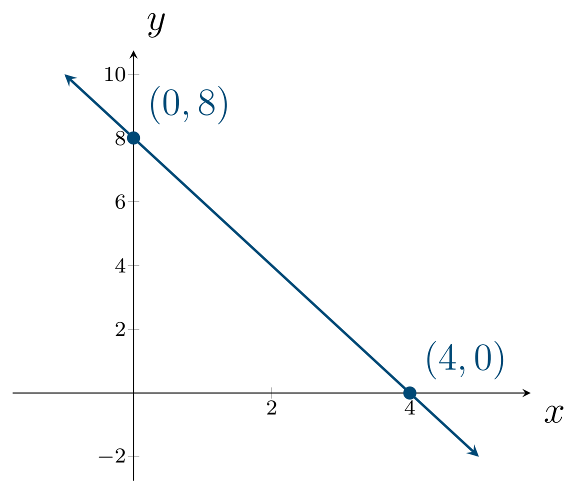
1.3 Linear Functions

Definition.
A **linear function** is a function of the form

$$y = f(x) = ax + b$$

where a and b are constants.

Example. $y = -2x + 8$



A linear function can be uniquely determined using only *two* distinct points.

Definition.
The point(s) where a graph intersects the axes are called intercepts. The x-coordinate of the point where the function intersects the x-axis is called the **x-intercepts**. The y-coordinate of the point where the function intersects the x-axis is called the **y-intercepts**.

- To solve for the y-intercept:
  - Set $x = 0$,
  - Solve for y.
- To solve for the x-intercept:
  - Set $y = 0$,
  - Solve for x.

Example. Find the intercepts and graph the following lines:

$3x + 2y = 12$

x-intercept:

set $y=0$, find x.

$3x + 2(0) = 12$

$\frac{3x}{3} = \frac{12}{3}$

$\boxed{x = 4} \to (4, 0)$

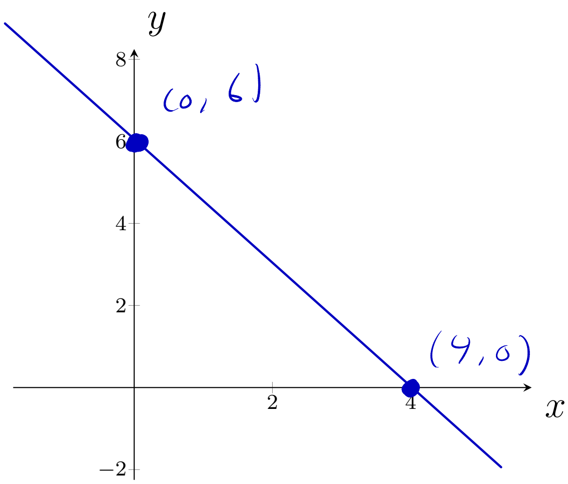
y-intercept:

set $x=0$, find y.

$3(0) + 2y = 12$

$2y = 12$

$\boxed{y = 6} \to (0, 6)$



$x = 4y$

x-intercept:

set $y=0$, find x.

$x = 4(0)$

$\boxed{x = 0} \to (0, 0)$

y-intercept:

set $x=0$, find y.

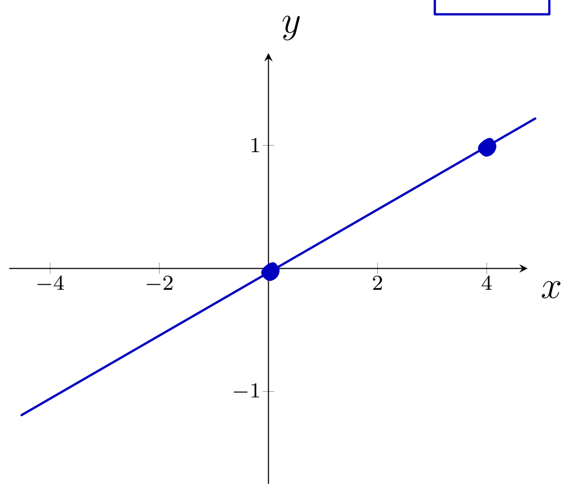
$0 = 4y$

$\boxed{y = 0} \to (0, 0)$

To graph this, choose another point:

$\underline{y=1} \to$ $x = 4(1)$

$\boxed{x = 4} \to (4, 1)$



> **Definition.**
> If a nonvertical line passes through the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$, its **slope**, denoted by m, is found using
>
> $$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$
>
> Δy is "delta y", and represents the change in y
> Δx is "delta x", and represents the change in x

Note: The slope of a vertical line is undefined.

Example. Find the slope of the line passing through the points $(-2, 1)$ and $(5, 3)$.

(x_1, y_1) (x_2, y_2)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 1}{5 - (-2)} = \boxed{\frac{2}{7}}$$

> *Note:*
>
> - Two distinct nonvertical lines are *parallel* if and only if their slopes are *equal*.
> - Two distinct nonvertical lines are *perpendicular* if and only if their slopes are *negative reciprocals*:
>   e.g. If ℓ_1 has a nonzero slope m, then ℓ_2 is perpendicular if its slope is $-1/m$.

Point-slope form

Definition.
The equation of the line passing through the point (x_1, y_1) with slope m can be written in the point-slope form:

$$y - y_1 = m(x - x_1)$$

Example. Find the equation of each line that passes through the point $(-3, 4)$ and has

a slope of $m = \frac{1}{4}$

$y - 4 = \frac{1}{4}(x - (-3))$

$+4 + y - 4 = \frac{x}{4} + \frac{3}{4} + 4$

$y = \frac{x}{4} + \frac{3}{4} + 4\left(\frac{4}{4}\right)$

$y = \frac{x}{4} + \frac{3}{4} + \frac{16}{4} \Rightarrow \boxed{y = \frac{x}{4} + \frac{19}{4}}$

the point $(-2, 1)$ on the line

(x_2, y_2) (x_1, y_1)

$m = \frac{4-1}{-3-(-2)} = \frac{3}{-1} = -3$

$y - 4 = -3(x - (-3))$

$y = -3x - 9 + 4$

$\boxed{y = -3x - 5}$

$y - 1 = -3(x - (-2))$

$y = -3x - 6 + 1$

$\boxed{y = -3x - 5}$

a slope of zero (horizontal)

$y - 4 = 0(x - (-3))$

$\boxed{y = 4}$

an undefined slope (vertical)

$\boxed{x = -3}$

Slope-intercept form

Definition.
The slope-intercept form of the equation of a line with slope m and y-intercept b is

$$y = mx + b$$

Example (Example 7, p.82)**.** The population of U.S. males, y (in thousands), projected from 2015 to 2060 can be modeled by

$$y = 1125.9x + 142,960$$

where x is the number of years after 2000.

- Find the slope and y-intercept of the graph of this function.

set $x=0$ $\longrightarrow$ $y = 1125.9(0) + 142,960$

$$\boxed{y = 142,960}$$

- What does the y-intercept tell us abut the population of U.S. males?

Zero years after 2015, the population of U.S. males is 142,960.

- Interpret the slope as a rate of change.

Each year, the population of U.S. males is expected to increase by 1125.9.

Forms of Linear Equations

$$\begin{aligned} \text{General form: } & ax + by + c = 0 \\ \text{Point-slope form: } & y - y_1 = m(x - x_1) \\ \text{Slope-intercept form: } & y = mx + b \\ \text{Vertical line: } & x = a \\ \text{Horizontal line: } & y = b \end{aligned}$$