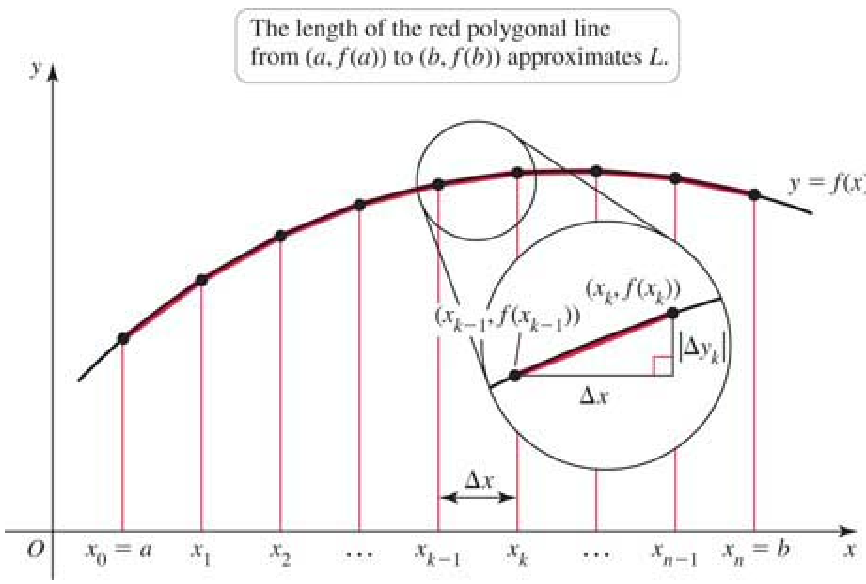
6.5: Length of Curves

Definition. (Arc Length for $y = f(x)$)
Let f have a continuous first derivative on the interval $[a, b]$. The length of the curve from $(a, f(a))$ to $(b, f(b))$ is

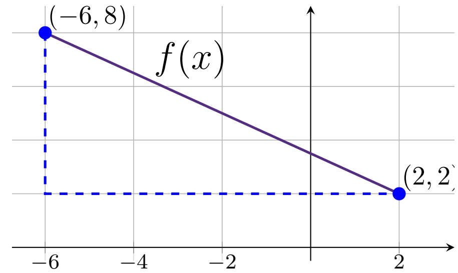
$$L = \int_a^b \sqrt{1 + f'(x)^2}\, dx.$$



Definition. (Arc Length for $x = g(y)$)
Let g have a continuous first derivative on the interval $[c, d]$. The length of the curve from $(g(c), c)$ to $(g(d), d)$ is

$$L = \int_c^d \sqrt{1 + g'(y)^2}\, dy.$$

Example. Using a geometric argument, we can see that the length of $f(x) = -\dfrac{3}{4}x + \dfrac{7}{2}$ on the interval $[-6, 2]$ is $L = 10$. Compute this using the arc-length formula.



Example. Find the arc length of the curve $y = \frac{1}{4}x^2 - \frac{1}{2}\ln(x)$, for $1 \leq x \leq 2$.

Example. Find the arc length of the curve $y = \dfrac{1}{3}x^{3/2}$ on $[0, 12]$.

Example. Find a curve that passes through $(1, 2)$ on $[2, 6]$ whose arc length is computed using

$$\int_2^6 \sqrt{1 + 16x^{-2}}\, dx.$$

Example. Suppose f has length L on $[a, b]$. Evaluate

$$\int_{a/c}^{b/c} \sqrt{1 + f'(cx)^2}\, dx.$$