CS6890 HW02

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Problem 1

Transform the following LP into the standard form.

- minimize $z = 2x_1 - 3x_2 + 5x_3 + x_4$ subject to
  1. $-x_1 + 3x_2 - x_3 + 2x_4 \leq -12$
  2. $5x_1 + x_2 + 4x_3 - x_4 \geq 10$
  3. $3x_1 - 2x_2 + x_3 - x_4 = -8$
  4. $x_1, x_2, x_3, x_4 \geq 0$

Only $\leq$ inequalitites. Multiply 2. by -1. $-5x_1 - x_2 - 4x_3 + x_4 \leq 10$

To transform a minimzation problem to a maximization problem, multiply the objective function by -1. $z = -2x_1 + 3x_2 - 5x_3 - x_4$

Transform = inequality by turning it into two inequalitites with 3..
$3x_1 - 2x_2 + x_3 - x_4 \leq -8$ and $-3x_1 + 2x_2 - x_3 + x_4 \leq 8$

Becomes:

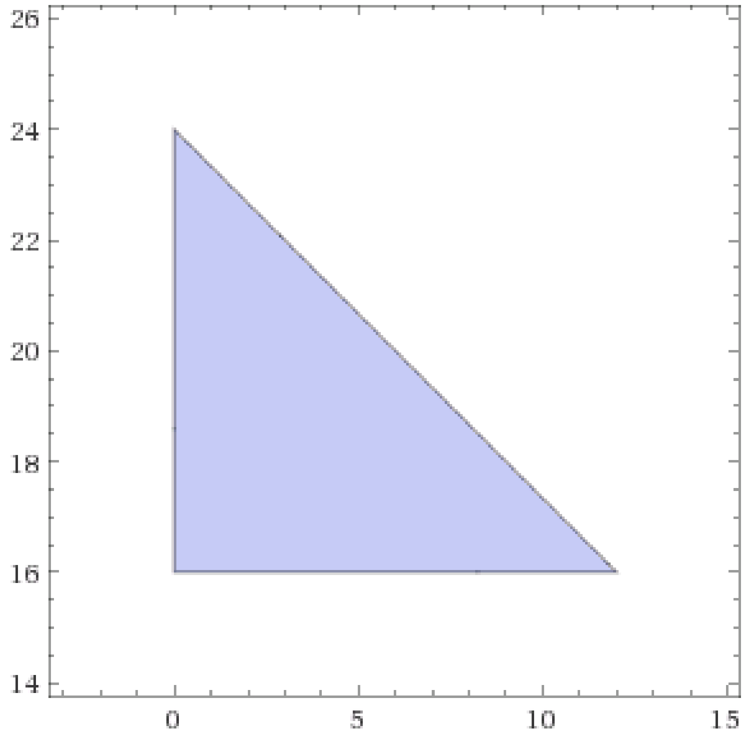
- maximize $z = -2x_1 + 3x_2 - 5x_3 - x_4$ subject to
  1. $-x_1 + 3x_2 - x_3 + 2x_4 \leq -12$
  2. $-5x_1 - x_2 - 4x_3 + x_4 \leq 10$
  3. $3x_1 - 2x_2 + x_3 - x_4 \leq -8$
  4. $-3x_1 + 2x_2 - x_3 + x_4 \leq 8$
  5. $x_1, x_2, x_3, x_4 \geq 0$

Problem 2

Nick's Furniture, LLC produces two types of wooden chairs - A and B. The manufacture of chair A requires 2 hours of assembly time and 4 hours of finishing. Chair B requires 3 hours to assemble and 3 hours to finish. The company estimates that next week 72 hours will be available for assembly operations and 108 hours for finishing. The unit profits for chairs A and B are $10 and $9, respectively. It is also estimated that the maximum demand for chair B will be 16. Formulate an LP model and solve it graphically to answer the question of what is the optimal product mix for the company next week.

