

2 Функции на $P\Gamma$

Пример 1: $X - P\Gamma$ — метр. Косос.
 $P\Gamma$ со метрикой $d_{\text{Хаусдорфа}}$ и метрикой d_F .
 Метрику d_F — метрику

$$\mathcal{F} = \{ \varphi_\alpha : U_\alpha \rightarrow V_\alpha \} \quad U_\alpha \subset X, V_\alpha \subset \mathbb{C}:$$

$$1) \bigcup_\alpha U_\alpha = X$$

$$2) \varphi_\alpha - \text{сложн.} : \forall \alpha, \beta. T_{\alpha\beta} = \varphi_\beta \circ \varphi_\alpha^{-1}$$

— сложн. на $U_\alpha \cap U_\beta$ (если $U_\alpha \cap U_\beta \neq \emptyset$)

Пример 2:

$$1) \text{ Сфера } S^2 \subset \mathbb{R}^3. \quad \mathbb{C}_\infty$$

$$2) \text{ Метр. } \mathbb{C}/L \text{ где } L \subset \mathbb{C} - \text{прям.} \\ L = \langle w_1, w_2 \rangle$$



$$g = 1$$

(классы

1 053.

Case 4.

$$g > 0$$

— Choice

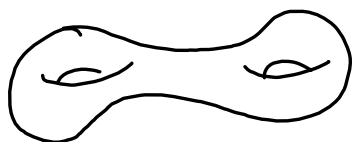
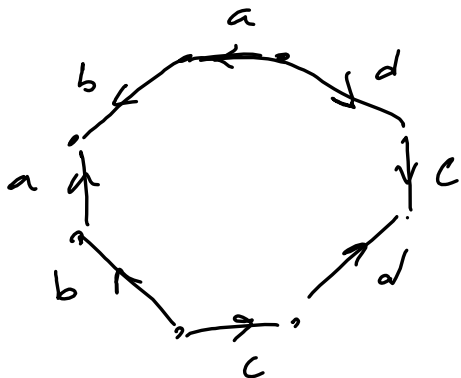
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успехом

9 2 2



g) $1p^1 \rightarrow 1p^1(2)$

9) 21.05.2022 by 06:00 in the lab

$$X = \{ (x, y, z) \in \mathbb{R}^3 : F(x, y, z) = 0 \}$$

$$F \in C[x, y, z]_d - \text{smooth point.}$$

Опр: $X - P \cap$, $p \in X$, $W \subset X$ - окр p
 $f: W \rightarrow \mathbb{C}$ - $q \sim p$ has. W has q rows p ,

еще \exists has $\varphi: U \rightarrow V$, $p \in U$:

$f \circ \varphi^{-1}$ has 1 $\varphi(p)$.

f - has W , еще W has.

$\forall q \in W$.

Лемма: X , $p \in X$, $W \subset X$, $f: W \rightarrow \mathbb{C}$
 - has W has W has

1) f has 1 $p \in W$ \forall has $\varphi: U \rightarrow V$

$p \in U$: $f \circ \varphi^{-1}$ has 1 $\varphi(p)$

2) f has W \exists $(\varphi_x: U_x \rightarrow V_x)$

$W \subset \cup U_x$ $\forall x$ $f \circ \varphi_x^{-1}$ has $W \cap U_x$

3) f has 1 $p \in W$ f has 1 окр p

□ 1) φ - map \Rightarrow sup.

$$f \circ \varphi^{-1} = (f \circ \varphi^{-1}) \circ (\varphi \circ \varphi^{-1}) = \text{rel.}$$

rel. rel.

2) φ : 1) \Rightarrow 2) , 1) \Leftarrow 2)

Lemma : Every $f \in \mathcal{G}$ - rel. \in $\mathcal{P}(W)$

no $f \notin \mathcal{G}$, $f \in \mathcal{G}$ - rel. \in $\mathcal{P}(W)$

□ 5.2 \mathbb{R}

Def : $\mathcal{O}_{X,W} = \mathcal{O}_W = \{ f : W \rightarrow \mathbb{C} - \text{rel.} \}$

has rel. $\varphi = \psi$.

Proposition .

