CS6890 HW01

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0.1 Problem 1

Solve the linear system with the Gauss method

$$\begin{cases} x_1 - 2x_2 + x_3 - x_4 = 4 \\ 2x_1 - 3x_2 + 2x_3 - 3x_4 = -1 \\ 3x_1 - 5x_2 + 3x_3 - 4x_4 = 3 \\ -x_1 + x_2 - x_3 + 2x_4 = 5 \end{cases}$$

$$\left(\begin{array}{cccc|c} 1 & -2 & 1 & -1 & 4 \\ 2 & -3 & 2 & -3 & -1 \\ 3 & -5 & 3 & -4 & 3 \\ -1 & 1 & -1 & 2 & 5 \end{array}\right) \begin{array}{c} \xrightarrow{-2r_1+r_2->r_2} \\ \xrightarrow{-3r_1+r_3->r_3} \\ \xrightarrow{r_1+r_4->r_4} \end{array} \left(\begin{array}{cccc|c} 1 & -2 & 1 & -1 & 4 \\ 0 & 1 & 0 & -1 & -9 \\ 0 & 1 & 0 & -1 & -9 \\ 0 & -1 & 0 & 1 & 9 \end{array}\right) \begin{array}{c} \xrightarrow{-r_2+r_3->r_3} \\ \xrightarrow{r_2+r_4->r_4} \\ \xrightarrow{r_1-r_2->r_1} \end{array} \left(\begin{array}{cccc|c} 1 & -3 & 1 & 0 & 13 \\ 0 & 1 & 0 & -1 & -9 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\begin{cases} x_1 - 3x_2 + x_3 = 13 \\ x_2 - x_4 = -9 \end{cases}$$

0.2 Problem 2

Find the inverse of the matrix if it exists.

$$\begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & -1 & -3 & 4 \\ 1 & 0 & -1 & 2 \\ -3 & 0 & 0 & -1 \end{bmatrix}$$

determinate = 1
Thus inverse exists.

$$\left(\begin{array}{rrrr|rrrr} 1 & 0 & 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & -3 & 4 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 2 & 0 & 0 & 1 & 0 \\ -3 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \end{array}\right)$$

Row reduce until identity is swapped

$$\left(\begin{array}{rrrr|rrrr} 1 & 0 & 0 & 0 & -1 & 0 & -1 & -1 \\ 0 & 1 & 0 & 0 & -3 & -1 & 0 & -1 \\ 0 & 0 & 1 & 0 & 5 & 0 & 4 & 3 \\ 0 & 0 & 0 & 1 & 3 & 0 & 4 & 2 \end{array}\right)$$

Thus identity is

$$\begin{bmatrix} -1 & 0 & -1 & -1 \\ -3 & -1 & 0 & -1 \\ 5 & 0 & 4 & 3 \\ 3 & 0 & 4 & 2 \end{bmatrix}$$

0.3 Problem 3

Let

$$A^{-1} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 3 & 1 \\ 4 & 1 & 2 \end{bmatrix}$$

if Possible find a matrix C such that

$$ACA = \begin{bmatrix} 2 & 1 & 3 \\ -1 & 2 & 2 \\ 2 & 1 & 4 \end{bmatrix}$$

$$C = A^{-1}(ACA)A^{-1}$$

$$C = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 3 & 1 \\ 4 & 1 & 2 \end{bmatrix} * \begin{bmatrix} 2 & 1 & 3 \\ -1 & 2 & 2 \\ 2 & 1 & 4 \end{bmatrix} * \begin{bmatrix} 1 & 2 & 1 \\ 0 & 3 & 1 \\ 4 & 1 & 2 \end{bmatrix}$$

$$C = \begin{bmatrix} 2 & 6 & 11 \\ -1 & 7 & 10 \\ 11 & 8 & 22 \end{bmatrix} * \begin{bmatrix} 1 & 2 & 1 \\ 0 & 3 & 1 \\ 4 & 1 & 2 \end{bmatrix}$$

$$C = \begin{bmatrix} 46 & 33 & 30 \\ 39 & 29 & 26 \\ 99 & 68 & 63 \end{bmatrix}$$

0.4 Problem 4

Determine if the set of all invertible 4x4 matrices is closed under addition and scalar multiplication.

0.5 Problem 5

Determine if b = (7, 6, 1) lies in sp(v1, v2, v3), where v1 = (1, 2, 3), v2 = (-2, -5, 2), and v3 = (1, 2, -1).

$$\left(\begin{array}{ccc|c} 1 & -2 & 1 & 7 \\ 2 & -5 & 2 & 6 \\ 3 & 2 & -1 & 1 \end{array}\right)$$

Row reduction results in

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right)$$

Thus b lies in sp(v1,v2,v3)

0.6 Problem 6

Let v1 = (1, 3, 4), v2 = (2, 7, 2), and v3 = (-1, 2, 1). Determine if sp(v1, v2, v3) = R3

$$\begin{pmatrix} 1 & 3 & 4 \\ 2 & 7 & 2 \\ -1 & 2 & 1 \end{pmatrix}$$

Row reduction yields

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Thus these vectors are in R3

0.7 Problem 7

Determine if {(1, 4, -1, 3),(-1, 5, 6, 2),(1, 13, 4, 7)} is dependent or independent in R4.

$$\left(\begin{array}{rrr|r} 1 & -1 & 1 & 0 \\ 4 & 5 & 13 & 0 \\ -1 & 6 & 4 & 0 \\ 3 & 2 & 7 & 0 \end{array}\right)$$

Row reduction yields $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ Since this contains only trivial solutions ie,

$$c_1, c_2, c_3 = 0$$

then that makes this set linearly independent.

0.8 Problem 8

Find a basic for the nullspace of A, where

$$A = \begin{bmatrix} 3 & 1 & 9 \\ 1 & 2 & -2 \\ 2 & 1 & 5 \end{bmatrix}$$

Row reduce to

$$A = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix}$$

The system has infinitely many solutions since row 3 is all zeros.

$$\begin{cases} x_1 + 4x_3 & = 0 \\ x_2 - 3x_3 & = 0 \\ & + 0 = 0 \end{cases}$$

Thus the null space has a basis formed by this set

$$\left\{ \begin{matrix} \text{-4} \\ 3 \\ 1 \end{matrix} \right\}$$

0.9 Problem 9

Find a basis for the column space of A, where

$$A = \begin{bmatrix} 1 & -2 & 2 & -1 \\ -3 & 6 & 1 & 10 \\ 1 & -2 & -4 & -7 \end{bmatrix}$$

Row Reducing this matrix gives

$$A = \begin{bmatrix} 1 & -2 & 0 & 3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This implies that

$$< 1, -2, 0, 3 > and < 0, 0, 1, 1 >$$

Forms the basis of the column space of A

0.10 Problem 10

Compute det(A), where

$$A = \begin{bmatrix} 3 & 2 & 4 \\ 0 & 1 & 2 \\ 1 & 4 & 1 \end{bmatrix}$$

The determinate is = -21