IOQM 2025

Basic Mathematics

DPP-01

1. If the real number x, y, z are such that $x^2 + 4y^2 + 16z^2 = 48$ and $xy + 4yx + 2zx = 24$, what is the value of $x^2 + y^2 + z^2$?

2. The value of

$$\frac{\left(4a^2-9b^2\right)^3+\left(9b^2-16c^2\right)^3+\left(16c^2-4a^2\right)^3}{(2a-3b)^3+(3b-4c)^3+(4c-2a)^3}$$

is

(A) $3\,abc$

(B) $3(2a + 3b)(3b + 4c)(4c + 2a)$

(C) $(2a + 3b)(3b + 4c)(4c + 2a)$

(D) 11

3. Let a, b and c be such that $a + b + c = 0$ and $P=\frac{a^2}{2a^2+bc}+\frac{b^2}{2b^2+ca}+\frac{c^2}{2c^2+ab}$ is defined. What is the value of P?

4. The expression $(5a - 3b)^3 + (3b - 7c)^3 - (5a - 7c)^3$ is divisible by

(A) $(5a + 3b + 7c)$

(B) $(5a - 3b - 7c)$

(C) $(3b - 7c)$

(D) $(7c + 5a)$

5. If $a^2 + 2b = 7$, $b^2 + 4c = -7$ and $c^2 + 6a = -14$, then the value of $(a^2 + b^2 + c^2)$ is:

(A) 14 (B) 25

(C) 36 (D) 47

6. If $\left(x+\frac{1}{x}\right)=5$, then $\left(x^3+\frac{1}{x^3}\right)-5\left(x^2+\frac{1}{x^2}\right)+\left(x+\frac{1}{x}\right)$, is equal to

(A) 0 (B) 5

(C) -5 (D) 10

7. If $x-\frac{1}{x}=3$ then value of $x^7-\frac{1}{x^7}$ is

(A) 4287

(B) 4283

(C) 4285

(D) 4281

8. Find the numerical value of

$\sqrt[3]{2+\sqrt{5}}+\sqrt[3]{2-\sqrt{5}}$.

9. Value of $\sqrt{10+\sqrt{24}+\sqrt{40}+\sqrt{60}}$ is

(A) $1+\sqrt{2}+\sqrt{3}$

(B) $\sqrt{2}+\sqrt{3}+\sqrt{5}$

(C) $\sqrt{3}+\sqrt{5}+2$

(D) $\sqrt{6}+2+\sqrt{7}$

10. If $\frac{4}{2+\sqrt{3}+\sqrt{7}}=\sqrt{a}+\sqrt{b}-\sqrt{c}$, then which of following can be true?

(A) $a=1,\ b=\frac{4}{3},\ c=\frac{7}{3}$

(B) $a=1,\ b=\frac{2}{3},\ c=\frac{7}{9}$

(C) $a=\frac{2}{3},\ b=1,\ c=\frac{7}{3}$

(D) $a=\frac{7}{9},\ b=\frac{4}{3},\ c=1$

Answer Key

DPP-01

1. (21)

$x^2 + 4y^2 + 16z^2 = 48$ ...(1)

$xy + 4yz + 2zx = 24$ ... (2)

Let $N = x^2 + y^2 + z^2$

Multiply equation (2) by (2) and subtract from equation (1):

$x^2 + 4y^2 + 16z^2 - 2xy - 8yz - 4xz = 0$

$(x)^2 + (2y)^2 + (4z)^2 - x(2y) - (2y)(4z) - (x)(4z) = 0$

We know that:

$a^2 + b^2 + c^2 - ab - bc - ca = 0$

$$\frac{1}{2}\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right] = 0$$

$a = b = c$

so $x = 2y = 4z$

Let $x = 2y = 4z = k$

$x^2 + 4y^2 + 16z^2 = 48$

$$k^2 + 4\times\frac{k^2}{4} + 16\times\frac{k^2}{16} = 48$$

$k = 4$

So $$x^2 + y^2 + z^2 = 4^2 + \left(\frac{4}{2}\right)^2 + \left(\frac{4}{4}\right)^2$$

$= 16 + 4 + 1 = 21$

2. (C)

