Equations for Moller-Plesset Theory of Order 4

PW Borthwick

January 2023

1 Diagram Details

- number of nodes (order) is 4
- number of diagrams in order is 39
- number of node pairs per diagram is 6
- number of connections (lines) per diagram is 8

2 Equations

$$(\mathbf{0\ \ 1\ \ 3\ \ 3\ \ 1\ \ 0}) \rightarrow (\mathbf{0\ \ 0\ \ 2\ \ 2\ \ 0\ \ 0}) \uparrow \qquad (\mathbf{0\ \ 1\ \ 1\ \ 1\ \ 1\ \ 0}) \downarrow$$

$$(-1)^{4+1}\ \ (2)^{-2}\ \ \frac{\langle ab\|ij\rangle\ \langle cd\|kl\rangle\ \langle ik\|cd\rangle\ \langle jl\|ab\rangle}{(\epsilon_i+\epsilon_j-\epsilon_a-\epsilon_b)(\epsilon_i+\epsilon_j+\epsilon_k+\epsilon_l-\epsilon_c-\epsilon_d-\epsilon_a-\epsilon_b)(\epsilon_j+\epsilon_l-\epsilon_a-\epsilon_b)}$$

$$(\mathbf{0\ \ 1\ \ 3\ \ 3\ \ 1\ \ 0}) \rightarrow (\mathbf{0\ \ 1\ \ 1\ \ 1\ \ 1\ \ 0}) \uparrow \qquad (\mathbf{0\ \ 0\ \ 2\ \ 2\ \ 0\ \ 0}) \downarrow$$

$$(-1)^{4+1}\ \ (2)^{-2}\ \ \frac{\langle ab\|ij\rangle\ \langle cd\|kl\rangle\ \langle kl\|ac\rangle\ \langle ij\|bd\rangle}{(\epsilon_i+\epsilon_j-\epsilon_a-\epsilon_b)(\epsilon_i+\epsilon_j+\epsilon_k+\epsilon_l-\epsilon_a-\epsilon_c-\epsilon_b-\epsilon_d)(\epsilon_i+\epsilon_j-\epsilon_b-\epsilon_d)}$$

$$(\mathbf{0\ \ 2\ \ 2\ \ 2\ \ 2\ \ 0}) \rightarrow (\mathbf{0\ \ 0\ \ 2\ \ 2\ \ 0\ \ 0}) \uparrow \qquad (\mathbf{0\ \ 2\ \ 0\ \ 0\ \ 2\ \ 0}) \downarrow$$

$$(-1)^{4+2}\ \ (2)^{-4}\ \ \frac{\langle ab\|ij\rangle\ \langle cd\|kl\rangle\ \langle ij\|cd\rangle\ \langle kl\|ab\rangle}{(\epsilon_i+\epsilon_j-\epsilon_a-\epsilon_b)(\epsilon_i+\epsilon_j+\epsilon_k+\epsilon_l-\epsilon_c-\epsilon_d-\epsilon_a-\epsilon_b)(\epsilon_k+\epsilon_l-\epsilon_a-\epsilon_b)}$$

$$(\mathbf{0\ \ 2\ \ 2\ \ 2\ \ 2\ \ 0}) \rightarrow (\mathbf{0\ \ 1\ \ 1\ \ 1\ \ 1\ \ 0}) \uparrow \qquad (\mathbf{0\ \ 1\ \ 1\ \ 1\ \ 1\ \ 0}) \downarrow$$

$$(-1)^{4+4}\ \ (2)^{0}\ \ \frac{\langle ab\|ij\rangle\ \langle cd\|kl\rangle\ \langle ik\|ac\rangle\ \langle jl\|bd\rangle}{(\epsilon_i+\epsilon_j-\epsilon_a-\epsilon_b)(\epsilon_i+\epsilon_j+\epsilon_k+\epsilon_l-\epsilon_a-\epsilon_c-\epsilon_b-\epsilon_d)(\epsilon_j+\epsilon_l-\epsilon_b-\epsilon_d)}$$

$$(\mathbf{0\ \ 2\ \ 2\ \ 2\ \ 2\ \ 0}) \rightarrow (\mathbf{0\ \ 2\ \ 0\ \ 0\ \ 2\ \ 0}) \uparrow \qquad (\mathbf{0\ \ 0\ \ 2\ \ 2\ \ 0\ \ 0}) \downarrow$$

