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FREE AND SLAVE LABOR IN THE ANTEBELLUM SOUTH: PERFECT SUBSTITUTES OR DIFFERENT INPUTS?

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Abstract—The substitutability between free and slave labor is examined and the permissibility of aggregating the two into a single labor variable is investigated, using a translog production function. Slaves on large cotton farms worked in gangs; free labor was not observed to do so. Despite this, previous research has aggregated free and slave labor and employed functional forms imposing strong restrictions on substitution. Estimation of the translog function shows that simple additive aggregation is not acceptable; on large farms, slaves and free labor were complements, while on small, nongang farms, they were substitutes.

HISTORICAL evidence suggests that, in the antebellum South, larger cotton-growing slave plantations relied almost exclusively on the work gang system of labor organization (Fogel and Engerman, 1974a). The gang system was adopted only beyond some threshold level of slaveholding (about 15 slaves) which made this technology practical to implement. Free labor was not observed working in gangs. This implies that on large farms free and slave labor were different inputs and thus could not be viewed as perfect substitutes in the production process. Free and slave labor would have been less substitutable on these farms than on small farms, which did not rely on the work gang system. However, in virtually all investigations of the economics of American slavery, free and slave labor are added together to form an aggregate labor variable, thereby assuming perfect substitutability between the two.

This issue has important implications for previous studies of the characteristics of agricultural production under slavery, both those which investigated the substitution characteristics directly, and those which concentrated on the variation of efficiency with farm size and slaveholding. Zepp (1976) did make an attempt to determine the elasticity of substitution between slaves and all other inputs. However, while Zepp did disaggregate slave labor, he aggregated all other inputs. Furthermore, the production function used was the constant elasticity of substitution (CES) production function, which assumes complete strong separability, thereby preventing the value of the elasticity from varying. Other works aggregate free and slave labor, and most use either a CES form or a Cobb-Douglas, which assumes a unitary elasticity of substitution.

The use of untested restrictions on separability and substitutability casts some doubt on the validity of the conclusions of these works. Clearly, if these restrictions are tested and not accepted, estimates of elasticities of substitution obtained using the restrictions are not acceptable. There may also be implications for investigations of the relative efficiency of slave agriculture. Berndt and Christensen (1974) have shown that estimates of marginal product may be biased if unacceptable restrictions are placed on separability, although estimates of output are generally not. The "economies of scale in slaveholding" postulated by Fogel and Engerman (1974a) involve a one-time jump in efficiency associated with the implementation of the gang system. This could mean a jump in output, a jump in the marginal product of slave labor, or both. If free and slave labor are aggregated, it is impossible to distinguish the marginal product of slave labor. Even if they are disaggregated, the use of CES or Cobb–Douglas may produce biased estimates of marginal product.

This paper tests several hypotheses in order to determine (1) the validity of earlier studies of agricultural production under slavery; (2) whether the degree of substitution between free and slave labor was less on large farms than on small farms; and (3) whether on large farms, which presumably employed the gang system, free and slave labor, far from being perfect substitutes, were different, complementary inputs.

Separate flexible form production functions are estimated for large and small farms. A comparison

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between a measure of substitutability estimated for each regime will indicate whether differences exist in the relationship between free and slave labor on small and large farms. Secondly, the two production functions will be tested for separability in order to determine:

(1) Whether adding free and slave labor to form an aggregate labor variable is permissible; and
(2) whether Cobb-Douglas and/or CES functions may be used to approximate production technology.[1]

The form of the production function chosen is the translog. The measure of substitutability used for testing for differences in substitutability is the Hicks elasticity of complementarity. Factor price elasticities are also presented.

The paper is organized as follows. The data source used in this study is described in section I, as well as variable definitions. The methodology employed to test the hypotheses described above is presented in section II. The empirical results are interpreted in section III. Section IV summarizes the major findings and presents conclusions.

[1] This approach can easily be used to test the relative efficiency of gang and non-gang farms. The presence of ordinary economies of scale under the two regimes can be investigated, and the behavior of the two production functions at the threshold for implementation of the gang system examined. Using this method, I found a significant increase in output and marginal product of slave labor at the threshold; on average, large farms had 24% more output than if they had used the gang system (Field, 1988).

