Properties of Continuous Functions

1. The constant function $f(x) = c$ is continuous everywhere.
2. The identify function $f(x) = x$ is continuous everywhere.

If f and g are continuous at $x = a$, then

> $[f(x)]^n$, where n is a real number, is continuous at $x = a$ whenever it is defined at that number
>
> $f \pm g$ is continuous at $x = a$
>
> fg is continuous at $x = a$
>
> f/g is continuous at $x = a$ provided that $g(a) \neq 0$

Polynomial and Rational Functions

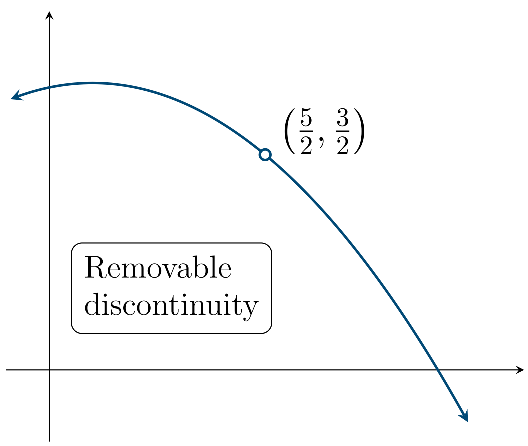
1. A polynomial function is continuous for all x.
2. A rational function (a function of the form $\frac{p}{q}$, where p and q are polynomials) is continuous for all x for which $q(x) \neq 0$.

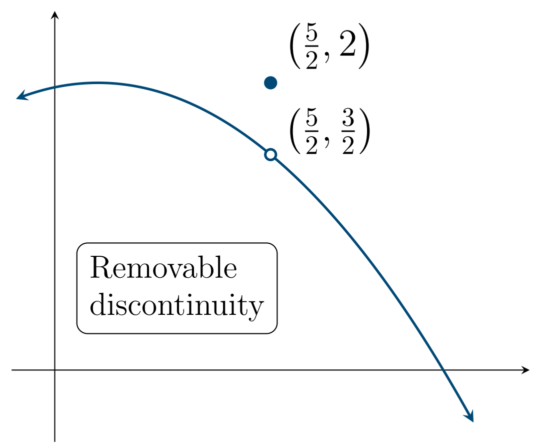
Definition.

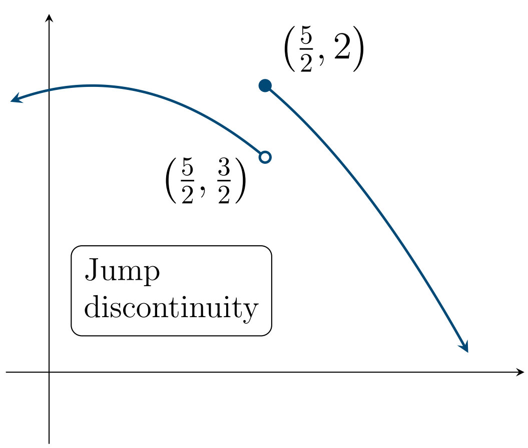
A **removable discontinuity** at $x = a$ is one that disappears when the function becomes continuous after defining $f(a) = \lim_{x \to a} f(x)$.

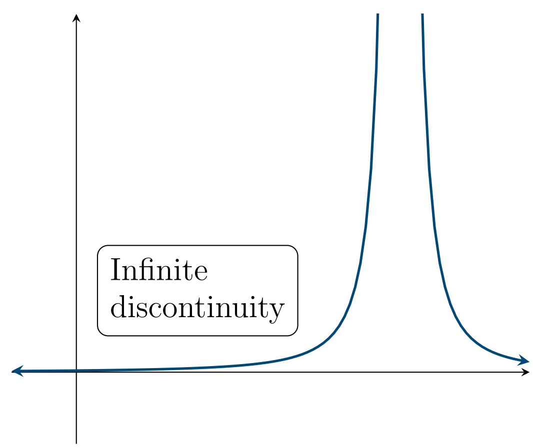
A **jump discontinuity** is one that occurs whenever $\lim_{x \to a^-} f(x)$ and $\lim_{x \to a^+} f(x)$ both exist, but $\lim_{x \to a^-} f(x) \neq \lim_{x \to a^+} f(x)$.

A **vertical discontinuity** occurs whenever $f(x)$ has a vertical asymptote.

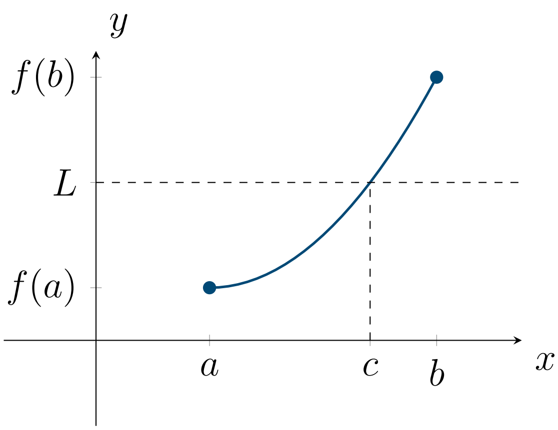


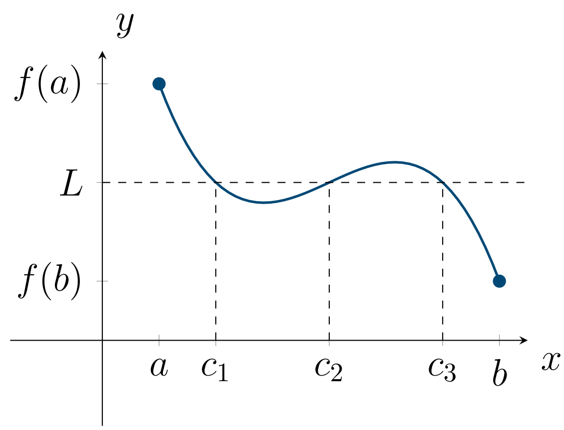




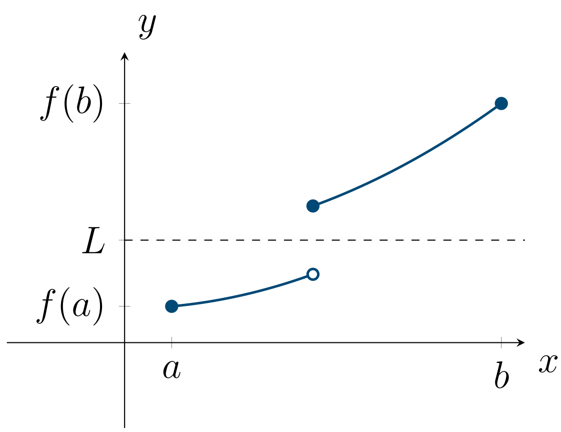


Theorem 4: Intermediate Value Theorem
Suppose f is continuous on the interval $[a, b]$ and L is a number strictly between $f(a)$ and $f(b)$. Then there exists at least one number c in (a, b) satisfying $f(c) = L$.





Note: It is important that the function be continuous on the interval $[a, b]$:



Theorem 5: Existence of Zeros of a Continuous Function
If f is a continuous function on a closed interval $[a, b]$, and if $f(a)$ and $f(b)$ have opposite signs, then there is at least one solution of the equation $f(x) = 0$ in the interval (a, b).