Composition

$$(f \circ g)(x) = f(g(x))$$

Example. $f(x) = \sqrt{x}, \ g(x) = x + 1$

$(f \circ g)(x) =$ $f(g(x)) = f(x+1) = \sqrt{x+1}$

$(g \circ f)(x) =$ $g(f(x)) = g(\sqrt{x}) = \sqrt{x} + 1$

4.7 Intersection of curves and simultaneous solutions

Example.

$y = x^2 - 4$

$x + y = 8 \longrightarrow y = -x + 8$

$\Rightarrow -x + 8 = x^2 - 4$

$0 = x^2 + x - 12$

$= (x+4)(x-3)$

$\left.\begin{matrix} x = -4 \\ x = 3 \end{matrix}\right\} \Rightarrow \begin{matrix} y = 12 \\ y = 5 \end{matrix}$

Example.

$$2x + 3y = 7$$
$$-3x + 7y = 11 \rightarrow$$

$7y = 3x + 11$

$y = \frac{3}{7}x + \frac{11}{7}$

$\rightarrow 2x + 3\left(\frac{3}{7}x + \frac{11}{7}\right) = 7$

$\left(2x + \frac{9}{7}x + \frac{33}{7} = 7\right) 7$

$14x + 9x + 33 = 49$

$23x = 16$

$\boxed{x = \frac{16}{23}}$

$y = \frac{3}{7}\left(\frac{16}{23}\right) + \frac{11}{7}\left(\frac{23}{23}\right)$

$y = \frac{48}{7 \cdot 23} + \frac{253}{7 \cdot 23}$

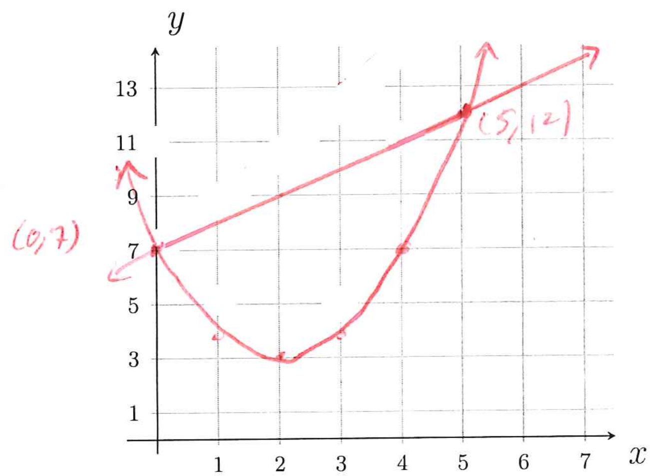
$y = \frac{301}{7 \cdot 23} = \boxed{\frac{43}{23}}$

$23 \overline{)301}$ 13; 23; 71; 69; 2

$7 \overline{)301}$ 43; 28; 21

Example.

$$y = x + 7$$
$$y = (x-2)^2 + 3$$



$x + 7 = (x-2)^2 + 3$

$0 = x^2 - 4x + 4 - 4$

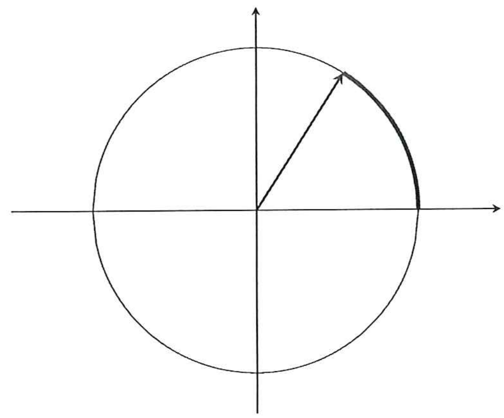
$0 = x^2 - 5x$

$= x(x-5)$

5.1 Angles

Definition. The **unit circle** is the circle of radius 1 that is centered at the origin.

Definition. The angle corresponding to an arc length of 1 on a unit circle is called a **radian**.



