1 15.3: Partial Derivatives

Definition. (Partial Derivatives)
The **partial derivative of f with respect to x at the point (a, b) is**

$$f_x(a, b) = \lim_{h \to 0} \frac{f(a + h, b) - f(a, b)}{h}.$$

The **partial derivative of f with respect to y at the point (a, b) is**

$$f_y(a, b) = \lim_{h \to 0} \frac{f(a, b + h) - f(a, b)}{h},$$

provided these limits exist.

Theorem 15.4: (Clairut) Equality of Mixed Partial Derivatives Assume f is defined on an open set D of $\mathbb{R}^2$, and that f_{xy} and f_{yx} are continuous throughout D. Then $f_{xy} = f_{yx}$ at all points of D.

Definition. (Differentiability)
The function $z = f(x, y)$ is **differentiable at** (a, b) provided $f_x(a, b)$ and $f_y(a, b)$ exist and the change $\Delta z = f(a + \Delta x, b + \Delta y) - f(a, b)$ equals

$$\Delta z = f_x(a, b)\Delta x + f_y(a, b)\Delta y = \varepsilon_1 \Delta x + \varepsilon_2 \Delta y,$$

where for fixed a and b, ε_1 and ε_2 are functions that depend only on Δx and δy, with $(\varepsilon_1, \varepsilon_2) \to (0, 0)$ as $(\Delta x, \Delta y) \to (0, 0)$. A function is **differentiable** on an open set R if it is differentiable at every point of R.

Theorem 15.5: Conditions for Differentiability
Suppose the function f has partial derivatives f_x and f_y defined on an open set containing (a, b), with f_x and f_y continuous at (a, b). Then f is differentiable at (a, b).

Theorem 15.6: Differentiable Implies Continuous
If a function f is differentiable at (a, b), then it is continuous at (a, b).