

# Analyzing the Role of the DSO in Electricity Trading of VPPs via a Stackelberg Game Model

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**Abstract**—The increasing penetration of distributed energy resources (DER) has sparked interest in promoting their participation in the power market. Here we consider a setting in which different virtual power plants (VPPs) with certain flexible resources take part in electricity trading, either by direct participation in the wholesale power market, or interfaced by the Distribution System Operator (DSO). Our goal is to examine the role and influence of the DSO as a stakeholder, for which we formulate a Stackelberg game via a bilevel optimization model: the DSO maximizes profits at the upper level, while VPPs minimize operating costs at the lower level. To solve this problem, we use the Karush-Kuhn-Tucke optimality conditions of the convex lower-level problems to achieve a single-level mixed-integer nonlinear program. The results show that the role of the DSO as an intermediary agent leads to a decrease in operating costs for the VPPs, while guaranteeing a profit for the DSO.

**Index Terms**—Electricity market; bilevel optimization; virtual power plants; distribution system operator

## NOMENCLATURE

### Indices and Sets

- $\Omega_j$  Set of distributed energy resources in VPP<sub>j</sub>  
 $i \in \Omega_j$  Index of  $i$ -th distributed energy resource in VPP<sub>j</sub>

### Parameters

- $T$  Length of market horizon  
 $M$  Constant for ‘big-M’ method (MW)  
 $\lambda^{\text{CEP}}$  Contractual electricity purchase price (€/MWh)  
 $\lambda^{\text{CES}}$  Contractual electricity sale price (€/MWh)  
 $P_{\text{max}}^{\text{VPP,p}}$  Maximum power purchase volume of VPPs (MWh)<sup>1</sup>  
 $P_{\text{max}}^{\text{VPP,s}}$  Maximum power sale volume of VPPs (MWh)  
 $P^{\text{D}}$  Fixed demand in VPPs (MW)  
 $a, b, c$  Cost coefficients for micro turbines (€/MWh<sup>2</sup>, €/MWh, €)  
 $e$  Cost coefficient for battery storage (€/MWh<sup>2</sup>)  
 $P_{\text{max}}^{\text{MT}}$  Rated power of micro turbine (MW)  
 $P_{\text{down}}^{\text{MT}}$  Downwards ramp rate of micro turbine (MW/h)  
 $P_{\text{up}}^{\text{MT}}$  Upwards ramp rate of micro turbine (MW/h)  
 $P_{\text{max}}^{\text{BS}}$  Rated power of battery storage device (MW)  
 $P_{\text{max}}^{\text{WT}}$  Available power from wind turbines (MW)  
 $E_{\text{max}}$  Capacity of battery storage device (MWh)

<sup>1</sup>An hourly clearing of the energy market is assumed for simplicity ( $\Delta t = 1\text{h}$ ), therefore MW and MWh are used interchangeably.

### Decision variables

- $\lambda^{\text{ES}}$  Electricity sale price for VPPs (€/MWh)  
 $\lambda^{\text{EP}}$  Electricity purchase price for VPPs (€/MWh)  
 $P^{\text{DSO,s}}$  Electricity sold by DSO to the wholesale power market (MWh)  
 $P^{\text{DSO,p}}$  Electricity purchased by DSO from the wholesale power market (MWh)  
 $P^{\text{VPP,p}}$  Electricity purchased by VPPs from the DSO (MWh)  
 $P^{\text{VPP,s}}$  Electricity sold by VPPs to the DSO (MWh)  
 $P^{\text{MT}}$  Output of micro turbine (MW)  
 $P^{\text{BS}}$  Output of battery storage device (MW)  
 $P^{\text{WT}}$  Output of wind turbines (MW)  
 $\text{SoC}$  State of charge of battery storage device (%)  
 $z$  Binary variable for ‘big-M’ method

## I. INTRODUCTION

Virtual power plants (VPPs) are an emerging option for aggregating distributed energy resources (DER) to the power grid, forming a virtual entity to participate in electricity trading. In a context where different aggregators participate in the power market, each stakeholder pursues the maximization of its own profit. Some studies have used game theory to analyze the equilibrium problem between stakeholders [1], [2].

In previous studies, the role and profit achieved by the Distribution System Operator (DSO) as the intermediary agent between the aggregators and the wholesale market are considered differently [1], [3]. In [1], the DSO acts as a central controller to manage VPPs in the game, without gaining profits. The benefits of VPPs are determined by affordable power output and fuel cost. In [3], VPPs can reduce the risks faced by the DSO from uncertainty in the price of the spot market and fluctuations in power consumption, which means that the DSO may benefit indirectly from them.

When the DSO acts as a stakeholder, it should not incur in economic losses, therefore its revenues and costs must be considered in each hourly clearing. In order to study the interactions between the DSO and the different VPPs, we assume that the DSO is an intermediary agent for aggregators and sets trading prices for them. The goal is to analyze the behavior of the different selfish agents in this setting, to understand the role of the DSO.

