8.2: Integration by Parts

Integraton by Parts
Suppose u and v are differentiable functions. Then

$$\int u\,dv = uv - \int v\,du.$$

A good mnemonic is ILATE.

Example. Evaluate $\int xe^{-\frac{x}{2}}\,dx$.

Example. Find the area of the region between the x-axis and $f(x) = \dfrac{\ln(x)}{x^2}$ on $[1, e]$.

Example. Evaluate $\int x^2 \cos(2x)\, dx$.

Example. Evaluate $\int e^{-x} \sin(3x)\, dx$.

Example. Evaluate $\int e^{4x} \cos(3x)\, dx$.

Example. Derive the integral formula

$$\int \ln(x)\,dx + x\ln(x) - x + C$$

Example. Evaluate $\int 10\cos(\sqrt{x})\,dx$

Integration by Parts for Definite Integrals
Let u and v be differentiable. Then

$$\int_a^b u(x)v'(x)\,dx = u(x)v(x)\Big|_a^b - \int_a^b v(x)u'(x)\,dx$$

Example. Evaluate $\displaystyle\int_1^e \ln(2x)\,dx$.