

### A-1.5.2 Approximation

horner scheme approx. method

Prinzipiell:  $\mathbb{Z}[x] = \{a+bi, a, b \in \mathbb{Z}\}$   
 $x = (1+i)(-i)$  .  $(Q(i))$

Opp's LEC - numer. eval. & - horner

$f(x) \in Q[x]$ .

Anzahl. d. Wurz. verlängert, da

$\Rightarrow S \in \mathbb{Z}[x]$  :  $y(x) = 0$

D: i - numer. meth. roots  $x^L + 1 = 0$

Rechts:  $r \in Q$  - numer. meth. ( $\approx$ ) reell

□ ..

Fix  $\alpha_1, \dots, \alpha_l$  - arr. which

$V = \left\{ \sum_{i=1}^l r_i \alpha_i \mid r_i \in \mathbb{Q} \right\}$  - basis.  $\frac{\mathbb{Z}}{\mathbb{Q}}$

$$\dim_Q V \leq l$$

unrest.

Kette: 1)  $\alpha_1, \dots, \alpha_l \rightarrow V$  - basis bilden  
Es ist  $\lambda \in \mathbb{Z} \in \mathbb{C}$  dann  $\forall v \in V \quad \lambda v \in V$ .

$\mu = \lambda - \text{arr. versch}$

2) Basis arr. versch. nach

$\square$  1)  $\alpha_i \in V \quad 1 \leq i \leq l \quad \lambda \alpha_i \in V$

$\lambda \alpha_i = \sum_{j=1}^l a_{ij} \alpha_j \quad a_{ij} \in \mathbb{Q} \quad 1 \leq i \leq l$   
 $\Leftrightarrow \sum_{j=1}^l (a_{ij} - \delta_{ij} \lambda) \alpha_j = 0, \quad \delta_{ij} \in \{0, 1\}$

2)  $\det(a_{ij} - \delta_{ij} \lambda) = 0 \quad , \quad \lambda \in \mathbb{Q}^{2 \times 3}$

$\deg \lambda \leq l$

2)  $d_1, d_2 - \text{arr. recen}$   
 $d_1^n + r_1 d_1^{n+1} + \dots + r_n d_1^n = 0$   $d_1^n + s_1 d_1^{n+1} + \dots + s_n d_1^n = 0$

$V = \cup \subset \text{nozero}$   $d_1^i d_2^j$   $0 \leq i \leq n$   
 $0 \leq j \leq k$

$\gamma \in V$   $d_1, d_2, \gamma \in V$

$\gamma = \bigcup b_{ij} d_1^i d_2^j$   $d_1, \gamma = \bigcup b_{ij} d_1^{i+1} d_2^j$   
 $d_1^n = \dots = d_1^{n+1} = \dots$

2)  $d_1, \gamma \in d_1, \gamma \in V \Leftrightarrow (d_1 \prec d_1, \gamma \in V)$

2)  $d_1 \prec d_1 - \text{arr. recen}$

Arrangement  $\leftarrow d_1, d_2, \gamma \in V \Rightarrow d_1 \prec d_1 - \text{arr.}$

$d_1 \cdot \text{arr. } d_1^n + r_1 d_1^{n+1} + \dots + r_n d_1^n = 0 \Rightarrow d_1^n = 0$

$1 + r_1 d_1 + \dots + r_n d_1^n = 0$

2)  $d_1^n = \text{arr. } \widehat{\text{arr.}} \text{ hole. } \blacksquare$

Qz: Für  $R$  - Menge (höher  $\in$ -es.)

$\mu$  wa. homogen hat  $R$  über

$\mu$  - as. grupp in  $\in$ -es. unters.

Dann  $\mu$  ist  $\mu$  in  $R$ :

$$1) \quad r(x+s) = rx + rs, \quad r \in R, \quad x, s \in K$$

$$2) \quad (r+s)x = rx + sx, \quad r, s \in R, \quad x \in K$$

$$3) \quad (rs)x = r(sx) \quad 4) \quad 1 \cdot x = x. \quad x \in K$$

Qz:  $\mu$  - homogen hat homom.

~~Wiederholung~~  $\exists \lambda_1, \dots, \lambda_k \in A \subset K$ :

$$x = \sum_{i=1}^k a_i \lambda_i \quad a_i \in R$$

Sei  $w_1, \dots, w_n$  - versch versch. reell

no  $w = \sum_{i=1}^n r_i w_i : r_i \in \mathbb{Z}$  - homom.  
wegen hom  $\mathbb{Z}$

$\square$  q.e.d.

