1 16.4: Triple Integrals

Theorem 16.5: Triple Integrals
Let f be continuous over the region

$$D = \{(x, y, z) : a \leq x \leq b,\ g(x) \leq y \leq h(x),\ G(x, y) \leq z \leq H(x, y)\},$$

where g, h, G, and H are continuous functions. Then f is integrable over D and the triple integral is evaluated as the iterated integral

$$\iiint_D f(x, y, z)\, dV = \int_a^b \int_{g(x)}^{h(x)} \int_{G(x,y)}^{H(x,y)} f(x, y, z)\, dz\, dy\, dx.$$

Definition. (Average Value of a Function of Three Variables)
If f is continuous on a region D of $\mathbb{R}^3$, then the **average value** of f over D is

$$\bar{f} = \frac{1}{\text{volume of } D} \iiint_D f(x, y, z)\, dV.$$