1 1.1: Solutions of Linear Equations and Inequalities in One Variable

Definition. (Functions)
A **function** f is a special relation between x and y such that each input x results in *at most* one y. The symbol $f(x)$ is read "f of x" and is called the **value of f at x**

Example. Let $f(x) = \frac{x^2}{2} + x$. Evaluate the following:

$f(1)$

$f\left(\frac{1}{2}\right)$

$f(-2)$

$f(0)$

$f(f(x))$

Composite Functions:

Let f and g be functions of x. Then, the **composite functions** g of f (denoted $g \circ f$) and f of g (denoted $f \circ g$) are defined as:

$$(g \circ f)(x) = g(f(x))$$
$$(f \circ g)(x) = f(g(x))$$

Example. Let $g = x - 1$. Find:

$(g \circ f)(x)$

$(f \circ g)(x)$

Operations with Functions:

Let f and g be functions of x and define the following:

Sum	$(f+g)(x) = f(x) + g(x)$
Difference	$(f-g)(x) = f(x) - g(x)$
Product	$(f \cdot g)(x) = f(x) \cdot g(x)$
Quotient	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ if $g(x) \neq 0$

Definition.

An **expression** is a meaningful string of numbers, variables and operations:

$$3x - 2$$

An **equation** is a statement that two quantities or algebraic expressions are equal:

$$3x - 2 = 7$$

A **solution** is a value of the variable that makes the equation true:

$$\begin{aligned} 3(3) - 2 &= 7 \\ 9 - 2 &= 7 \\ 7 &= 7 \end{aligned}$$

A **solution set** is the set of ALL possible solutions of an equation:

$3x - 2 = 7$ only has the solution $x = 3$,

$2(x-1) = 2x - 2$ is true for all possible values of x.

Properties of Equality:

Substitution Property: The equation formed by substituting one expression for an equal expression is equivalent to the original equation:

$$
\begin{aligned}
3(x-3) - \frac{1}{2}(4x-18) &= 4 \\
3x - 9 - 2x + 9 &= 4 \\
x &= 4
\end{aligned}
$$

Addition Property: The equation formed by adding the same quantity to both sides of an equation is equivalent to the original equation:

$$
\begin{gathered}
x - 4 = 6 \\
x - 4 + 4 = 6 + 4 \\
x = 10
\end{gathered}
\qquad\qquad
\begin{gathered}
x + 5 = 12 \\
x + 5 + (-5) = 12 + (-5) \\
x = 7
\end{gathered}
$$

Multiplication Property: The equation formed by multiplying both sides of an equation by the same *nonzero* quantity is equivalent to the original equation:

$$
\begin{gathered}
\frac{1}{3}x = 6 \\
3\left(\frac{1}{3}x\right) = 3(6) \\
x = 18
\end{gathered}
\qquad\qquad
\begin{gathered}
5x = 20 \\
\frac{5x}{5} = \frac{20}{5} \\
x = 4
\end{gathered}
$$

Solving a linear equation:

Using the properties of equality above, we can solve any linear equation in 1 variable:

Example. Solve $\frac{3x}{4}+3=\frac{x-1}{3}$

1. Eliminate fractions: $$12\left(\frac{3x}{4}+3\right)=12\left(\frac{x-1}{3}\right)$$
2. Remove/evaluate parenthesis: $$9x+36=4x-4$$
3. Use addition property to isolate the variable to one side: $$9x+36-36-4x=4x-4-36-4x$$
4. Use multiplication property to isolate variable: $$\frac{5x}{5}=\frac{-40}{5}$$
5. Verify solution via substitution: $$\underbrace{\frac{3(-8)}{4}+3}_{-6+3=-3}\stackrel{?}{=}\underbrace{\frac{(-8)-1}{3}}_{\frac{-9}{3}=-3}$$

Example. Solve the following:

$\frac{3x+1}{2}=\frac{x}{3}-3$

$\frac{2x-1}{x-3}=4+\frac{5}{x-3}$