1) $\varphi : U \rightarrow V$ - rel. . no $\varphi = \psi$

rel. $\varphi = \psi$

2) $X = \mathbb{C}_\infty$ $f(z)$ - rel. \in $\mathcal{O}_{X,\infty}$
(\Rightarrow)

(2) $f(\frac{1}{z})$ holomorphisch in $z \neq 0$.

Es sei $f(z) = \frac{p(z)}{q(z)}$ — rat. Fkt. ($p, q \in \mathbb{C}[z]$)

f holomorphisch in ∞ (2) dann $\deg p \leq \deg q$

3) $X = \mathbb{P}^1$, $p(z, w), q(z, w) \in \mathbb{C}[w, d]$

$p = [z_0 : w_0]$, $q(z_0, w_0) \neq 0$, dann

$f([z_0 : w_0]) = \frac{p(z, w)}{q(z, w)}$ — holomorphisch in z

$p = [z_0 : w_0]$

4) $X = \mathbb{C}/L$, $\pi: \mathbb{C} \rightarrow \mathbb{C}/L$; $z \mapsto z \bmod L$

f holomorphisch in p (2) \exists umkehrf. z

dann $p: f \circ \pi^{-1}$ holomorphisch in z

5) $X \subset \mathbb{P}^2$ — messbar geschlossene Menge

Функция (характеристика) области может

1) быть непрерывной: $\forall n < \infty \quad C_n = 0$

2) быть разрывной: $C_n \neq 0$ где n — натуральное число, $n < \infty$

т.е. $f(z) = \sum_{n=-\infty}^{\infty} C_n (z - z_0)^n$

3) быть функцией области: \exists точка.

Пусть $n < \infty$: $C_n \neq 0$

Одн. $X = P \cap D$, $P \in X$, w — вып. P

1) P — вып. отс., если \exists вып. φ :

$\varphi(P)$ — вып. отс. $\text{в } f \cdot \varphi^{-1}$

2) P — норм., если $\exists \varphi$: $\varphi(P)$ — норм. $\text{в } f \cdot \varphi^{-1}$

3) P — вып. отс., если $\varphi(P)$ — вып. отс.

Lemma : f unique graph. J.T. number.
 $1, 2, 3$ (2) \forall hyper φ , $\varphi(p)$ - unique.
 owl. work $f \circ \varphi^{-1}$ need $1, 2, 3$

□ Def (1)

Def : $X = p \cap$, $f : X \rightarrow \mathbb{C}$, $p \in X$

f max. hyperbolic groups 1 p , each
 min. f - value 1 p , and \rightarrow when
 given. $2i$ when h work 1 p
 f max. hyper. W , even f hyper
 1 2 $\forall 2 \in W$

Lemma : f, g - hyper. 1 p (in W),
 no $f \neq g$, f, g - hyper.; even $g \neq 0$
 no $\frac{f}{g}$ - hyper. 1 p (in W)

1) Def. $T_f = \frac{f}{g}$; Teorema de Weierstrass :
 $f, g \in \mathbb{C}$ h.e. $W \subset \mathbb{C}$ - komp. bere.
 no photo images in notebook group. [2]
Def. : $M_{X,W} = M_W = \{ f: W \rightarrow \mathbb{C} - \text{h.e.} \}$
 pers. versch. h.e. p-5

Proposition:

1) $X = \mathbb{C}$ - off. h.e. whole \mathbb{C}
 whole ab.

2) $X = \mathbb{C}_\infty$: f h.e. $1 \leq \infty$ (2)

$f(\frac{1}{z})$ h.e. $1 \leq \infty$

$\sum_{n=0}^{\infty} f(z) = \frac{p(z)}{q(z)}$ - pers. op. in

no h.e. $1 \leq \infty$

Th. 0. pers. op. in $\in M_{\mathbb{C}_\infty}$

$$3) \quad X = P^1 \quad f([z:w]) = \frac{p(z,w)}{q(z,w)}, \quad p, q \in \mathbb{C}[z,w]$$