Becomes:

- maximize $z = 10x_1 + 9x_2$ subject to
  1. $2x_1 + 3x_2 \leq 72$
  2. $4x_1 + 3x_2 \leq 108$
  3. $x_1 \geq 0, x_2 \geq 16$

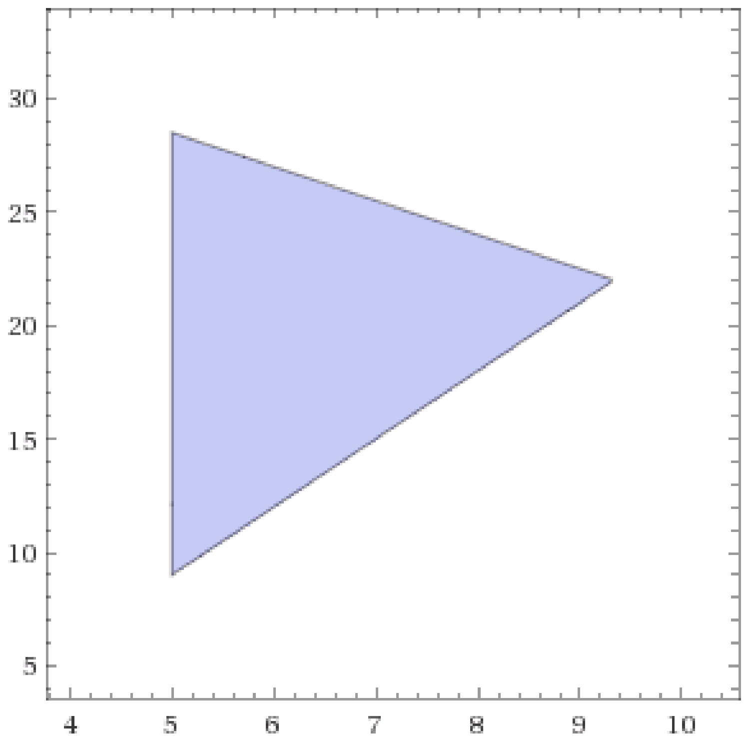


With corner points being: (0,24)=216,(0,16)=144,($\frac{40}{3}$, 16)=$\frac{832}{3}$≈277

Thus the maximum solution is: $x_1 = \frac{40}{3}, x_2 = 16$

Problem 3

- minimize $z = 4x_1 + 5x_2$ subject to
  1. $3x_1 + 2x_2 \leq 24$
  2. $x_1 \geq 5$
  3. $3x_1 - x_2 \leq 6$
  4. $x_1, x_2 \geq 0$

