5.6: Exponential Functions As Mathematical Models

Example. Consider the exponential function

$$Q(t) = Q_0 e^{kt}$$

What does Q_0 represent?

What does k represent?

Show that the rate of increase of $Q(t)$ is proportional to the quantity $Q(t)$.

Definition.
$Q(t)$ is said to exhibit **Exponential Growth**.

Example. Under ideal laboratory conditions, the number of bacteria in a culture grows in accordance with the law $Q(t) = Q_0 e^{kt}$, where Q_0 denotes the number of bacteria initially present in the culture, k is a constant determined by the strain of bacteria under consideration and other factors, and t is the elapsed time measured in hours. Suppose $10,000$ bacteria are present initially in the culture and $60,000$ are present 2 hours later.

How many bacteria will there be in the culture at the end of 4 hours?

What is the rate of growth of the population after 4 hours?

Example. Radioactive substances decay exponentially. For example, the amount of radium present at any time t obeys the law $Q(t) = Q_0 e^{-kt}$, where Q_0 is the initial amount present and k is a specific positive constant. The **half-life of a radioactive substance** is the time required for a given amount to be reduced by one-half. It is known that the half-life of radium is approximately 1600 years. Suppose initially there are 200 milligrams of pure radium.

What is the amount left after t years? What about 800 years?

How fast is the amount of radium decaying after t years? What about 800 years?

Example. Carbon 14, a radioactive isotope of carbon, has a half-life of 5730 years. What is its decay constant?

Example. The Camera Division of Eastman Optical produces a 35-mm single-lens reflex camera. Eastman's training department determines that after completing the basic training program, a new, previously inexperienced employee will be able to assemble

$$Q(t) = 50 - 30e^{-0.5t}$$

model F cameras per day t months after the employee starts work on the assembly line.

How many model F cameras can a new employee assemble per day after basic training?

How many model F cameras can an employee with 1 month of experience assemble per day? What about 2 months? 6 months?

How many model F cameras can the average experienced employee ultimately be expected to assemble per day?

Example. The number of soldiers at Fort MacArthur who contracted influenza after t days during a flu epidemic is approximated by the *logistic model*

$$Q(t) = \frac{5000}{1 + 1249e^{-kt}}$$

If 40 soldiers contracted the flu by day 7, find how many soldiers contracted the flu by day 15.

At what rate is the number of soldiers contracting the flu changing on day 15?