If $A + B + C = 0$

then $A^3 + B^3 + C^3 = 3ABC$

$$\frac{\left(4a^2-9b^2\right)^3 + \left(9b^2-16c^2\right)^3 + \left(16c^2-4a^2\right)^3}{(2a-3b)^3 + (3b-4c)^3 + (4c-2a)^3}$$

$$= \frac{3\left(4a^2-9b^2\right)\left(9b^2-16c^2\right)\left(16c^2-4a^2\right)}{3(2a-3b)(3b-4c)(4c-2a)}$$

$= (2a + 3b)(3b + 4c)(4c + 2a)$

3. (1)

$a + b + c = 0$

$\because a + b + c = 0$

$\therefore a^3 + b^3 + c^3 = 3abc$

$$P = \frac{4a^2b^2c^2 + 2a^3b^3 + 2a^2c^3 + a^4bc + 4a^2b^2c^2 + 2b^3c^3 + b^4ac + 4a^2b^2c^2 + 2a^3c^3 + 2b^3c^3 + c^4ab}{\left(8a^2b^2c^2 + 4\left(a^3b^3 + a^3c^3 + b^3c^3\right) + 2a^4bc + 2ab^4c + 2abc^4 + a^2b^2c^2\right)}$$

$$P = \frac{12a^2b^2c^2 + 4a^3b^3 + 4a^3c^3 + 4b^3c^3 + abc\left(a^3+b^3+c^3\right)}{9a^2b^2c^2 + 4\left(a^3b^3 + b^3c^3 + c^3a^3\right) + 2abc(3abc)}$$

Put $a^3 + b^3 + c^3 = 3abc$

$$P = \frac{15a^2b^2c^2 + 4\left(a^3b^3 + b^3c^3 + a^3c^3\right)}{15a^2b^2c^2 + 4\left(a^3b^3 + b^3c^3 + c^3a^3\right)}$$

$= 1$

4. (C)

$(5a - 3b)^3 + (3b - 7c)^3 - (5a - 7c)^3$

$= (5a - 3b)^3 + (3b - 7c)^3 + (7c - 5a)^3$

Let $x = 5a - 3b$

$y = 3b - 7c$

$z = 7c - 5a$

So $x + y + z = 0$

We know that when $x + y + z = 0$,

$x^3 + y^3 + z^3 = 3abc$

$(5a - 3b)^3 + (3b - 7c)^3 + (7c - 5a)^3$

$= 3(5a - 3b)(3b - 7c)(7c - 5a)$

So It has a factor as $(3b - 7c)$

5. (A)

$a^2 + 2b = 7$ …(1)

$b^2 + 4c = -7$ …(2)

$c^2 + 6a = -14$ …(3)

By adding all these equations:

$a^2 + b^2 + c^2 + 2b + 4c + 6a = -14$

$a^2 + 6a + 9 + b^2 + 2b + 1 + c^2 + 4c + 4 - 14 = -14$

$(a + 3)^2 + (b + 1)^2 + (c + 2)^2 = 0$

We know that: when $x^2 + y^2 + z^2 = 0$

then $x = y = z = 0$

So $a + 3 = 0 \Rightarrow a = -3$

$b + 1 = 0 \Rightarrow b = -1$

$c + 2 = 0 \Rightarrow c = -2$

$a^2 + b^2 + c^2 = (-3)^2 + (-1)^2 + (-2)^2 = 14$

6. (A)

$$\left(x + \frac{1}{x}\right) = 5 \quad \text{…(1)}$$

Let $$\left(x^3 + \frac{1}{x^3}\right) - 5\left(x^2 + \frac{1}{x^2}\right) + \left(x + \frac{1}{x}\right) = K$$

