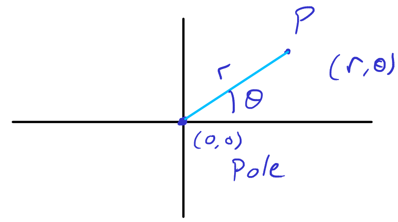
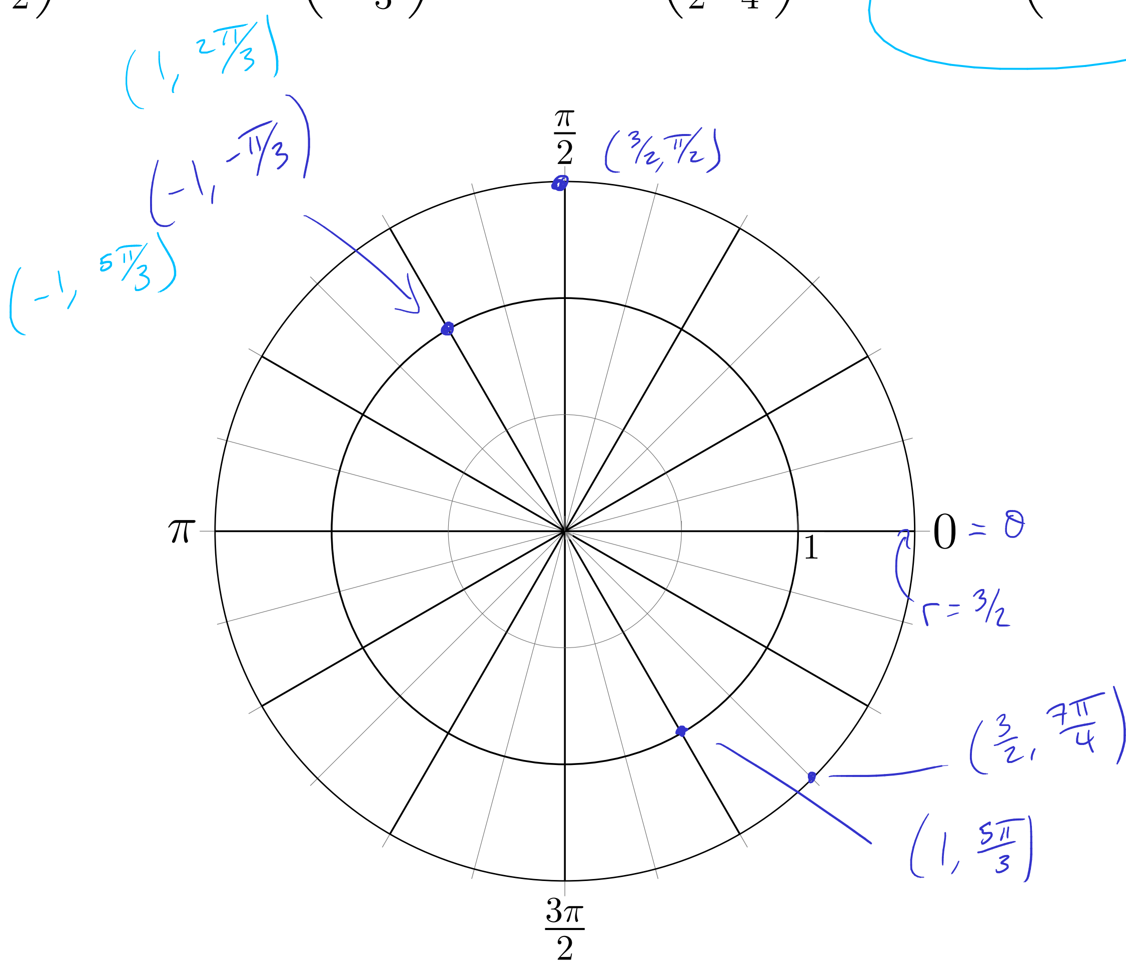


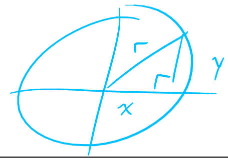
12.2: Polar Coordinates

Defining Polar Coordinates When using polar coordinates, the origin of the coordinate system is called the **pole**, and the positive x-axis is called the **polar axis**. The polar coordinates for a point P are of the form (r, θ).
The **radial coordinate** r describes the *signed* (*directed*) distance from the origin to P. The **angular coordinate** θ describes an angle whose initial side is the positive x-axis and whose terminal side lies on the ray passing through the origin and P.

