10.5: Comparison Tests

Theorem 10.14: Comparison Test
Let $\sum a_k$ and $\sum b_k$ be series with positive terms where $a_k \leq b_k$.

1. If $\sum b_k$ converges, then $\sum a_k$ converges.
2. If $\sum a_k$ diverges, then $\sum b_k$ diverges.

Example. Use the comparison test to determine if the series $\displaystyle\sum_{k=1}^{\infty} \frac{k^2}{k^3 - 3}$ converges or diverges.

Theorem 10.15: Limit Comparison Test
Let $\sum a_k$ and $\sum b_k$ be series with positive terms and let

$$\lim_{k\to\infty} \frac{a_k}{b_k} = L.$$

1. If $0 < L < \infty$ (that is, L is a finite positive number), then $\sum a_k$ and $\sum b_k$ either both converge or both diverge.
2. If $L = 0$ and $\sum b_k$ converges, then $\sum a_k$ converges.
3. If $L = \infty$ and $\sum b_k$ diverges, then $\sum a_k$ diverges.

Example. Using either the Comparison Test or the Limit Comparison Test, determine if the series

$$\sum_{k=1}^{\infty} \frac{4k^2 - k}{k^3 + 9}$$

converges or diverges.

Example. Determine if the following series converge or diverge.

$$\sum_{k=1}^{\infty} \frac{1}{k\sqrt{k^2+1}}$$

$$\sum_{k=1}^{\infty} \frac{\ln(k)}{k^2}$$

$$\sum_{k=1}^{\infty} \left(1 + \frac{2}{k}\right)^k$$

$$\frac{1}{14^3} + \frac{2}{15^3} + \frac{3}{16^3} + \dots$$

$$\sum_{k=1}^{\infty} \frac{\sin\left(\frac{\pi}{k}\right)}{k^3}$$

$$\sum_{k=1}^{\infty} \frac{\sqrt[3]{k^2+4}}{\sqrt{k^3+9}}$$