Lemma: If  $w \in \omega - \omega_c - \text{year}$  then  
 $w = \bigcup_{i \geq 1} r_i w_i : r_i \in \mathbb{R}$ . See week

below  $\forall \delta \in \mathbb{R} \quad w \otimes \delta w$ .

$w$  - year wrt. mult.

2) Year after year of nature

1) Annual yes,  $\exists \alpha \in \mathbb{R}$

Annual rel. growth  $\alpha$   
never ends.

Lemma: For  $\alpha - \text{growth}$ :  $g(\alpha) = 0$   
you get  $Q^{(1)} = 1 + \alpha$  year.

$\forall \alpha \in \mathbb{R}$   $\exists \alpha \in Q^{(1)}$   $g(Q^{(1)}) = g(\alpha) = 1 + \alpha$

Def:  $\forall$   $w \in \omega$  then  $\text{mult. } L$   
 $\text{def } L = \inf \{ \alpha \in \omega : \text{mult. } \alpha \leq w \}$

Lemma:  $\mathcal{I}$  - blb. know. adm. &  
 $\deg \mathcal{I} = n$ , the  $\mathcal{I}$  mean  $n$  pos  
 hapless.

Q.M.:  $\mathcal{I} \cap -$  wgn help. thm.  
 know  $\mathcal{I} \in Q\{\mathbf{x}\}$  as  $\mathcal{I} \ni$  res  
 only.

Sch:  $\mathcal{I} -$  adm. mean

$$Q(\mathcal{I}) = \left\{ \frac{g(\mathcal{I})}{h(\mathcal{I})} \mid g \in Q\{\mathbf{x}\}, h(\mathcal{I}) \neq 0 \right\}$$

$$Q\{\mathcal{I}\} = \{ g(\mathcal{I}) \mid g \in Q\{\mathbf{x}\} \}$$

- v-ecc in hor. to

Lemma:  $\mathcal{I}$  - same adm.

$$Q(\mathcal{I}) = Q\{\mathcal{I}\} \quad \square - \square$$

- $\mathbb{Q}(\alpha, \beta)/\mathbb{Q}$  - Galois nur  
char.,  $\mathcal{I}$  - elem. nur anz. 2  
 $\deg f = n$ ,  $\bar{m} = [\mathbb{Q}(\alpha, \beta) : \mathbb{Q}] = n$
- Och.:  $\overline{\text{Norm}}(F) \subset \mathbb{C}$  verz.  
 ein  $\sum F : \mathbb{Q}) < \infty$
- Def.:  $R$  . reelle Zahl  $\Rightarrow A \times CR$   
 $L$ . anz. char.  $\leq [F : \mathbb{Q}]$ .
- Och.:  $F$  . reelle Zahl  
 $D = D_F = \{ \alpha \in F : f(\alpha) = 0 \in \mathbb{Z}[x] \}$   
 $f$  - prim.
- $D_F \subset F$  nur reelle Zahlen nur  
 17:  $D = \mathbb{Z}[i]$  - nur in  $\mathbb{Q}(i)$ .

Lemma:  $\forall \beta \in F \exists b \in \mathbb{Z}_{\neq 0} : b\beta \in \mathcal{D}_F$

$\square$   $\beta -$  null. num  $\beta$

$$\beta = b_0 + b_1 \beta^{n-1} + \dots + b_n \underset{b_i \in \mathbb{Q}}{=} 0$$

$$a_0 \beta^n + a_1 \beta^{n-1} + \dots + a_n \underset{\begin{array}{l} a_i \in \mathbb{Z} \\ \times a_0^{n-1} \end{array}}{=} 0$$

$$(a_0 \beta)^n + a_1 (a_0 \beta)^{n-1} + a_2 a_0 (a_0 \beta)^{n-2} + \dots = 0$$

$a_0 \beta$  - num  $g \in \mathbb{Z} \setminus \{0\}$  - gen.

Lemma:  $F -$  null. num,  $\mathcal{D} \subset \mathcal{D}_F$

$\mathbb{P} \subset \mathcal{D}$  - null. For  $P \subset \mathbb{P}$  csg. num.

Satz  $F / \mathbb{Q}$ .