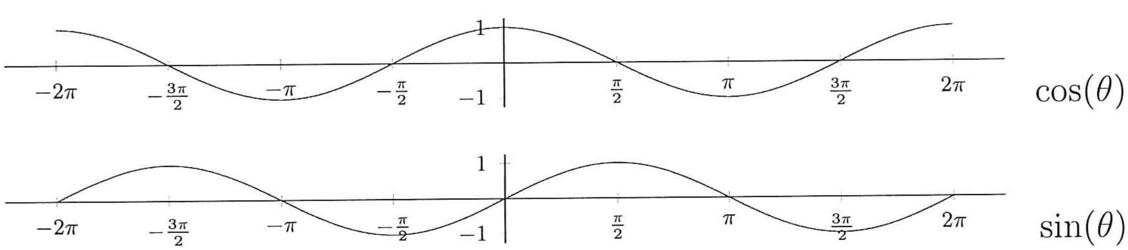
A circle is 2π radians or $360°$. Thus:

$$2\pi = 360° \Longrightarrow 1 = \frac{180°}{\pi} = \frac{\pi}{180°}$$

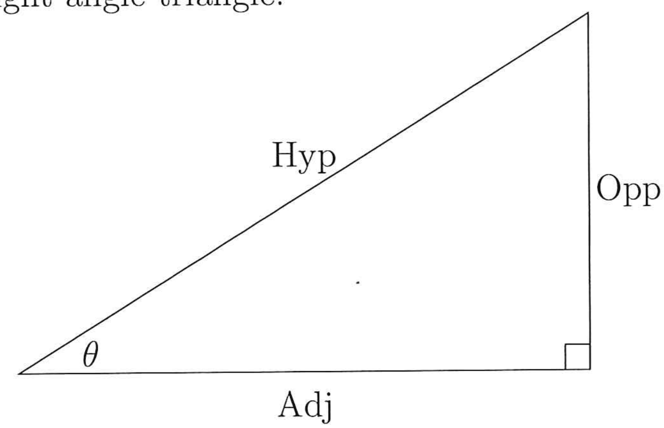
5.2 Definition of $\sin(\theta)$ and $\cos(\theta)$

Definition. The coordinates of a unit circle are given by $\Big(\cos(\theta), \sin(\theta)\Big)$ for each θ.

Definition. The $\sin(\theta)$ and $\cos(\theta)$ functions are **periodic** since these functions repeat themselves over a fixed interval



Definition. Alternatively, $\cos(\theta)$ and $\sin(\theta)$ can be consider the ratio of the sides of a right angle triangle.



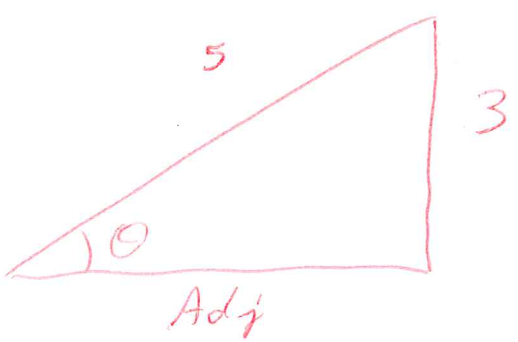
$$\sin\theta = \frac{\text{Opp}}{\text{Hyp}}$$

$$\cos\theta = \frac{\text{Adj}}{\text{Hyp}}$$

$$\tan\theta = \frac{\text{Opp}}{\text{Adj}}$$

e.g. If $\sin\theta = \frac{3}{5}$, find $\cos\theta$ and $\tan\theta$

$\cos\theta = \frac{Adj}{Hyp}$ $\tan\theta = \frac{Opp}{Adj}$ $\sin\theta = \frac{Opp}{Hyp} = \frac{3}{5}$



