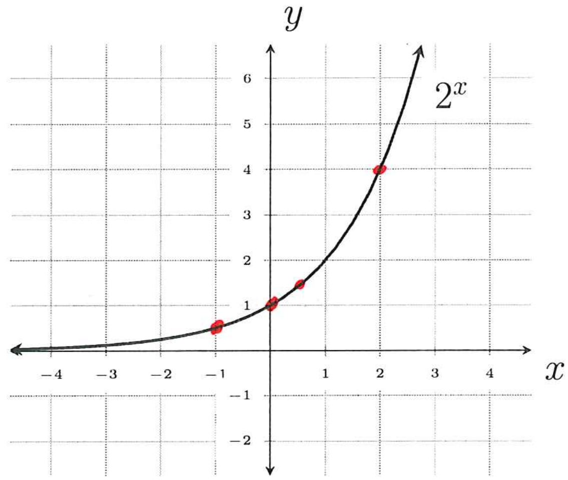
6.1 The Family of Exponential Functions

Definition. An **exponential function** has the form $f(x) = a^x$, where $a > 0$. The number a is called the **base**.

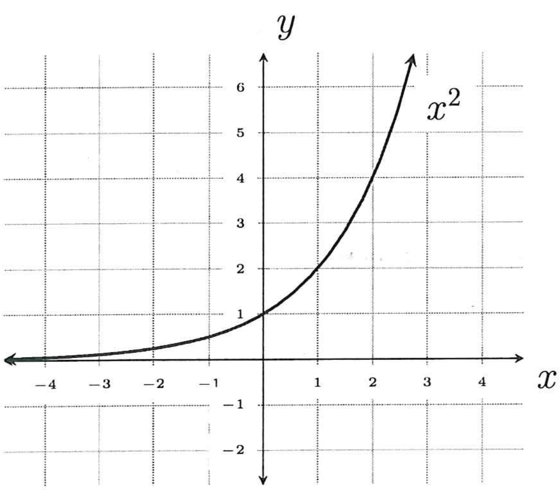
Example.

x	$f(x)$	
-1	2^{-1}	$= 1/2$
0	2^0	$= 1$
$1/2$	$2^{1/2}$	$= \sqrt{2}$
2	2^2	$= 4$
3.2	$2^3 \cdot 2^{1/5}$	$= 8\sqrt[5]{2}$

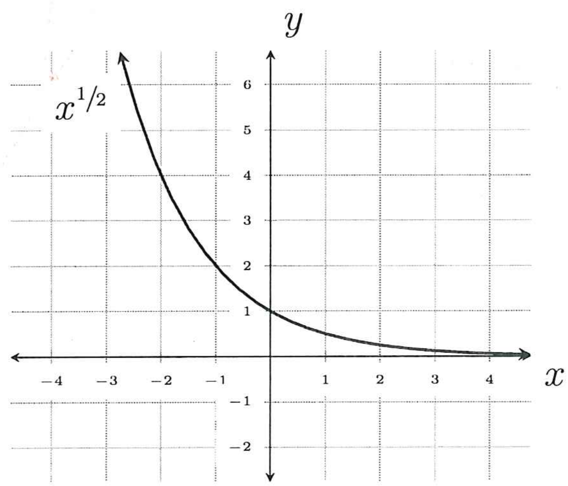


The base a determines if a^x increases with exponential growth or decreases with exponential decay:

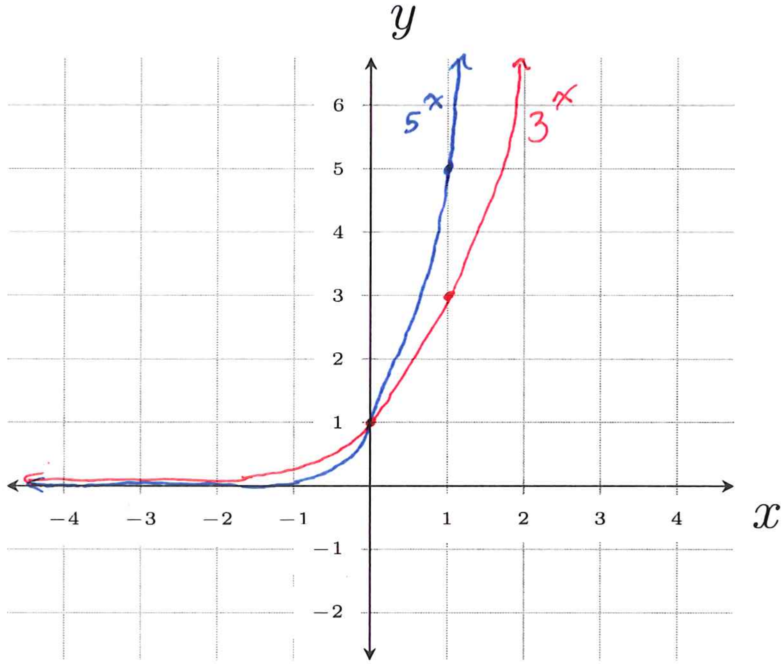
$a > 1$



$0 < a < 1$



Example. Graph 3^x and 5^x on the axes provided below:

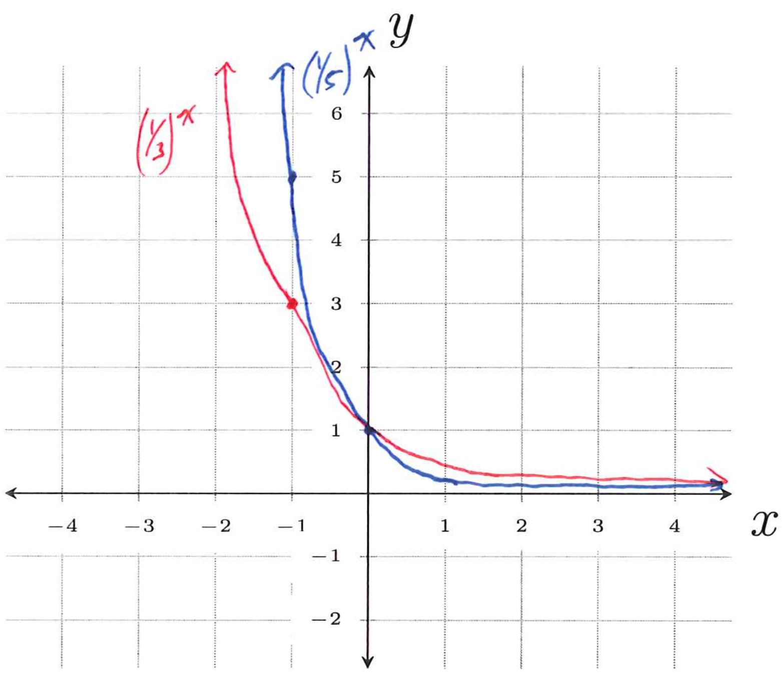


Example. Graph $\left(\frac{1}{3}\right)^x$ and $\left(\frac{1}{5}\right)^x$ on the axes provided below:

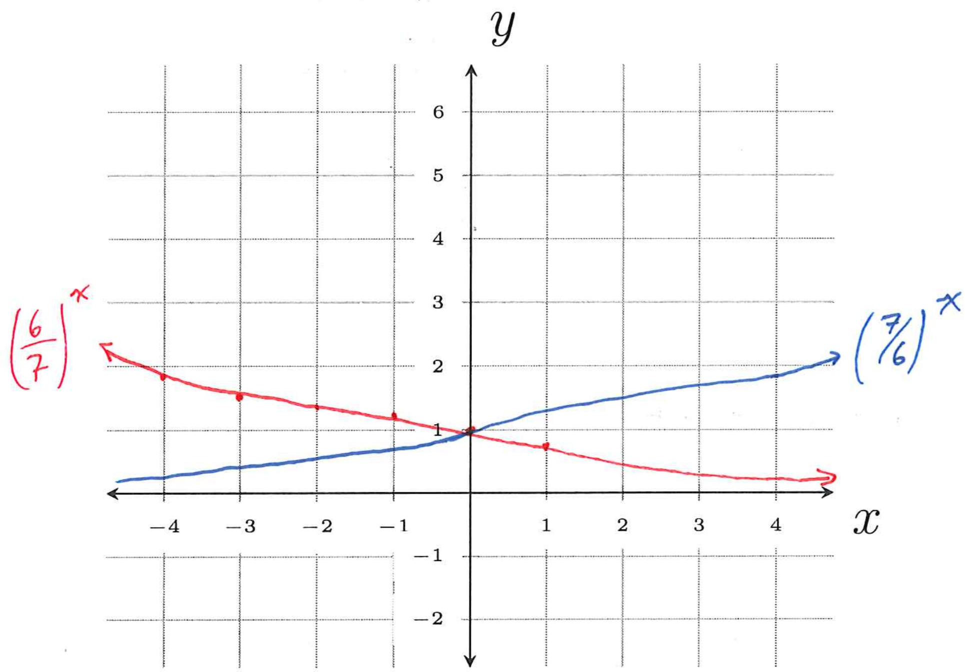
Note:

$\left(\frac{1}{3}\right)^x = 3^{-x}$

$\left(\frac{1}{5}\right)^x = 5^{-x}$



Example. Graph $\left(\frac{6}{7}\right)^x$ and $\left(\frac{7}{6}\right)^x$:



6.2 The Function e^x

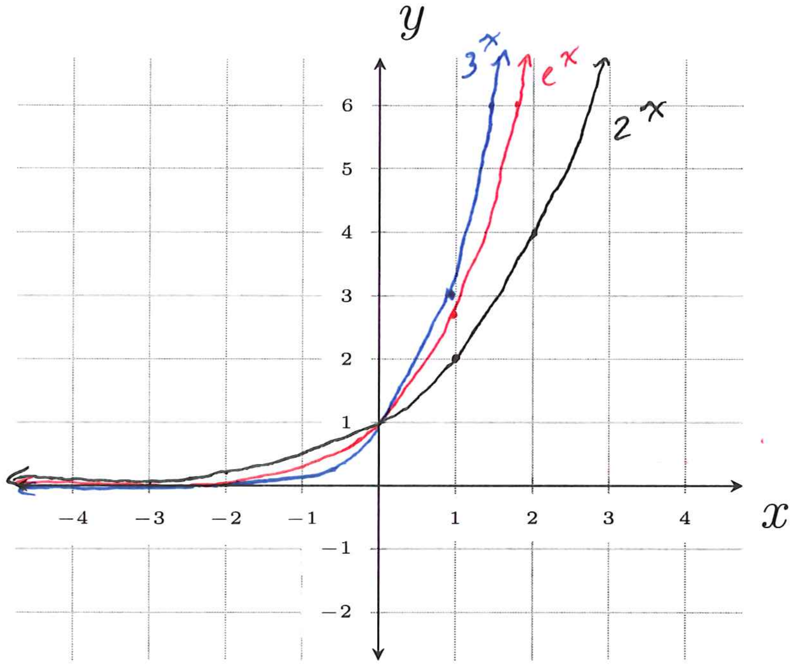
The number e is an irrational number whose exact form is

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n \approx 2.718281828459045\ldots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots = \sum_{n=0}^{\infty}\frac{x^n}{n!}$$

