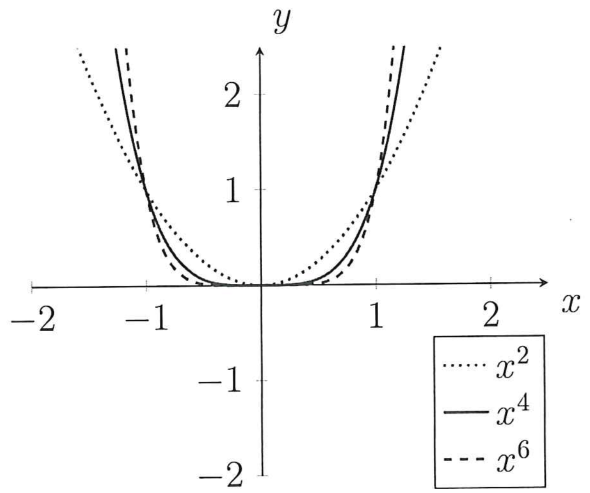
4.3 Power Functions

Definition. Functions of the form $f(x) = x^r$, where r is a constant, are called **power functions**.

A.



- These are **even** functions

  $f(-x) = f(x)$

- Symmetry about the y-axis

  $(-x, y)$, (x, y)

e.g. $f(x) = x^2$

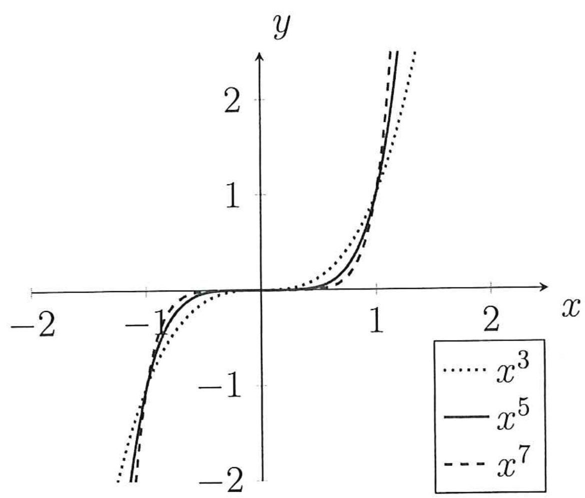
$\longrightarrow f(-x) = (-x)^2 = x^2 = f(x)$

$g(x) = |x| + 1$

$\longrightarrow g(-x) = |-x| + 1 = |x| + 1 = g(x)$

$(-x, y)$ (x, y)

B.



- These are **odd** functions

  $f(x) = f(-x)$

- Symmetry about the origin

  $(-x, -y)$, (x, y)

eg $f(x) = x^3$

$\longrightarrow f(-x) = (-x)^3 = -(x)^3 = -f(x)$

$g(x) = 2x^3 + 3x$

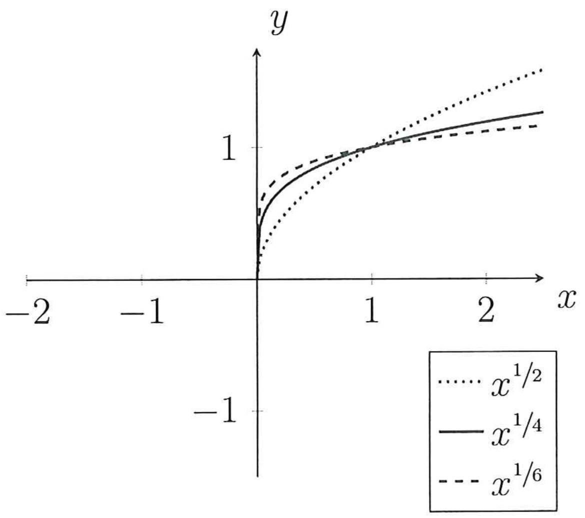
$\longrightarrow g(-x) = 2(-x)^3 + 3(-x)$

$= -2(x)^3 - 3(x)$

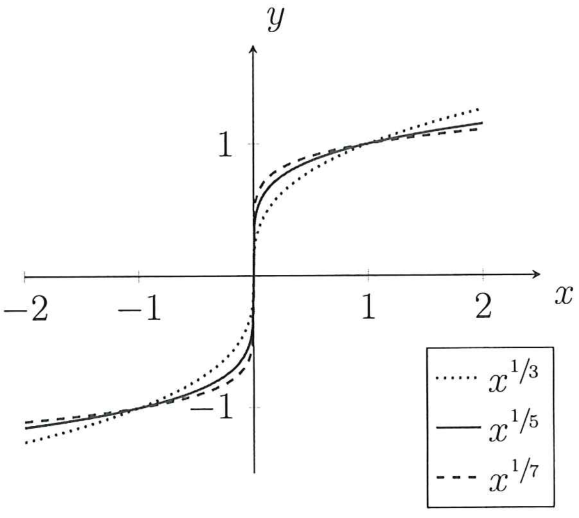
$= -g(x)$.

