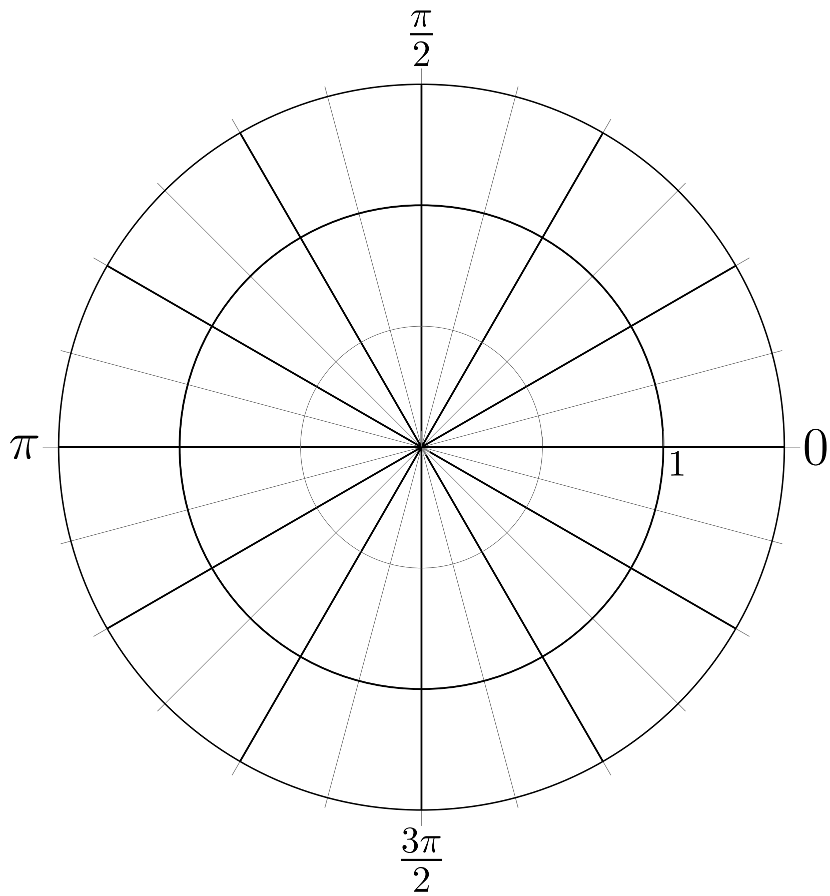
12.2: Polar Coordinates

Defining Polar Coordinates When using polar coordinates, the origin of the coordinate system is called the **pole**, and the positive x-axis is called the **polar axis**. The polar coordinates for a point P are of the form (r, θ).
The **radial coordinate** r describes the *signed* (*directed*) distance from the origin to P. The **angular coordinate** θ describes an angle whose initial side is the positive x-axis and whose terminal side lies on the ray passing through the origin and P.

Example (LC 33.4)**.** Graph the following polar coordinates

A) $\left(\frac{3}{2}, \frac{\pi}{2}\right)$ B) $\left(1, \frac{5\pi}{3}\right)$ C) $\left(\frac{3}{2}, \frac{7\pi}{4}\right)$ D) $\left(-1, \frac{-\pi}{3}\right)$



$\frac{\pi}{2}$
π
0
1
$\frac{3\pi}{2}$

Procedure: Converting Coordinates
A point with polar coordinates (r, θ) has Cartesian coordinates (x, y), where

$$x = r\cos\theta \qquad \text{and} \qquad y = r\sin\theta.$$

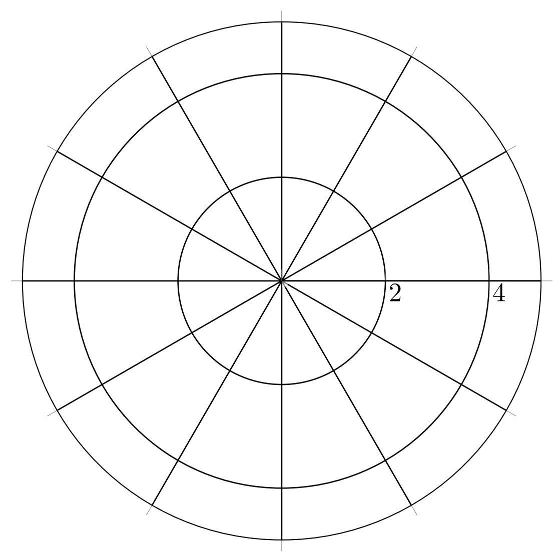
A point with Cartesian coordinates (x, y) has polar coordinates (r, θ), where

