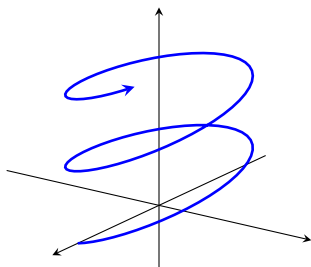


## 1 14.1: Vector-Valued Functions

Vector-valued functions are functions of the form  $\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$ , where  $x(t)$ ,  $y(t)$ , and  $z(t)$  are parametric equations dependent on  $t$ .



### Curves in Space

Consider

$$\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k},$$

where  $f$ ,  $g$ , and  $h$  are defined for  $a \leq t \leq b$ . The **domain** of  $\mathbf{r}$  is the largest set of  $t$  for which all of  $f, g$ , and  $h$  are defined.

**Example.** What plane does the curve  $\mathbf{r}(t) = t\mathbf{i} + 6t^3\mathbf{k}$  lie?

**Example** (Lines as vector-valued functions). Find a vector function for the line that passes through the points  $P(5, 2, -4)$  and  $Q(5, 5, -2)$ . What about the line segment that connects  $P$  and  $Q$ ?

**Example.** Find the domain of

$$\mathbf{r}(t) = \sqrt{16 - t^2}\mathbf{i} + \sqrt{t}\mathbf{j} + \frac{4}{\sqrt{3 + t}}\mathbf{k}$$

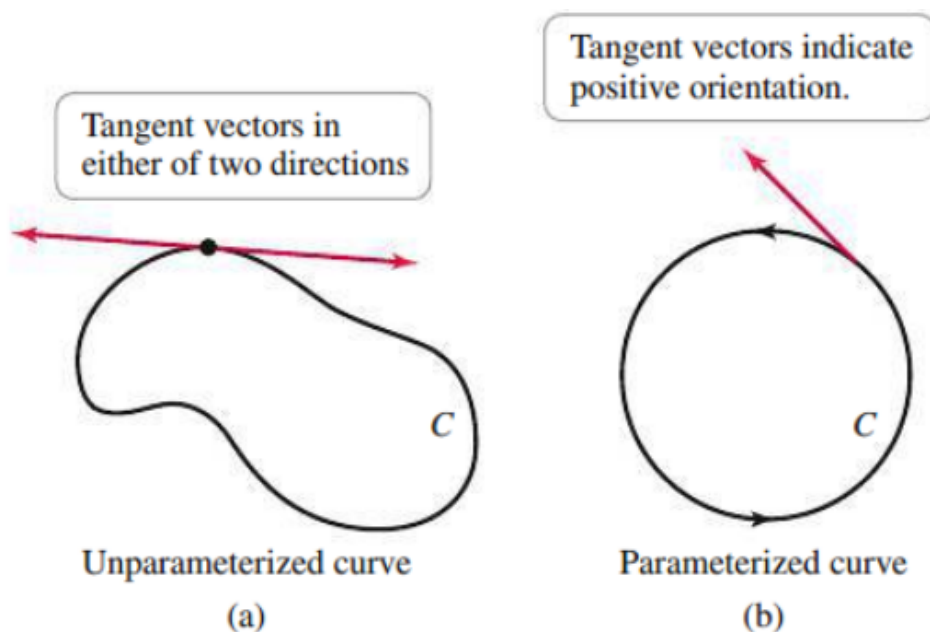
**Example.** Find the point  $P$  on

$$\mathbf{r}(t) = t^2\mathbf{i} + 2t\mathbf{j} + 2t\mathbf{k},$$

closest to  $P_0(4, 17, 10)$ . What is the distance between  $P$  and  $P_0$ ?

## Orientation of Curves

- A **unparameterized curve** is a smooth curve  $C$  with no specified direction and the tangent vector can be drawn in two directions.
- A **parameterized curve** is a smooth curve  $C$  described by a function  $\mathbf{r}(t)$  for  $a \leq t \leq b$  and has a direction referred to as its **orientation**.
- The *positive* orientation is the direction of the curve generated when  $t$  increases from  $a$  to  $b$ .
- The tangent vector of a parameterized curve points in the positive orientation of the curve.



**Example.** Graph the curve described by the equation

$$\mathbf{r}(t) = 4 \cos(t)\mathbf{i} + \sin(t)\mathbf{j} + \frac{t}{2\pi}\mathbf{k},$$

where  $0 \leq t \leq 2\pi$ . Indicate the positive orientation of this curve.

## Limits and Continuity for Vector-Valued Functions

The properties of limits extend to vector-valued functions naturally. In particular, for  $\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle$ , if

$$\lim_{t \rightarrow a} f(t) = L_1, \quad \lim_{t \rightarrow a} g(t) = L_2, \quad \lim_{t \rightarrow a} h(t) = L_3$$

then

$$\lim_{t \rightarrow a} \mathbf{r}(t) = \left\langle \lim_{t \rightarrow a} f(t), \lim_{t \rightarrow a} g(t), \lim_{t \rightarrow a} h(t) \right\rangle = \langle L_1, L_2, L_3 \rangle.$$

### Definition. (Limit of a Vector-Valued Function)

A vector-valued function  $\mathbf{r}$  approaches the limit  $\mathbf{L}$  as  $t$  approaches  $a$ , written  $\lim_{t \rightarrow a} \mathbf{r}(t) = \mathbf{L}$ , provided  $\lim_{t \rightarrow a} |\mathbf{r}(t) - \mathbf{L}| = 0$ .

A function  $\mathbf{r}(t)$  is **continuous** at  $t = a$ , provided  $\lim_{t \rightarrow a} \mathbf{r}(t) = \mathbf{r}(a)$ .

**Example.** Evaluate the following limits:

$$\lim_{t \rightarrow \pi} \left( \cos(t) \mathbf{i} - 7 \sin \left( -\frac{t}{2} \right) \mathbf{j} + \frac{t}{\pi} \mathbf{k} \right)$$

$$\lim_{t \rightarrow \infty} \left( \frac{t}{t-3} \mathbf{i} + \frac{40}{1+19e^{-t}} \mathbf{j} + \frac{1}{2t} \mathbf{k} \right)$$