CS 6890: Linear and Integer Programming
Homework 5

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Learning Objectives

1. Network Simplex Algorithm

Problem 1 (10 point)

Implement the following network simplex algorithm discussed in Lecture 10.

1. Given a feasible basis represented by a rooted spanning tree, compute the flows by starting at the end nodes of the tree and working toward the root and compute the node potentials by starting at the root node and working toward the end nodes of the tree.

2. Check for optimality (i.e., $z_{ij} - c_{ij} \leq 0$, for all i, j). If the solution is optimal, stop. If not, determine the entering arc (i, j).

3. Add the entering arc to the spanning tree and induce the flow around the formed cycle to determine Δ. If no departing arc can be found, the problem has no solution. Otherwise, let (p, q) be the departing arc.

4. Remove the departing arc (p, q) from the tree. Set the flow of the entering arc (i, j) to Δ and update the flows of the other arcs around the cycle. Go to step 2.

We have not finished discussing the flow induction principle. We will do it in Lecture 11. In the meantime, you can start working on steps 1 and 2. You can package your solution as $NetSimplex.java/py$. The class $NetSimplex$ should have a method $NetSimplex.doSimplex(G, T)$, where G is a network graph and T is a spanning tree of G.

Problem 2 (10 points)

Let $G(N, A)$ be a flow network where $N = \{1, 2, 3, 4, 5\}$ and $A = \{(1, 2), (1, 3), (1, 4), (2, 3), (2, 5), (3, 4), (3, 5), (4, 5), (5, \infty)\}$. The arc $(5, \infty)$ simply means that 5 is the root node with the arc from 5 going into space.

Let $b(i)$ be the balance of node i. $G(N, A)$ has the following balances:

1. $b(1) = 12$;
2. $b(2) = 7$;
3. $b(3) = 3$;
4. $b(4) = -8$;
5. $b(5) = -14$.

Let c_{ij} be the cost of the arc (i, j). The network's arcs have the following costs:

- $c_{12} = 2$;
- $c_{13} = 3$;
- $c_{14} = 4$;
- $c_{23} = 4$;
- $c_{25} = 3$;
- $c_{34} = 1$;
- $c_{35} = 5$;
- $c_{45} = 6$

Assume that a starting basic feasible solution is given by the following rooted spanning tree $T(N, A)$, where $N = \{1, 2, 3, 4, 5\}$ and $A = \{(1, 3), (1, 4), (2, 3), (3, 5), (5, \infty)\}$. Use your implementation of the network simplex algorithm to find an optimal flow for this network.

Implement your solution in the method $NetSimplex.problem2(G, T)$ where $G = G(N, A)$ is the network graph defined above and $T = T(N, A)$. The output of this method should have the following format (the actual flow values for this problem are different):

```
x12 = 10
x14 = 15
x25 = 20
x14 = 78
z   = 89
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What To Submit

Submit your source as $NetSimplex.java/py$.