6.3: Volume by Slicing

General Slicing Method

Suppose a solid object extends from $x = a$ to $y = b$, and the cross section of the solid perpendicular to the x-axis has an area given by a function A that is integrable on $[a, b]$. The volume of the solid is

$$V = \int_a^b A(x)\,dx.$$

Disk Method about the x-Axis

Let f be continuous with $f(x) \geq 0$ on the interval $[a, b]$. If the region R bounded by the graph of f, the x-axis, and the lines $x = a$ and $x = b$ is revolved about the x-axis, the volume of the resulting solid of revolution is

$$V = \int_a^b \pi\, \underbrace{f(x)}_{\substack{\text{disk}\\ \text{radius}}}{}^2 dx.$$

Washer Method about the x-Axis

Let f and g be continuous functions with $f(x) \geq g(x) \geq 0$ on $[a, b]$. Let R be the region bounded by $y = f(x)$, $y = g(x)$, and the lines $x = a$ and $x = b$. When R is revolved about the x-axis, the volume of the resulting solid of revolution is

$$V = \int_a^b \pi\left(\underbrace{f(x)}_{\substack{\text{outer}\\ \text{radius}}}{}^2 - \underbrace{g(x)}_{\substack{\text{inner}\\ \text{radius}}}{}^2 \right) dx.$$

Disk and Washer Methods about the y-Axis

Let p and q be continuous functions with $p(y) \geq q(y) \geq 0$ on $[c, d]$. Let R be the region bounded by $x = p(y)$, $x = q(y)$, and the lines $y = c$ and $y = d$. When R is revolved around the y-axis, the volume of the resulting solid of revolution is given by

$$V = \int_c^d (\underbrace{p(y)}_{\substack{\text{outer} \\ \text{radius}}}{}^2 - \underbrace{q(y)}_{\substack{\text{inner} \\ \text{radius}}}{}^2)\, dy.$$

If $q(y) = 0$, the disk method results:

$$V = \int_c^d \underbrace{p(y)}_{\substack{\text{outer} \\ \text{radius}}}{}^2 dy.$$