$\square$   $(\beta_i)_{1 \leq i \leq n} -$  Sgr  $F / \mathbb{Q}$ ,  $\sum \beta_i \in \mathbb{Q}$  zsh

$\exists b \in \mathbb{Z}_{\neq 0} : b\beta_i \in \mathcal{D}_F$ ,  $\leq^1$  zsh

$\lambda \in I$  zu  $b, \rho: \lambda \in ?$

$b, \rho: \lambda$  - abh. versch. /  $\emptyset$

zu ~ Sym  $\text{def}$

Hausaufgabe:  $I \subset D$  - was ist  $\lim_{n \rightarrow \infty} a_n$  in  $\mathbb{R}$   
in  $\mathbb{C}$

? für  $\lambda \in \underline{I} \subset D$ :  $\lambda^{m-a_1} \lambda^{m-a_2} \dots = 0$

- abh. davon,  $a_n \neq 0$

zu  $a_n = \lambda^{m-a_1} \lambda^{m-a_2} \dots \in ?$   
 $\in \mathbb{C}$

$a_n \in I \cap \mathbb{Z}$   $\text{def}$ .

Ces, Lien symmetrische Matrix

$Lik - \text{Matrix}, \sum_{i=1}^n L_{ii} = 1$

$L_1, \dots, L_n - \text{Symm. Lien}$

$\forall L \in L \quad L^T = [a_{ij}]$

Oz.:  $N_{Lik}(L) \geq \det(A)$

$\operatorname{Tr}_{Lik}(L) = \operatorname{tr}(A)$

Rechen: Oz. ist vst. der Matrix

Spur

$D = \dots$   $\begin{pmatrix} & \\ & \end{pmatrix}$

Oz.:  $Lik - \text{Cesarmatr., da } \operatorname{Tr}_{Lik} \neq 0$

Oz.:  $\operatorname{Gal}(Lik) \cong \{ \sigma: L \rightarrow L : \sigma(x) = x \quad \forall x \in k \}$

Körper:  $L/K$  - norm. csn. per se  
 $[L:K] \geq 4$ , da  $|Gal(L/K)| \geq 4$

Qz.:  $L/K$  ist fakt. Triv. da  
 $[L:K] = |Gal(L/K)|$

Körper:  $L/K$  - norm. csn.  
 $Gal(L/K) = \{0, \dots, 0_n\}$ . Es ist  
 $\nu_{L/K}(\alpha) = \bigcap_{i=1}^n \sigma_i(\alpha)$   $Tr_{L/K}(\alpha) = \sum_{i=1}^n \sigma_i(\alpha)$

Körper:  $L/K$  - csgular, da  
 $\forall \alpha \in L$  mit min. d  $L(\alpha)$  in einer  
Hyper. aufges.

Körper:  $L/K$  - csg., non  $\Delta \subset L$

$f$  - mult. mon.  $\Leftrightarrow f(x, z) = a_0 + a_1 x^1 + \dots + a_n x^n$

$\text{Gal}(L/K)_2 \subset \langle \sigma_1, \dots, \sigma_n \rangle$

$$f(x, z) = \prod_{i=1}^n (x - \sigma_i(\alpha))$$

$$\mu_{L/K}(\alpha) = (-1)^n a_0, \quad \text{Tr}_{L/K}(\alpha) = -a_{n-1}$$

zsh:  $\alpha^{(i)} = \sigma_i(\alpha)$  - each.  $\in L$

17:  $Q(i) \quad a+bi \mapsto a-bi$  - each.

$$\mu = a^2 + b^2 = (a+bi)(a-bi)$$

Out:  $\alpha_1, \dots, \alpha_n \in L$

$$\Delta(\alpha_1, \dots, \alpha_n) = \det(\text{Tr}_{L/K}(\alpha_i \alpha_j))$$

has such behaviour  $\alpha_1, \dots, \alpha_n$

Lemma:  $\sum \Delta(\lambda_1, \dots, \lambda_n) \neq 0$   $\forall n \in \mathbb{N}$

$\lambda_1, \dots, \lambda_n$  - Syst. L/H.

Oft. eine L/H - ceh.  $\wedge (\lambda_i) - \text{sym}$   
mehr  $\Delta(\lambda_1, \dots, \lambda_n) \neq 0$

Lemma:  $\sum (\lambda_{i_1} \dots \lambda_{i_n})_{i_1, \dots, i_n} = \text{gle Sym L/H}$ ,  $\lambda_i = \sum_{j=1}^n a_{ij} \beta_j$   $i \leq i \leq n$

$$\Delta(\lambda_1, \dots, \lambda_n) = \det(a_{ij})^2 \Delta(\beta_1, \dots, \beta_n)$$

Lemma: L/H - ceh.

$$\Delta(\lambda_1, \dots, \lambda_n) = (\det(\lambda_i^{(j)}))^2$$

Lemma:  $\delta$  - vbl. mon.  $\beta \in L$   
L/H - fun.  $\alpha, \beta, \gamma \in \mathbb{R}^{n-1}$  - eine Wurzel.