Example (LC 33.4)**.** Graph the following polar coordinates

A) $\left(\frac{3}{2}, \frac{\pi}{2}\right)$ B) $\left(1, \frac{5\pi}{3}\right)$ C) $\left(\frac{3}{2}, \frac{7\pi}{4}\right)$ D) $\left(-1, \frac{-\pi}{3}\right)$





$\frac{x}{r} = \cos\theta$

Procedure: Converting Coordinates

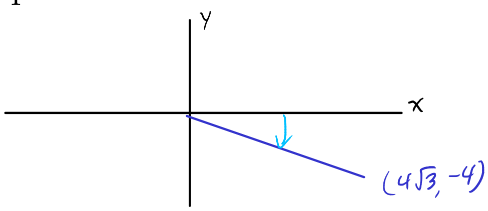
A point with polar coordinates (r, θ) has Cartesian coordinates (x, y), where

$$x = r\cos\theta \qquad \text{and} \qquad y = r\sin\theta.$$

A point with Cartesian coordinates (x, y) has polar coordinates (r, θ), where

$$r^2 = x^2 + y^2 \qquad \text{and} \qquad \tan\theta = \frac{y}{x}.$$

Example (LC 33.5)**.** Consider the Cartesian coordinate $\left(4\sqrt{3}, -4\right)$. Rewrite this point in polar coordinates. *Note*: There are infinitely many polar representations



$r = \sqrt{x^2+y^2}$

$= \sqrt{48+16}$

$= \sqrt{64}$

$= 8$

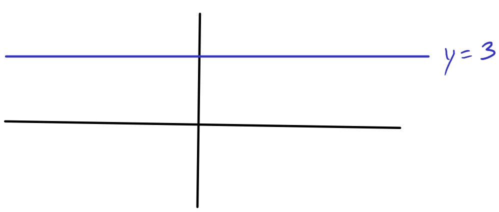
$\cos\theta = \frac{x}{r} = \frac{4\sqrt{3}}{8} = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{-\pi}{6}$

$\sin\theta = \frac{y}{r} = \frac{-4}{8} = -\frac{1}{2} \Rightarrow \theta = \frac{-\pi}{6}$

Polar: $\left(8, -\frac{\pi}{6}\right), \left(8, \frac{11\pi}{6}\right)$

$\left(-8, \frac{5\pi}{6}\right)$

Example (LC 33.6)**.** Rewrite $y = 3$ in terms of polar coordinates.



$3 = r\sin\theta$

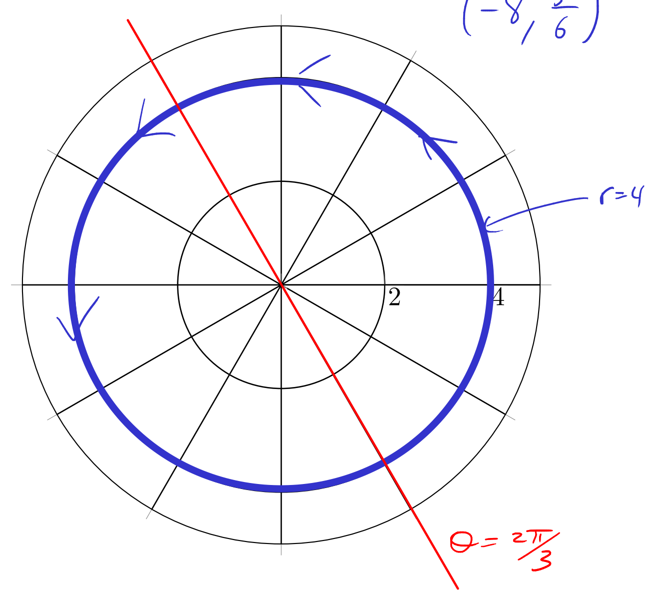
$y = r\sin\theta \rightarrow r = \frac{3}{\sin\theta}$

