

1 6.1: Velocity and Net Change

Definition. (Position, Velocity, Displacement, and Distance)

1. The **position** of an object moving along a line at time t , denoted $s(t)$, is the location of the object relative to the origin.
2. The **velocity** of an object at time t is $v(t) = s'(t)$.
3. The **displacement** of the object between $t = a$ and $t = b > a$ is

$$s(b) - s(a) = \int_a^b v(t) dt.$$

4. The **distance traveled** by the object between $t = a$ and $t = b > a$ is

$$\int_a^b |v(t)| dt$$

where $|v(t)|$ is the **speed** of the object at time t .

Theorem 6.1: Position from Velocity

Given the velocity $v(t)$ of an object moving along a line and its initial position $s(0)$, the position function of the object for future times $t \geq 0$ is

$$\underbrace{s(t)}_{\text{position at } t} = \underbrace{s(0)}_{\text{initial position}} + \underbrace{\int_0^t v(x) dx}_{\text{displacement over } [0, t]}.$$

Theorem 6.2: Velocity from Acceleration

Given the acceleration $a(t)$ of an object moving along a line and its initial velocity

$v(0)$, the velocity of the object for future times $t \geq 0$ is

$$v(t) = v(0) + \int_0^t a(x) dx.$$

Theorem 6.3: Net Change and Future Value

Suppose a quantity Q changes over time at a known rate Q' . Then the **net change** in Q between $t = a$ and $t = b > a$ is

$$\underbrace{Q(b) - Q(a)}_{\text{net change in } Q} = \int_a^b Q'(t) dt.$$

Given the initial value $Q(0)$, the **future value** of Q at time $t \geq 0$ is

$$Q(t) = Q(0) + \int_0^t Q'(x) dx.$$

Velocity-Displacement Problems

Position $s(t)$

Velocity: $s'(t) = v(t)$

Displacement: $s(b) - s(a) = \int_a^b v(t) dt$

Future position: $s(t) = s(0) + \int_0^t v(x) dx$

General Problems

Quantity $Q(t)$ (such as volume or population)

Rate of change: $Q'(t)$

Net change: $Q(b) - Q(a) = \int_a^b Q'(t) dt$

Future value of Q : $Q(t) = Q(0) + \int_0^t Q'(x) dx$
