

1.3: The Language of Relations and Functions

Definition.

Let A and B be sets. A **relation** R from A to B is a subset of $A \times B$. Given an ordered pair (x, y) , x **is related to** y by R , written $x R y$, if, and only if, (x, y) is in R . The set A is called the **domain** of R and the set B is called its **co-domain**.

The notation for a relation R may be written symbolically as follows:

$$x R y \text{ means that } (x, y) \in R.$$

The notation $x \not R y$ means that x is not related to y by R :

$$x \not R y \text{ means that } (x, y) \notin R.$$

Example. Let $A = \{1, 2\}$ and $B = \{1, 2, 3\}$ and define a relation R from A to B as follows; Given any $(x, y) \in A \times B$,

$$(x, y) \in R \text{ means that } \frac{x - y}{2} \text{ is an integer.}$$

State explicitly which ordered pairs are in $A \times B$ and which are in R

$$A \times B = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3)\} \quad R = \{(1, 1), (1, 3), (2, 2)\}$$

Is $1 R 3$?

Yes

Is $2 R 3$?

No

Is $2 R 2$?

Yes

What are the domain and co-domain of R ?

$$\text{Domain} = \{1, 2\}$$

$$\text{Co-domain} = \{1, 2, 3\}$$

Example. Define a relation C from \mathbb{R} to \mathbb{R} as follows: For any $(x, y) \in \mathbb{R} \times \mathbb{R}$,

$$(x, y) \in C \text{ means that } x^2 + y^2 = 1.$$

Is $(1, 0) \in C$?

$$1^2 + 0^2 = 1$$

Yes

Is $(0, 0) \in C$?

$$0^2 + 0^2 = 0$$

No

Is $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right) \in C$?

$$\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= \frac{1}{4} + \frac{3}{4} = 1$$

Yes

Is $-2 \in C$?

$$(-2)^2 + 0^2 = 4$$

No

Is $0 \in C$?

$$0^2 + (-1)^2 = 1$$

Yes

Is $1 \in C$?

$$1^2 + 1^2 = 2$$

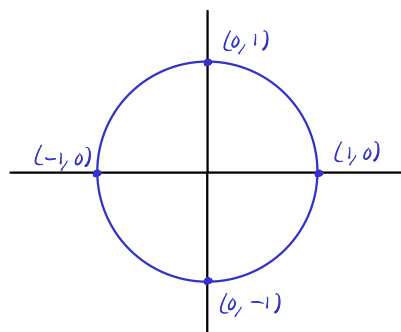
No

What are the domain and co-domain of C ?

Domain: \mathbb{R}

Co-domain: \mathbb{R}

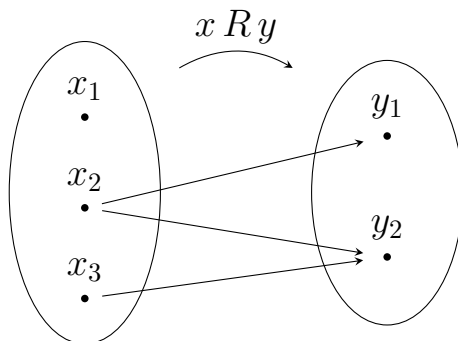
Draw a graph for C by plotting the points of C in the Cartesian plane.



Definition.

Suppose R is a relation from set A to a set B . The **arrow diagram for R** is obtained as follows:

1. Represent the elements of A as points in one region and the elements of B as points in another region.
2. For each x in A and y in B , draw an arrow from x to y if, and only if, x is related to y by R .

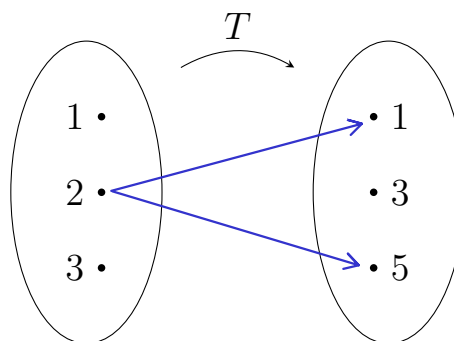
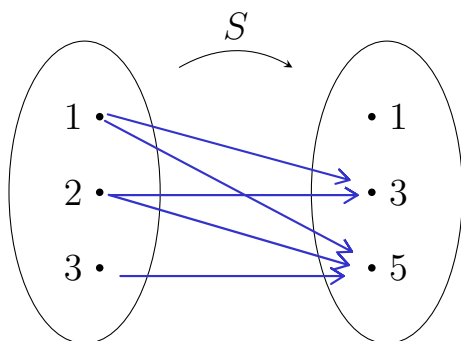


Example. Let $A = \{1, 2, 3\}$ and $B = \{1, 3, 5\}$ and define relations S and T from A to B as follows: For every $(x, y) \in A \times B$,

$(x, y) \in S$ means that $x < y$

$T = \{(2, 1), (2, 5)\}$.