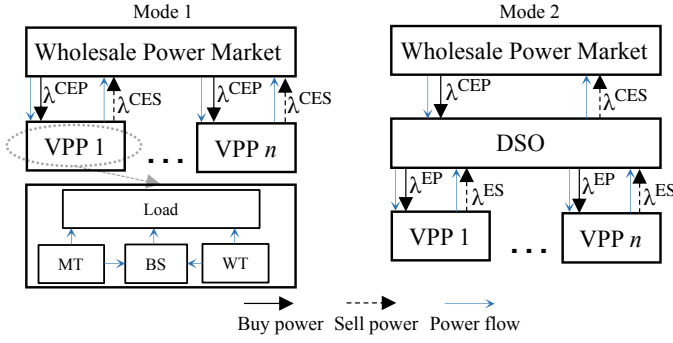


Fig. 1: Diagram of energy and cash flows in the two market settings considered.

The DSO and VPPs form a Stackelberg game, also known as a bilevel optimization problem. A bilevel model may be expressed as a mathematical program with equilibrium constraints (MPEC) under certain conditions [4], [5], in which the lower-level problems are transformed to be constraints of the upper level. Karush-Kuhn-Tucker (KKT) conditions are the main method of the problem transformation [6], valid if the lower-level problems are convex. This is the method used in the present paper for solving the Stackelberg game.

## II. STACKELBERG GAME MODEL FOR DSO AND VPPs

To minimize operating costs, the internal power balance of a single VPP may not be achieved, showing a surplus or shortage of electricity to the outside grid. We consider two different trading possibilities for balancing energy consumption within VPPs, as depicted in Fig. 1:

- ‘Mode 1’, where VPPs trade directly with the wholesale power market at contract prices  $\lambda^{\text{CEP}}$ ,  $\lambda^{\text{CES}}$ , therefore no game is played. Mathematically, this is a simple setting in which each VPP simply minimizes its own cost, independent of other VPPs.
- ‘Mode 2’, where the DSO plays the role of an intermediary agent between the VPPs and the wholesale power market, therefore VPPs sell excess energy to the DSO at price  $\lambda^{\text{ES}}$  and buy energy from the DSO at price  $\lambda^{\text{EP}}$ . Prices are set by the DSO via a Stackelberg game. The DSO also trades with the non-strategic wholesale power market at fixed contract prices  $\lambda^{\text{CES}}$ ,  $\lambda^{\text{CEP}}$ .

In Mode 2, the DSO acts as a leader who trades with the wholesale power market and all VPPs, while VPPs are followers who only trade with the DSO. The leader and the followers play a Stackelberg game as described in more detail in Fig. 2. The DSO first agrees with the VPPs on the volume of electricity to be sold or purchased, and then trades with the wholesale power market to realize this traded quantity. The DSO, as the strategic agent, sets the prices  $\lambda^{\text{ES}}$  and  $\lambda^{\text{EP}}$  for trading with the VPPs, while guaranteeing that it will not incur a loss when eventually trading with the wholesale market at contract prices  $\lambda^{\text{CEP}}$ ,  $\lambda^{\text{CES}}$ . VPPs take the prices and solve their own optimization problem, deciding the output of their DER in order to minimize overall operating costs, therefore eventually setting the volume of electricity traded with the DSO.

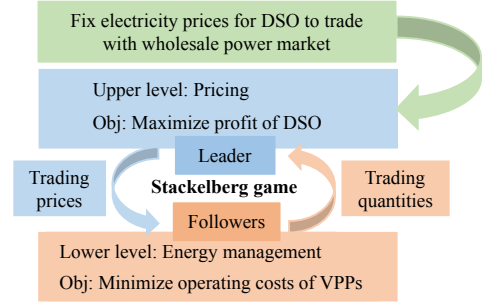


Fig. 2: Stackelberg game framework for DSO and VPPs.

Each VPP is considered here to contain dispatchable generation in the form of micro turbines (MTs), weather-driven generation such as wind turbines (WTs), battery storage devices (BS) and a fixed hourly demand representing the energy needs of consumers, as shown in Fig. 1.

### A. Game model of leader-DSO for pricing

The game strategy adopted by DSO is to set  $\lambda_t^{\text{ES}}$  and  $\lambda_t^{\text{EP}}$  for VPPs, then trade with them. The strategy spaces  $\lambda^{\text{ES}}$  and  $\lambda^{\text{EP}}$  are, respectively:

$$\begin{aligned} \lambda^{\text{ES}} &= \{\lambda_1^{\text{ES}}, \lambda_2^{\text{ES}}, \dots, \lambda_T^{\text{ES}}\} \\ \lambda^{\text{EP}} &= \{\lambda_1^{\text{EP}}, \lambda_2^{\text{EP}}, \dots, \lambda_T^{\text{EP}}\} \end{aligned} \quad (1)$$

1) *Utility function*: The utility function of DSO is to maximize its profits, which comprises of the cost and revenue from trading with the wholesale market and VPPs:

$$\begin{aligned} \max \quad C^{\text{DSO}} &= \sum_{t=1}^T (\lambda_t^{\text{CES}} \cdot P_t^{\text{DSO},s} - \lambda_t^{\text{CEP}} \cdot P_t^{\text{DSO},p} + \\ &\quad \lambda_t^{\text{EP}} \cdot \sum_{j=1}^J P_{j,t}^{\text{VPP},p} - \lambda_t^{\text{ES}} \cdot \sum_{j=1}^J P_{j,t}^{\text{VPP},s}) \end{aligned} \quad (2)$$

Where  $\lambda_t^{\text{CES}} \cdot P_t^{\text{DSO},s}$  and  $\lambda_t^{\text{EP}} \cdot \sum_{j=1}^J P_{j,t}^{\text{VPP},p}$  are the income by trading with the wholesale market and VPPs, respectively;  $\lambda_t^{\text{CEP}} \cdot P_t^{\text{DSO},p}$  and  $\lambda_t^{\text{ES}} \cdot \sum_{j=1}^J P_{j,t}^{\text{VPP},s}$  are the corresponding transaction costs.

2) *Constraints*: The premise for aggregators to trade with the DSO is that they can take more favorable prices. Otherwise, for VPPs, this is no different from trading energy directly with the wholesale market. Therefore, the prices set by the DSO for VPPs must meet eq. (3):

$$\begin{cases} \lambda_t^{\text{CES}} \leq \lambda_t^{\text{ES}} \leq \lambda_t^{\text{CEP}} \\ \lambda_t^{\text{CES}} \leq \lambda_t^{\text{EP}} \leq \lambda_t^{\text{CEP}} \end{cases} \quad (3)$$

For transactions with the wholesale market, it is impossible for the DSO to buy and sell electricity at the same unit time, and only one trading decision can be made (buy, sell, or no transaction). We use  $P_t^{\text{DSO}}$  to represent the trading for the DSO, based on the demand from each VPP in a unit time. Therefore  $P_t^{\text{DSO},p}$  and  $P_t^{\text{DSO},s}$  must follow eqs. (4)-(6):

$$P_t^{\text{DSO}} = \sum_{j=1}^J (P_{j,t}^{\text{VPP},p} - P_{j,t}^{\text{VPP},s}) \quad (4)$$

$$P_t^{\text{DSO},p} = \begin{cases} P_t^{\text{DSO}}, & \text{if } P_t^{\text{DSO}} \geq 0 \\ 0, & \text{if } P_t^{\text{DSO}} < 0 \end{cases} \quad (5)$$

$$P_t^{\text{DSO},s} = \begin{cases} -P_t^{\text{DSO}}, & \text{if } P_t^{\text{DSO}} < 0 \\ 0, & \text{if } P_t^{\text{DSO}} \geq 0 \end{cases} \quad (6)$$

Where  $P_t^{\text{DSO}}$  is the quantity of electricity traded by the DSO with the wholesale market, equal to the quantity traded with the VPPs. Eq. (4) means that the DSO first clears the market in the lower level by trading with the different VPPs, and then trades with the wholesale market based on the electricity deficit or surplus in the lower level.  $P_t^{\text{DSO}} \geq 0$  indicates that electricity is purchased from the wholesale market by the DSO; while  $P_t^{\text{DSO}} < 0$  indicates the opposite.

Constraints (5)-(6) involve 'if-then' logical statements. A 'big-M' method can be used to transform them into indicator constraints containing a binary variable, thus making them suitable for optimization solvers. Consider two generic conditional constraints:

$$\begin{cases} \text{if } (x \geq \alpha), \text{ then } y = A \cdot x \\ \text{else if } (x < \alpha), \text{ then } y = B \cdot x \end{cases} \quad (7)$$

These can be reformulated via the 'big-M' method into the following indicator constraints, where it is necessary to include an auxiliary binary variable 'z':

$$\begin{cases} -M \cdot (1 - z) \leq x - \alpha \leq M \cdot z \\ -M \cdot (1 - z) \leq y - A \cdot x \leq M \cdot (1 - z) \\ -M \cdot z \leq y - B \cdot x \leq M \cdot z \end{cases} \quad (8)$$

Where if  $z = 1$ ,  $y = A \cdot x$ ; else if  $z = 0$ ,  $y = B \cdot x$ . Parameter 'M' is a sufficiently large constant used to relax the above constraints appropriately. Its value should be set as small as possible to avoid numerical issues when solving the optimization [7], while ensuring that it is also sufficiently high to relax the above constraints appropriately.

For reformulating (5) in the same way, simply set  $A = 1$ ,  $B = 0$ ,  $z_1$ ,  $\alpha = 0$ ,  $x = P_t^{\text{DSO}}$ ,  $y = P_t^{\text{DSO},p}$ :

$$\begin{cases} -M \cdot (1 - z_1) \leq P_t^{\text{DSO}} - 0 \leq M \cdot z_1 \\ -M \cdot (1 - z_1) \leq P_t^{\text{DSO},p} - P_t^{\text{DSO}} \leq M \cdot (1 - z_1) \\ -M \cdot z_1 \leq P_t^{\text{DSO},p} - 0 \leq M \cdot z_1 \end{cases} \quad (9)$$

Equivalently, using  $A = 0$  and  $B = -1$  for eq. (6):

$$\begin{cases} -M \cdot (1 - z_2) \leq P_t^{\text{DSO}} - 0 \leq M \cdot z_2 \\ -M \cdot (1 - z_2) \leq P_t^{\text{DSO},s} - 0 \leq M \cdot (1 - z_2) \\ -M \cdot z_2 \leq P_t^{\text{DSO},s} + P_t^{\text{DSO}} \leq M \cdot z_2 \end{cases} \quad (10)$$

In order for eqs. (9) and (10) to simultaneously hold at any given unit time,  $z_1$  and  $z_2$  must also satisfy:

$$z_1 = z_2, \quad \forall t \quad (11)$$

Note that index 't' has been dropped from the binary variables for reasons of clarity in the mathematical expressions.

### B. Game model of followers-VPPs for managing energy

The game strategy of VPPs is the operating plan, including the volume of electricity traded with the DSO, the output of MT, and the charging and discharging power of BS, all considering the available output of WT and the fixed demand to be satisfied. The strategy space of VPP<sub>j</sub> is then defined as:

$$P_j = \{P_{j,t}^{\text{VPP},s}, P_{j,t}^{\text{VPP},p}, P_{i,t}^{\text{MT}}, P_{i,t}^{\text{BS}}, P_{i,t}^{\text{WT}}\} \quad (12)$$

1) *Utility function*: The utility function of VPPs is to minimize operating costs, including the cost of purchasing

electricity from DSO, the cost of MT power production given by function ' $C_{i,t}^{\text{MT}}$ ', and the degradation cost of BS given by function ' $C_{i,t}^{\text{BS}}$ ' due to cycling. Wind power is considered as a completely free resource. This function is given by eq. (13):

$$\min C_j^{\text{VPP}} = \sum_{t=1}^T [\lambda_t^{\text{EP}} \cdot P_{j,t}^{\text{VPP},p} - \lambda_t^{\text{ES}} \cdot P_{j,t}^{\text{VPP},s} + \sum_{i \in \Omega_j} (C_{i,t}^{\text{MT}} + C_{i,t}^{\text{BS}})] \quad (13)$$

$$C_{i,t}^{\text{MT}} = a_i \cdot (P_{i,t}^{\text{MT}} \cdot \Delta t)^2 + b_i \cdot P_{i,t}^{\text{MT}} \cdot \Delta t + c_i \quad (14)$$

$$C_{i,t}^{\text{BS}} = e_i \cdot (P_{i,t}^{\text{BS}} \cdot \Delta t)^2 \quad (15)$$

Where eq. (15) is composed of the square of charge or discharge, a setting from [8], which makes  $C_{i,t}^{\text{BS}}$  not change sign in (13) when  $P_{i,t}^{\text{BS}}$  is negative.

2) *Constraints*: When VPPs respond to prices, each of them must meet constraints which are shown in eqs. (16)-(21).

$$P_{j,t}^{\text{VPP}} = P_{j,t}^{\text{VPP},p} - P_{j,t}^{\text{VPP},s} \quad (16)$$

$$\begin{cases} 0 \leq P_{j,t}^{\text{VPP},p} \leq P_{\max}^{\text{VPP},p} \\ 0 \leq P_{j,t}^{\text{VPP},s} \leq P_{\max}^{\text{VPP},s} \end{cases} \quad (17)$$

$$P_{j,t}^{\text{VPP}} + \sum_{i \in \Omega_j} (P_{i,t}^{\text{MT}} + P_{i,t}^{\text{BS}} + P_{i,t}^{\text{WT}}) \cdot \Delta t = \sum_{i \in \Omega_j} P_{i,t}^{\text{D}} \cdot \Delta t \quad (18)$$

$$\begin{cases} 0 \leq P_{i,t}^{\text{MT}} \leq P_{i,\max}^{\text{MT}} \\ P_{i,\text{down}}^{\text{MT}} \cdot \Delta t \leq P_{i,t+1}^{\text{MT}} - P_{i,t}^{\text{MT}} \leq P_{i,\text{up}}^{\text{MT}} \cdot \Delta t \end{cases} \quad (19)$$

$$\begin{cases} -P_{i,\max}^{\text{BS}} \leq P_{i,t}^{\text{BS}} \leq P_{i,\max}^{\text{BS}} \\ \text{SoC}_{i,t} = \text{SoC}_{i,t-1} - \frac{\Delta t}{E_{i,\max}} \cdot P_{i,t}^{\text{BS}} \\ \text{SoC}_{i,\min} \leq \text{SoC}_{i,t} \leq \text{SoC}_{i,\max} \\ \text{SoC}_{i,0} = \text{SoC}_{i,T} \end{cases} \quad (20)$$

$$0 \leq P_{i,t}^{\text{WT}} \leq P_{i,t,\max}^{\text{WT}} \quad (21)$$

The internal power balance of VPP<sub>j</sub> is represented by eqs. (16)-(18). Within a limited transaction volume, VPPs sell or purchase electricity in each unit time to meet the surplus or shortage of electricity. The output and ramp rates of MT are limited by eq. (19). The charge and discharge range of the BS and boundary of the SoC are captured by eq. (20). We constrain BS to be consistent at the start and end of the cycle to ensure that it has sufficient electricity left to operate in the next cycle. The output of WT is shown in eq. (21), indicating that it will not exceed the upper limit of its available resources.

### C. Stackelberg game model

Based on the above sections, the Stackelberg game model between the DSO (leader) and multiple aggregators/VPPs (followers) is established in eq. (22):

$$\begin{aligned} & \max_{\lambda^{\text{ES}}, \lambda^{\text{EP}}} C^{\text{DSO}}(\lambda^{\text{ES}}, \lambda^{\text{EP}}, \sum_j P_j) \\ & \text{s.t.} \quad (3), (4), (9) - (11) \end{aligned} \quad (22)$$

$$\left\{ \begin{array}{l} \min_{P_j} \sum_j C_j^{\text{VPP}}(\lambda^{\text{ES}}, \lambda^{\text{EP}}, P_j) \\ \text{s.t.} \quad (16) - (21) \end{array} \right\}$$

Where DSO and VPPs formulate strategies with the goal of maximizing profits and minimizing operating costs, respectively. The DSO profit is related to  $\lambda^{\text{ES}}$ ,  $\lambda^{\text{EP}}$  and the transaction quantities  $P_{j,t}^{\text{VPP},s}$ ,  $P_{j,t}^{\text{VPP},p}$  in  $P_j, \forall j$  (as defined in (12)). The responses of VPPs to electricity prices affect the profit of DSO: high  $\lambda^{\text{EP}}$  results in a low purchased quantity by VPPs, while low  $\lambda^{\text{ES}}$  implies a low volume of electricity sold by VPPs. Therefore, the DSO has to consider these movements and find the Nash equilibrium solution as the optimal prices. The definition and proof of the equilibrium solution for the Stackelberg game are explained in [4].

### III. CASE STUDY

The above game model in (22) is a bilevel problem, where the upper level is a mixed-integer nonlinear program. Its non-linearity comes from the products of decision variables  $\lambda_t^{\text{EP}} \cdot \sum_{j=1}^J P_{j,t}^{\text{VPP},p}$  and  $\lambda_t^{\text{ES}} \cdot \sum_{j=1}^J P_{j,t}^{\text{VPP},s}$  in (2). The lower level, i.e., eqs. (13)-(21), is a convex nonlinear program. Therefore, the lower level can be exactly replaced by its corresponding KKT optimality conditions [4], transforming the original bilevel problem into a single-level MPEC:

$$\begin{aligned} \max \quad & C^{\text{DSO}}(\cdot) \\ \text{s.t.} \quad & \left\{ (3), (4), (9) - (11), \text{KKT}^{(j)} \forall j \right\} \end{aligned} \quad (23)$$

Detailed derivations and explanations on MPEC and KKT conditions can be found in [4]. Here, `BilevelJuMP.jl` is used to compile the KKT conditions, in order to avoid the need to write them down manually. This is a Julia package that enables the user to describe both upper- and lower-level problems using the `JuMP.jl` algebraic syntax [9]. In the simulation, we choose `SCIP.jl` as the solver due to its compatibility with mixed-integer nonlinear programs [10], which leads to feasible solutions.

The code used to run the simulations is available on a public repository [11]. Note that this work can be easily extended to larger-scale problems because of the structural independence of each stakeholder, such as those involving more aggregators and DER, or different market mechanisms (like demand response, time-of-use tariff, etc.).

#### A. Test system setting

The test system comprises three VPPs of similar size. The primary differences among the VPPs lie in their distinct operating parameters of the DER as shown in Table I, and the internal demand profiles and available wind power, as shown in Fig. 3. For MT, each VPP has unique cost coefficients (a, b, c), output upper limits ( $P_{\max}^{\text{MT}}$ ), and ramp rates ( $P_{\text{up}}^{\text{MT}}$ ,  $P_{\text{down}}^{\text{MT}}$ ). In terms of BS, the variations include the capacity ( $E_{\max}$ ) and the upper limits for charge and discharge ( $P_{\max}^{\text{BS}}$ ). Furthermore, the power available from the WT ( $P_{\max}^{\text{WT}}$ ) differs between VPPs. Therefore, each VPP may have different dispatch strategies based on these various system settings.

The maximum transaction volume ( $P_{\max}^{\text{VPP},p}$ ,  $P_{\max}^{\text{VPP},s}$ ) of each single VPP is 10 MWh, i.e., the upper bound of the trading volumes for three VPPs is 30 MWh. Therefore, a tight value for the ‘big-M’ method is provided in (9)-(11) of  $M = 30$ .

TABLE I  
PARAMETERS SETTING OF EACH VPP

Parameters	VPP <sub>1</sub>	VPP <sub>2</sub>	VPP <sub>3</sub>
$P_{\max}^{\text{VPP},p}, P_{\max}^{\text{VPP},s}$	10, 10	10, 10	10, 10
a, b, c	0.08, 0.90, 1.20	0.1, 0.6, 1.0	0.15, 0.50, 0.80
e	0.05	0.05	0.05
$P_{\max}^{\text{MT}}, P_{\text{down}}^{\text{MT}}, P_{\text{up}}^{\text{MT}}$	6.0, -3.5, 3.5	5, -3, 3	4, -2, 2
$P_{\max}^{\text{BS}}$	0.6	0.6	1.2
SoC <sub>0</sub> , SoC <sub>min</sub> , SoC <sub>max</sub>	40, 20, 90	40, 20, 90	40, 20, 90
$E_{\max}$	1	1	2

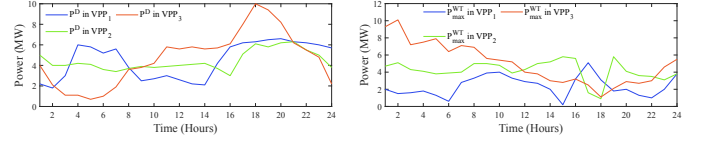


Fig. 3: Load profiles and available wind power in each VPP.

#### B. Analysis of trading results through the Stackelberg game

Here we examine the strategic behaviors of the DSO and VPPs under the two market modes in Section II. The profits of the DSO, and costs of each VPP, are shown in Table II.

In Mode 2, compared with Mode 1, the DSO earns profits by setting hourly prices  $\lambda_t^{\text{ES}}$ ,  $\lambda_t^{\text{EP}}$ , as shown in Fig. 4, and trading with VPPs. The costs of all VPPs decrease (0.13%, 2.94%, and 0.78%, respectively). The reason is that the revenues flowing into the wholesale market are 22.68% lower, which is almost equal to the sum of the decreased costs of the VPPs and the profits of the DSO. This result shows that selfish DSO and VPPs strategically expand their benefits in the game, by decreasing the net economic flows to the non-strategic wholesale power market.

To clarify how the inflow of money to the wholesale power market decreases, it is necessary to analyze the differences in the solutions for the two market modes, i.e.,  $\{P_j, \lambda^{\text{ES}}, \lambda^{\text{EP}}\}_{\text{Mode 2}} - \{P_j, \lambda^{\text{CES}}, \lambda^{\text{CEP}}\}_{\text{Mode 1}}, \forall j$  (shown as ‘Trade diff’ and ‘output diff’ in Tables III to V). We individualize the analysis for each VPP in the following subsections.

1) *Cost reduction analysis for VPP<sub>1</sub>*: The operations of WT and MT are the same in the two market modes, which means the cost reduction for VPP<sub>1</sub> comes from the difference in volume of electricity traded, operation of BS, and prices. Table III only lists the hours which show a different solution between modes 1 and 2, omitting hours with no change at all.

Combined with Fig. 4, it can be found that VPP<sub>1</sub> responds to trading prices to reduce operating costs. The sale price  $\lambda_{11}^{\text{ES}}$  (where ‘11’ means  $t = 11:00$ ), set by DSO, is higher than  $\lambda_{11}^{\text{CES}}$ , so more electricity is sold. But in Mode 1, without DSO, VPP<sub>1</sub> chooses to sell more electricity at 09:00 and 10:00 when the sale price is cheaper. The purchase price  $\lambda_{23}^{\text{EP}}$  is higher than  $\lambda_{24}^{\text{EP}}$ , so less electricity is bought at 23:00 in Mode 2. VPP<sub>1</sub> strategically reduces costs by charging and discharging the BS: purchasing electricity at a lower price, the SoC increases; selling electricity at a higher price, SoC decreases.

2) *Cost reduction analysis for VPP<sub>2</sub>*: All DER show operating differences under the two modes. Table IV in the

TABLE II  
COSTS AND PROFITS OF STAKEHOLDERS IN TWO MODES

Stakeholders	Mode 1 (k€)	Mode 2 (k€)
VPP <sub>1</sub>	-3.947	-3.942
VPP <sub>2</sub>	-0.918	-0.891
VPP <sub>3</sub>	-3.587	-3.559
DSO	—	1.134
Wholesale market	5.370	4.152

TABLE III  
COMPARISON OF SOLUTIONS UNDER DIFFERENT MODES FOR VPP<sub>1</sub>

Time (Hours)	9	10	11	...	23	24
Trade diff (MWh)						
Purchase	—	—	—	...	-0.10	0.10
Sale	-0.12	-0.12	0.23	...	—	—
SoC (%)						
Mode 2	55	90	90	...	20	40
Mode 1	43	67	90	...	30	40

Appendix lists the different solutions between the two modes. The purchase prices  $\lambda_{11}^{EP}$ - $\lambda_{14}^{EP}$ , set by the DSO, are lower than the contract price  $\lambda^{CEP}$ , and the purchase quantities by VPP<sub>2</sub> under the two modes are the same, the costs are thereby reduced in Mode 2. At 17:00 and 18:00, VPP<sub>2</sub> in Mode 2 chooses to purchase more electricity to sell, at an expensive price, from 19:00 to 22:00, to increase revenue. The price  $\lambda_{24}^{EP}$  is lower than  $\lambda_{23}^{EP}$ , so VPP<sub>2</sub> chooses to purchase more energy.

The costs reduction for VPP<sub>2</sub> requires the strategic operation of the internal DER. From 02:00 to 07:00, the operations of MT and BS under the two modes are the same, the output of WT directly affects the transactions. From 19:00 to 22:00, VPP<sub>2</sub> trades with the DSO by discharging the BS and increasing the MT output, thus increasing income. At 23:00, under Mode 1, more electricity is purchased to increase SoC, but this is not as cost-effective as trading at 24:00 in Mode 2.

3) *Cost reduction analysis for VPP<sub>3</sub>*: Table V in the Appendix lists the different solutions between the two modes. VPP<sub>3</sub> strategically purchases more electricity from the DSO when prices  $\lambda_{11}^{EP}$ - $\lambda_{14}^{EP}$  are cheaper than contract prices, and reduces the purchased volume in the following four hours, when purchase prices rise. From 11:00-14:00, MT reduces output, and internal power balance is maintained mainly by the purchased energy.

4) *Profit analysis for wholesale market and DSO*: The revenue and cost for the wholesale market are shown in Fig. 5. The income in Mode 2 (selling electricity to DSO) is significantly less than that in Mode 1 (selling electricity directly to VPPs). This leads to a reduction in money flowing into the wholesale market, where the loss is caused by the trading quantity (as the contract prices are fixed, not variables). This same volume of electricity is instead traded in the lower level between the VPPs.

The volumes that the DSO and VPPs trade with the wholesale market respectively in Mode 2 and Mode 1 are shown in Fig. 6. The negative half-axis represents the sale of electricity to the wholesale market, and the positive half-axis represents the purchase of electricity from the wholesale market.

As the intermediary agent for all aggregators, the DSO trades with the wholesale market at contract prices ( $\lambda^{CEP}$ ,  $\lambda^{CES}$ ) on the one hand, and sets more favorable ‘internal

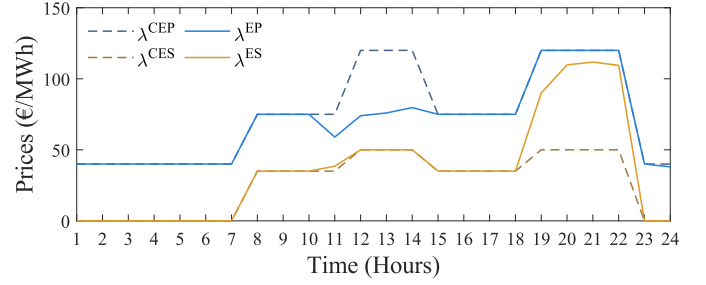


Fig. 4: Trading prices in two modes.

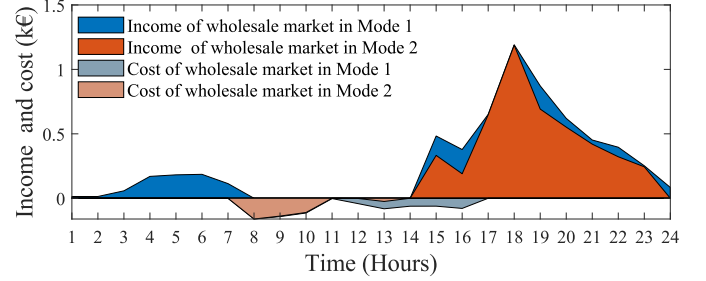


Fig. 5: Profits changing of the wholesale power market.

prices’ ( $\lambda^{EP}$ ,  $\lambda^{ES}$ ) based on the operation of each VPP, on the other hand. The DSO directly trades with the VPPs, that is, the lower level forms a trading cluster, resulting in a decrease in volumes traded with the wholesale market. Compared with Mode 1, the DSO promotes power sharing in (4) between VPPs at the ‘internal prices’, making a profit in the process. This profit results in lower revenue flowing into the wholesale market, as can be seen in Fig. 6: Before 08:00 and after 15:00, the results are significantly different for the cases in which VPPs trade directly with the wholesale market vs. when the DSO is in between. The selfish aggregators take the prices set by the DSO and prioritize trading with other aggregators, resulting in energy balancing happening mostly at DSO level.