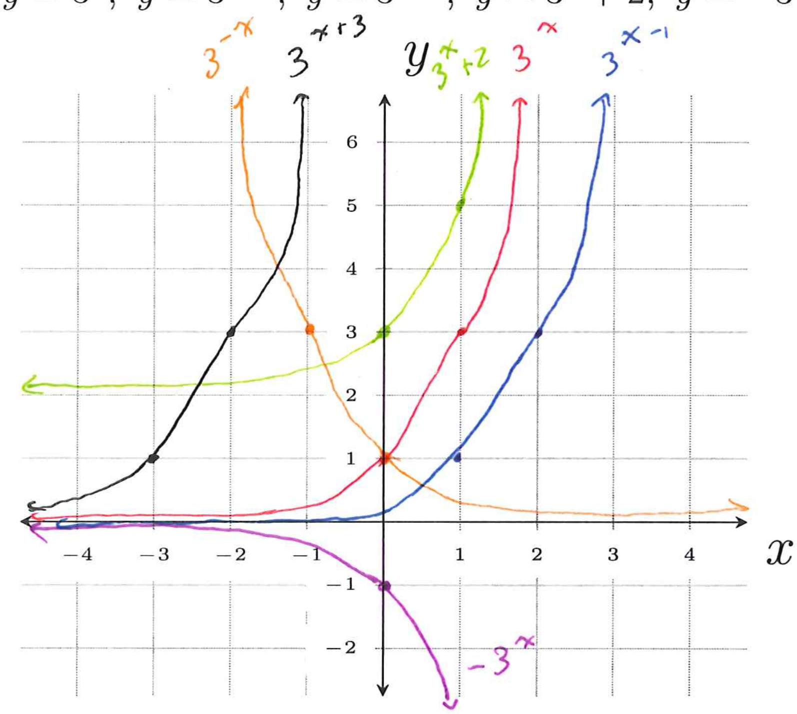
This exponential function has a 45° tangent at $x = 0$. This function shows in many applications, and each function a^x can be written as e^{kx}.

Example. Graph $2^x, e^x$ and 3^x:



All exponential functions follow the all the typical rules when performing transformation of functions:

Example. Graph $y = 3^x,\ y = 3^{x-1},\ y = 3^{x+3},\ y = 3^x + 2,\ y = -3^x,\ y = 3^{-x}$



Example.
Solve:

a) $2^{x-3} = 64 = 2^6$

$\Rightarrow x - 3 = 6 \Rightarrow \boxed{x = 9}$

b) $4^{2x-3} = 64 = 4^3$

$\Rightarrow 2x - 3 = 3$

$\boxed{x = 3}$

c) $10^{\sin x} = 1 = 10^0$

$\Rightarrow \sin x = 0$

$\Rightarrow \boxed{x = k\pi,\ k\text{: an integer}}$

d) $5^{x^2+2x} = 125 = 5^3$

$\Rightarrow x^2 + 2x = 3$

$x^2 + 2x - 3 = 0$

$(x-3)(x+1) = 0$

$\boxed{x = 3,\ x = -1}$

e) $\left(\dfrac{1}{4}\right)^{2-x} = 16^x = (4^2)^x$

$(4^{-1})^{2-x}$

$\Rightarrow -(2-x) = 2x$

$\Rightarrow x - 2 = 2x$

$\Rightarrow \boxed{-2 = x}$

Properties of exponents

1. $a^m \cdot a^n = a^{m+n}$
2. $\dfrac{a^m}{a^n} = a^{m-n}$
3. $(a^m)^n = a^{mn}$
4. $(ab)^n = a^n b^n$
5. $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}, b \neq 0$
6. $a^{-n} = \frac{1}{a^n}, a \neq 0$
7. $a^0 = 1, a \neq 0$

Example. Simplify:

a) $(e^x)^3$

e^{3x}

b) $\dfrac{e^{2x}}{e^x}$

$e^{2x-x} = e^x$

c) $\dfrac{e^{2x} - 1}{e^x - 1} = \dfrac{(e^x - 1)(e^x + 1)}{e^x - 1}$

$= e^x + 1$