1 6.7: Physical Applications

Definition. (Mass of a One-Dimensional Object)
Suppose a thin bar or wire is represented by the interval $a \leq x \leq b$ with a density function ρ (with units of mass per length). The **mass** of the object is

$$m = \int_a^b \rho(x)\, dx.$$

Example. A thin bar, represented by the interval $0 \leq x \leq 4$, has density in units of kg/m given by $\rho(x) = 5e^{-2x}$. What is the mass of the bar?

Definition. (Work)
The work done by a variable force F moving an object along a line from $x = a$ to $x = b$ in the direction of the force is

$$W = \int_a^b F(x)\,dx.$$

Example. According to **Hooke's Law**, the force required to keep a spring in a compressed or stretched position x units from the equilibrium position is $F(x) = kx$, where the positive spring constant k measures the stiffness of the spring.

Suppose a force of $40N$ is required to stretch a spring $0.1m$ from its equilibrium position. Assuming the spring obeys Hooke's Law, how much work is required to stretch the spring $0.4m$ beyond is equilibrium position?

Example. Imagine a chain of length L meters with constant density ρ kg/m is hanging vertically. Using g to represent the force due to gravity, the work required to lift the chain is

$$W = \int_0^L \rho g(L - y)\, dy$$

A 50 meter long chain hangs vertically from a cylinder attached to a winch. Assume there is no friction in the system and the chain has a density of 3 kg/m. How much work is required to wind the entire chain onto the cylinder if a 60-kg load is attached to the end of the chain? Use g for the acceleration due to gravity.

Example. A 30-meter long rope hangs freely from a ledge. The rope has a density of 5 kg/m. How much work is done if the top 1/3 of the rope is pulled up to the ledge? Use g for the acceleration due to gravity.

Procedure: Solving Pumping Problems

1. Draw a y-axis in the vertical direction (parallel to gravity) and choose a convenient origin. Assume the interval $[a, b]$ corresponds to the vertical extent of the fluid.
2. For $a \leq y \leq b$, find the cross-sectional area $A(y)$ of the horizontal slices and the distance $D(y)$ the slices must be lifted.
3. The work required to lift the water is

$$W = \int_a^b \rho g A(y) D(y)\, dy.$$

Procedure: Solving Force-on-Dam Problems

1. Draw a y-axis on the face of the dam in the vertical direction and choose a convenient origin (often taken to be the base of the dam).
2. Find the width function $w(y)$ for each value of y on the face of the dam.
3. If the base of the dam is at $y = 0$ and the top of the dam is at $y = a$, then the total force on the dam is

$$F = \int_a^b \rho g \underbrace{(a - y)}_{\text{depth}} \underbrace{w(y)}_{\text{width}}\, dy.$$