$$(-1)^{4+2}\ \ (2)^{-4}\ \ \frac{\langle ab\|ij\rangle\ \langle cd\|kl\rangle\ \langle kl\|ab\rangle\ \langle ij\|cd\rangle}{(\epsilon_i+\epsilon_j-\epsilon_a-\epsilon_b)(\epsilon_i+\epsilon_j+\epsilon_k+\epsilon_l-\epsilon_a-\epsilon_b-\epsilon_c-\epsilon_d)(\epsilon_i+\epsilon_j-\epsilon_c-\epsilon_d)}$$

$$(\mathbf{0\ \ 3\ \ 1\ \ 1\ \ 3\ \ 0}) \rightarrow (\mathbf{0\ \ 1\ \ 1\ \ 1\ \ 1\ \ 0}) \uparrow \qquad (\mathbf{0\ \ 2\ \ 0\ \ 0\ \ 2\ \ 0}) \downarrow$$

$$(-1)^{4+3}\ \ (2)^{-2}\ \ \frac{\langle ab\|ij\rangle\ \langle cd\|kl\rangle\ \langle ij\|ac\rangle\ \langle kl\|bd\rangle}{(\epsilon_i+\epsilon_j-\epsilon_a-\epsilon_b)(\epsilon_i+\epsilon_j+\epsilon_k+\epsilon_l-\epsilon_a-\epsilon_c-\epsilon_b-\epsilon_d)(\epsilon_k+\epsilon_l-\epsilon_b-\epsilon_d)}$$

$$(0\ \ 3\ \ 1\ \ 1\ \ 3\ \ 0) \to (0\ \ 2\ \ 0\ \ 0\ \ 2\ \ 0)\uparrow \qquad (0\ \ 1\ \ 1\ \ 1\ \ 1\ \ 0)\downarrow$$

$$(-1)^{4+3}\ \ (2)^{-2}\ \ \frac{\langle ab\|ij\rangle\ \langle cd\|kl\rangle\ \langle ik\|ab\rangle\ \langle jl\|cd\rangle}{(\epsilon_i+\epsilon_j-\epsilon_a-\epsilon_b)(\epsilon_i+\epsilon_j+\epsilon_k+\epsilon_l-\epsilon_a-\epsilon_b-\epsilon_c-\epsilon_d)(\epsilon_j+\epsilon_l-\epsilon_c-\epsilon_d)}$$

$$(1\ \ 0\ \ 3\ \ 3\ \ 0\ \ 1) \to (0\ \ 0\ \ 2\ \ 1\ \ 0\ \ 0)\uparrow \qquad (1\ \ 0\ \ 1\ \ 2\ \ 0\ \ 1)\downarrow$$

$$(-1)^{5+1}\ \ (2)^{-2}\ \ \frac{\langle ab\|ij\rangle\ \langle ic\|kl\rangle\ \langle kl\|cm\rangle\ \langle jm\|ab\rangle}{(\epsilon_i+\epsilon_j-\epsilon_a-\epsilon_b)(\epsilon_j+\epsilon_k+\epsilon_l-\epsilon_c-\epsilon_a-\epsilon_b)(\epsilon_j+\epsilon_m-\epsilon_a-\epsilon_b)}$$

$$(1\ \ 0\ \ 3\ \ 3\ \ 0\ \ 1) \to (1\ \ 0\ \ 1\ \ 2\ \ 0\ \ 1)\uparrow \qquad (0\ \ 0\ \ 2\ \ 1\ \ 0\ \ 0)\downarrow$$

$$(-1)^{3+1}\ \ (2)^{-2}\ \ \frac{\langle ab\|ij\rangle\ \langle cd\|ak\rangle\ \langle ke\|cd\rangle\ \langle ij\|be\rangle}{(\epsilon_i+\epsilon_j-\epsilon_a-\epsilon_b)(\epsilon_i+\epsilon_j+\epsilon_k-\epsilon_c-\epsilon_d-\epsilon_b)(\epsilon_i+\epsilon_j-\epsilon_b-\epsilon_e)}$$

