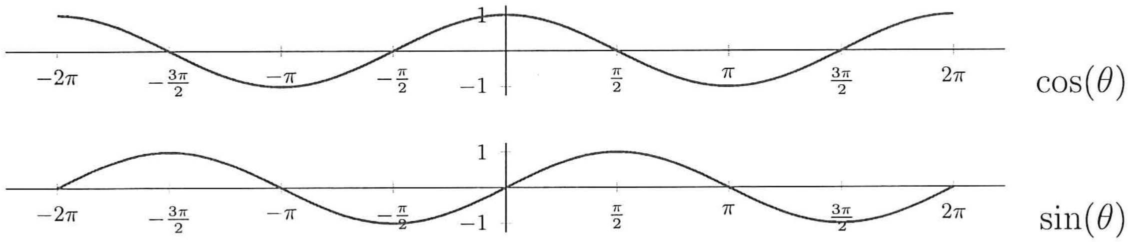
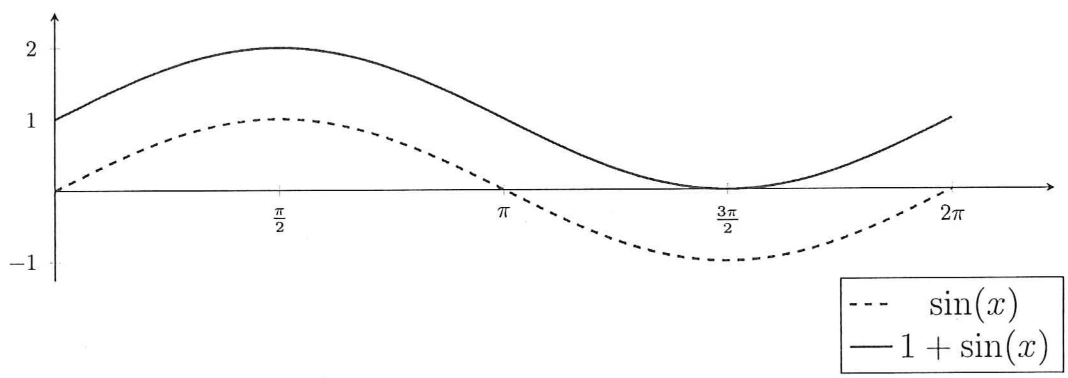
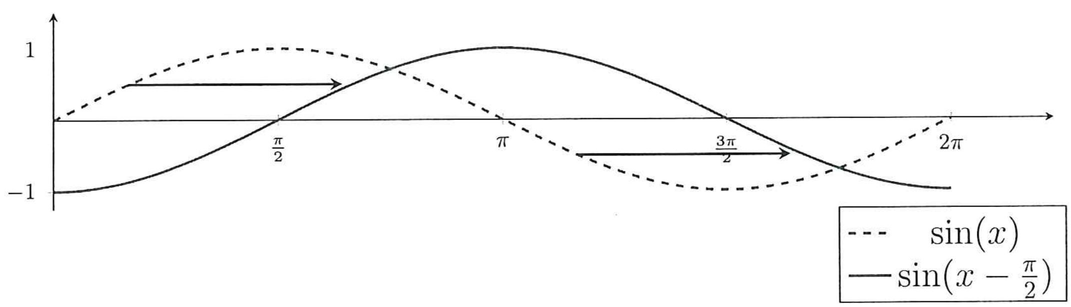
5.4 Graphs involving $\sin x$ and $\cos x$



Example. On $[0, 2\pi]$, graph $\sin x$, $1 + \sin x$ and $\sin\left(x - \dfrac{\pi}{2}\right)$.





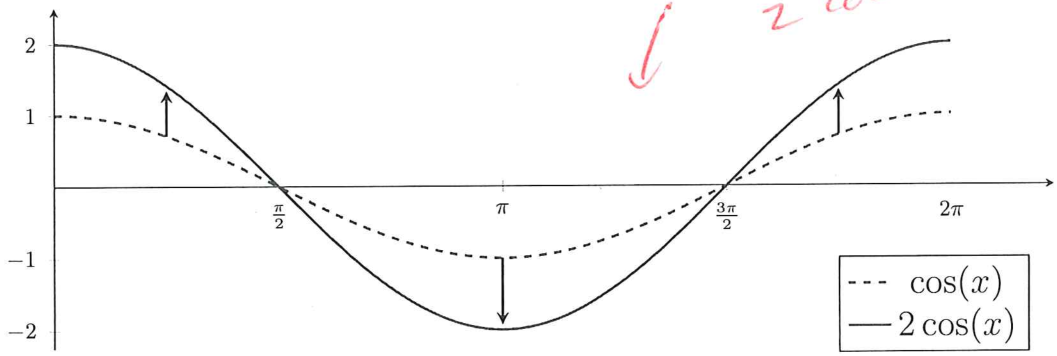
Definition. A shift to the left or right of a wave shaped graph, such as $\sin x$ or $\cos x$, is called a **phase shift**.

