## 5.2: Logarithmic Functions and Their Properties

#### Definition.

For a > 0 and  $a \neq 1$ , the **logarithmic function** 

$$y = \log_a(x)$$

(logarithmic form)

has domain x > 0, base a, and is defined by

$$a^y = x$$

(exponential form)

Example. Rewrite the following in exponential form

$$4 = \log_2(16)$$

$$5 = \log_{10}(100, 000)$$

$$\frac{1}{2} = \log_{16}(4)$$

$$-4 = \log_3\left(\frac{1}{81}\right)$$

$$-\frac{1}{4} = \log_{625}\left(\frac{1}{5}\right)$$

$$-\frac{5}{3} = \log_{\frac{1}{8}}(32)$$

**Example.** Simplify the following:

$$\log_3(9)$$

$$\log_4(2)$$

**Example.** Solve the following:

$$\log_5(x) = 4$$

$$\log_8(x) = 1$$

$$\log_{81}(x) = -\frac{1}{4}$$

$$\log_{10}(x+4) = 3$$

Common logarithms:  $\log(x) = \log_{10}(x)$ Natural logarithms:  $\ln(x) = \log_e(x)$ 

**Example** (The Rule of 70). If P is invested for t years at interest rate r, compounded continuously, then the future value of the investment is given by

$$S = Pe^{rt}$$
.

Find the value of t when the investment doubles.

Change of base formula:

If a > 0, b > 0 with  $a \neq 1$  and  $b \neq 1$ , then

$$\log_b(x) = \frac{\log_a(x)}{\log_b(x)}.$$

Note: This works for any valid base!

Base 
$$e: \log_b(x) = \frac{\ln(x)}{\ln(b)}$$

Base 10: 
$$\log_b(x) = \frac{\log(x)}{\log(b)}$$

Example. Solve the following

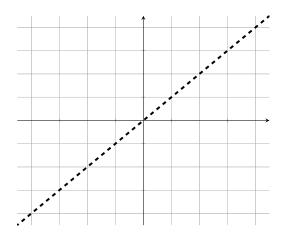
$$3^x = 10$$

$$6.5^x = 5$$

**Example.** Fill in the tables below and graph  $a^x$  and  $\log_a(x)$  on the same axes.

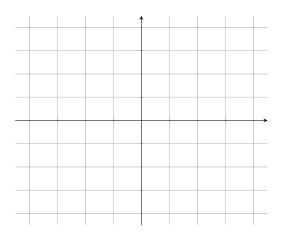
2

$x  y = a^x$	$\underline{x  y = \log_a(x)}$
-2	-2
-1	-1
0	0
1	1

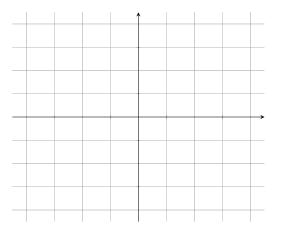


**Example.** Graph  $\log(-x)$ 

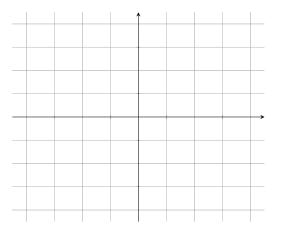
2



# **Example.** Graph ln(x)



# **Example.** Graph $-\log_2(-x)$



**Example.** Evaluate the following:

$$f(x) = \ln(x); \quad f(e^{-3x})$$
  $f(x) = 5^x; \quad f(\log_5(10))$ 

### Properties of exponents and logarithms: Assume a > 0:

$$a^{y} = x \qquad \log_{a}(x) = y$$

$$a^{1} = a \qquad \log_{a}(a) = 1$$

$$a^{0} = 1 \qquad \log_{a}(1) = 0$$

$$a^{x}a^{y} = a^{x+y} \qquad \log_{a}(xy) = \log_{a}(x) + \log_{a}(y)$$

$$\frac{a^{x}}{a^{y}} = a^{x-y} \qquad \log_{a}\left(\frac{x}{y}\right) = \log_{a}(x) - \log_{a}(y)$$

$$a^{xy} = (a^{x})^{y} \qquad \log_{a}(x^{y}) = y \log_{a}(x)$$

$$a^{\log_{a}(x)} = x \qquad \log_{a}(a^{x}) = x$$