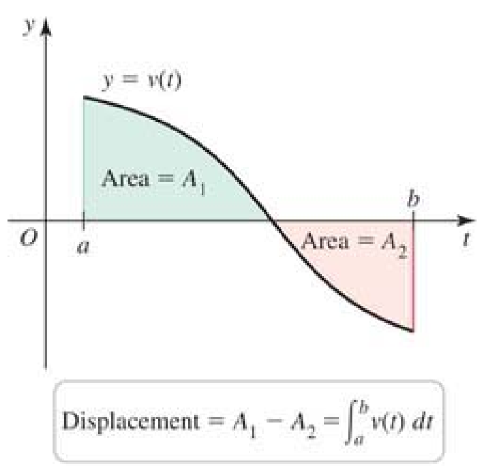
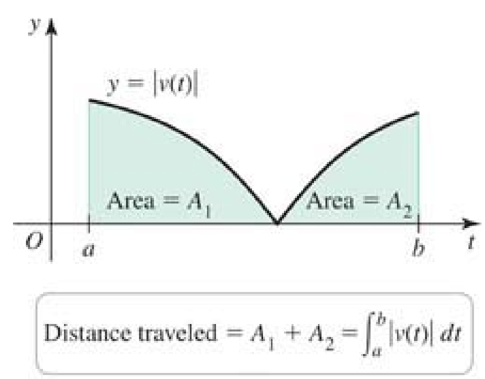
6.1: Velocity and Net Change

Definition. (Position, Velocity, Displacement, and Distance)

1. The **position** of an object moving along a line at time t, denoted $s(t)$, is the location of the object relative to the origin.
2. The **velocity** of an object at time t is $v(t) = s'(t)$.
3. The **displacement** of the object between $t = a$ and $t = b > a$ is
$$s(b) - s(a) = \int_a^b v(t)\, dt.$$
4. The **distance traveled** by the object between $t = a$ and $t = b > a$ is
$$\int_a^b |v(t)|\, dt$$
where $|v(t)|$ is the **speed** of the object at time t.



(a)

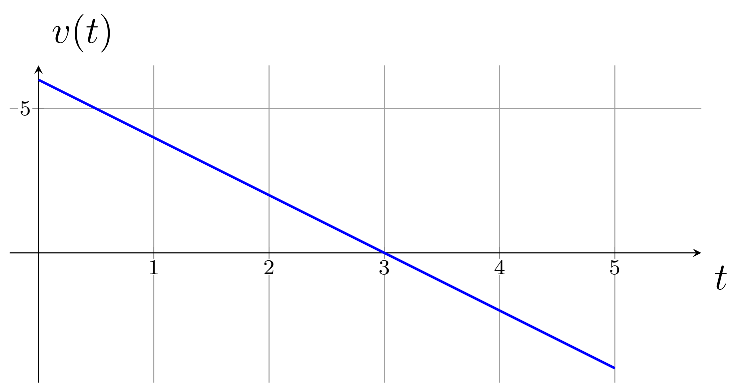


(b)

Example. Suppose an object moves along a line with velocity (in ft/s) $v(t) = 6 - 2t$, for $0 \le t \le 5$, where t is measured in seconds.

- Find the displacement of the object on the interval $0 \le t \le 5$.

- Find the distance traveled by the object on the interval $0 \le t \le 5$.



Example. A cyclist rides down a long straight road at a velocity (in m/min) given by $v(t) = 400 - 20t$, for $0 \leq t \leq 10$.

- How far does the cyclists travel in the first 5 minutes?

- How far does the cyclists travel in the first 10 minutes?

- How far has the cyclist traveled when her velocity is 250 m/min?

Example. The population of a community of foxes is observed to fluctuate on a 10-year cycle due to variations in the availability of prey. When population measurements began ($t = 0$), the population was 35 foxes. The growth rate in units of foxes/year was observed to be:

$$P'(t) = 5 + 10 \sin\left(\frac{\pi t}{5}\right)$$

- Find $P(t)$.

- Find the population of foxes after the first 5 years, rounded to the nearest whole number of foxes.

Theorem 6.1: Position from Velocity
Given the velocity $v(t)$ of an object moving along a line and its initial position $s(0)$, the position function of the object for future times $t \geq 0$ is

$$\underbrace{s(t)}_{\substack{\text{position} \\ \text{at } t}} = \underbrace{s(0)}_{\substack{\text{initial} \\ \text{position}}} + \underbrace{\int_0^t v(x)\,dx}_{\substack{\text{displacement} \\ \text{over } [0,t]}}.$$

Theorem 6.2: Velocity from Acceleration
Given the acceleration $a(t)$ of an object moving along a line and its initial velocity $v(0)$, the velocity of the object for future times $t \geq 0$ is

$$v(t) = v(0) + \int_0^t a(x)\,dx.$$

Example. At $t = 0$, a train approaching a station begins decelerating from a speed of 80 miles/hour according to the acceleration function $a(t) = -1280(1 + 8t)^{-3}$, where $t \geq 0$ is measured in hours. The units of acceleration are mi/hr^2.

- Find the velocity of the train at $t = 0.25$.

- How far does the train travel in the first 15 minutes (1/4 hour)?

- How long does it take the train to travel 9 miles?

Theorem 6.3: Net Change and Future Value
Suppose a quantity Q changes over time at a known rate Q'. Then the **net change** in Q between $t = a$ and $t = b > a$ is

$$\underbrace{Q(b) - Q(a)}_{\text{net change in } Q} = \int_a^b Q'(t)\,dt.$$

Given the initial value $Q(0)$, the **future value** of Q at time $t \geq 0$ is

$$Q(t) = Q(0) + \int_0^t Q'(x)\,dx.$$

Velocity-Displacement Problems	**General Problems**
Position $s(t)$	Quantity $Q(t)$ (such as volume or population)
Velocity: $s'(t) = v(t)$	Rate of change: $Q'(t)$
Displacement: $s(b) - s(a) = \int_a^b v(t)\,dt$	Net change: $Q(b) - Q(a) = \int_a^b Q'(t)\,dt$
Future position: $s(t) = s(0) + \int_0^t v(x)\,dx$	Future value of Q: $Q(t) = Q(0) + \int_0^t Q'(x)\,dx$