$r = 3\csc\theta$

Example (LC 33.7)**.** Graph $r = 4$ and $\theta = \frac{2\pi}{3}$

$(4, \theta)$

$\left(r, \frac{2\pi}{3}\right)$



$x = r\cos\theta$

$y = r\sin\theta$

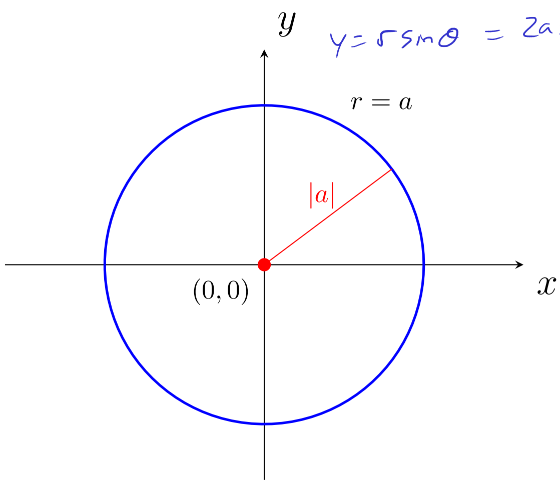
Summary: Circles in Polar Coordinates

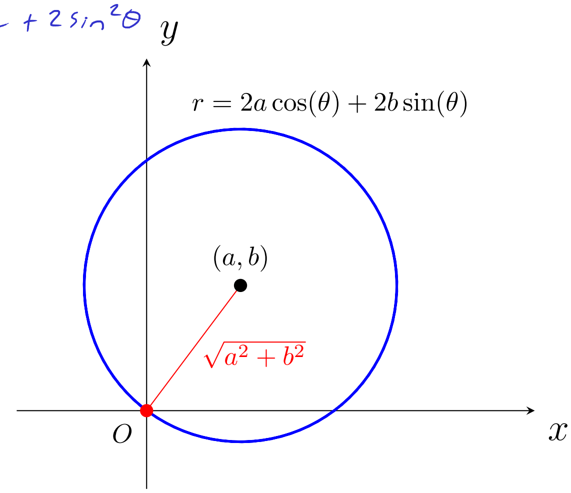
The equation $r = a$ describes a circle of radius $|a|$ centered at $(0,0)$.

The equation $r = 2a\cos\theta + 2b\sin\theta$ describes a circle of radius $\sqrt{a^2+b^2}$ centered at (a,b).

$x = r\cos\theta = 2a\cos^2\theta + 2b\sin\theta\cos\theta$

$y = r\sin\theta = 2a\sin\theta\cos\theta + 2\sin^2\theta$





Example. Rewrite the following in either polar coordinates or Cartesian coordinates

$r = 5\cos(\theta) + 12\sin(\theta)$

Circle centered @ $(5/2, 6)$

radius $= \sqrt{5^2+12^2} = 13$

$$\boxed{\left(x-\tfrac{5}{2}\right)^2 + (y-6)^2 = 13^2}$$

$x = \dfrac{3}{y}$

$r\cos\theta = \dfrac{3}{r\sin\theta}$

$r^2 = \dfrac{3}{\cos\theta\sin\theta}$

$$\boxed{r = \pm\sqrt{3\sec\theta\csc\theta}}$$

$r\cos(\theta) = \sin(2\theta)$

$x = 2\sin\theta\cos\theta$

$= 2\,\dfrac{y}{r}\,\dfrac{x}{r} \to y = \dfrac{r^2}{2}$

$x = r\cos\theta$

$\dfrac{x}{r} = \cos\theta$

tbd

$y = x^2$

$r\sin\theta = (r\cos\theta)^2 \to r\sin\theta = r^2\cos^2\theta$

$\dfrac{\sin\theta}{\cos^2\theta} = r$

$$\boxed{r = \sec\theta\tan\theta}$$

Procedure: Cartesian-to-Polar Method for Graphing $r = f(\theta)$

1. Graph $r = f(\theta)$ as if r and θ were Caresian coordinates with θ on the horizontal axis and r on the vertical axis. Be sure to choose an interval for θ on which the entire polar curve is produced.
2. Use the Cartesian graph that you created in Step 1 as a guide to sketch the points (r, θ) on the final *polar* curve.

