# 1.5: Solutions of Systems of Linear Equations

$$+ + + + = 18$$
 $+ + + = 14$ 
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#### Definition.

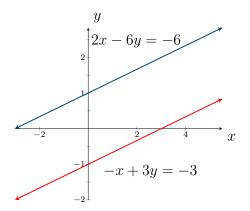
A system of equations is 2 (or more) equations. The ordered pairs (x, y) that satisfies *all* equations in the system are the solutions of the system.

When solving a system of linear equations, there are three possible outcomes:

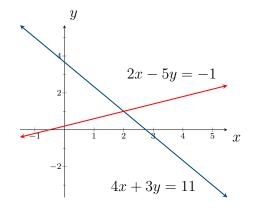
- 1. No solution (*Inconsistent*),
- 2. Exactly one solution,
- 3. Infinitely many solutions (Dependent).

**Example.** Use graphing to find the solutions to the following systems

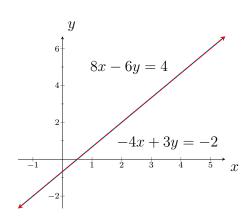
$$2x - 6y = -6$$
$$-x + 3y = -3$$



$$4x + 3y = 11$$
$$2x - 5y = -1$$



$$-4x + 3y = -2$$
$$8x - 6y = 4$$



### Equivalent systems result when

- 1. One expression is replaced by an equivalent expression.
- 2. Two equations are interchanged.
- 3. A multiple of one equation is added to another equation.
- 4. An equation is multiplied by a nonzero constant.

## Substitution Method

**Example.** Solve the system  $\begin{cases} 2x + 3y = 4 \\ x - 2y = 3 \end{cases}$ 

- 1. Solve one equation for either one of the variables in terms of the other.
- 2. Substitute this expression into the other equation to give the equation in one unknown.
- 3. Solve this equation for the unknown.
- 4. Substitute solution into the equation in Step 1.
- 5. Check the solution (x, y).

$$x = 2y + 3$$

$$2(2y+3) + 3y = 4$$

$$4y + 6 + 3y = 4$$
$$7y = -2 \Rightarrow y = -\frac{2}{7}$$

$$x = 2\left(-\frac{2}{7}\right) + 3 \Rightarrow x = \frac{17}{7}$$

$$2\left(\frac{17}{7}\right) + 3\left(-\frac{2}{7}\right) = 4$$
$$\left(\frac{17}{7}\right) - 2\left(-\frac{2}{7}\right) = 3$$

**Example.** Use the substitution method to solve the system

$$4x + 5y = 18 \tag{1}$$

$$3x - 9y = -12 (2)$$

#### **Elimination Method**

**Example.** Solve the system  $\begin{cases} 2x - 5y = 4 \\ x + 2y = 3 \end{cases}$ 

- 1. Multiply one or both equations by a nonzero number so the coefficients of one of the variables may cancel.
- 2. Add or subtract the equations to eliminate one of the variables.
- 3. Solve for the remaining variable.
- 4. Substitute solution in one of the original equations and solve for the other variable.
- 5. Check the solution (x, y)

$$\Rightarrow \begin{cases} 2x - 5y = 4 \\ -2x - 4y = -6 \end{cases}$$

$$0x - 9y = -2$$

$$\Rightarrow y = \frac{2}{9}$$

$$2x - 5\left(\frac{2}{9}\right) = 4 \implies x = \frac{23}{9}$$

$$2\left(\frac{23}{9}\right) - 5\left(\frac{2}{9}\right) = 4$$
$$\left(\frac{23}{9}\right) + 2\left(\frac{2}{9}\right) = 3$$

**Example.** Use the elimination method to solve the following systems:

$$2x - 6y = -6$$
$$-x + 3y = -3$$

$$4x + 3y = 11$$
$$2x - 5y = -1$$

$$-4x + 3y = -2$$
$$8x - 6y = 4$$

**Example.** A woman has \$500,000 invested in two rental properties. One yields an annual return of 10% on her investment, and the other returns 12% per year on her investment. Her total annual return from the two investments is \$53,000. Let x represent the amount of the 10% investment and y represent the amount of the 12% investment.

- Write an equation that states that the sum of investments is \$500,000.
- What is the annual return on the 10% investment? What about the 12% investment?
- Write an equation that states the sum of the annual return is \$53,000.
- Solve these two equations simultaneously to find how much is invested in each property.

**Example.** A nurse has two solutions that contain different concentrations of a certain medication. One is a 12.5% concentration, and the other is a 5% concentration. How many cubic centimeters of each should she mix to obtain 20 cubic centimeters of an 8% concentration?

**Example.** Using U.S. Bureau of Labor Statistics data for selected years from 1950 and projected to 2050, the number of men M and women W in the workforce (both in millions) can be modeled by the functions

$$M(t) = 0.591t + 37.3$$
 and  $W(t) = 0.786t + 13.1$ 

where t is the number of years after 1940. Find the year these functions predict that there will be equal numbers of men and women in the U.S. workforce.

