6.1: Probability Distributions Are Models of Random Experiments

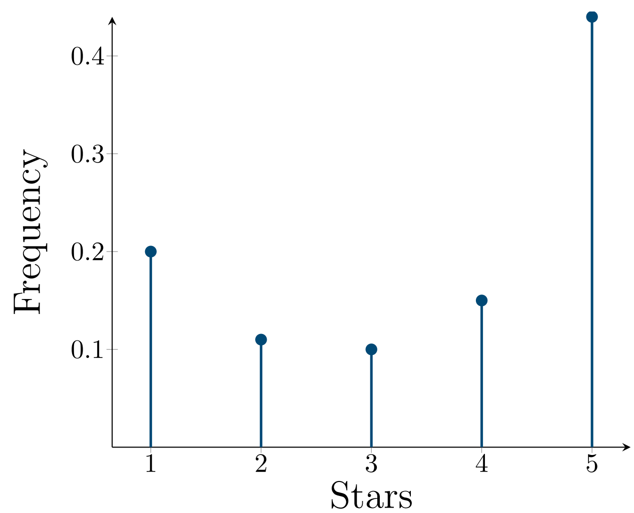
Definition.
The **probability distribution** describes

- all possible outcomes of a random experiment, and
- the probability of each outcome.

This is sometimes also referred to as the **probability distribution function (pdf)**.

Example. Suppose we are reading Amazon reviews of a particular product. In total, the product has $3,901$ reviews, distributed as shown below.

Stars	Frequency
5	0.44
4	0.15
3	0.10
2	0.11
1	0.20

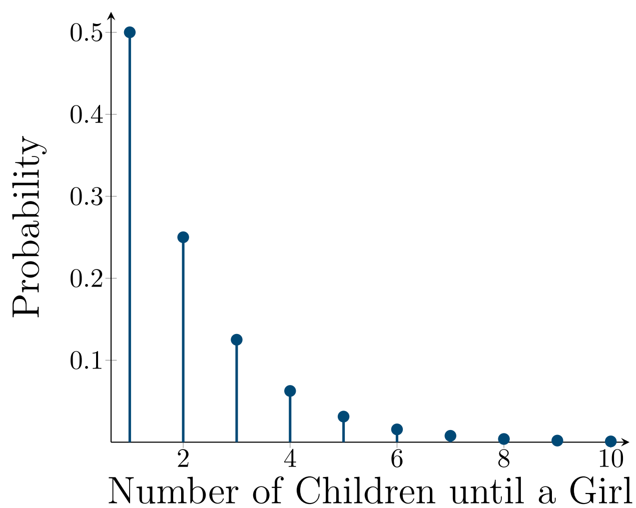


If we pick a reviewer at random, what is the probability that they give a 5 star review? What about a 1 star review?

What is the sum of the probabilities?

> *Note*: Valid probability distributions:
>
> - Have probabilities between 0 and 1,
> - The sum of the probabilities is *exactly* 1.

Example. Suppose a couple decides they will keep having children until they have a girl. Assuming that the likelihood of having a boy or girl is equally likely, the probability of having x children can be given by $(1/2)^x$, and is represented by the graph below.



What is the maximum number of children possible?

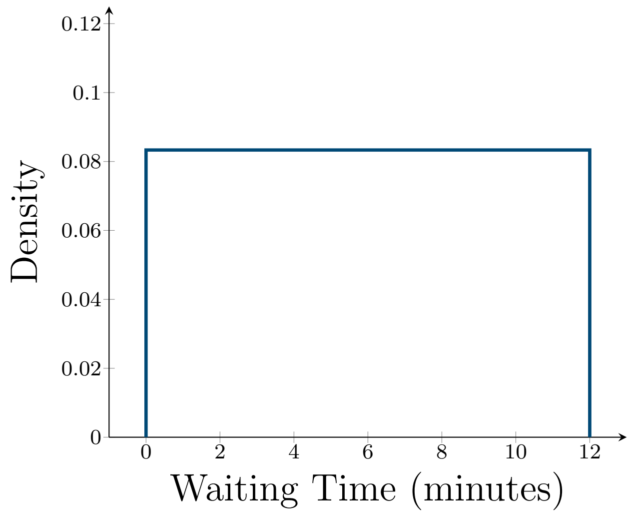
Do the probabilities sum to 1?

Finding the probabilities for continuous outcomes:

- is represented as area under a curve,
- is in the context of a range of values, and
- the probability of hitting an exact value is 0

Example. Suppose a coffee shop has done extensive research and knows each customer is helped in under 5 minutes. The shaded area of the graph represents the probability that a customer will wait less than 2 minutes.

Example. Suppose a bus arrives at the bus stop every 12 minutes. If you arrive at the bus stop at a randomly chosen time, then the probability distribution for the number of minutes you must wait is shown in the graph below:



Find the probability that you will have to wait less than 5 minutes.

