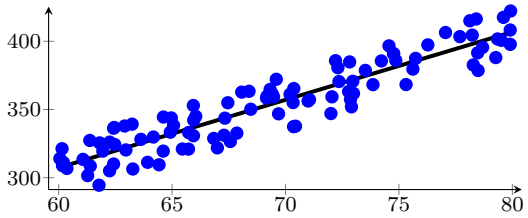
## 4.1: Visualizing Variability with a Scatterplot

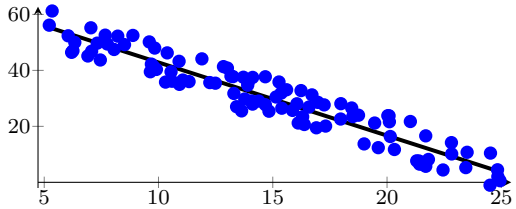
**Definition.**
A **scatterplot** is used when examining the relationship between two *numerical* variables where each point represents an observation. With scatterplots, we examine the

- **trend**: general tendency of the scatter plot going from left to right
- **strength**: strong associations have little vertical variation
- **shape**: is the scatterplot linear or nonlinear?

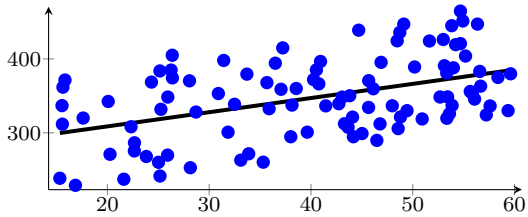
Positive trend



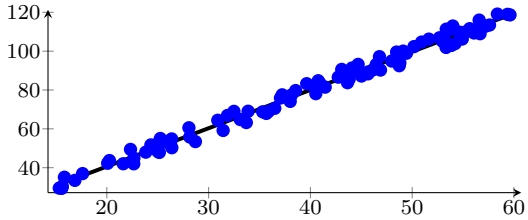

Negative trend



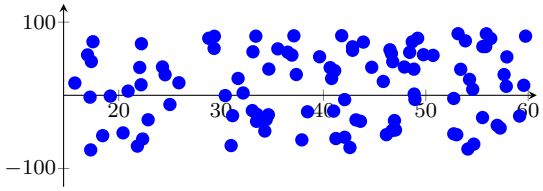

Weak association



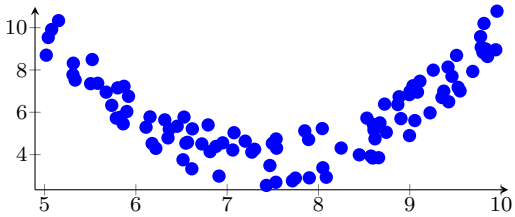

Strong association



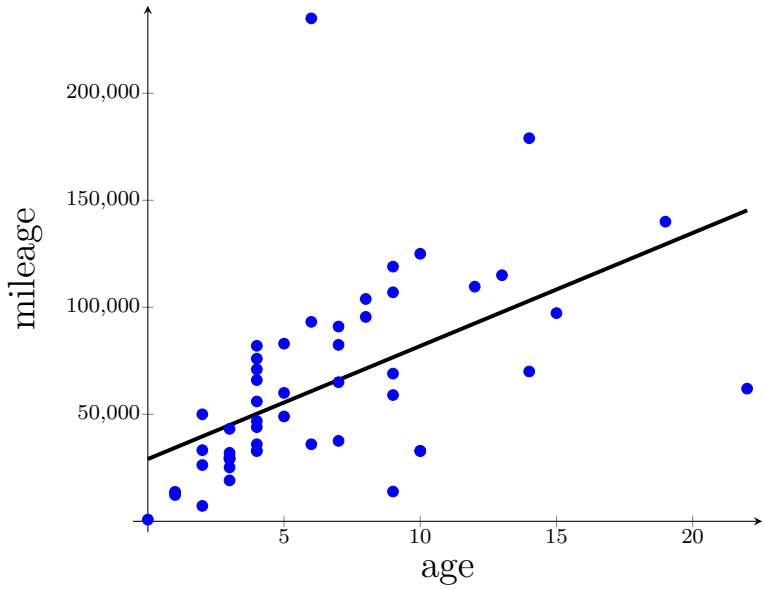

No trend




Quadratic/nonlinear shape



**Example.** The scatterplot below shows the age and corresponding mileage for a sample of used cars.




What is the association between the variables? Identify the trend, its shape, and how strong the relationship is.

