11.4: Working with Taylor Series

Limits by Taylor Series

Example (LC 31.1-31.2)**.** Evaluate the following limit using its Taylor series:

$$\lim_{x \to 0} \frac{12x - 8x^3 - 6\sin(2x)}{x^5}$$

Example. Evaluate the following limit using its Taylor series:

$$\lim_{x \to \infty} 2x^2 \left(e^{-2/x^2} - 1 \right)$$

Differentiating Power Series

Example (LC 31.3-31.4)**.** The differential equation

$$y'(t) + 4y = 8; \quad y(0) = 0$$

is satisfied by the function

$$y(t) = \sum_{k=1}^{\infty} \frac{8(-4)^{k-1} t^k}{k!}$$

Find $y'(t)$ as a power series.

Identify the function $y(t)$ represented by this power series.

$$e^x = 1 + \sum_{k=1}^{\infty} \frac{x^k}{k!}$$

Integrating Power Series

Example (LC 31.5-31.6)**.** Given that

$$x\cos\left(x^3\right) = \sum_{k=0}^{\infty} \frac{(-1)^k x^{6k+1}}{(2k)!}, \text{ for } |x| < \infty$$

Evalute $\displaystyle\int_0^1 x\cos\left(x^3\right)\,dx$ as an infinite series

Using the Alternating Series Estimation Theorem, what is the bound on $|R_3|$?

Representing Real Numbers

Example (LC 31.7)**.** Given that $\tan^{-1}(x) = \displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{2k+1}$, for $|x| \leq 1$, can we approximate $\dfrac{\pi}{3}$ using $x = \sqrt{3}$?

Example (LC 31.8)**.** Evaluate $\displaystyle\sum_{k=0}^{\infty} \frac{(\ln(2))^k}{k!}$.

Example. Let $f(x) = \begin{cases} \frac{e^x - 1}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$. Using $f(x)$ and $f'(x)$, evaluate

$$\sum_{k=1}^{\infty} \frac{k\,2^{k-1}}{(k+1)!}$$

Representing Functions as Power Series

Example (LC 31.9-31.10)**.** Consider the following Taylor series:

$$\sum_{k=1}^{\infty}(-1)^{k+1}\frac{3^k}{k\,5^k}$$

What function is being represented by this power series?

What does the sum of the series equal?

Example. Identify the function represented by

$$\sum_{k=0}^{\infty}(-1)^k\frac{x^{5k}}{3^k}$$