Note: The leading exponent does not guarantee even or odd

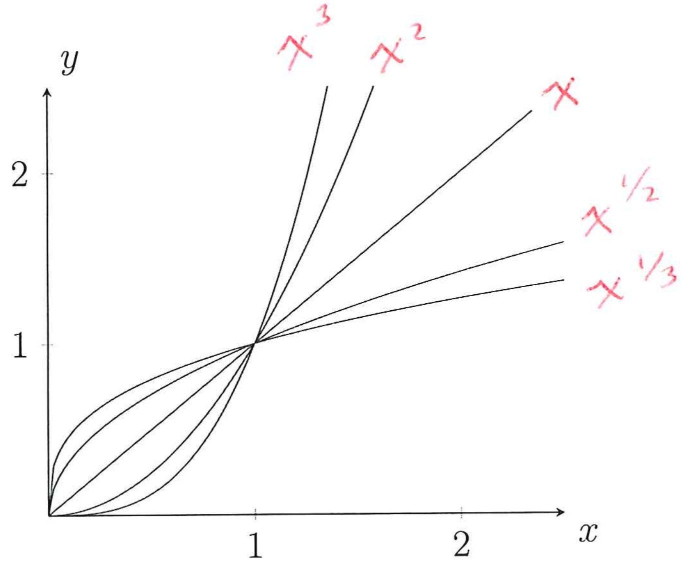
$h(x) = x^2 + x + 1$ is neither even or odd!

C. 

y
1
-1
-2
-1
1
2
x
$x^{1/2}$
$x^{1/4}$
$x^{1/6}$

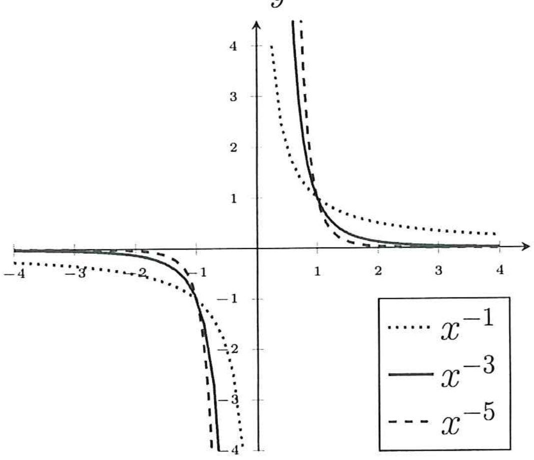


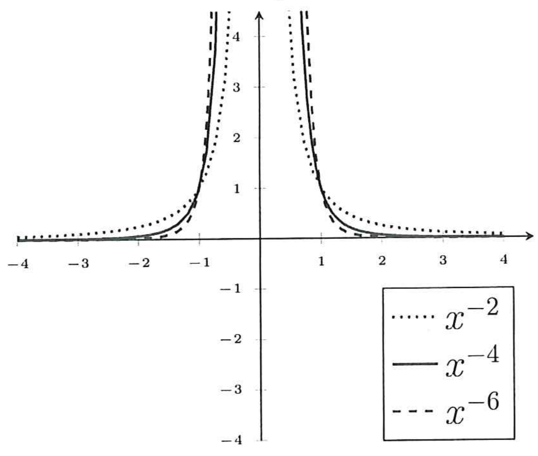
y
1
-1
-2
-1
1
2
x
$x^{1/3}$
$x^{1/5}$
$x^{1/7}$



x^3
x^2
x
$x^{1/2}$
$x^{1/3}$
y
2
1
1
2
x

D.





Example. Determine if the following functions are symmetric about the y-axis, x-axis or the origin.

a) $f(x) = 3x^5 + 2x^3 - x$

$f(-x) = 3(-x)^5 + 2(-x)^3 - (-x)$
$= -3x^5 - 2x^3 + x$
$= -(3x^5 + 2x - x) = -f(x)$
odd $\rightarrow$ sym about origin

