Math 211 Class notes
Fall 2024

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1.1: What Are Data?

Statistics rests on two major concepts:

a) Data

b) Variation

Statistics is the science of:

- Collecting
- Organizing
- Summarizing
- Analyzing Data

For the purpose of:

- Answering questions and/or
- Drawing conclusions

Context is important! Some questions you can ask:

- Who, or what, was observed?
- How were they measured?
- Who collected the data?
- Where/when/why were the data collected?
- What variables were measured?
- What are the units of measurement?
- How did they collect the data?

1.2: Classifying and Storing Data

- The collection of data is called a **data set** or a **sample**. The **population** refers to the set or group that contains everything relevant to the data.
- When we collect data, the characteristics of that data (e.g. gender, weight, temperature) are called **variables**.
- Variables can be categorized into two groups:
  - Numerical variables
  - Categorical variables

Example. The following table contains data crash-test dummy studies.

- How many variables does this table have?
- How many observations does this table have?
- For each variable, identify whether it is numerical or categorical:

Make	Model	Doors	Weight	Head Injury
Acura	Integra	2	2350	599
Chevrolet	Camaro	2	3070	733
Chevrolet	S-10 Blazer 4X4	2	3518	834
Ford	Escort	2	2280	551
Ford	Taurus	4	2390	480
Hyundai	Excel	4	2200	757
Mazda	626	4	2590	846
Volkswagen	Passat	4	2990	1182
Toyota	Tercel	4	2120	1138

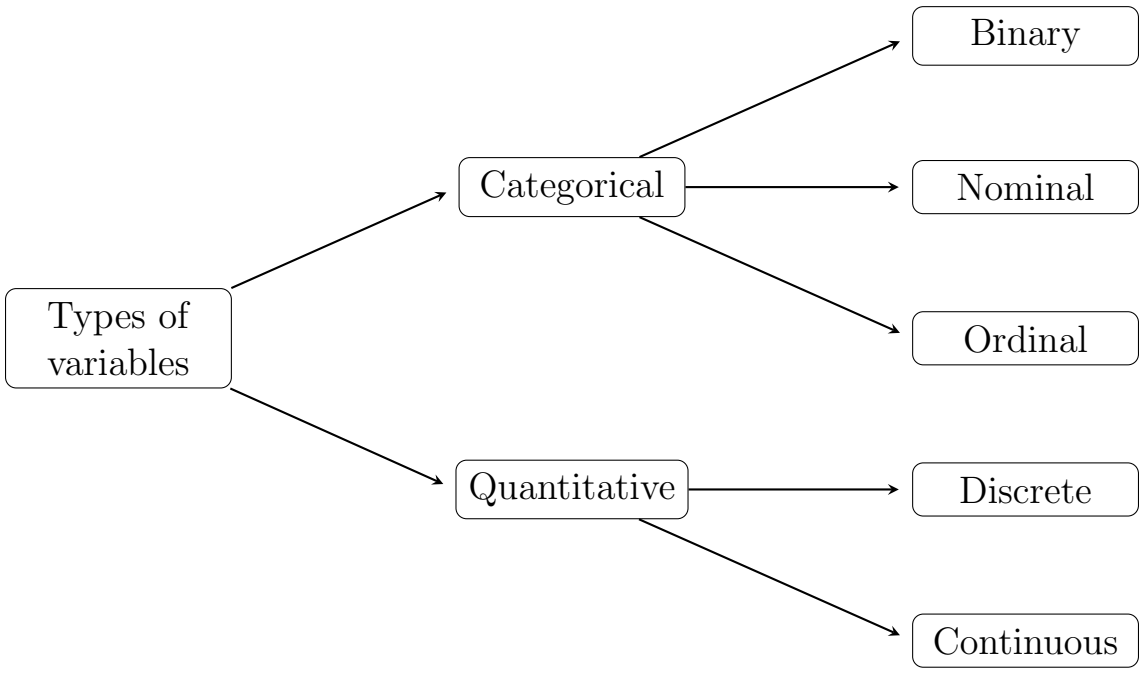
Coding categorical data using numbers:

Weight	Gender	Smoke
7.69	Female	No
0.88	Male	Yes
6.00	Female	No
7.19	Female	No
8.06	Female	No
7.94	Female	No

$\longrightarrow$

Weight	Female	Smoke
7.69	1	0
0.88	0	1
6.00	1	0
7.19	1	0
8.06	1	0
7.94	1	0

We can further break down variables into five types:



Example. Suppose a local store was interested in whether a new product would sell or not. The manager decided to take a random sample of 100 customers over a two-week period and asked each person whether they would buy the product or not and how many times would they buy the product over a six month period.

a) What is the population?

b) What is the sample?

c) What are the variables?

d) Classify each variable as numerical or categorical.

