10.5 Rationalizing Numerators or Denominators Using Conjugates

Definition. A **conjugate** is formed by changing the sign between two terms in a binomial.

Example. Rationalize the denominator of $\dfrac{x^2 - 3}{x + \sqrt{3}}$. $\left(\dfrac{x - \sqrt{3}}{x - \sqrt{3}}\right)$

$$= \frac{(x^2-3)(x-\sqrt{3})}{x^2-3}$$

$$\approx x - \sqrt{3}$$

Example. Write $\dfrac{1}{\sqrt{x + h}} - \dfrac{1}{\sqrt{x}}$ as one fraction, and rationalize the resulting numerator.

$$\frac{\sqrt{x} - \sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}\left(\frac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}}\right) = \frac{x - (x+h)}{\sqrt{x}\sqrt{x+h}\left(\sqrt{x} + \sqrt{x+h}\right)}$$

$$= \frac{-h}{x\sqrt{x+h} + \sqrt{x}(x+h)}$$

10.6 Extracting Factors from Radicals

Example. Simplify $\sqrt[3]{250x^4y^3}$

$$= \sqrt[3]{2 \cdot \underbrace{125}_{5^3}\, x^3 \cdot x\, y^3}$$

$$= 5xy\sqrt[3]{2x}$$

Example. Simplify $\sqrt{x^2y^6 + 3x^5y^4} = \sqrt{x^2y^4(y^2 + 3x^3)}$

$$= |x|y^2\sqrt{y^2 + 3x^3}$$