$$\Delta(1, \beta, -\beta^{n-1}) = (-1)^{\frac{n(n-1)}{2}} \mu(\mathcal{L}'(\lambda)).$$

Hyperbolische u reell . merkmale u  
uh hor reel

$$K/\mathbb{Q} \quad [K:\mathbb{Q}] = 4$$

klar :   $\mathcal{L} \in D_K \quad \Rightarrow \mu_{K/\mathbb{Q}}(-) \cdot \text{Tr}_{K/\mathbb{Q}}(\mathcal{L}) \in \mathbb{Z}$

$\lambda_1, -\lambda_4 \in D_K$  - dann  $K/\mathbb{Q} \quad \Rightarrow$

$$\Delta(\lambda_1, -\lambda_4) \in \mathbb{Z}$$

Klar :   $I \subset D_K$  - merkmale

$\lambda_1, -\lambda_4 \in I$  - sys. merkmale uh

$$|\Delta(\lambda_1, -\lambda_4)| = \text{uh}.$$

$$I \cong \mathbb{Z}\lambda_1 + \dots + \mathbb{Z}\lambda_n$$

Oct 1 Taken Sun ( $\omega^1$ ) for  
 you all.  
 E.g.  $(d_1, \rho_1) - \text{you sun}$   
 Sun:  $L_1 = [a_1: 1]$   
 $(a_1) - \text{you}, d_1 = 1$   
 $D(d_1 - d_2) = D(\rho_1 - \rho_2)$   
 now Sun more &  $\sigma_{\rho_2} \delta_K$ .

Observation post-closure  
 E.g.  $\mathbb{Z}[\omega] - \text{you less.}$   
 Other now: closure  $\mathbb{Z}[1 - \sqrt{5}]$   
 $L_1 = S \cdot T = (1 + 2\sqrt{-5})(1 - 2\sqrt{-5})$   
 " up, he also — (24h.)

$\kappa$  . metoder når  $D = D_K$  - høst  
svaret (  $Z \in \mathbb{S}$  ,  $Z \in \sqrt{\mathbb{S}}$  )

$P \subset D$  - yderligere  $\Leftrightarrow$   
 $\lambda_1 \in P =, \lambda \in P \vee \beta \in P$

Kern :  $I \subset D$  - kern .  $\bar{K} = \text{ker}$   
 $D/I$  - hovedring .  $\bar{D}$  er ikke ring  
er  $I = (a)$ ,  $a \in \mathbb{Z}_{>0}$ .  $|D/(a)| = a^n$

$\square I \subset D$ ,  $I \cap D \neq \emptyset \neq 0$

$\exists a \in I \cap \mathbb{Z}$ ,  $a \neq 0$ ,  $a \in \mathbb{Z}$

$(a) \subset I =, D/I \subset D/(a)$

$\exists (\omega)_i \in I$  - siger  $\kappa$ ,

$$\mathcal{D} = \mathbb{Z}\omega_1 + \dots + \mathbb{Z}\omega_n$$

$$\text{Jac. locus } \quad S = \{ [c_i \omega_i] : \begin{array}{l} 0 \leq c_i < a \\ c_i \in \mathbb{Z} \end{array} \}$$

$$V \sim \epsilon \mathcal{D} \quad \omega = \sum_{i=1}^n m_i \omega_i \quad m_i \in \mathbb{Z}$$

$$V : \quad m_i = q_i a + r_i \quad 0 \leq r_i < a$$

$$\omega \in [c_i \omega_i] (a)$$

$$\Leftrightarrow [c_i \omega_i] \in [c'_i \omega_i] (a) \quad 0 \leq c_i, c'_i < a$$

$$[c_i - c'_i, \omega_i] \geq 0 (a)$$

$$\Leftrightarrow \gamma_a, \quad \gamma \in \mathcal{D}$$

$$\gamma = [b_i \omega_i]$$

$$\gamma_a = [b_i a \omega_i] \Rightarrow c_i - c'_i = b_i a$$
$$c_i \geq c'_i (a) \Rightarrow c_i \geq c'_i$$

$$\mathcal{D}^1(a) \cong S, \quad |\mathcal{D}^1(a)| = a^m \quad \boxed{\text{B}}$$

Oz:  $R$  als höheren  
oder  $I_1 \subset I_2 \subset \dots \subset I_n \subset \dots$   
oder  $I_1 = I_{n+1}$

Codimension:  $D$  - rektifiz.

$\forall$   $P$  einer  $\in D$  schr. horiz.

