1 MATH 1040 Review

For the following functions, find their derivatives:

$y = \sqrt[7]{x^3} - \pi e^x + x^e + 3e^{-x}$

$f(x) = \left(\frac{1-\sin(x)}{1+\cos(x)}\right)$

$g(x) = \left(\frac{x^2+3x+1}{e^x}\right)$

$h(y) = -5\cot\left(3e^{4y}\right) + e^{\pi}$

Find $f''(x)$ for $f(x) = \tan(x)$

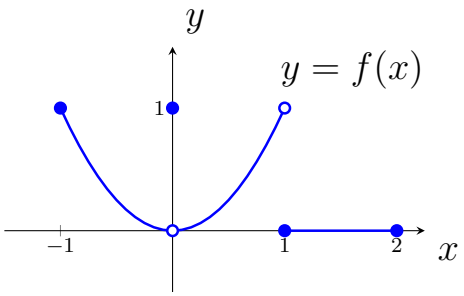
Find the equation of the line tangent to $\ell(x) = x\sqrt{5 - x^2}$ at the point $(1, 2)$.

Where is the tangent line of $u = \dfrac{1}{\sqrt{x}}$ parallel to the line $y = -4x - 3$?

Note: Limits will be on Test 1 and the final exam.

Example.

Using the graph below, evaluate each limit:



$\lim_{x\to -1^+} f(x)$ $\quad$ $\lim_{x\to 2^-} f(x)$

$\lim_{x\to 0^-} f(x)$ $\quad$ $\lim_{x\to 0^+} f(x)$ $\quad$ $\lim_{x\to 0} f(x)$

$\lim_{x\to 1^-} f(x)$ $\quad$ $\lim_{x\to 1^+} f(x)$ $\quad$ $\lim_{x\to 1} f(x)$

State the intervals of continuity on $[-1, 2]$.

Example. Algebraically, evaluate the following limits

$$\lim_{x\to 0} \left(\sin^2 x + \sec x\right) \qquad \lim_{y\to 0} \frac{5y^3 + 8y^2}{3y^4 - 16y^2}$$

$$\lim_{x \to \frac{1}{2}^-} \frac{4x - 2}{|2x^3 - x^2|}$$

$$\lim_{x \to 0} \frac{1 - \cos x}{\cos^2 x - 3\cos x + 2}$$

$$\lim_{x \to 0} \frac{x}{\sqrt{5x + 1} - 1}$$

$$\lim_{x \to 0} \frac{e^{4x} - 1}{e^x - 1}$$

$$\lim_{x \to 0} \frac{\sin(x)}{x}$$

$$\lim_{x \to 0} \frac{\tan(3x)}{5x}$$

$$\lim_{x\to\infty} \frac{4x^3+1}{2x^3+\sqrt{16x^6+1}}$$

$$\lim_{x\to-\infty} \frac{4x^3+1}{2x^3+\sqrt{16x^6+1}}$$

$$\lim_{x\to-\infty} \left(x+\sqrt{x^2+2x}\right)$$

$$\lim_{t \to -2^-} \frac{t^3 - 5t^2 + 6t}{t^4 - 4t^2}$$

$$\lim_{t \to -2^+} \frac{t^3 - 5t^2 + 6t}{t^4 - 4t^2}$$

$$\lim_{x \to -\infty} \frac{3x + 7}{x^2 - 4}$$

Find the equation of the slant (oblique) asymptote of $f(x) = \dfrac{3x^5 + x^4 + 2x^2 + 1}{x^4 + 3}$.

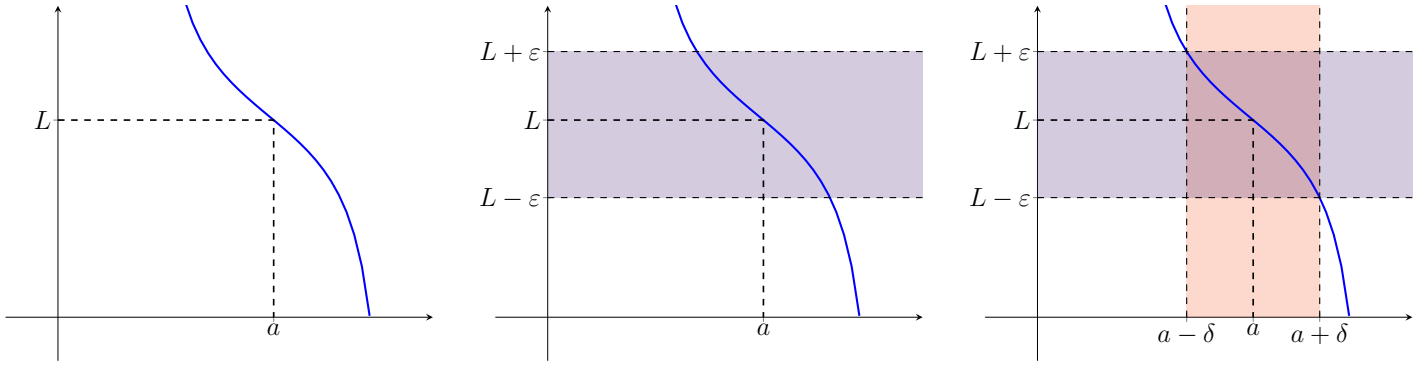
Find k such that $f(x)$ is continuous at $x = 1$:

$$f(x) = \begin{cases} k \tan\left(\frac{\pi x}{3}\right), & x \geq 1 \\ x - 2, & x < 1 \end{cases}$$