$$(2\ \ 0\ \ 2\ \ 2\ \ 0\ \ 2) \to (0\ \ 0\ \ 2\ \ 0\ \ 0\ \ 0)\uparrow \qquad (2\ \ 0\ \ 0\ \ 2\ \ 0\ \ 2)\downarrow$$

$$(-1)^{6+2}\ \ (2)^{-4}\ \ \frac{\langle ab\|ij\rangle\ \langle ij\|kl\rangle\ \langle kl\|mn\rangle\ \langle mn\|ab\rangle}{(\epsilon_i+\epsilon_j-\epsilon_a-\epsilon_b)(\epsilon_k+\epsilon_l-\epsilon_a-\epsilon_b)(\epsilon_m+\epsilon_n-\epsilon_a-\epsilon_b)}$$

$$(2\ \ 0\ \ 2\ \ 2\ \ 0\ \ 2) \to (1\ \ 0\ \ 1\ \ 1\ \ 0\ \ 1)\uparrow \qquad (1\ \ 0\ \ 1\ \ 1\ \ 0\ \ 1)\downarrow$$

$$(-1)^{4+4}\ \ (2)^{0}\ \ \frac{\langle ab\|ij\rangle\ \langle ic\|ak\rangle\ \langle kd\|cl\rangle\ \langle jl\|bd\rangle}{(\epsilon_i+\epsilon_j-\epsilon_a-\epsilon_b)(\epsilon_j+\epsilon_k-\epsilon_c-\epsilon_b)(\epsilon_j+\epsilon_l-\epsilon_b-\epsilon_d)}$$

$$(2\ \ 0\ \ 2\ \ 2\ \ 0\ \ 2) \to (2\ \ 0\ \ 0\ \ 2\ \ 0\ \ 2)\uparrow \qquad (0\ \ 0\ \ 2\ \ 0\ \ 0\ \ 0)\downarrow$$

$$(-1)^{2+2}\ \ (2)^{-4}\ \ \frac{\langle ab\|ij\rangle\ \langle cd\|ab\rangle\ \langle ef\|cd\rangle\ \langle ij\|ef\rangle}{(\epsilon_i+\epsilon_j-\epsilon_a-\epsilon_b)(\epsilon_i+\epsilon_j-\epsilon_c-\epsilon_d)(\epsilon_i+\epsilon_j-\epsilon_e-\epsilon_f)}$$

$$(3\ \ 0\ \ 1\ \ 1\ \ 0\ \ 3) \to (1\ \ 0\ \ 1\ \ 0\ \ 0\ \ 1)\uparrow \qquad (2\ \ 0\ \ 0\ \ 1\ \ 0\ \ 2)\downarrow$$

$$(-1)^{5+3}\ \ (2)^{-2}\ \ \frac{\langle ab\|ij\rangle\ \langle ij\|ak\rangle\ \langle kc\|lm\rangle\ \langle lm\|bc\rangle}{(\epsilon_i+\epsilon_j-\epsilon_a-\epsilon_b)(\epsilon_k-\epsilon_b)(\epsilon_l+\epsilon_m-\epsilon_b-\epsilon_c)}$$

$$(3\ \ 0\ \ 1\ \ 1\ \ 0\ \ 3) \to (2\ \ 0\ \ 0\ \ 1\ \ 0\ \ 2)\uparrow \qquad (1\ \ 0\ \ 1\ \ 0\ \ 0\ \ 1)\downarrow$$

$$(-1)^{3+3}\ \ (2)^{-2}\ \ \frac{\langle ab\|ij\rangle\ \langle ic\|ab\rangle\ \langle de\|ck\rangle\ \langle jk\|de\rangle}{(\epsilon_i+\epsilon_j-\epsilon_a-\epsilon_b)(\epsilon_j-\epsilon_c)(\epsilon_j+\epsilon_k-\epsilon_d-\epsilon_e)}$$

$$(1\ \ 3\ \ 0\ \ 0\ \ 3\ \ 1) \to (0\ \ 2\ \ 0\ \ 0\ \ 1\ \ 1)\uparrow \qquad (1\ \ 1\ \ 0\ \ 0\ \ 2\ \ 0)\downarrow$$

