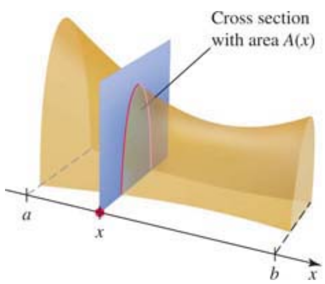
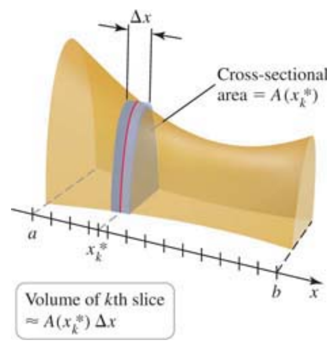
6.3: Volume by Slicing

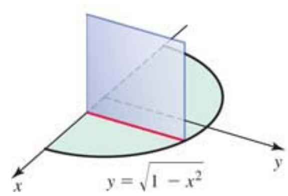
General Slicing Method
Suppose a solid object extends from $x = a$ to $y = b$, and the cross section of the solid perpendicular to the x-axis has an area given by a function A that is integrable on $[a, b]$. The volume of the solid is

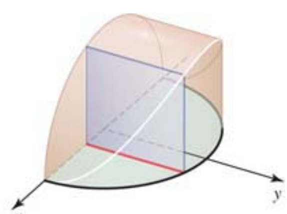
$$V = \int_a^b A(x)\,dx.$$





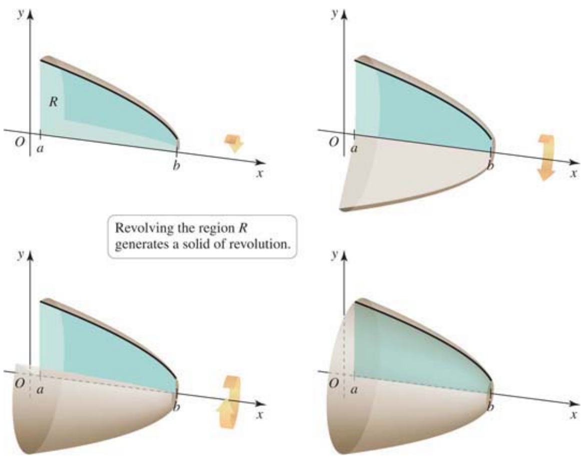
Example. Use the general slicing method to find the volume of the solid whose base is the region bounded by the semicircle $y = \sqrt{1 - x^2}$ and the x-axis, and whose cross sections through the solid perpendicular to the x-axis are squares.

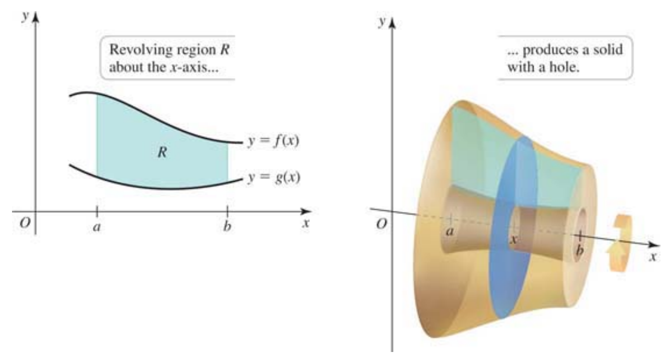




Disk Method about the x-Axis
Let f be continuous with $f(x) \geq 0$ on the interval $[a, b]$. If the region R bounded by the graph of f, the x-axis, and the lines $x = a$ and $x = b$ is revolved about the x-axis, the volume of the resulting solid of revolution is

$$V = \int_a^b \pi \underbrace{f(x)}_{\text{disk radius}}{}^2 dx.$$

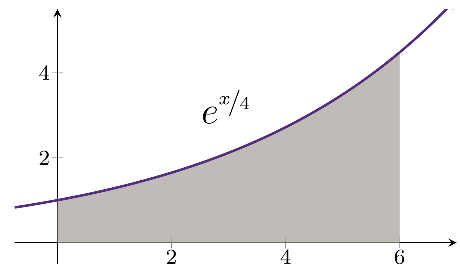




Washer Method about the x-Axis
Let f and g be continuous functions with $f(x) \geq g(x) \geq 0$ on $[a, b]$. Let R be the region bounded by $y = f(x)$, $y = g(x)$, and the lines $x = a$ and $x = b$. When R is revolved about the x-axis, the volume of the resulting solid of revolution is

$$V = \int_a^b \pi(\underbrace{f(x)}_{\text{outer radius}}{}^2 - \underbrace{g(x)}_{\text{inner radius}}{}^2)\, dx.$$

Example. Consider the region bounded by $y = e^{x/4}$, $y = 0$, $x = 0$, and $x = 6$. Find the volume of the solid generated by rotating the region about the x-axis.