1.4: Organizing Categorical Data

Definition.
In the context of statistics, **frequency** is the number of times a value of a variable is observed in a data set.
Relative frequency (proportion) is a ratio of the frequency of a variable to the total frequency of the group desired. This can be left as a fraction, decimal, or percentage.

Example. The following **two-way table** contains the results of a national survey that asks American youths whether they wear a seat belt while driving or riding in a car:

	Male	Female	Total
Not Always	2	3	
Always	3	7	
Total			

a) Find the total number of males, females, and total participants in this survey.

b) Identify the frequencies, and compute the percentages below:

	Male	Female	Total
Not Always			
Always			
Total			100%

c) Are males or females more likely to take the risk of not wearing a seat belt?

d) Should we use the frequencies or the relative frequencies to make comparisons?

1.5: Collecting Data to Understand Causality

Definition.

- In an **observational study**, we observe individuals and measure variables of interest but do not attempt to influence the responses. (Observe but do not disturb)
- In a **controlled experiment**, we deliberately impose some treatment on (that is, do something to) individuals in order to observe their responses. Researchers assign subjects to a treatment group or control group.
- **Anecdotal evidence** is a story based on someone's experience.

- In an **observational study**, the researcher observes values of the response variable for the sampled subjects, without anything being done to the subjects (such as imposing a treatment).
- In short, an *observational study* merely observes rather than experiments with the study subjects.

Note: Anecdotal evidence and observational studies:

- NEVER point to causality (cause-and effect).
- Only point to an association between variables!

To establish cause-and-effect: Use a controlled experiment!

Definition.
Differences between two groups that could explain different experiences/outcomes are called **confounding variables** or **confounding factors**.

How to design a good experiment ("Gold standard" in experiments):

- Random allocation – participants randomly allocated to treatment and control group
- Use of a placebo if appropriate
  - A **placebo** is a fake treatment (e.g. sugar pill).
  - The **Placebo-Effect** is reacting to a treatment you haven't received.
- Blinding the study – used to avoid bias
  - Single blind – Researcher is unaware of treatment group
  - Double blind – Researcher and subjects are both unaware of treatment group
- Large sample size – accounts for variability

2.1: Visualizing Variation in Numerical Data

Definition.
The **distribution of a sample** of data is a way of organizing the data by recording the

- values that were observed, and
- the frequencies of these values.

Example. Below are the number of goals scored by first year NCAA female soccer players in Division III in the 2016-17 season:

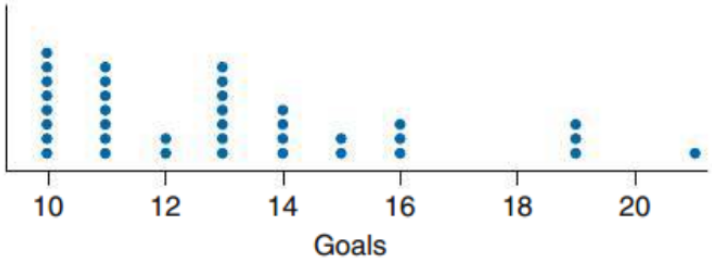
$$11, 14, 16, 13, 13, 10, 13, 11, 16, 21, 13, 19, 10, 10, 14, 13, 10, 13,$$

$$15, 10, 15, 13, 11, 19, 11, 11, 16, 10, 12, 11, 14, 11, 10, 14, 10, 19, 12$$

The **distribution** lists the values *and* the frequencies:

Value	Frequency
10	8
11	7
12	2
13	7
14	4
15	2
16	3
17	0
18	0
19	3
20	0
21	1

A **dotplot** represents the data by using a dot where each value occurs:

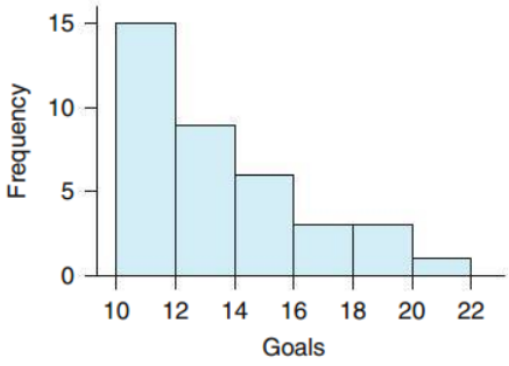


▲ **FIGURE 2.2** Dotplot of the number of goals scored by first-year women soccer players in NCAA Division III, 2016–17. Each dot represents a soccer player. Note that the horizontal axis begins at 10.

