

8.1: Relations on Sets

Definition.

A relation R from A to B is called a **binary relation** because it is a subset of a Cartesian product of two sets.

Example. Define a relation L from \mathbb{R} to \mathbb{R} :

$$\forall x, y \in \mathbb{R}, x L y \Leftrightarrow x < y.$$

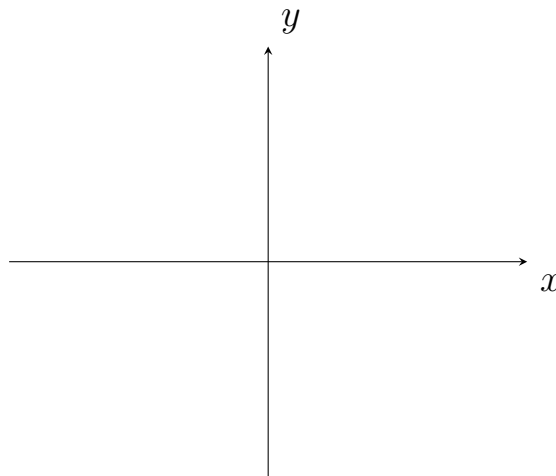
Is $57 L 53$?

Is $(-17) L (-14)$?

Is $143 L 143$?

Is $(-35) L 1$?

Draw the graph of L as a subset of the Cartesian plane $\mathbb{R} \times \mathbb{R}$.



Definition.

Two integers m and n are **congruent modulo 2** if, and only if, $m \bmod 2 = n \bmod 2$.

Example. Define a relation E from \mathbb{Z} to \mathbb{Z} :

$$\forall (m, n) \in \mathbb{Z} \times \mathbb{Z}, m E n \Leftrightarrow m - n \text{ is even.}$$

Is $4 E 0$? Is $2 E 6$? Is $3 E (-3)$? Is $5 E 2$?

Prove that if n is any odd integer, then $n E 1$.

Example. Let $X = \{a, b, c\}$. Define a relation \mathbf{S} from $\mathcal{P}(X)$ to $\mathcal{P}(X)$ as follows:

$$\forall A, B \in \mathcal{P}(X), A \mathbf{S} B \Leftrightarrow A \text{ has at least as many elements as } B.$$

Is $\{a, b\} \mathbf{S} \{b, c\}$?

Is $\{a\} \mathbf{S} \emptyset$?

Is $\{b, c\} \mathbf{S} \{a, b, c\}$?

Is $\{c\} \mathbf{S} \{a\}$?

Definition.

Let R be a relation from A to B . Define the inverse relation R^{-1} from B to A as follows:

$$R^{-1} = \{(y, x) \in B \times A \mid (x, y) \in R\}.$$

Example. Let $A = \{2, 3, 4\}$ and $B = \{2, 6, 8\}$, and let R be the “divides” relation from A to B :

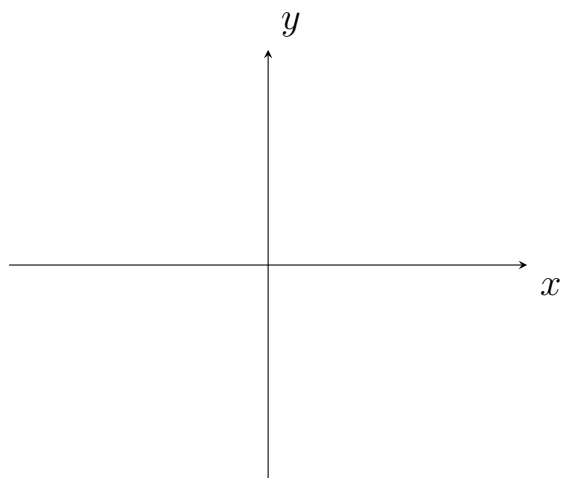
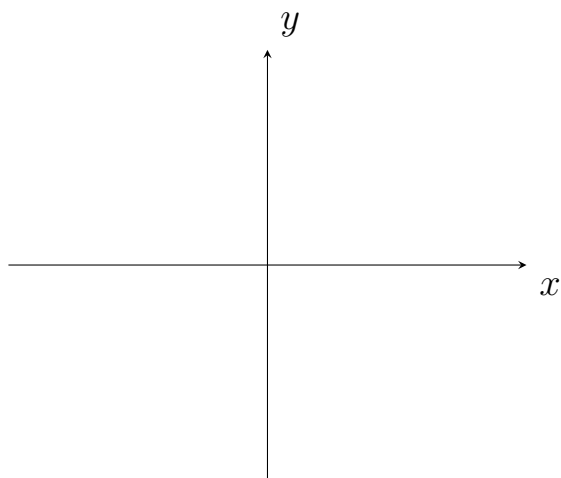
$$\forall (x, y) \in A \times B, \ x R y \Leftrightarrow x \mid y$$

Explicitly state which ordered pairs are in R and R^{-1} . Draw arrow diagrams for both.

Example. Define a relation R from \mathbb{R} to \mathbb{R} as follows:

$$\forall (x, y) \in \mathbb{R} \times \mathbb{R}, x R y \Leftrightarrow y = 2|x|.$$

Draw the graphs of R and R^{-1} in the Cartesian plane. Is R^{-1} a function?



Definition.

A **relation on a set** A is a relation from A to A .

A **graph** G consists of two finite sets:

- a nonempty set $V(G)$ of **vertices** and
- a set $E(G)$ of **edges**,

where each edge is associated with a set consisting of either one or two vertices called its endpoints.

A **directed graph** is a graph whose edges are directional.

Example. Let $A = \{3, 4, 5, 6, 7, 8\}$ and define a relation R on A as follows

$$\forall x, y \in A, \ x R y \Leftrightarrow 2 \mid (x - y).$$

Draw the directed graph of R .

Definition.

Given sets A_1, A_2, \dots, A_n , an **n-ary relation** R on $A_1 \times A_2 \times \dots \times A_n$ is a subset of $A_1 \times A_2 \times \dots \times A_n$. The special case of 2-ary, 3-ary, and 4-ary relations are called **binary**, **ternary**, and **quaternary relations**, respectively.