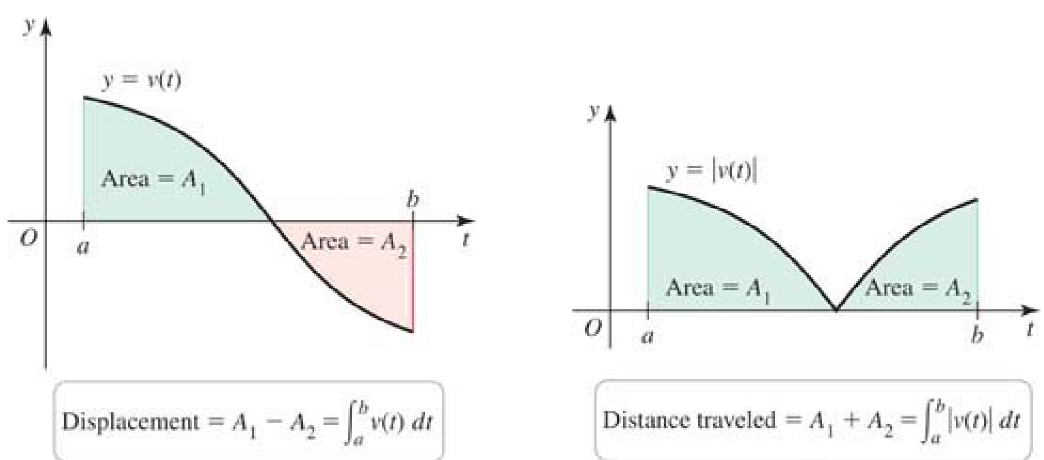
1 6.1: Velocity and Net Change

Definition. (Position, Velocity, Displacement, and Distance)

1. The **position** of an object moving along a line at time t, denoted $s(t)$, is the location of the object relative to the origin.
2. The **velocity** of an object at time t is $v(t) = s'(t)$.
3. The **displacement** of the object between $t = a$ and $t = b > a$ is
$$s(b) - s(a) = \int_a^b v(t)\,dt.$$
4. The **distance traveled** by the object between $t = a$ and $t = b > a$ is
$$\int_a^b |v(t)|\,dt$$
where $|v(t)|$ is the **speed** of the object at time t.

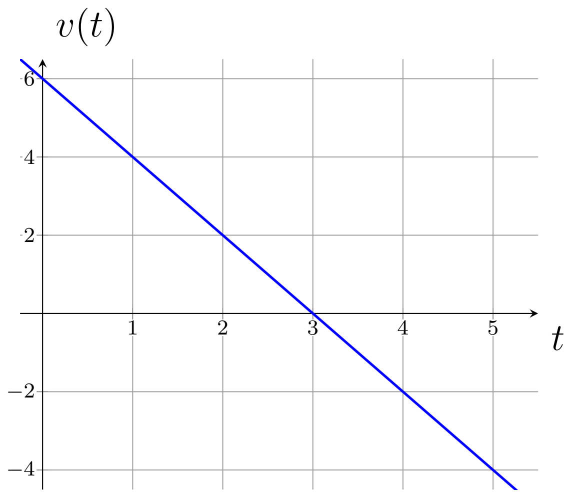


(a) (b)

Example. Suppose an object moves along a line with velocity (in ft/s) $v(t) = 6 - 2t$, for $0 \leq t \leq 5$, where t is measured in seconds.

- Find the displacement of the object on the interval $0 \leq t \leq 5$.

- Find the distance traveled by the object on the interval $0 \leq t \leq 5$.



Theorem 6.1: Position from Velocity
Given the velocity $v(t)$ of an object moving along a line and its initial position $s(0)$, the position function of the object for future times $t \geq 0$ is

$$\underbrace{s(t)}_{\substack{\text{position}\\ \text{at } t}} = \underbrace{s(0)}_{\substack{\text{initial}\\ \text{position}}} + \underbrace{\int_0^t v(x)\, dx}_{\substack{\text{displacement}\\ \text{over } [0,t]}} .$$

Theorem 6.2: Velocity from Acceleration
Given the acceleration $a(t)$ of an object moving along a line and its initial velocity $v(0)$, the velocity of the object for future times $t \geq 0$ is

$$v(t) = v(0) + \int_0^t a(x)\,dx.$$

Theorem 6.3: Net Change and Future Value
Suppose a quantity Q changes over time at a known rate Q'. Then the **net change** in Q between $t = a$ and $t = b > a$ is

$$\underbrace{Q(b) - Q(a)}_{\text{net change in } Q} = \int_a^b Q'(t)\,dt.$$

Given the initial value $Q(0)$, the **future value** of Q at time $t \geq 0$ is

$$Q(t) = Q(0) + \int_0^t Q'(x)\,dx.$$

Velocity-Displacement Problems	**General Problems**
Position $s(t)$	Quantity $Q(t)$ (such as volume or population)
Velocity: $s'(t) = v(t)$	Rate of change: $Q'(t)$
Displacement: $s(b) - s(a) = \int_a^b v(t)\,dt$	Net change: $Q(b) - Q(a) = \int_a^b Q'(t)\,dt$
Future position: $s(t) = s(0) + \int_0^t v(x)\,dx$	Future value of Q: $Q(t) = Q(0) + \int_0^t Q'(x)\,dx$