Multiplicative Marketing Response Function

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Section 1

There are two general types of equations that are used for Marketing Mix analysis. These are additive and multiplicative. Additive models multiply a coefficient with each marketing variable and then add up the terms.

$$Sales = const \ \& \ other \ terms + \sum(\beta_i * M_i) \ , \quad (1)$$

where, *i*, is an index over the coefficients, β, and the marketing variables, M.

Multiplicative models raise each marketing variable by a coefficient and multiply everything together.

$$Sales = (const \ \& \ other \ terms) \ \prod({M_i}^{\alpha_i}) \ , \quad (2)$$

Where, α, is the coefficient. The capital pi, Π, is like the capital sigma, Σ, but for multiplication. Capital sigma means that all the terms are added while capital pi means that all the terms are multiplied.

Typically, with both the additive and multiplicative models, the marketing variable are transformed in some way. Transformations almost always include adstocking to account for lagged impacts. Since the additive model as presented does not incorporate any diminishing return, this concept is introduced through transformations. One way to do this is with something called a sigmoid function which gives an S shaped response curve.

The multiplicative model structurally does incorporate diminishing return and it is often used as presented above but with adstocking.

Section 2

Fitting the multiplicative model is done by transforming the equation with the logarithm function.

$$\log(sales) = \log\left((const \ \& \ other \ terms) \prod({M_i}^{\alpha_i})\right) \quad (3)$$

$$\log(sales) = \log\left((const \; \& \; other \; terms) + \sum(M_i^{\alpha_i}\right) \quad (4)$$

$$\log(sales) = \log\left((const \; \& \; other \; terms) + \sum(\alpha_i * \log(\mathrm{M}_i)\right) \quad (5)$$

In this form, the multiplicative model looks just like the additive model except that everything is transformed by the logarithm. After the transformation the equation is linear and can be fit with linear regression or other techniques for fitting linear equations.

Section 3

The coefficients of the multiplicative model are marketing elasticities. In economics, the concept of elasticity is a common way to understand the relationship between two variables since it is not dependent on level. It is defined as the percentage change in one variable to the percentage change in another. Here, marketing elasticities measure the percentage change in sales to the percentage change in marketing investment. Typical values for marketing elasticities are around 2% to 20%.

While marketing elasticies are meaningful in the range of normal spend, problems emerge at the extremes, for both very high and very low levels of spend. On the low end, the multiplicative model predicts zero sales if any marketing channel in the model has a spend of zero. This is huge problem for modeling since few if any marketing channels are always on and at reasonably high levels. The multiplicative model also suggests increased sales at any arbitrary high level of spend. This is unrealistic since at some level, increased marketing ceases to be effective.

To address these two concerns, we need an expression for elasticity that is zero when marketing spend is zero and is head back to zero for high levels of spend. In between, it is positive.

The equation for elasticity we use is,

$$marketing \; elaticity = \gamma * \frac{x}{x+c} * \frac{m}{m+x} \;, \quad (6)$$

where x is the spend for the marketing channel, gamma, γ, is a regression coefficient, m and c are parameters. The second factor, $\frac{x}{x+c}$, makes the equation zero when x is zero, and the third factor, $\frac{m}{m+x}$, brings the equation to zero as spend increases.

Integrating this elasticity equation to an equation for sales gives,

$$Sales = (const \; \& \; other \; terms) \; \prod(T_i^{\alpha_i}) \;,$$

or equivalently,

$$\log(sales) = \log\left((const \;\&\; other\ terms) + \sum(\alpha_i * \log(\mathrm{T}_i)\right) \quad , \qquad (7)$$

Where, $T_i = \frac{\left(1+\frac{x_i}{c_i}\right)}{\left(1+\frac{x_i}{m_i}\right)}$.

The parameters c and m are specific to each marketing variable and for each geography.