathcal{D}_1(a)$$

^
wobei.

\mathcal{I} ist linear. linear
wobei $\sup M \geq a$

$$M: \mathcal{I} = (\mathcal{P})$$

$$\mathcal{I} \sim \mathcal{J} \quad \mathcal{J} - \text{wobei linear}$$

$$M. a \quad |\mathcal{D}| \sim 1 \quad \text{wobei} \quad n < \infty$$

Lemma: $\mathcal{I} \subset \mathcal{D} - \text{wobei} \mathcal{D}, n \geq k_n$

$$\exists n \quad 1 \leq k \leq k_n \quad : \quad \mathcal{I}^n - \text{wobei}$$

$$\mathcal{I}^i \quad 1 \leq i \leq k_n \quad \exists j < k_n$$

$$\mathcal{I}^{j_1} \sim \mathcal{I}^{j_2} \Leftrightarrow \exists \alpha, \alpha' \in \mathcal{D} \quad (\alpha) \mathcal{I}^{j_1} \subset (\alpha') \mathcal{I}^{j_2}$$

$$A \subseteq B \subseteq C, \quad B \subseteq C$$

$$(A) \subseteq B \subseteq C \subseteq D \Rightarrow (A, B) \subseteq C \Rightarrow (A, B) \subseteq C \subseteq D$$

$$\Rightarrow \frac{A}{B} \in D \Rightarrow (A) \subseteq B \subseteq C \subseteq D$$

$$\in \omega$$

$$\Rightarrow B \subseteq C$$

$$B \subseteq C$$

□

Lemma 1

$$1) A, B, C \text{ — sets} \quad AB \subseteq AC \Rightarrow$$

$$2) B \subseteq C$$

$$2) A, B \text{ — sets} \quad B \supset A \Rightarrow \exists C \text{ — set}$$

$$A \subseteq B \subseteq C$$

$$\square \quad \supset) \quad \exists k. A^k \subseteq (\alpha)$$

$$A \cap \subseteq \subseteq C \quad \times \quad A^{k-1}$$

$$(\alpha) \cap \subseteq (\alpha) \cap C \quad \supset) \quad \cap \subseteq C$$

$$(\cap \in \cap \quad \alpha \cap \in (\alpha) \cap \quad \exists \omega \in \mathcal{D} \quad \gamma \in C : \\$$

$$\alpha \cap \subseteq \omega \subseteq \gamma \quad \supset) \quad \cap = \omega \gamma$$

$$\supset) \quad \cap \in C \quad)$$

$$2) \quad \exists k \quad \cap^k \subseteq (\cap)$$

$$A \subseteq \cap \quad \cap^{k-1} A \subseteq \cap^k \subseteq (\cap)$$

$$C \subseteq \bigcap_{\cap} \cap^{k-1} A \subseteq \mathcal{D} - \omega$$

$$\cap C \subseteq \bigcap_{\cap} (\cap) A \subseteq A \quad \square$$

Lemma 1: A - column. $\Rightarrow D (A \neq D)$

$A \approx \Pi P$, P - sq. \Rightarrow

$\Rightarrow \exists$ some P_1 $A \subset P_1$

2) $A \approx P_1 B_1$, $B_1 \subsetneq D \Rightarrow P_2$,
 $B_1 \subset P_2$

$A \approx P_1 P_2 B_2 \dots$

$A \subset B_1 \subset B_2 \subset \dots \subset B_t$

$A \approx P_1 \dots P_t$ \square

Def 1: P - sq. A - sq. $V_P(A) = V$:

$A \subset P^V$, $A \not\subset P^{V+1}$

Lemma: 1) $V_P(P) = 1$

2) $Q \neq P \quad V_P(Q) = 0$

3) $V_P(AB) = V_P(A) + V_P(B)$ yes \square or \square \square

Theorem: $A \subset D_K \quad \exists$ only Lemma

$A = \bigcap_P I^{a(P)} \quad a(P) = V_P(A)$

\square 1 non, new 1 new \geq new

$D \supset D_K$

$P \cap \mathbb{Z} \neq \emptyset \quad P \cap \mathbb{Z} = (P)$

$\mathbb{Z} \subset P - \mathbb{Z}$

$(P) - \mathbb{Z} \subset \mathbb{Z}$

$P \cap P \quad V_P(P)$

D/P - new

Lemma: $P \neq \emptyset \Rightarrow P$ has \Rightarrow

D/P - mod.

$\square - \square$

$$\mathbb{F}_p \cong \mathbb{Z}/p\mathbb{Z} \subset D/P \cong \mathbb{F}_{p^d}$$

Def: $e = v_P(p)$ - valuation level. w.

f - degree when

$$\text{over } (p) \cong p_1^{e_1} \cdots p_g^{e_g}$$

Lemma: $P \subset D$ - w. $D/P \cong \mathbb{F}_{p^d}$

$$\text{then } |D/P^e| \cong p^{ef}$$

\square w. g. to e :