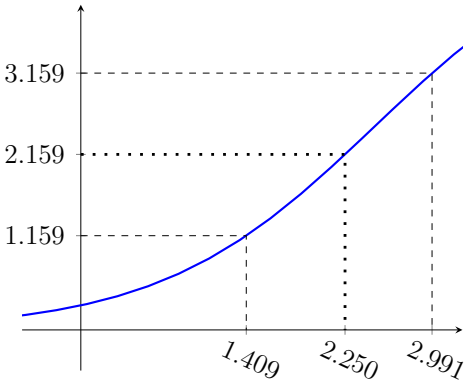
Find c such that $f(x)$ is continuous:

$$f(x) = \begin{cases} \dfrac{\sin^2 3x}{x^2}, & x \neq 0 \\ c, & x = 0 \end{cases}$$

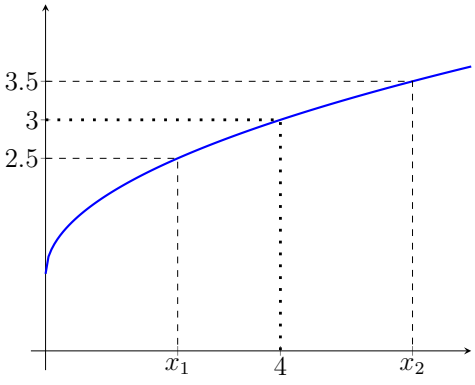
δ — ε proofs:



Example. Use the graph of f below to find a number δ such that if $0 < |x - 2.25| < \delta$ then $|f(x) - 2.159| < 1$.



Example. Use the graph of $g(x) = \sqrt{x}+1$ to help find a number δ such that if $|x - 4| < \delta$ then $\left|\left(\sqrt{x}+1\right) - 3\right| < \dfrac{1}{2}$.



Example. Algebraically, prove the following limits:

$$\lim_{x\to 3}(10-3x)=1$$

$$\lim_{x\to 14}\left(2-\frac{2}{7}x\right)=-2$$

$$\lim_{x\to 3}\frac{x^2+x-12}{x-3}=7$$

Rates of change

Example. Find the average rate of change of $f(x) = 3x^2 - 4x$ over the interval $[-1, 4]$ and the instantaneous rate of change at $x = 3$.

Limit definition of the derivative Recall the following definition:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Example. Use the limit definition of the derivative to find $f'(x)$ when $f(x) = -\frac{1}{x^2}$ and then evaluate $f'(3)$.

Example. Use the limit definition of the derivative to find $f'(x)$ when $f(x) = \frac{1-x}{2x}$.

Function	Derivative
Increasing	
Decreasing	
Max/Min	
Inflection point	
Constant	
Linear	
Quadratic	

A function is not differentiable wherever it has a

1. _____
2. _____
3. _____

The Chain Rule and Product Rule

Example. Find the derivatives of the following functions

$y = \cos\left(2x^5 + 7x\right)$ $\qquad$ $p(x) = \sqrt{2}x + \sqrt{3x}$

$y = x^{2e} - e^{\frac{3x-2}{x^2-3x}}$ $\qquad$ $y = f\left(\sqrt[3]{g(4x^3)}\right)$

$\frac{d}{dx}\left[\frac{f(x)-3g(x)}{2}\right]$

$\frac{d}{dx}\left[\frac{x[g(x)]^2}{h(x)}\right]$

$h(\theta) = \sqrt[3]{-\theta + \cot(9+2\theta)}$

$y(\theta) = \tan^2(\cot(3\theta))$

$y = 3x^2(e^{-x} + 2)^4 \tan(3x + 2)$ $\qquad\qquad$ $h(x) = \frac{-1}{2\sqrt[5]{\csc^2(4x)}}$

Example. Let $f(1) = 3, f'(1) = 4, g(1) = 2, g'(1) = 6, g(3) = 5$ and $g'(3) = 2$.
Now, let $H(x) = (g \circ f))(x) = g(f(x))$ and find $H'(1)$.

Example. Find $\dfrac{d^2}{d\theta^2}\left[\sin^2(3\theta)\right]$.