Histograms:

A **histogram** represents the data by using bars to indicate how much data lies in each *bin* (also called *interval* or *class*):

▶ **FIGURE 2.3** Histogram of number of goals for female first-year soccer players in NCAA Division III, 2016–17. The first bar, for example, tells us that 15 players scored between 10 and 12 goals during the season.

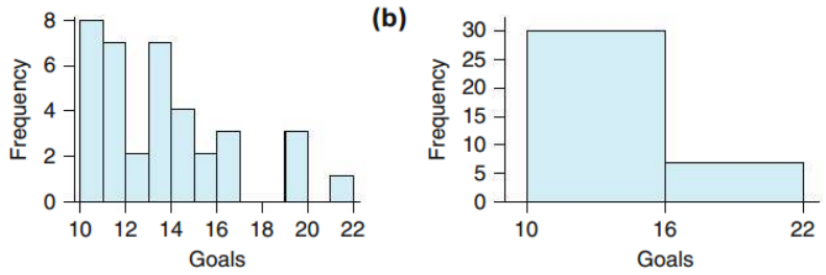


Q: Where do we place data points that lie on a boundary?

Note: Bin size plays a significant role in how the data is represented in a histogram. A bin width that is:

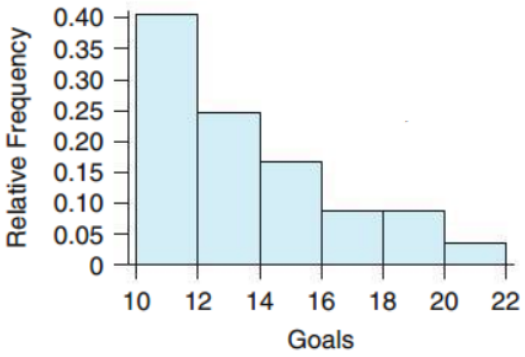
- too narrow shows too much detail.
- too wide hides detail.

▶ **FIGURE 2.4** Two more histograms of goals scored in one season, the same data as in Figure 2.3. **(a)** This histogram has narrow bins and is spiky. **(b)** This histogram has wide bins and offers less detail.



A **relative frequency histogram** changes the units on the vertical axis to represent relative frequencies:

▶ **FIGURE 2.5** Relative frequency histogram of goals scored by first-year women soccer players in NCAA Division III, 2016–17.



Stemplots:

Definition.
A **stemplot** divides each observation into a *stem* and *leaf*. The **leaf** is the last digit in the observation, and the **stem** contains all the digits preceding the leaf.

Example. A collection of college students who said that they drink alcohol were asked how many alcoholic drinks they had consumed in the last seven days. Their answers were:

$$1, 1, 1, 1, 1, 2, 2, 2, 3, 3, 3, 3, 3, 4, 5, 5, 5, 6, 6, 6, 8, 10, 10, 15, 17, 20, 25, 30, 30, 40$$

Stem	Leaves
0	11111222333334555666 8
1	0057
2	05
3	00
4	0

Example. Below is a stemplot for exam grades. How many grades are between 40% and 59%?