$\cap$   $\text{gr. } \boxed{\text{B}}$

Lemma:  $I, J$  - ideal  $D$ .  $I \subset J$

1)  $\exists n \in \mathbb{Z} : I = J^n$ ,  $\text{no } J = D$

2)  $\exists n \in \mathbb{Z} : \omega \in D \setminus (I^n \cup J^n)$ , und  
 $(\omega)^2 \subset J$

$\square$  1)  $\alpha_1, \dots, \alpha_s \in I$  -  $\text{Satz } D$

$I = I \cap A : \alpha_i \in \bigcap_{j=1}^s \beta_{ij}, \alpha_i, \beta_{ij} \in J$

2)  $\det(\beta_{ij} - \delta_{ij}) = 0$

$\Rightarrow \exists t \in \{(-1)^{\beta_{ij}} = 0 \mid i \in I\}$

$J \neq \emptyset$

2) ( $\omega$ )  $I = J \cap$

$\rho \in J \quad \rho \alpha_i = \omega \alpha_i \quad (\frac{\Delta}{\omega}) I \subset I$

$\omega_1, \dots, \omega_s - \text{Satz } D \quad l \in D \in$   
 $= \mathbb{Z}\omega_1 + \dots + \mathbb{Z}\omega_s$

$I - \text{max. } / \mathbb{Z}$

$\frac{\Delta}{\omega} \cdot x \in I \quad \vee \quad x \in I \quad \Rightarrow \quad \frac{\Delta}{\omega} \in D$

$J \subset (\omega) \quad \frac{1}{\omega} J \subset D - \text{max.}$

$$(\omega) \exists \geq \exists ! \quad \exists \geq (\omega') \exists ! \stackrel{1)}{\Rightarrow} \frac{1}{\omega} \exists = \omega \\ \Leftrightarrow \exists \geq (\omega)$$

Og: known  $P, C \subset D$  when

which , can  $\exists x. P(x) \in D$ :

$$(2) \exists \geq (\lambda) \exists \quad ((\frac{\lambda}{x}) \exists \geq \exists )$$

Koen: generic domain .  $\exists \geq \exists$ .

$$\exists \dots \exists$$

Og: even  $|D| \sim |$  somehow .

$$\text{or, } n_k = |D| \sim | \quad (\text{as } n_k = \infty)$$

Koen:  $n_k = 1 \Leftrightarrow D = K^P K$ .

Koen :  $K = \text{new value} , D = D_n$

- $\exists n \in \mathbb{Z}_{>0}$  (number) : A L. is D.  
 $\rho \neq 0$   $\exists t \quad 1 \leq t \leq n \quad \exists w \in D$   
 $|\mu(t\omega - w\rho)| < |\mu(w)| \quad \mu = \mu_{n,Q}$   
 $\square \quad \gamma < \frac{\rho}{\rho} \in K$  .  $\gamma$  is a rational number  
 $\exists \mu \quad \exists t \quad \exists w \in D \quad |\mu(t\gamma - w\rho)| < 1$   
 $\text{by } w_1 - w_n - \text{sign } D$   
 $\gamma = [c, \omega], \quad c_i \in \mathbb{Q}$   
 $|\mu\gamma| = |\prod_j \gamma^{(j)}| = |\prod_j ([c_i, \omega_i]^{(j)})| \leq$   
 $\leq C (\max_i |c_i|)^n$   
 $C = \prod_j ([\omega_i^{(j)}])$

$$z_m < \sqrt{c} \quad m = k^n$$

$$c_i \in Q \quad c_i = [c_i] + [c_i] \\ "a_i" \qquad \qquad \qquad "b_i" \quad 0 \leq b_i < 1 \\ c_i \notin$$

$$\gamma = \bigcup c; \omega, \quad \{\gamma\} = \bigcup a; \omega: \quad \gamma \in \{\gamma\} + \gamma$$

$$\{\gamma\} = \bigcup b; \omega,$$

Such.  $\varphi: K \rightarrow \mathbb{R}^n$  [ $c: v \mapsto (c_1, \dots, c_n)$ ]

$$\psi(\gamma) = \psi(\{\gamma\}) + \psi(\{>\gamma\})$$

Es ist ~~wie~~  $U =$

$$z \in \{x_{\min} - x_2 : 0 \leq x \leq 1\}$$

Taylor & he down < later

$\frac{1}{m}$  ,  $m^4$   $\rightarrow m$

$$1 \leq i \leq n+1 \quad \varphi(z_j y) \in U$$

$$z) \Rightarrow j_1, j_2 \in \{1, \dots, n\} \quad 1 \leq j_2 < j_1 \leq n+1$$

$\{j_1, \delta\}, \{j_2, \delta\} \subset \text{open sets}$

$$j_1 \delta - j_2 \delta = \underbrace{\{j_1, \delta\} - \{j_2, \delta\}}_{w \in D} - \underbrace{\{j_1, \delta\} - \{j_1, \delta\}}_{\delta \in U}$$

