

Theorem 2.9: Continuity Rules

If f and g are continuous at a , then the following functions are also continuous at a . Assume c is a constant and $n > 0$ is an integer.

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| a) $f + g$ | b) $f - g$ |
| c) cf | d) fg |
| e) f/g , provided that $g(a) \neq 0$. | f) $(f(x))^n$ |

Theorem 2.1:0: Polynomial and Rational Functions

- a) A polynomial function is continuous for all x .
- b) A rational function (a function of the form $\frac{p}{q}$, where p and q are polynomials) is continuous for all x for which $q(x) \neq 0$.

Theorem 2.1:1: Continuity of Composite Functions at a Point

If g is continuous at a and f is continuous at $g(a)$, then the composite function $f \circ g$ is continuous at a .

Theorem 2.1:2: Limits of Composite Functions

1. If g is continuous at a and f is continuous at $g(a)$, then

$$\lim_{x \rightarrow a} f(g(x)) = f\left(\lim_{x \rightarrow a} g(x)\right).$$

2. If $\lim_{x \rightarrow a} g(x) = L$ and f is continuous at L , then

$$\lim_{x \rightarrow a} f(g(x)) = f\left(\lim_{x \rightarrow a} g(x)\right).$$

Theorem 2.1:3: Continuity of Functions with Roots

Assume n is a positive integer. If n is an odd integer, then $(f(x))^{1/n}$ is continuous at all points at which f is continuous.

If n is even, then $(f(x))^{1/n}$ is continuous at all points a at which f is continuous at $f(a) > 0$.

Theorem 2.1:4: Continuity of Inverse Functions

If a function f is continuous on an interval I and has an inverse on I , then its inverse f^{-1} is also continuous (on the interval consisting of the points $f(x)$, where x is in I).

Theorem 2.1:5: Continuity of Transcendental Functions

The following functions are continuous at all points of their domains.

Trigonometric

$\sin x$ $\cos x$
 $\tan x$ $\cot x$
 $\sec x$ $\csc x$

Inverse Trigonometric

$\sin^{-1} x$ $\cos^{-1} x$
 $\tan^{-1} x$ $\cot^{-1} x$
 $\sec^{-1} x$ $\csc^{-1} x$

Exponential

b^x e^x

Logarithmic

$\log_b x$ $\ln x$