$$r^2 = x^2 + y^2 \qquad \text{and} \qquad \tan\theta = \frac{y}{x}.$$

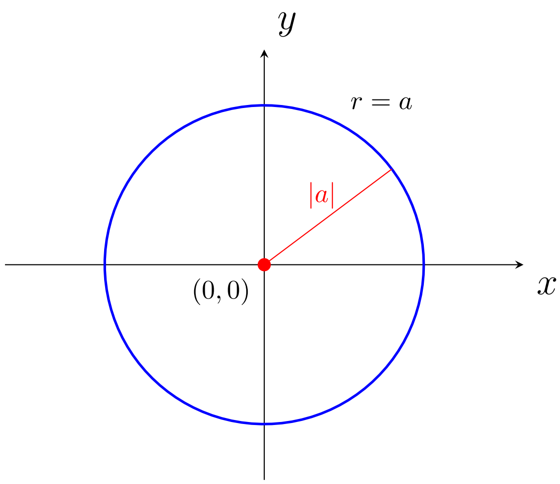
Example (LC 33.5)**.** Consider the Cartesian coordinate $\left(4\sqrt{3}, -4\right)$. Rewrite this point in polar coordinates. *Note*: There are infinitely many polar representations

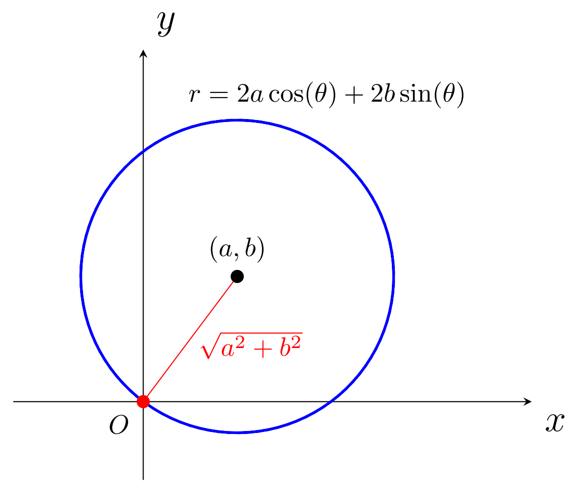
Example (LC 33.6)**.** Rewrite $y = 3$ in terms of polar coordinates.

Example (LC 33.7)**.** Graph $r = 4$ and $\theta = \frac{2\pi}{3}$



Summary: Circles in Polar Coordinates
The equation $r = a$ describes a circle of radius $|a|$ centered at $(0, 0)$.
The equation $r = 2a\cos\theta + 2b\sin\theta$ describes a circle of radius $\sqrt{a^2 + b^2}$ centered at (a, b).





Example. Rewrite the following in either polar coordinates or Cartesian coordinates

$r = 5\cos(\theta) + 12\sin(\theta)$

$x = \dfrac{3}{y}$

$r\cos(\theta) = \sin(2\theta)$

$y = x^2$

Procedure: Cartesian-to-Polar Method for Graphing $r = f(\theta)$

1. Graph $r = f(\theta)$ as if r and θ were Caresian coordinates with θ on the horizontal axis and r on the vertical axis. Be sure to choose an interval for θ on which the entire polar curve is produced.
2. Use the Cartesian graph that you created in Step 1 as a guide to sketch the points (r, θ) on the final *polar* curve.

Summary: Symmetry in Polar Equations

Symmetry about the x-axis occurs if the point (r, θ) is on the graph whenever $(r, -\theta)$ is on the graph.

Symmetry about the y-axis occurs if the point (r, θ) is on the graph whenever $r, \pi - \theta) = (-r, -\theta)$ is on the graph.

Symmetry about the origin occurs if the point (r, θ) is on the graph whenever $(-r, \theta) = (r, \theta + \pi)$ is on the graph.

Example (LC 33.8-33.9). Consider the polar curve $r = 2\sin(\theta) - 1$

Complete the table below

θ	0	$\pi/6$	$\pi/4$	$\pi/2$	π	$3\pi/2$
$r = 2\sin(\theta) - 1$						

Graph the polar curve $r = 2\sin(\theta) - 1$