$\cos\theta = \frac{4}{5}$

$\tan\theta = \frac{3}{5}$

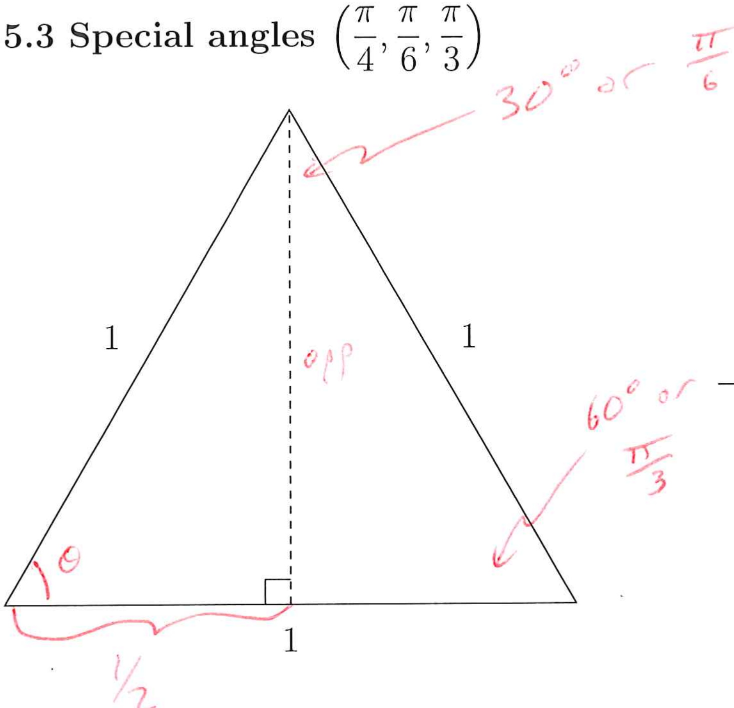
Hint: This is handy for double angle formula questions!