b) $f(x) = 2|x|$

$f(-x) = 2|-x| = 2|x| = f(x)$
even $\rightarrow$ sym about y-axis

c) $x^3 - y^5 = 0$

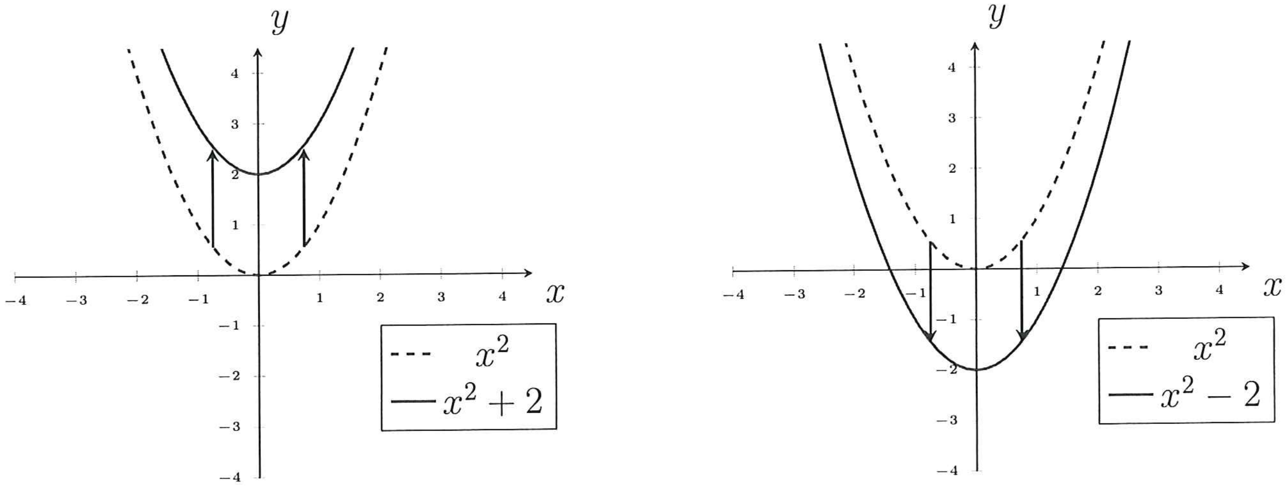
$(-x)^3 - (-y)^5 = -x^3 + y^5 = (-1)(\underbrace{x^3 - y^5}_{0}) = 0$
$\Rightarrow$ we have the points $(-x, -y)$ and $(x, -y)$
$\rightarrow$ sym about x-axis
$(-x)^3 - (y)^5 = -x^3 - y^5 \rightarrow$ No sym about y-axis

d) $f(x) = x|x|$

$f(-x) = -x|-x| = -x|x| \neq -f(x)$
$\neq f(x)$
No symmetry!

