8/31

Example. For $m(\omega) = 40\omega - 5\omega^2$, find

Domain of $m(\omega)$:

$(-\infty, \infty)$

$ax^2 + bx + c = y$

Range of $m(\omega)$:

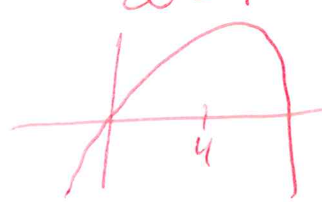
vertex at $\frac{-b}{2a}$

maximum/Minimum at $\omega = 0$?

$\Rightarrow$ max at $\omega = 4$

$m(4) = 40(4) - 5(4)^2 = 160 - 80 = 80$

$(-\infty, 80$



Example. A cylindrical water tower with a radius of 10m and a height of 50m is filled to a height of h. The volume V of water (in cubic meters) is given by the function $g(h) = 100\pi h$. Identify the independent and dependent variables of for this function, and then determine an appropriate domain.

Domain: $[0, 50]$

Range: $[0, 5000\pi]$.

uses 2 points

4.2 Lines And Their Equations

Definition. The **slope** of a line is $m = \dfrac{\Delta y}{\Delta x} = \dfrac{y_1 - y_2}{x_1 - x_2}$.

The slope is the rate at which y increases or decreases with respect to x.

Definition. The **point-slope** formula of the line with slope m going through point (x_0, y_0) is

$$y - y_0 = m(x - x_0)$$

Example. Find the equation of the line with slope $m = -3$ that goes through the point $P_1 = (2, -5)$.

x_0 y_0

$y - y_0 = m(x - x_0)$

$y - (-5) = -3(x-2)$

$y = -3x + 6 - 5$

$y = -3x + 1$

Check

$-3(2)+1 = -6+1 = -5$

Example. Find the equation of the line that goes through $P_1 = (1, -2)$ and $P_2 = (-2, 3)$.

Find slope

$m = \dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{3-(-2)}{-2-1} = -\dfrac{5}{3}$

$y - y_0 = m(x - x_0)$

$y - 3 = -\frac{5}{3}(x+2)$

$y = -\frac{5}{3}x - \frac{10}{3} + 3\left(\frac{3}{3}\right) = -\frac{5}{3}x - \frac{1}{3}$

Check $-\frac{5}{3}(1) - \frac{1}{3} = -\frac{6}{3} = -2$

$-\frac{5}{3}(-2) - \frac{1}{3} = \frac{10}{3} - \frac{1}{3} = \frac{9}{3} = 3$

Definition. The **slope-intercept form** of the line with slope m and intercept b is

$$y = mx + b$$

Example. Find the equation of the line with slope $m = 3$ that with intercept $b = -1$.

$y = mx + b$

$y = 3x - 1$

Example. Find the equation of the line that goes through $P_1 = (0, 1/2)$ and $P_2 = (4, -1/2)$.

intercept

$m = \dfrac{-1/2 - 1/2}{4 - 0} = \dfrac{-1}{4}$

$y - y_0 = m(x - x_0)$

$\Rightarrow y - (-1/2) = -\frac{1}{4}(x - 4)$

$y = -\frac{1}{4}x + 1 - 1/2 = -\frac{1}{4}x + \frac{1}{2}$

$y = mx + b$

$y = -\frac{1}{4}x + \frac{1}{2}$

Check

$-1/4(0) + 1/2 = \frac{1}{2}$ ✓

$-1/4(4) + 1/2 = -1 + 1/2 = -1/2$ ✓

Definition. Two lines, with slopes m_1 and m_2 are **parallel** when $m_1 = m_2$. Lines are **perpendicular** when $m_1 = -\dfrac{1}{m_2}$

Example. Find the line parallel to $f(x) = -1/2x + 4$ that goes through the point $P_1 = (1, 4)$. Also find the line perpendicular to $f(x)$ that goes through the point $P_2 = (2, -3)$.

$\longrightarrow m_1 = -\frac{1}{2} \longrightarrow y_1 - 4 = -\frac{1}{2}(x-1)$

$y_1 = -\frac{1}{2}x + \frac{1}{2} + 4 = -\frac{1}{2}x + \frac{9}{2}$

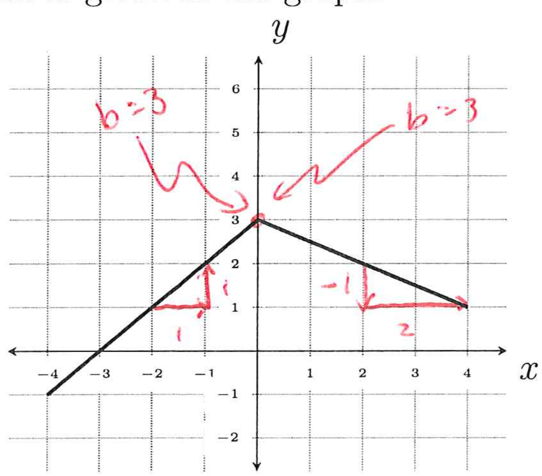
Check: $-\frac{1}{2}(1) + \frac{9}{2} = -\frac{1}{2} + \frac{9}{2} = \frac{8}{2} = 4$ ✓

$m_2 = -\frac{1}{-1/2} = 2 \longrightarrow y_2 - (-3) = 2(x-2)$

$y_2 = 2x - 4 - 3 = 2x - 7$

Check $2(2) - 7 = 4 - 7 = -3$ ✓

Example. (Briggs: 1.2.7) Write a definition of the piecewise linear function $y = f(x)$ that is given in the graph.