These results can be presented in terms of the total factor productivity approach used by Fogel and Engerman; they tend to reinforce Fogel and Engerman's conclusions.

TOTAL FACTOR PRODUCTIVITY INDICES

		F-E (1)	Cobb-Douglas, With Corrected Inputs and Outputs (2)	Translog (3)
(a)	Free Farms	100.0	100.0	100.0
(b)	Slave Farms Not Using the Gang System (1–15 slaves)	100.8	113.9	136.7
(c)	Slave Farms Using the Gang System (16 + Slaves)	140.3	174.4	203.2
(d)	Advantage to Gang System ((c) ÷ (b))	37.6	53.1	48.7

(1) Figures are taken from Fogel and Engerman (1980), p. 674.

(2) See section I for a discussion of the variables. Separate cost shares were used for free and slave farms.

(3) Separate cost shares from translog were used for free and slave farms, evaluated at mean input values.

I. The Data Set and Variable Definitions

The data set used is the Parker-Gallman sample of farms in the "Cotton South," drawn from manuscript censuses from the 1860 census for farms in eleven states. The Parker-Gallman sample provides information on the number of improved and unimproved acres per farm, as well as the value of the acreage, the value of farm machinery, the number of livestock, the farm's crop outputs, and data on the population resident on the farm (free and slave, by age and sex) for each farm.[2]

My input and output definitions are largely based on Fogel and Engerman's with some modifications. The output definition was changed to reflect a problem with double-counting of feed allowances, which led to an underestimation of final output. In addition, free and slave labor were disaggregated in order to test for the existence of a consistent aggregate labor index, and to examine the pattern of substitutability between the two types of labor. Fogel and Engerman used a weighted labor input with weights for laborers of different ages and sexes. The capital input is an annualized capital input, assuming a 10% rate of return. Finally, the land input consists of improved acreage times a soil quality index derived from information on soil types found in the Parker-Gallman sample. This procedure was followed to avoid including locational components as would occur if land values were used.

II. Methodology—A Translog Production Function

In order to investigate the substitutability of free and slave labor, it is necessary to use a flexible form production function which does not

[2] The data were made available in part by the Interuniversity Consortium for Political and Social Research. The data for the 1860 Cotton Sample were originally collected by William N. Parker and Robert E. Gallman under a grant from the National Science Foundation. Neither the original collectors of the data nor the consortium bear any responsibility for the analyses or interpretations presented here.

impose restrictions on separability a priori. The transcendental logarithmic or translog production function is the form chosen.

The empirical model estimated is specified as follows:

$$\begin{aligned}\ln Q = a_0 &+ a_F \ln F + a_s \ln S + a_K \ln K \\ &+ a_L \ln L + \tfrac{1}{2} b_{FF} (\ln F)^2 \\ &+ \tfrac{1}{2} b_{SS} (\ln S)^2 + \tfrac{1}{2} b_{KK} (\ln K)^2 \\ &+ \tfrac{1}{2} b_{LL} (\ln L)^2 + b_{FS} \ln F \ln S \\ &+ b_{FK} \ln F \ln K + b_{FL} \ln F \ln L \\ &+ b_{SK} \ln S \ln K + b_{SL} \ln S \ln L \\ &+ b_{LK} \ln L \ln K \qquad (1)\end{aligned}$$

where Q = output, F = prime age male equivalent free labor, S = prime age male equivalent slave labor, K = annual capital input, and L = the land input. This is the physical production function. It is estimated using ordinary least squares methods, assuming that all inputs are exogenous, that factor prices are endogenous and that input and output values are not chosen simultaneously. This may not be a good assumption if the unit of observation is the farm. However, given the lack of data on factor prices, this assumption will be made.

Fogel and Engerman (1980) cite historical evidence that indicates that use of the gang system was instituted above a threshold of about fifteen slaves. They theorize that the gang system represented a different agricultural production technology. Therefore, one would expect a switch in regimes at about fifteen slaves. The data set was split and a Chow test performed. The null hypothesis of no switch was rejected. Production technology followed different regimes on small and large farms.

