10.1 Common Factors

Example. Factor $3x^2y^3 + 15xy^4 - 21x^3y^2$

10.2 Special Formulas

1. $x^2 - y^2 = (x + y) \cdot (x - y)$
2. $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$
3. $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$
4. $x^2 + 2xy + y^2 = (x + y)^2$
5. $x^2 - 2xy + y^2 = (x - y)^2$
6. $acx^2 + (bc + ad)x + bd = (ax + b)(cx + d)$

Example. Factor:

- $z^2 - 9$ $= z^2 - (3)^2 = (z-3)(z+3)$
- $x^4 - y^4$ $= (x^2)^2 - (y^2)^2 = (x^2-y^2)(x^2+y^2) = (x-y)(x+y)(x^2+y^2)$
- $(x - y)^2 - 4y^2$ $= ((x-y)+2y)((x-y)-2y) = (x+y)(x-3y)$
- $a^3 + 8b^3$ $= (a+2b)(a^2 - 2ab + 4b^2)$
- $27x^3 + 64y^3z^6$ $= (3x)^3 + (4yz^2)^3 = (3x+4yz^2)(9x^2 - 12xyz^2 + 16y^2z^4)$
- $x^2 + 5x + 6$ $= (x+2)(x+3)$

10.3 Grouping

Example. Factor:

- $10xy + 15y + 4x + 6$ $= 5y(2x+3) + 2(2x+3)$ } Factor by grouping
  $= (5y+2)(2x+3)$

- $6x^2 - 11x - 7$ $= 6x^2 - 14x + 3x - 7$
  $= 2x(3x-7) + 1(3x-7)$
  $= (2x+1)(3x-7)$

  Diff signs
  Sign of larger number

  $6 \times 7 = 42 \cdot 1$
  $21 \cdot 2$
  $14 \cdot 3$ ← diff of 11
  $7 \cdot 6$

- $3x^2 + 10x + 8$

  $3 \cdot 8 = 24 \cdot 1$
  $12 \cdot 2$
  $8 \cdot 3$
  $6 \cdot 4$ ← sum of 10

  $3x^2 + 10x + 8 = 3x^2 + 6x + 4x + 8$
  $= 3x(x+2) + 4(x+2)$
  $= (3x+4)(x+2)$

  largest is pos
  same sign

- $6ax + 3ay - 4bx - 2by + 10x + 5y$

  $= 3a(2x+y) - 2b(2x+y) + 5(2x+y)$

  $= (3a - 2b + 5)(2x+y)$

10.4 The Factor Theorem and Long Division

Theorem (The Factor Theorem). *Let $P(x)$ be a polynomial. Let a be any real number. Then $x - a$ is a factor of $P(x)$ if and only if $P(a) = 0$.*

Example. Factor $P(x) = x^3 - 2x^2 - 5x + 6$.

$P(1) = 1 - 2 - 5 + 6 = 7 - 7 = 0$

-7

7

$$x^2 - x - 6$$
$$x-1 \;\overline{)\; x^3 - 2x^2 - 5x + 6}$$
$$-(x^3 - x^2)$$
$$-x^2 - 5x$$
$$-(-x^2 + x)$$
$$-6x + 6$$
$$-(-6x + 6)$$
$$0$$

$\Rightarrow x^3 - 2x^2 - 5x + 6 = (x-1)(x^2 - x - 6)$

$= (x-1)(x-3)(x+2)$

Definition. The **Quadratic Formula** is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where $ax^2 + bx + c = 0$.

Example. Factor

- $2x + 3x - 2$

$x = \frac{-3 \pm \sqrt{(3)^2 - 4(2)(-2)}}{2(2)} = \frac{-3 \pm \sqrt{9+16}}{4} = \frac{-3 \pm 5}{4} \searrow -2$, $\nearrow \frac{1}{2}$

$\hookrightarrow 2(x+2)(x-\frac{1}{2})$

- $x^2 + x + 1$

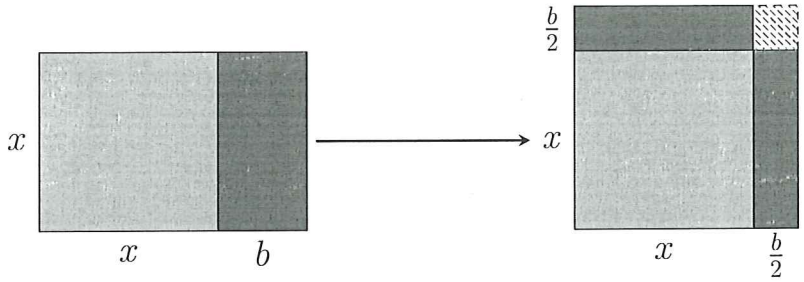
$x = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(1)}}{2} = \frac{-1 \pm \sqrt{-3}}{2}$ ← No Real roots!

2.1 Completing the Square

horizontal shift

Vertical shift

$$ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a} = a(x - h)^2 + k$$



Example. Complete the square for $f(x) = x^2 - 3x + 4$

$$f(x) = \left(x^2 - 3x + \left(\tfrac{3}{2}\right)^2\right) - \left(\tfrac{3}{2}\right)^2 + 4$$

$$= \left(x - \tfrac{3}{2}\right)^2 + \frac{7}{4}$$

Example. Complete the square for $f(x) = 4x^2 + 20x - 100$

$$f(x) = 4\left(x^2 + 5x + \left(\tfrac{5}{2}\right)^2 - \left(\tfrac{5}{2}\right)^2\right) - 100$$

$$= 4\left(x^2 + 5x + \left(\tfrac{5}{2}\right)^2\right) - 4\left(\tfrac{25}{4}\right) - 100$$

$$= 4\left(x + \tfrac{5}{2}\right)^2 - 125$$

Definition. The equation of a circle centered at (h, k) with radius r is given by

$$(x-h)^2 + (y-k)^2 = r^2$$

Example. Identify the center and radius of

$$x^2 - 2x + y^2 + 2y = 2$$

$(x^2 - 2x + \underline{\quad 1 \quad}) + (y^2 + 2y + \underline{\quad 1 \quad}) = 2 + 1 + 1$

$\left[\frac{1}{2}(-2)\right]^2$ $\left[\frac{1}{2}(2)\right]^2$

$(x^2 - 2x + 1) + (y^2 + 2y + 1) = 4$

$(x-1)^2 + (y+1)^2 = 2^2$

This is the circle centered at $(1, -1)$ with radius 2.