$$(-1)^{4+1}\ \ (2)^{-2}\ \ \frac{\langle ab\|ij\rangle\ \langle ic\|kl\rangle\ \langle jd\|ab\rangle\ \langle kl\|cd\rangle}{(\epsilon_i+\epsilon_j-\epsilon_a-\epsilon_b)(\epsilon_j+\epsilon_k+\epsilon_l-\epsilon_a-\epsilon_b-\epsilon_c)(\epsilon_k+\epsilon_l-\epsilon_c-\epsilon_d)}$$

$$(1\ \ 3\ \ 0\ \ 0\ \ 3\ \ 1) \to (1\ \ 1\ \ 0\ \ 0\ \ 2\ \ 0)\uparrow \qquad (0\ \ 2\ \ 0\ \ 0\ \ 1\ \ 1)\downarrow$$

$$(-1)^{4+1}\ \ (2)^{-2}\ \ \frac{\langle ab\|ij\rangle\ \langle cd\|ak\rangle\ \langle ij\|bl\rangle\ \langle kl\|cd\rangle}{(\epsilon_i+\epsilon_j-\epsilon_a-\epsilon_b)(\epsilon_i+\epsilon_j+\epsilon_k-\epsilon_b-\epsilon_c-\epsilon_d)(\epsilon_k+\epsilon_l-\epsilon_c-\epsilon_d)}$$

$$(\mathbf{2\ \ 2\ \ 0\ \ 0\ \ 2\ \ 2}) \rightarrow (\mathbf{0\ \ 2\ \ 0\ \ 0\ \ 0\ \ 2}) \uparrow \qquad (\mathbf{2\ \ 0\ \ 0\ \ 0\ \ 2\ \ 0}) \downarrow$$

$$(-1)^{4+2}\ \ (2)^{-4}\ \ \frac{\langle ab\|ij\rangle\ \langle ij\|kl\rangle\ \langle cd\|ab\rangle\ \langle kl\|cd\rangle}{(\epsilon_i+\epsilon_j-\epsilon_a-\epsilon_b)(\epsilon_k+\epsilon_l-\epsilon_a-\epsilon_b)(\epsilon_k+\epsilon_l-\epsilon_c-\epsilon_d)}$$

$$(\mathbf{2\ \ 2\ \ 0\ \ 0\ \ 2\ \ 2}) \rightarrow (\mathbf{1\ \ 1\ \ 0\ \ 0\ \ 1\ \ 1}) \uparrow \qquad (\mathbf{1\ \ 1\ \ 0\ \ 0\ \ 1\ \ 1}) \downarrow$$

$$(-1)^{4+4}\ \ (2)^{0}\ \ \frac{\langle ab\|ij\rangle\ \langle ic\|ak\rangle\ \langle jd\|bl\rangle\ \langle kl\|cd\rangle}{(\epsilon_i+\epsilon_j-\epsilon_a-\epsilon_b)(\epsilon_j+\epsilon_k-\epsilon_b-\epsilon_c)(\epsilon_k+\epsilon_l-\epsilon_c-\epsilon_d)}$$

$$(\mathbf{2\ \ 2\ \ 0\ \ 0\ \ 2\ \ 2}) \rightarrow (\mathbf{2\ \ 0\ \ 0\ \ 0\ \ 2\ \ 0}) \uparrow \qquad (\mathbf{0\ \ 2\ \ 0\ \ 0\ \ 0\ \ 2}) \downarrow$$

$$(-1)^{4+2}\ \ (2)^{-4}\ \ \frac{\langle ab\|ij\rangle\ \langle cd\|ab\rangle\ \langle ij\|kl\rangle\ \langle kl\|cd\rangle}{(\epsilon_i+\epsilon_j-\epsilon_a-\epsilon_b)(\epsilon_i+\epsilon_j-\epsilon_c-\epsilon_d)(\epsilon_k+\epsilon_l-\epsilon_c-\epsilon_d)}$$

$$(\mathbf{3\ \ 1\ \ 0\ \ 0\ \ 1\ \ 3}) \rightarrow (\mathbf{1\ \ 1\ \ 0\ \ 0\ \ 0\ \ 2}) \uparrow \qquad (\mathbf{2\ \ 0\ \ 0\ \ 0\ \ 1\ \ 1}) \downarrow$$

