1   15.4: The Chain Rule

Theorem 15.7: Chain Rule (One Independent Variable)

Let z be a differentiable function of x and y on its domain, where x and y are differentiable functions of t on an interval I. Then

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}.$$

Theorem 15.8: Chain Rule (Two Independent Variables)

Let z be a differentiable function of x and y, where x and y are differentiable functions of s and t. Then

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s} \quad \text{and} \quad \frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}.$$

Theorem 15.9: Implicit Differentiation

Let F be differentialbe on its domain and suppose $F(x, y) = 0$ defines y as a differentiable function of x. Provided $F_y \neq 0$,

$$\frac{dy}{dx} = -\frac{F_x}{F_y}.$$