1.1: Variables

Definition.
A **variable** is a placeholder for something which may or may not be unknown.

Example. Is there a number with the following property: doubling it and adding 3 gives the same result as squaring it?

$\rightarrow$ Is there a number x with the property that $2x + 3 = x^2$?
$\rightarrow$ Is there a number $\square$ with the property that $2 \cdot \square + 3 = \square^2$?

Example. No matter what number might be chosen, if it is greater than 2, then its square is greater than 4.

$\rightarrow$ No matter what number n might be chosen, if n is greater than 2, then n^2 is greater than 4.

Example. Use variables to rewrite the following sentences:

> Are there numbers with the property that the sum of their squares equals the square of their sum?

> Given any real number, its square is nonnegative.

Definition.

- A **universal statement** says that a certain property is true for all elements in a set.
- A **conditional statement** says that if one thing is true, then some other thing also has to be true.
- Given a property that may or may not be true, an **existential statement** says that there is at least one thing for which the property is true.

Definition.
A **universal conditional statement** is both universal and conditional:

For every animal a, if a is a dog, then a is a mammal.

Conditional statements can be rewritten in ways that make them appear more to be purely universal or purely conditional:

If a is a dog, then a is a mammal.
All dogs are mammals

Example. Rewrite the following universal condition statement:

For every real number x, if x is nonzero then x^2 is positive.

If a real number is nonzero, then its square ______________________.

For every nonzero real number x, ______________________.

If x ______________________, then ______________________.

The square of any nonzero real number is ______________________.

All nonzero real numbers have ______________________.

Definition.
A **universal existence statement** is a statement that is universal because its first part says that a certain property is true for all objects of a given type, and it is existential because its second part asserts the existence of something:

Every real number has an additive inverse.

In the above example, note that the particular additive inverse depends on the given real number:

For every real number r, there is an additive inverse for r.

Example. Rewrite the following universal existence statement:

Every pot has a lid

All pots ____________________.

For ever pot P, there is ____________________.

For every pot P, there is a lid L such that ____________________.

Definition.
An **existential universal statement** is a statement that is existential because its first part asserts that a certain object exists and is universal because its second part says that the object satisfies a certain property for all things of a certain kind:

> There is a positive integer that is less than or equal to every positive integer.

The number one satisfies the above statement, which can also be rewritten:

> There is a positive integer m that is less than or equal to every positive integer.

Example. Rewrite the following existence universal statement:

> There is a person in my class who is at least as old as every person in my class.

Some ____________________ is at least as old as ____________________.

There is a person p in my class such that p is ____________________.

There is a person p in my class with the property that for every person q in my class, p is ____________________.