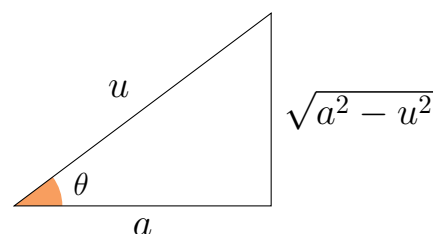
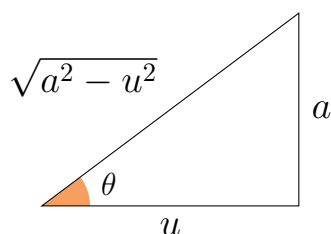
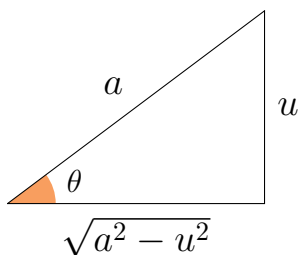
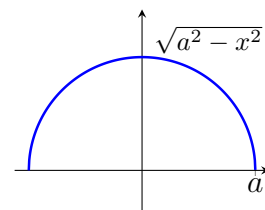


8.4: Trigonometric Substitutions

Example. Verify the formula for the area of a circle with radius a by finding the area under $f(x) = \sqrt{a^2 - x^2}$.



$a^2 - u^2$	$u = a \sin(\theta), \quad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}, \quad \text{for } u \leq a$	$a^2 + a^2 \sin^2(\theta) = a^2 \cos^2(\theta)$
-------------	---	---

$a^2 + u^2$	$u = a \tan(\theta), \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2},$	$a^2 - a^2 \tan^2(\theta) = a^2 \sec^2(\theta)$
-------------	--	---

$u^2 - a^2$	$u = a \sec(\theta), \quad \begin{cases} 0 \leq \theta < \frac{\pi}{2}, & \text{for } u \geq a \\ \frac{\pi}{2} < \theta \leq \pi, & \text{for } u \leq -a \end{cases}$	$a^2 \sec^2(\theta) - a^2 = a^2 \tan^2(\theta)$
-------------	---	---

Example. $\int \frac{\sqrt{x^2 - 4}}{x^3} dx$

Example. $\int \frac{\sqrt{16 - x^2}}{x} dx$

Example. $\int \frac{x^3}{(25 - 4x^2)^{3/2}} dx$

Example. $\int_0^{1/3} \frac{dx}{(9x^2 + 1)^{3/2}}$

Example. $\int \frac{x}{\sqrt{x^2 - 2x + 10}}$