The data set was split at several alternate cutoff points (S, the adjusted slave labor force, = 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 15, and 20) in order to determine the most likely point for the switch in regimes. Of the values tested, the maximum likelihood value was an adjusted slave labor force of six at the cutoff. This corresponds to an average of fifteen slaves held. Thus, the statistical procedure and historical evidence reinforce each other. Although there is no direct evidence on which farms in the sample employed the gang system, the fact that a switch occurred at about fifteen slaves suggests strongly that the shift in regimes was due to implementation of the gang technology.

Using the two production functions, the question of substitutability of inputs, in particular between free and slave labor, can be considered, and substitution characteristics on small and large farms compared. Two measures of substitutability, Hicks elasticities of complementarity (HEC) and factor price elasticities, will be employed. The Allen elasticity of substitution is a more common measure; it is the measure that Zepp (1976) and Schmitz and Schaefer (1978) used. However, using AESs estimated from the physical translog production function may not be desirable because their derivation from the production function involves inverting the matrix of estimated coefficients. If one coefficient has a large standard error, this will affect all estimates of AESs (Hamermesh and Grant, 1979).

HECs between all pairs of inputs will be calculated for small and large farms. The relative size of the HEC between free and slave labor is of particular interest. If this statistic is larger on large farms, this would imply that free and slave labor were more complementary or less substitutable under the gang system, as has been hypothesized. Tests will be performed in order to see whether this difference is statistically significant. In addition, factor price elasticities between free and slave labor will be examined.

The substitutability of inputs is closely related to the existence of functional separability. Specific restrictions which imply separability will be tested. The CES and Cobb-Douglas production functions make implicit assumptions of separability. Tests will be performed in order to see whether the translog form can be considered an approximation to either of these functional forms, or whether use of these forms is inappropriate.

III. Empirical Results

A. *Substitutability of Inputs*

To investigate the pattern of substitutability between all pairs of inputs, and in particular between free and slave labor, Hicks elasticities of complementarity were calculated for small and large farms. For the translog function, the own HEC is defined as

$$h_{ii} = \left(b_{ii} + M_i^2 - M_i\right)/M_i^2 \qquad (2)$$

TABLE 1.—HICKS ELASTICITIES OF COMPLEMENTARITY

	Small Farms	Large Farms
h_{FF}	−6.80	−1227.69
	(0.04)	(188.18)
h_{SS}	0.46	−2.85
	(0.01)	(0.10)
h_{KK}	−9.43	−1.11
	(0.34)	(0.09)
h_{LL}	−1.77	−1.49
	(0.02)	(0.09)
h_{FS}	0.30	7.03
	(0.03)	(1.27)
h_{FK}	5.34	−20.94
	(0.11)	(3.50)
h_{FL}	−0.31	−14.45
	(0.03)	(2.57)
h_{SK}	1.73	2.06
	(0.03)	(0.06)
h_{SL}	1.41	0.77
	(0.01)	(0.02)
h_{LK}	1.91	0.84
	(0.02)	(0.03)

Note: Standard errors are presented in parentheses.

and the cross elasticity of complementarity as

$$h_{ij} = (b_{ij} + M_i M_j)/M_i M_j \tag{3}$$

where M_i is the logarithmic marginal product, $\partial \ln Q/\partial \ln X_i$. Both the M_i and the HECs are functions of input values. Mean input values for each group of farms were used to evaluate the HECs. For small farms, these were $F = 2.02$, $S = 2.13$, $K = \$162.50$, and $L = 243.79$. For large farms, the mean values were $F = 1.68$, $S = 12.60$, $K = \$587.89$, and $L = 930.85$. The mean HECs and their standard errors are presented in table 1.[3]

The Hicks elasticity of complementarity between free and slave labor is positive for both small and large farms, indicating that free and slave labor were "q-complements," or that the two factors worked together to increase output. However, h_{FS} is smaller on small farms. Free and slave labor appear to have been more complementary or less substitutable on large farms.

