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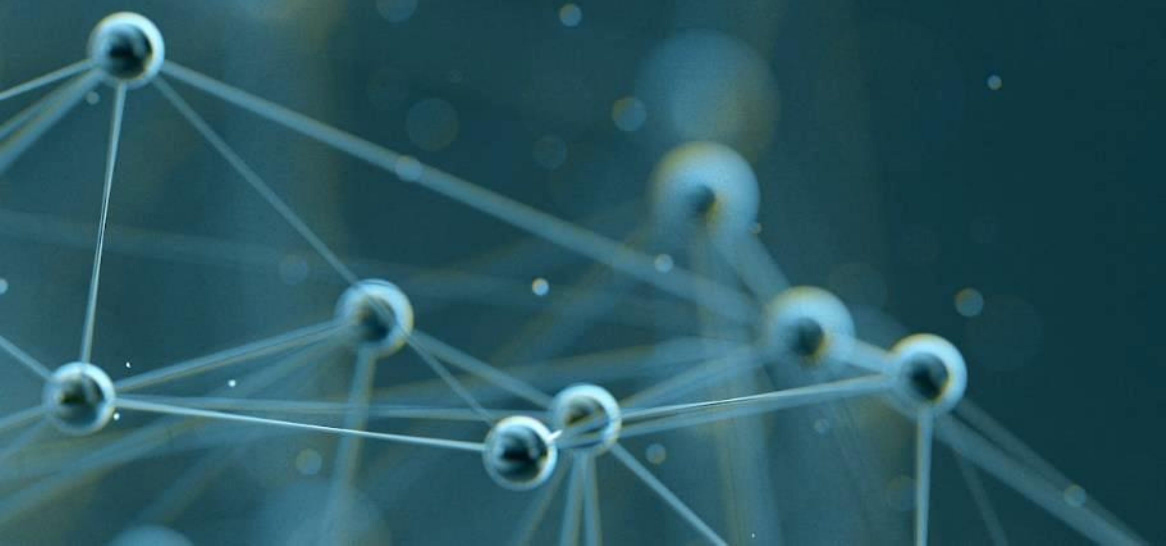
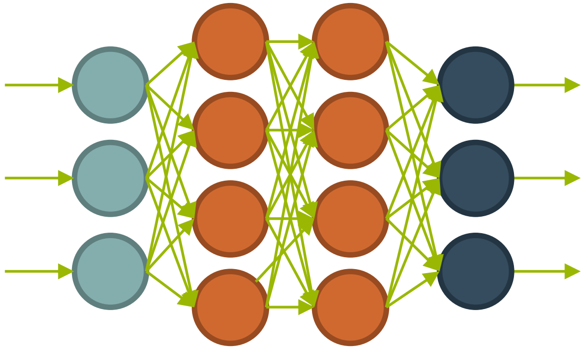
INTRODUCTION TO NEURAL NETS

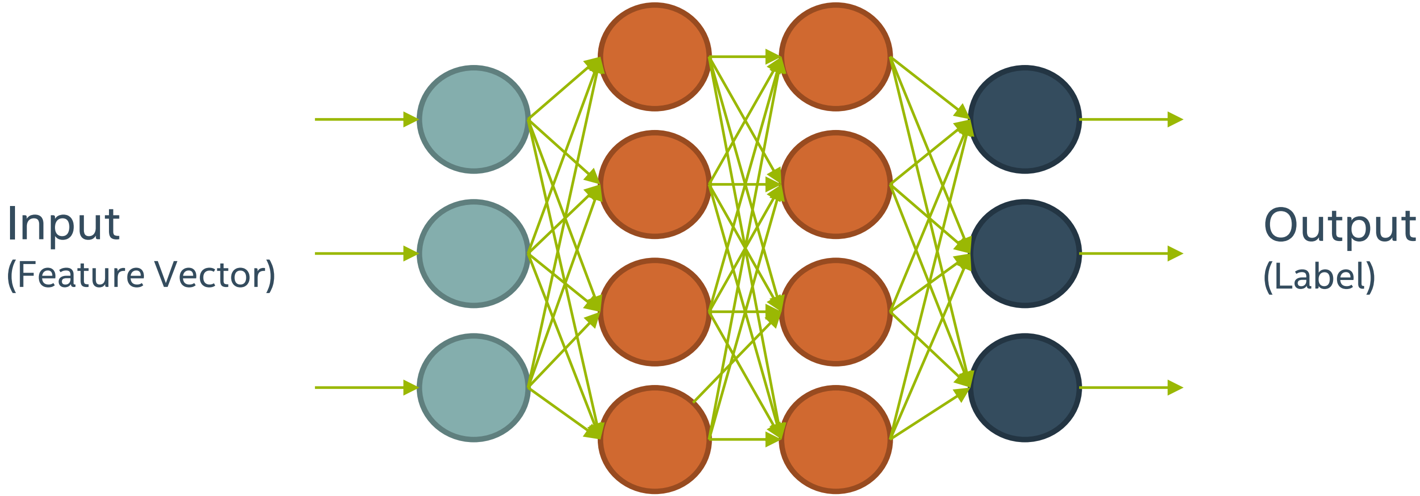


MOTIVATION FOR NEURAL NETS

- Use biology as inspiration for mathematical model
- Get signals from previous neurons
- Generate signals (or not) according to inputs
- Pass signals on to next neurons
- By layering many neurons, can create complex model

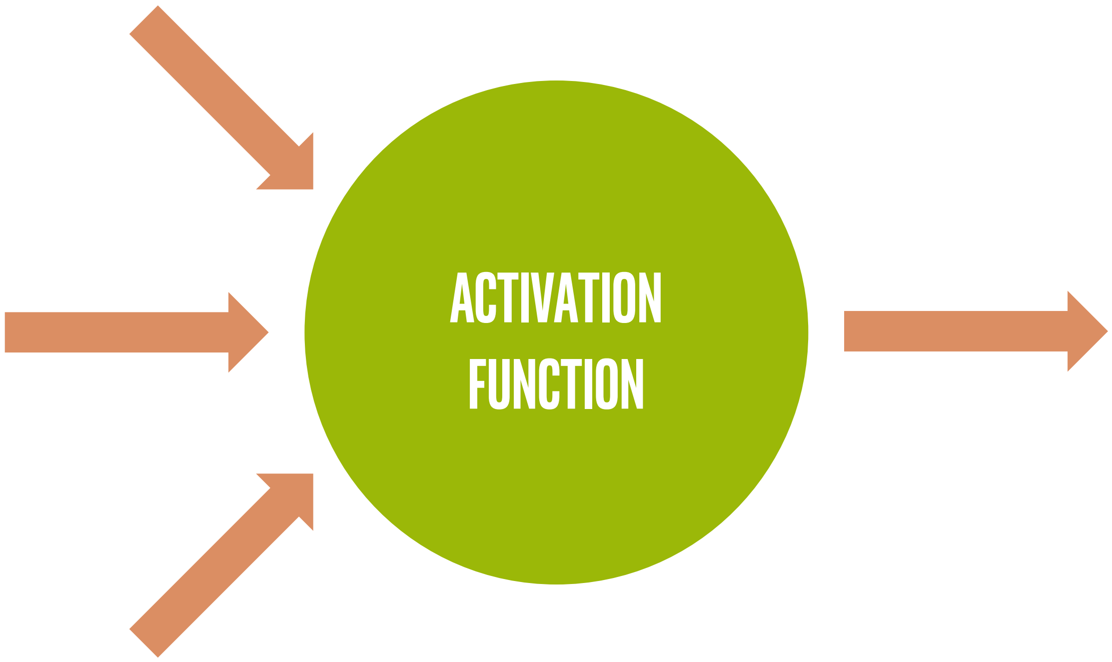


NEURAL NET STRUCTURE



- Can think of it as a complicated computation engine
- We will ”train it” using our training data
- Then (hopefully) it will give good answers on new data

BASIC NEURON VISUALIZATION



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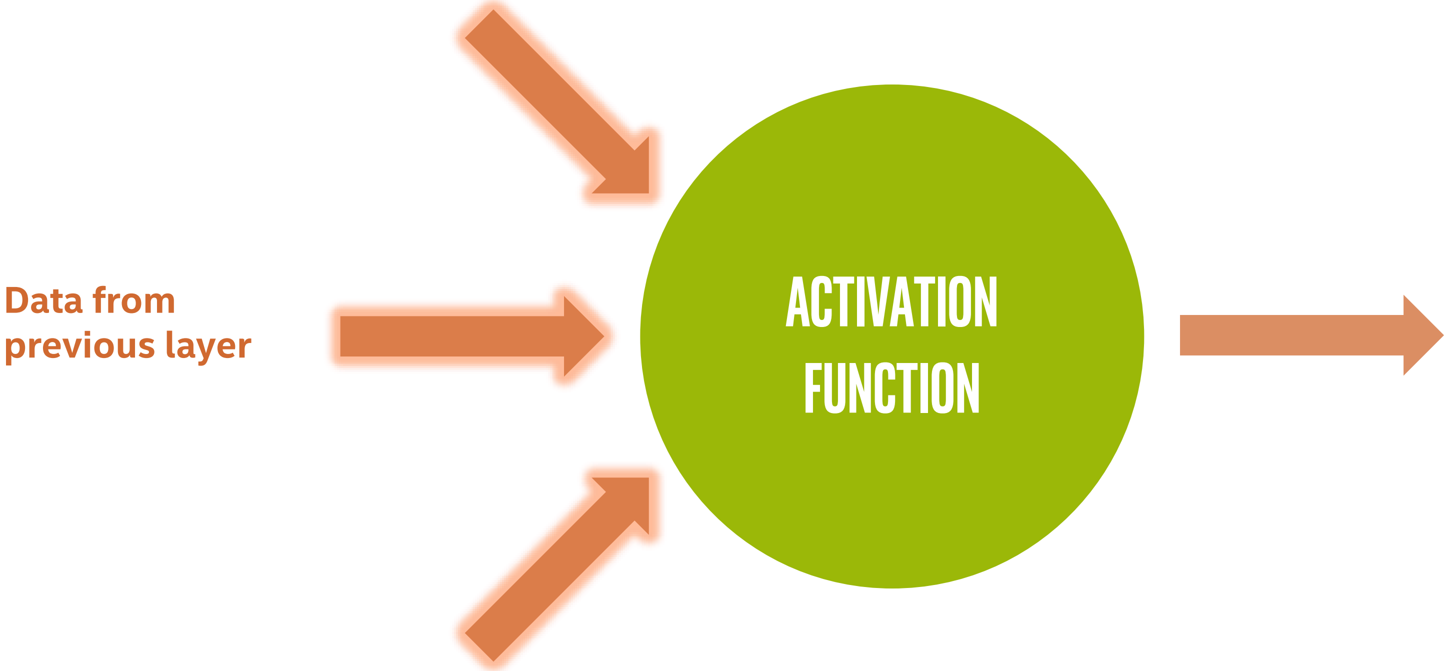
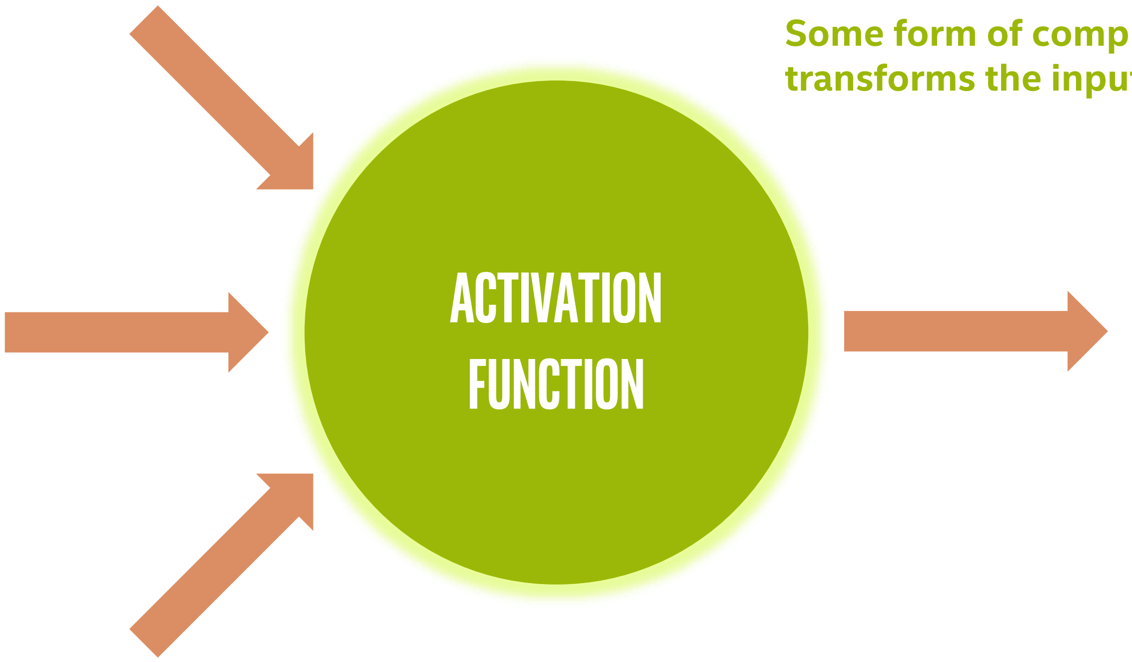
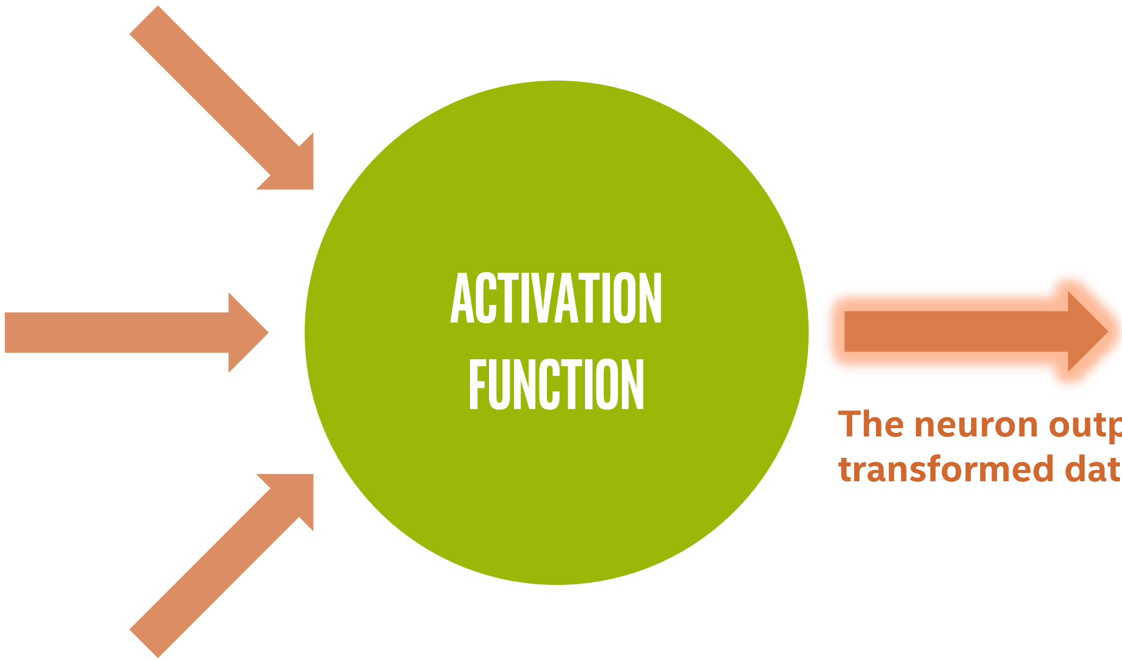


