6.7: Physical Applications

Definition. (Mass of a One-Dimensional Object)
Suppose a thin bar or wire is represented by the interval $a \leq x \leq b$ with a density function ρ (with units of mass per length). The **mass** of the object is

$$m = \int_a^b \rho(x)\,dx.$$

Definition. (Work)
The work done by a variable force F moving an object along a line from $x = a$ to $x = b$ in the direction of the force is

$$W = \int_a^b F(x)\,dx.$$

Procedure: Solving Pumping Problems

1. Draw a y-axis in the vertical direction (parallel to gravity) and choose a convenient origin. Assume the interval $[a, b]$ corresponds to the vertical extent of the fluid.
2. For $a \leq y \leq b$, find the cross-sectional area $A(y)$ of the horizontal slices and the distance $D(y)$ the slices must be lifted.
3. The work required to lift the water is

$$W = \int_a^b \rho g A(y) D(y)\,dy.$$

Procedure: Solving Force-on-Dam Problems

1. Draw a y-axis on the face of the dam in the vertical direction and choose a convenient origin (often taken to be the base of the dam).
2. Find the width function $w(y)$ for each value of y on the face of the dam.
3. If the base of the dam is at $y = 0$ and the top of the dam is at $y = a$, then the total force on the dam is

$$F = \int_a^b \rho g \underbrace{(a - y)}_{\text{depth}} \underbrace{w(y)}_{\text{width}} \, dy.$$