1 13.5: Lines and Planes in Space

Equation of a Line

A **vector equation of the line** passing through the point $P_0(x_0, y_0, z_0)$ in the direction of the vector $\mathbf{v} = \langle a, b, c \rangle$ is $\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}$, or

$$\langle x, y, z \rangle = \langle x_0, y_0, z_0 \rangle + t\langle a, b, c \rangle, \quad \text{for} \quad -\infty < t < \infty$$

Equivalently, the corresponding **parametric equations of the line** are

$$x = x_0 + at, \quad y = y_0 + bt, \quad z = z_0 + ct, \quad \text{for} \quad -\infty < t < \infty$$

Distance Between a Point and a Line

The distance d between the point Q and the $\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}$ is

$$d = \frac{\left|\mathbf{v} \times \overrightarrow{PQ}\right|}{|\mathbf{v}|},$$

where P is any point on the line and $\mathbf{v}$ is a vector parallel to the line.

Definition. (Plane in $\mathbb{R}^3$)

Given a fixed point P_0 and a nonzero **normal vector n**, the set of points P in $\mathbb{R}^3$ for which $\overrightarrow{P_0P}$ is orthogonal to $\mathbf{n}$ is called a **plane** (Figure 13.72)

General Equation of a Plane in $\mathbb{R}^3$

The plane passing through the point $P_0(x_0, y_0, z_0)$ with a nonzero normal vector $\mathbf{n} = \langle a, b, c \rangle$ is described by the equation

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0 \quad \text{or} \quad ax + by + cz = d,$$

where $d = ax_0 + by_0 + cz_0$.

Definition. (Parallel and Orthogonal Planes)

Two distinct planes are **parallel** if their respective normal vectors are parallel (that is, the normal vectors are scaling multiples of each other). Two plans are **orthogonal** if their respective normal vectors are orthogonal (that is, the dot product of the normal vectors is *zero*).