1 16.1: Double Integrals over Rectangular Regions

Definition. (Double Integrals)
A function f defined on a rectangular region R in the xy-plane is **integrable** on R if $\lim_{\Delta \to 0} \sum_{k=1}^{n} f(x_k^*, y_k^*)\Delta A_k$ exists for all partitions of R and for all choices of (x_k^*, y_k^*) within those partitions. The limit is the **double integral of f over R**, which we write

$$\iint_R f(x,y)\,dA = \lim_{\Delta \to 0} \sum_{k=1}^{n} f(x_k^*, y_k^*)\,\Delta A_k.$$

Theorem 16.1: (Fubini) Double Integrals over Rectangular Regions
Let f be continuous on the rectangular region $R = \{(x,y) : a \le x \le b, c \le y \le d\}$. The double integral of f over R may be evaluated by either of the two iterated integrals:

$$\iint_R f(x,y)\,dA = \int_c^d \int_a^b f(x,y)\,dx\,dy = \int_a^b \int_c^d f(x,y)\,dy\,dx.$$

Definition. (Average Value of a Function over a Plane Region)
The **average value** of an integrable function f over a region R is

$$\bar{f} = \frac{1}{\text{area of } R} \iint_R f(x,y)\,dA.$$