CS 6890: Linear and Integer Programming

Vladimir Kulyukin
Department of Computer Science
Utah State University

July 1, 2017

Learning Objectives

1. Linear Systems
2. Row Echelon Form
3. Matrix Invertibility
4. Spans and Bases
5. Determinants

Introduction

This assignment is a workout for you to better understand the linear algebra concepts that we will use many times in this course. This is a pencil and paper assignment. Try to solve these problems by hand without writing any code or using calculators to improve your intuition about these concepts.

Problem 1 (1 point)

Solve the linear system with the Gauss method

1. $x_1 - 2x_2 + x_3 - x_4 = 4$
2. $2x_1 - 3x_2 + 2x_3 - 3x_4 = -1$
3. $3x_1 - 5x_2 + 3x_3 - 4x_4 = 3$
4. $-x_1 + x_2 - x_3 + 2x_4 = 5.$

Problem 2 (1 point)

Find the inverse of the matrix if it exists.

$$\begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & -1 & -3 & 4 \\ 1 & 0 & -1 & 2 \\ -3 & 0 & 0 & -1 \end{bmatrix}$$

Problem 3 (1 point)

Let $A^{-1} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 3 & 1 \\ 4 & 1 & 2 \end{bmatrix}$

If possible find a matrix $\mathbf{C}$ such that

$$ACA = \begin{bmatrix} 2 & 1 & 3 \\ -1 & 2 & 2 \\ 2 & 1 & 4 \end{bmatrix}.$$

Problem 4 (1 point)

Determine if the set of all invertible 4×4 matrices is closed under addtion and scalar multiplication.

Problem 5 (1 point)

Determine if $\mathbf{b} = (7, 6, 1)$ lies in $sp(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$, where $\mathbf{v}_1 = (1, 2, 3)$, $\mathbf{v}_2 = (-2, -5, 2)$, and $\mathbf{v}_3 = (1, 2, -1)$.

Problem 6 (1 point)

Let $\mathbf{v}_1 = (1, 3, 4)$, $\mathbf{v}_2 = (2, 7, 2)$, and $\mathbf{v}_3 = (-1, 2, 1)$. Determine if $sp(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3) = \mathcal{R}^3$.

Problem 7 (1 point)

Determine if $\{(1, 4, -1, 3), (-1, 5, 6, 2), (1, 13, 4, 7)\}$ is dependent or independent in $\mathcal{R}^4$.

Problem 8 (1 point)

Find a basic for the nullspace of A, where

$$A = \begin{bmatrix} 3 & 1 & 9 \\ 1 & 2 & -2 \\ 2 & 1 & 5 \end{bmatrix}.$$

Problem 9 (1 point)

Find a basis for the column space of A, where

$$A = \begin{bmatrix} 1 & -2 & 2 & -1 \\ -3 & 6 & 1 & 10 \\ 1 & -2 & -4 & -7 \end{bmatrix}.$$

Problem 10 (1 point)

Compute $det(A)$, where

$$A = \begin{bmatrix} 3 & 2 & 4 \\ 0 & 1 & 2 \\ 1 & 4 & 1 \end{bmatrix}.$$

What To Submit

Type your answers in your favorite math-friendly editor and submit your answers in a pdf document through Canvas.