

### 1.3: Linear Functions

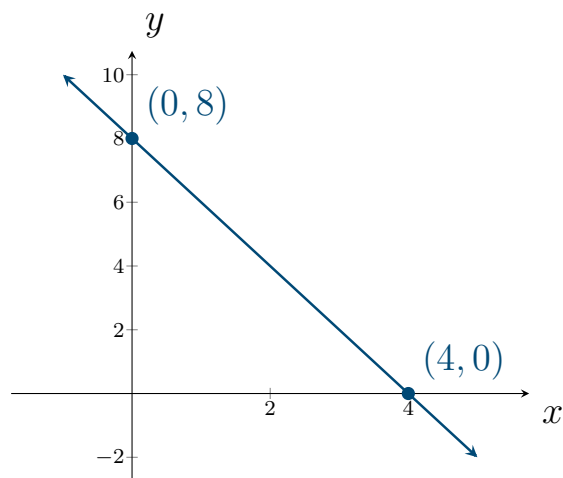
**Definition.**

A **linear function** is a function of the form

$$y = f(x) = mx + b$$

where  $m$  and  $b$  are constants.

**Example.**  $y = -2x + 8$



A linear function can be uniquely determined using only *two* distinct points.

**Definition.**

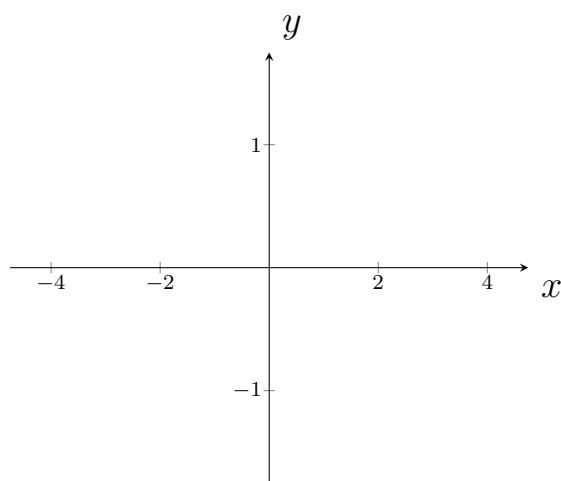
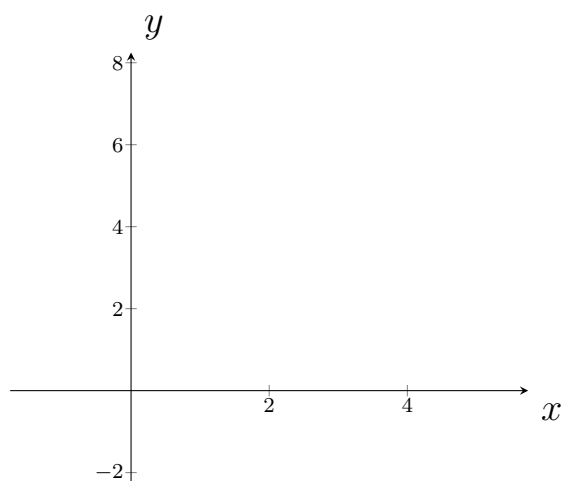
The point(s) where a graph intersects the axes are called intercepts. The  $x$ -coordinate of the point where the function intersects the  $x$ -axis is called the  **$x$ -intercepts**. The  $y$ -coordinate of the point where the function intersects the  $y$ -axis is called the  **$y$ -intercepts**.

- To solve for the  $y$ -intercept:
  - Set  $x = 0$ ,
  - Solve for  $y$ .
- To solve for the  $x$ -intercept:
  - Set  $y = 0$ ,
  - Solve for  $x$ .

**Example.** Find the intercepts and graph the following lines:

$$3x + 2y = 12$$

$$x = 4y$$



**Definition.**

If a nonvertical line passes through the points  $P_1(x_1, y_1)$  and  $P_2(x_2, y_2)$ , its **slope**, denoted by  $m$ , is found using

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$

$\Delta y$  is “delta  $y$ ”, and represents the change in  $y$

$\Delta x$  is “delta  $x$ ”, and represents the change in  $x$

*Note:* The slope of a vertical line is undefined.

**Example.** Find the slope of the line passing through the points  $(-2, 1)$  and  $(5, 3)$ .

*Note:*

- Two distinct nonvertical lines are *parallel* if and only if their slopes are *equal*.
- Two distinct nonvertical lines are *perpendicular* if and only if their slopes are *negative reciprocals*:  
e.g. If  $\ell_1$  has a nonzero slope  $m$ , then  $\ell_2$  is perpendicular if its slope is  $-1/m$ .

## Point-slope form

**Definition.**

The equation of the line passing through the point  $(x_1, y_1)$  with slope  $m$  can be written in the point-slope form:

$$y - y_1 = m(x - x_1)$$

**Example.** Find the equation of each line that passes through the point  $(-3, 4)$  and has

a slope of  $m = \frac{1}{4}$

the point  $(-2, 1)$  on the line

a slope of zero (horizontal)

an undefined slope (vertical)

## Slope-intercept form

**Definition.**

The slope-intercept form of the equation of a line with slope  $m$  and  $y$ -intercept  $b$  is

$$y = mx + b$$

**Example** (Example 7, p.82). The population of U.S. males,  $y$  (in thousands), projected from 2015 to 2060 can be modeled by

$$y = 1125.9x + 142,960$$

where  $x$  is the number of years after 2000.

- Find the slope and  $y$ -intercept of the graph of this function.
- What does the  $y$ -intercept tell us about the population of U.S. males?
- Interpret the slope as a rate of change.

**Example.** Each day, a young person should sleep 8 hours plus  $\frac{1}{4}$  hour for each year the person is under 18 years of age. Assuming that the relation is linear, write the equation relating hours of sleep  $y$  and age  $x$

## Forms of Linear Equations

General form:  $Ax + By = C$

Point-slope form:  $y - y_1 = m(x - x_1)$

Slope-intercept form:  $y = mx + b$

Vertical line:  $x = a$

Horizontal line:  $y = b$