# 1 15.8: Lagrange Multipliers

**Definition. (Parallel Gradients)**
Let $f$ be a differentiable function in a region of $\mathbb{R}^2$ that contains the smooth curve $C$ given by $g(x, y) = 0$. Assume $f$ has a local extreme value on $C$ at a point $P(a, b)$. Then $\nabla f(a, b)$ is orthogonal to the line tangent to $C$ at $P$. Assuming $\nabla g(a, b) \neq \mathbf{0}$, it follows that there is a real number $\lambda$ (called a **Lagrange multiplier**) such that $\nabla f(a, b) = \lambda \nabla g(a, b)$.

**Procedure- Lagrange Multipliers: Absolute Extrema on Closed and Bounded Constraint Curves**
Let the objective function $f$ and the constraint function $g$ be differentiable on a region $\mathbb{R}^2$ with $\nabla g(x, y) \neq \mathbf{0}$ on the curve $g(x, y) = 0$. To locate the absolute maximum and minimum values of $f$ subject to the constraint $g(x, y) = 0$, carry out the following steps.

1. Find the values of $x$, $y$, and $\lambda$ (if they exist) that satisfy the equations
$$\nabla f(x, y) = \lambda \nabla g(x, y) \text{ and } g(x, y) = 0.$$
2. Evaluate $f$ at the values $(x, y)$ in Step 1 and at the endpoints of the constraint curve (if they exist). Select the largest and smallest corresponding function values. These values are the absolute maximum and minimum values of $f$ subject to the constraint.

**Procedure- Lagrange Multipliers: Absolute Extrema on Closed and Bounded Constraint Surfaces**
Let $f$ and $g$ be differentiable on a region of $\mathbb{R}^3$ with $\nabla g(x, y, z) \neq \mathbf{0}$ on the surface $g(x, y, z) = 0$. To locate the absolute maximum and minimum values of $f$ subject to

the constraint $g(x, y, z) = 0$, carry out the following steps.

1. Find the values of $x$, $y$, $z$, and $\lambda$ that satisfy the equations

$$\nabla f(x, y, z) = \lambda \nabla g(x, y, z) \text{ and } g(x, y, z) = 0.$$

2. Among the points $(x, y, z)$ found in Step 1, select the largest and smallest corresponding function values. These values are the absolute maximum and minimum values of $f$ subject to the constraint.