

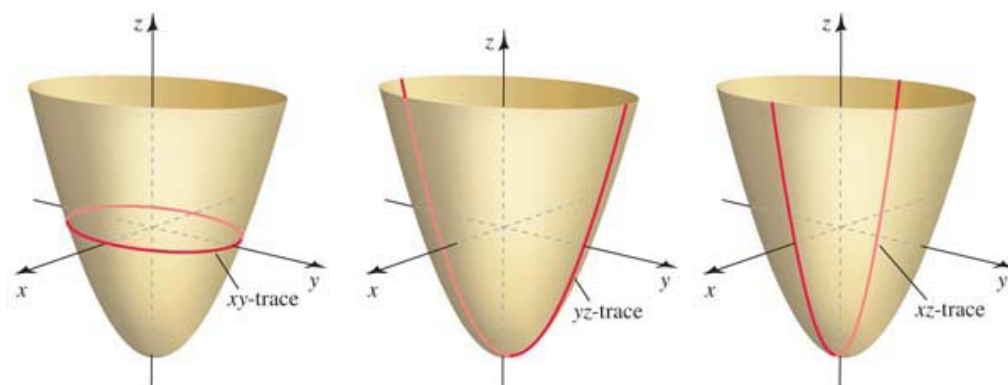
1 13.6: Cylinders and Quadric Surfaces

Cylinders and Traces:

When talking about three-dimensional surfaces, a *cylinder* refers to a surface that is parallel to a line. When considering surfaces that is parallel to one of the coordinate axes, that the associated variable is missing (e.g. $3y^2 + z^2 = 8$ is parallel to the x -axis).

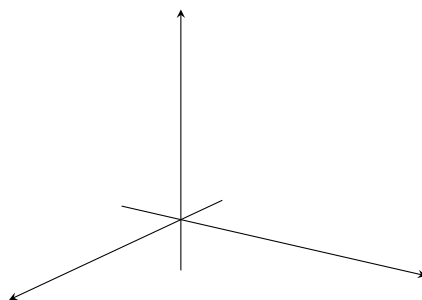
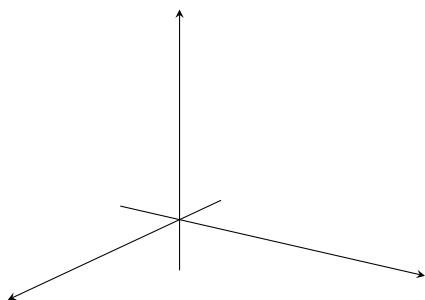
Definition. (Trace)

A **trace** of a surface is the set of points at which the surface intersects a plane that is parallel to one of the coordinate planes. The traces in the coordinate planes are called the **xy -trace**, the **yz -trace**, and the **xz -trace** (Figure 13.80).



Example. Roughly sketch the following functions:

1. $x^2 + 4y^2 = 16$
2. $x - \sin(z) = 0$



Quadric Surfaces:

Quadric surfaces are described by the general quadratic (second-degree) equation in three variables,

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0,$$

Where the coefficients A, \dots, J and not all zero.

Example (An ellipsoid). The surface defined by the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. Graph $a = 3$, $b = 4$ and $c = 5$.

Example (An elliptic paraboloid). The surface defined by the equation $z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$. Graph the elliptic paraboloid with $a = 4$ and $b = 2$.

Example (A hyperboloid of one sheet).

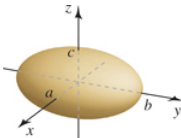
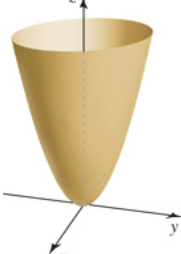
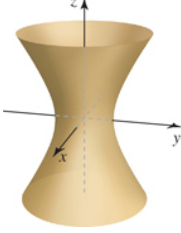
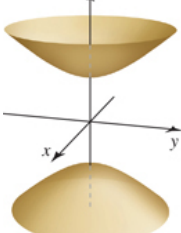
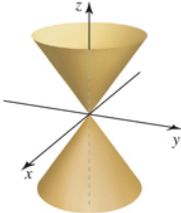
Graph the surface defined by the equation $\frac{x^2}{4} + \frac{y^2}{9} - z^2 = 1$.

Example (A hyperboloid of two sheets). Graph the surface defined by the equation $-16x^2 - 4y^2 + z^2 + 64x - 80 = 0$.

Example (Elliptic cones). Graph the surface defined by the equation $\frac{y^2}{4} + z^2 = 4x^2$.

Example (A hyperbolic paraboloid).

Graph the surface defined by the equation $z = x^2 - \frac{y^2}{4}$.

Name	Standard Equation	Features	Graph
Ellipsoid	$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$	All traces are ellipses.	
Elliptic paraboloid	$z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$	Traces with $z = z_0 > 0$ are ellipses. Traces with $x = x_0$ or $y = y_0$ are parabolas.	
Hyperboloid of one sheet	$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$	Traces with $z = z_0$ are ellipses for all z_0 . Traces with $x = x_0$ or $y = y_0$ are hyperbolas.	
Hyperboloid of two sheets	$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$	Traces with $z = z_0$ with $ z_0 > c $ are ellipses. Traces with $x = x_0$ and $y = y_0$ are hyperbolas.	
Elliptic cone	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$	Traces with $z = z_0 \neq 0$ are ellipses. Traces with $x = x_0$ or $y = y_0$ are hyperbolas or intersecting lines.	
Hyperbolic paraboloid	$z = \frac{x^2}{a^2} - \frac{y^2}{b^2}$	Traces with $z = z_0 \neq 0$ are hyperbolas. Traces with $x = x_0$ or $y = y_0$ are parabolas.	