

8.5: Partial Fractions

Example. Simplify $f(x) = \frac{1}{x-2} + \frac{2}{x+4}$ by finding a common denominator.

Procedure: Partial Fractions with Simple Linear Factors

Suppose $f(x) = p(x)/q(x)$, where p and q are polynomials with no common factors and with the degree of P less than the degree of q . Assume q is the product of simple linear factors. The partial fraction decomposition is obtained as follows.

Step 1: Factor the denominator q in the form $(x - r_1)(x - r_2) \dots (x - r_n)$

Step 2: Partial fraction decomposition

$$\frac{p(x)}{q(x)} = \frac{A_1}{(x - r_1)} + \frac{A_2}{(x - r_2)} + \dots + \frac{A_n}{(x - r_n)}.$$

Step 3: Clear denominators Multiply both sides of the equation in Step 2 by $q(x) = (x - r_1)(x - r_2) \dots (x - r_n)$

Step 4: Solve for coefficients Equate like powers of x in Step 3 to solve for the undetermined coefficients A_1, \dots, A_n .

Example. Perform partial fraction decomposition on $f(x) = \frac{3x}{x^2 + 2x - 8}$.

Example. $\int \frac{28x^3 - 56x^2 + 9}{x^2 - 2x}$

Procedure: Partial Fractions for Repeated Linear Factors

Suppose the repeated linear factor $(x - r)^m$ appears in the denominator of a proper rational function in reduced form. The partial fraction decomposition has a partial fraction for each power of $(x - r)$ up to and including the m th power; that is, the partial fraction decomposition contains the sum

$$\frac{A_1}{(x - r)} + \frac{A_2}{(x - r)^2} + \frac{A_3}{(x - r)^3} + \cdots + \frac{A_m}{(x - r)^m}$$

where A_1, \dots, A_m are constants to be determined.

Example. Setup the partial fraction decomposition for $f(x) = \frac{x^3 - 8x + 19}{x^4 + 3x^3}$.

Example. Setup the partial fraction decomposition for $g(x) = \frac{2}{x^5 - 6x^4 + 9x^3}$.

Example. Evaluate $\int \frac{x^2 + 1}{(2x - 3)(x - 2)^2} dx$.

Example. Evaluate $\int \frac{8}{3x^3 + 7x^2 + 4x} dx$.

Example. Perform partial fraction decomposition on the following fractions or identify them as irreducible.

$$\frac{1}{x^2 - 13x + 43}$$

$$\frac{x^2}{(x - 4)(x + 5)}$$

Example. Perform partial fraction decomposition on the following fractions or identify them as irreducible.

$$\frac{7}{(x^2 + 1)^2}$$

$$\frac{1}{x^2 + 11x + 28}$$

Example. Evaluate $\int \frac{4x}{(x+1)(x^2+1)} dx$

Example. Evaluate $\int \frac{3x^2 + 2x + 12}{(x^2 + 4)^2} dx$

Example. Evaluate $\int \frac{1}{x\sqrt{1+2x}} dx$ using the substitution $u = \sqrt{1+2x}$.