Math1080

Unit 1 Formula reference sheet

Note: Let $f(x)$ and $g(y)$ represent the area between 2 functions where appropriate.

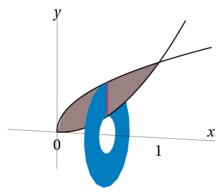
Velocity

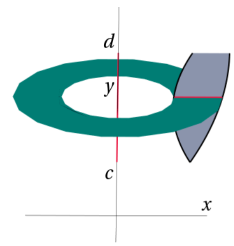
$$F(b) - F(a) = \int_a^b f(x)\,dx \quad \Longrightarrow \quad v(t) = v(0) + \int_0^t a(x)\,dx$$

Disk/Washer

πr^2

x − axis, $y = 0$	$\int_a^b \pi\left(R^2 - r^2\right) dx$	y − axis, $x = 0$	$\int_c^d \pi\left(R^2 - r^2\right) dy$
$y = c$	$\int_a^b \pi\left((R-c)^2 - (r-c)^2\right) dx$	$x = c$	$\int_c^d \pi\left((R-c)^2 - (r-c)^2\right) dy$

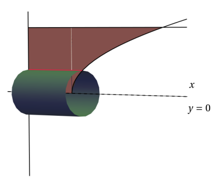


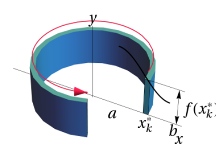


Shell method

$2\pi r \cdot h$

x − axis, $y = 0$	$\int_c^d 2\pi y \cdot g(y)\,dy$	y − axis, $x = 0$	$\int_a^b 2\pi x \cdot f(x)\,dx$
$c < a$, $y = c$	$\int_c^d 2\pi(y - c) \cdot g(y)\,dy$	$c < a$, $x = c$	$\int_a^b 2\pi(x - c) \cdot g(x)\,dx$
$b < c$, $y = c$	$\int_c^d 2\pi(c - y) \cdot g(y)\,dy$	$b < c$, $x = c$	$\int_a^b 2\pi(c - x) \cdot g(x)\,dx$





Arc length

$\sqrt{\left(\frac{dx}{dx}\right)^2 + \left(\frac{dy}{dx}\right)^2}$

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\,dx$$

Surface area

$2\pi r \cdot h$

$$S = \int_a^b 2\pi f(x)\sqrt{1 + [f'(x)]^2}\,dx$$

Mass

$\rho \cdot d$

$$m = \int_a^b \rho\,dx$$

Work

$F \cdot d$

$$W = \int_a^b F(x)\,dx$$

Chain with load

$$W = \int_0^L \rho g(L - y)\,dy + mgL$$

Pumping

$$W = \int_a^b \rho g A(y)(h - y)\,dy$$

Force-on-dam

$$F = \int_0^a \rho g \underbrace{(a - y)}_{\text{depth}}\underbrace{w(y)}_{\text{width}}\,dy$$

Integration by parts

$$\int u\,dv = uv - \int v\,du$$