10.3: Infinite Series

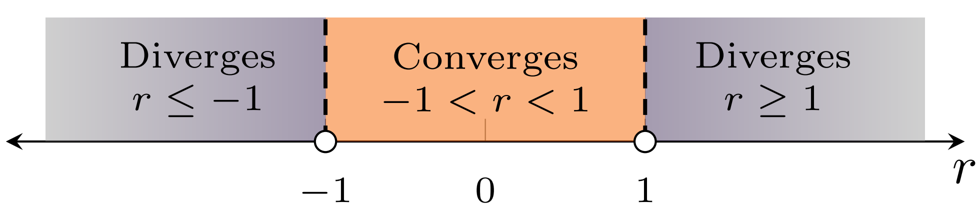
A **Geometric sum** with n terms has the form

$$S_n = a + ar + ar^2 + \cdots + ar^{n-1} = \sum_{k=0}^{n-1} ar^k$$

Derivation of partial sum formula:

Theorem 10.7: Geometric Series

Let $a \neq 0$ and r be real numbers. If $|r| < 1$, then $\displaystyle\sum_{k=0}^{\infty} ar^k = \frac{a}{1-r}$. If $|r| \geq 1$, then the series diverges.



Example. Evaluate the following geometric series or state that the series diverges

$$\sum_{k=0}^{\infty} 1.1^k$$

$$\sum_{k=0}^{\infty} e^{-k}$$

$$\sum_{k=2}^{\infty} 3(-0.75)^k$$

$$\sum_{k=1}^{\infty} \frac{7}{10^k}$$

Telescoping Series:

Example. Evaluate the following series

$$\sum_{k=1}^{\infty} \cos\left(\frac{1}{k^2}\right) - \cos\left(\frac{1}{(k+1)^2}\right)$$

$$\sum_{k=3}^{\infty} \frac{1}{(k-2)(k-1)}$$

Theorem 10.8: Properties of Convergent Series

1. Suppose $\sum a_k$ converges to A and c is a real number. The series $\sum ca_k$ converges, and $\sum ca_k = c\sum a_k = cA$.
2. Suppose $\sum a_k$ diverges. Then $\sum ca_k$ also diverges, for any real number $c \neq 0$.
3. Suppose $\sum a_k$ converges to A and $\sum b_k$ converges to B. The series $\sum (a_k \pm b_k)$ converges and $\sum (a_k \pm b_k) = \sum a_k \pm \sum b_k = A \pm B$.
4. Suppose $\sum a_k$ diverges and $\sum b_k$ converges. Then $\sum (a_k \pm b_k)$ diverges.
5. If M is a positive integer, then $\displaystyle\sum_{k=1}^{\infty} a_k$ and $\displaystyle\sum_{k=M}^{\infty} a_k$ either both converge or both diverge. In general, *whether* a series converges does not depend on a finite number of terms added to or removed from the series. However, the *value* of a convergent series does change if nonzero terms are added or removed.

Example. Evaluate

$$\sum_{k=1}^{\infty} \left[\frac{1}{2}\left(\frac{2}{5}\right)^k + \frac{2}{3}\left(\frac{1}{6}\right)^k \right]$$