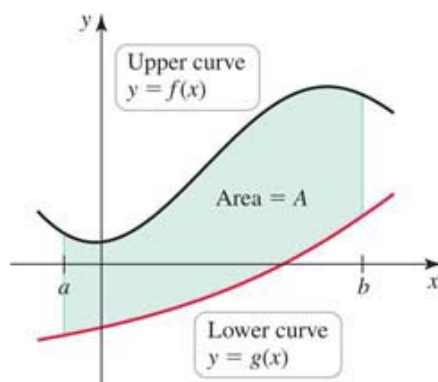


6.2: Regions Between Curves

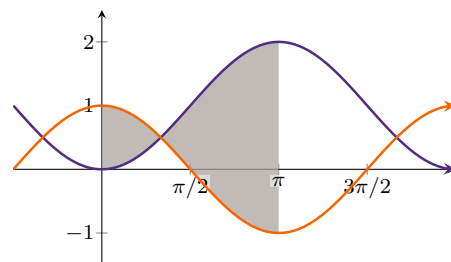
Definition. (Area of a Region Between Two Curves)

Suppose f and g are continuous functions with $f(x) \geq g(x)$ on the interval $[a, b]$. The area of the region bounded by the graphs of f and g on $[a, b]$ is

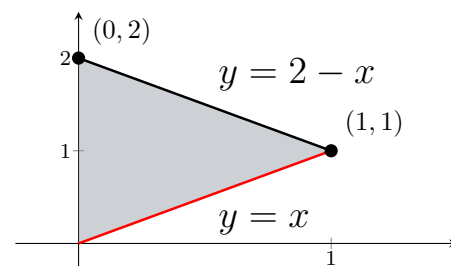
$$A = \int_a^b (f(x) - g(x)) \, dx.$$



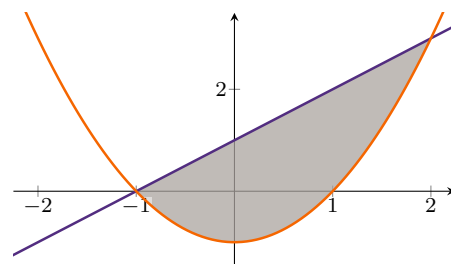
Example. Consider the region bounded by the curves $y = \cos(x)$ and $y = 1 - \cos(x)$, $0 \leq x \leq \pi$. Set up the integral(s) representing the area of this region.



Example. Find the area of the region by integrating with respect to x .



Example. Find the volume of the solid whose base is bounded by the graphs of $y = x + 1$ and $y = x^2 - 1$, with the cross sections in the shape of rectangles of height 2 taken perpendicular to the x -axis.



Definition. (Area of a Region Between Two Curves with Respect to y)

Suppose f and g are continuous functions with $f(y) \geq g(y)$ on the interval $[c, d]$. The area of the region bounded by the graphs $x = f(y)$ and $x = g(y)$ on $[c, d]$ is

$$A = \int_c^d (f(y) - g(y)) dy.$$

Example. Find the area of the region bounded by $x = 3y$, and $x = y^2 - 10$

by integrating with respect to x

by integrating with respect to y

Example. Find the area of the region bounded by $y = x^3$, and $y = \sqrt{x}$
by integrating with respect to x

by integrating with respect to y

Example. Find the area of the region bounded by $y = 4\sqrt{2x}$, $y = 2x^2$, and $y = -4x + 6$