$m = \frac{1}{1}$ $\qquad$ $m = \frac{-1}{2}$

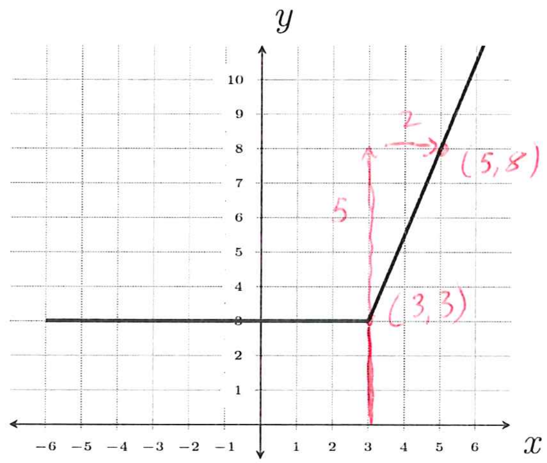
$$f(x) = \begin{cases} x + 3, & x \leq 0 \\ -\frac{1}{2}x + 3, & x > 0 \end{cases}$$

Note: Only one interval contains zero!

e.g. $x \leq 0$, $x \geq 0$ Not allowed!

(Brigs: 1.2.25, 1.2.26) Write a definition of the function whose graph is given.

Example.



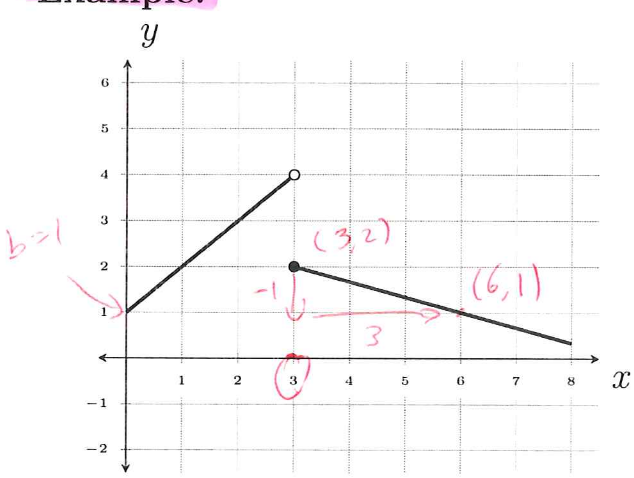
$$g(x)=\begin{cases} 3, & x\le 3 \\ \frac{5}{2}x-\frac{9}{2}, & x>3 \end{cases}$$

$m=\frac{8-3}{5-3}=\frac{5}{2}$

$y-3=\frac{5}{2}(x-3)$

$y=\frac{5}{2}x-\frac{15}{2}+3=\frac{5}{2}x-\frac{9}{2}$

Example.



$$h(x)=\begin{cases} x+1, & x<3 \\ -\frac{1}{3}x+3, & x\ge 3 \end{cases}$$ Notice intervals!

$m=\frac{2-1}{3-6}=\frac{1}{-3}=-\frac{1}{3}$

$y-1=-\frac{1}{3}(x-6)$

$y=-\frac{1}{3}x+3$

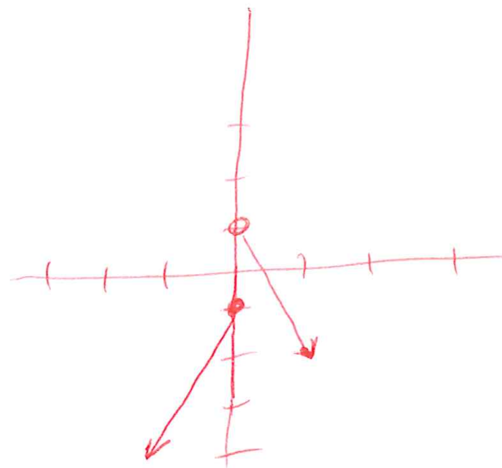
Check $-\frac{1}{3}(6)+3=-\frac{6}{3}+3$

$=-2+3$

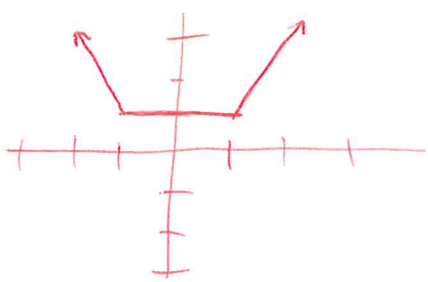
$=1$ ✓

(Briggs: 1.2.31, 1.2.33, 1.2.34) Graph the following functions

Example. $f(x) = \begin{cases} 3x - 1 & \text{if } x \leq 0 \\ -2x + 1 & \text{if } x > 0 \end{cases}$



Example. $f(x) = \begin{cases} -2x - 1 & \text{if } x \leq -1 \\ 1 & \text{if } -1 \leq x \leq 1 \\ 2x - 1 & \text{if } x > 1 \end{cases}$



Example. $f(x) = \begin{cases} 2x + 2 & \text{if } x < 0 \\ x + 2 & \text{if } 0 \leq x \leq 2 \\ 3 - \frac{x}{2} & \text{if } x > 2 \end{cases}$