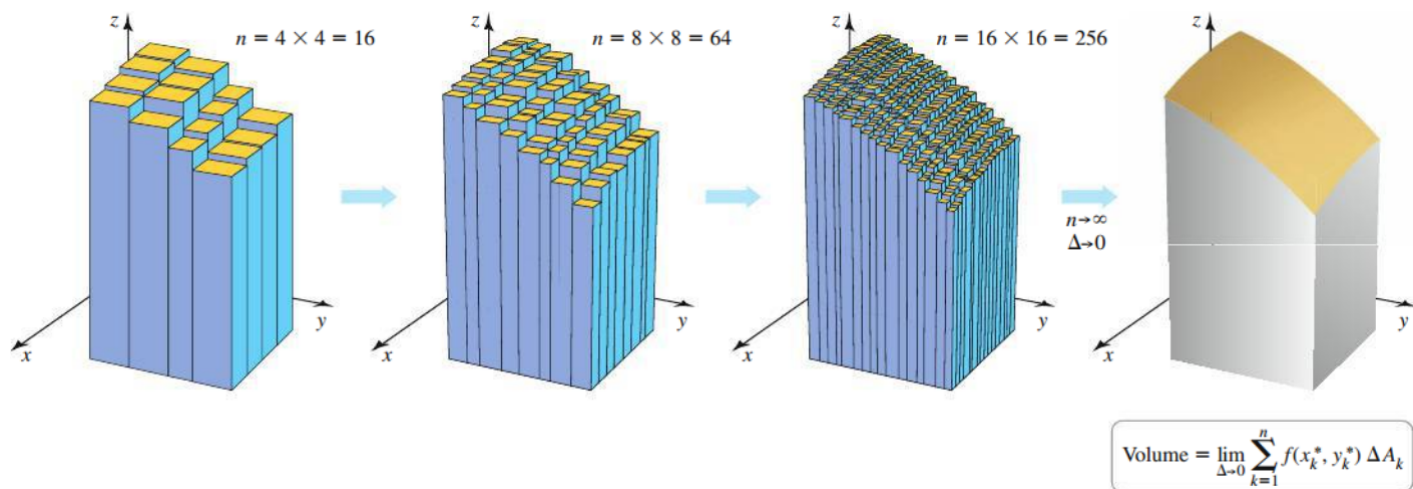


16.1: Double Integrals over Rectangular Regions



Definition. (Double Integrals)

A function f defined on a rectangular region R in the xy -plane is **integrable** on R if $\lim_{\Delta \rightarrow 0} \sum_{k=1}^n f(x_k^*, y_k^*) \Delta A_k$ exists for all partitions of R and for all choices of (x_k^*, y_k^*) within those partitions. The limit is the **double integral of f over R** , which we write

$$\iint_R f(x, y) dA = \lim_{\Delta \rightarrow 0} \sum_{k=1}^n f(x_k^*, y_k^*) \Delta A_k.$$

Example. Compute the following integral: $\int_0^1 \int_0^2 (6 - 2x - y) dy dx$

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Theorem 16.1: (Fubini) Double Integrals over Rectangular Regions

Let f be continuous on the rectangular region $R = \{(x, y) : a \leq x \leq b, c \leq y \leq d\}$. The double integral of f over R may be evaluated by either of the two iterated integrals:

$$\iint_R f(x, y) \, dA = \int_c^d \int_a^b f(x, y) \, dx \, dy = \int_a^b \int_c^d f(x, y) \, dy \, dx.$$

Example. Find the volume of the solid bounded by the surface $f(x, y) = 4 + 9x^2y^2$ over the region $R = \{(x, y) : -1 \leq x \leq 1, 0 \leq y \leq 2\}$. Integrate with respect to x first, then with respect to y first.

Example. Evaluate $\iint_R ye^{xy} dA$, where $R = \{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq \ln(2)\}$.

Definition. (Average Value of a Function over a Plane Region)

The **average value** of an integrable function f over a region R is

$$\bar{f} = \frac{1}{\text{area of } R} \iint_R f(x, y) \, dA.$$

Example. Find the average value of $f(x, y) = 2 - x - y$ over the region $R = \{(x, y) : 0 \leq x \leq 2, 0 \leq y \leq 2\}$.