$$\underbrace{\{j_1, \dots, j_n, \delta\}}_{t \leq n}$$

hence  $w \in U$   
 $< \frac{1}{n}$

$$\mu_S \leq C \frac{1}{n^2} < 1 \quad \text{QED}$$

Lemma:  $h_n < \infty$

$\square \quad I \subset D - \text{near } z, \quad z \in I \setminus \{0\}$

$|n \omega| \in \mathbb{Z}_{>0}, \quad n \in I \setminus \{0\} - \text{neither}$   
 $\text{negative}$

$\exists \mu \quad \forall \alpha \in \mathbb{I} \quad \exists \gamma, \leq \alpha \in \mu \quad \exists \omega \in \omega$

$$|\mu(\ell\alpha - \omega\beta)| < (\mu(\beta))$$

$\alpha, \beta \in \mathbb{I} \quad \ell\alpha - \omega\beta \in \mathbb{I} \quad \Rightarrow \ell\alpha - \omega\beta = 0$

$$\ell\alpha = \omega\beta$$

$$\bar{\mu} \cdot 0 \quad \mu' \alpha = \omega'\beta \quad \mu' \alpha \in (\beta)$$

$$\Rightarrow \mu' \cdot \mathbb{I} \subset (\beta)$$

$$\exists \bar{\beta} \in \mu' \cdot \mathbb{I} : \mu' \bar{\beta} \subset (\beta) \quad \forall \alpha \in \mu' \cdot \mathbb{I} \quad \exists \gamma \in \mathcal{D} : \alpha = \beta\gamma$$

$\mathcal{J} \subset \mathcal{D}$  - use in

$$\mu' \cdot \mathbb{I} = (\beta, \mathcal{J})$$

$$\beta \in \mathbb{I} \quad \mu' \beta \in \mu' \mathbb{I} \quad \mu' \beta \in (\beta, \mathcal{J}) \\ \Rightarrow \mu' \in \mathcal{J} \quad (\mu' \beta = \omega\gamma)$$

$a \in \mathbb{Z}$     $a \in \mathbb{I} \Leftrightarrow (a_1 \in \mathbb{I} \Leftrightarrow \mathcal{D}/I \subset \mathcal{D}_+(a))$

Even zeros. Then  
marked  $\rightarrow n! \geq a$

$$n! \in \mathbb{I} = (\mathbb{P})$$

$\mathbb{I} \sim \mathbb{J}$     $\mathbb{J} - \text{nonzero zero}$   
 $\bar{n}, a$     $|\mathcal{D}| \sim 1$     $\text{marked } \approx h_u < \omega$

Then:  $\mathbb{P} \subset \mathcal{D}$  - non  $\mathcal{D}$ ,  $n \neq h_u$   
 $\exists a$     $1 \leq n \leq h_u$  :  $\mathbb{P}'$  - marked  
 $\mathbb{P} \subset \mathbb{P}'$ .    $1 \leq j \leq h_{\mathbb{P}'} \Rightarrow \exists j < b$

$\mathbb{P}^{j_1} \sim \mathbb{P}^{j_2} \Leftrightarrow \exists \alpha, \beta \in \mathcal{D}$   
 $(\alpha) \mathbb{P}^{j_1} = (\alpha) \mathbb{P}^{j_2}$

$$n = j_0 - j_1, \quad j = \overline{j}^{\top}$$

$$(\alpha) \quad \overline{j}^{j_1} = (\alpha, j)^{j_0} = \left( \frac{\alpha}{\alpha}, j \right)^{j_0} \subset \overline{j}^{j_1}$$

$$\begin{aligned} & \Rightarrow \frac{\alpha}{\alpha} \in \mathcal{D} & \Rightarrow (\omega) \overline{j}^{j_1} = j \overline{j}^{j_1} \\ & \in \omega & \Rightarrow j = (\omega) \\ & & " \overline{j}^{j_0} \quad \text{□} \end{aligned}$$

