PW Measurement

Our Multiplicative Marketing Mix Response Function

Phil Weissman
phil@pwmeasurement.com

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Section 1

There are two general types of equations that are used for Marketing Mix analysis, additive and multiplicative. Additive models multiply a coefficient with each marketing variable and then add up the terms.

$$Sales = const \ \& \ other \ terms + \sum(\beta_i * x_i) \ , \quad (1)$$

Multiplicative models raise each marketing variable by a coefficient and multiply everything together.

$$Sales = (const \ \& \ other \ terms) * \prod({x_i}^{\alpha_i}) \ , \quad (2)$$

Where the α_i terms are the coefficients.

Typically, with both the additive and multiplicative models, the marketing variable are transformed in some way. Transformations almost always include adstocking to account for lagged media impacts and leading holiday impacts. Since the additive model as presented does not incorporate any diminishing return, diminishing returns is introduced through transformations.

The multiplicative model structurally incorporates diminishing return and it has been the primary engine of some marketing consulting firms as presented above, but with adstocking.

Section 2

Fitting the multiplicative model is done by transforming the equation with the logarithm function.

$$\log(sales) = \log\left((const \ \& \ other \ terms) \prod({x_i}^{\alpha_i})\right) \quad (3)$$

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$$\log(sales) = \log\left((const \ \& \ other \ terms) + \sum(x_i^{\alpha_i}\right) \quad (4)$$

$$\log(sales) = \log\left((const \ \& \ other \ terms) + \sum(\alpha_i * \log(\mathrm{x}_i)\right) \quad (5)$$

In this form, with all the variables logged, the multiplicative model is additive and can be fit with linear techniques.

Section 3

The coefficients of the multiplicative model as presented above are constant marketing elasticities for all levels of spend. An elasticity expresses the percentage change of one variable relative to another. If we work out the partial derivative of (2) and transform it into and elasticity, we find,

$$\frac{\partial Sales}{\partial x_i} * \frac{x_i}{Sales} = Elasticity \ of \ i = \alpha_i \ . \quad (6)$$

While marketing elasticies are meaningful in the range of normal spend and for non-granular analysis, problems emerge at the extremes, for both very high and very low levels of spend. On the low end, the multiplicative model predicts zero sales if any marketing channel in the model has a spend of zero. Transformed into logs as presented by equation (5), the problem is even more significant since log(0) is undefined.

This is a big problem for marketing mix since few if any marketing channels are always on and at reasonably high levels. Additionally, the multiplicative model also suggests increased sales at any arbitrary high level of spend. This is unrealistic since at some level, increased marketing must cease to be effective.

A primary function of marketing mix is optimization. A model that behaves reasonably at high and low levels of spend is of vital importance. To address this concern, we need an expression for elasticity that is zero when marketing spend is zero and goes back to zero for high levels of spend. In between, it is positive.

Section 4

At PW Measurement, we use a form of the multiplicative model transformed to address the problems of elasticity at 0 and at high levels of spend. Development of this model starts with an expression for elasticity,

$$marketing \ elaticity = \gamma * \frac{x}{x+C} * \frac{M}{M+x} \ , \quad (7)$$

where x is the spend for the marketing channel, gamma, γ, is a regression coefficient, m and c are hyperparameters. The second factor, $\frac{x}{x+C}$, makes the equation zero when x is zero, and the third factor, $\frac{M}{M+x}$, brings the equation to zero as spend increases to high levels.

Integrating this elasticity into an equation for sales yields,

$$Sales = (const \; \& \; other \; terms) \prod (T_i{}^{\alpha_i}), \quad (8)$$

or equivalently,

$$\log(sales) = \log\left((const \; \& \; other \; terms) + \sum(\alpha_i * \log(T_i)\right) \quad , \quad (9)$$

Where, $T_i = \frac{\left(1+\frac{x_i}{c_i}\right)}{\left(1+\frac{x_i}{m_i}\right)}$.

Note: As written, Eq (8) and (9) require a small change to Eq (7).

The hyperparameters C and M are specific to each marketing variable and for each geography. The *C* hyperparameter influences how quickly the elasticity increases from 0 while M influences how quickly the elasticity moves back down towards zero. The max elasticity is found at, x = sqrt(M*C).

Section 5

To find the hyperparameters, we first find a somewhat arbitrary average value, $\bar{x}$, for each marketing variable *x*. *M* and *C* are then defined relative to this average,

$$C = c * \bar{x} \quad (10)$$
$$M = m * \bar{x} \quad ,$$

Note that the scaled hyperparemters are lower case.

The original hyperpareters, *C* and *M*, are in the units of the media variables x, while the scaled versions, *c* and *m*, are unitless and are multiplied by the average value of *x*.

The hyperparameter that controls the lower-spend side of the function is, *C,* and it is less than, $\bar{x}$. Lower case, *c,* tends to be in the range [0.1, 0.8].

The hyperparameter which controls the higher-spend side of the function is, *M,* and M is greater than, $\bar{x}$. Lower case, *m,* tends to be in the range [10, 80].

Specific values for *c* and *m* are found through a grid search.

Section 6

Modeling at a granular level in terms of time and geography often result in few outcome events per period. When this is the case, it's best to use a distribution of count such Poisson, quasi-Poisson, or negative binomial. This prevents problems with negative estimates and allows for zero response. When using a count distribution, the response variable needs to be a "count" of something. It cannot be dollar sales. Instead of sales, variables such as units sold, or leads are appropriate. While this is sometimes a better choice of response variable regardless of response distribution, a count response variable isn't always easy or appropriate to find. For a paint retailer, we didn't use a "count" model but we wanted a "count" response variable, deciding to use gallons of paint sold.

In Eq (9), if we replace Sales with Leads, we can then use a count distribution. Leveraging the fact that the expectation of Leads is equal to the expectation of λ, the count parameter,

$$\log(\lambda) = \log\left(\left(const \,\&\, other\ terms\right) + \sum(\alpha_i * \log(\mathrm{T}_i)\right) \quad . \quad (11)$$

Eq (11) is in standard form for Poisson regression since all packages use the logarithm for the link function, the functions that connects the regression equation to the parameter.

Eq (11) powers PW Measurement's marketing mix analysis.