

# Math 123 Class notes

## Fall 2025

To accompany *Applied Calculus* by *Tan*

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## 1.4: Straight Lines

### Definition. (Slope of a Nonvertical Line)

If  $(x_1, y_1)$  and  $(x_2, y_2)$  are any two distinct points on a nonvertical line  $L$ , then the slope  $m$  of  $L$  is given by

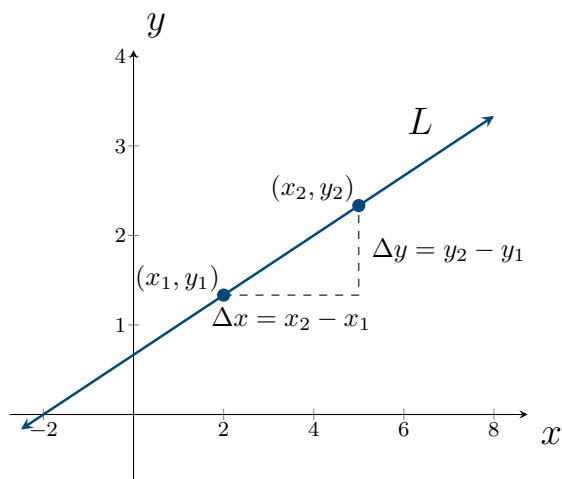
$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

**Example.** Compute the slope of the line passing through the points

$$(x_1, y_1) = (1, 1) \text{ and } (x_2, y_2) = (4, 2)$$

$$(x_1, y_1) = (3, 2) \text{ and } (x_2, y_2) = (-1, 2)$$

$$(x_1, y_1) = (4, 1) \text{ and } (x_2, y_2) = (4, 4)$$



**Definition. (Point-Slope Form of an Equation of a Line)**

An equation of the line that has slope  $m$  and passes through the point  $(x_1, y_1)$  is given by

$$y - y_1 = m(x - x_1)$$

**Example.** Find the equation of the line going through the points

$$(x_1, y_1) = (-2, 1) \text{ and } (x_2, y_2) = (3, -2)$$

$$(x_1, y_1) = (3, 4) \text{ and } (x_2, y_2) = (-1, 4)$$

$$(x_1, y_1) = (2, 0) \text{ and } (x_2, y_2) = (2, 1)$$

**Definition. (Slope-Intercept Form of an Equation of a Line)**

An equation of the line that has slope  $m$  and intersects the  $y$ -axis at the point  $(0, b)$  is given by

$$y = mx + b$$

**Example.** Rewrite the equations in the previous example in slope-intercept form.

**Definition. (Parallel and Perpendicular lines)**

Let  $L_1$  and  $L_2$  be lines with slopes  $m_1$  and  $m_2$  respectively. If  $L_1$  and  $L_2$  are *parallel*, then

$$m_1 = m_2.$$

If  $L_1$  and  $L_2$  are *perpendicular*, then

$$m_1 = -\frac{1}{m_2}.$$

**Example.**

Find the line *parallel* to  $y = \frac{3}{2}x + 1$  that passes through the point  $(-4, 10)$ .

Find the line *perpendicular* to  $y = \frac{3}{2}x + 1$  that passes through the point  $(-3, 4)$ .

## Forms of Linear Equations

General form:  $Ax + By = C$

Point-slope form:  $y - y_1 = m(x - x_1)$

Slope-intercept form:  $y = mx + b$

Vertical line:  $x = a$

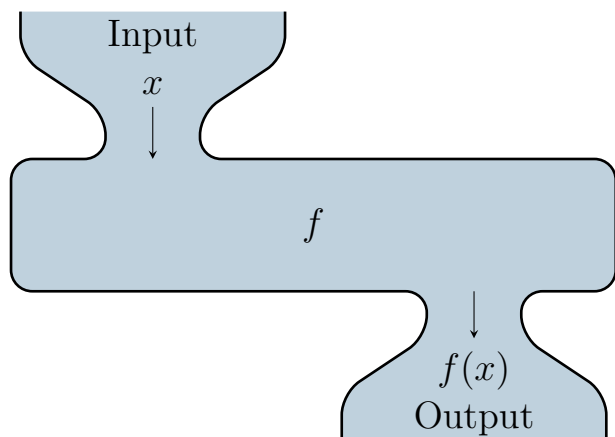
Horizontal line:  $y = b$

## 2.1: Functions and Their Graphs

### Definition.

A **function** is a rule that assigns to each element in a set  $A$  one and only one element in a set  $B$ .

In the context above, the set  $A$  is called the **domain**, and the set  $B$  is called the **range**.



**Example.** Let  $f(x) = 2x^2 - 2x + 1$ . Evaluate the following

$$f(1)$$

$$f(-2)$$

$$f(a)$$

$$f(a + h)$$



**Example.** Find the domain and range of the following functions:

$$f(x) = x$$

$$A = \pi r^2$$

$$y = \sqrt{x - 1}$$

$$y = \frac{1}{x^2 - 4}$$

**Definition. (Vertical Line Test)**

A curve in the  $xy$ -plane is the graph of a function  $y = f(x)$  (an explicit function) if and only if each vertical line intersects it in at most one point

**Example.** Use the vertical line test on the following graphs to determine which graphs may represent an explicit function:

