Linear Programming Code

1. Implementation of the basic simplex algorithm

Code

```
function [Solution,BasicVar,Status] = AxEqualsb(A, b, c, BasicVariables)
    [rows, ~] = size(b);
    if (all(b >= 0) && rank(A)==rows)
        [Solution,BasicVar,Status]=basicsimplex(A,b,c,BasicVariables);
    else
        disp('b must be nonnegative and rank(A) == size of b');
    end
end
```

Interpretation

- The function works for Contraint $Ax = b$.
- It checks whether b is non-nagative using `all(b >= 0)`.
- It also checks the matrix A is full rank using `rank(A)==rows`.
- If all the inputs are valid, we feed all the parameters into the `basicsimplex()` function and return the results.

Results

I use the following code to test the algorithm. I set `A = [[1,0,1];[0,1,1]]`, `b = [1;2]`, and `c = [-1;-1;-3]`. The result I expected to get is `[0;1;1]`.

Running this to test:

```
A = [[1,0,1];[0,1,1]];
b = [1;2];
c = [-1;-1;-3];

[Solution,BasicVar,Status] = AxEqualsb(A, b, c, [1, 2])
```

```
Solution =

     0
     1
     1


BasicVar =

     3     2


Status =

     0
```

We can see the basic variables are x_2 and x_3 with values 1 and 1 respectively. `Status = 0` showing the results are valid.

2. Implementation of general linear programming algorithm

Code

```
function [Solution,BasicVar,Status] = AxSmallerThanb(A_hat, b_hat, c_hat)
    [rows, ~] = size(b_hat);
    if (all(b_hat >= 0) && rank(A_hat)==rows)
        I = eye(rows);
        c_new = [c_hat;zeros(rows,1)];
        A_new = [A_hat I];
        [~, A_cols] = size(A_hat);
        BasicVariables = (A_cols+1):(rows+A_cols);
        [Solution,BasicVar,Status]=basicsimplex(A_new,b_hat,c_new,BasicVariables);
    else
        disp('b_hat must be nonnegative and rank(A) == size of b_hat');
    end
end
function [Solution,BasicVar,Status] = AxGreaterThanb(A_tilda, b_tilda, c_tilda)
    [rows, ~] = size(b_tilda);
    if (all(b_tilda >= 0) && rank(A_tilda)==rows)
        I = eye(rows);
        A_phaseII = [A_tilda -I];
        c_phaseII = [c_tilda;zeros(rows, 1)];
        A_phaseI = [A_phaseII I];
        [~, A_cols] = size(A_tilda);
        PhaseI_BasicVariables = (A_cols+1+rows):(2*rows+A_cols);
        c_phaseI = [zeros(A_cols+rows, 1);ones(rows, 1)];
        [SolutionI,BasicVarI,StatusI]=basicsimplex(A_phaseI,b_tilda,c_phaseI,PhaseI_BasicVariables);
        if (StatusI==0)
            % Phase II
            [Solution,BasicVar,Status]=basicsimplex(A_phaseII,b_tilda,c_phaseII,BasicVarI);
        else
            Solution=SolutionI;
            BasicVar=BasicVarI;
            Status=StatusI;
        end
    else
        disp('b_tilda must be nonnegative and rank(A) == size of b_tilda');
    end
end
```

Interpretation

- The first function `AxSmallerThanb()` works for Contraint $Ax \leq b$.
  - It checks whether b is non-nagative using `all(b >= 0)`.
  - It also checks the matrix A is full rank using `rank(A)==rows`.
  - If all the inputs are valid, we are going to add slack variables to make the problem standard.
  - Finally, we feed all the parameters into the `basicsimplex()` function and return the results.
- The second function `AxGreaterThanb()` works for Contraint $Ax \geq b$.
  - It checks whether b is non-nagative using `all(b >= 0)`.
  - It also checks the matrix A is full rank using `rank(A)==rows`.
  - If all the inputs are valid, we are going to first add slack variables to make the problem standard, and call it `A_phaseII`.
  - Second, we are going to add phase I variables so that we have an intial basic solution.
  - Third, change the objective function based on the phase I variables .
  - Next, we feed all the parameters into the `basicsimplex()` function and return the results.
  - Then, do phase II simplex method based on the `BasicVarI` returned from phase I.
  - Finally, the results are returned.

