6.3: Future Values of Annuities

Definition.

- An **annuity** is a financial plan characterized by regular payments (e.g. mortgages, student loans, etc.).
- The sum of all the payments and the interest earned is called the **future value of the annuity** or its **future value**.
- An **ordinary annuity** or (**annuity immediate**) is an annuity in which payments are made at the *end of each of the equal payment intervals*.

Example. Suppose that we invest \$100 at the end of each year for 5 years in an account that pays 10% compounded annually. How much money will you have at the end of the 5 years?

Definition.
If $\$R$ is deposited at the *end of each period* for n periods in an annuity that earns interest at a rate of i per period, the **future value of the annuity** will be

$$S = R \cdot S_{\overline{n}|i} = R\left[\frac{(1+i)^n - 1}{i}\right]$$

The notation $S_{\overline{n}|i}$ represents the future value of an ordinary annuity of \$1 per period for n periods with an interest rate of i per period.

Example. Suppose a pair of twins take different steps to save for retirement. Both regularly make investments of $\$2,000$ into accounts that earn 10%, compounded annually. Starting at age 21:

Find the future value if twin A makes his payments for 8 years, and then lets his investment accrue compound interest every year for 36 years.

Find the future value if twin B waits 8 years before making regular payments for the following 36 years.

Example. Suppose that you wish to have $\$50,000$ saved up in 5 years. To do this, you want to make regularly monthly payments. What is the amount of the monthly payments if the interest rate is 5%? What if the interest rate is 15%?

Example. A small business invests $\$1,000$ at the end of each month in an account that earns 6% compounded monthly. How long will it take until the business has $\$100,000$ toward the purchase of its own office building?

Definition.
An **annuity due** differs from an ordinary annuity in that the payments are made at the *beginning of each period*.

If $\$R$ is deposited at the *beginning of each period* for n periods in an annuity that earns interest at a rate of i per period, the **future value of the annuity** will be

$$S_{\text{due}} = R \cdot S_{\overline{n}|i}(1+i) = R\left[\frac{(1+i)^n - 1}{i}\right](1+i)$$

Example. Find the future value of an investment if \$150 is deposited at the beginning of each month for 9 years at an interest rate of 7.2% compounded monthly.