$$(-1)^{4+3}\ \ (2)^{-2}\ \ \frac{\langle ab\|ij\rangle\ \langle ij\|ak\rangle\ \langle cd\|bl\rangle\ \langle kl\|cd\rangle}{(\epsilon_i+\epsilon_j-\epsilon_a-\epsilon_b)(\epsilon_k-\epsilon_b)(\epsilon_k+\epsilon_l-\epsilon_c-\epsilon_d)}$$

$$(\mathbf{3\ \ 1\ \ 0\ \ 0\ \ 1\ \ 3}) \rightarrow (\mathbf{2\ \ 0\ \ 0\ \ 0\ \ 1\ \ 1}) \uparrow \qquad (\mathbf{1\ \ 1\ \ 0\ \ 0\ \ 0\ \ 2}) \downarrow$$

$$(-1)^{4+3}\ \ (2)^{-2}\ \ \frac{\langle ab\|ij\rangle\ \langle ic\|ab\rangle\ \langle jd\|kl\rangle\ \langle kl\|cd\rangle}{(\epsilon_i+\epsilon_j-\epsilon_a-\epsilon_b)(\epsilon_j-\epsilon_c)(\epsilon_k+\epsilon_l-\epsilon_c-\epsilon_d)}$$

$$(\mathbf{1\ \ 1\ \ 2\ \ 2\ \ 1\ \ 1}) \rightarrow (\mathbf{0\ \ 0\ \ 2\ \ 1\ \ 0\ \ 0}) \uparrow \qquad (\mathbf{1\ \ 1\ \ 0\ \ 1\ \ 1\ \ 1}) \downarrow$$

$$(-1)^{5+1}\ \ (2)^{-1}\ \ \frac{\langle ab\|ij\rangle\ \langle ic\|kl\rangle\ \langle jk\|cm\rangle\ \langle lm\|ab\rangle}{(\epsilon_i+\epsilon_j-\epsilon_a-\epsilon_b)(\epsilon_j+\epsilon_k+\epsilon_l-\epsilon_c-\epsilon_a-\epsilon_b)(\epsilon_l+\epsilon_m-\epsilon_a-\epsilon_b)}$$

$$(\mathbf{1\ \ 1\ \ 2\ \ 2\ \ 1\ \ 1}) \rightarrow (\mathbf{0\ \ 1\ \ 1\ \ 0\ \ 1\ \ 0}) \uparrow \qquad (\mathbf{1\ \ 0\ \ 1\ \ 2\ \ 0\ \ 1}) \downarrow$$

$$(-1)^{5+3}\ \ (2)^{-1}\ \ \frac{\langle ab\|ij\rangle\ \langle ic\|kl\rangle\ \langle kl\|am\rangle\ \langle jm\|bc\rangle}{(\epsilon_i+\epsilon_j-\epsilon_a-\epsilon_b)(\epsilon_j+\epsilon_k+\epsilon_l-\epsilon_a-\epsilon_b-\epsilon_c)(\epsilon_j+\epsilon_m-\epsilon_b-\epsilon_c)}$$

$$(\mathbf{1\ \ 1\ \ 2\ \ 2\ \ 1\ \ 1}) \rightarrow (\mathbf{0\ \ 1\ \ 1\ \ 1\ \ 0\ \ 1}) \uparrow \qquad (\mathbf{1\ \ 0\ \ 1\ \ 1\ \ 1\ \ 0}) \downarrow$$

$$(-1)^{4+3}\ \ (2)^{0}\ \ \frac{\langle ab\|ij\rangle\ \langle ic\|kl\rangle\ \langle kd\|ac\rangle\ \langle jl\|bd\rangle}{(\epsilon_i+\epsilon_j-\epsilon_a-\epsilon_b)(\epsilon_j+\epsilon_k+\epsilon_l-\epsilon_a-\epsilon_c-\epsilon_b)(\epsilon_j+\epsilon_l-\epsilon_b-\epsilon_d)}$$

$$(\mathbf{1\ \ 1\ \ 2\ \ 2\ \ 1\ \ 1}) \rightarrow (\mathbf{1\ \ 0\ \ 1\ \ 1\ \ 1\ \ 0}) \uparrow \qquad (\mathbf{0\ \ 1\ \ 1\ \ 1\ \ 0\ \ 1}) \downarrow$$

