2.3: Business Applications Using Quadratics

Recall the following:

Definition.
Profit is the difference between the revenue and total cost:

$$P(x) = R(x) - C(x)$$

where

$$\begin{aligned} P(x) &= \text{profit from sale of } x \text{ units,} \\ R(x) &= \text{total revenue from sale of } x \text{ units,} \\ C(x) &= \text{total cost from production and sale of } x \text{ units.} \end{aligned}$$

In general, **total revenue** is

$$\text{Revenue} = (\text{price per unit})(\text{number of units})$$

The **total cost** is composed of fixed cost and variable cost:

- **Fixed costs** (FC) remain constant regardless of the number of units produced.
- **Variable costs** (VC) are directly related to the number of units produced.

The total cost is given by

$$\text{Cost} = \text{variable costs} + \text{fixed costs}$$

Example. Suppose that a company's cost include a fixed cost of $\$1,200$, and a variable cost per unit of $\frac{x}{4} + 18$ dollars, where x is the total number of units produced. If the selling price of their product is $(156 - \frac{3x}{4})$ dollars per unit, then

How many units should be sold to maximize the revenue?

$R(x) = (156 - \frac{3x}{4})x$

$= -\frac{3}{4}x^2 + 156x$

$a = -3/4 < 0$

$\Rightarrow$ vertex is a maximum

$\frac{-b}{2a} = -\frac{156}{2(-3/4)} = \boxed{104 \text{ units}}$

Find the profit function.

$P(x) = R(x) - C(x) = \left[-\frac{3}{4}x^2 + 156x\right] - \left[1200 + \left(\frac{x}{4} + 18\right)x\right]$

$= -x^2 + 138x - 1200$

How many units should be sold to maximize the profit?

$a = -1 < 0$

$\Rightarrow$ vertex is a maximum

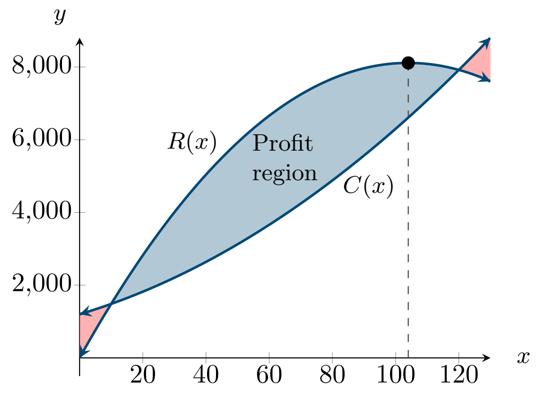
$\frac{-b}{2a} = \frac{-138}{2(-1)} = \boxed{69 \text{ units}}$

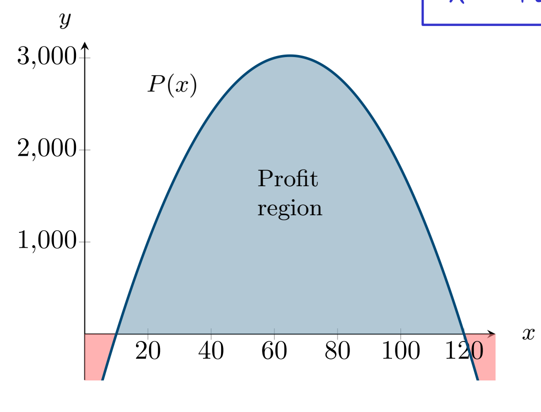
Find the **break-even point** (e.g. where $R(x) = C(x)$ and $P(x) = 0$).

$0 = -x^2 + 138x - 1200$

$x = \frac{-138 \pm \sqrt{(138)^2 - 4(-1)(-1200)}}{2(-1)} = 69 \pm \sqrt{3516}$

$\boxed{\begin{array}{l} x = 9.3259 \\ x = 128.6741 \end{array}}$





Example. Suppose that the demand function for a commodity is given by the equation

$$p^2 + 7q = 1900,$$

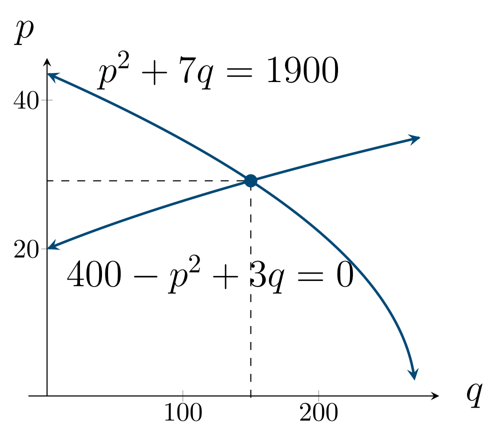
and the supply function is given by the equation

$$400 - p^2 + 3q = 0.$$

Find the **market equilibrium**

$$p^2 + 7q = 1900$$
$$+\,(-p^2 + 3q = -400)$$
$$0p^2 + 10q = 1500$$
$$\frac{10q}{10} = \frac{1500}{10}$$
$$\boxed{q = 150}$$

$$p^2 + 7(150) = 1900$$
$$-1050 + p^2 + 1050 = 1900 - 1050$$
$$p^2 = 850$$
$$p = \pm\sqrt{850} \approx \pm 29.1548$$



Example. If the supply and demand functions for a commodity are given by $p - q = 10$ and $q(2p - 10) = 2100$, what is the equilibrium price and what is the corresponding number of units supplied and demanded?

$$+q + p - q = 10 + q$$

$$-10 + p = 10 + q - 10$$

$$p - 10 = q$$

$$q(2p-10) = 2100$$

$$(p-10)(2p-10) = 2100$$

$$p(2p-10) - 10(2p-10) = 2100$$

$$-2100 + 2p^2 - 10p - 20p + 100 = 2100 - 2100$$

$$2p^2 - 30p - 2000 = 0$$

$$\Rightarrow p = \frac{30 \pm \sqrt{(-30)^2 - 4(2)(-2000)}}{2(2)} = \frac{30 \pm \sqrt{16900}}{4}$$

$$= \frac{30 \pm 130}{4} \quad \begin{cases} p = -25 \\ \boxed{p = 40} \end{cases}$$

$$p = 40 \rightarrow p - q = 10$$

$$40 - q = 10$$

$$\boxed{30 = q}$$