Lemma 1

$$\begin{aligned} & z, A, B, C - \text{wörter} \quad AB = AC \Rightarrow \\ & z, B = C \end{aligned}$$

$$\begin{aligned} & z, A, B - \text{wörter} \quad B \supset A \Rightarrow \exists C - \text{wo} \\ & A = BC \end{aligned}$$

$$\boxed{1} \quad \exists u \cdot A^u = (\alpha)$$

$$A^u = A \subset \times A^{u-1}$$

$$(\alpha)B = (\alpha)C \Rightarrow B = C$$

$$(\beta \in B \quad \alpha \beta \in (\alpha)C \quad \exists \gamma \in D \quad \gamma \in C : \\ \alpha \beta = \gamma \alpha \gamma \Rightarrow \beta = \gamma) \\ \Rightarrow \beta \in C \quad )$$

$$2) \exists u \quad B^u = (\beta)$$

$$A \subset B \quad B^{u-1}A \subset B^u = (\beta)$$

$$C = \bigcup_{\beta} B^{u-1}A \subset D - \{\gamma\}$$

$$B^u = \bigcup_{\beta} (\beta)A = A \quad \text{□}$$

Lemma 1  $A$  - closed  $\Leftrightarrow \mathcal{D} \cap A = \mathcal{D}$  ( $A \neq \mathcal{D}$ )

$A = \bigcap P$ ,  $P$  - cl. ns.

$\exists$   $\text{succ. } P, A \subset P,$

$\Rightarrow A = P \setminus B, B \subsetneq \mathcal{D} \Rightarrow P_L, B \subset P_L$

$A = P_1 P_2 B_1 \dots$

$A \subset B_1 \subset B_2 \subset \dots \subset B_t$

$A = P_1 \cup P_2 \quad \text{~~cl~~}$

Obs:  $P$  - cl.  $A$  - ns.  $V_P(A) = V$ :

$A \subset P^v, A \not\subset P^{v+1}$

Axiom: 1)  $v_p(p) = 1$

2)  $Q \neq p \quad v_p(Q) = 0$

3)  $v_p(A \cap B) = v_p(A) + v_p(B)$   $\square$  direkt  $\square$

Theorem:  $A \in \mathcal{D}_k \quad \exists \text{ ord. } \lambda$

$A = \bigcap_P P^{a(P)}$   $a(P) = v_p(A)$

$\square$  1. non. ver 1. lehr  $\leq$   $\square$

$\mathcal{D} = \mathcal{D}_k \quad P \cap \mathbb{Z} \neq 0 \quad P \cap \mathbb{Z} = (P)$

$\mathbb{Z}$   $\overset{j}{\rightarrow}$   $P - u$   $(P) - u \rightsquigarrow \ell \mathbb{Z}$

$P \models p \quad v_p(p)$

$\mathcal{D}/P$  - wozu

Lemma:  $P - \gamma \rightarrow P \cdot \text{hanc.} \Rightarrow$   
 $DIP$  - wobei  
 $\square - \square$

$$IF_p = ZIRZ \subset DIP = IF_p^d$$

Zy.:  $e = \nu_p(p)$  - result level. us.  
A - charact. num

$$\text{char. } (p) = p_1^{e_1} \cdots p_g^{e_g}$$

Lemma:  $P \subset D$ . us  $DIP \cong IF_p^d$

$$\text{from } |DIP^e| = p^{et}$$

$\square$  wusg  $\Leftarrow e$ :

$e = 1$ . num

$$e > 1$$

$$\underline{P}^e \subset \underline{P}^{e-1} \quad \exists \omega \in \underline{P}^{e-1} \setminus \underline{P}^e$$

$$(\omega_1 + P^e) \subset P^{e-1} \quad \underline{P}^{e-1} \setminus (\omega_1 + P^e)$$

$$P^e \subset (\omega_1 + P^e) \quad (\omega_1 + P^e) \setminus P^e$$

$$(\omega_1 + P^e) = P^i \quad i \leq e$$

$$2. \quad \underline{P}^{e-1} \setminus P^i \quad i = e-1$$

$$\text{Th. 2. } (\omega_1 + P^e) = P^{e-1}$$

$$\text{Teach } \varphi: D \rightarrow \underline{P}^{e-1} \setminus P^e$$

$$\varphi(\sigma) = \sigma\omega_1 + P^e \subset \underline{P}^{e-1} \quad \forall \sigma \in D$$

$\varphi$  - smooth - why?