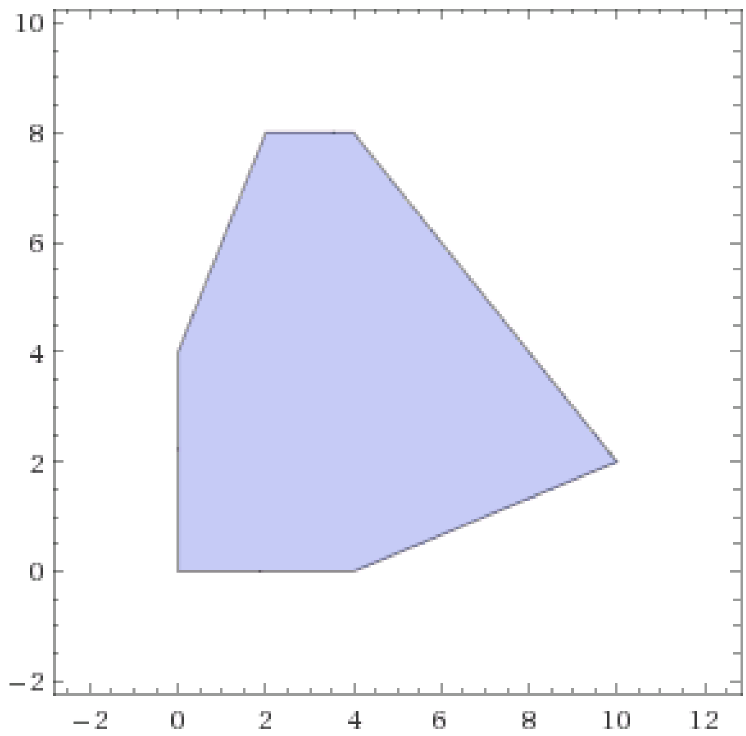


With corner points being: (5,9)=65,(4,6)=46,$(5, \frac{9}{2})$=$\frac{85}{2}$=22.5

Thus the minimum solution is: $x_1 = 5, x_2 = \frac{9}{2}$

Problem 4

- minimize $z = x_1 - 4x_2$ subject to
  1. $x_1 + x_2 \leq 12$
  2. $-2x_1 + x_2 \leq 4$
  3. $x_2 \leq 8$
  4. $x_1 - 3x_2 \leq 4$
  5. $x_1, x_2 \geq 0$



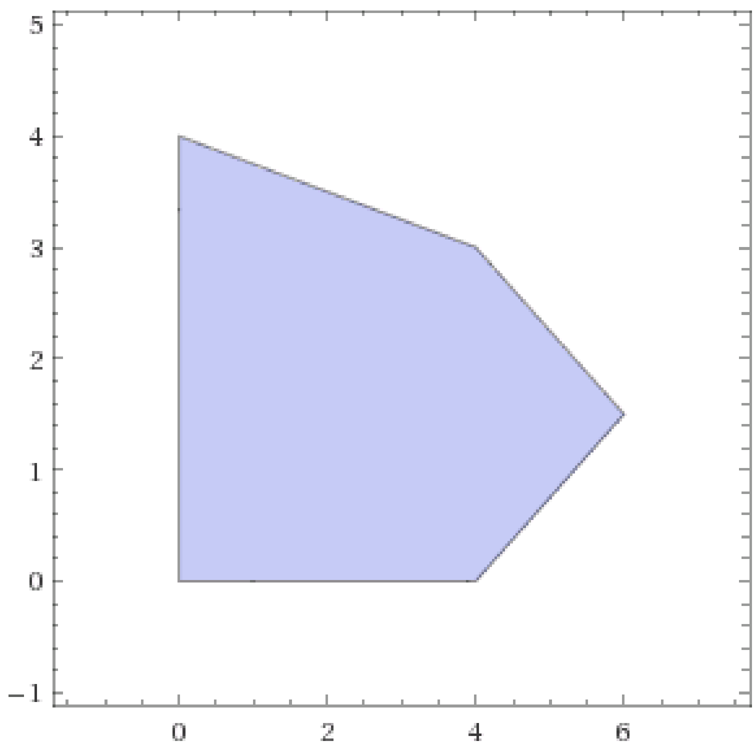
With corner points being: (0,0)=0,(4,0)=4,(0,4)=-16,(2,8)=-30,(4,8)=-28,(10,2)=2

Thus the minimum solution is: $x_1 = 2, x_2 = 8$

Problem 5

- maximize $z = 6x_1 + 8x_2$ subject to

    1. $x_1 + 4x_2 \leq 16$
    2. $3x_1 + 4x_2 \leq 24$
    3. $3x_1 - 4x_2 \leq 12$
    4. $x_1, x_2 \geq 0$

