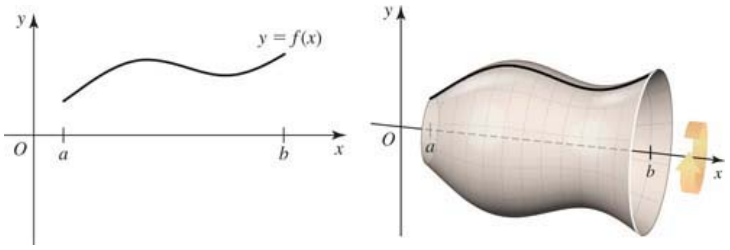
6.6: Surface Area

Definition. (Area of a Surface of Revolution)
Let f be a nonnegative function with a continuous first derivative on the interval $[a, b]$. The area of the surface generated when the graph of f on the interval $[a, b]$ is revolved around the x-axis is

$$S = \int_a^b 2\pi f(x)\sqrt{1 + f'(x)^2}\, dx.$$



Example. Find the exact area of the surface obtained by rotating the curve $y = x^3$, $0 \le x \le 2$ about the x-axis.

Example. Find the exact area of the surface obtained by rotating the curve $y = \sqrt{8x - x^2}$, $1 \leq x \leq 7$ about the x-axis.

Example. Find the exact area of the surface obtained by rotating the curve $y = \frac{1}{2}(e^x + e^{-x})$, $-\ln(2) \leq x \leq \ln(2)$ about the x-axis.