Trend: Positive trend

Shape: Linear

Strength: Medium strength

## 4.2: Measuring Strength of Association with Correlation

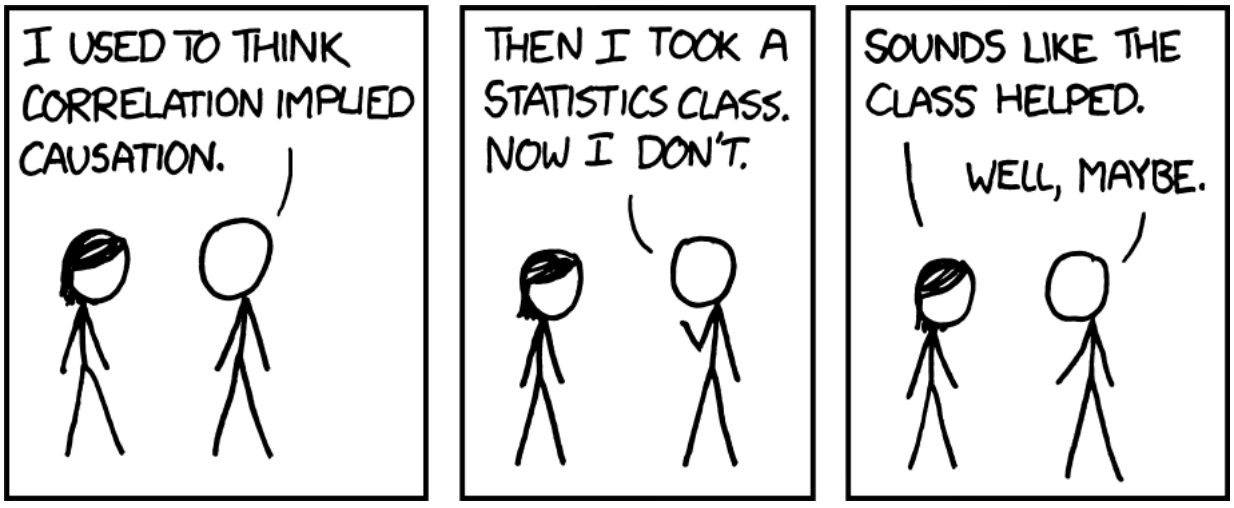
**Definition.**
The **correlation coefficient** is a number that measures the strength of the linear association between two numerical variables. The correlation coefficient is between $-1$ and 1:

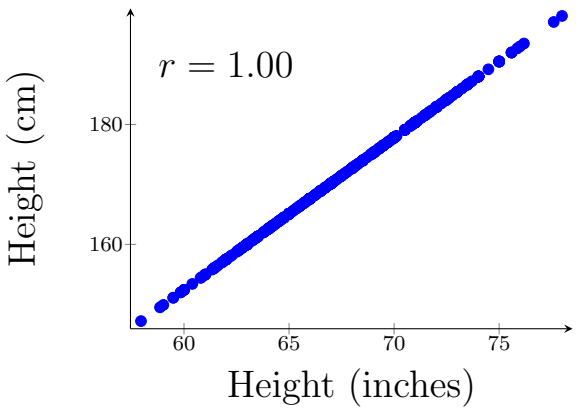
| | |
|---|---|
| $1 \rightarrow$ | Strong association and positive trend |
| $0 \rightarrow$ | Weak or no association |
| $-1 \rightarrow$ | Strong association and negative trend |

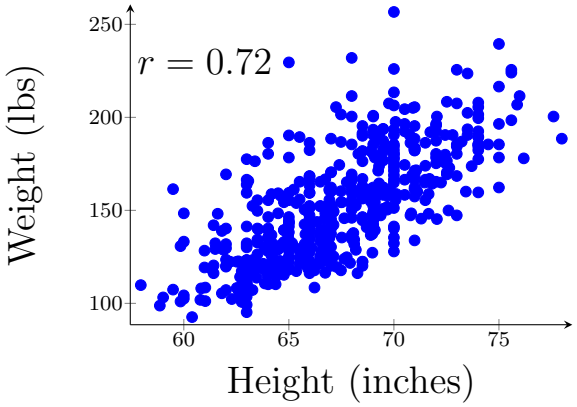
*The correlation coefficient only makes sense if the trend is linear and both variables are numerical!!*

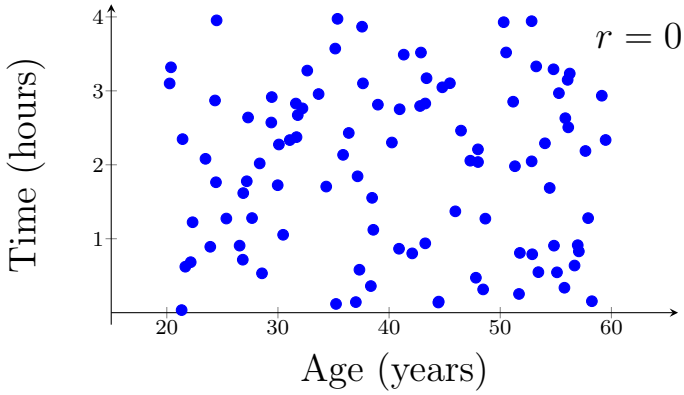
Note: *Correlation does not mean causation!*