BASIC NEURON VISUALIZATION

Some form of computation transforms the inputs

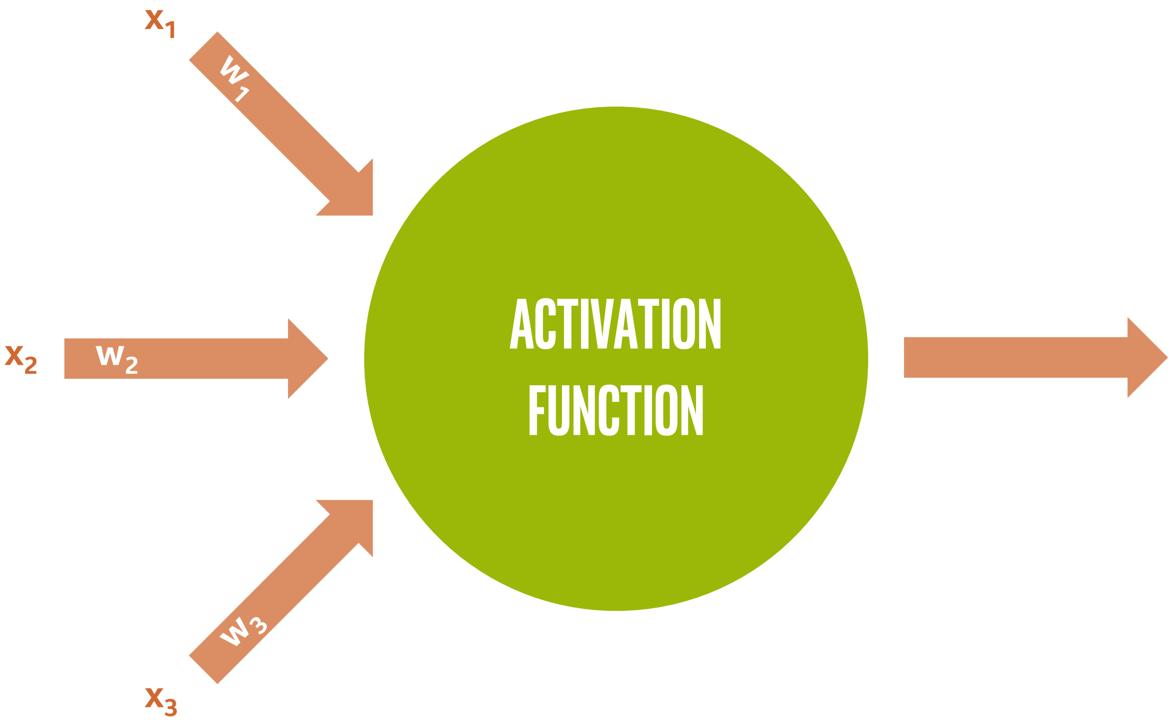


BASIC NEURON VISUALIZATION

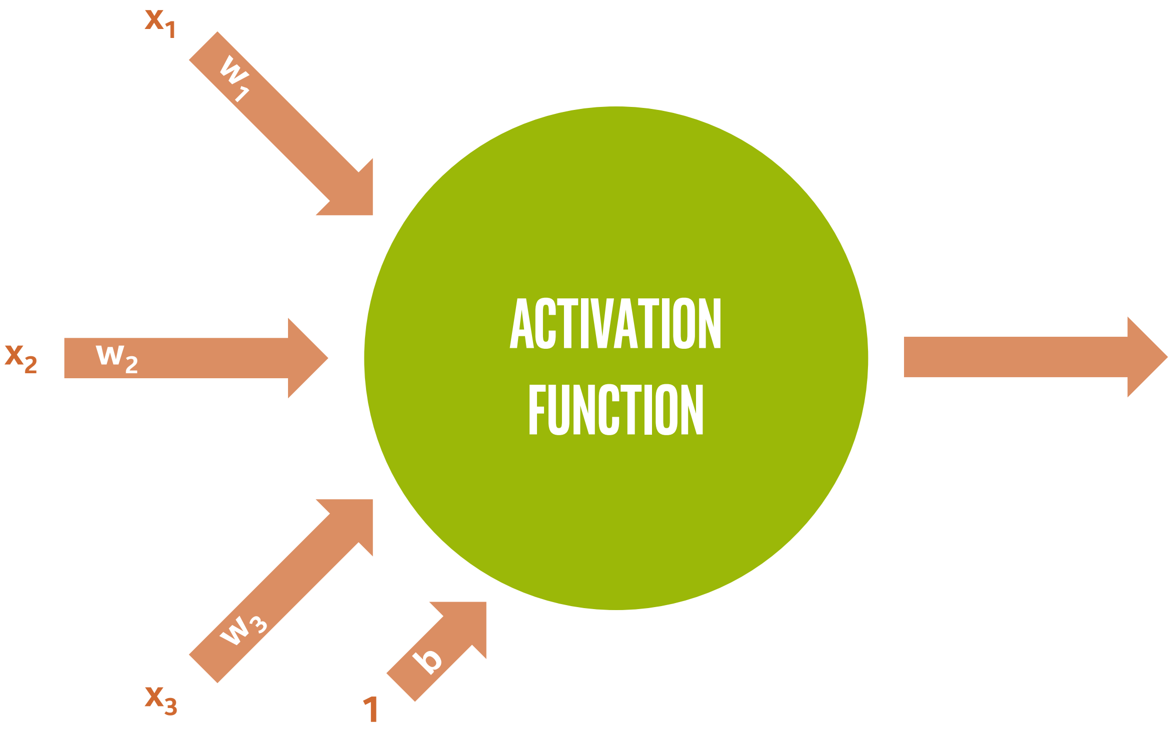


The neuron outputs the transformed data

BASIC NEURON VISUALIZATION

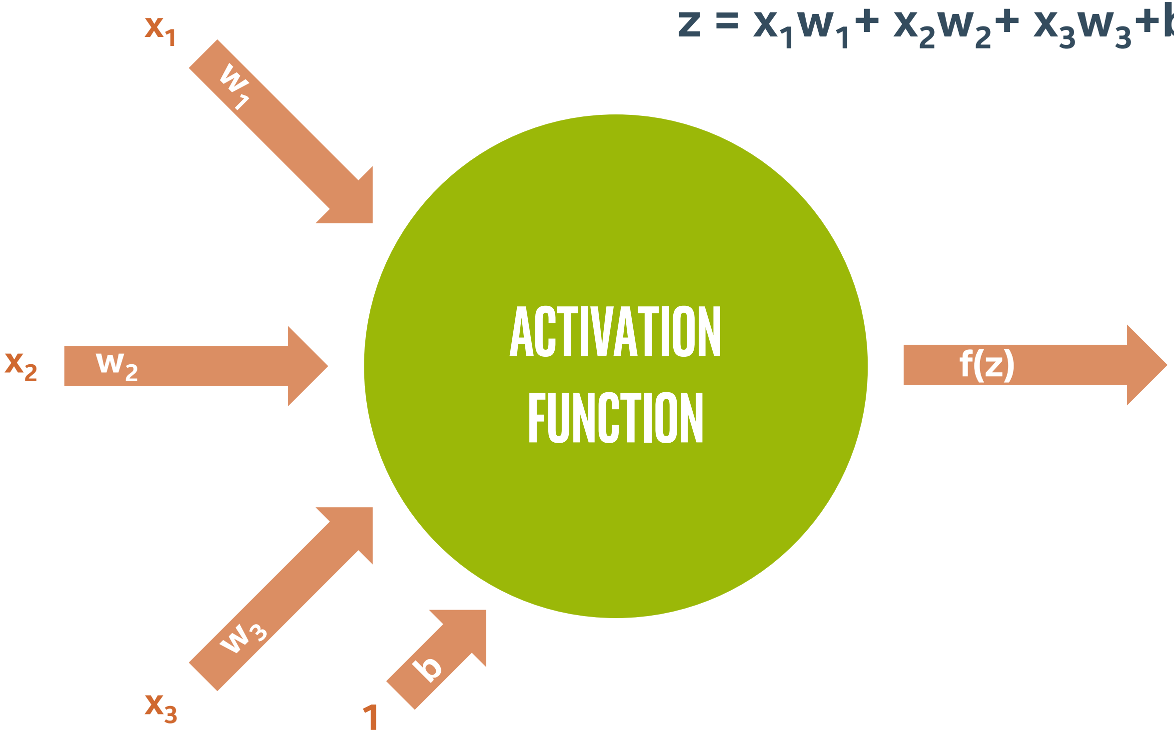


BASIC NEURON VISUALIZATION



BASIC NEURON VISUALIZATION

$$z = x_1w_1 + x_2w_2 + x_3w_3 + b$$



IN VECTOR NOTATION

z = "net input"

b = "bias term"

f = activation function

a = output to next layer

$$z = b + \sum_{i=1}^{m} x_i w_i$$

$$z = b + x^T w$$

$$a = f(z)$$

RELATION TO LOGISTIC REGRESSION

When we choose: $f(z) = \frac{1}{1+e^{-z}}$

$$z = b + \sum_{i=1}^{m} x_i w_i = x_1 w_1 + x_2 w_2 + \cdots + x_m w_m + b$$

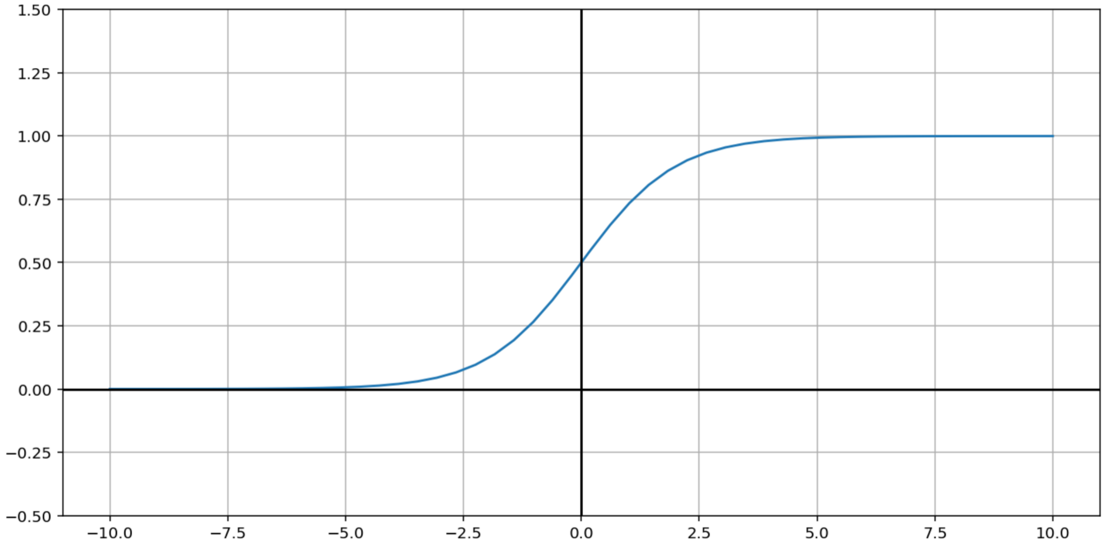
Then a neuron is simply a "unit" of logistic regression!

weights ⇔ coefficients **inputs ⇔ variables**

bias term ⇔ constant term

RELATION TO LOGISTIC REGRESSION

This is called the "sigmoid" function: $\sigma(z) = \dfrac{1}{1 + e^{-z}}$



NICE PROPERTY OF SIGMOID FUNCTION

$$\sigma(z) = \frac{1}{1+e^{-z}}$$

Quotient rule

$$\frac{d}{dx} \cdot \frac{f(x)}{g(x)} = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$

$$\sigma'(z) = \frac{0-(-e^{-z})}{(1+e^{-z})^2} = \frac{e^{-z}}{(1+e^{-z})^2}$$

$$= \frac{1+e^{-z}-1}{(1+e^{-z})^2} = \frac{\cancel{1+e^{-z}}}{(1+e^{-z})^{\cancel{2}}} - \frac{1}{(1+e^{-z})^2}$$

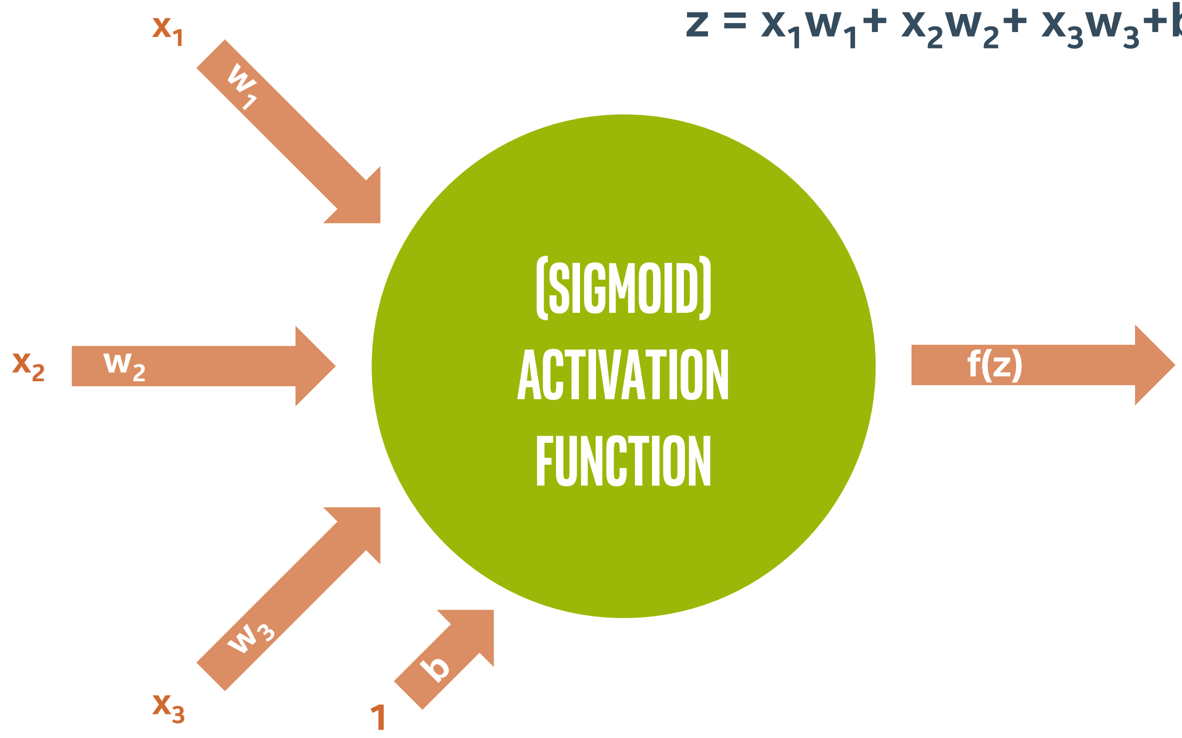
$$= \frac{1}{1+e^{-z}} - \frac{1}{(1+e^{-z})^2} = \frac{1}{1+e^{-z}}\left(1 - \frac{1}{1+e^{-z}}\right)$$

$$\sigma'(z) = \sigma(z)(1-\sigma(z))$$

This will be helpful!

