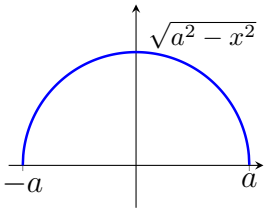
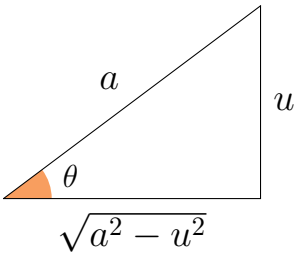
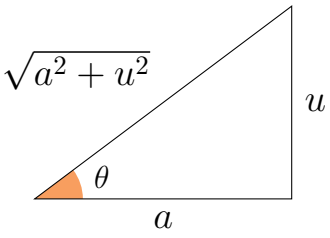
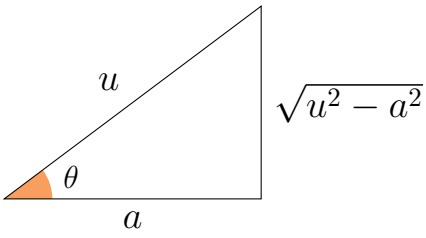
8.4: Trigonometric Substitutions

Example. Verify the formula for the area of a circle with radius a by finding the area under $f(x) = \sqrt{a^2 - x^2}$.









$a^2 - u^2$	$u = a\sin(\theta)$,	$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$, for $\lvert u\rvert \le a$	$a^2 - a^2\sin^2(\theta) = a^2\cos^2(\theta)$
$a^2 + u^2$	$u = a\tan(\theta)$,	$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$,	$a^2 + a^2\tan^2(\theta) = a^2\sec^2(\theta)$
$u^2 - a^2$	$u = a\sec(\theta)$,	$\begin{cases} 0 \le \theta < \frac{\pi}{2}, & \text{for } u \ge a \\ \frac{\pi}{2} < \theta \le \pi, & \text{for } u \le -a \end{cases}$	$a^2\sec^2(\theta) - a^2 = a^2\tan^2(\theta)$

Example. $\displaystyle\int \frac{\sqrt{x^2-4}}{x^3}\,dx$

Example. $\displaystyle\int \frac{\sqrt{16-x^2}}{x}\,dx$

Example. $\displaystyle\int \frac{x^2}{(25-4x^2)^{3/2}}\,dx$

Example. $\displaystyle\int_0^{1/3} \frac{dx}{(9x^2+1)^{3/2}}$

Example. $\displaystyle\int \frac{x}{\sqrt{x^2-2x+10}}$