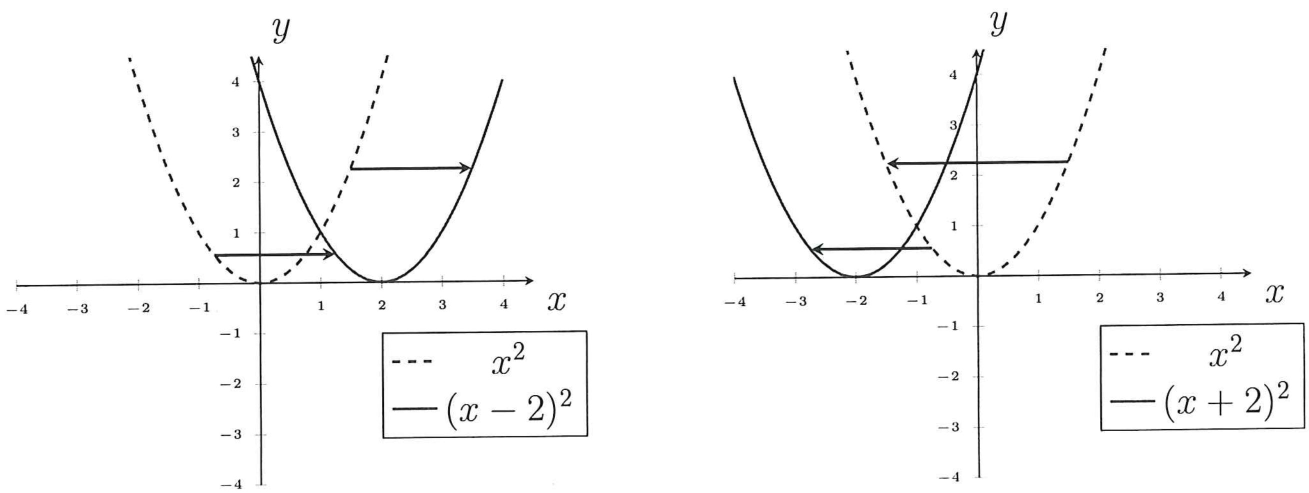
4.4 Shifting up and down

$$f(x) \text{ vs. } f(x) + c$$



4.5 Shifting left and right

$$f(x) \text{ vs. } f(x - c)$$

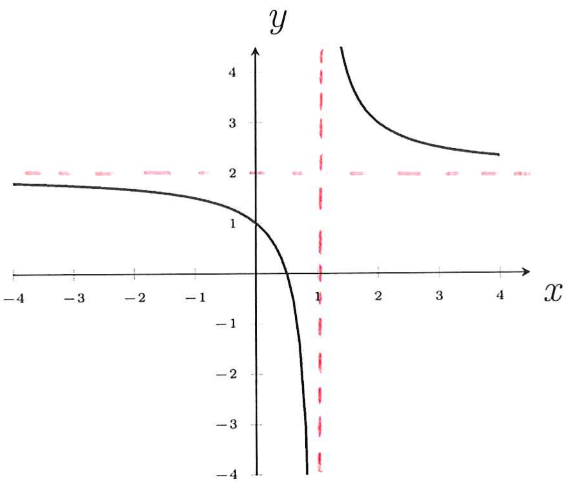


4.6 Translations

Example.

Graph $\dfrac{1}{x-1}+2$

1. shift right by 1
2. shift up by 2



Example. "Completing the square"

Graph $g(x) = x^2 - 6x + 5$

$g(x) = x^2 - 6x + 5$

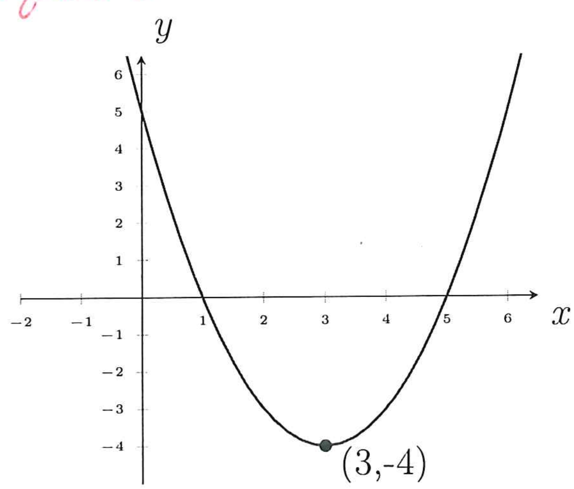
$= (x^2 - 6x + \_\_\_) + (5 - \_\_\_)$

$\left[\frac{1}{2}(-6)\right]^2 = [-3]^2 = 9$

$= (x^2 - 6x + 9) + (5 - 9)$

$= (x-3)^2 - 4$

$\Rightarrow$ ① Right by 3, ② Down by 4



Example.

Graph $f(t) = 40t - 5t^2$

$$\left[\tfrac{1}{2}(-8)\right]^2$$

$$f(t) = -5t^2 + 40t$$

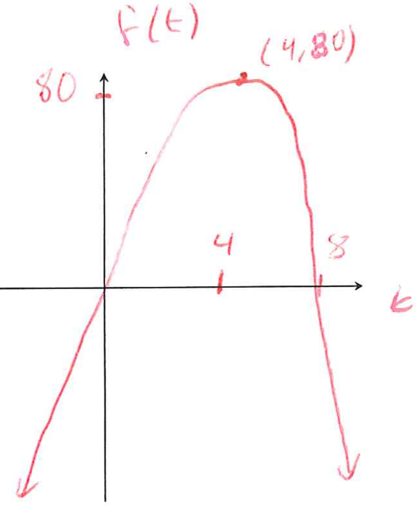
$$= -5(t^2 - 8 + 16) - 5(-16)$$

$$= -5(t - 4)^2 + 80$$

→ ① shift right 4

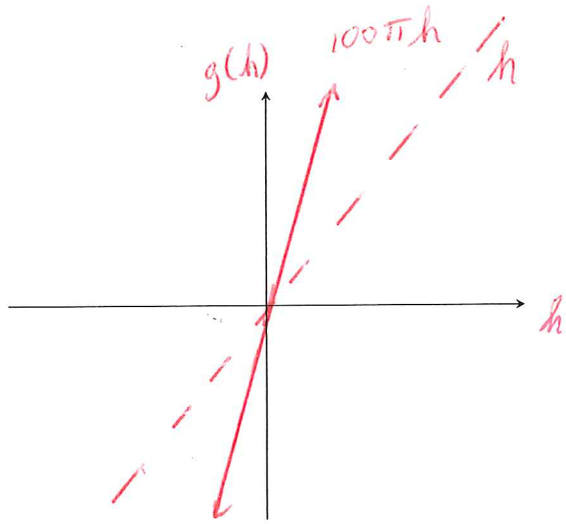
② scale by -5

③ shift up 80



Example.

Graph $g(h) = 100\pi h$



Composition

$$(f \circ g)(x) = f(g(x))$$

Example. $f(x) = \sqrt{x}, \;\; g(x) = x + 1$

$(f \circ g)(x) =$ $f(g(x)) = f(x+1) = \sqrt{x+1}$

$(g \circ f)(x) =$ $g(f(x)) = g(\sqrt{x}) = \sqrt{x} + 1$