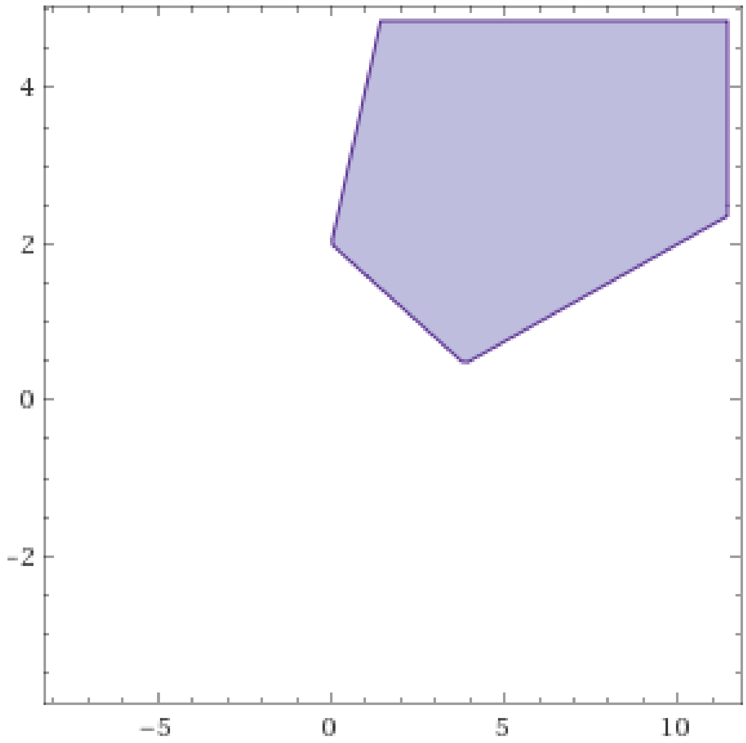


With corner points being: (0,0)=0,(0,4)=32,(4,0)=24,(4,3)=48,(6,$\frac{1}{2}$)=40

Thus the maximize solution is: $x_1 = 4, x_2 = 3$

Problem 6

- maximize $z = x_1 + 2x_2$ subject to
  1. $-2x_1 + x_2 \leq 2$
  2. $2x_1 + 5x_2 \geq 10$
  3. $x_1 - 4x_2 \leq 2$
  4. $x_1, x_2 \geq 0$



With corner points being: (0,2)=4,$(\frac{50}{13}, \frac{6}{13}) = \frac{62}{13}$,$(\infty, \infty) = \infty$

Thus the maximize solution is: $x_1 = \infty, x_2 = \infty$

Problem 7

Given the polyhedral set $S = (x1, x2)|x1 + x2 \leq 10, -x1 + x2 \leq 6, x1 - 4x2 \leq 0.$

- Find all extreme points of S.
- Represent the point x = (2, 4) as a convex combination of the extreme points.

$\left(\begin{array}{cc|c} 1 & 1 & 10 \\ -1 & 1 & 6 \end{array}\right)$ Then $(x_1, x_2) = (2, 8)$

$\left(\begin{array}{cc|c} 1 & 1 & 10 \\ 1 & 4 & 0 \end{array}\right)$ Then $(x_1, x_2) = (8, 2)$

$\left(\begin{array}{cc|c} -1 & 1 & 6 \\ 1 & 4 & 0 \end{array}\right)$ Then $(x_1, x_2) = (-8, -2)$

$(2, 4) = x_1(2, 8) + x_2(8, 2) + x_3(-8, -2)$

$$\begin{cases} 2x_1 + 8x_2 - 8x_3 = 2 \\ 8x_1 + 2x_2 - 2x_3 = 4 \\ x_1 + x_2 + x_3 = 1 \end{cases}$$

Row reducing leads to $(x_1, x_2, x_3) = (\frac{7}{15}, \frac{1}{3}, \frac{1}{5})$
The point as a convex combination of the exteme points is:
$(2, 4) = \frac{7}{15}(2, 8) + \frac{1}{3}(8, 2) + \frac{1}{5}(-8, -2)$

Problem 8

Let S1 and S2 be convex sets. Is S1 $\cap$ S2 convex? Is S1 $\cup$ S2 convex? You can either state your answers as proofs or, if you do not feel comfortable with proofs, justify your answers with a few sentences.

Yes for intersection and no for union.

No for union because imagine 2 circles (which are convex sets) union then there will exist a piece that can't connect on top of each circle.