- hkf. φ^{-1} :

$$(\quad \varphi = \varphi_1 : U \rightarrow \mathbb{C} \quad \varphi([z:w]) = \frac{z}{w} \\ \{w \neq 0\})$$

$$\varphi^{-1}(a) = \{a:1\}$$

$$f \circ \varphi^{-1}(a) = f(\varphi^{-1}(a)) = \frac{p(a,1)}{q(a,1)} \quad \text{hkf. \& a} \\ \text{hkf.}$$

$$z \neq 0 \quad - \text{ (hkf. hkf. hkf.)}$$

$$4) \quad X = \mathbb{C}/L \quad \pi: \mathbb{C} \rightarrow \mathbb{C}/L \quad f \text{ hkf. \& } v$$

$$\Leftrightarrow \quad g = f \circ \pi \text{ hkf. \& } \pi^{-1}(w)$$

$$g(z+w) = g(z) \quad \forall w \in L$$

$$5) \quad X: F(x,y,t) = 0, \quad G, H \in \mathbb{C}[x,y,t]$$

$$H \neq 0 \quad \frac{G(x,y,t)}{H(x,y,t)} = \text{hkf. \& } F$$

Def. X — P \cap , \nexists $p \in X$

$$z \approx \varphi(x) - \text{const}, \quad \varphi(r) = z_0$$

$$f(\varphi''(z)) = \sum_{n \geq k} c_n (z - z_0)^n = p \wedge, \quad c_n \neq 0$$

μ μ_2 μ_0 μ_1 μ_2 μ_3 μ_4 μ_5 μ_6 μ_7 μ_8 μ_9 μ_{10} μ_{11} μ_{12} μ_{13} μ_{14} μ_{15} μ_{16} μ_{17} μ_{18} μ_{19} μ_{20} μ_{21} μ_{22} μ_{23} μ_{24} μ_{25} μ_{26} μ_{27} μ_{28} μ_{29} μ_{30} μ_{31} μ_{32} μ_{33} μ_{34} μ_{35} μ_{36} μ_{37} μ_{38} μ_{39} μ_{40} μ_{41} μ_{42} μ_{43} μ_{44} μ_{45} μ_{46} μ_{47} μ_{48} μ_{49} μ_{50} μ_{51} μ_{52} μ_{53} μ_{54} μ_{55} μ_{56} μ_{57} μ_{58} μ_{59} μ_{60} μ_{61} μ_{62} μ_{63} μ_{64} μ_{65} μ_{66} μ_{67} μ_{68} μ_{69} μ_{70} μ_{71} μ_{72} μ_{73} μ_{74} μ_{75} μ_{76} μ_{77} μ_{78} μ_{79} μ_{80} μ_{81} μ_{82} μ_{83} μ_{84} μ_{85} μ_{86} μ_{87} μ_{88} μ_{89} μ_{90} μ_{91} μ_{92} μ_{93} μ_{94} μ_{95} μ_{96} μ_{97} μ_{98} μ_{99} μ_{100} μ_{101} μ_{102} μ_{103} μ_{104} μ_{105} μ_{106} μ_{107} μ_{108} μ_{109} μ_{110} μ_{111} μ_{112} μ_{113} μ_{114} μ_{115} μ_{116} μ_{117} μ_{118} μ_{119} μ_{120} μ_{121} μ_{122} μ_{123} μ_{124} μ_{125} μ_{126} μ_{127} μ_{128} μ_{129} μ_{130} μ_{131} μ_{132} μ_{133} μ_{134} μ_{135} μ_{136} μ_{137} μ_{138} μ_{139} μ_{140} μ_{141} μ_{142} μ_{143} μ_{144} μ_{145} μ_{146} μ_{147} μ_{148} μ_{149} μ_{150} μ_{151} μ_{152} μ_{153} μ_{154} μ_{155} μ_{156} μ_{157} μ_{158} μ_{159} μ_{160} μ_{161} μ_{162} μ_{163} μ_{164} μ_{165} μ_{166} μ_{167} μ_{168} μ_{169} μ_{170} μ_{171} μ_{172} μ_{173} μ_{174} μ_{175} μ_{176} μ_{177} μ_{178} μ_{179} μ_{180} μ_{181} μ_{182} μ_{183} μ_{184} μ_{185} μ_{186} μ_{187} μ_{188} μ_{189} μ_{190} μ_{191} μ_{192} μ_{193} μ_{194} μ_{195} μ_{196} μ_{197} μ_{198} μ_{199} μ_{200} μ_{201} μ_{202} μ_{203} μ_{204} μ_{205} μ_{206} μ_{207} μ_{208} μ_{209} μ_{210} μ_{211} μ_{212} μ_{213} μ_{214} μ_{215} μ_{216} μ_{217} μ_{218} μ_{219} μ_{220} μ_{221} μ_{222} μ_{223} μ_{224} μ_{225} μ_{226} μ_{227} μ_{228} μ_{229} μ_{230} μ_{231} μ_{232} μ_{233} μ_{234} μ_{235} μ_{236} μ_{237} μ_{238} μ_{239} μ_{240} μ_{241} μ_{242} μ_{243} μ_{244} μ_{245} μ_{246} μ_{247} μ_{248} μ_{249} μ_{250} μ_{251} μ_{252} μ_{253} μ_{254} μ_{255} μ_{256} μ_{257} μ_{258} μ_{259} μ_{260} μ_{261} μ_{262} μ_{263} μ_{264} μ_{265} μ_{266} μ_{267} μ_{268} μ_{269} μ_{270} μ_{271} μ_{272} μ_{273} μ_{274} μ_{275} μ_{276} μ_{277} μ_{278} μ_{279} μ_{280} μ_{281} μ_{282} μ_{283} μ_{284} μ_{285} μ_{286} μ_{287} μ_{288} μ_{289} μ_{290} μ_{291} μ_{292} μ_{293} μ_{294} μ_{295} μ_{296} μ_{297} μ_{298} μ_{299} μ_{300} μ_{301} μ_{302} μ_{303} μ_{304} μ_{305} μ_{306} μ_{307} μ_{308} μ_{309} μ_{310} μ_{311} μ_{312} μ_{313} μ_{314} μ_{315} μ_{316} μ_{317} μ_{318} μ_{319} μ_{320} μ_{321} μ_{322} μ_{323} μ_{324} μ_{325} μ_{326} μ_{327} μ_{328} μ_{329} μ_{330} μ_{331} μ_{332} μ_{333} μ_{334} μ_{335} μ_{336} μ_{337} μ_{338} μ_{339} μ_{340} μ_{341} μ_{342} μ_{343} μ_{344} μ_{345} μ_{346} μ_{347} μ_{348} μ_{349} μ_{350} μ_{351} μ_{352} μ_{353} μ_{354} μ_{355} μ_{356} μ_{357} μ_{358} μ_{359} μ_{360} μ_{361} μ_{362} μ_{363} μ_{364} μ_{365} μ_{366} μ_{367} μ_{368} μ_{369} μ_{370} μ_{371} μ_{372} μ_{373} μ_{374} μ_{375} μ_{376} μ_{377} μ_{378} μ_{379} $\mu_{380}</$