Draw arrow diagrams for S and T



Definition.

A **function** F from a set A to a set B is a relation with domain A and co-domain B that satisfies the following two properties:

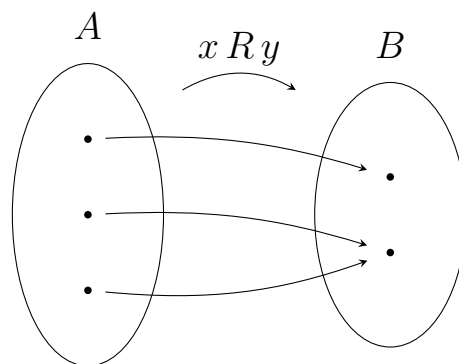
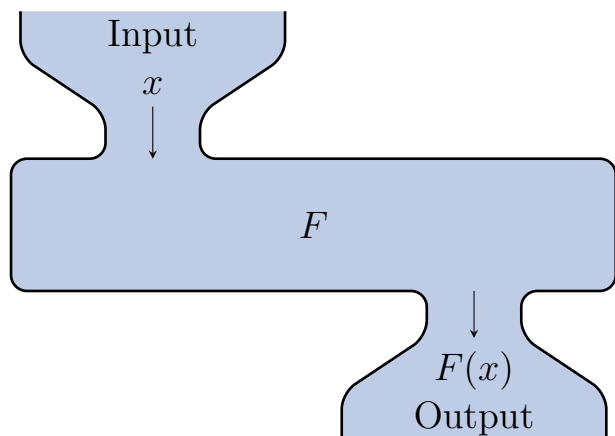
1. For every element x in A , there is an element y in B such that $(x, y) \in F$.
2. For all elements x in A and y and z in B ,

if $(x, y) \in F$ and $(x, z) \in F$, then $y = z$.

Note: A relation from A to B is a function if, and only if,

1. Every element of A is the first element of an ordered pair of F
2. No two distinct ordered pairs in F have the same first element.

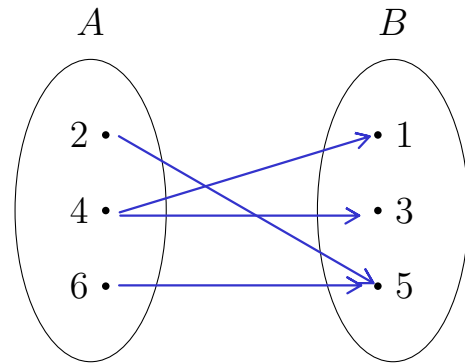
Note: If A and B are sets and F is a function from A to B , then given any element x in A , the unique element in B that is related to x by F is denoted $F(x)$, which is read “ F of x ”.



Example. Let $A = \{2, 4, 6\}$ and $B = \{1, 3, 5\}$. Which of the relations R , S , and T defined below are functions from A to B ?

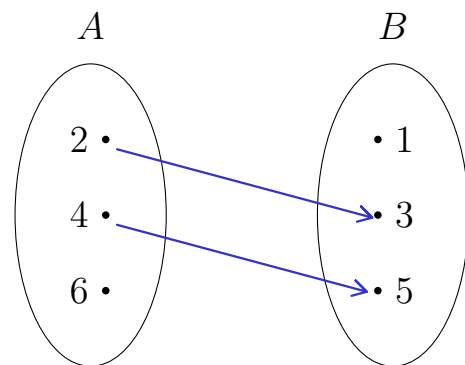
$$R = \{(2, 5), (4, 1), (4, 3), (6, 5)\}$$

Not a function since 4 maps to 2 distinct outputs



For every $(x, y) \in A \times B$, $(x, y) \in S$ means that $y = x + 1$.

Not a function since 6 maps to an element outside of B



T is defined by the arrow diagram

Is a function;
Each element in A maps to a distinct element in B

