CS 6890: Linear and Integer Programming

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Learning Objectives

1. Linear Program Formulation
2. Converting Linear Programs to Standard Form
3. Solving LPs Graphically
4. Convex and Polyhedral Sets

Introduction

This is a pencil and paper assignment. When a problem asks you to solve an LP problem graphically, make sure that your graph is legible and the extreme points are clearly marked.

Problem 1 (1 point)

Transform the following LP into the standard form

- minimize $z = 2x_1 - 3x_2 + 5x_3 + x_4$ subject to
  1. $-x_1 + 3x_2 - x_3 + 2x_4 \leq -12$
  2. $5x_1 + x_2 + 4x_3 - x_4 \geq 10$
  3. $3x_1 - 2x_2 + x_3 - x_4 = -8$
  4. $x_1, x_2, x_3, x_4 \geq 0$.

Problem 2 (2 points)

Nick's Furniture, LLC produces two types of wooden chairs - A and B. The manufacture of chair A requires 2 hours of assembly time and 4 hours of finishing. Chair B requires 3 hours to assemble and 3 hours to finish. The company estimates that next week 72 hours will be available for assembly operations and 108 hours for finishing. The unit profits for chairs A and B are \$10 and \$9, respectively. It is also estimated that the maximum demand for chair B will be 16. Formulate an LP model and solve it graphically to answer the question of what is the optimal product mix for the company next week.

Problem 3 (1 point)

Solve the following LP graphically.

- minimize $z = 4x_1 + 5x_2$ subject to
    1. $3x_1 + 2x_2 \leq 24$
    2. $x_1 \geq 5$
    3. $3x_1 - x_2 \leq 6$
    4. $x_1, x_2 \geq 0$.

Problem 4 (1 point)

Solve the following LP graphically.

- minimize $z = x_1 - 4x_2$ subject to
    1. $x_1 + x_2 \leq 12$
    2. $-2x_1 + x_2 \leq 4$
    3. $x_2 \leq 8$
    4. $x_1 - 3x_2 \leq 4$
    5. $x_1, x_2 \geq 0$.

Problem 5 (1 point)

Solve the following LP graphically.

- maximize $z = 6x_1 + 8x_2$ subject to
  1. $x_1 + 4x_2 \leq 16$
  2. $3x_1 + 4x_2 \leq 24$
  3. $3x_1 - 4x_2 \leq 12$
  4. $x_1, x_2 \geq 0$.

Problem 6 (1 point)

Solve the following LP graphically.

- maximize $z = x_1 + 2x_2$ subject to
  1. $-2x_1 + x_2 \leq 2$
  2. $2x_1 + 5x_2 \geq 10$
  3. $x_1 - 4x_2 \leq 2$
  4. $x_1, x_2 \geq 0$.

Problem 7 (2 points)

Given the polyhedral set $S = \{(x_1, x_2) | x_1 + x_2 \leq 10, -x_1 + x_2 \leq 6, x_1 - 4x_2 \leq 0\}$.

1. Find all extreme points of S.
2. Represent the point $\mathbf{x} = (2, 4)$ as a convex combination of the extreme points.

Problem 8 (1 point)

Let S_1 and S_2 be convex sets. Is $S_1 \cap S_2$ convex? Is $S_1 \cup S_2$ convex? You can either state your answers as proofs or, if you do not feel comfortable with proofs, justify your answers with a few sentences.

What To Submit

Type your answers in your favorite math-friendly editor and submit your answers in a pdf document through Canvas. You can also write your answers legibly and submit their images through Canvas.