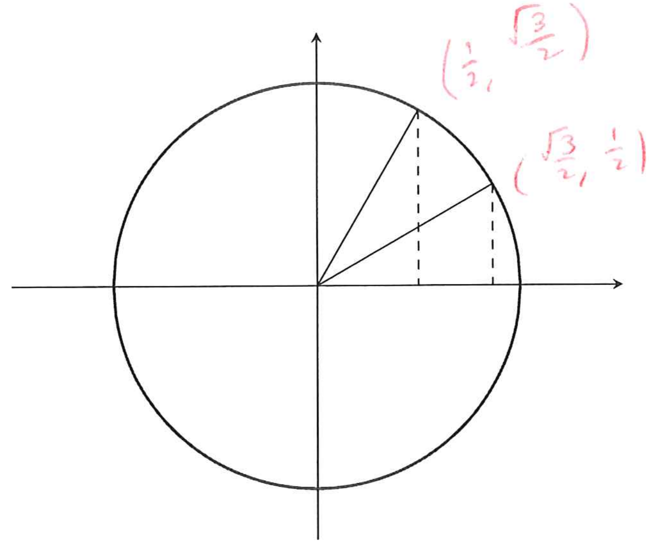
$(Adj)^2 + 3^2 = 5^2$

$(Adj)^2 = 25 - 9$

$Adj = \sqrt{16} = 4$

5.3 Special angles $\left(\frac{\pi}{4}, \frac{\pi}{6}, \frac{\pi}{3}\right)$

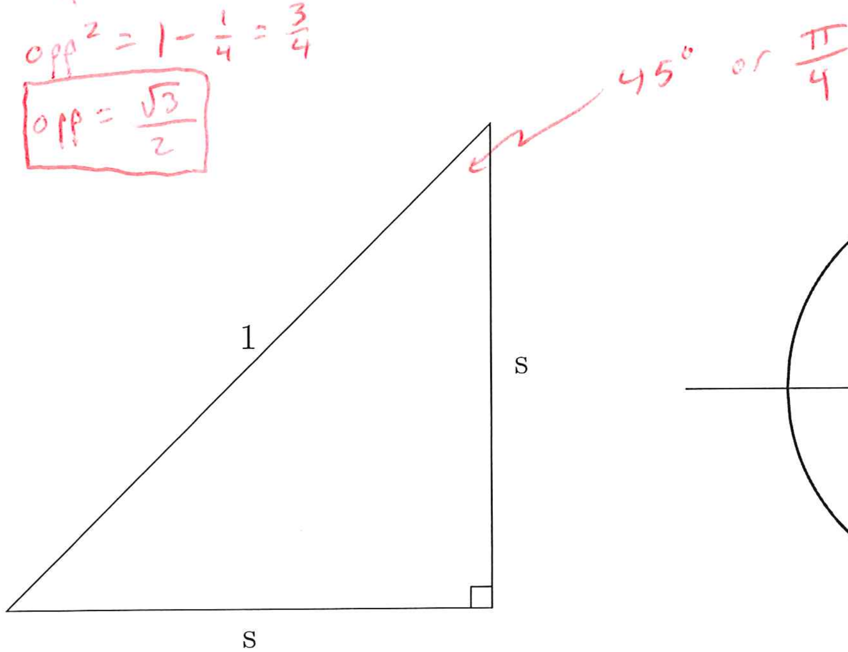


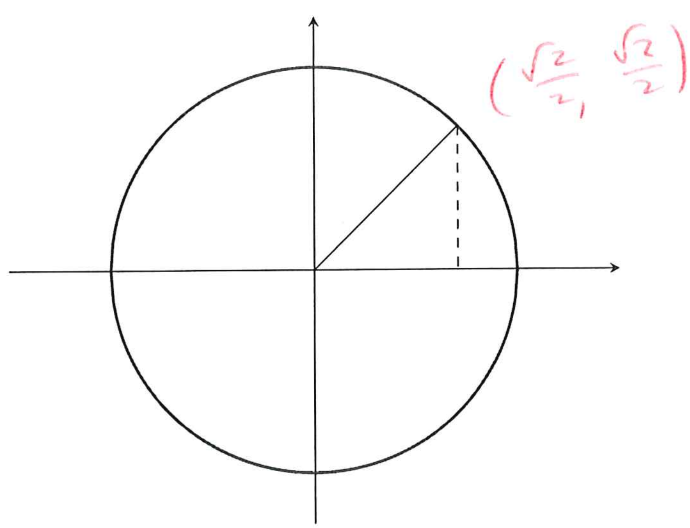


$\left(\frac{1}{2}\right)^2 + (opp)^2 = 1$

$opp^2 = 1 - \frac{1}{4} = \frac{3}{4}$

$\boxed{opp = \frac{\sqrt{3}}{2}}$

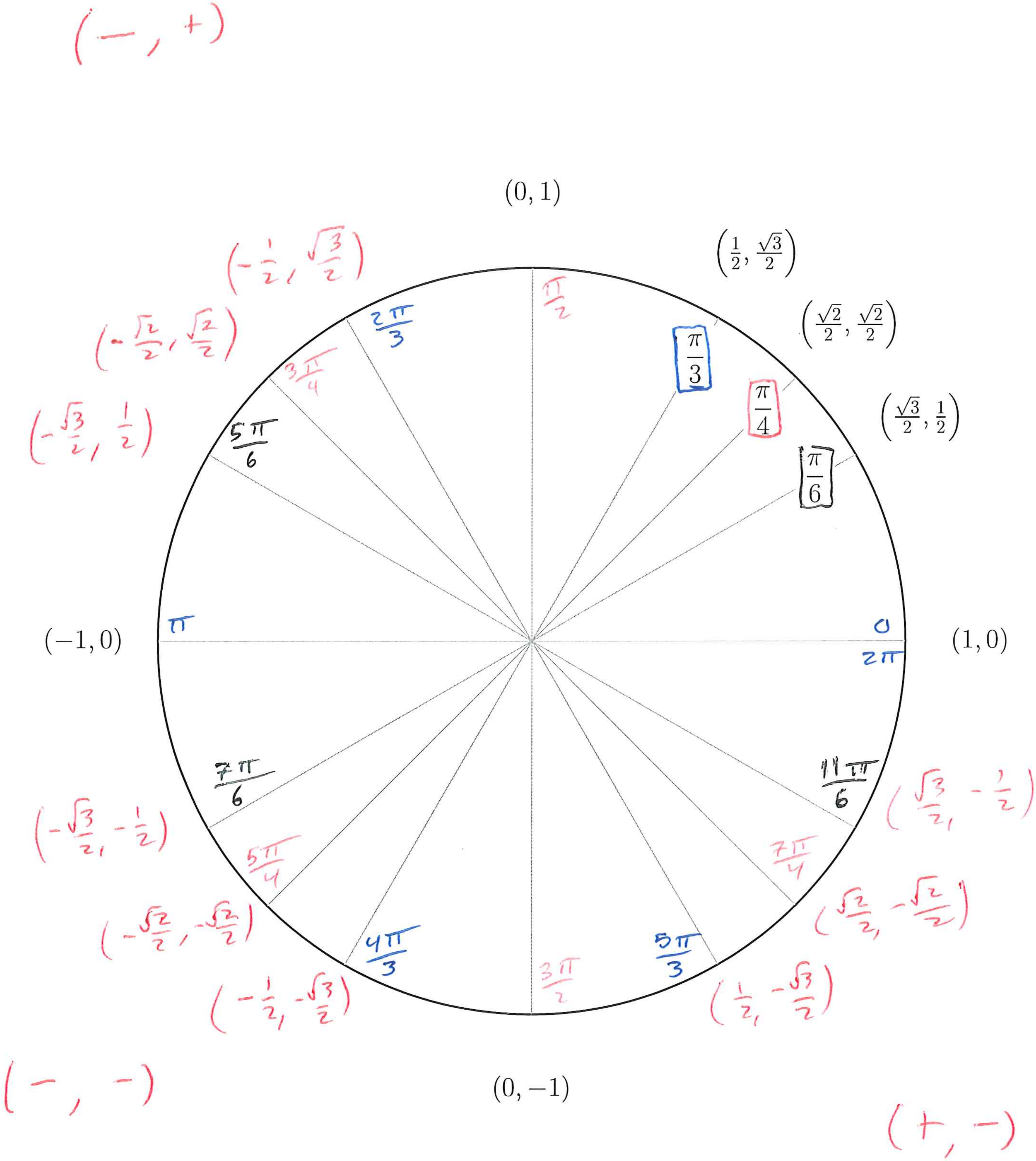




$s^2 + s^2 = 1 \rightarrow 2s^2 = 1$

$s^2 = \frac{1}{2}$

$s = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \boxed{\frac{\sqrt{2}}{2}}$



$(-,+)$
$(0,1)$
$\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$
$\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$
$\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$
$\left(-\frac{1}{2},\frac{\sqrt{3}}{2}\right)$
$\left(-\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$
$\left(-\frac{\sqrt{3}}{2},\frac{1}{2}\right)$
$\frac{\pi}{2}$
$\frac{2\pi}{3}$
$\frac{3\pi}{4}$
$\frac{5\pi}{6}$
$\boxed{\frac{\pi}{3}}$
$\boxed{\frac{\pi}{4}}$
$\boxed{\frac{\pi}{6}}$
π
$(-1,0)$
0
2π
$(1,0)$
$\frac{7\pi}{6}$
$\frac{5\pi}{4}$
$\frac{4\pi}{3}$
$\frac{3\pi}{2}$
$\frac{5\pi}{3}$
$\frac{7\pi}{4}$
$\frac{11\pi}{6}$
$\left(-\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$