$$V_p(A) = \mu - 0.5 \sigma_y \mu.$$

Решение: $v_r(A)$ — искомое.

\mathbb{C}^n $\pi_{\mathbb{C}^n}$ ψ $-$ $g_{\mathbb{C}^n}$ $\psi_{\mathbb{C}^n}$ $\psi = \psi(x)$

$\psi(p) = \psi_0$, $\psi \sim \psi$ $T = \psi \circ \psi^{-1}$

$$T(w) = z \quad : \quad z \approx T(w) \approx z_0 + \frac{\sum_{k \geq 1} a_k (w - w_0)^k}{z - z_0}$$

$a_i \neq 0$ (1. $\forall i \in \overline{1, n}$) , n ist

$$\sum_{m \geq n} c_m' (w - w_n)^m = A(\psi^{-1}(w)) = A(\tilde{\varphi}^{-1}(\tilde{T}(w))) =$$

$$= \left[C_n \left(\sum_{k=1}^n a_k (w - w_0)^k \right) \right]^n = C_n (a_1 (w - w_0))^n$$

$$\Rightarrow \mu \geq \nu, \quad C_n \geq C_\mu a_1^n \neq 0 \quad \square$$

Lemma: $\bar{A} \supset \cup_{p \in X} \{p\}$ — help. $p \in X$

1) f — value p (\Rightarrow) $v_p \geq 0$

2) $f(p) \neq 0$ (\Rightarrow) $v_p(f) > 0$

3) p — value f ($f(p) = \infty$) (\Rightarrow) $v_p(f) < 0$

4) p — не имеет в \bar{A} — help (\Rightarrow) $v_p(f) = 0$

\square 5-я \square

Lemma: 1) $v_p(f \cdot g) = v_p(f) + v_p(g)$

2) $v_p(\frac{1}{f}) = -v_p(f)$, $v_p(\frac{f}{g}) = v_p(f) - v_p(g)$

3) $v_p(f \pm g) \geq \min(v_p(f), v_p(g))$

\square 6-я \square | \bar{A} — help. $0 < p < 1$
 $v_p(f) = p \cdot v_p(f)$ — help. help. help.

