Implicit Differentiation:

1. Differentiate both sides of the equation with respect to x, treating y as a differentiable function of x.
2. Collect the terms with ${}^{dy}\!/_{dx}$ on one side of the equation.
3. Solve for ${}^{dy}\!/_{dx}$.

Example. Find the derivatives of the following by rewriting each function explicitly before taking the derivative, and by using implicit differentiation. Compare the results.

$y^2 = x$

$\sqrt{x} + \sqrt{y} = 4$

Example. Find $\dfrac{dy}{dx}$ given the equation

$$y^3 - y + 2x^3 - x = 8$$

Example. Consider the equation $x^2 + y^2 = 4$.

Find $\dfrac{dy}{dx}$ by implicit differentiation.

Find the slope of the tangent line to the graph of the function $y = f(x)$ at the point $(1, \sqrt{3})$.

Find an equation of the tangent line.

Related Rates:
Related rates are problems that use a mathematical relationship between two or more objects under specific constraints. From this, we can differentiate this relationship and examine how each variable changes with respect to time.

The volume of a cone with radius r and height h is given by

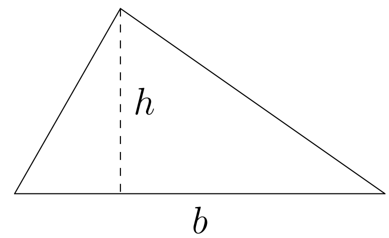
$$V = \frac{1}{3}\pi r^2 h$$

Find dV/dt when r and h are changing.

Find dV/dt when r is constant and h is changing.

Find dV/dt when r is changing and h is constant.

Example. The altitude of a triangle is increasing at a rate of $1\,cm/min$ while the area of the triangle is increasing at a rate of $2\,cm^2/min$. How fast is the base of the triangle changing when the altitude is $10\,cm$ and the area is $100\,cm^2$.



Example. The base of a 13-ft ladder leaning against a wall begins to slide away from the wall. At the instant of time when the base is 12 ft from the wall, the base is moving at a rate of 8 ft/sec. How fast is the top of the ladder sliding down the wall at that instant of time?

Example. At noon, ship A is 100 km west of ship B. Ship A is sailing south at 35 km/h and ship B is sailing north at 25 km/h. How fast is the distance between the ships changing at 4pm?