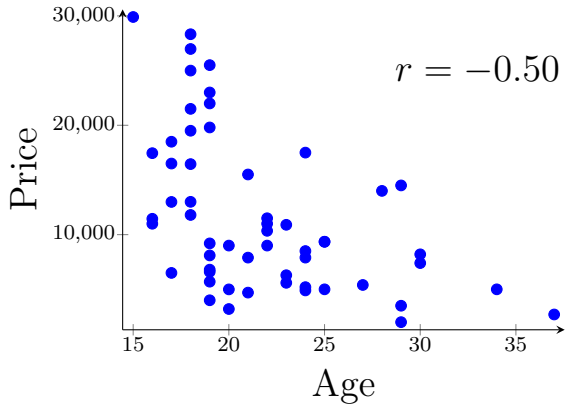
Take a few minutes to look at the graphs at this link: Spurious correlations

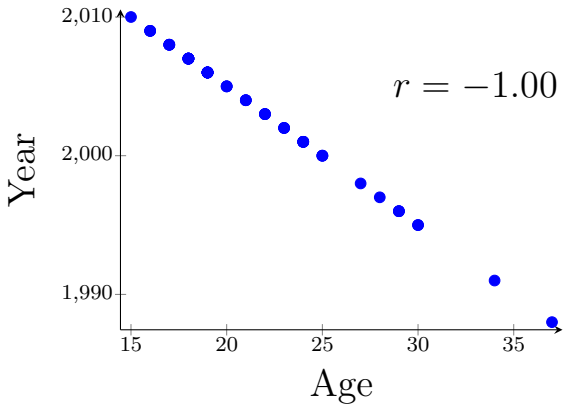



$r = 1.00$
Height (cm)
180
160
60
65
70
75
Height (inches)


$r = 0.72$
Weight (lbs)
250
200
150
100
60
65
70
75
Height (inches)


$r = 0$
Time (hours)
4
3
2
1
20
30
40
50
60
Age (years)


$r = -0.50$
Price
30,000
20,000
10,000
15
20
25
30
35
Age


$r = -1.00$
Year
2,010
2,000
1,990
15
20
25
30
35
Age

**Definition.**
The formula for the correlation coefficient between two variables $x$ and $y$ is

$$r = \frac{\sum z_x z_y}{n-1}$$

where $z_x$ and $z_y$ are the $z$ scores for each entry in the $x$ and $y$ lists.

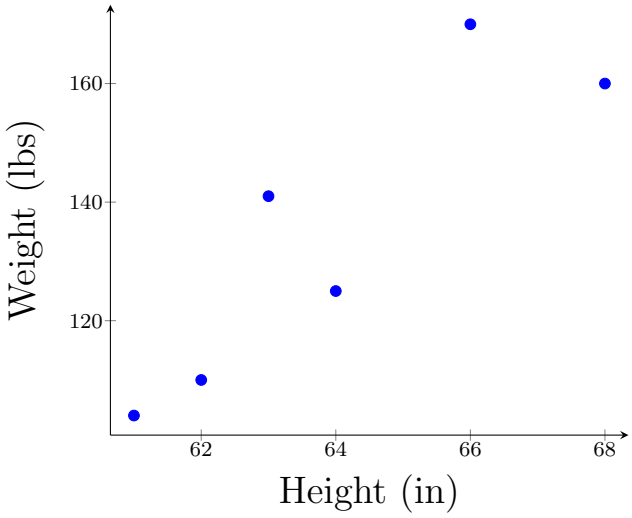
$$z_x = \frac{x - \overline{x}}{s_x} \qquad z_y = \frac{y - \overline{y}}{s_y}$$

**Example.** Below are the heights and weights of six women:

| | | | | | | |
|---|---|---|---|---|---|---|
| Heights | 61 | 62 | 63 | 64 | 66 | 68 |
| Weights | 104 | 110 | 141 | 125 | 170 | 160 |

Compute the correlation coefficient by hand. Then, graph the scatterplot and compute the correlation coefficient using StatCrunch.

| | x | y | zx | zy | zxzy |
|---|---|---|---|---|---|
| | 61 | 104 | -1.1504 | -1.1598 | 1.3343 |
| | 62 | 110 | -0.7670 | -0.9353 | 0.7174 |
| | 63 | 141 | -0.3835 | 0.2245 | -0.0861 |
| | 64 | 125 | 0.0000 | -0.3741 | 0.0000 |
| | 66 | 170 | 0.7670 | 1.3095 | 1.0043 |
| | 68 | 160 | 1.5339 | 0.9353 | 1.4347 |
| | | | | sum | 4.4047 |
| mean | 64 | 135 | | sum/(n-1) | 0.8809 |
| std dev | 2.60768 | 26.7283 | | | |



**Understanding the Correlation Coefficient:**

- Changing the order of the variables does not change $r$
- Adding a constant or multiplying by a positive constant does not affect $r$
- The correlation coefficient is unitless
- None of this makes any sense if the relationship between the variables is not linear!