

### 3.3: The Chain Rule

**Example.** Let  $f(x) = (x^3 + x + 1)^2$ . Find  $f'(x)$   
using the product rule

by expanding

What about  $\frac{d}{dx} \left[ (x^3 + x + 1)^{100} \right]$ ?

## Composite Functions:

Let  $f$  and  $g$  be functions of  $x$ . Then, the **composite functions**  $g$  of  $f$  (denoted  $g \circ f$ ) and  $f$  of  $g$  (denoted  $f \circ g$ ) are defined as:

$$(g \circ f)(x) = g(f(x))$$

$$(f \circ g)(x) = f(g(x))$$

**Example.** 'Break-down' the following composite functions:

$$\frac{1}{x+3}$$

$$(x^4 + 3x - 8)^3$$

$$\left(\frac{1-x}{x^3+1}\right)^4$$

$$\frac{3}{\sqrt{(x+1)^2-1}}$$

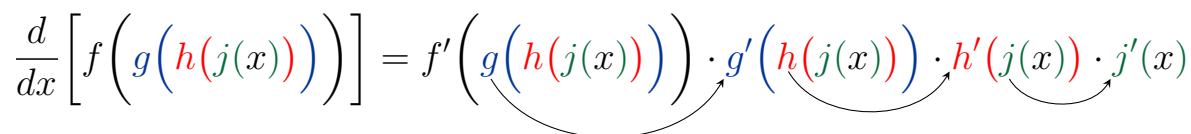
### Rule 7: The Chain Rule

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$$

If  $y = f(u)$  and  $u = g(x)$ , then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

*Note:*

$$\frac{d}{dx} \left[ f \left( g \left( h \left( j(x) \right) \right) \right) \right] = f' \left( g \left( h \left( j(x) \right) \right) \right) \cdot g' \left( h \left( j(x) \right) \right) \cdot h' \left( j(x) \right) \cdot j'(x)$$


### The General Power Rule

$$\frac{d}{dx}[(f(x))^n] = n(f(x))^{n-1}f'(x)$$

**Example.** Use the chain rule to show

$$\frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

**Example.** Find the derivative of the following functions

$$F(x) = (x^3 + x + 1)^{100}$$

$$G(t) = (3t + 1)^2$$

$$H(u) = \sqrt{u^2 + 1} - 3$$

$$J(\nu) = \nu^2(2\nu + 3)^5$$

$$\kappa(x) = (2x^2 + 3)^4(3x - 1)^5$$

$$\tau(x) = \frac{1}{(4x^2 - 7)^2}$$

**Example.** Find the equation of the line tangent to  $f(x)$  at  $\left(0, \frac{1}{8}\right)$

$$f(x) = \left(\frac{2x + 1}{3x + 2}\right)^3$$

**Example.** The membership of The Fitness Center, which opened a few years ago, is approximated by the function

$$N(t) = 100(64 + 4t)^{2/3} \quad (0 \leq t \leq 52)$$

where  $N(t)$  gives the number of members at the beginning of week  $t$ .

Find  $N'(t)$

How fast was the center's membership increasing initially ( $t = 0$ )?

How fast was the membership increasing at the beginning of the 40th week?

What was the membership when the center first opened? At the beginning of the 40th week?

**Rule 1: Derivative of a Constant**

$$\frac{d}{dx}[c] = 0$$

**Rule 2: The Power Rule**

$$\frac{d}{dx}[x^n] = nx^{n-1}$$

**Rule 3: Derivative of a Constant Multiple of a Function**

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)]$$

**Rule 4: The Sum Rule**

$$\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$$

**Rule 5: The Product Rule**

$$\frac{d}{dx}[f(x) \cdot g(x)] = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

**Rule 6: The Quotient Rule**

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

**Rule 7: The Chain Rule**

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$$