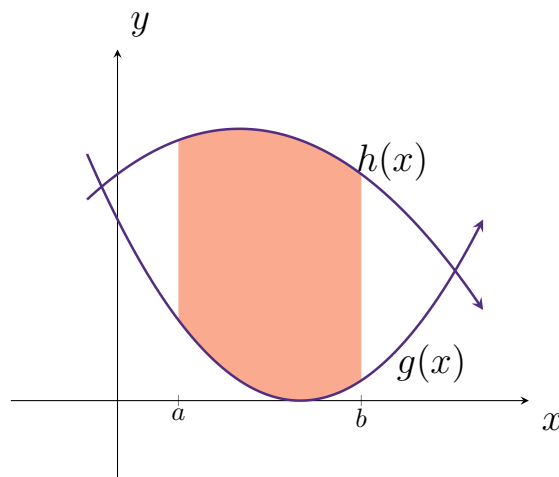


## 16.2: Double Integrals over General Regions

In this section, we consider double integrals over non-rectangular regions. For instance, my domain for  $x$  and  $y$  can be constrained where  $a \leq x \leq b$  and  $g(x) \leq y \leq h(x)$ :



### Theorem 16.2: Double Integrals over Nonrectangular Regions

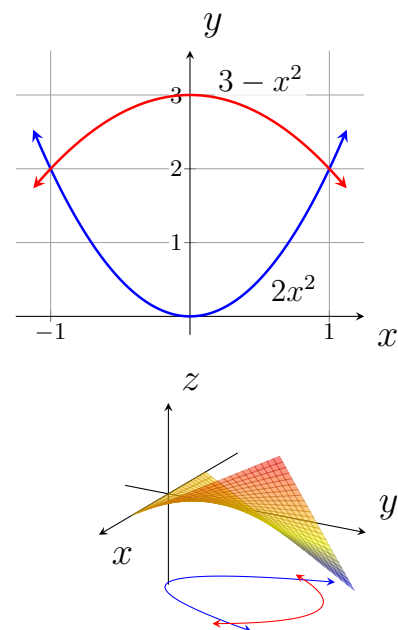
Let  $R$  be a region bounded below and above by the graphs of the continuous functions  $y = g(x)$  and  $y = h(x)$ , respectively, and by the lines  $x = a$  and  $x = b$ . If  $f$  is continuous on  $R$ , then

$$\iint_R f(x, y) dA = \int_a^b \int_{g(x)}^{h(x)} f(x, y) dy dx.$$

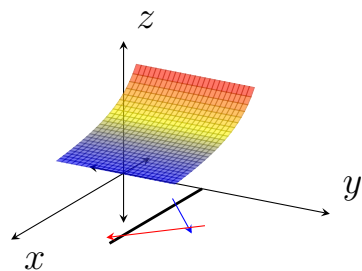
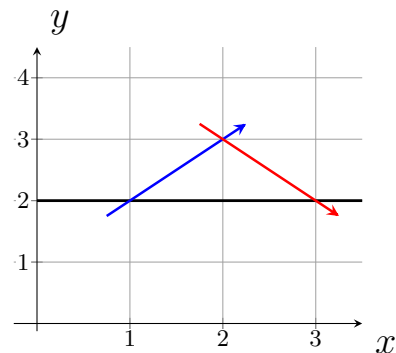
Let  $R$  be a region bounded on the left and right by the graphs of the continuous functions  $x = g(y)$  and  $x = h(y)$ , respectively, and the lines  $y = c$  and  $y = d$ . If  $f$  is continuous on  $R$ , then

$$\iint_R f(x, y) dA = \int_c^d \int_{g(y)}^{h(y)} f(x, y) dx dy.$$

**Example.** Consider the surface generated by the function  $f(x, y) = 3xy$ . Find the volume of the solid generated by  $f(x, y)$  over the region bounded by  $2x^2$  and  $3 - x^2$ .



**Example.** Find the area under  $f(x, y) = \frac{1}{x} + 1$  over the region formed by the lines  $x = 2$ ,  $y = 1 + x$ , and  $y = 5 - x$ .



**Example.** Find the volume of the tetrahedron in the first octant bounded by the plane  $z = c - ax - by$  and the coordinate planes ( $x = 0$ ,  $y = 0$ , and  $z = 0$ ). Assume  $a$ ,  $b$ , and  $c$  are positive real numbers.

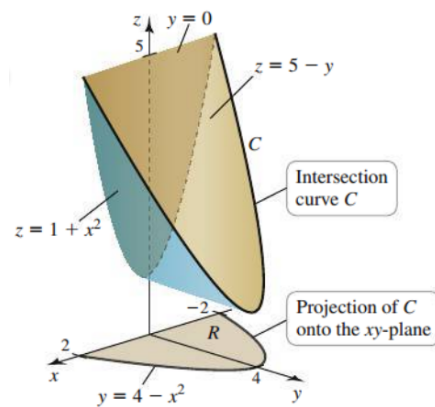
**Example.** For the following problems, reverse the order of integration

- $\int_0^2 \int_0^{2x} f(x, y) \, dy \, dx$

- $\int_0^1 \int_{x^3}^{\sqrt{x}} f(x, y) \, dy \, dx$

- $\int_{-3}^4 \int_{2x^2}^{2x+24} f(x, y) \, dy \, dx$

**Example.** Find the volume between  $f(x, y) = 5 - y$  and  $g(x, y) = 1 + x^2$  over the region  $R = \{(x, y) : 0 \leq y \leq 4 - x^2, -2 \leq x \leq 2\}$ .



### Areas of Regions by Double Integrals

Let  $R$  be a region in the  $xy$ -plane. Then

$$\text{area of } R = \iint_R dA.$$

**Example.** Find the area of the region  $R$  bounded by  $y = x^2$ ,  $y = 6 - x$ , and  $y = 6 + 5x$ .