u-sub
IBP ← Product of functions
PFD
Trig sub ⟸ u^2-a^2, u^2+a^2, a^2-u^2
Trig int

8.6: Integration Strategies

Example. What integration methods can be used to evaluate the functions below? (No need to evaluate the integral)

$$\int \frac{1}{1-x^2}\,dx$$

$u = 1-x^2$
$du = -2x\,dx$

PFD: $\frac{1}{1-x^2} = \frac{A}{1+x} + \frac{B}{1-x}$

Trig sub: $x = 1\sin\theta$

$$\int x\sec^2(x)\,dx$$

$u = x$
$du = dx$

IBP

$$\int \frac{x}{\sqrt{64-x^2}}\,dx$$

u-sub: $u = 64-x^2$
$du = -2x\,dx$

trig sub: $x = 8\sin\theta$
a^2-u^2 $dx = 8\cos\theta\,d\theta$

$-8 \le x \le 8$

IBP $u = x$
$du = dx$
$v =$
$dv = \frac{1}{\sqrt{64-x^2}}\,dx$

$$\int \frac{x^3}{\sqrt{64-x^2}}\,dx$$

$\int \frac{u}{2\sqrt{64-u}}\,du$

u-sub: $u = x^2$
$du = 2x\,dx$

trig sub
$-8 \le x \le 8$
$x = 8\sin\theta$
$dx = 8\cos\theta\,d\theta$

$$\int \frac{8^3\sin^3\theta}{\sqrt{64-64\sin^2\theta}}\,8\cos\theta\,d\theta$$

Example. Identify ~~two~~ 3 integration techniques which can be used to evaluate

$$\int \frac{4-3x^2}{x(x^2-4)}\,dx.$$

u-sub
IBP
PFD
Trig sub
Trig int

PFD: $\frac{4-3x^2}{x(x^2-4)} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{x-2}$

Trig sub: u^2-a^2 $x=2\sec\theta$ $x>2$

$dx = 2\sec\theta\tan\theta\,d\theta$ $x<-2,\ x=-2\sec\theta$

$\theta = \sec^{-1}\left(\frac{x}{2}\right)$ $-2<x<2$

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u-sub: $u = x^3-4x$

$du = 3x^2-4\,dx \rightarrow -du = 4-3x^2\,dx$

Example. Perform a substitution of variables to rewrite $\int x\sin(\sqrt{x})\,dx$.

Let $t=\sqrt{x}$

$dt = \frac{1}{2\sqrt{x}}\,dx \rightarrow 2dt = \frac{1}{\sqrt{x}}\,dx$

$= \int \frac{x^{3/2}\sqrt{x}\sin(\sqrt{x})}{\sqrt{x}}\,dx$

$= \int 2t^3\sin(t)\,dt$

IBP

ILATE

$u = 2t^3$ $v = -\cos(t)$

$du = 6t^2\,dt$ $dv = \sin(t)\,dt$

Example. $\displaystyle\int_1^3 \frac{\tan^{-1}(\sqrt{x})}{x^{1/2}+x^{3/2}}\,dx$

$\frac{x^{3/2}}{x^{1/2}} = x$

$u = x^{1/2}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$x=1,\ u=1$

$x=3,\ u=\sqrt{3}$

$= \displaystyle\int_1^3 \frac{\tan^{-1}(\sqrt{x})}{\sqrt{x}\,(1+x)}\,dx$

$= 2\displaystyle\int_1^{\sqrt{3}} \frac{\tan^{-1}(u)}{(1+u^2)}\,du$

$v = \tan^{-1}(u)$

$dv = \frac{1}{1+u^2}\,du$

$u=1,\ v=\pi/4$

$u=\sqrt{3},\ v=\pi/3$

$= 2\displaystyle\int_{\pi/4}^{\pi/3} v\,dv$

$= \frac{2v^2}{2}\Big|_{\pi/4}^{\pi/3} = \left(\frac{\pi}{3}\right)^2 - \left(\frac{\pi}{4}\right)^2 = \boxed{\frac{7\pi^2}{144}}$