EXAMPLE NEURON COMPUTATION

$$z = x_1 w_1 + x_2 w_2 + x_3 w_3 + b$$



EXAMPLE NEURON COMPUTATION

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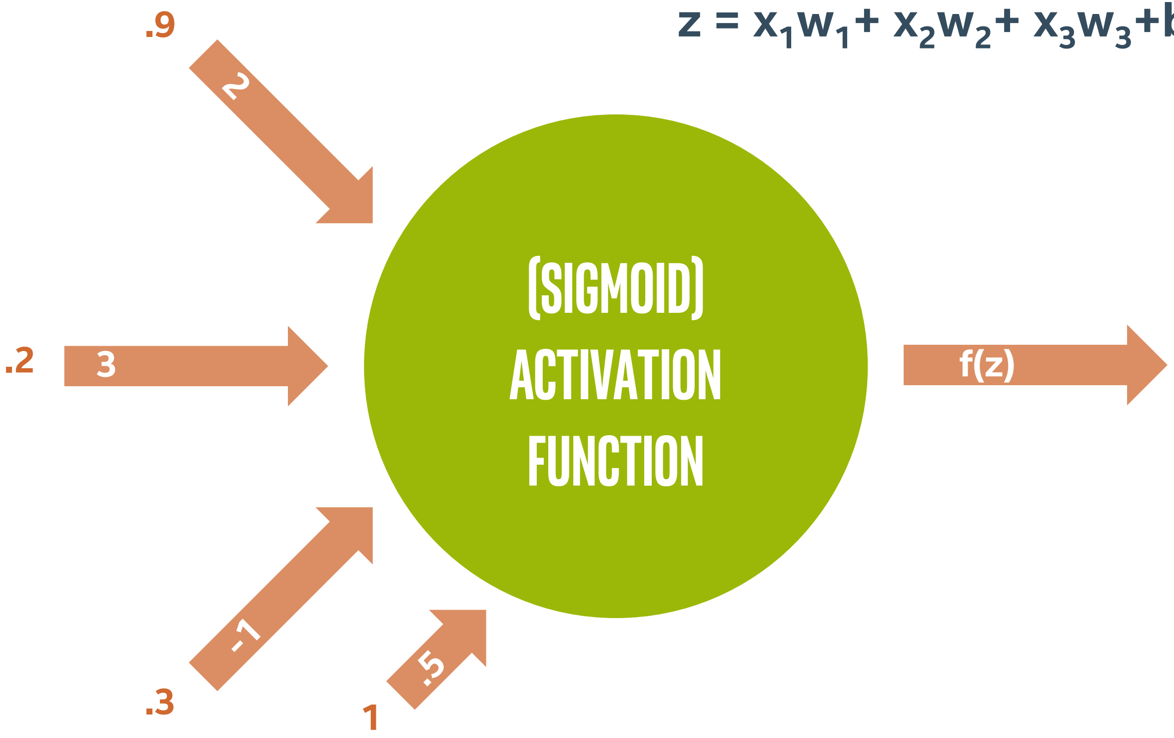
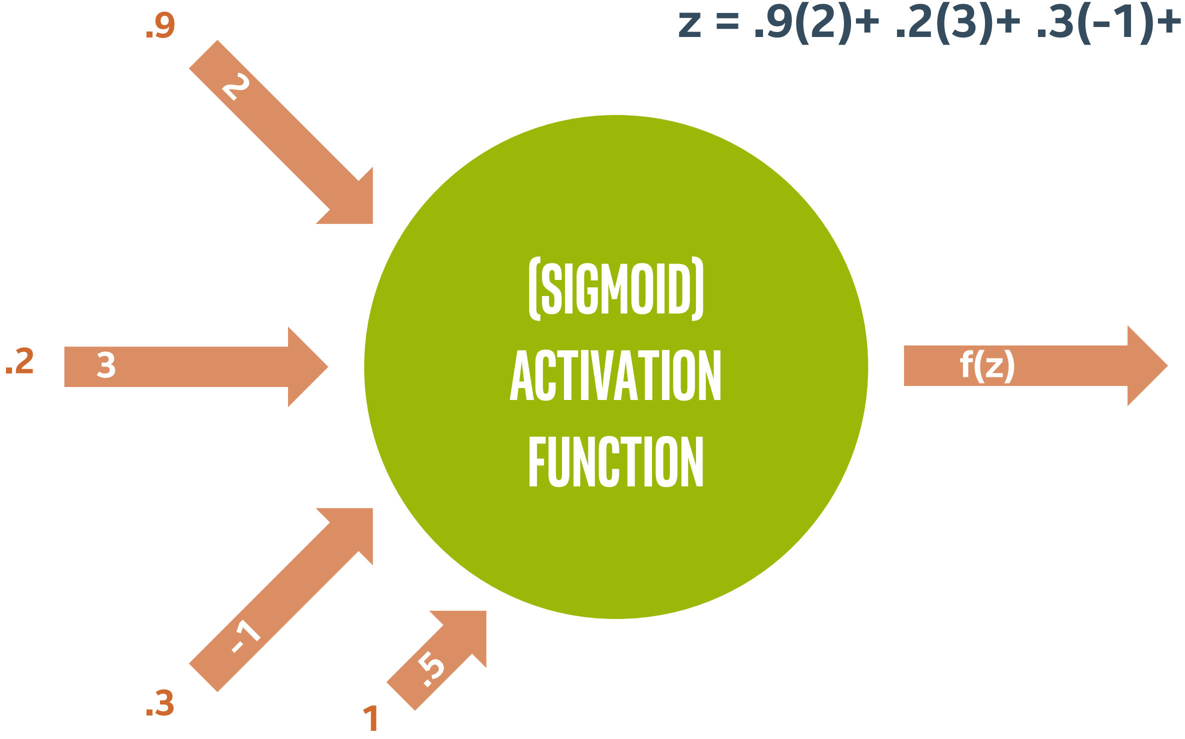


EXAMPLE NEURON COMPUTATION

$$z = .9(2) + .2(3) + .3(-1) + .5 = 2.6$$



EXAMPLE NEURON COMPUTATION

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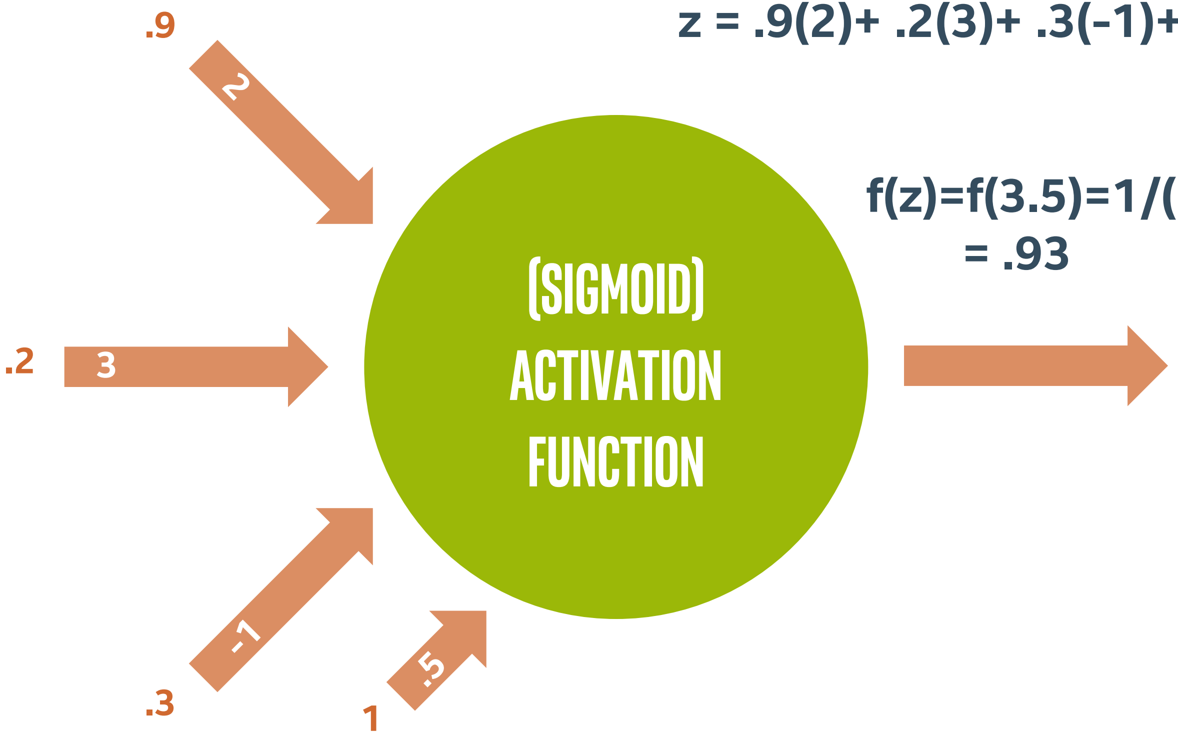
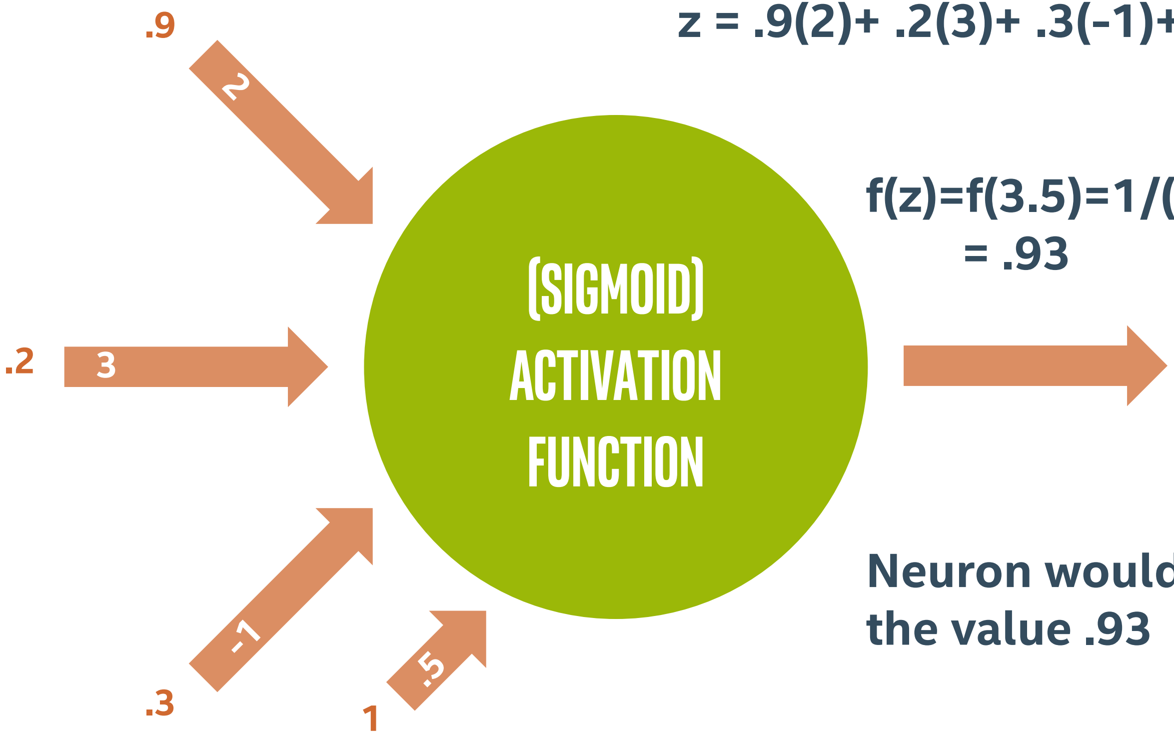


$$f(z) = f(3.5) = 1/(1+\exp(-2.6)) = .93$$

EXAMPLE NEURON COMPUTATION



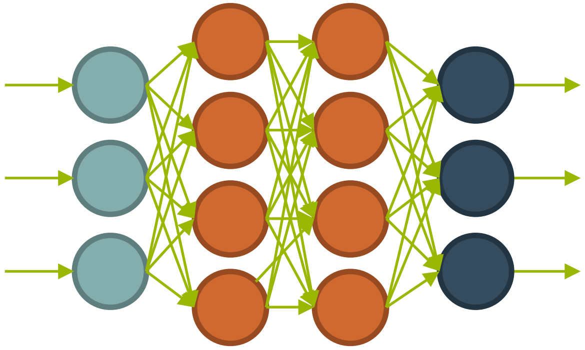
z = .9(2)+ .2(3)+ .3(-1)+.5 = 2.6

f(z)=f(3.5)=1/(1+exp(-2.6))
= .93

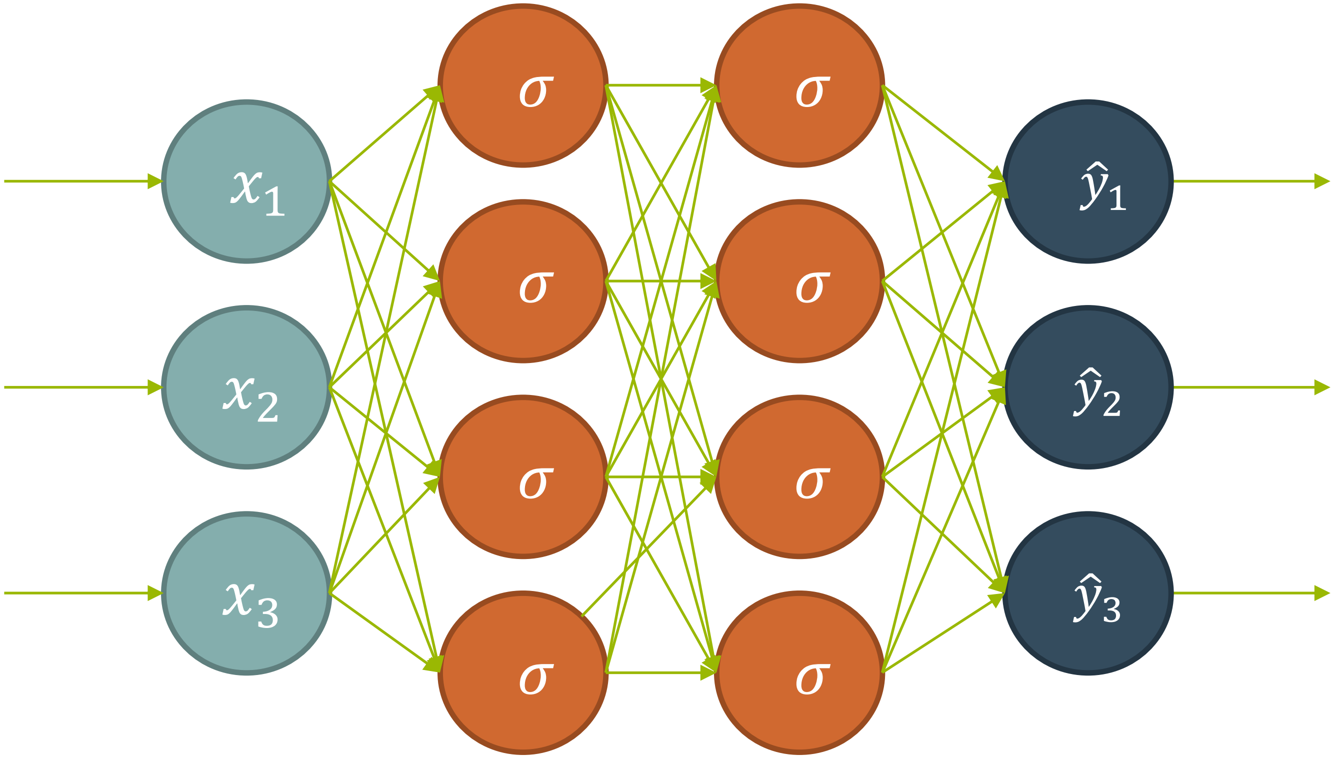
Neuron would output the value .93

WHY NEURAL NETS?

