

Math 121 Class notes Fall 2024

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Definition. (Functions)

A **function** f is a special relation between x and y such that each input x results in *at most* one y . The symbol $f(x)$ is read “ f of x ” and is called the **value of f at x**

Example. Let $f(x) = \frac{x^2}{2} + x$. Evaluate the following:

$$f(1)$$

$$f\left(\frac{1}{2}\right)$$

$$f(-2)$$

$$f(0)$$

$$f(f(x))$$

Composite Functions:

Let f and g be functions of x . Then, the **composite functions** g of f (denoted $g \circ f$) and f of g (denoted $f \circ g$) are defined as:

$$(g \circ f)(x) = g(f(x))$$

$$(f \circ g)(x) = f(g(x))$$

Example. Let $g = x - 1$. Find:

$$(g \circ f)(x)$$

$$(f \circ g)(x)$$

Operations with Functions:

Let f and g be functions of x and define the following:

Sum	$(f + g)(x) = f(x) + g(x)$
Difference	$(f - g)(x) = f(x) - g(x)$
Product	$(f \cdot g)(x) = f(x) \cdot g(x)$
Quotient	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ if $g(x) \neq 0$

Definition.

An **expression** is a meaningful string of numbers, variables and operations:

$$3x - 2$$

An **equation** is a statement that two quantities or algebraic expressions are equal:

$$3x - 2 = 7$$

A **solution** is a value of the variable that makes the equation true:

$$3(3) - 2 = 7$$

$$9 - 2 = 7$$

$$7 = 7$$

A **solution set** is the set of ALL possible solutions of an equation:

$3x - 2 = 7$ only has the solution $x = 3$,

$2(x - 1) = 2x - 2$ is true for all possible values of x .

Properties of Equality:

Substitution Property: The equation formed by substituting one expression for an equal expression is equivalent to the original equation:

$$\begin{aligned}3(x - 3) - \frac{1}{2}(4x - 18) &= 4 \\3x - 9 - 2x + 9 &= 4 \\x &= 4\end{aligned}$$

Addition Property: The equation formed by adding the same quantity to both sides of an equation is equivalent to the original equation:

$$\begin{array}{ll}x - 4 = 6 & x + 5 = 12 \\x - 4 + 4 = 6 + 4 & x + 5 + (-5) = 12 + (-5) \\x = 10 & x = 7\end{array}$$

Multiplication Property: The equation formed by multiplying both sides of an equation by the same *nonzero* quantity is equivalent to the original equation:

$$\begin{array}{ll}\frac{1}{3}x = 6 & 5x = 20 \\3\left(\frac{1}{3}x\right) = 3(6) & \frac{5x}{5} = \frac{20}{5} \\x = 18 & x = 4\end{array}$$

Solving a linear equation:

Using the properties of equality above, we can solve any linear equation in 1 variable:

Example. Solve $\frac{3x}{4} + 3 = \frac{x-1}{3}$

1. Eliminate fractions:

$$12\left(\frac{3x}{4} + 3\right) = 12\left(\frac{x-1}{3}\right)$$

2. Remove/evaluate parenthesis:

$$9x + 36 = 4x - 4$$

3. Use addition property to isolate the variable to one side:

$$9x + 36 - 36 - 4x = 4x - 4 - 36 - 4x$$

4. Use multiplication property to isolate variable:

$$\frac{5x}{5} = \frac{-40}{5}$$

5. Verify solution via substitution:

$$\underbrace{\frac{3(-8)}{4} + 3}_{-6 + 3 = -3} \stackrel{?}{=} \underbrace{\frac{(-8) - 1}{3}}_{\frac{-9}{3} = -3}$$

Example. Solve the following:

$$\frac{3x+1}{2} = \frac{x}{3} - 3$$

$$\frac{2x-1}{x-3} = 4 + \frac{5}{x-3}$$