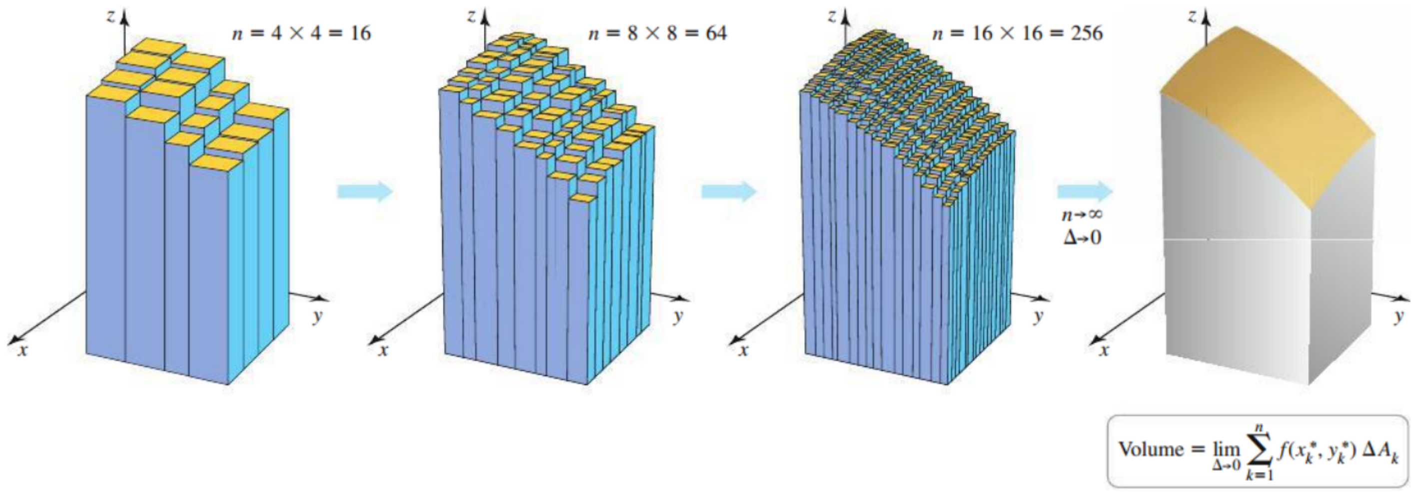
1 16.1: Double Integrals over Rectangular Regions



Definition. (Double Integrals)
A function f defined on a rectangular region R in the xy-plane is **integrable** on R if $\lim_{\Delta \to 0} \sum_{k=1}^{n} f(x_k^*, y_k^*)\,\Delta A_k$ exists for all partitions of R and for all choices of (x_k^*, y_k^*) within those partitions. The limit is the **double integral of f over R**, which we write

$$\iint_R f(x,y)\,dA = \lim_{\Delta \to 0} \sum_{k=1}^{n} f(x_k^*, y_k^*)\,\Delta A_k.$$

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Theorem 16.1: (Fubini) Double Integrals over Rectangular Regions
Let f be continuous on the rectangular region $R = \{(x, y) : a \le x \le b,\ c \le y \le d\}$. The double integral of f over R may be evaluated by either of the two iterated integrals:

$$\iint\limits_R f(x, y)\, dA = \int_c^d \int_a^b f(x, y)\, dx\, dy = \int_a^b \int_c^d f(x, y)\, dy\, dx.$$

Example. Find the volume of the solid bounded by the surface $f(x, y) = 4 + 9x^2y^2$ over the region $R = \{(x, y) : -1 \leq x \leq 1,\ 0 \leq y \leq 2\}$. Integrate with respect to x first, then with respect to y first.

Example. Evaluate $\displaystyle\iint_R ye^{xy}\,dA$, where $R = \{(x, y) : 0 \le x \le 1, 0 \le y \le \ln(2)\}$.

Definition. (Average Value of a Function over a Plane Region)
The **average value** of an integrable function f over a region R is

$$\bar{f} = \frac{1}{\text{area of } R} \iint\limits_R f(x,y)\,dA.$$

Example. Find the average value of $f(x,y) = 2 - x - y$ over the region $R = \{(x,y) : 0 \le x \le 2,\ 0 \le y \le 2\}$.