$$(-1)^{4+3}\ \ (2)^{0}\ \ \frac{\langle ab\|ij\rangle\ \langle cd\|ak\rangle\ \langle ik\|cl\rangle\ \langle jl\|bd\rangle}{(\epsilon_i+\epsilon_j-\epsilon_a-\epsilon_b)(\epsilon_i+\epsilon_j+\epsilon_k-\epsilon_c-\epsilon_b-\epsilon_d)(\epsilon_j+\epsilon_l-\epsilon_b-\epsilon_d)}$$

$$(\mathbf{1\ \ 1\ \ 2\ \ 2\ \ 1\ \ 1}) \rightarrow (\mathbf{1\ \ 0\ \ 1\ \ 2\ \ 0\ \ 1}) \uparrow \qquad (\mathbf{0\ \ 1\ \ 1\ \ 0\ \ 1\ \ 0}) \downarrow$$

$$(-1)^{3+3}\ \ (2)^{-1}\ \ \frac{\langle ab\|ij\rangle\ \langle cd\|ak\rangle\ \langle ie\|cd\rangle\ \langle jk\|be\rangle}{(\epsilon_i+\epsilon_j-\epsilon_a-\epsilon_b)(\epsilon_i+\epsilon_j+\epsilon_k-\epsilon_c-\epsilon_d-\epsilon_b)(\epsilon_j+\epsilon_k-\epsilon_b-\epsilon_e)}$$

$$(\mathbf{1\ \ 1\ \ 2\ \ 2\ \ 1\ \ 1}) \rightarrow (\mathbf{1\ \ 1\ \ 0\ \ 1\ \ 1\ \ 1}) \uparrow \qquad (\mathbf{0\ \ 0\ \ 2\ \ 1\ \ 0\ \ 0}) \downarrow$$

$$(-1)^{3+1}\ \ (2)^{-1}\ \ \frac{\langle ab\|ij\rangle\ \langle cd\|ak\rangle\ \langle ke\|bc\rangle\ \langle ij\|de\rangle}{(\epsilon_i+\epsilon_j-\epsilon_a-\epsilon_b)(\epsilon_i+\epsilon_j+\epsilon_k-\epsilon_b-\epsilon_c-\epsilon_d)(\epsilon_i+\epsilon_j-\epsilon_d-\epsilon_e)}$$

$$(\mathbf{1\ \ 2\ \ 1\ \ 1\ \ 2\ \ 1}) \rightarrow (\mathbf{0\ \ 1\ \ 1\ \ 0\ \ 1\ \ 0}) \uparrow \qquad (\mathbf{1\ \ 1\ \ 0\ \ 1\ \ 1\ \ 1}) \downarrow$$

$$(-1)^{5+1}\ \ (2)^{0}\ \ \frac{\langle ab\|ij\rangle\ \langle ic\|kl\rangle\ \langle jk\|am\rangle\ \langle lm\|bc\rangle}{(\epsilon_i+\epsilon_j-\epsilon_a-\epsilon_b)(\epsilon_j+\epsilon_k+\epsilon_l-\epsilon_a-\epsilon_b-\epsilon_c)(\epsilon_l+\epsilon_m-\epsilon_b-\epsilon_c)}$$

$$(\mathbf{1\ \ 2\ \ 1\ \ 1\ \ 2\ \ 1}) \rightarrow (\mathbf{0\ \ 1\ \ 1\ \ 1\ \ 0\ \ 1}) \uparrow \qquad (\mathbf{1\ \ 1\ \ 0\ \ 0\ \ 2\ \ 0}) \downarrow$$

$$(-1)^{4+2}\ \ (2)^{-1}\ \ \frac{\langle ab\|ij\rangle\ \langle ic\|kl\rangle\ \langle jd\|ac\rangle\ \langle kl\|bd\rangle}{(\epsilon_i+\epsilon_j-\epsilon_a-\epsilon_b)(\epsilon_j+\epsilon_k+\epsilon_l-\epsilon_a-\epsilon_c-\epsilon_b)(\epsilon_k+\epsilon_l-\epsilon_b-\epsilon_d)}$$