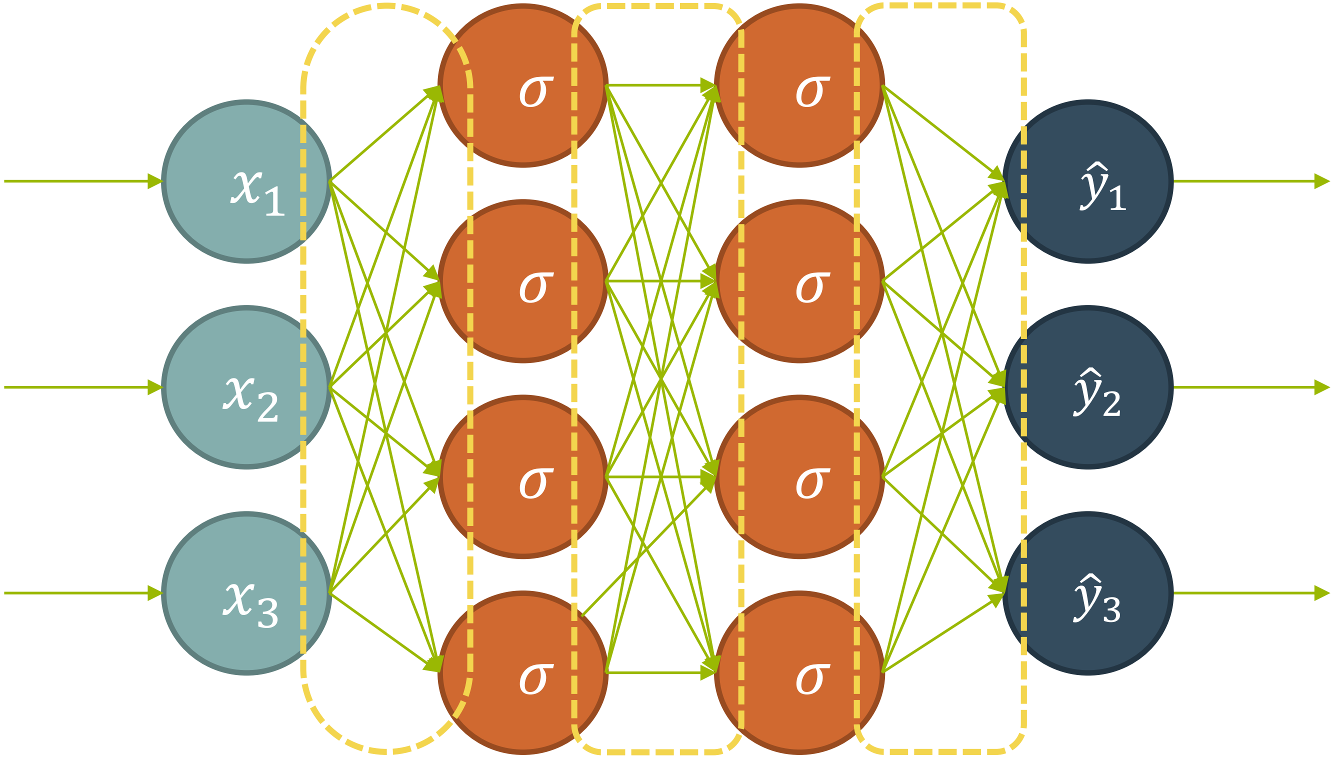
- Why not just use a single neuron? Why do we need a larger network?
- A single neuron (like logistic regression) only permits a linear decision boundary.
- Most real-world problems are considerably more complicated!



FEEDFORWARD NEURAL NETWORK

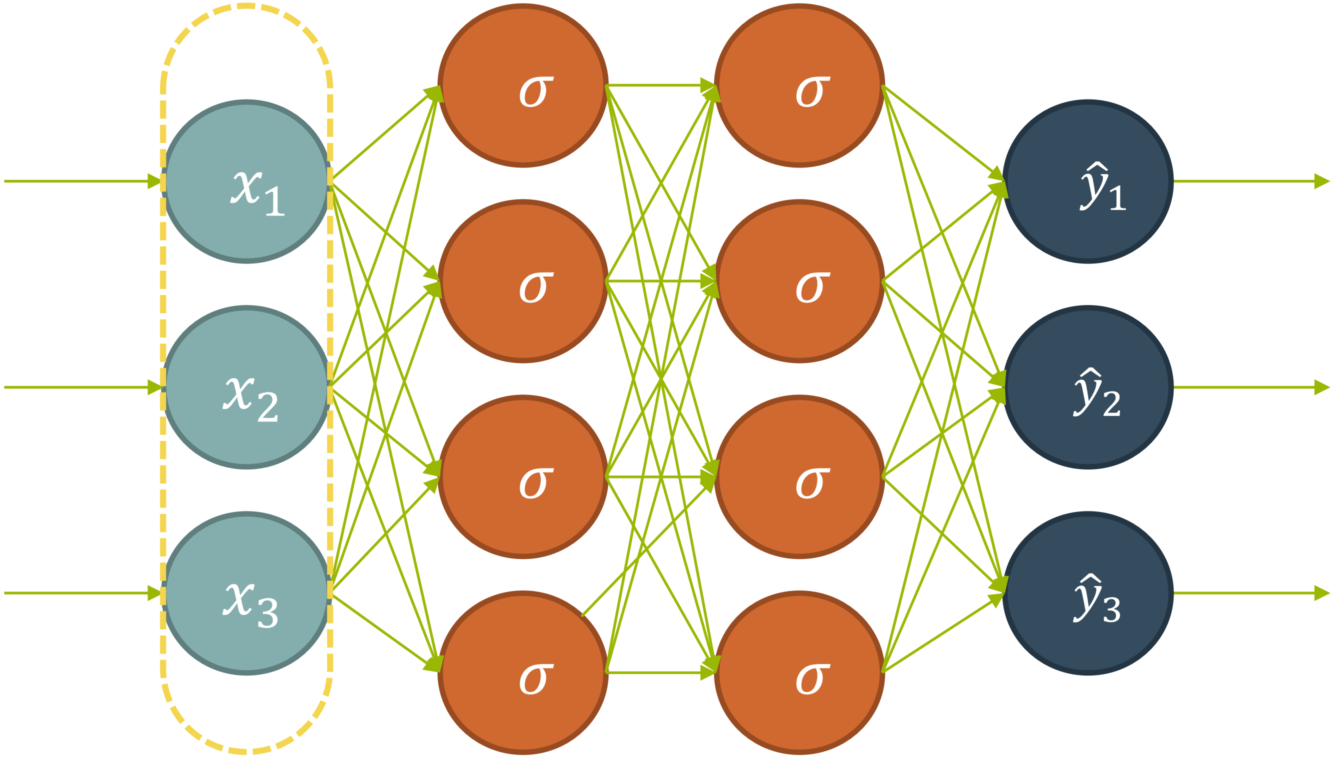


WEIGHTS



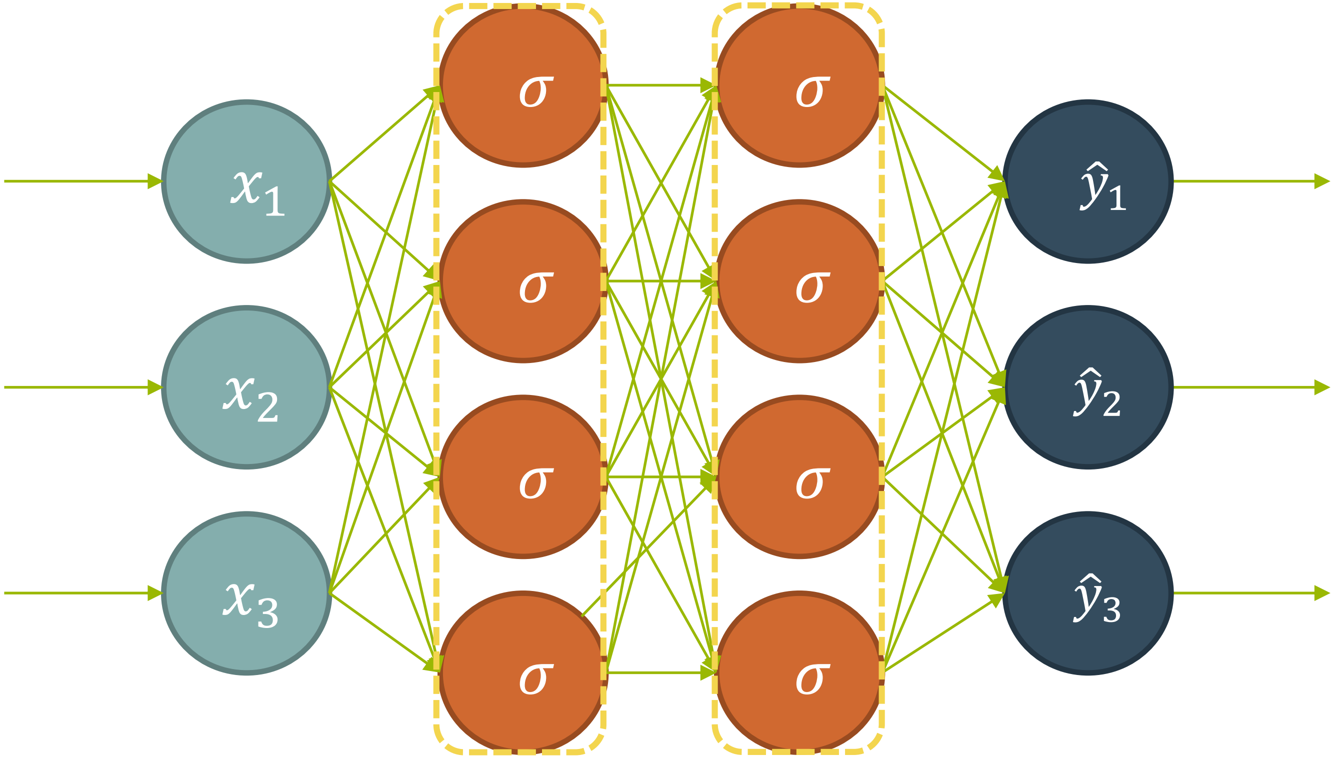
x_1
x_2
x_3
σ
σ
σ
σ
σ
σ
σ
σ
$\hat{y}_1$
$\hat{y}_2$
$\hat{y}_3$