$e \geq 1$ - when

$$e > 1$$

$$P^e \subsetneq P^{e-1}$$

$$\exists \alpha \in P^{e-1} \setminus P^e$$

$$(\alpha) + P^e \subset P^{e-1}$$

$$P^{e-1} \mid (\alpha) + P^e$$

$$P^e \subset (\alpha) + P^e$$

$$(\alpha) + P^e \mid P^e$$

$$(\alpha) + P^e = P^i \quad 1 \leq i \leq e$$

$$2. \quad P^{e-1} \mid P^i$$

$$i \geq e-1$$

$$\text{h.o.} \quad (\alpha) + P^e = P^{e-1}$$

$$\text{Track} \quad \varphi: D \rightarrow P^{e-1} \mid P^e$$

$$\varphi(\sigma) = \sigma\alpha + P^e \subset P^{e-1} \quad \forall \sigma \in D$$

$$\varphi - \text{surjective} \quad \text{why?}$$

$$\ker \varphi \supset \delta \quad : \quad \gamma \in P^e \quad (2, \quad \nu_P(\gamma) \geq e$$

$$\nu_P(\gamma) \geq \nu_P(\delta) + \nu_P(2) = \nu_P(\delta) + e + 1 \geq e$$

$$\gamma \in \ker \varphi \quad (2, \quad \nu_P(\gamma) \geq 1 \quad (2, \quad \gamma \in P$$

$$\ker \varphi = P,$$

$$\Rightarrow \quad \mathcal{D}/P \cong P^{e+1}, P^e$$

$$P^e \subset P^{e+1} \subset \mathcal{D} \quad \mathcal{D}/P^{e+1} \cong$$

$$\cong (\mathcal{D}/P^e)/(P^{e+1}/P^e)$$

$$(\mathcal{D}/P^e)/(P^{e+1}/P^e) \cong (\mathcal{D}/P^{e+1}) \cong P^{e+1}$$

QED

Lemma: R - unital c.c.s

A_1, \dots, A_g - c.s. $A_i \cdot A_j = R$

$\forall i \neq j \quad A_i \cdot A_j = 0$

$$R/A \cong R/A_1 \oplus \dots \oplus R/A_g$$

\square Also $K \cong$ b.c. \square

Lemma: P, Q - u.c.c.s. $P \neq Q$

$\forall u, v \in \mathbb{Z} \Rightarrow P^u + Q^v = 0$

\square See \square $D = D_K$

Theorem 1 $(P) = P_1^{e_1} \dots P_g^{e_g}$, $n = \sum_{i=1}^g k_i Q_i$
 A_1, A_g - u.c.c.s. P_i

$$\sum_{i=1}^g e_i A_i = n$$

$$\square \quad p_i^{e_i} \in p_i^{e_i} \in \mathcal{D}$$

$$\xrightarrow{h^0} \quad \mathcal{D}(p) \simeq \mathcal{D}(p_i^{e_i}) \oplus \dots \oplus \mathcal{D}(p_0^{e_0})$$

$$\text{then } \nearrow p^{\vee} \quad \nearrow p \in \mathcal{E}_i \mathcal{I}_i$$

$$\simeq \quad p \in \mathcal{E}_i \mathcal{I}_i \quad \square$$

$$\underline{\text{Lemma}} \quad p \in \mathcal{E} - \mathcal{H} \quad p_1, p_2 - \text{wh}$$

$$\text{has } p \in \mathcal{D} \quad (p) \subset p, \quad (p) \in p_1$$

$$p_1 \mid p \quad p_2 \mid p$$

$$\exists \sigma \in \text{Gal}(K/Q) : \sigma p_1 = p_2$$

$$\square \quad (p) \subset p_0 : p_0 \notin \sum \sigma p_i : \sigma \in \text{Gal}$$

$$k \neq 0 \quad \exists \mathcal{L} \subset \mathcal{D} :$$

$$\mathcal{L} \not\subseteq 0(P_0)$$

$$\mathcal{L} \not\subseteq 1(\sigma P_1)$$

$$\forall \sigma \in \text{hcl}$$



$$(\mathcal{L}) \subset P_0$$

$$\mu(\mathcal{L}) = \bigcap_{\sigma} \sigma(\mathcal{L}) = \mathcal{L} \cdot \bigcap_{\sigma} \sigma(\mathcal{L}) \in (\mathcal{L}) \subset P_0$$

$$\mu(\mathcal{L}) \in \mathbb{Z} \quad \text{z}, \quad \mu(\mathcal{L}) \in P_0 \cap \mathbb{Z} = (P)$$

$$(P) \subset P_1 \quad \text{z}, \quad \mu(\mathcal{L}) \in P_1$$

$$P_1 \mid \mu(\mathcal{L}) = \bigcap_{\sigma} \sigma \mathcal{L} \quad \text{z}, \quad \exists \sigma : P_1 \mid \sigma \mathcal{L}$$

$$\sigma \mathcal{L} \not\subseteq 0(P_1) \quad (\text{z}), \quad \mathcal{L} \not\subseteq 0(\sigma^{-1} P_1)$$

$$- > < \quad \text{图}$$

Theorem: K/Q - tower, $p \in \mathcal{A} - \mathcal{A}$

$$(P) = P_1^{e_1} \dots P_g^{e_g}$$

$$\text{Then } e_1 \dots e_g \geq e$$

$$f_1 \dots f_g \geq f$$

$$e f_g \geq h$$

$$\square \quad P_i \quad \forall i \quad \exists \sigma \quad \sigma P_i = P_i$$

$$D/P_i \cong D/\sigma P_i = D/P_i$$

$$\geq 1, f_i \geq f_i = f$$

$$(P) = \sigma(P) = (\sigma P_1)^{e_1} \dots (\sigma P_g)^{e_g}$$

$$= P_1^{e_1} \dots P_g^{e_g}$$

$$\sigma P_i = P_i$$

$$e_i \geq e_i = e$$

$$h \geq \sum_{i=1}^g e_i f_i \geq e f \geq e f_g \quad \square$$

$$\mathbb{Q}(\sqrt{2})$$

$$x^2 - d = 0$$

$$\mathcal{D}_{\mathbb{Q}(\sqrt{2})}$$

$$\neq \mathbb{Z}[\sqrt{2}]$$

$$\mathbb{Q}(\zeta)$$

$$, \quad \zeta^4 = 1$$

$$\mathcal{D}_{\mathbb{Q}(\zeta)}$$