As shown in Table II and the previous analysis, the reduced economic flows into the wholesale market are equal to the profits gained by the DSO, plus the cost reduction for the VPPs. This may in fact be advantageous for the overall power system, represented by the wholesale market: while the revenues flowing into other agents of the wholesale market have been reduced, weather-based DER and their inherent uncertainty would be managed by the VPPs locally in this setting, potentially saving some operating costs in the form of reserves to the overall grid. However, to fully understand whether the local management of DER uncertainty would be economically acceptable for VPPs, future work should explicitly model it.

#### IV. CONCLUSION

This work provides a detailed analysis of the interactions between the DSO and VPPs in electricity trading through a Stackelberg game. The findings highlight that adopting a bilevel model allows the DSO, acting as the intermediary agent, to set trading prices that reduce operating costs for VPPs, while ensuring profitability for the DSO.



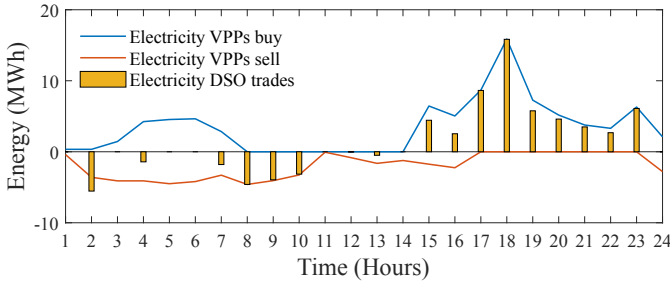


Fig. 6: Quantities DSO and VPPs trade with wholesale power market respectively.

While these findings are based on results for a particular setting, i.e., a small test case with three VPPs, the promising trends identified justify carrying out additional theoretical work in the future. The advantages of DSO-VPP trading in a general market setting should be further investigated via mathematical proofs, in order to enhance interest in market participation of these resource aggregators.

In future work, we will investigate market settings for ancillary services, exploring the coupling relationship between the electricity market and the stable operation of the system.

#### V. ACKNOWLEDGEMENT

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#### APPENDIX

Tables IV and V list the different solutions between two modes, for VPP<sub>2</sub> and VPP<sub>3</sub>, respectively.

TABLE IV

COMPARISON OF SOLUTIONS UNDER DIFFERENT MODES FOR VPP<sub>2</sub>

Time (Hours)		2	3	...	6	7	...	17	18	19	20	21	22	23	24
Trade diff (MWh)	Purchase	—	—	...	—	—	...	<b>0.21</b>	<b>0.21</b>	—	—	—	—	<b>-0.10</b>	<b>0.10</b>
	Sale	-0.04	0.01	...	0.01	0.19	...	—	—	<b>1.50</b>	<b>0.57</b>	<b>0.27</b>	<b>0.62</b>	—	—
MT output diff (MW)		0	0	...	0	0	...	0	0	1.07	0.51	0.41	0.53	0	0
WT output diff (MW)		<b>-0.04</b>	<b>0.01</b>	...	<b>0.01</b>	<b>0.19</b>	...	0	0	0	0	0	0	0	0
SoC (%)	Mode 2	48	75	...	63	90	...	90	90	90	72	34	20	20	40
	Mode 1	48	75	...	63	90	...	69	47	90	78	26	20	30	40

TABLE V

COMPARISON OF SOLUTIONS UNDER DIFFERENT MODES FOR VPP<sub>3</sub>

Time (Hours)		1	2	3	4	5	6	7	...	11	12	13	14	15	16	17	18	...	24
Trade diff (MWh)	Purchase	—	—	—	—	—	—	—	...	<b>0.30</b>	<b>0.80</b>	<b>1.14</b>	<b>1.23</b>	<b>-0.26</b>	<b>-0.26</b>	<b>-0.26</b>	<b>-0.26</b>	...	—
	Sale	-0.06	2.32	-2.67	1.56	0.04	0.43	1.16	...	—	—	—	—	—	—	—	—	...	-0.50
MT output diff (MW)		0	0	0	0	0	0	0	...	<b>-0.30</b>	<b>-0.68</b>	<b>-0.72</b>	<b>-0.74</b>	0	0	0	0	...	0
WT output diff (MW)		-0.06	2.32	-2.67	1.56	0.04	0.43	1.16	...	0	0	0	0	0	0	0	0	...	-0.50
SoC (%)	Mode 2	47	54	61	69	76	83	90	...	90	90	90	71	76	81	85	90	...	40
	Mode 1	47	54	61	69	76	83	90	...	90	84	63	20	37	55	73	90	...	40

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