INPUT LAYER

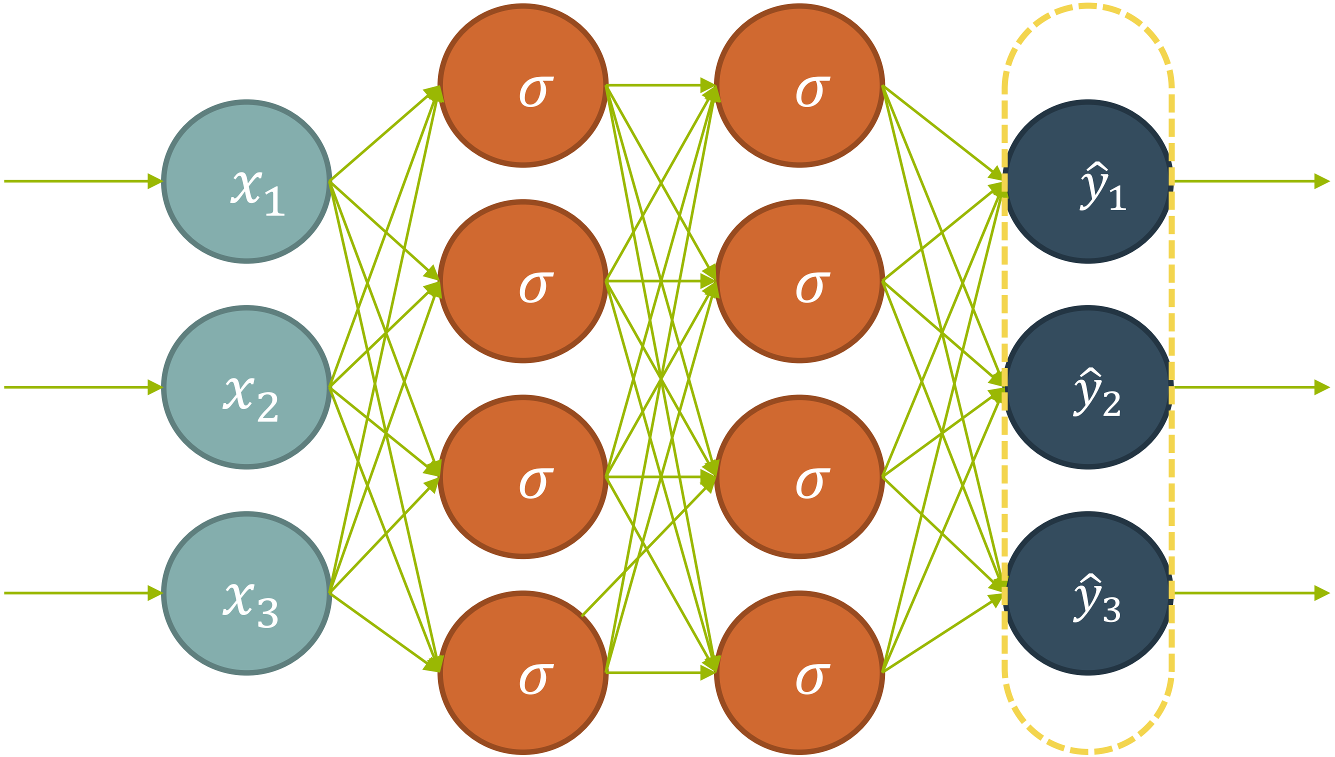


x_1
x_2
x_3
σ
σ
σ
σ
σ
σ
σ
σ
$\hat{y}_1$
$\hat{y}_2$
$\hat{y}_3$

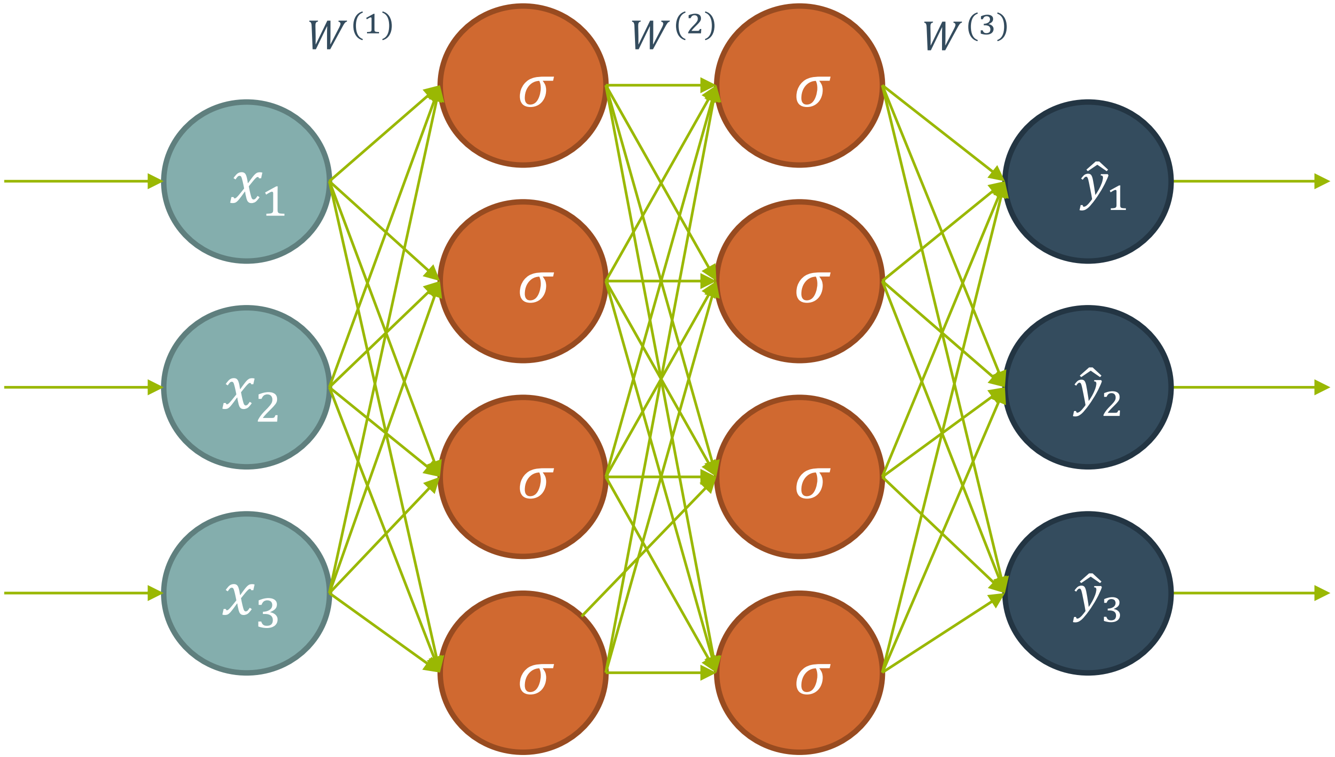
HIDDEN LAYERS



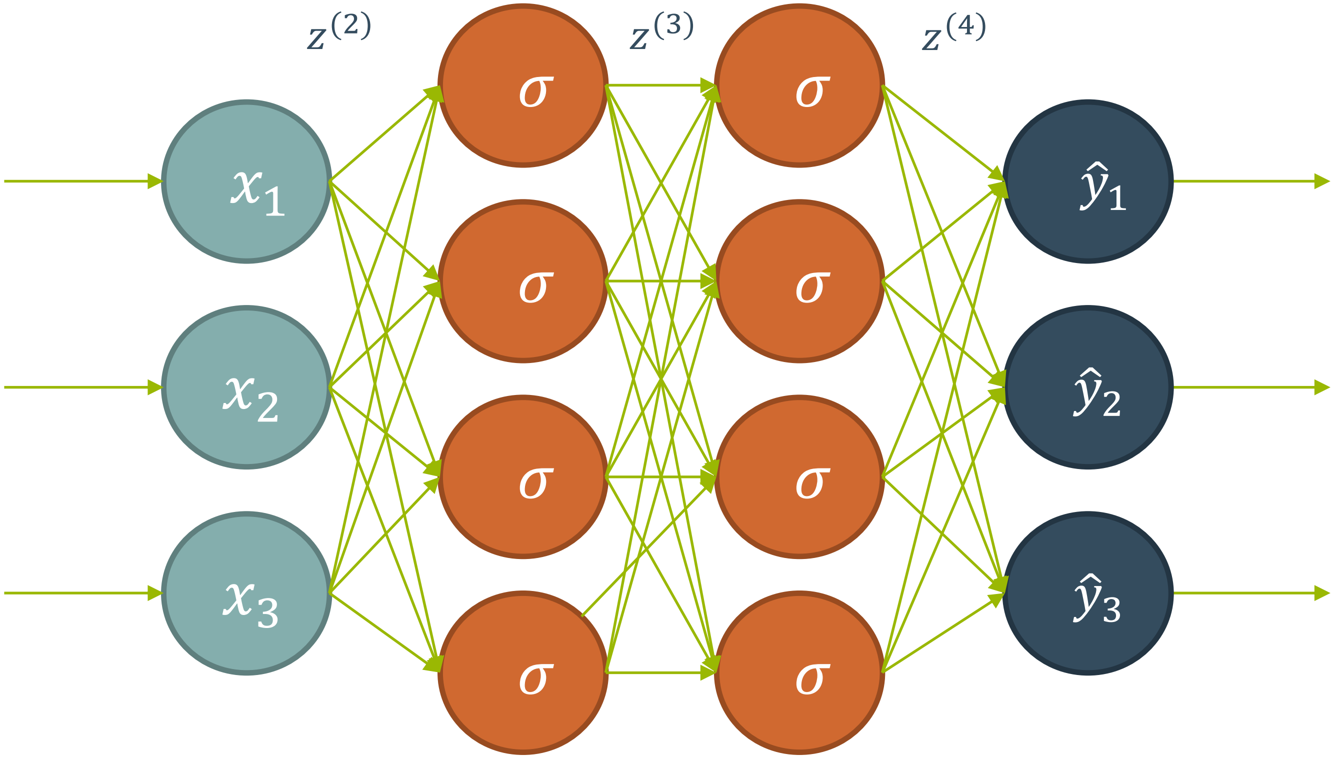
OUTPUT LAYER



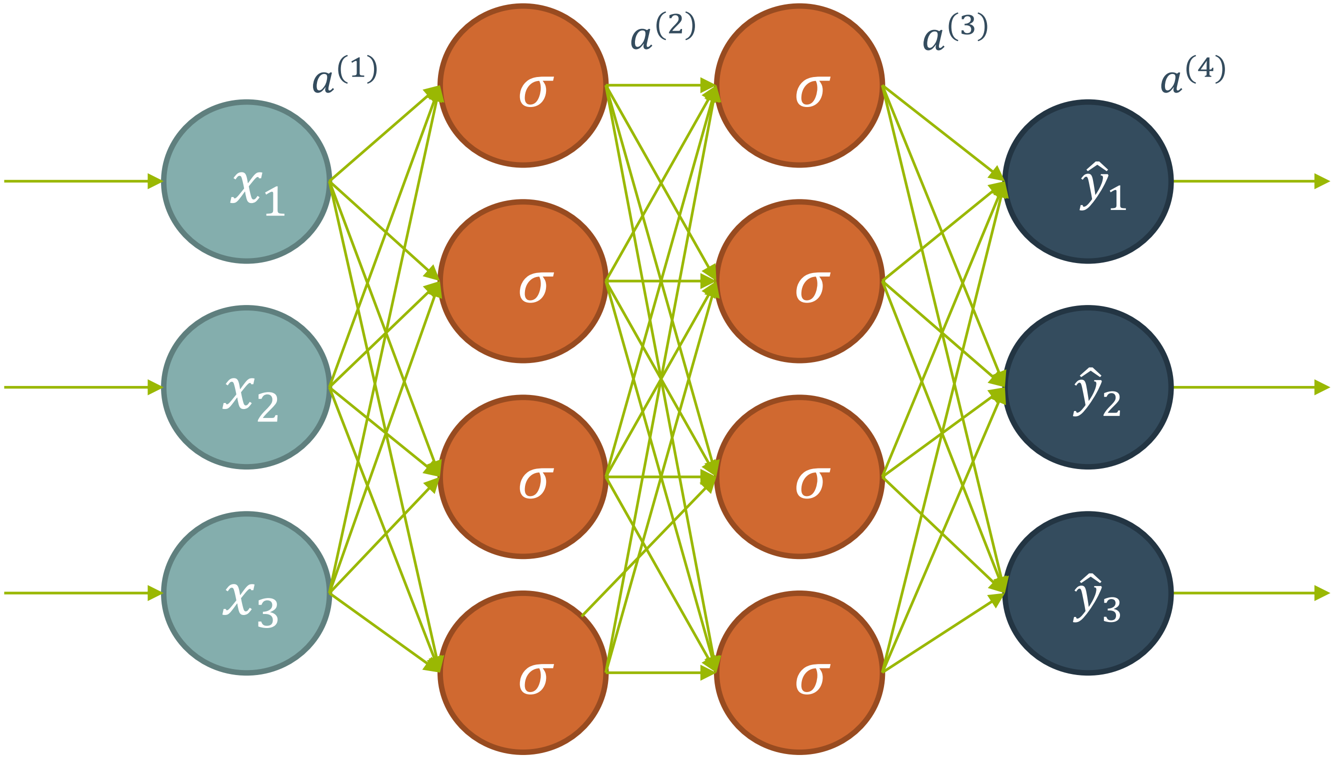
WEIGHTS (REPRESENTED BY MATRICES)



NET INPUT (SUM OF WEIGHTED INPUTS, BEFORE ACTIVATION FUNCTION)



ACTIVATIONS (OUTPUT OF NEURONS TO NEXT LAYER)



MATRIX REPRESENTATION OF COMPUTATION

x

$(x = a^{(1)})$

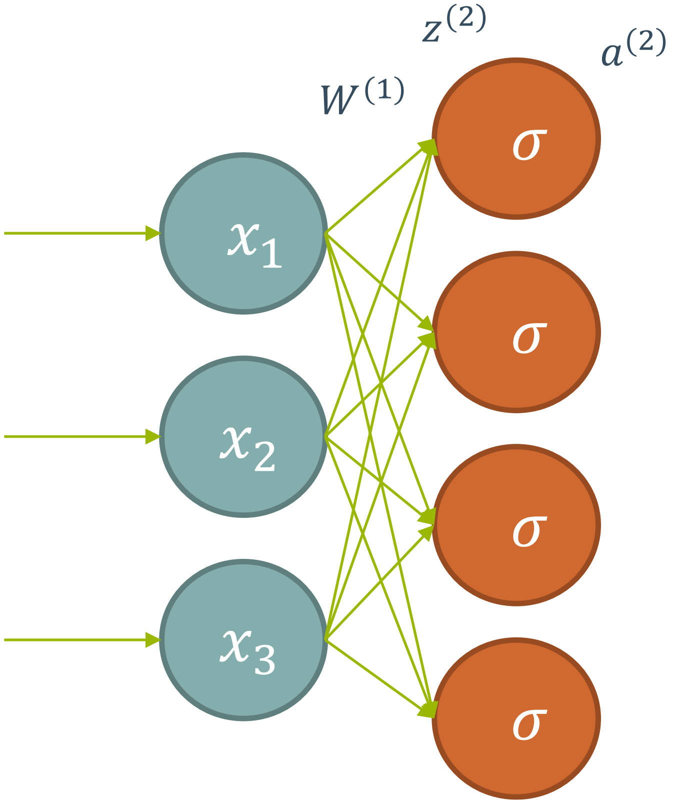
$z^{(2)} = xW^{(1)}$

$a^{(2)} = \sigma(z^{(2)})$

$W^{(1)}$ is a 3x4 matrix

$z^{(2)}$ is a 4-vector

$a^{(2)}$ is a 4-vector



CONTINUING THE COMPUTATION

For a single training instance (data point)

Input: vector x (a row vector of length 3)

Output: vector $\hat{y}$ (a row vector of length 3)

$$z^{(2)} = xW^{(1)} \qquad a^{(2)} = \sigma(z^{(2)})$$

$$z^{(3)} = a^{(2)}W^{(2)} \qquad a^{(3)} = \sigma(z^{(3)})$$

$$z^{(4)} = a^{(3)}W^{(3)} \qquad \hat{y} = softmax(z^{(4)})$$

MULTIPLE DATA POINTS

In practice, we do these computation for many data points at the same time, by "stacking" the rows into a matrix. But the equations look the same!

Input: matrix x (an nx3 matrix) (each row a single instance)

Output: vector $\hat{y}$ (an nx3 matrix) (each row a single prediction)

$$z^{(2)} = xW^{(1)} \qquad a^{(2)} = \sigma(z^{(2)})$$

$$z^{(3)} = a^{(2)}W^{(2)} \qquad a^{(3)} = \sigma(z^{(3)})$$

$$z^{(4)} = a^{(3)}W^{(3)} \qquad \hat{y} = softmax(z^{(4)})$$

Now we know how feedforward NNs do Computations.

Next, we will learn how to adjust the weights to learn from data.

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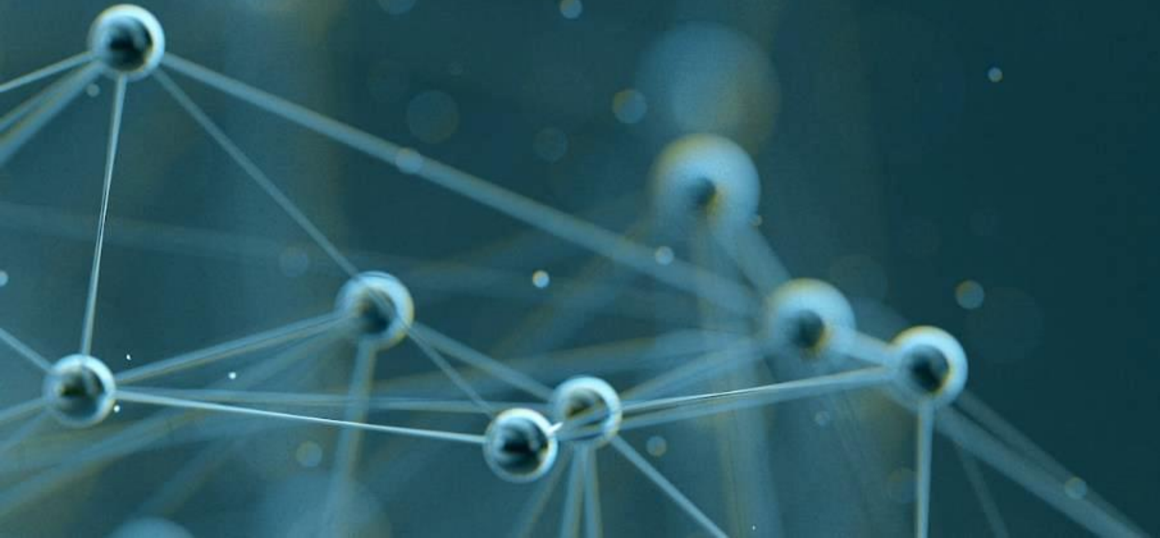
BACKPROPAGATION IN NEURAL NETS



HOW TO TRAIN A NEURAL NET?

- Put in training inputs, get the output
- Compare output to correct answers: look at loss function J
- Adjust and repeat!
- Backpropagation tells us how to make a single adjustment using calculus.

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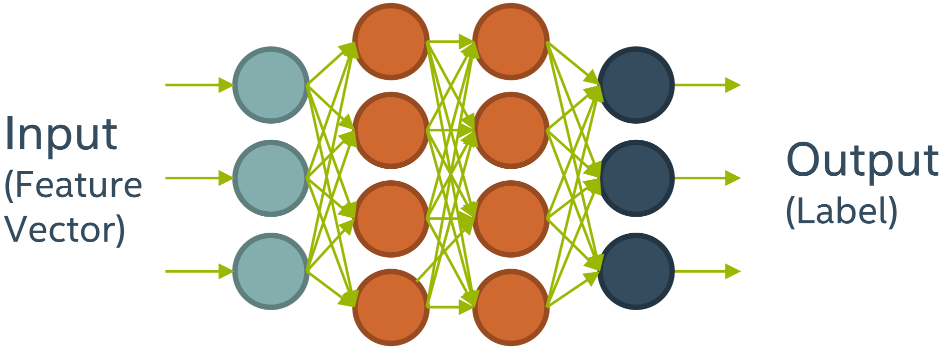

HOW HAVE WE TRAINED BEFORE?

Gradient Descent!

1. Make prediction
2. Calculate Loss
3. Calculate gradient of the loss function w.r.t. parameters
4. Update parameters by taking a step in the opposite direction
5. Iterate

HOW HAVE WE TRAINED BEFORE?

