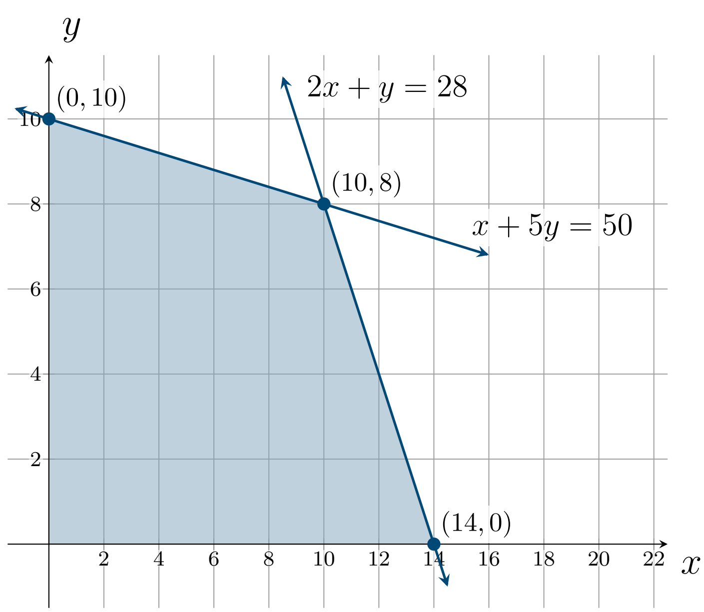
4.2: Linear Programming: Graphical Methods

Definition.
Linear programming is an optimization technique that can be used to solve linearly constrained problems:

$$\begin{aligned} \max F = 3x + \ y & \\ \text{subject to} \quad x + 5y &\leq 50 \\ 2x + \ y &\leq 28 \\ x \geq 0, y &\geq \ 0 \end{aligned}$$

The **constraints** of a linear program (LP) may be limitations or requirements of the variables. The **objective function** is the function that we wish to optimize (e.g. maximize profit *or* minimize cost).



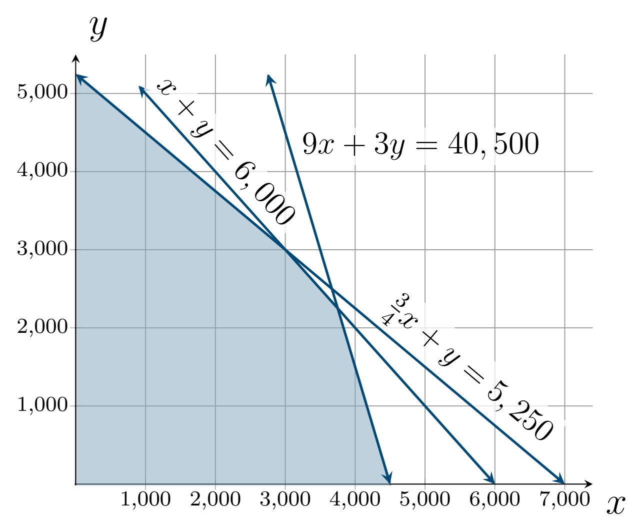
Linear programming (graphical method)

1. Write the objective function and constraint inequalities from the problem.
2. Graph the solution of the constraint system.
3. Find the corners of the resulting feasible region.
4. Evaluate the objective function at each corner.
5. If two corners give the optimal value, then the entire boundary joining these two points optimizes the function.

Example. A farm co-op has 6000 acres available on which to plant corn and soybeans. The following table summarizes each crop's requirement for fertilizer/herbicide, harvesting labor hours, and the available amounts of these resources.

	Corn	**Soybeans**	**Available**
Fertilizer/herbicide	9 gal/acre	3 gal/acre	40,500 gal
Harvesting labor	3/4 hr/acre	1 hr/acre	5,250 hr

Setup the system of inequalities that represents the constraints.

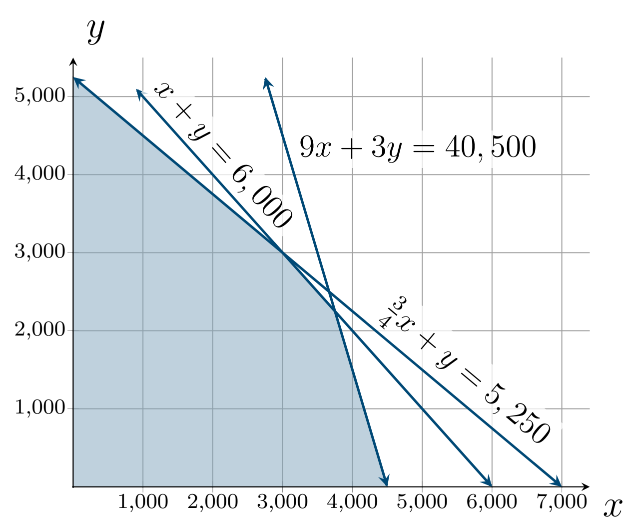


Example. Using the linear constraints from above, suppose the co-ops profits per acre are \$240 for corn and \$160 for soybeans. This gives us the following linear program:

$$\begin{aligned} \max P = 240x+160y & \\ \text{subject to} \qquad x + \quad y &\le 6,000 \\ 9x + \ 3y &\le 40,500 \\ \frac{3}{4}x + \quad y &\le 5,250 \\ x \ge 0, y &\ge 0 \end{aligned}$$

1. Find the "corners" of the feasible region
2. Evaluate the profit function at the corners

(x, y)	$P = 240x + 160y$
$(0, 0)$	\$0
$(0, 5250)$	\$840,000
$(3000, 3000)$	\$1,200,000
$(3750, 2250)$	\$1,260,000
$(4500, 0)$	\$1,080,000

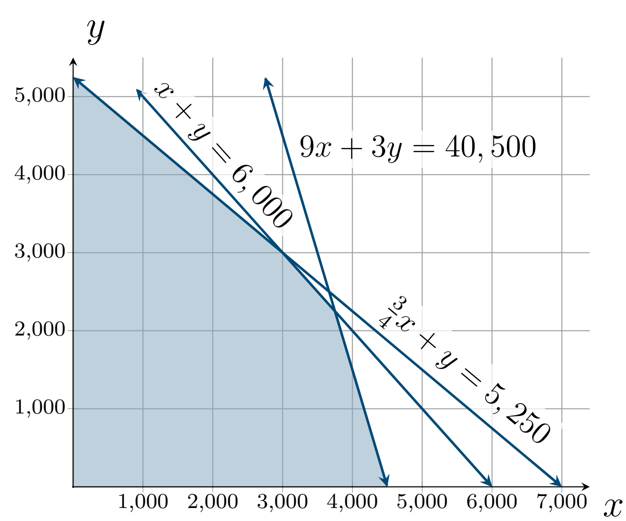


Example. Suppose the profits per acre are instead \$300 for corn, and \$100 for soybeans. This gives us the following linear program:

$$\begin{aligned} &\max P = 300x + 100y \\ \text{subject to} \quad & x + y \le 6,000 \\ & 9x + 3y \le 40,500 \\ & \frac{3}{4}x + y \le 5,250 \\ & x \ge 0, y \ge 0 \end{aligned}$$

Evaluate the profit function at the corners. What combination of corn and soy-beans maximizes the profit?

(x, y)	$P = 300x + 100y$
$(0, 0)$	\$0
$(0, 5250)$	\$525,000
$(3000, 3000)$	\$1,200,000
$(3750, 2250)$	\$1,350,000
$(4500, 0)$	\$1,350,000



Example.