6.5: Evaluating Definite Integrals

The Fundamental Theorem of Calculus
Let f be continuous on $[a, b]$. Then,

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

where F is any antiderivative of f; that is, $F'(x) = f(x)$.

Example. Let R be the region under the graph of $f(x) = x$ on the interval $[1, 3]$. Find the area of R Graph

using geometry

using the Fundamental Theorem of Calculus

Properties of the Definite Integral

Let f and g be integrable functions; then,

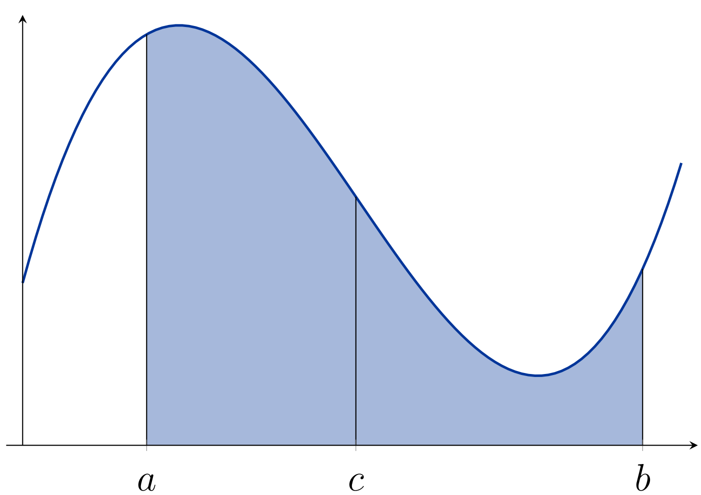
1. $\displaystyle\int_a^a f(x)\,dx = 0$

2. $\displaystyle\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

3. $\displaystyle\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$ (c constant)

4. $\displaystyle\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

5. $\displaystyle\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ ($a < c < b$)



Example. Evaluate the following definite integrals

$$\int_0^4 x\sqrt{9+x^2}\,dx$$

$$\int_0^2 xe^{2x^2}\,dx$$

Example. Find the area of each region R described below: Graphs

Under $f(x) = \sqrt{x}$ from $x = 1$ to $x = 4$

Under $f(x) = e^{x/2}$ from $x = -1$ to $x = 1$