

## 6.1: Simple Interest

### Definition.

The **simple interest**  $I$  is given by

$$I = Prt$$

where

$I$  = interest (in dollars)

$P$  = principal (in dollars)

$r$  = annual interest rate (as a decimal)

$t$  = time (in years)

**Note:** The time measurements of  $r$  and  $t$  must agree

From this, the **future value** of simple interest is

$$S = P + I = P + Prt = P(1 + rt)$$

## Example.

If \$8,000 is invested for 2 years at an annual rate of 9%, how much interest will be received at the end of the 2-year period? What will the future value be?

$$P = \$8,000$$

$$t = 2 \text{ yr}$$

$$r = 9\%/\text{yr} = 0.09/\text{yr}$$

$$I = P \cdot r \cdot t = 8000(0.09) \cdot 2 = \$1440$$

$$S = P + I = 8000 + 1440 = \$9440$$

- or -

$$S = P(1+r \cdot t) = 8000(1.18) = \$9440$$

If \$4,000 is borrowed for 39 weeks at an annual interest rate of 15%, how much interest is due at the end of the 39 weeks?

$$P = \$4,000$$

$$t = 39/52 \text{ yr}$$

$$r = 15\%/\text{yr} = 0.15/\text{yr}$$

$$I = P \cdot r \cdot t = 4000(0.15) \frac{39}{52} = \$450$$

An investor wants to have \$20,000 in 9 months. If the best available simple interest rate is 6.05% per year, how much must be invested now?

$$S = \$20,000$$

$$t = 9/12 \text{ yr} = \frac{3}{4} \text{ yr}$$

$$r = 6.05\%/\text{yr} = 0.0605/\text{yr}$$

$$S = P(1+r \cdot t)$$

$$\frac{20000}{(1+0.0605(\frac{3}{4}))} = P \cdot \frac{1+0.0605(\frac{3}{4})}{(1+0.0605(\frac{3}{4}))}$$

$$P = \frac{20000}{(1+0.0605(\frac{3}{4}))} = \$19,131.89$$

## Definition.

The **return on investment** (ROI) is the ratio between the gain and cost of an investment:

$$ROI = \frac{\text{Gains on investment}}{\text{Cost of investment}}$$

The **earned (effective) interest rate** is the equivalent interest rate of the investment when all the fees and dividends are included.

**Example.** Mary Spaulding bought Wind-Gen Electric stock for \$6,125.00. After 6 months, the value of her shares had risen by \$138.00 and dividends totaling \$144.14 had been paid.

Find the return on investment on this investment.

$$ROI = \frac{138.00 + 144.14}{6125.00} = 0.0461 = 4.61\%$$

Find the simple interest rate she earned on this investment if she sold the stock at the end of the 6 months.

$$I = 138.00 + 144.14$$

$$P = 6125$$

$$t = \frac{6}{12} \text{ yr.} = \frac{1}{2} \text{ yr}$$

$$I = P r t$$

$$\frac{(138 + 144.14)}{6125(\frac{1}{2})} = \frac{6125 r (\frac{1}{2})}{6125(\frac{1}{2})}$$

$$\frac{2(138 + 144.14)}{6125} = r = 0.0921 \\ = 9.21\%$$

## Example.

To buy a Treasury bill (T-bill) that matures to \$10,000 in 6 months, you must pay \$9,750. What annual simple interest rate does this earn?

$$S = 10000 = P + I \Rightarrow I = 250$$

$$P = 9750$$

$$t = 6/12 \text{ yr} = \frac{1}{2} \text{ yr}$$

$$S = P + I = P + P_r t$$

$$\Rightarrow X + I = X + P_r t$$

$$I = P_r t$$

$$\frac{250}{9750(\frac{1}{2})} = \frac{9750(r)(\frac{1}{2})}{9750(\frac{1}{2})}$$

$$\frac{2(250)}{9750} = r = \frac{0.0513}{= 5.13\%}$$

If the bank charges a fee of \$40 to buy a T-bill, what is the actual interest rate you earn?

The \$40 is removed from your gains

$$\Rightarrow S = 10000 - 40 = P + I \Rightarrow I = 210$$

$$P = 9750$$

$$t = 6/12 \text{ yr} = \frac{1}{2} \text{ yr}$$

$$I = P_r t$$

$$\frac{210}{9750(\frac{1}{2})} = \frac{9750(r)(\frac{1}{2})}{9750(\frac{1}{2})}$$

$$\frac{2(210)}{9750} = r = \frac{0.0431}{= 4.31\%}$$