$$(\mathbf{1\ \ 2\ \ 1\ \ 1\ \ 2\ \ 1}) \rightarrow (\mathbf{0\ \ 2\ \ 0\ \ 0\ \ 1\ \ 1}) \uparrow \qquad (\mathbf{1\ \ 0\ \ 1\ \ 1\ \ 1\ \ 0}) \downarrow$$

$$(-1)^{4+2}\ \ (2)^{-1}\ \ \frac{\langle ab\|ij\rangle\ \langle ic\|kl\rangle\ \langle kd\|ab\rangle\ \langle jl\|cd\rangle}{(\epsilon_i+\epsilon_j-\epsilon_a-\epsilon_b)(\epsilon_j+\epsilon_k+\epsilon_l-\epsilon_a-\epsilon_b-\epsilon_c)(\epsilon_j+\epsilon_l-\epsilon_c-\epsilon_d)}$$

$$(\mathbf{1\ \ 2\ \ 1\ \ 1\ \ 2\ \ 1}) \rightarrow (\mathbf{1\ \ 0\ \ 1\ \ 1\ \ 1\ \ 0}) \uparrow \qquad (\mathbf{0\ \ 2\ \ 0\ \ 0\ \ 1\ \ 1}) \downarrow$$

$$(-1)^{4+2}\ \ (2)^{-1}\ \ \frac{\langle ab\|ij\rangle\ \langle cd\|ak\rangle\ \langle ij\|cl\rangle\ \langle kl\|bd\rangle}{(\epsilon_i+\epsilon_j-\epsilon_a-\epsilon_b)(\epsilon_i+\epsilon_j+\epsilon_k-\epsilon_c-\epsilon_b-\epsilon_d)(\epsilon_k+\epsilon_l-\epsilon_b-\epsilon_d)}$$

$$(\mathbf{1\ \ 2\ \ 1\ \ 1\ \ 2\ \ 1}) \rightarrow (\mathbf{1\ \ 1\ \ 0\ \ 0\ \ 2\ \ 0}) \uparrow \qquad (\mathbf{0\ \ 1\ \ 1\ \ 1\ \ 0\ \ 1}) \downarrow$$

$$(-1)^{4+2}\ \ (2)^{-1}\ \ \frac{\langle ab\|ij\rangle\ \langle cd\|ak\rangle\ \langle ik\|bl\rangle\ \langle jl\|cd\rangle}{(\epsilon_i+\epsilon_j-\epsilon_a-\epsilon_b)(\epsilon_i+\epsilon_j+\epsilon_k-\epsilon_b-\epsilon_c-\epsilon_d)(\epsilon_j+\epsilon_l-\epsilon_c-\epsilon_d)}$$

$$(\mathbf{1\ \ 2\ \ 1\ \ 1\ \ 2\ \ 1}) \rightarrow (\mathbf{1\ \ 1\ \ 0\ \ 1\ \ 1\ \ 1}) \uparrow \qquad (\mathbf{0\ \ 1\ \ 1\ \ 0\ \ 1\ \ 0}) \downarrow$$

$$(-1)^{3+1}\ \ (2)^{0}\ \ \frac{\langle ab\|ij\rangle\ \langle cd\|ak\rangle\ \langle ie\|bc\rangle\ \langle jk\|de\rangle}{(\epsilon_i+\epsilon_j-\epsilon_a-\epsilon_b)(\epsilon_i+\epsilon_j+\epsilon_k-\epsilon_b-\epsilon_c-\epsilon_d)(\epsilon_j+\epsilon_k-\epsilon_d-\epsilon_e)}$$

$$(\mathbf{2\ \ 1\ \ 1\ \ 1\ \ 1\ \ 2}) \rightarrow (\mathbf{0\ \ 1\ \ 1\ \ 0\ \ 0\ \ 1}) \uparrow \qquad (\mathbf{2\ \ 0\ \ 0\ \ 1\ \ 1\ \ 1}) \downarrow$$