$$\text{Теорема: } X \subset \mathbb{C}_\infty \quad f \sim \frac{p}{q}$$

$$f \in \mathbb{C} \cap \prod (z - \lambda_i)^{e_i}, \quad \lambda_i - \text{zeros of } f$$

$$v_{\lambda_i}(f) = e_i, \quad v_\infty(p) = \deg q - \deg p = -\sum e_i$$

$$\Rightarrow \sum_{p \in X} v_p(f) = \sum e_i - \sum e_i = 0$$

$$\text{Corollary } \mathbb{C}_\infty$$

$$\text{Теорема: } \forall f \in \mathcal{M}_{\mathbb{C}_\infty} \quad f \sim \frac{p}{q} - \text{res. form}$$

\square $\mathcal{M}_{\mathbb{C}_\infty}$ is a vector space and \mathbb{C}_∞ is a subalgebra, i.e. zero div.

$\Rightarrow f$ is a rational function with poles at $0, \infty$ and $(\lambda_i)_{1 \leq i \leq k}$.

$$v_{\lambda_i}(z) = e_i, \quad \text{Then } r(z) = \prod_{i=1}^k (z - \lambda_i)^{e_i}$$

r - ~~max~~ g.p., (where $0 < \infty < \infty$)
~~order~~ C - ∞ - ∞ $\in C_\infty \setminus \infty \subset \mathbb{C}$)

$$\text{Then } g(z) = \frac{f(z)}{r(z)} - \text{h.p. g.p.}$$

for $0 < \infty < \infty$ $\in C_\infty \setminus \infty \subset \mathbb{C}$

2) g - ~~zero~~ $h.p.$ $\in \mathbb{C}$, h.c.

$$g(z) = \sum_{n=0}^{\infty} c_n z^n$$

$$g \text{ h.p. } \in \infty \quad g\left(\frac{1}{z}\right) = g(w), \quad w = \frac{1}{z}$$

$$g(w) = \sum_{n=0}^{\infty} c_n w^{-n} - \text{h.p. } \in \infty : \quad \Delta_n \neq 0$$

$$2) \quad g \in \mathbb{C}[z]$$

$$\text{Let } g \neq 0 \quad \exists z_0 : \quad g(z_0) = 0 \quad - \infty$$

$$2) \quad g(z) = c \quad \omega < z$$

$$f = c \cdot v \quad \text{②}$$

$$\underline{\text{Claim}}: \quad \forall f \in M_{C_\infty}$$

$$\sum_{p \in C_\infty} v_p(f) = 0$$

$$\underline{\text{Proposition.}} \quad \text{Lefschetz} \quad \text{③}$$

$$\text{Sei} \quad p, q \in (\{z, w\}^d, \quad q \neq 0$$

$$f = \frac{p}{q} \quad : \quad f(z, w) = w^d f\left(\frac{z}{w}, 1\right) =$$

$$= w^d c \prod \left(\frac{z}{w} - \lambda_i\right)^{e_i} = \prod (b_i z - a_i w)^{e_i}$$

$$\underline{\text{Theorem}} \quad \forall f \in M_{\mathbb{P}^1} \quad f = \frac{p}{q},$$

$$p, q \in (\{z, w\}^d$$

$$\square \quad \text{Sei} \quad \text{unimod.} \quad C_\infty \quad \text{④}$$

$$\underline{\text{Claim}}: \quad \forall f \in M_{\mathbb{P}^1} \quad \sum v_p(f) = 0$$

Thm 8.1

$$L \cong L(1, \omega) \cong L_\omega, \quad \omega \in H \cong \{z: \operatorname{Im} z > 0\}$$

$$\theta(\omega, z) \cong \theta_\omega(z) \cong \prod_{\ell \in \mathbb{Z}} e^{i(2\ell z + \ell^2 \omega)}$$

$$\theta(\omega) \cong \theta(\omega, 0)$$

$$\theta_\omega(z+1) \cong \theta_\omega(z), \quad \theta_\omega(z+\omega) \cong e^{-i(2z+\omega)} \theta_\omega(z) \quad \forall z \in \mathbb{C}$$

