

1. Make sure you're using the correct rule:

Function	Derivative rule	Example
c^b	Constant	$\frac{d}{dx}[\pi^e] = 0$
x^c	Power	$\frac{d}{dx}[x^3] = 3x^2$
c^x	Exponent	$\frac{d}{dx}[c^x] = \ln(c) c^x$
$\log_b(x)$	Logarithm	$\frac{d}{dx}[\log_2(x)] = \frac{d}{dx}\left[\frac{\ln(x)}{\ln(2)}\right] = \frac{1}{\ln(2)x}$

2. Make sure you're familiar with the laws of logarithms! Simplifying before taking a derivative can make the problem much easier!!

$$\begin{aligned} &\frac{d}{dx}\left[\ln\left(\frac{(42x^2+1)^2(33x-15)^3(110x^{-1}-1)^4}{(2x+7)^3\cos(x)e^{-x}}\right)\right] \\ &= \frac{d}{dx}\left[2\ln(42x^2+1)+3\ln(33x-15)+4\ln(110x^{-1}-1)-3\ln(2x+7)-\ln(\cos(x))+x\right] \\ &= 2\frac{84x}{42x^2+1}+3\frac{33}{33x-15}+4\frac{-110x^{-2}}{110x^{-1}-1}-3\frac{2}{2x+7}-\frac{-\sin(x)}{\cos(x)}+1 \end{aligned}$$

3. Logarithmic differentiation can be used to “break apart” functions of the form $f(x)^{g(x)}$. Notice that this is when the base AND the exponent of the function contain a variable:

$$\begin{aligned} f(x) &= (3x^2+1)^{\cos(x)} \Rightarrow \ln(f(x)) = \ln\left((3x^2+1)^{\cos(x)}\right) \\ &\Rightarrow \ln(f(x)) = \cos(x) \ln(3x^2+1) \end{aligned}$$

The next step uses chain rule on the left-hand side and product rule on the right:

$$\Rightarrow \frac{1}{f(x)}f'(x) = \frac{d}{dx}[\cos(x)] \ln(3x^2+1) + \cos(x)\frac{d}{dx}[\ln(3x^2+1)]$$

Use the derivative of $\cos(x)$ and $\ln(f(x))$ (chain-rule)

$$\begin{aligned} \Rightarrow \frac{f'(x)}{f(x)} &= -\sin(x) \ln(3x^2+1) + \cos(x)\frac{1}{3x^2+1} \cdot \frac{d}{dx}[3x^2+1] \\ \Rightarrow \frac{f'(x)}{f(x)} &= -\sin(x) \ln(3x^2+1) + \cos(x)\frac{6x}{3x^2+1} \end{aligned}$$

Since the original function was only in terms of x , we need to write the derivative solely in terms of x . We do this by multiplying both sides by $f(x)$ and then substituting the original function back in:

$$\begin{aligned} \Rightarrow f'(x) &= f(x)\left(-\sin(x) \ln(3x^2+1) + \cos(x)\frac{6x}{3x^2+1}\right) \\ \Rightarrow \boxed{f'(x) &= (3x^2+1)^{\cos(x)}\left(-\sin(x) \ln(3x^2+1) + \cos(x)\frac{6x}{3x^2+1}\right)} \end{aligned}$$

4. MyLab Math has examples where you can rewrite a function using e^x and $\ln(x)$. This method is equivalent:

$$\begin{aligned}f(x) &= (3x^2 + 1)^{\cos(x)} = e^{\ln((3x^2+1)^{\cos(x)})} = e^{\cos(x) \ln(3x^2+1)} \\f'(x) &= e^{\cos(x) \ln(3x^2+1)} \frac{d}{dx} [\cos(x) \ln(3x^2 + 1)] \\f'(x) &= e^{\cos(x) \ln(3x^2+1)} \left(\frac{d}{dx} [\cos(x)] \ln(3x^2 + 1) + \cos(x) \frac{d}{dx} [\ln(3x^2 + 1)] \right) \\f'(x) &= e^{\cos(x) \ln(3x^2+1)} \left(-\sin(x) \ln(3x^2 + 1) + \cos(x) \frac{6x}{3x^2 + 1} \right) \\f'(x) &= (3x^2 + 1)^{\cos(x)} \left(-\sin(x) \ln(3x^2 + 1) + \cos(x) \frac{6x}{3x^2 + 1} \right)\end{aligned}$$