5.4: Differentiation of Exponential Functions

Rule 1: Derivative of the Exponential Function

$$\frac{d}{dx}[e^x] = e^x$$

Example. Find the derivative of the following functions

$f(x) = x^2 e^x$

$$f'(x) = \frac{d}{dx}\left[x^2\right] e^x + x^2 \frac{d}{dx}\left[e^x\right]$$

$$= 2x e^x + x^2 e^x$$

$$= (2x + x^2)\, e^x$$

$g(t) = (e^t + 2)^{3/2}$

$$g'(t) = \frac{3}{2}(e^t + 2)^{1/2} e^t$$

Rule 2: The Chain Rule for Exponential Functions
If $f(x)$ is a differentiable function, then

$$\frac{d}{dx}\left[e^{f(x)}\right] = e^{f(x)}\, f'(x)$$

Example. Find the derivative of the following functions

$f(x) = e^{2x}$

$$f'(x) = e^{2x} \frac{d}{dx}[2x] = 2e^{2x}$$

$y = e^{-3x}$

$$y' = e^{-3x} \frac{d}{dx}[-3x] = -3e^{-3x}$$

$g(t) = e^{2t^2+t}$

$$g'(t) = e^{2t^2+t} \frac{d}{dt}\left[2t^2+t\right] = (4t+1)e^{2t^2+t}$$

$y = xe^{-2x}$

$$y' = \frac{d}{dx}\left[x\right]e^{-2x} + x\frac{d}{dx}\left[e^{-2x}\right]$$

$$= 1e^{-2x} + x\ e^{-2x}(-2)$$

$$= (1-2x)e^{-2x}$$

Example. Find the inflection points of the function $f(x) = e^{-x^2}$.

$$f'(x) = e^{-x^2} \cdot (-2x) = -2x\, e^{-x^2}$$

$$f''(x) = -2e^{-x^2} - 2x\, e^{-x^2}(-2x) = (-2+4x^2)\, e^{-x^2}$$

Solve $f''(x) = 0$

$$\underbrace{(-2+4x)}_{=0}\, e^{-x^2} = 0$$

$$-2+4x = 0$$
$$x = 1/2$$

		$1/2$	
$-2+4x$	$-$		$+$
e^{-x^2}	$+$		$+$
	$-$		$+$
	$\cap$		$\cup$

$\Rightarrow$ Inflection point at $(1/2, f(1/2)) \rightarrow \left(\frac{1}{2}, e^{-1/4}\right)$