## 1.1: Variables

**Definition.**
A **variable** is a placeholder for something which may or may not be unknown.

**Example.** Is there a number with the following property: doubling it and adding 3 gives the same result as squaring it?

$\rightarrow$ Is there a number $x$ with the property that $2x + 3 = x^2$?
$\rightarrow$ Is there a number $\square$ with the property that $2 \cdot \square + 3 = \square^2$?

**Example.** No matter what number might be chosen, if it is greater than 2, then its square is greater than 4.

$\rightarrow$ No matter what number $n$ might be chosen, if $n$ is greater than 2, then $n^2$ is greater than 4.

**Example.** Use variables to rewrite the following sentences:

Are there numbers with the property that the sum of their squares equals the square of their sum?

Do there exist numbers a & b such that $(a+b)^2 = a^2+b^2$?

Given any real number, its square is nonnegative.

For any real number $r$, $r^2 > 0$

> **Definition.**
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> - A **universal statement** says that a certain property is true for all elements in a set.
> - A **conditional statement** says that if one thing is true, then some other thing also has to be true.
> - Given a property that may or may not be true, an **existential statement** says that there is at least one thing for which the property is true.

**Definition.**
A **universal conditional statement** is both universal and conditional:

> For every animal $a$, if $a$ is a dog, then $a$ is a mammal.

Conditional statements can be rewritten in ways that make them appear more to be purely universal or purely conditional:

> If $a$ is a dog, then $a$ is a mammal.
> All dogs are mammals

**Example.** Rewrite the following universal condition statement:

> For every real number $x$, if $x$ is nonzero then $x^2$ is positive.

If a real number is nonzero, then its square is positive.

For every nonzero real number $x$, $x^2$ is positive.

If $x$ is a nonzero real number, then $x^2$ is positive.

The square of any nonzero real number is positive.

All nonzero real numbers have positive squares.

**Definition.**
A **universal existence statement** is a statement that is universal because its first part says that a certain property is true for all objects of a given type, and it is existential because its second part asserts the existence of something:

Every real number has an additive inverse.

In the above example, note that the particular additive inverse depends on the given real number:

For every real number $r$, there is an additive inverse for $r$.

**Example.** Rewrite the following universal existence statement:

Every pot has a lid

Not every lid will fit every pot

All pots have lids.

For ever pot $P$, there is a lid.

For every pot $P$, there is a lid $L$ such that $L$ is a lid for $P$.

**Definition.**
An **existential universal statement** is a statement that is existential because its first part asserts that a certain object exists and is universal because its second part says that the object satisfies a certain property for all things of a certain kind:

> There is a positive integer that is less than or equal to every positive integer.

The number one satisfies the above statement, which can also be rewritten:

> There is a positive integer $m$ that is less than or equal to every positive integer.

**Example.** Rewrite the following existence universal statement:

> There is a person in my class who is at least as old as every person in my class.

Some ___person in my class___ is at least as old as ___every person in my class___.

There is a person $p$ in my class such that $p$ is ___at least as old as every person in my class___.

There is a person $p$ in my class with the property that for every person $q$ in my class, $p$ is ___at least as old as $q$___.