$$(-1)^{5+3}\ \ (2)^{-1}\ \ \frac{\langle ab\|ij\rangle\ \langle ij\|kl\rangle\ \langle kc\|am\rangle\ \langle lm\|bc\rangle}{(\epsilon_i+\epsilon_j-\epsilon_a-\epsilon_b)(\epsilon_k+\epsilon_l-\epsilon_a-\epsilon_b)(\epsilon_l+\epsilon_m-\epsilon_b-\epsilon_c)}$$

$$(\mathbf{2\ \ 1\ \ 1\ \ 1\ \ 1\ \ 2}) \rightarrow (\mathbf{1\ \ 0\ \ 1\ \ 0\ \ 1\ \ 0}) \uparrow \qquad (\mathbf{1\ \ 1\ \ 0\ \ 1\ \ 0\ \ 2}) \downarrow$$

$$(-1)^{5+3}\ \ (2)^{-1}\ \ \frac{\langle ab\|ij\rangle\ \langle ic\|ak\rangle\ \langle jk\|lm\rangle\ \langle lm\|bc\rangle}{(\epsilon_i+\epsilon_j-\epsilon_a-\epsilon_b)(\epsilon_j+\epsilon_k-\epsilon_b-\epsilon_c)(\epsilon_l+\epsilon_m-\epsilon_b-\epsilon_c)}$$

$$(\mathbf{2\ \ 1\ \ 1\ \ 1\ \ 1\ \ 2}) \rightarrow (\mathbf{1\ \ 0\ \ 1\ \ 1\ \ 0\ \ 1}) \uparrow \qquad (\mathbf{1\ \ 1\ \ 0\ \ 0\ \ 1\ \ 1}) \downarrow$$

$$(-1)^{4+3}\ \ (2)^{0}\ \ \frac{\langle ab\|ij\rangle\ \langle ic\|ak\rangle\ \langle jd\|cl\rangle\ \langle kl\|bd\rangle}{(\epsilon_i+\epsilon_j-\epsilon_a-\epsilon_b)(\epsilon_j+\epsilon_k-\epsilon_c-\epsilon_b)(\epsilon_k+\epsilon_l-\epsilon_b-\epsilon_d)}$$

$$\begin{pmatrix} \mathbf{2} & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{2} \end{pmatrix} \rightarrow \begin{pmatrix} \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{1} \end{pmatrix} \uparrow \qquad \begin{pmatrix} \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{1} \end{pmatrix} \downarrow$$

$$(-1)^{4+3} \quad (2)^{0} \quad \frac{\langle ab \| ij \rangle \ \langle ic \| ak \rangle \ \langle kd \| bl \rangle \ \langle jl \| cd \rangle}{(\epsilon_i + \epsilon_j - \epsilon_a - \epsilon_b)(\epsilon_j + \epsilon_k - \epsilon_b - \epsilon_c)(\epsilon_j + \epsilon_l - \epsilon_c - \epsilon_d)}$$

$$\begin{pmatrix} \mathbf{2} & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{2} \end{pmatrix} \rightarrow \begin{pmatrix} \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{2} \end{pmatrix} \uparrow \qquad \begin{pmatrix} \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} \end{pmatrix} \downarrow$$

$$(-1)^{3+3} \quad (2)^{-1} \quad \frac{\langle ab \| ij \rangle \ \langle ic \| ak \rangle \ \langle de \| bc \rangle \ \langle jk \| de \rangle}{(\epsilon_i + \epsilon_j - \epsilon_a - \epsilon_b)(\epsilon_j + \epsilon_k - \epsilon_b - \epsilon_c)(\epsilon_j + \epsilon_k - \epsilon_d - \epsilon_e)}$$

$$\begin{pmatrix} \mathbf{2} & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{2} \end{pmatrix} \rightarrow \begin{pmatrix} \mathbf{2} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{1} \end{pmatrix} \uparrow \qquad \begin{pmatrix} \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{pmatrix} \downarrow$$

$$(-1)^{3+3} \quad (2)^{-1} \quad \frac{\langle ab \| ij \rangle \ \langle cd \| ab \rangle \ \langle ie \| ck \rangle \ \langle jk \| de \rangle}{(\epsilon_i + \epsilon_j - \epsilon_a - \epsilon_b)(\epsilon_i + \epsilon_j - \epsilon_c - \epsilon_d)(\epsilon_j + \epsilon_k - \epsilon_d - \epsilon_e)}$$