$\sin\left(x + \frac{\pi}{2}\right) = \cos(x)$

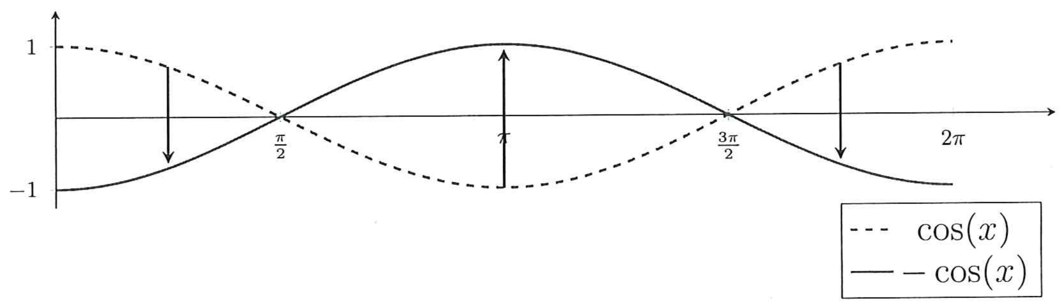
$\cos\left(x - \frac{\pi}{2}\right) = \sin(x)$

Example. Vertical scaling:

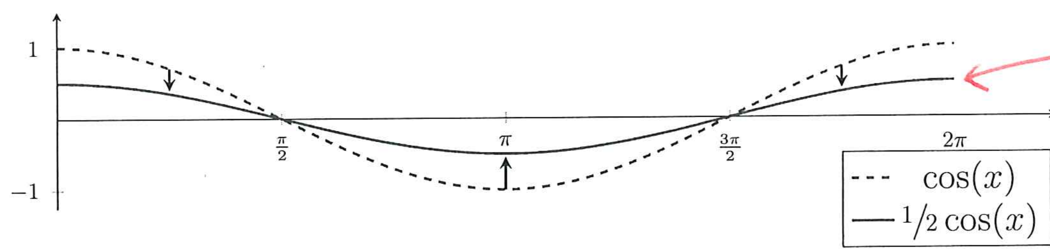
amplitude of $2\cos(x)$ is 2



Definition. The **amplitude** of a sinusoidal graph is equal to ½ of the distance from the top to the bottom of the waves.



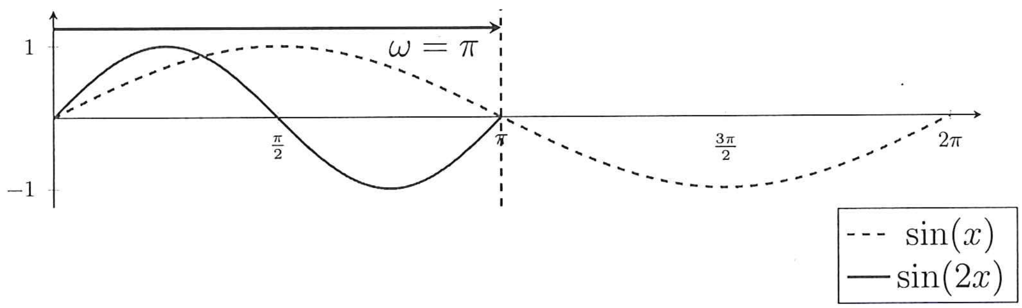
amp = 1

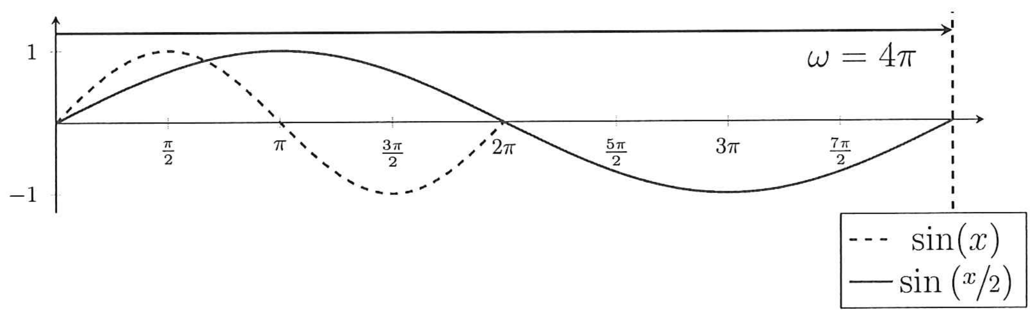


amp = 1/2

Definition.

- The **period** of the oscillating function is the length of a cycle.
- For functions of the form $\sin(ax)$ the **period** is given by $\omega = \frac{2\pi}{a}$. The same holds for $\cos(ax), \sec(ax)$ and $\csc(ax)$ since these 4 functions all have a **period** of 2π.
- When using $\tan(ax), \cot(ax)$ we need to divide a by the functions original period: $\pi \Rightarrow$ **period** is $\omega = \dfrac{\pi}{a}$.





Definition. The **frequency** is given by $\lambda = \dfrac{1}{\omega}$

$3\cos\left(\frac{\pi}{6}x\right) - 1$

amplitude: $A = 3$

Range: $[-4, 2]$

Period: $\omega = \dfrac{2\pi}{\pi/6} = 12$

Frequency: $\lambda = \dfrac{1}{\omega} = \dfrac{1}{12}$.

$\frac{1}{2}\tan(2\pi x)$

Amplitude: $A =$ N/A

Period: $\omega = \dfrac{\pi}{2\pi} = \dfrac{1}{2}$

Frequency: $\lambda = \dfrac{1}{\omega} = \dfrac{1}{1/2} = 2$.