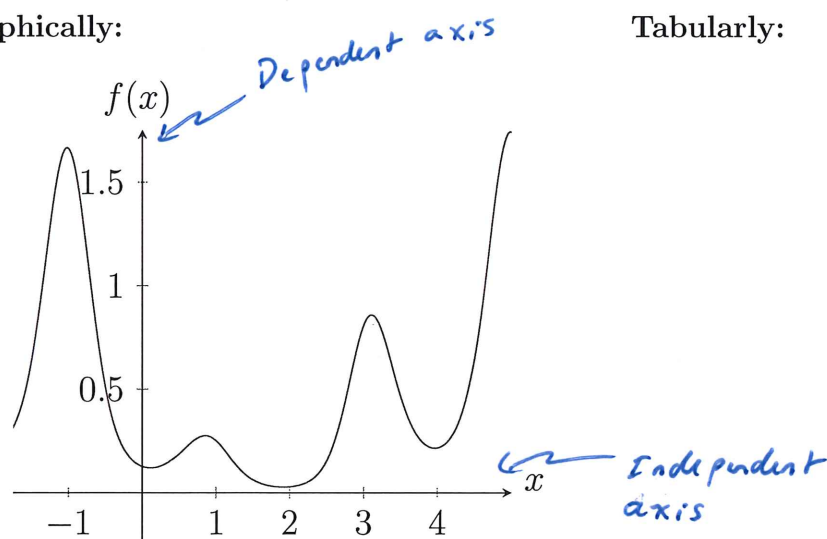


4.1 Functions And Their Graphs

- A way to relate two quantities to each other.

Graphically:



Tabularly:

x	$f(x)$
-1	$3/2$
0	0
1	$1/2$
2	1
3	$1/4$

Definition.

A **function** f defined from a set A to a set B is a rule that associates with each element of the set A one, and only one, element of set B .

Definition.

The **domain** of a function is the set of all input values.

$$f(x) = x + 1 \rightarrow \text{all real numbers}$$

$$h(x) = 1/x \rightarrow x \neq 0$$

$$g(x) = \sqrt{x} \rightarrow x \geq 0$$

Definition.

The **range** of a function is the set of all output values.

$$f(x) = x + 1 \rightarrow (-\infty, \infty)$$

$$g(x) = x^2 \rightarrow [0, \infty)$$

Example. For $f(x) = 2x^2$, find

$$f(4) = 2(4)^2 = 2 \cdot 16 = 32$$

$$f(-3) = 2(-3)^2 = 2 \cdot 9 = 18$$

$$f(4+h) = 2(4+h)^2 = 2(16+8h+2h^2) = 32+16h+2h^2$$

$$f(x+\Delta x) = 2x^2 + 4x \cdot \Delta x + (\Delta x)^2$$

$$f\left(\sqrt{\frac{x}{2}}\right) = 2\left(\sqrt{\frac{x}{2}}\right)^2 = \frac{2x}{2} = x$$

Domain of $f(x)$: $(-\infty, \infty)$ -or- \mathbb{R}

Range of $f(x)$: $[0, \infty)$

Example. For $g(x) = \sqrt{x} + 1$, find

Domain of $g(x)$: $x \geq 0$ -or- $[0, \infty)$

Range of $g(x)$: $g(x) \geq 0$ -or- $[0, \infty)$

Example. For $h(x) = \sqrt{3-x} - 2$, find

Domain of $h(x)$: $3-x \geq 0$ -or- $[3, \infty)$
 $3 \geq x$

Range of $h(x)$: $h(x) \geq -2$ -or- $[-2, \infty)$

Example. For $j(x) = \sqrt[3]{3-x} - 2$, find

Domain of $j(x)$: $(-\infty, \infty)$ -or- \mathbb{R}

Can take odd roots of negative numbers

Range of $j(x)$: $(-\infty, \infty)$ -or- \mathbb{R}

Example. For $\kappa(\nu) = \frac{\nu^2 - 1}{\nu - 1}$, find

Domain of $\kappa(\nu)$: $\nu \neq 1$ -or- $(-\infty, 1) \cup (1, \infty)$

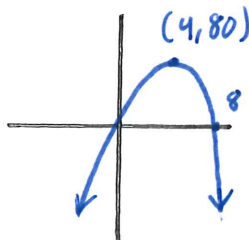
Range of $\kappa(\nu)$: $\kappa(\nu) = \frac{(\nu-1)(\nu+1)}{(\nu-1)} = \nu+1, \nu \neq 1 \Rightarrow \kappa(\nu) \neq 2$

\Rightarrow Range: $(-\infty, 2) \cup (2, \infty)$

Example. For $\ell(t) = 40t - 5t^2$, find

Domain of $\ell(t)$: $(-\infty, \infty)$

Range of $\ell(t)$: $(-\infty, 80]$



Example. For $m(\omega) = 40\omega - 5\omega^2$, find

Domain of $m(\omega)$: $(-\infty, \infty)$

Range of $m(\omega)$: $(-\infty, 80]$

Example. A cylindrical water tower with a radius of 10m and a height of 50m is filled to a height of h . The volume V of water (in cubic meters) is given by the function $g(h) = 100\pi h$. Identify the independent and dependent variables of for this function, and then determine an appropriate domain.

Independent variable: h

Dependent variable: $g(h)$

Domain: $0 \leq h \leq 50$ -or- $[0, 50]$