Lab 8 - Phoebe Spratt

3. For all natural numbers, $n \geq 7$, $3^n < n!$.

<u>Base case:</u> (n = 7)

$3^7 = 2187$

$7! = 5040$

So $3^7 < 7!$

<u>Induction step:</u>

Assume that for some natural number k with $k \geq 7$, $3^k < k!$

$(k + 1)! = k!(k + 1)$

$> 3^k(k + 1)$ (by IH)

$> 3^k(8 + 1)$

$> 3^k \cdot 3$

$= 3^{k+1}$

So $3^{k+1} < (k + 1)!$ ▫

4. For all natural numbers $n \geq 1$, $\sum_{i=1}^{n} 2i = n(n + 1)$.

<u>Base case:</u> (n = 1)

$\sum_{i=1}^{1} 2i = 2$

$1(1 + 1) = 2$

So $\sum_{i=1}^{1} 2i = 1(1 + 1)$

<u>Induction step:</u>

Assume for some $k \in N$ $\sum_{i=1}^{k} 2i = k(k + 1)$

$$\sum_{i=1}^{k+1} 2i = \sum_{i=1}^{k} 2i + 2(k + 1)$$

$= k(k + 1) + 2(k + 1)$ (by IH)

$= (k + 1)(k + 2)$

$= (k + 1)(k + 1 + 1)$

Therefore $\sum_{i=1}^{k+1} 2i = (k + 1)(k + 1 + 1)$ ▫

5. For all $n \in N$, $n^2 - 3n$ is even.

Base case: (n = 0)

$0^2 - 3(0) = 0 - 0 = 0 \in N$

0 is even because there exists an integer k such that $0 = 2k$

Induction step:

Assume for some natural number k and $k^2 - 3k$ is even

Since $k^2 - 3k$ is even, there must exist some integer j that $k^2 - 3k = 2j$

$(k + 1)^2 - 3(k + 1) = k^2 + 2k + 1 - 3k - 3$

$= k^2 - 3k + 2k + 1 - 3$

$= 2j + 2k - 2$ (by IH)

$= 2(j + k - 1)$

j, k, and -1 are integers, so $j + k - 1$ is too

Therefore $(k + 1)^2 - 3(k + 1)$ is even ▫