Summary: Symmetry in Polar Equations

Symmetry about the x-axis occurs if the point (r, θ) is on the graph whenever $(r, -\theta)$ is on the graph.

Symmetry about the y-axis occurs if the point (r, θ) is on the graph whenever $r, \pi - \theta) = (-r, -\theta)$ is on the graph.

Symmetry about the origin occurs if the point (r, θ) is on the graph whenever $(-r, \theta) = (r, \theta + \pi)$ is on the graph.

$r = 2a\cos\theta + 2b\sin\theta$

Example (LC 33.8-33.9). Consider the polar curve $r = 2\sin(\theta) - 1$

Complete the table below

θ	0	$\pi/6$	$\pi/4$	$\pi/2$	π	$3\pi/2$
$r = 2\sin(\theta) - 1$	-1	0	$\sqrt{2}-1$	1	-1	-3

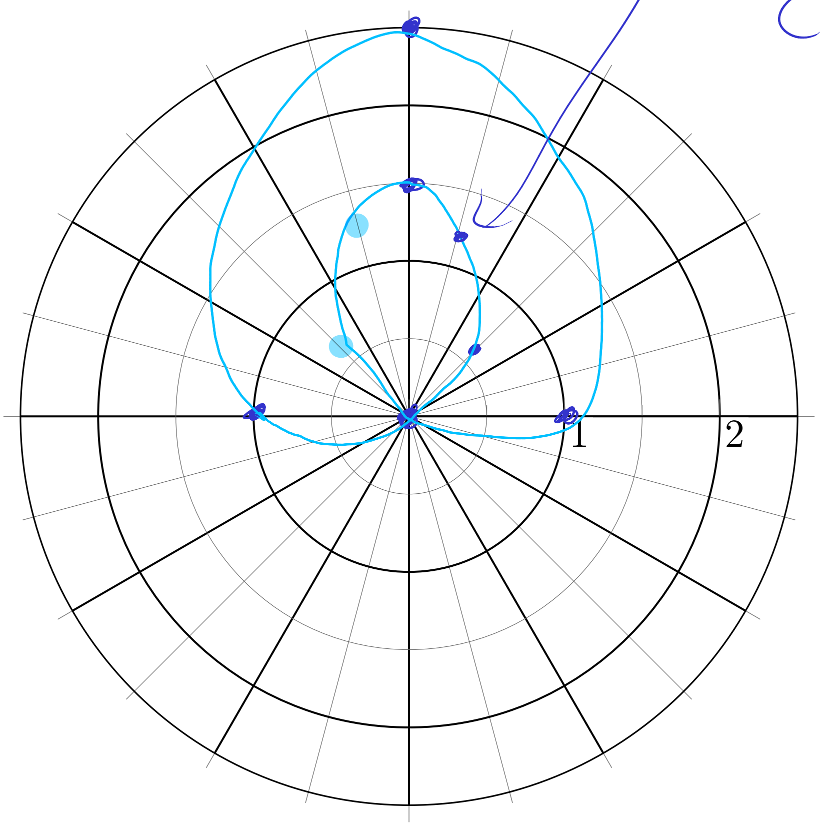
$2\sin(\pi/3) - 1$

$2\left(\frac{\sqrt{3}}{2}\right) - 1 = \sqrt{3} - 1$

$(\pi/3, \sqrt{3}-1)$

Graph the polar curve $r = 2\sin(\theta) - 1$

Cardiod



$\theta = \pi/3$

$r = \sqrt{3} - 1$