

1 14.4: Length of Curves

Definition. (Arc Length for Vector Functions)

Consider the parameterized curve $\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle$, where f' , g' , and h' are continuous, and the curve is traversed once for $a \leq t \leq b$. The **arc length** of the curve between $(f(a), g(a), h(a))$ and $(f(b), g(b), h(b))$ is

$$L = \int_a^b \sqrt{f'(t)^2 + g'(t)^2 + h'(t)^2} dt = \int_a^b |\mathbf{r}'(t)| dt.$$

Theorem 14.3: Arc Length as a Function of a Parameter

Let $\mathbf{r}(t)$ describe a smooth curve, for $t \geq a$. The arc length is given by

$$s(t) = \int_a^t |v(u)| du,$$

where $|\mathbf{v}| = |\mathbf{r}'|$. Equivalently, $\frac{ds}{dt} = |\mathbf{v}(t)|$. If $|\mathbf{v}(t)| = 1$, for all $t \geq a$, then the parameter t corresponds to arc length.