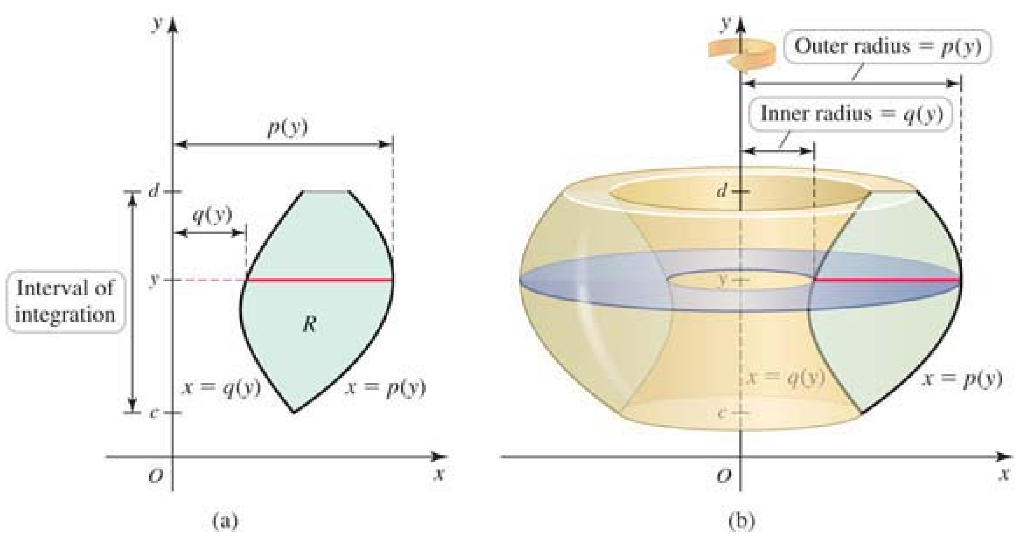
Disk and Washer Methods about the y-Axis

Let p and q be continuous functions with $p(y) \geq q(y) \geq 0$ on $[c, d]$. Let R be the region bounded by $x = p(y)$, $x = q(y)$, and the lines $y = c$ and $y = d$. When R is revolved around the y-axis, the volume of the resulting solid of revolution is given by

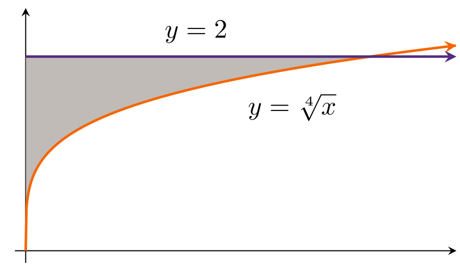
$$V = \int_c^d (\underbrace{p(y)}_{\substack{\text{outer}\\\text{radius}}}{}^2 - \underbrace{q(y)}_{\substack{\text{inner}\\\text{radius}}}{}^2\ )\, dy.$$

If $q(y) = 0$, the disk method results:

$$V = \int_c^d \underbrace{p(y)}_{\substack{\text{disk}\\\text{radius}}}{}^2 dy.$$



Example. Consider the region bounded between $y = \sqrt[4]{x}$, $y = 2$, and $x = 0$.

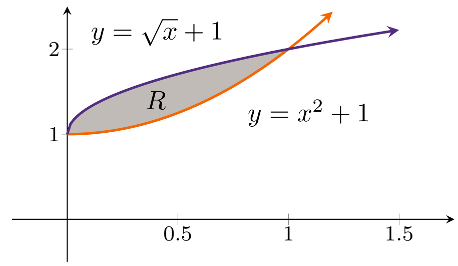


Setup the integral with respect to x that gives the area of the region.

Setup the integral with respect to y that gives the area of the region.

Use the disk/washer method to setup the that represents the volume of the solid generated by rotating the region about the x-axis.

Example. Consider the region R between $y = \sqrt{x}+1$ and $y = x^2+1$. Setup the integrals which find the volume of the solid obtained by rotating the region R as indicated below.



about the y-axis

about the x-axis

about the line $x = 1$

about the line $y = -1$