However, before concluding this, it must be established that the difference is statistically significant. The hypothesis that h_{FS} (large) $\leq h_{FS}$ (small) was therefore tested, treating the HECs as independent random variables. The value of the t-statistic was 5.27; the null hypothesis was rejected. In addition, factor price elasticities were examined. These were computed by multiplying the HEC by the appropriate logarithmic marginal product. Thus,

$$\Theta_{SF} = (h_{FS})(m_F) = \partial \ln p_S/\partial \ln F,$$

and

$$\Theta_{FS} = (h_{FS})(m_S) = \partial \ln p_F/\partial \ln S.^4$$

On small farms, $\Theta_{SF} = 0.054$ and $\Theta_{FS} = 0.097$. On large farms, $\Theta_{SF} = 0.098$, and $\Theta_{FS} = 2.85$. So, both factor price elasticities were larger on large farms, confirming the evidence of the HECs. The differences proved to be statistically significant. For Θ_{SF}, the t-statistic was 3.95; for Θ_{FS}, $t = 5.41$. It may be concluded that free and slave labor were more complementary on farms employing the gang system. Thus, estimation of the elasticities bears out what intuition suggests—that free and slave labor were not freely substitutable. Instead they were complementary inputs under the gang system.

That is the expected result under a system that organized slave labor into work gangs where free laborers would not work. Free labor was managerial; slave labor worked in the fields.

These results are dependent on the cutoff point chosen, as well as on the variable definitions used. The robustness of the results was therefore checked by calculating HECs at several alternate cutoff points ($S = 4$, 8, and 15) and using Fogel and Engerman's original input and output definitions. The HEC between free and slave labor was greater on large farms no matter what the cutoff point; this was also true when capital was defined as the value of machinery and equipment and land as the value of the farm. When the old capital definition and the new land definition were used, the estimated HEC was negative on large farms, however. Finally, when the original output definition was used, estimates of the marginal product of free labor were negative on small farms; HECs were not calculated. On the whole, then, the results seem fairly robust.

B. *Functional Separability*

Testing for functional separability of inputs is an extremely important step in the examination of

[3] All of the own HECs except that for slave labor on small farms are negative, which is consistent with convexity. The fact that h_{SS} is positive for small farms may indicate that these farms were not in a situation of equilibrium, but were evolving and growing with an eye toward implementation of the gang system.

[4] This assumes that, in equilibrium, Q/X_i can be taken as p_i.

the pattern of substitutability between free and slave labor. The tests for functional separability determine the possibilities for aggregation of inputs, since the acceptance of a separability restriction implies the existence of some aggregate index of a subset of the inputs. Based on these tests, one can establish that the aggregation of free and slave labor into a single labor variable is unacceptable. The tests also demonstrate the inappropriateness of particular functional forms, specifically CES and Cobb-Douglas. Since the aggregation of the two types of labor and the use of CES and Cobb-Douglas characterize previous work on slavery, the results of tests of functional separability are extremely significant.

Denny and Fuss (1977) derive approximate tests for varying degrees of separability for the translog function with three inputs. These tests have been generalized to the n input case by Taylor (1982). The testing procedure is a nested sequential procedure starting with the least restrictive form, weak separability, proceeding to strong separability, then complete strong separability (which implies the CES form) and finally the Cobb-Douglas form.[5] Whenever the first separability hypothesis is rejected, the testing sequence ends. The significance level of each test depends on the significance level of previous tests in the sequence. The levels adopted by Denny and Fuss were used in this analysis: a 0.01 significance level for weak separability tests, 0.025 for strong separability, and 0.05 for logarithmic strong separability.

The test results are presented in tables 2 and 3. The tests show that on both small and large farms, weak separability cannot be rejected. In particular, weak separability of F and S cannot be rejected on either class of farm. The pattern of substitutability revealed by examination of the HECs, however, is not consistent with free and slave labor being perfect substitutes. Some type of aggregation may be permissible; simple addition is not.

Table 2.—Tests for Separability
Small Farms

A. Weak Separability

Type	F-Statistic	Reject if $\alpha = 0.01$?
$(F, S) - K, L$	1.33	No
$(F, K) - S, L$	1.20	No
$(F, L) - K, S$	0.76	No
$(S, K) - F, L$	0.69	No
$(S, L) - F, K$	0.76	No
$(L, K) - F, S$	1.64	No
$(F, S, K) - L$	1.00	No
$(F, S, L) - K$	1.04	No
$(S, K, L) - F$	6.28	Yes

