3.1 Equations of Degree 1 (Linear equations)

Definition. The expression $ax + b$, with $a \neq 0$, is a polynomial of degree 1, and so the equation $ax + b = 0$ is called an **equation of degree 1**. Since the graph of the function $ax + b$ is a straight line, the equation $ax + b = 0$ is also called a **linear equation**.

Example. Solve for x : $$2x + 5y = 3x + y + 1$$

$$\boxed{4y - 1 = x}$$

Example. Solve for y : $$\frac{2}{3}y + 2x - 1 = \frac{3}{4}y + x - \frac{1}{2}$$

$$\left(\frac{2}{3} - \frac{3}{4}\right)y = -x + \frac{1}{2}$$

$$\frac{2}{3} - \frac{3}{4} = \frac{8-9}{12} = -\frac{1}{12}$$

$$-\frac{1}{12}y = -x + \frac{1}{2}$$

$$\boxed{y = 12x - 6}$$

3.2 Equations of Degree 2 (Quadratic equations)

Definition. The expression $ax^2 + bx + c$ with $a \neq 0$ is a polynomial of degree 2, and the equation $ax^2 + bx + c = 0$ is called an **equation of degree 2** or a **quadratic equation**. The roots of a **quadratic equation** can be found using the **quadratic formula**:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example. Solve for s : $s^2 + 4s + 4 = 0.$

$$s = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(4)}}{2} = \frac{-4 \pm \sqrt{16-16}}{2} = \frac{-4}{2} = -2 \Rightarrow \boxed{s = -2}$$

Also

$$s^2 + 4s + 4 = (s+2)(s+2) = 0 \Rightarrow \boxed{s = -2}$$

Definition. In the quadratic formula, if $b - 4ac < 0$ (called the **discriminant**), then the equation contains no Real roots. If we define $i = \sqrt{-1}$, which is an **imaginary number**, then we have a root that's a **complex number**, $a + bi$.

Example. Solve for y : $y^2 + 2y + 2 = 0.$

$$y = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(2)}}{2} = \frac{-2 \pm \sqrt{4-8}}{2} = \frac{-2 \pm \sqrt{-4}}{2} = \frac{-2 \pm i\sqrt{4}}{2}$$

$$= \frac{-2 \pm 2i}{2}$$

$$\Rightarrow \boxed{y = -1 \pm i.}$$

3.3 Solving Other Types of Equations

Example. Solve for x : $\dfrac{1}{x-5}+\dfrac{1}{x+5}=\dfrac{10}{x^2-25}$. Does your solution make sense?

Domain : $x \neq -5,\ x \neq 5 \Rightarrow (-\infty,-5)\cup(-5,5)\cup(5,\infty)$

$(x^2-25)\left(\dfrac{1}{x-5}+\dfrac{1}{x+5}\right)=\left(\dfrac{10}{x^2-25}\right)(x^2-25)$

$(x+5)+(x-5)=10$

$2x=10$

$x=5 \leftarrow$ Notice that $x=5$ is NOT in our domain!

$\boxed{\text{No solutions!}}$

Example. Solve for x : $x^4-5x^2-36=0$ *Hint:* Let $y=x^2$.

$y^2-5y-36=0$

larger number negative

difference of 5

$1\cdot 36$

$2\cdot 18$

$3\cdot 12$

$4\cdot 9$

$6\cdot 6$

$y^2-9y+4y-36=0$

$y(y-9)+4(y-9)=0$

$(y+4)(y-9)=0$

$(x^2+4)(x^2-9)=0$

$(x^2+4)(x-3)(x+3)=0 \Rightarrow$ $x=3,\ x=-3 \leftarrow$ Real

$x=\pm 2i \leftarrow$ Imaginary (complex)

Example. Solve for x : $x^6 + 6x^3 - 16 = 0$

Let $y = x^3$

$1 \cdot 16$
$2 \cdot 8$
$4 \cdot 4$ Diff of 6

$$y^2 + 6y - 16 = 0$$
$$y^2 + 8y - 2y - 16 = 0$$
$$y(y+8) - 2(y+8) = 0$$
$$(y-2)(y+8) = 0$$
$$(x^3 - 2)(x^3 + 8) = 0$$

$\Rightarrow \quad x^3 - 2 = 0 \qquad \Rightarrow \quad x^3 + 8 = 0$

$x^3 = 2 \qquad x^3 = -8$

$\boxed{x = \sqrt[3]{2}} \qquad \boxed{x = -2}$

Both valid

Example. Solve for x : $x + \sqrt{x} - 6 = 0$

Let $y = \sqrt{x}$

$1 \cdot 6$
$2 \cdot 3$ Diff of 1

$$y^2 + y - 6 = 0$$
$$y^2 + 3y - 2y - 6 = 0$$
$$y(y+3) - 2(y+3) = 0$$
$$(y-2)(y+3) = 0$$
$$(\sqrt{x} - 2)(\sqrt{x} + 3) = 0$$

Domain
$\sqrt{x} \rightarrow x \geq 0$

$\sqrt{x} = 2 \qquad \sqrt{x} = -3 \leftarrow$ Not in domain!

$\boxed{x = 4}$

Real