$\left(-\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}\right)$
$\left(-\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$
$\left(\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$
$\left(\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}\right)$
$\left(\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$
$(-,-)$
$(0,-1)$
$(+,-)$

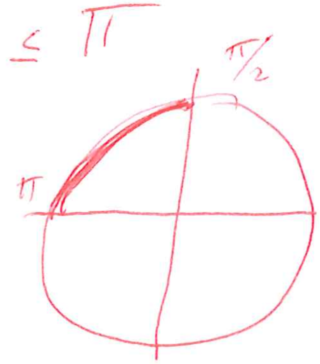
5.5 The other trigonometric functions

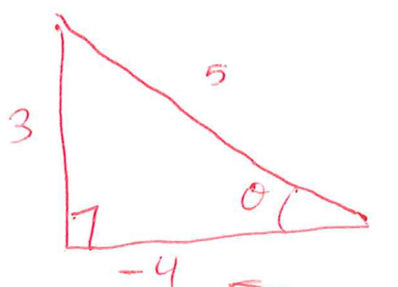
$$\left.\begin{aligned} \tan\theta &= \frac{\sin\theta}{\cos\theta} \\ \cot\theta &= \frac{\cos\theta}{\sin\theta} \end{aligned}\right\} \text{Range: } (-\infty, \infty)$$

$$\left.\begin{aligned} \sec\theta &= \frac{1}{\cos\theta} \\ \csc\theta &= \frac{1}{\sin\theta} \end{aligned}\right\} \text{Range: } (-\infty, -1] \cup [1, \infty)$$

e.g. If $\sin\theta = \frac{3}{5}$, $\frac{\pi}{2} \le \theta \le \pi$

Find the other 5 trig functions





negative!

$\Rightarrow \cos\theta = \frac{-4}{5} \Longrightarrow$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{3/5}{-4/5} = \boxed{-\frac{3}{4}}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{-4/5}{3/5} = \boxed{-\frac{4}{3}}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{-4/5} = \boxed{-5/4}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{3/5} = \boxed{5/3}$$