$\ker \psi \supseteq \gamma : \gamma \in P^e \iff \nu_p(\gamma) \geq e$

$$\nu_p(\gamma \cdot \gamma) = \nu_p(\gamma - \nu_p(\gamma)) = \nu_p(\gamma) + e-1 \geq e$$

$\gamma \in \ker \psi \iff \nu_p(\gamma) \geq 1 \iff \gamma \in P$

$\ker \psi = P$ ,

$$\therefore D/P \cong P^{e-1}/P^e$$

$$P^e \subset P^{e-1} \subset D$$

$$D/P^{e-1} =$$
$$\cong (D, P^e)/(P^{e-1}/P^e)$$

$$(D, P^e)/(P^{e-1}, P^e) \cong (D/P^{e-1}) \cong r^{d(e-1)}$$

$\sim P^e$

□

- Kern:  $R = \text{ker} \varphi \subset \mathfrak{c}_x$   
 $A_1 - A_0$  - us  $A_1 - A_0 \cong R$
- $\forall i \neq j \quad P \cong A_i - A_0$
- $R/A \cong R/A_1 \oplus R/A_0$
12. Das  $\kappa \mapsto s$  & rechts  $\oplus$
- Kern:  $P, Q$  -  $\mathbb{Z}$ . us.  $P \neq Q$
- $\forall u, v \in D \Rightarrow P^u \cdot Q^v \in D$
- $\square$  Um  $\oplus$  rechts  $D = D_A$
- Frage:  $(P) = P_1^{e_1} \cdots P_g^{e_g}, \quad n = \text{lcm. } P;$   
 $f_1, f_g - \text{rest. } P;$   
 $\sum_{i=1}^g e_i d_i = n$

$$\square \quad P_i^{e_i} \prec P_j^{e_j} = D$$

$$\xrightarrow{h^{\otimes 0}} D'(P) \cong D(P)^e \otimes - \otimes D(P)^{e_0}$$

$$\text{then } \xrightarrow{n} P^n \xrightarrow{P \in \mathcal{E}, d} P \in \mathcal{E}, d;$$

$$\therefore n \in \mathcal{E}, d; \quad \square$$

$$\text{Lemma: } p \in \mathbb{Z} - \mathbb{Q}. \quad P_1, P_2 = \text{nr}$$

$$\text{Mas } p \in D \quad ( \quad (p) \subset P_1, (r) \subset P_2 \\ P_1 \cap P_2 \quad P_2 \cap P_1$$

$$\exists \sigma \in \text{Gal}(K/\mathbb{Q}) : \sigma P_1 = P_2$$

$$\square \quad (p) \subset P_0 : P_0 \not\in \sigma P_1 : \sigma \in \text{Gal}$$

$\kappa \neq 0 \quad \exists \quad \lambda \in \mathcal{D} :$

$$\lambda \gtrsim_0 (P_0) \quad \lambda \gtrsim_1 (5 P_1)$$



$\forall \sigma \in \text{hol}$

$$(\lambda) \subset P_0$$

$$\mu(\lambda_1, \lambda_2) \underset{\sigma}{\cap} \sigma(\lambda) = \lambda \cdot \underset{\sim}{\cap} \sigma(\lambda_1) \in (\lambda) \subset P_0$$

$$\mu(\lambda_1) \in \mathbb{Z} \quad \Rightarrow \quad \mu(\lambda_1) \in P_0 \cap \mathbb{Z} = \{P\}$$

$$(P) \subset P_1 \quad \Rightarrow \quad \mu(\lambda) \in P_1$$

$$P_1 \mid \mu(\lambda) = \underset{\sigma}{\cap} \sigma \lambda \quad \Rightarrow \quad \exists \sigma : P_1 \mid \sigma \lambda$$

$$\sigma \lambda \gtrsim_0 (P_1) \quad (\approx) \quad \lambda \gtrsim_0 (5^{-1} P_1)$$

$- > <$  ?

Fraktion:  $\kappa / Q = \text{Trivariante}, P \in \mathbb{Z}[x]$

$$(P) = P_1^{e_1} \cdots P_g^{e_g}$$

Frak.  $\ell_i = \dots = e_i = e$        $e \neq 0 \neq n$   
 $f_i = \dots = d_i = d$

$$\square P, \quad \forall i: \exists \sigma \quad \sigma P_i = P_i$$

$$\mathcal{D}/P_i \cong \mathcal{D}/\sigma P_i = \mathcal{D}/P_i$$

$$z, f, z f_i = f$$

$$(P) = \sigma(P) = (\sigma P_1)^{e_1} \cdots (\sigma P_g)^{e_g}$$
$$= P_1^{e_1} - P_g^{e_g} \quad \sigma P_i = P_i$$

$$n = \sum_{i=1}^g e_i d_i = g d = e d \quad \square$$

$$Q(\sqrt{d}) \quad x^{\perp - d} = 0$$
$$\mathcal{D}_{Q(\sqrt{d})} \neq \mathbb{Z}[\sqrt{d}]$$

$$Q(s), \quad s^{\perp} = 1$$

$$\mathcal{D}_{Q(s)}$$