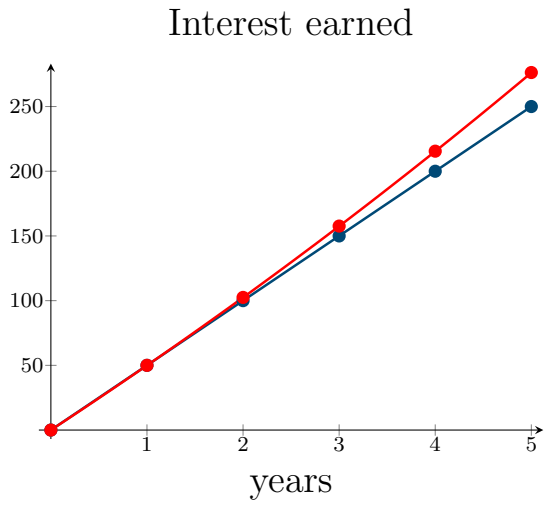
6.2: Compound Interest

Example. Suppose you invest \$1,000 at 5% annual interest. With simple interest, you can take 2 approaches:

1. Gain interest on *only* your initial investment
2. Reinvest the interest gained

Year	Simple interest	Simple interest reinvested
1	\$1,050.00	\$1,050.00
2	\$1,100.00	\$1,102.50
3	\$1,150.00	\$1,157.63
4	\$1,200.00	\$1,215.51
5	\$1,250.00	\$1,276.28



Definition.
Compound interest is a method where the interest for each period is added to the principal before interest is calculated for the next period.

Example. Using the example above, derive a formula for the future value of an investment compounded annually.

Definition.
When interest is compounded multiple times a year (e.g. quarterly, monthly, etc.), the **nominal interest rate** is the interest rate *per year*.

If $\$P$ is invested for t years at a nominal interest rate r compounded m times per year, then the **total number of compounding periods** is

$$n = mt$$

the interest rate per compounding period (**periodic interest rate**) is

$$i = \frac{r}{m}$$

and the future value is

$$S = P(1+i)^n = P\Big(1+\frac{r}{m}\Big)^{mt}$$

Example. If $\$3,000$ is invested for 5 years at 9% compounded 4 times a year, how much interest is earned?

Example. For the following, identify the annual interest rate, the length in years, the periodic interest rate, and the number of periods:

12% compounded monthly for 7 years

7.2% compounded quarterly for 11 quarters

Frequency	m
Annually	1
Semi-annually	2
Quarterly	4
Monthly	12
Weekly	52
Daily	365

Example. Ben and Taylor want to have $\$200,000$ in Arthur's college fund on his 18th birthday, and they want to know the impact on this goal of having $\$10,000$ invested at 9.8%, compounded quarterly, on his 1st birthday. To advise Ben and Taylor regarding this, find

the future value of the $\$10,000$ investment,

the amount of compound interest that the investment earns,

the impact this would have on their goal.

Example. What amount must be invested now to have $\$12,000$ after 3 years with an interest rate of 6%, compounded semi-annually?

Example. Three years after Google stock was first sold publicly, its share price had risen 500%. Google's 500% increase means that $\$10,000$ invested in Google stock at its initial public offering (I.P.O) was worth $\$60,000$ three years later. What interest rate compounded annually does this represent?

Example. Suppose we invest \$1 at a 100% interest rate for 1 year:

$$S = \left(1 + \frac{1.00}{m}\right)^m$$

Compute the future value

Annually

Semi-annually

Monthly

Weekly

Daily

Each minute ($m = 525,600$)

Definition.
If $\$P$ is invested for t years at a nominal rate r compounded continuously, then the future value is given by the exponential function

$$S = Pe^{rt}$$

Example. Which investment strategy is worth more: $\$3,000$ for 8 years at

9%, compounded annually

8%, compounded continuously

Example. Suppose you invest \$900 at 11.5%, compounded continuously. How long will it take to gain \$700 in interest?

Definition.
Let r represent the annual (nominal) interest rate for an investment. Then the **annual percentage yield (APY)** is:

Periodic compounding:

$$\text{APY} = \left(1 + \frac{r}{m}\right)^m - 1$$

Continuous compounding:

$$\text{APY} = e^r - 1$$