B. Partial Strong Separability

Type	F-Statistic	Reject if $\alpha = 0.025$?
$(F, S) - (K, L)$	1.64	No
$(F, K) - (S, L)$	1.40	No
$(F, L) - (K, S)$	3.41	Yes
$(F, S, K) - L$	3.32	Yes
$(F, S, L) - K$	2.51	No
$(S, K, L) - F$	1.46	No

C. Complete Strong Separability (CES)

F-Statistic	Reject if $\alpha = 0.05$?
8.87	Yes

Since some type of weak separability is accepted for both small and large farms, testing proceeded to the strong separability restrictions. Logarithmic partial strong separability could not be rejected either for small or large farms, and testing proceeded to the next level. However, complete strong separability is rejected both for small and large farms. The CES form, and by extension the Cobb-Douglas, is inappropriate. Since the CES form was rejected, testing stopped at this point.

IV. Summary and Conclusions

Based on a translog production model of antebellum slave cotton production, an investigation of the substitutability and separability characteristics of the production technology has some important implications for research into slave agriculture. The estimation results are summarized below. First, it can be shown that production

[5] The propositions on which the tests are based are

(1) The translog function is a quadratic approximation to an arbitrary weakly separable function $\ln Q = F[G(\ln X_1, \ldots, \ln X_t), \ln X_{t+1}, \ldots, \ln X_n]$ if $a_i/a_j = b_{ik}/b_{jk}$ for all $i, j \in (1, \ldots, t)$ and $k \in (t+1, \ldots, n)$ where the a's and b's are estimated parameters.

(2) The translog function is a quadratic approximation to an arbitrary logarithmic partially strongly separable function of the form $\ln Q = G(\ln X_1, \ldots, \ln X_t) + H(\ln X_{t+1}, \ldots, \ln X_n)$ if $b_{ik} = b_{jk} = 0$ for all $i, j \in (1, \ldots, t)$ and $k \in (t+1, \ldots, n)$.

(3) The homogeneous translog function is a quadratic approximation to an arbitrary CES function if $\Sigma_j b_{ij} = 0$ and $a_i/a_j = b_{ik}/b_{jk}$ for $i \neq j \neq k$.

(4) The homogeneous translog function is a quadratic approximation to an arbitrary Cobb-Douglas function if, in addition to the constraints of (3), $b_{ik} = b_{jk} = 0$.

TABLE 3.—TESTS FOR SEPARABILITY
LARGE FARMS

A. Weak Separability

Type	F-Statistic	Reject if $\alpha = 0.01$?
$(F, S) - K, L$	1.62	No
$(F, K) - S, L$	0.51	No
$(F, L) - K, S$	1.13	No
$(S, K) - F, L$	0.40	No
$(S, L) - F, K$	4.15	No
$(L, K) - F, S$	0.96	No
$(F, S, K) - L$	3.02	No
$(F, S, L) - K$	1.70	No
$(S, K, L) - F$	1.40	No

B. Partial Strong Separability

Type	F-Statistic	Reject if $\alpha = 0.025$?
$(F, S) - (K, L)$	2.05	No
$(F, K) - (S, L)$	2.04	No
$(F, L) - (K, S)$	0.78	No
$(F, S, K) - L$	0.44	No
$(F, S, L) - K$	2.18	No
$(S, K, L) - F$	0.91	No

C. Complete Strong Separability (CES)

	F-Statistic	Reject if $\alpha = 0.05$?
	4.11	Yes

technology followed two different regimes, one on large farms, which probably employed the gang system, and another on small farms. The cutoff between the two regimes occurred at about fifteen slaves. This cutoff is suggested both by the historical and statistical evidence. Second, investigation of functional separability also reveals that use of CES and Cobb-Douglas production functions to model antebellum cotton growing technology is not appropriate. Finally, free and slave labor were complementary inputs on large farms, both in the sense of the Hicks elasticity and the Allen elasticity. On small farms, they were complementary in the sense of the Hicks elasticity, although significantly less so than on large farms, while in terms of the Allen elasticity, they were substitutes. On both types of farm, investigation of the separability characteristics of the translog function reveals some aggregation of free and slave labor to have been permissible. However, simple additive aggregation is not acceptable. The findings support the contention that modelling Southern slave agriculture with a CES or Cobb-Douglas function, or treating free and slave labor as perfect substitutes, is unacceptable, and will not yield defensible results, although this has been the approach followed by most researchers.

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