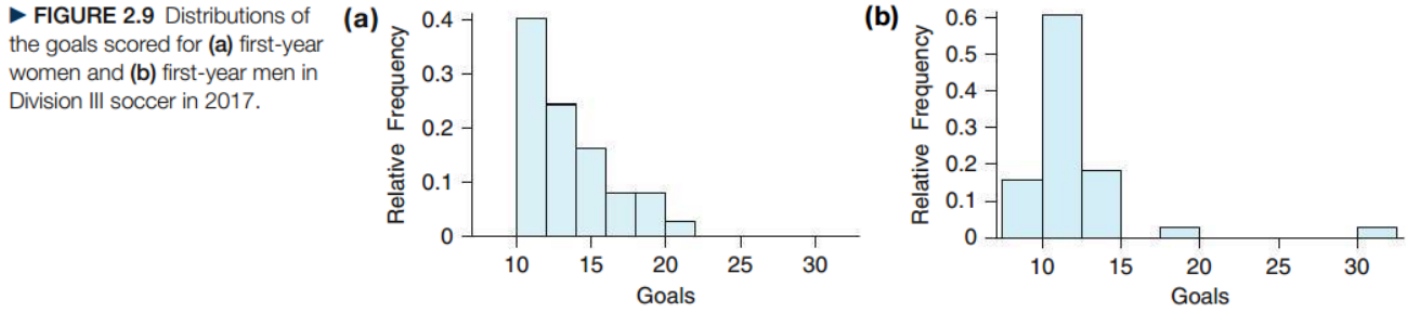
Stem	Leaves
3	8
4	
5	
6	0257
7	00145559
8	0023
9	0025568
10	00

2.2: Summarizing Important Features of a Numerical Distribution

Definition.
When examining a distribution:

- the **center** represents the typical or most common values, and
- the **spread** represents the variability in the data.

Example. Below are the histograms containing the number of goals scored by first year NCAA female (left) and male (right) soccer players in Division III in the 2016-17 season:



▶ **FIGURE 2.9** Distributions of the goals scored for **(a)** first-year women and **(b)** first-year men in Division III soccer in 2017.

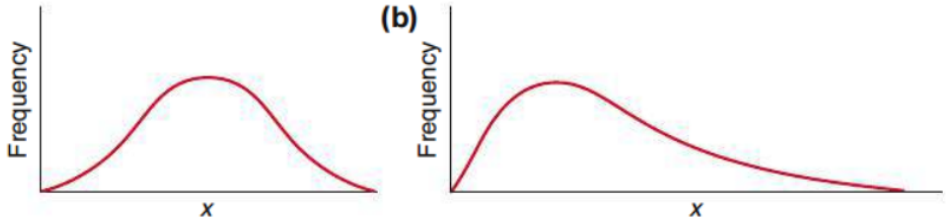
- Are their any notable differences in the shapes?

- What is the approximate center for each distribution?

- How do the spreads compare?

Three basic characteristics to consider when examining a distribution's shape:

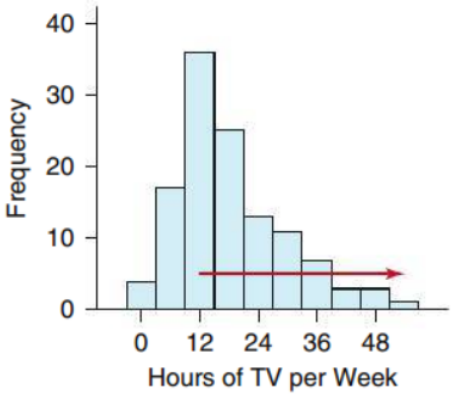
1. Is the distribution symmetric or skewed?
2. How many "mounds" appear?
3. Are unusually large or small values present?



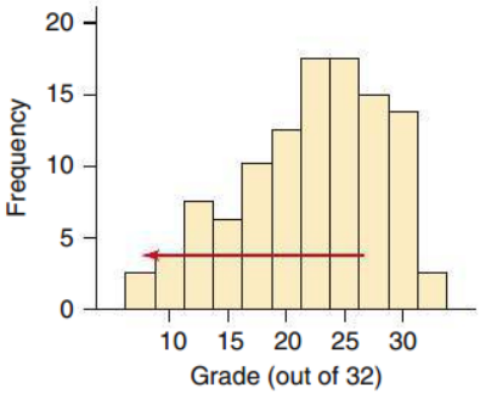
▶ **FIGURE 2.10** Sketches of **(a)** a symmetric distribution and **(b)** a right-skewed distribution.

Definition.

- A **right-skewed distribution** has a "tail" that extends towards the right.
- A **left-skewed distribution** has a "tail" that extends towards the left.
- A **symmetric** distribution has "tails" of approximately equal size.