Results

I'm going to use the same A, b, and c as part 1.

Running this to test:

```
[Solution,BasicVar,Status] = AxSmallerThanb(A, b, c)
[Solution,BasicVar,Status] = AxGreaterThanb(A, b, c)
```

```
Solution =

     0
     1
     1
     0
     0


BasicVar =

     3     2


Status =

     0


Solution =

     0
     0
     2
     1
     0


BasicVar =

     4     3


Status =

    -1
```

- We can see for the first case, the basic variables are x_2 and x_3 with values 1 and 1 respectively. `Status = 0` showing the results are valid.
- For the second case, it's straightforward that the solution will be infinity. Indeed, `Status = -1` proves that there's no solution for this case.

3. Study of L^1 versus L^2 approximation

Code

```
% Part 3 Apple stock vs Dow Jones Index
Apple = readtable('APPL_DATA.csv');
Apple = flipud(Apple);
Apple = Apple(1:253,[1,4]);
Apple = table2array(Apple(:,2));
DowJones = readtable('Dow_Jones.csv');
DowJones = DowJones(:,1:2);
DowJones = table2array(DowJones(:,2));

DowJones = str2double(DowJones);

X = transpose(DowJones)
Y = transpose(Apple)
n = 253;
```

```
% L1 Regression
[RegressionModel]=L1_MultilinearRegression(X,Y);
%
% Least square
%
Xhat=X-mean(X,2)*ones(1,n);
Yhat=Y-mean(Y);
Coef_LSQ=inv(Xhat*Xhat')*Xhat*Yhat';
Intersect_LSQ=mean(Y)-Coef_LSQ'*mean(X,2);
Prediction=Coef_LSQ'*X+Intersect_LSQ;
%
figure;
plot(Y,RegressionModel.Prediction,'o','MarkerSize',[8],'MarkerFaceColor','r','MarkerEdgeColor','r');
hold on
plot(Y,Prediction,'o','MarkerSize',[8],'MarkerFaceColor','b','MarkerEdgeColor','b');
%
figure;
plot(Y'-RegressionModel.Prediction,'o','MarkerSize',[8],'MarkerFaceColor','r','MarkerEdgeColor','r');
hold on
plot(Y-Prediction,'o','MarkerSize',[8],'MarkerFaceColor','b','MarkerEdgeColor','b');

RegressionModel.SRE
sum(abs(Y-Prediction))
```

Explanation

- For this part of the problem, I'm using Apple Stock Price data and Dow Jones Index Price data for the past year.
- First, I read the data from the `.csv` files and do some manipulations to them to a vector.
- Second, I feed both data into `L1_MultilinearRegression(X,Y)`, `X` is Dow Jones data and `Y` is Apple data. This should return a L^1 model.
- Third, I perform L^2 regression on the same set of data.
- Close to finish, I plot the regression model and the residual graph for both types of regression, shown in figures below.
- Finally, I return the sum of residuals for both models.