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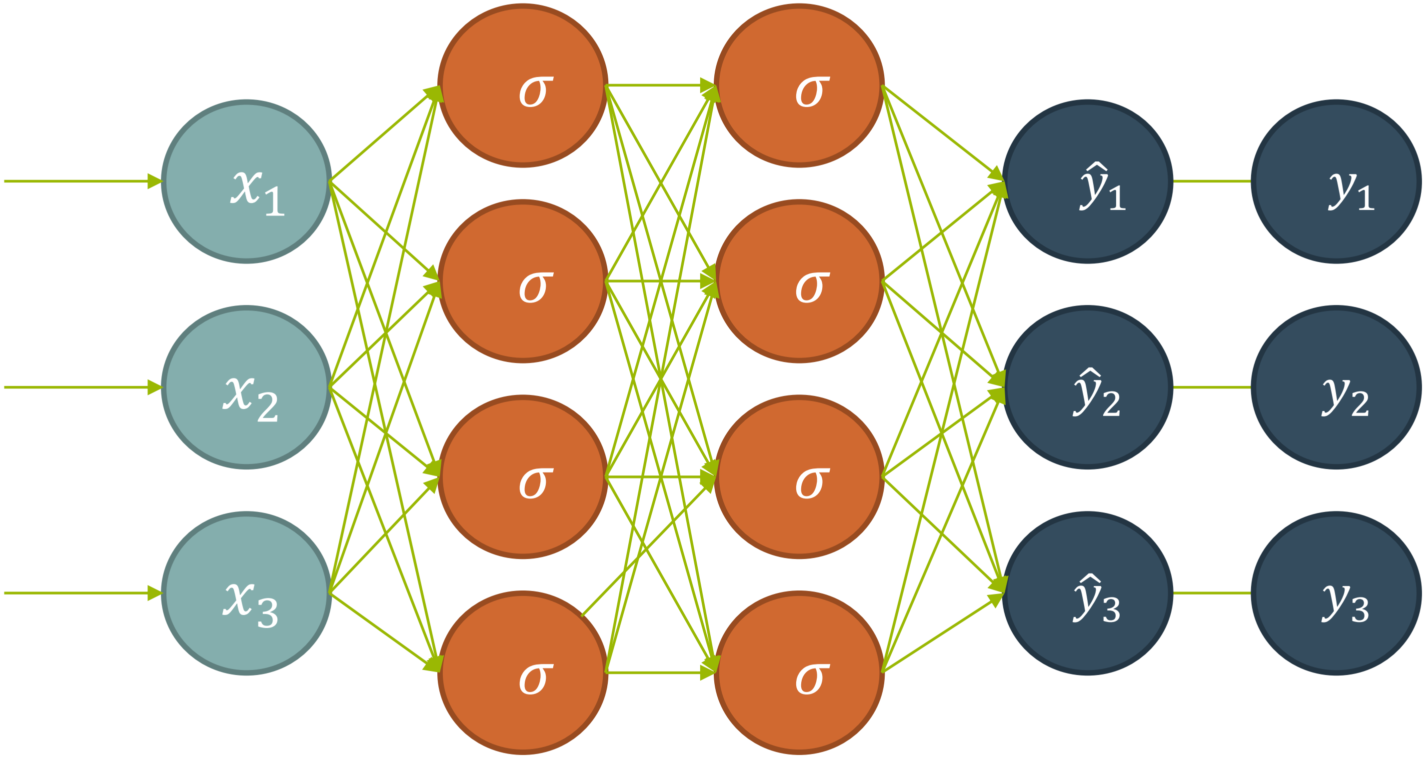
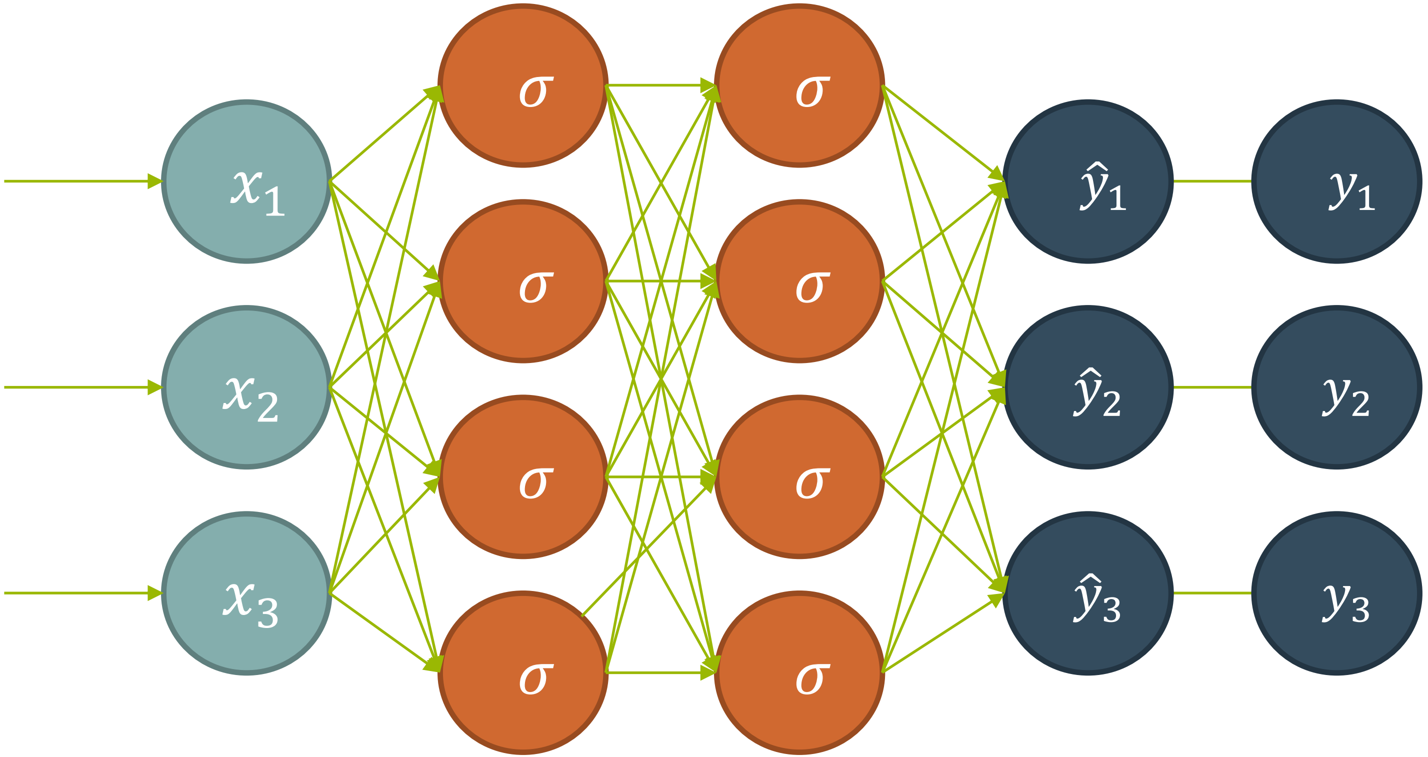
FEEDFORWARD NEURAL NETWORK



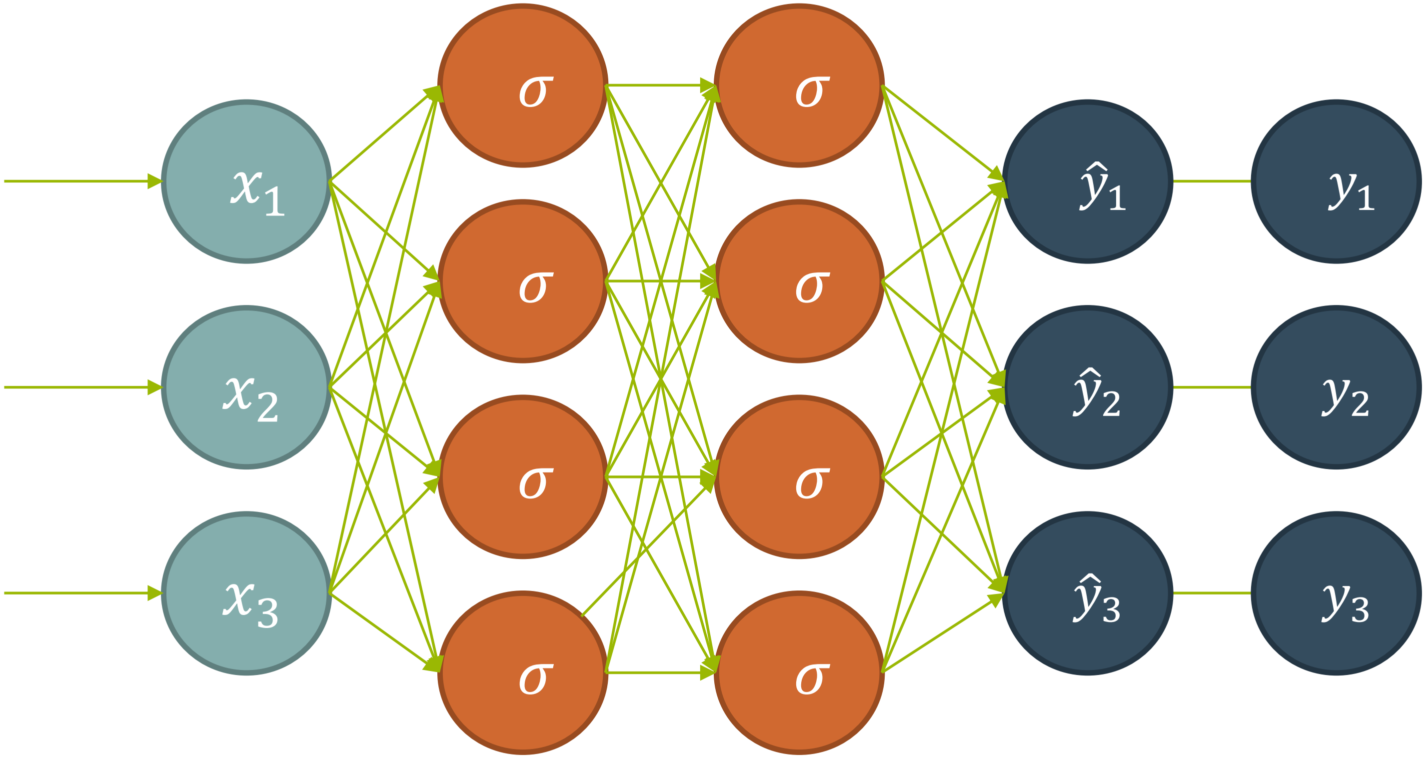
FORWARD PROPAGATION

Pass in Input

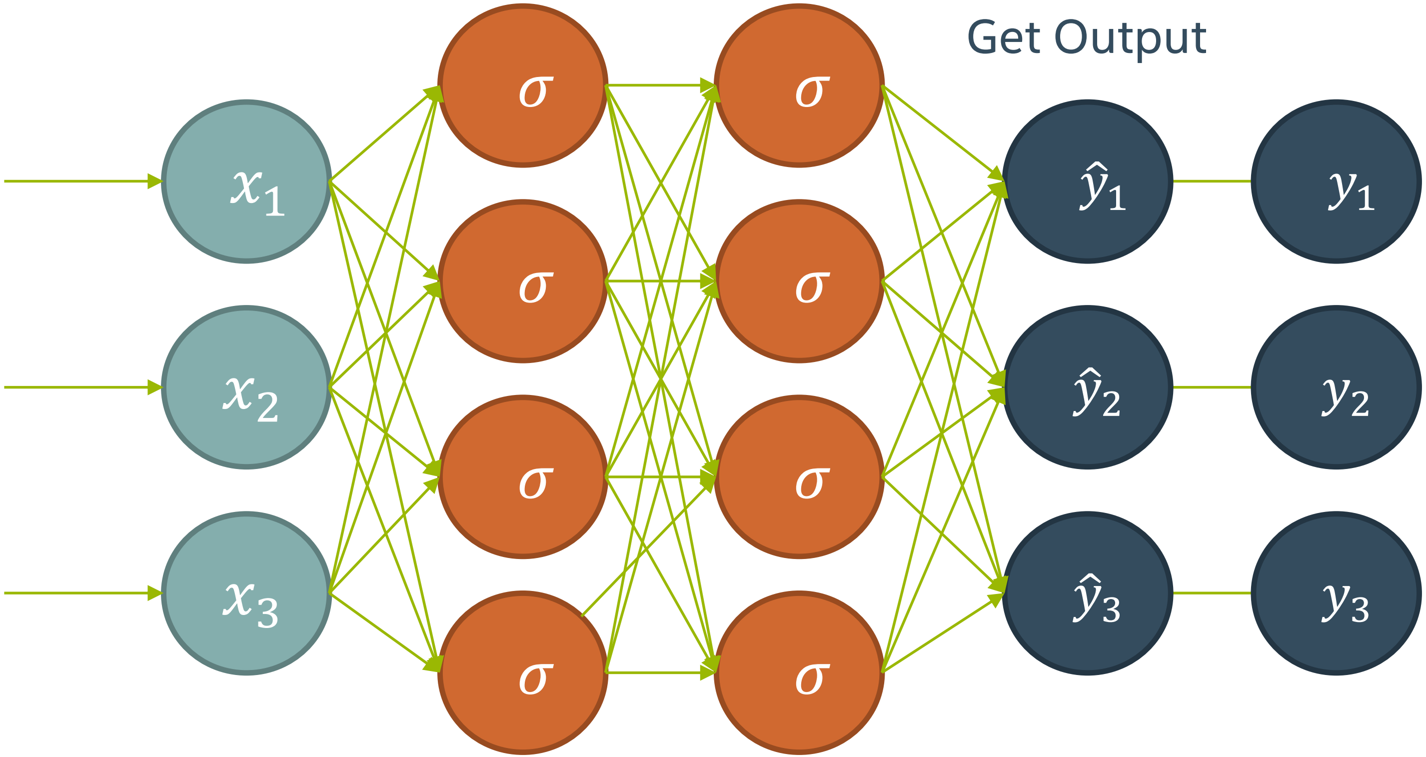


FORWARD PROPAGATION

Calculate each Layer

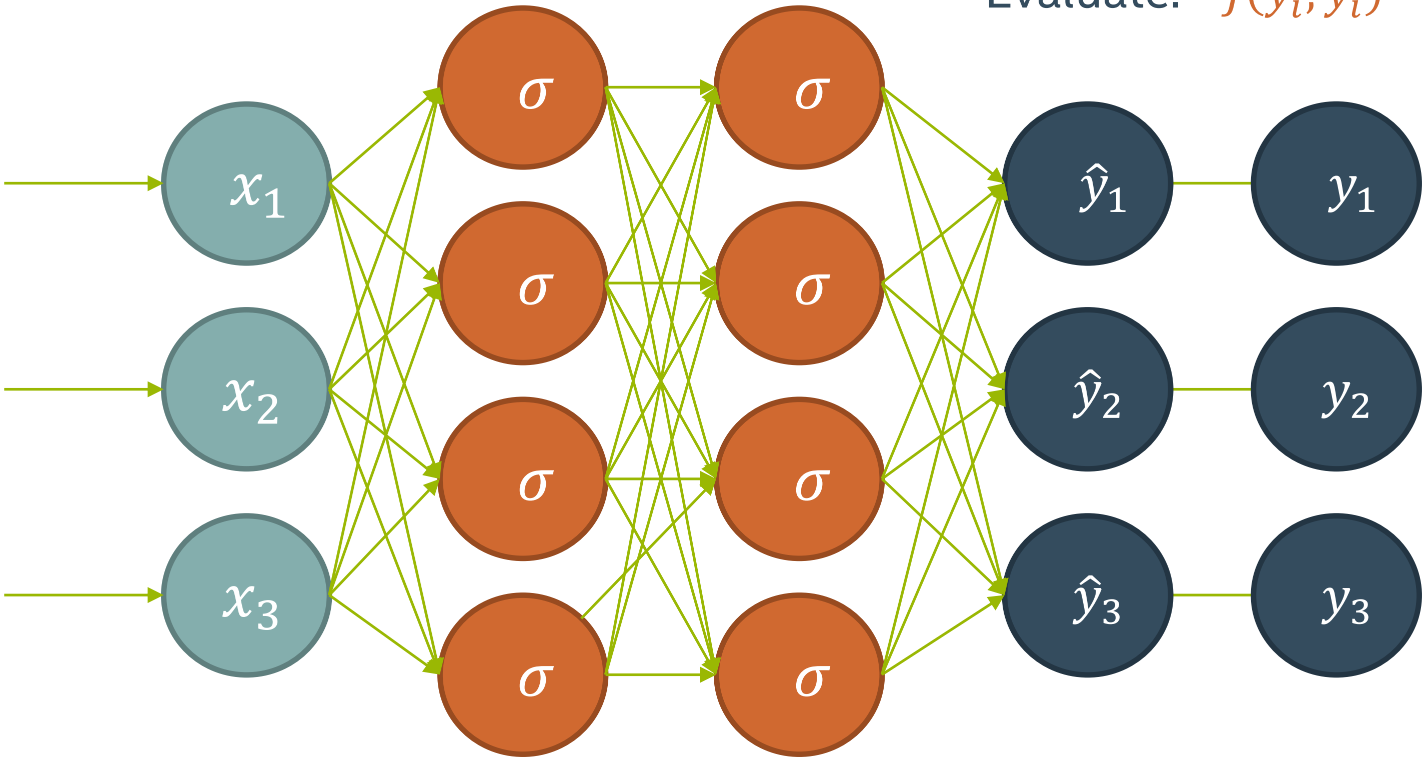


FORWARD PROPAGATION



FORWARD PROPAGATION

Evaluate: $J(y_i, \hat{y_i})$



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HOW TO CALCULATE GRADIENT?

Chain rule

HOW TO TRAIN A NEURAL NET?