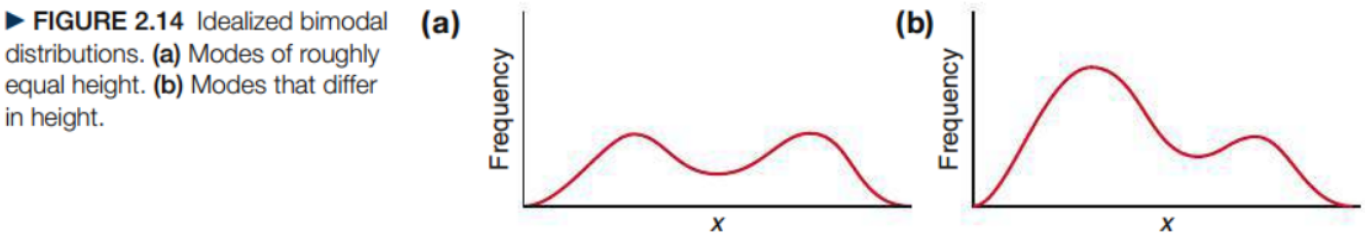
▲ **FIGURE 2.12** This data set on TV hours viewed per week is skewed to the right. (Source: Minitab Program)



▲ **FIGURE 2.13** This data set on test scores is skewed to the left.

Definition.

- A **unimodal distribution** has data grouped in a single "mound",
- a **bimodal distribution** has data grouped in two "mounds", and
- a **multimodal distribution** has data grouped in more than two "mounds".



▶ **FIGURE 2.14** Idealized bimodal distributions. **(a)** Modes of roughly equal height. **(b)** Modes that differ in height.

Example. In a 5k/10k race where all the runners start at the same time, what do we expect the shape of the distribution of the finishing times will look like?

Definition.
An **outlier** is an extreme value in a distribution of data. Outliers don't fit the pattern of the rest of the data.

Example. Consider the distribution of exam grades. What are possible explanations of any outliers?

Definition.
The most frequently occuring value is called the **mode**.

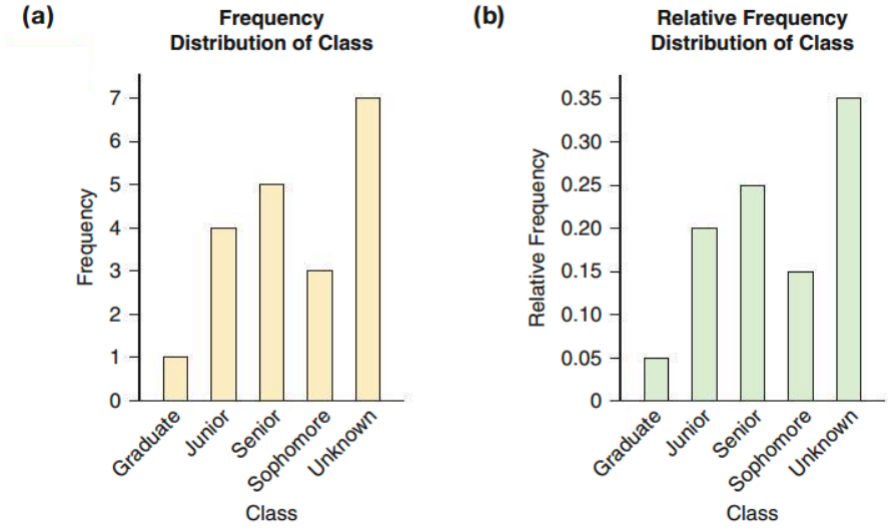
Why might the mode not be a reliable measure of center for numerical data?

2.3: Visualizing Variation in Categorical Variables

Definition.
A **bar chart** (also bar graph or bar plot) shows a bar for each observed category where the height of the bar is proportional to the frequency of that category.

Example. A summer introductory statistics course at UCLA has the following distribution of students across different years:

Class	Frequency
Unknown	7
Freshman	0
Sophomore	3
Junior	4
Senior	5
Graduate	1
Total	20



Bar Charts vs. Histograms:

- Bar charts are for categorical data
- Histograms are for numerical data

	Histogram	Bar Chart
Bars:	Should touch	May or may not touch
Bar width:	Corresponds to bin width	Can be any width (consistent)
Horizontal labels:	Numerical order	No inherent order

- A **Pareto chart** is a bar graph with bars arranged from tallest to shortest.

Definition.
A **pie chart** is a circle divided up into pieces where each area is proportional to the relative frequency of the category it represents.

Example.

Class	Frequency
Unknown	7
Freshman	0
Sophomore	3
Junior	4
Senior	5
Graduate	1
Total	20