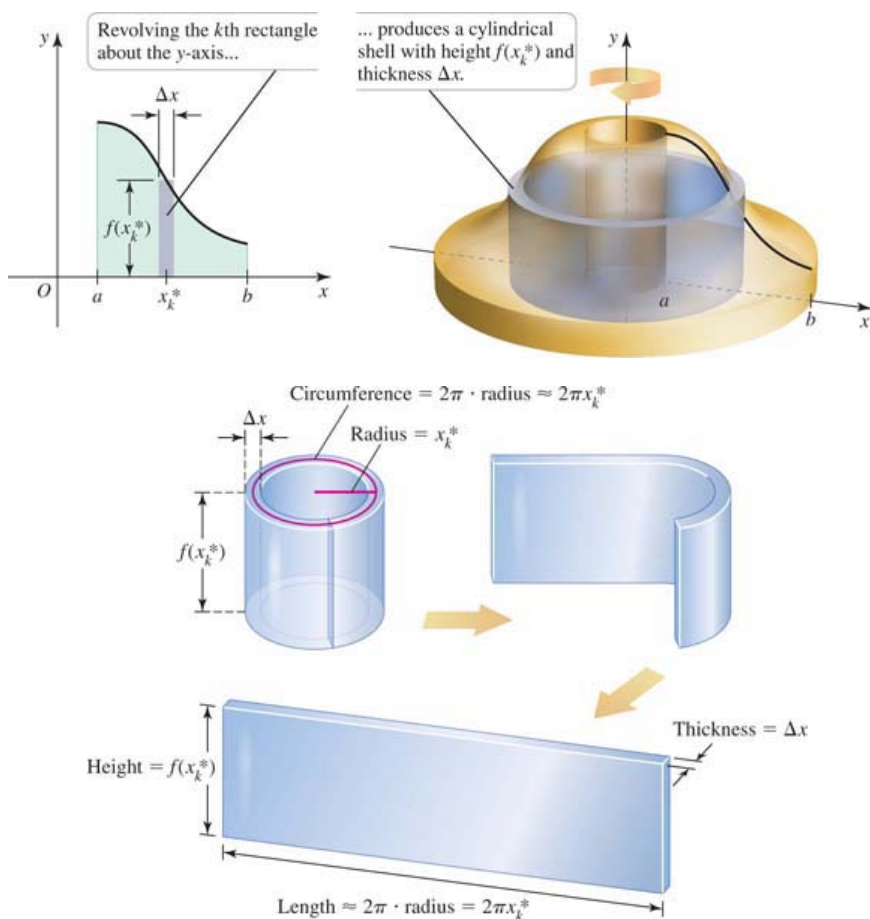## 6.4: Volume by Shells

Volume by the Shell Method

Let f and g be continuous functions with $f(x) \geq g(x)$ on $[a, b]$. If R is the region bounded by the curves $y = f(x)$ and $y = g(x)$ between the lines $x = a$ and $x = b$, the volume of the solid generated when R is revolved about the y-axis is

$$V = \int_a^b \underbrace{2\pi x}_{\substack{\text{shell} \\ \text{circumference}}} \underbrace{(f(x) - g(x))}_{\substack{\text{shell} \\ \text{height}}} dx.$$

Example. Consider a general region R revolved around the y-axis.

When using the **disk/washer** method, we integrate with respect to _____
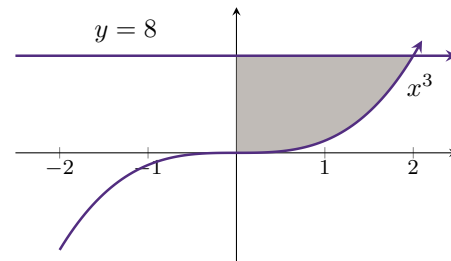
When using the **shell** method, we integrate with respect to _____

Example. Consider a general region R revolved around the x-axis.

When using the **disk/washer** method, we integrate with respect to _____

When using the **shell** method, we integrate with respect to _____

Example. Consider the region bounded between $y = x^3$, $y = 8$ and $x = 0$.

Use the disk/washer method to setup the integral that represents the volume of the solid generated by rotating the region about the x-axis.

about the y-axis.

Use the disk/washer method to setup the integral that represents the volume of the solid generated by rotating the region about the line $x = -1$.

about the line $y = 8$.

Example. Consider the region bounded by $y = \dfrac{1}{x+1}$ and $y = 1 - \dfrac{x}{3}$. Use both the disk/washer method and shell method to find the volume of the solid generated when R is rotated about the x-axis.

Example. Determine if the following statements are true.

When using the shell method, the axis of the cylindrical shells is parallel to the axis of revolution.

If a region is revolved about the y-axis, then the shell method must be used.

If a region is revolved about the x-axis, it is possible to use the disk/washer method and integrate with respect to x.

Example. Consider the region R bounded by $y = 4 - x^2$, $y = 2$, and $x = 1$. Use the shell method to setup the integral that represents the volume of the solid generated by rotating the region R about the indicated axis of rotation.
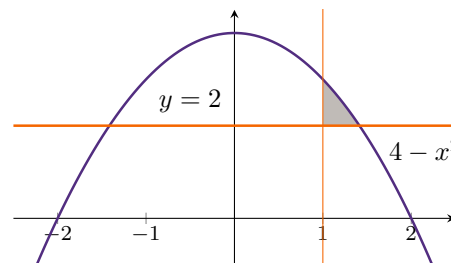
about x-axis,

about y-axis,

about the line $x = -2$,

about the line $y = 2$.