8.2: Integration by Parts

Integraton by Parts

Suppose u and v are differentiable functions. Then

$$\boxed{\int u\,dv = uv - \int v\,du.}$$

A good mnemonic is ILATE.

$$\int \frac{d}{dx}[u\,v]\,dx = \int \frac{du}{dx}\,v\,dx + \int u\,\frac{dv}{dx}\,dx \;\rightarrow\; uv = \int v\,du + \int u\,dv$$

Inverse trig
Logarithmic
Algebraic
Trig
Exponential

↑ precedence of taking the derivative

Example. Evaluate $\int xe^{-\frac{x}{2}}\,dx$.

algebraic, expo

Inverse trig
Logarithmic
→ Algebraic
Trig
→ Exponential

$$\int u\,dv = uv - \int v\,du$$

$u = x$ $\quad v = -2e^{-x/2}$

$du = dx$ $\quad dv = e^{-x/2}\,dx$

$$\int xe^{-x/2}\,dx = -2xe^{-x/2} - \int -2e^{-x/2}\,dx$$

$$= -2xe^{-x/2} + 2\int e^{-x/2}\,dx$$

$$= -2xe^{-x/2} - 4e^{-x/2} + C$$

$$\int u\,dv = uv - \int v\,du$$

Example. Find the area of the region between the x-axis and $f(x) = \dfrac{\ln(x)}{x^2}$ on $[1, e]$.

$$\int_1^e \frac{\ln(x)}{x^2}\,dx \qquad u = \ln(x) \qquad v = -x^{-1}$$

$$du = \frac{1}{x}\,dx \qquad dv = x^{-2}\,dx$$

Inverse trig
→ Logarithmic
→ Algebraic
Trig
Exponential

$$= \left[-\frac{\ln(x)}{x}\right]_1^e - \int_1^e -\frac{1}{x}\left(x^{-1}\right)dx$$

Example. Evaluate $\int x^2 \cos(2x)\, dx$.

Example. Evaluate $\int e^{-x} \sin(3x)\, dx$.

Example. Evaluate $\int e^{4x} \cos(3x)\, dx$.

Example. Derive the integral formula

$$\int \ln(x)\,dx + x\ln(x) - x + C$$

Example. Evaluate $\int 10\cos(\sqrt{x})\,dx$

Integration by Parts for Definite Integrals
Let u and v be differentiable. Then

$$\int_a^b u(x)v'(x)\,dx = u(x)v(x)\Big|_a^b - \int_a^b v(x)u'(x)\,dx$$

Example. Evaluate $\displaystyle\int_1^e \ln(2x)\,dx$.