7.4: Estimating the Population Proportion with Confidence Intervals

Definition.
Suppose that we wish to estimate a population proportion p based on a sample proportion $\hat{p}$.

- A **confidence interval** is an interval about the point estimate $\hat{p}$ that we can be confident contains the true population proportion p:

$$\hat{p} \pm m$$

- The **margin of error (ME)** is half the width of the confidence interval. When estimating a population proportion, the margin of error is

$$m = z^* SE$$

- The **confidence level** measures how often the estimation method is successful. A larger confidence level results in a larger margin of error.

Recall the standard error (SE) for population proportions is

$$SE = \sqrt{\frac{p(1-p)}{n}}$$

Some common values for the margin of error:

Confidence Level	Margin of Error
99.7%	$3.0 \cdot SE$
99%	$2.58 \cdot SE$
95%	$1.96 \cdot SE$
90%	$1.645 \cdot SE$
80%	$1.28 \cdot SE$

Example. In 2018, Gallup took a poll of 497 randomly selected adults who teach K–12 students and 42% of them said that digital devices (smartphones, tablets, computers) had "mostly helpful" effects on students' education.

Check that the conditions of the CLT apply.

Estimate the standard error.

Give the 95% confidence interval and interpret the result.

Example. After the Great Recession, the Pew Research Center noted there seemed to be a decline in households that rented their homes and were looking to purchase homes. However, Pew reported that in 2016 "a solid 72%" of renters reported that they wished to buy their own home. Pew reports that the "margin of error at 95% confidence level is plus-or-minus 5.4 points."

State the confidence interval in interval form and interpret the result.

8.1: The Essential Ingredients of Hypothesis Testing

Definition.

- A **hypothesis test** is a procedure that enables us to choose between two claims.
- The **null hypothesis**, H_0, represents the current belief, or status quo.
- The **alternative hypothesis**, H_a, is what we wish to test.

A hypothesis test has 4 steps:

1. Formulate your null and alternative hypotheses
2. Examine or collect data
3. Compare data to our expectations; is the result significant?
4. Interpret the results

Two-Sided	One-Sided (Left)	One-Sided (Right)
$H_0 : p = p_0$	$H_0 : p = p_0$	$H_0 : p = p_0$
$H_a : p \neq p_0$	$H_a : p < p_0$	$H_a : p > p_0$

Example. When flipping a coin, it is considered fair if both sides of the coin have an equally likely chance of appearing face up. Suppose we have a coin that we believe might be unfair. Let p be the proportion of times where heads appears face up. Formulate the null and alternative hypotheses.

Example. Historically, about 70% of all U.S. adults were married. A sociologist who asks whether marriage rates in the United States have declined will take a random sample of U.S. adults and record whether or not they are married.

Write the null and alternative hypotheses.

Example. An Internet retail business is trying to decide whether to pay a search engine company to upgrade its advertising. In the past 15% of customers who visited the company's web page by clicking on the advertisement bought something. The search engine company offers to do an experiment: for one day a random sample of customers will see the retail business's ad in a more prominent position to try and increase the proportion of customers who make a purchase.

Write the null and alternative hypotheses.

Definition.

- The **significance level**, denoted by α, is the probability of rejecting the null hypothesis when it is actually true (false positive).
- A **test statistic** is similar to a z score comparing the alternative hypothesis to the null hypothesis:

$$z = \frac{\hat{p} - p_0}{SE}, \quad \text{where} \quad SE = \sqrt{\frac{p_0(1-p_0)}{n}}$$

- The p**-value** is the probability that the null hypothesis is true. When the p-value is
  - greater than α, we fail to reject the null hypothesis
  - less than or equal to α, we reject the null hypothesis

Note: Hypothesis tests don't prove the null hypothesis!

8.2: Hypothesis Testing in Four Steps

1. **Hypothesize:** formulate your hypotheses
2. **Check conditions:**
   - **Random and Independent:** The sample must be randomly collected from the population, and observations are independent of each other
   - **Large Sample:** The sample size must be large enough for at least 10 successes, $np_0 \geq 10$, and 10 failures, $n(1-p_0) \geq 10$.
   - **Large Population:** If the sample is collected without replacement, the population of size N must be at least 10 times bigger than the sample: $N \geq 10n$

   If these conditions are met, we compute the test statistic for the One-Proportion z-Test which follows a z-distribution:

   $$z = \frac{\hat{p} - p_0}{SE}, \qquad \text{where} \qquad SE = \sqrt{\frac{p_0(1-p_0)}{n}}$$
3. **Compute:** Stating a significance level, compute the observed test statistic z and/or p-value.
4. **Interpret:** Decide whether to reject or fail to reject the null hypothesis.

Example. Unlike flipping a coin, spinning a coin leads to a biased outcome. Suppose we spun a coin 60 times, and saw a sample proportion of $\hat{p} = 0.35$.

Formulate the null and alternative hypotheses

Check the conditions required to perform a hypothesis test.

Find the test statistic and p-value

Using a significance level of $\alpha = 0.05$, decide whether to reject or fail to reject the null hypothesis.

Example. A group of medical researchers knew from previous studies that in the past, about 39% of all men between the ages of 45 and 59 were regularly active. Researchers were concerned that this percentage had declined over time. For this reason, they did selected a random sample, without replacement, of 1927 men in this age group and interviewed them. Out of this sample, 680 said they were regularly active.

Formulate the null and alternative hypotheses

Check the conditions required to perform a hypothesis test.

Find the test statistic and p-value

Using a significance level of $\alpha = 0.05$, decide whether to reject or fail to reject the null hypothesis.