Relationship to Featherstone's ABA (when $\dot{q}=0$, $a_{grav}=0$)

$F_i \tau = p_i^A$ ~ Articulated Bias Force

$P_i \tau = a_i$ ~ Spatial acceleration

Algorithm 1 - Pseudo code of the algorithm to directly compute the inverse of the joint space inertia matrix and which is inspired from ABA exposed by Featherstone [4, p. 132] and follows the same notations.

1 *First forward pass:*

2 **for** $i = 1$ **to** N_B **do**

3 $\quad [\boldsymbol{X}_J, \boldsymbol{S}_i] = \text{jcalc}(\text{jtype}(i), \boldsymbol{q}_i, \dot{\boldsymbol{q}}_i)$

4 $\quad {}^i\boldsymbol{X}_{\lambda(i)} = \boldsymbol{X}_J\, \boldsymbol{X}_T(i)$

5 $\quad \boldsymbol{I}_i^A = \boldsymbol{I}_i$

6 **end**

7 *Backward pass:*

8 **for** $i = N_B$ **to** 1 **do**

9 $\quad \boldsymbol{U}_i = \boldsymbol{I}_i^A \boldsymbol{S}_i$

10 $\quad \boldsymbol{D}_i = \boldsymbol{S}_i^T \boldsymbol{U}_i$

11 $\quad M_{\text{inv}}[i, i] = \boldsymbol{D}_i^{-1}$

12 $\quad M_{\text{inv}}[i, \text{subtree}(i)] = M_{\text{inv}}[i, \text{subtree}(i)] - \boldsymbol{D}_i^{-T} \boldsymbol{S}_i^T \boldsymbol{F}_i[:, \text{subtree}(i)]$

13 $\quad$ **if** $\lambda(i) \neq 0$ **then**

14 $\quad\quad \cancel{\boldsymbol{F}_{\lambda(i)}[:, \text{subtree}(i)] = \boldsymbol{F}_{\lambda(i)}[:, \text{subtree}(i)] + {}^{\lambda(i)}\boldsymbol{X}_i^* \boldsymbol{U}_i M_{\text{inv}}[i, \text{subtree}(i)]}$

15 $\quad\quad \boldsymbol{I}_i^a = \boldsymbol{I}_i^A - \boldsymbol{U}_i \boldsymbol{D}_i^{-1} \boldsymbol{U}_i^T$

16 $\quad\quad \boldsymbol{I}_{\lambda(i)}^A = \boldsymbol{I}_{\lambda(i)}^A + {}^{\lambda(i)}\boldsymbol{X}_i^* \boldsymbol{I}_i^a\, {}^i\boldsymbol{X}_{\lambda(i)}$

17 $\quad$ **end**

18 **end**

19 *Second forward pass:*

20 **for** $i = 1$ **to** N_B **do**

21 $\quad$ **if** $\lambda(i) \neq 0$ **then**

22 $\quad\quad M_{\text{inv}}[i, i:] = M_{\text{inv}}[i, i:] - \boldsymbol{D}_i^{-1} \boldsymbol{U}_i^T\, {}^i\boldsymbol{X}_{\lambda(i)} \boldsymbol{P}_{\lambda(i)}[i, i:]$

23 $\quad$ **end**

24 $\quad \boldsymbol{P}_i[i, i:] = \boldsymbol{S}_i M_{\text{inv}}[i, i:]$

25 $\quad$ **if** $\lambda(i) \neq 0$ **then**

26 $\quad\quad \boldsymbol{P}_i[i, i:] = \boldsymbol{P}_i[i, i:] + {}^i\boldsymbol{X}_{\lambda(i)} \boldsymbol{P}_{\lambda(i)}[i, i:]$

27 $\quad$ **end**

28 **end**

At this point in the algorithm $M_{inv}[i,:]$ is not its final value. Compared to ABA it satisfies:

$$M_{inv}[i,:]\,\tau = D_i^{-1} u_i$$

Replace w/ lines below

I_i^a is the portion of I_i^A transmitted across the joint

Replacement for line 14:

$$F_i^a[:, \text{subtree}(i)] = F_i[:, \text{subtree}(i)] + U_i M_{inv}[i, \text{subtree}(i)]$$

$$F_{\lambda(i)}[:, \text{subtree}(i)] = F_{\lambda(i)}[:, \text{subtree}(i)] + {}^{\lambda(i)}X_i^* F_i^a[:, \text{subtree}(i)]$$

- F_i^a is the portion of F_i transmitted across the joint
- Relates to ABA on pg. 132 of [4] as $F_i^a \tau = p_i^a$