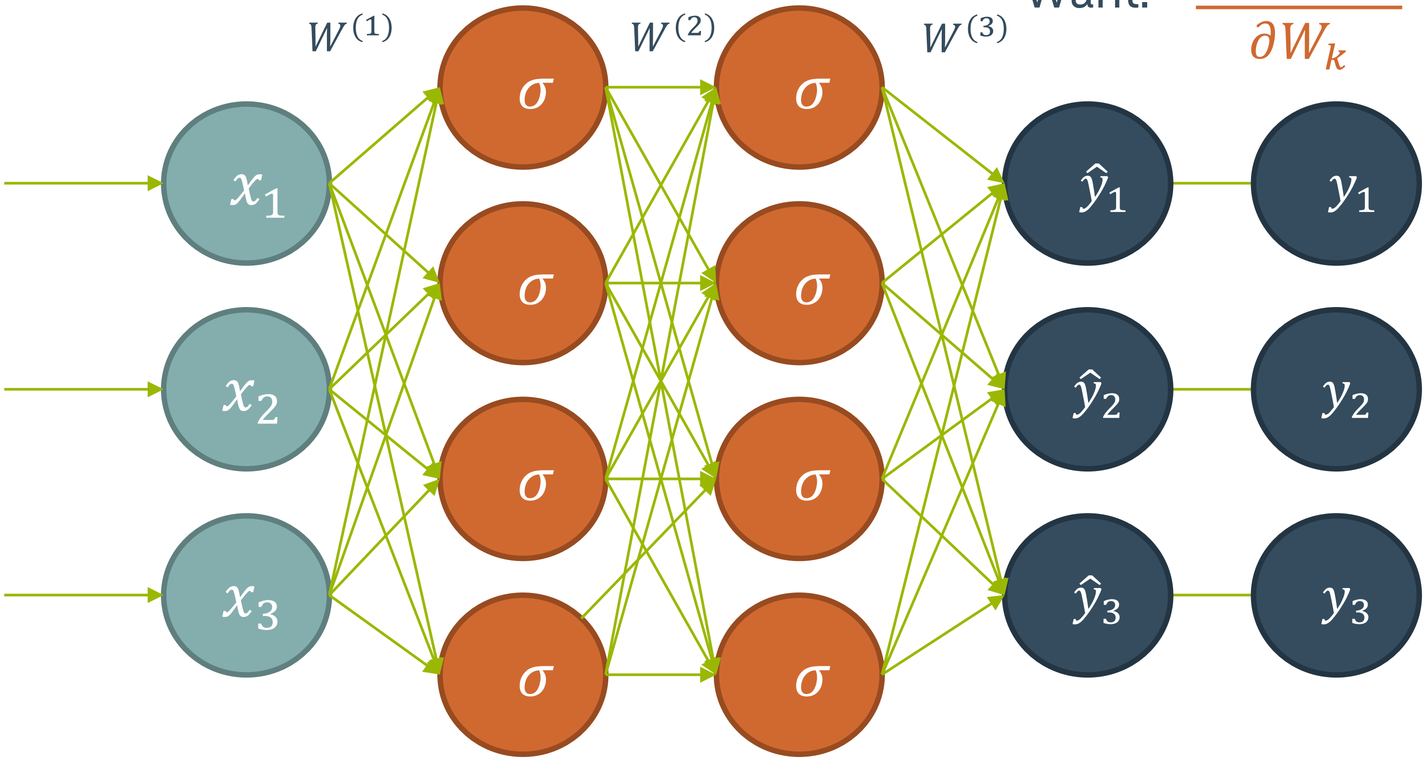
- How could we change the weights to make our Loss Function lower?
- Think of neural net as a function F: X -> Y
- F is a complex computation involving many weights W_k
- Given the structure, the weights "define" the function F (and therefore define our model)
- Loss Function is J(y,F(x))

HOW TO TRAIN A NEURAL NET?

- Get $\frac{\partial J}{\partial W_k}$ for every weight in the network.
- This tells us what direction to adjust each Wk if we want to lower our loss function.
- Make an adjustment and repeat!

FEEDFORWARD NEURAL NETWORK

Want: $\frac{\partial J(y_i, \hat{y}_i)}{\partial W_k}$



CALCULUS TO THE RESCUE

- Use calculus, chain rule, etc. etc.
- Functions are chosen to have “nice” derivatives
- Numerical issues to be considered

PUNCHLINE

$$\frac{\partial J}{\partial W^{(3)}} = (\hat{y} - y) \cdot a^{(3)}$$

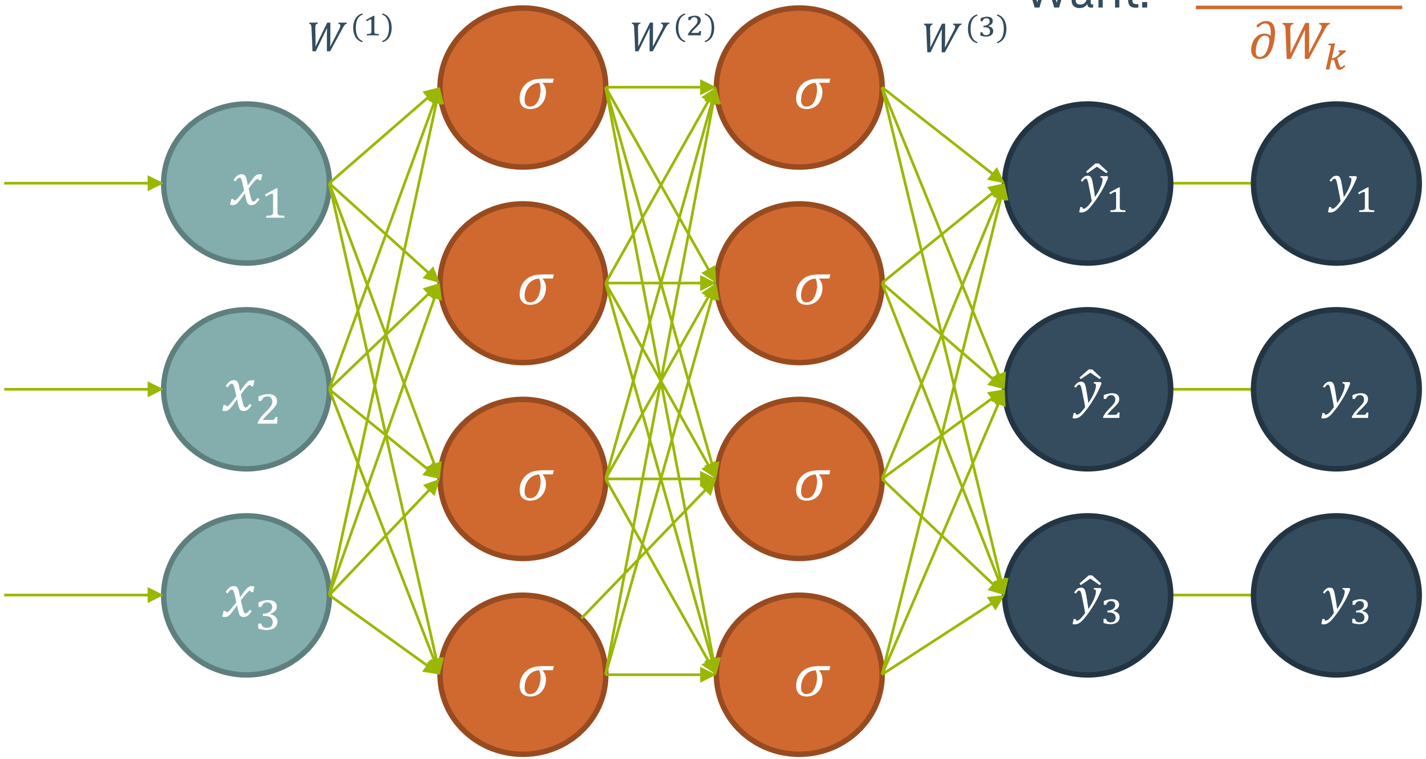
$$\frac{\partial J}{\partial W^{(2)}} = (\hat{y} - y) \cdot W^{(3)} \cdot \sigma'\left(z^{(3)}\right) \cdot a^{(2)}$$

$$\frac{\partial J}{\partial W^{(1)}} = (\hat{y} - y) \cdot W^{(3)} \cdot \sigma'\left(z^{(3)}\right) \cdot W^{(2)} \cdot \sigma'\left(z^{(2)}\right) \cdot X$$

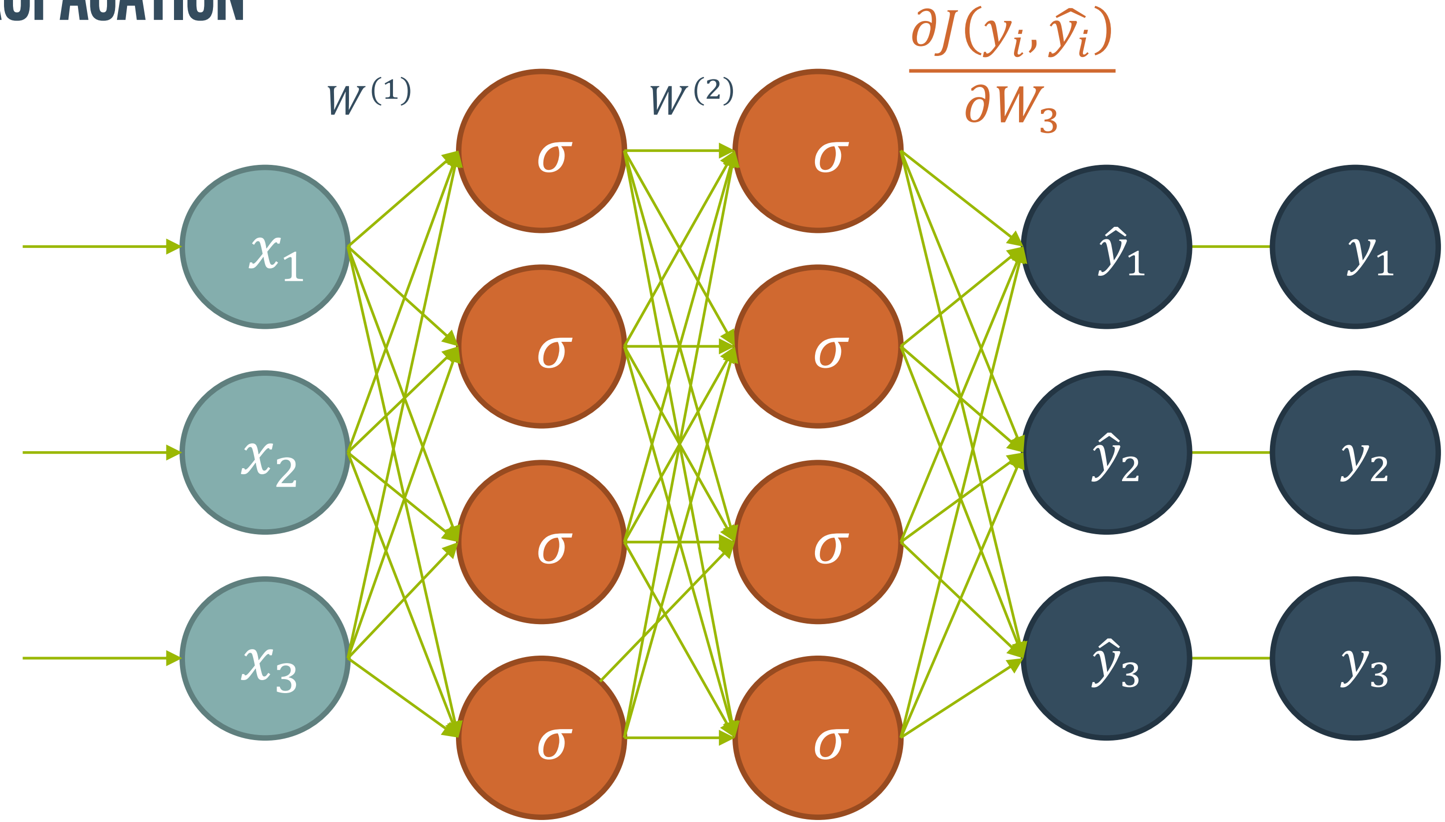
- Recall that: $\sigma'(z) = \sigma(z)(1-\sigma(z))$
- Though they appear complex, above are easy to compute!

BACKPROPAGATION

Want: $\dfrac{\partial J(y_i, \hat{y}_i)}{\partial W_k}$



BACKPROPAGATION



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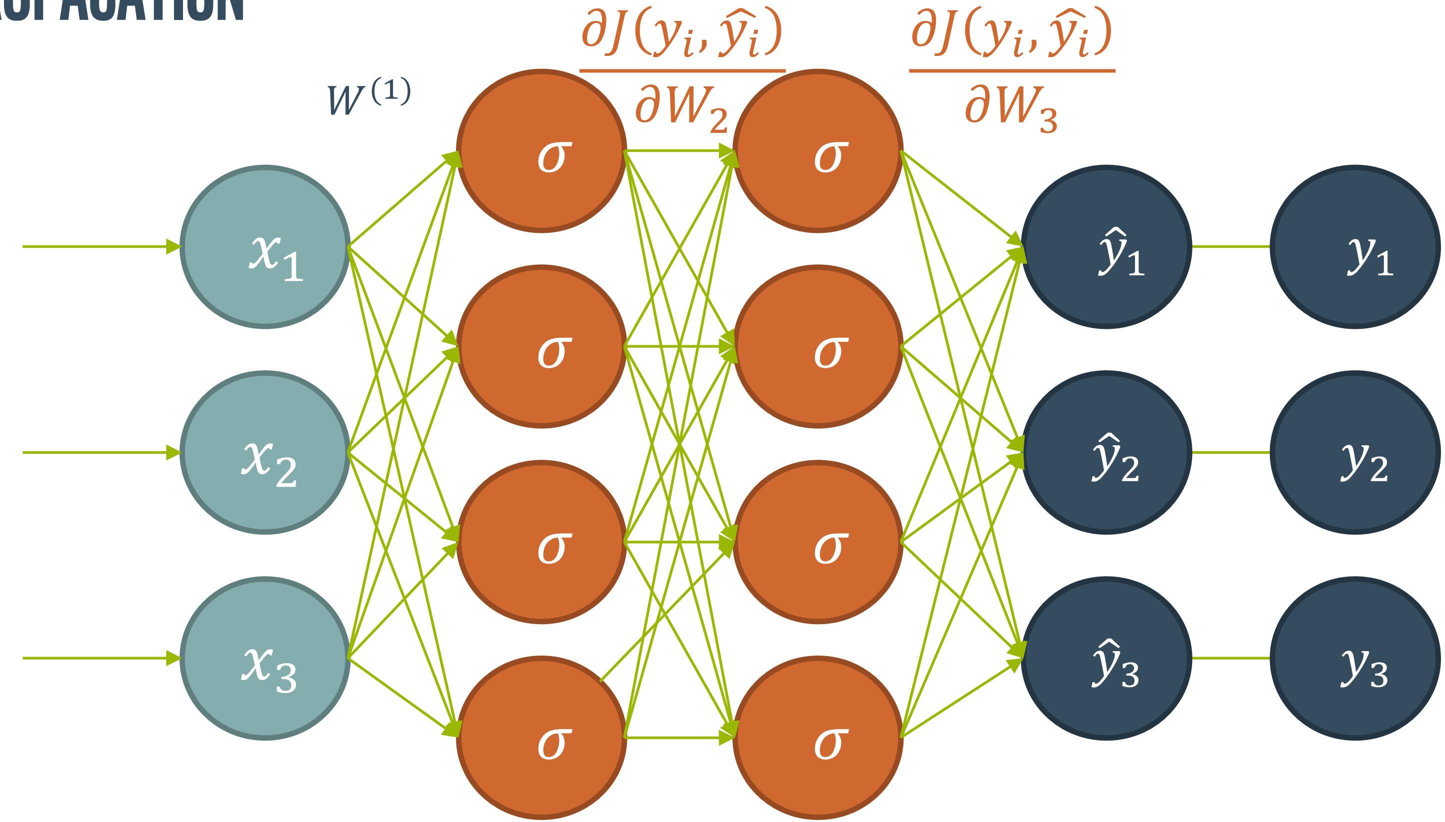