Lemma: 1) θ_ω - holom. on \mathbb{C}

$$2) \quad \theta_\omega(z_0) \neq 0 \iff \theta_\omega(z_0 + n + n\omega) \neq 0 \quad \forall n \in \mathbb{Z}$$

$$3) \quad \nu_{z_0}(\theta_\omega) = \nu_{z_0 + n + n\omega}(\theta_\omega)$$

4) Proof whenever holds

$$z_0 \neq \frac{1}{2} + \frac{\omega}{2} + n + n\omega, \quad \nu_{z_0}(\theta_\omega) \neq 1$$

$$\square \text{ } \zeta_L : \int_{p(1, \omega)} \frac{\Theta'(z)}{\Theta(z)} dz \quad (2)$$

Defn.

$$\Theta_{\omega}^{(x)}(z) = \Theta_{\omega}(z - \frac{1}{L} - \frac{\omega}{L} - x)$$

$$\text{Lemma: } \Theta_{\omega}^{(x)}(z+1) = \Theta_{\omega}^{(x)}(z)$$

$$\Theta_{\omega}^{(x)}(z+\omega) = -e^{-2\pi i (Lz-x)} \Theta_{\omega}^{(x)}(z)$$

$$\text{Theorem: } x_i, \zeta_i \in \mathbb{C} \quad 1 \leq i \leq d$$

$$\prod_{i=1}^d x_i - \prod_{i=1}^d \zeta_i \in \mathbb{Z}$$

$$\text{Proof: } f = \prod_{i=1}^d \Theta_{\omega}^{(x_i)}(z) / \prod_{i=1}^d \Theta_{\omega}^{(\zeta_i)}(z) \in \mathcal{M}_{0,L}$$

(Using f def. of \mathbb{C} in \mathbb{C} - hypothesis)

\square f - def. h.h. $\Theta_{\omega}^{(x)}$ def.

$$f(z+1) = f(z) \quad \text{as } \Theta_{\omega}^{(x)}$$

$$\begin{aligned}
 f(z+w) &= \prod_i \theta_w^{(x_i)}(z+w) \prod_j (\theta_w^{(y_j)}(z+w))^{-1} = \\
 &= \prod_i (-e^{-2\pi i(z-x_i)}) \theta_w^{(x_i)}(z) \prod_j (-e^{-2\pi i(z-y_j)}) \theta_w^{(y_j)}(z)^{-1} = \\
 &= e^{-2\pi i(\sum y_j - \sum x_i)} f(z) = f(z),
 \end{aligned}$$



Замечание: Если μ — меру на \mathbb{P}^1
 $\mathbb{P}^1 = \{[z:w] \in \mathbb{C}^2 \setminus \{0,0\} / \mathbb{C}^*$
 то \mathbb{C}^* — группа. На $\mathbb{C}_{\neq(0,0)}^2$ — группа.
 $(z, w) \rightarrow (\lambda z, \lambda w)$
 $f \in \mu_{\mathbb{P}^1}$ $f = \frac{p(z, w)}{q(z, w)}$ — рациональная функция.
 $f(\lambda z, \lambda w) = \frac{\lambda^d p(z, w)}{\lambda^d q(z, w)} = f(z, w)$

It is the yellow

Theorem: X - normed space. yellow
 $f(x, y) = 0$, $f \in C(X, \mathbb{R})$ - no labels.
 $p, q \in C(X, \mathbb{R})$, $f \neq q$. Then

$$h = \frac{p}{q} \in \mathcal{M}_X$$

Theorem: X - normed space. — — —
 $f(x, y, z) = 0$, $p, q \in C(X, \mathbb{R}, z)$.
 $f \neq q$, then $h = \frac{p}{q} \in \mathcal{M}_X$

Theorem (Tietze-Stern extension): Given
 $f \in C(X, \mathbb{R})$ - map. $g \in C(X, \mathbb{R})$.
 $g(x, y) = 0 \quad \forall (x, y) : f(x, y) = 0$, then
 $f \neq g$ (if continuous map).