Results

Figure for regression model:

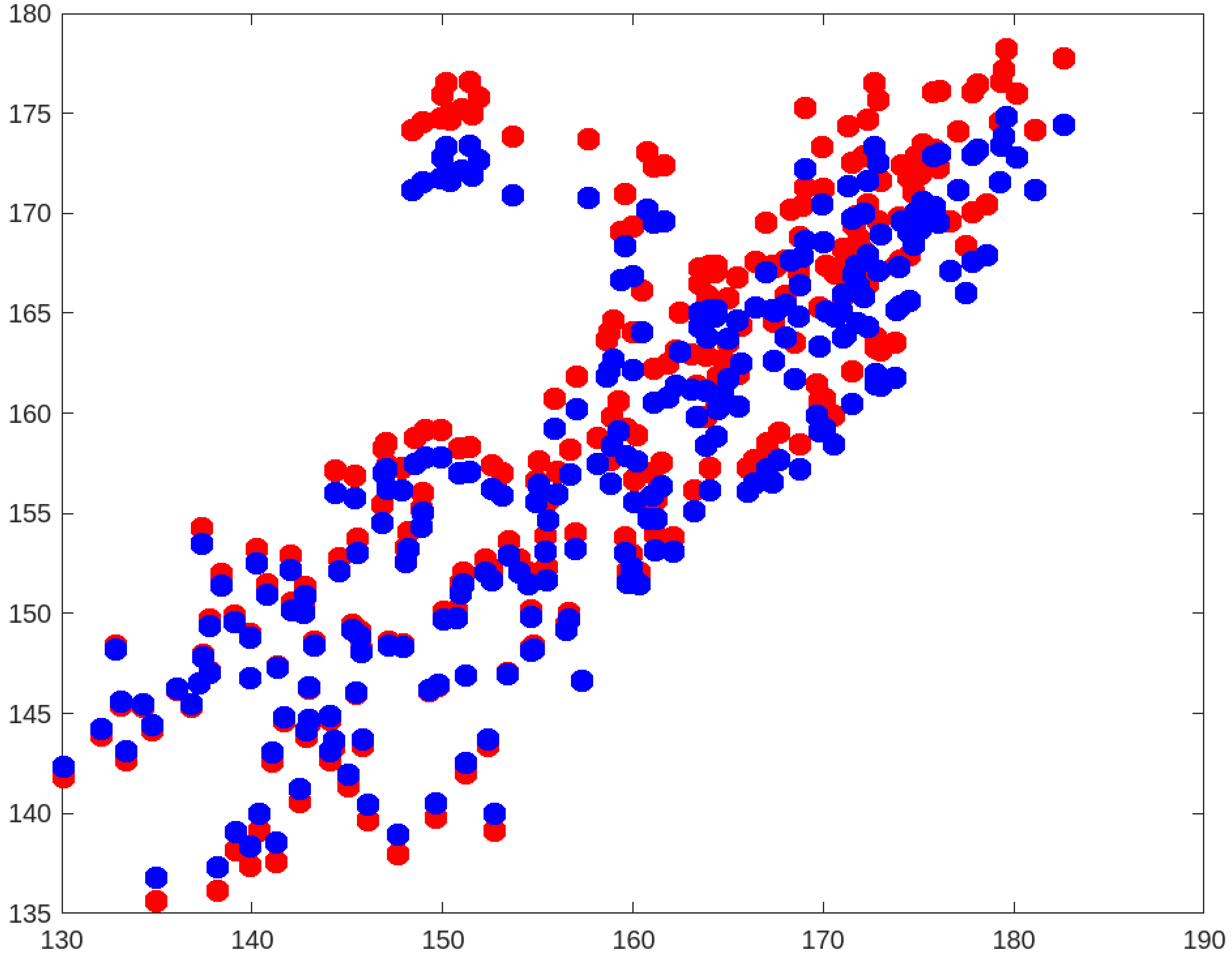
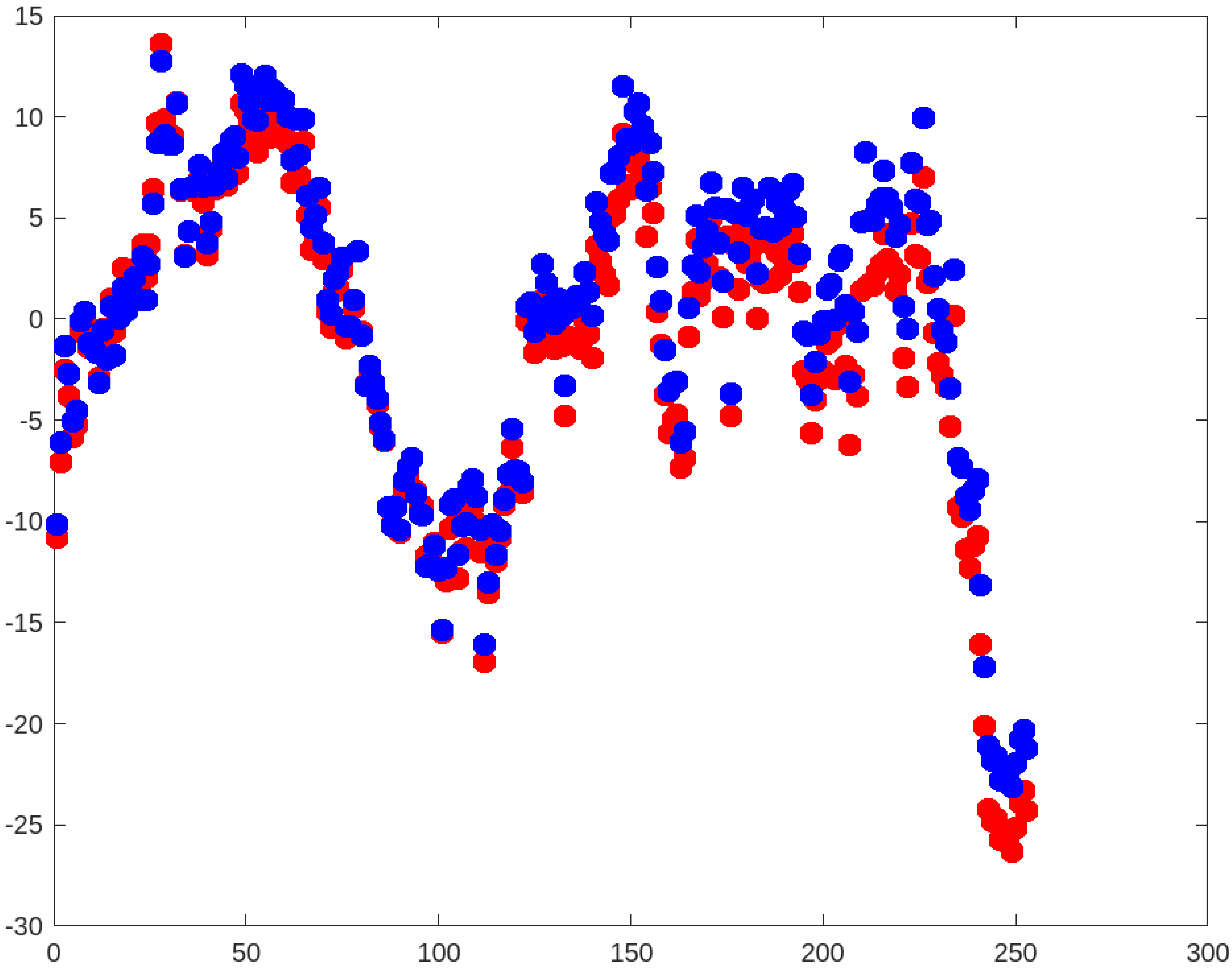


Figure for residual plot:



Sum of residuals (first one is for L^1, second one is for L^2):

```
ans =

   1.5284e+03


ans =

   1.5824e+03
```

Comparison of Results

The sum of L^2 residuals is a little higher than the sum of L^1 residuals. We also discover that L^2 typically guards against severe mistakes. The red dot in the picture above, which reflects L^1 predictions, is visible more frequently in areas farthest from $y = 0$. This is due to the tendency of L^2 regression to punish extreme spots (projections represented in blue dot).

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