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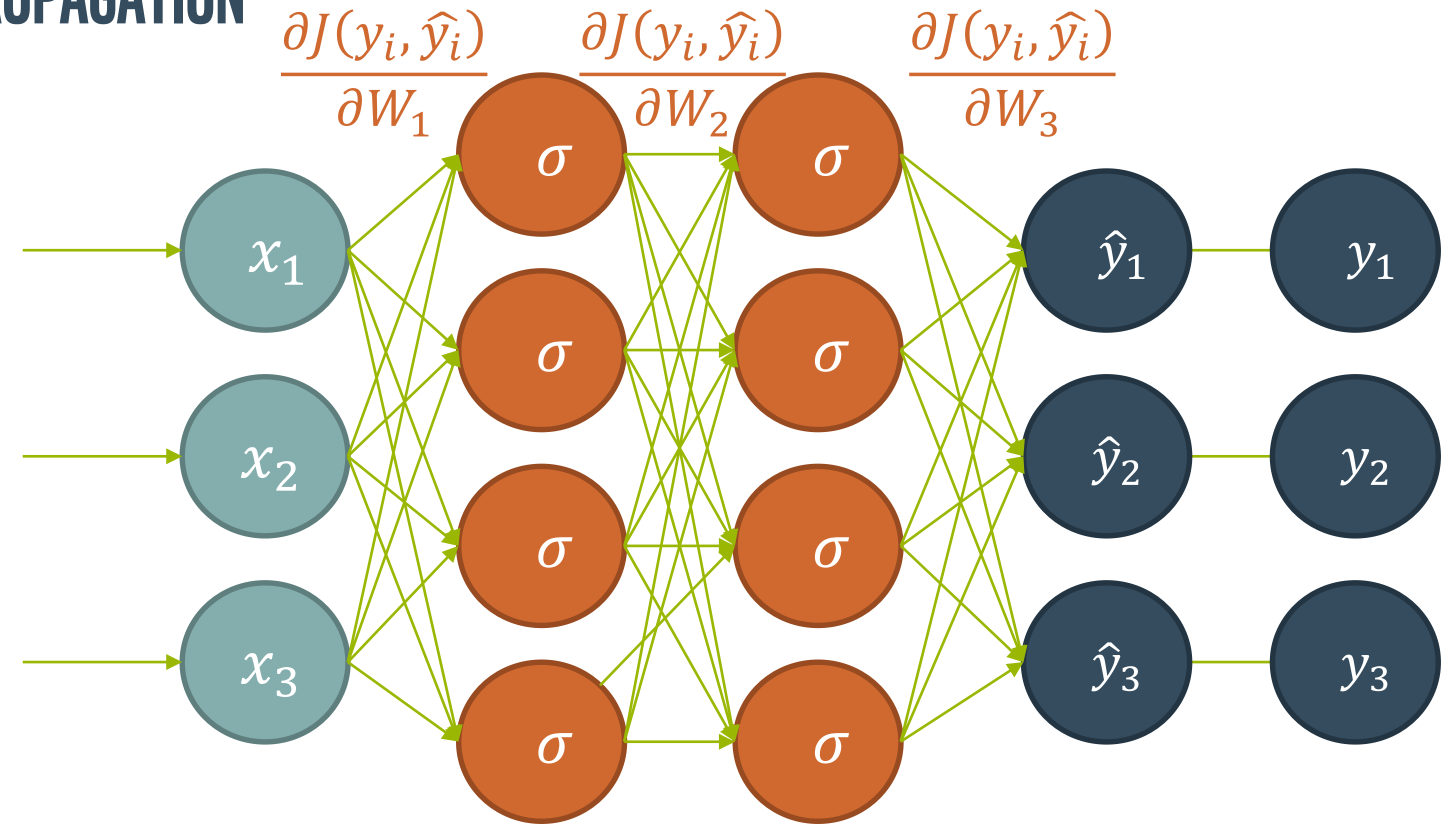

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Gradient Descent!

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VANISHING GRADIENTS

Recall that:

$$\frac{\partial J}{\partial W^{(1)}} = (\hat{y} - y) \cdot W^{(3)} \cdot \sigma'\left(z^{(3)}\right) \cdot W^{(2)} \cdot \sigma'\left(z^{(2)}\right) \cdot X$$

- Remember: $\sigma'(z) = \sigma(z)(1-\sigma(z)) \leq .25$
- As we have more layers, the gradient gets very small at the early layers.
- This is known as the "vanishing gradient" problem.
- For this reason, other activations (such as ReLU) have become more common.

OTHER ACTIVATION FUNCTIONS

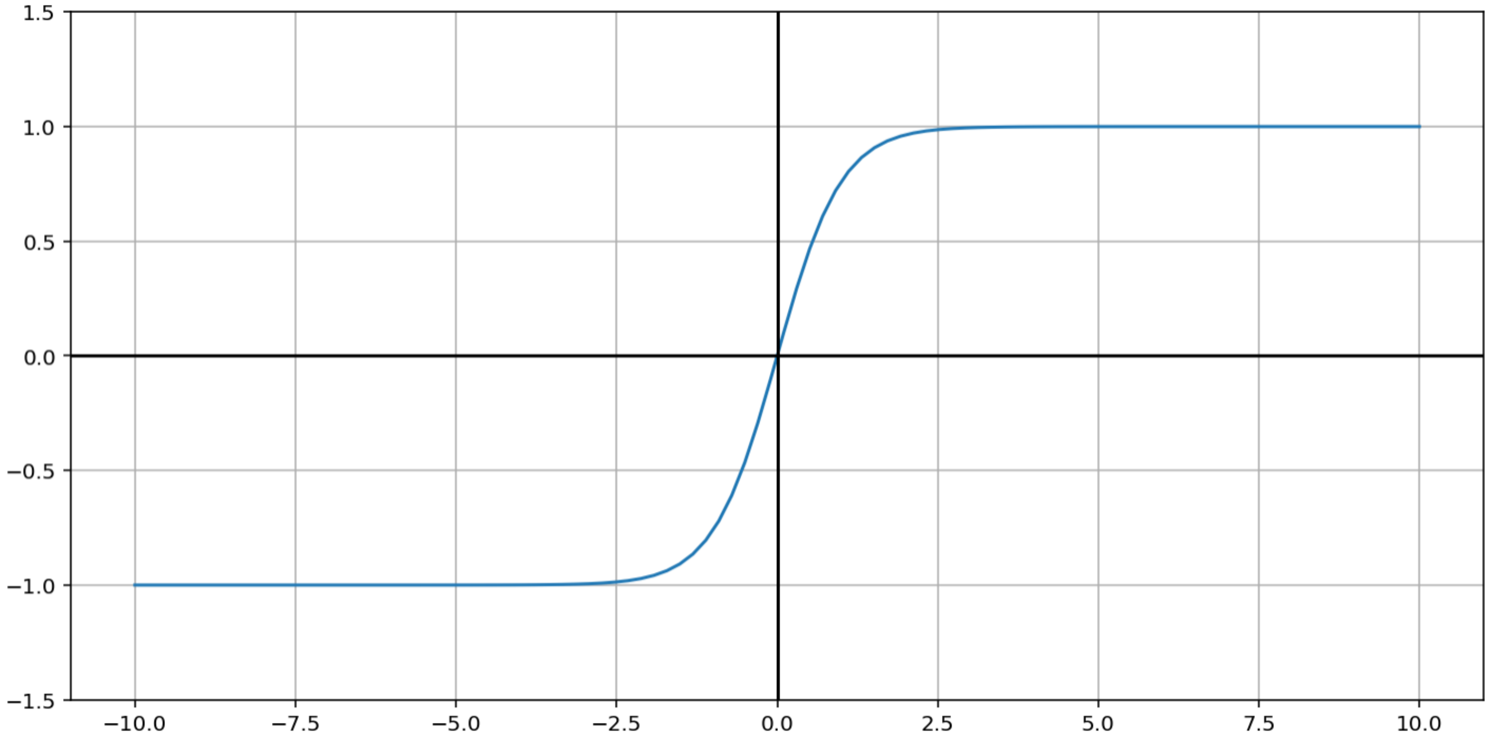
HYPERBOLIC TANGENT FUNCTION

- Hyperbolic tangent function
- Pronounced “tanch”

$$tanh(z) = \frac{\sinh(z)}{\cosh(z)} = \frac{e^{2x} - 1}{e^{2x} + 1}$$

$$tanh(0) = 0$$
$$tanh(\infty) = 1$$
$$tanh(-\infty) = -1$$

HYPERBOLIC TANGENT FUNCTION



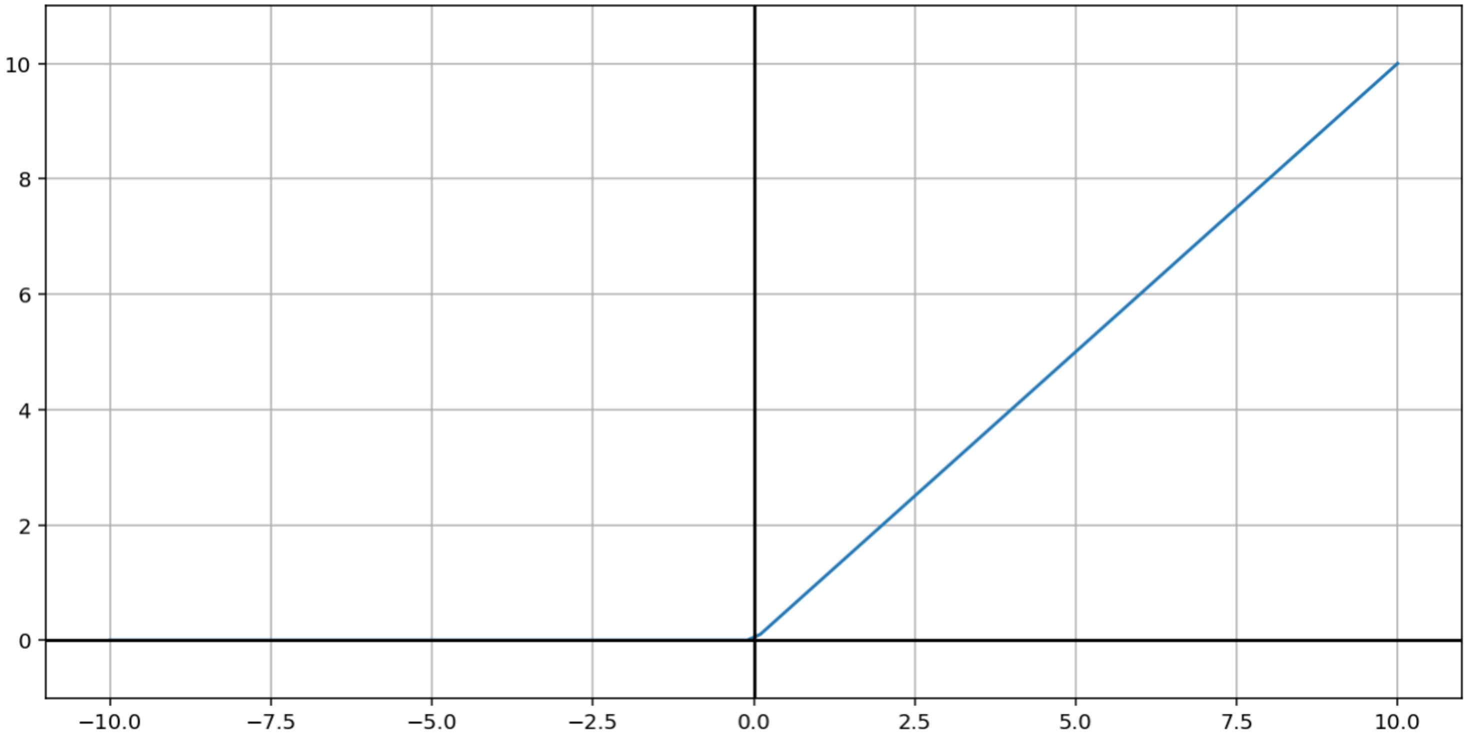
RECTIFIED LINEAR UNIT (RELU)

$$ReLU(z) = \begin{cases} 0, & z < 0 \\ z, & z \geq 0 \end{cases}$$

$$= \max(0, z)$$

$$ReLU(0) = 0$$
$$ReLU(z) = z \qquad \text{for } (z \gg 0)$$
$$ReLU(-z) = 0$$

RECTIFIED LINEAR UNIT (RELU)



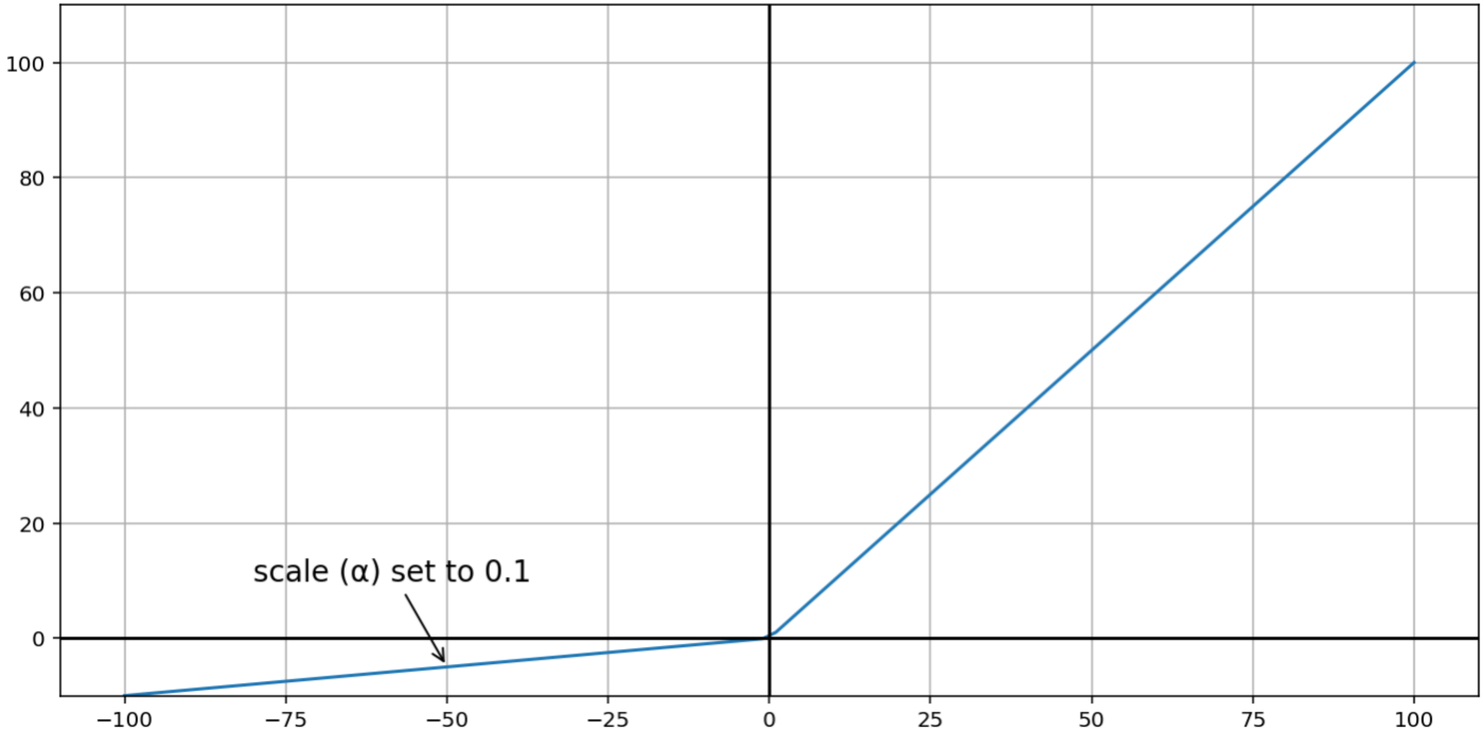
"LEAKY" RECTIFIED LINEAR UNIT (RELU)

$$LReLU(z) = \begin{cases} \alpha z, & z < 0 \\ z, & z \geq 0 \end{cases}$$

$$= \max(\alpha z, z) \qquad \text{for } (\alpha < 1)$$

$$LReLU(0) = 0$$
$$LReLU(z) = z \qquad \text{for } (z \gg 0)$$
$$LReLU(-z) = -\alpha z$$

"LEAKY" RECTIFIED LINEAR UNIT (RELU)



WHAT NEXT?

We now know how to make a single update to a model given some data.

But how do we do the full training?

We will dive into these details in the next lecture.

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