Machine Learning HW1

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		Q1	Q2	Q3	Q4	Q5	Total
Grade	Max	1	1	1	1	1	5
	Expected	1	1	1	1	1	5

Q1

Q1a

$P(X=1|Y=0) = \frac{P(X=1,Y=0)}{P(Y=0)} = \frac{0.20}{0.15+0.3+0.2} = 0.307$

Q1b

consider:
$P(X=1) = 0.05 + 0.2 = 0.25$
$P(x=1) \neq P(X=1|Y=1)$
so it is not independent.

Q1c

$E[5*X+3*Y*Y] = 5E[X] + 3E[Y^2]$

Q3

we are trying to find: $p(terrorist = true|detected = true)$
that is :
$p(terrorist = true|detected = true) = \frac{p(detected=true|terrorist=true)p(terrorist=true)}{p(detected=true)}$
$p(detected = true|terrorist = true) = 0.95$
$p(terrorist = true) = 0.01$
$p(detected = true) = p(detected = trueterrorist = true)p(terrorist = true) + p(detected = trueterrorist = false)p(terrorist = false) = 0.95(0.01) + 0.05(0.99) = 0.059$
$p(terrorist = true|detected = true) = \frac{0.95*0.01}{0.059} = 0.161$

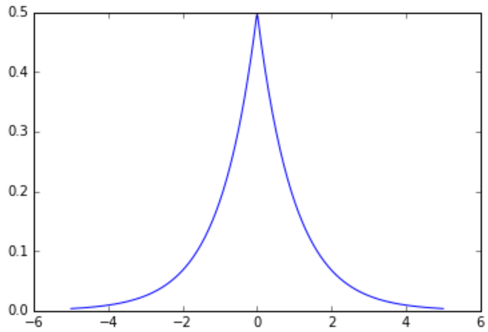
Q4

we need to choose i that has the maximum result for : $EU(\alpha_i|x) = \sum_k U_{ik}P(S_k|x)$

we have:
$EU(\alpha_1|x) = 5*0.7 + 3*0.2 + 1*0.1 = 4.2$
$EU(\alpha_2|x) = 0*0.7 + 4*0.2 + -2*0.1 = 0.6$
$EU(\alpha_3|x) = -3*0.7 + 0*0.2 + 10*0.1 = -1.1$
so the best decision is $i = 1$

Q5

Q5a



$p(x > 2) = \int_2^\infty \frac{1}{2}\mathrm{e}^{-|x|}\,\mathrm{d}x = \int_2^\infty \frac{1}{2}\mathrm{e}^{-x}\,\mathrm{d}x$
$= -0.5 * (0 - e^{-2}) = 0.0676$

Q5b

$$\begin{aligned} E[x] = \sum_{k=0}^{n} k\binom{n}{k}p^k(1-p)^{n-k} &= \sum_{k=0}^{n} n\binom{n-1}{k-1}p^k(1-p)^{n-k} \\ &= n\sum_{k=0}^{n}\binom{n-1}{k-1}p^k(1-p)^{n-k} \\ &= n\sum_{k=0}^{n-1}\binom{n-1}{k}p^{k+1}(1-p)^{n-k-1} \\ &= np\sum_{k=0}^{n-1}\binom{n-1}{k}p^{k}(1-p)^{n-k-1} \\ &= np\Big(p+(1-p)\Big)^{n-1} \\ &= np \end{aligned}$$

$$E[x^2] = E[X(X-1)] + E[X] = n(n-1)p^2 + np$$