By squaring equation (1):

$$\left(x+\frac{1}{x}\right)^3=x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)$$

$$125=x^3+\frac{1}{x^3}+3\times 5$$

$$x^3+\frac{1}{x^3}=110 \qquad \ldots(2)$$

By squaring both side of equation (1):

$$x^2+\frac{1}{x^2}=23 \qquad \ldots(3)$$

From equation (2) and (3):

k = 110 – 5 × 23 + 5

= 0

7. **(A)**

$$\left(x-\frac{1}{x}\right)^2=(3)^2 \Rightarrow x^2+\frac{1}{x^2}=11$$

Now, $\left(x^2+\frac{1}{x^2}\right)^2=11^2 \Rightarrow x^4+\frac{1}{x^4}=119$

Again, $\left(x-\frac{1}{x}\right)^3=(3)^3$

$$\Rightarrow x^3-\frac{1}{x^3}-3\left(x-\frac{1}{x}\right)=27$$

$$\Rightarrow x^3-\frac{1}{x^3}=36$$

$$\left(x^4+\frac{1}{x^4}\right)\left(x^3-\frac{1}{x^3}\right)=x^7-x+\frac{1}{x}-\frac{1}{x^7}$$

$$\Rightarrow 119\times 36=\left(x^7-\frac{1}{x^7}\right)-\left(x-\frac{1}{x}\right)$$

$$\Rightarrow 4284=\left(x^7-\frac{1}{x^7}\right)-3$$

$$\Rightarrow\left(x^7-\frac{1}{x^7}\right)=4287$$

8. **(1)**

Let $p=\sqrt[3]{2+\sqrt{5}}+\sqrt[3]{2-\sqrt{5}}$

$$\Rightarrow p^3=\left(2+\sqrt{5}\right)+\left(2-\sqrt{5}\right)+3\sqrt[3]{(2+\sqrt{5})(2-\sqrt{5})}$$

$$\left[\sqrt[3]{(2+\sqrt{5})}+\sqrt[3]{(2-\sqrt{5})}\right]$$

$$\Rightarrow p^3=4+3(\sqrt[3]{4-5})p$$

$\Rightarrow p^3 = 4 – 3p$

$\Rightarrow p^3 + 3p – 4 = 0$

$\Rightarrow p^3 – p + 4p – 4 = 0$

$\Rightarrow p(p^2 – 1) + 4(p – 1) = 0$

$\Rightarrow (p – 1)[p(p + 1) + 4] = 0$

$\Rightarrow p = 1$ or $p(p + 1) = – 4$

But p is a positive number.

So, $p(p + 1) \neq –4$

$$\therefore p=1 \Rightarrow \sqrt[3]{2+\sqrt{5}}+\sqrt[3]{2-\sqrt{5}}=1$$

9. **(B)**

$$10+\sqrt{24}+\sqrt{40}+\sqrt{60}$$

$$=10+2\sqrt{6}+2\sqrt{10}+2\sqrt{15}$$

$$=10+2(\sqrt{2}\sqrt{3}+\sqrt{3}\sqrt{5}+\sqrt{5}\sqrt{2})$$

$$=(\sqrt{2})^2+(\sqrt{3})^2+(\sqrt{5})^2+2\sqrt{2}\sqrt{3}$$
$$+2\sqrt{3}\sqrt{5}+2\sqrt{5}\sqrt{2}$$

$$=(\sqrt{2}+\sqrt{3}+\sqrt{5})^2$$

Now,

$$\sqrt{10+\sqrt{24}+\sqrt{40}+\sqrt{60}}$$

$$=\sqrt{(\sqrt{2}+\sqrt{3}+\sqrt{5})^2}$$

$$=\sqrt{2}+\sqrt{3}+\sqrt{5}$$

10. **(A)**

$$\frac{4}{(2+\sqrt{3}+\sqrt{7})}\times\frac{(2+\sqrt{3}-\sqrt{7})}{(2+\sqrt{3}-\sqrt{7})}$$

$$=\frac{4(2+\sqrt{3}-\sqrt{7})}{(2+\sqrt{3})^2-(\sqrt{7})^2}$$

$$=\frac{8+4\sqrt{3}-4\sqrt{7}}{4+3+4\sqrt{3}-7}$$

$$=\frac{8+4\sqrt{3}-4\sqrt{7}}{4\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$$

$$=\frac{8\sqrt{3}+12-4\sqrt{21}}{12}$$

$$=1+\frac{2\sqrt{3}}{3}-\frac{\sqrt{21}}{3}$$

$$=\sqrt{1}+\sqrt{\frac{4}{3}}-\sqrt{\frac{7}{3}}$$

$$=\sqrt{a}+\sqrt{b}-\sqrt{c}$$

On comparing both sides we get

$$a=1,\ b=\frac{4}{3},\ c=\frac{7}{3}$$