Find the probability that you will have to wait between 4 and 10 minutes.

What is the probability that you will have to wait *exactly* 12 minutes?

6.2: The Normal Model

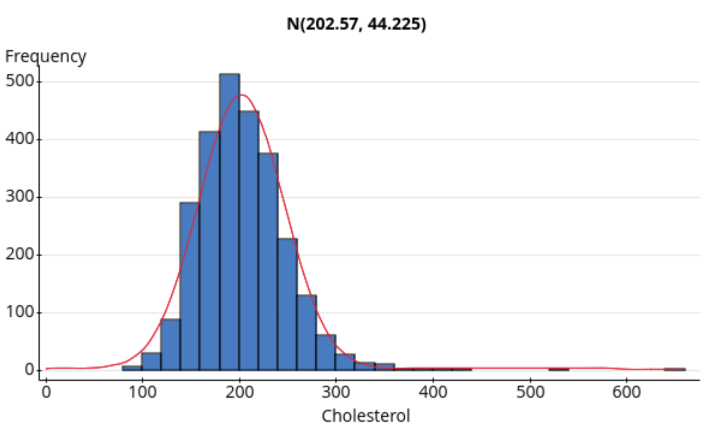
Definition.
The **Normal Distribution** is a symmetric, unimodal model that provides a very close fit for many numerical variables:

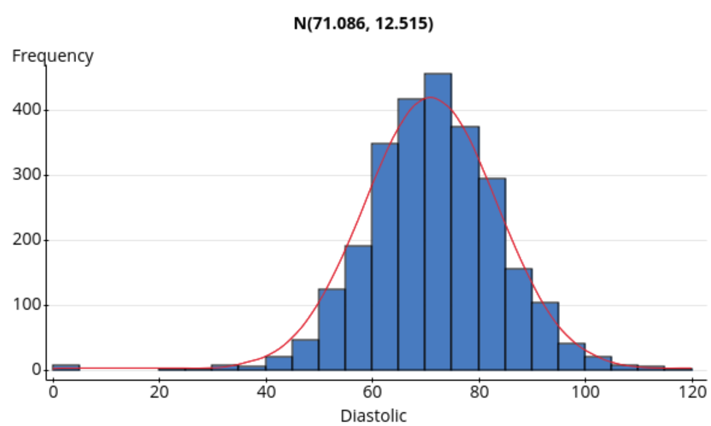
$$\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

We use $N(\mu, \sigma)$ to denote the Normal Distribution with mean μ and standard deviation σ.
Note:

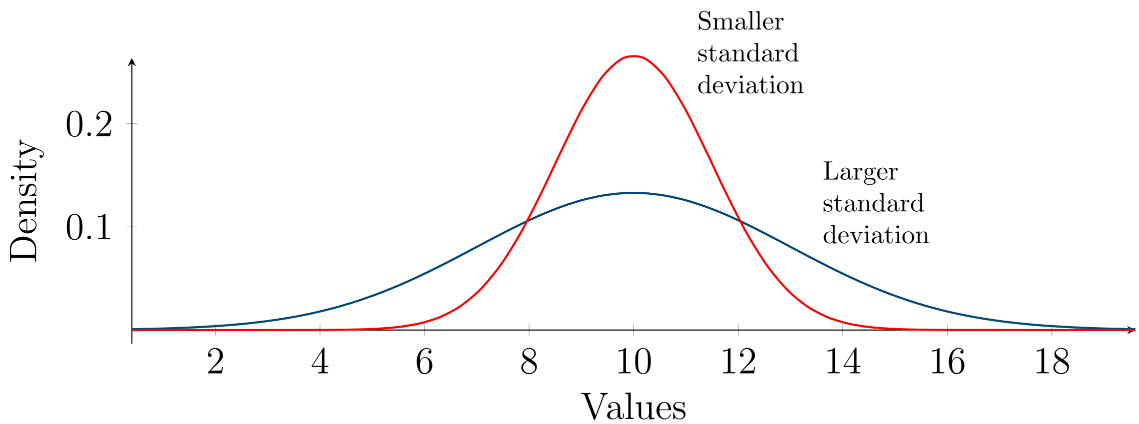
- μ and σ are used in the context of a probability distribution, whereas $\overline{x}$ and s are used for data.
- Other sources denote the Normal Distribution with mean μ and *variance* σ^2 as $N(\mu, \sigma^2)$ or $\mathcal{N}(\mu, \sigma^2)$.

Example. Below are some histograms from a dataset that show the measured cholesterol and diastolic blood pressure from 2,793 people. These histograms have the Normal Distribution with the corresponding mean μ and standard deviation σ overlayed:

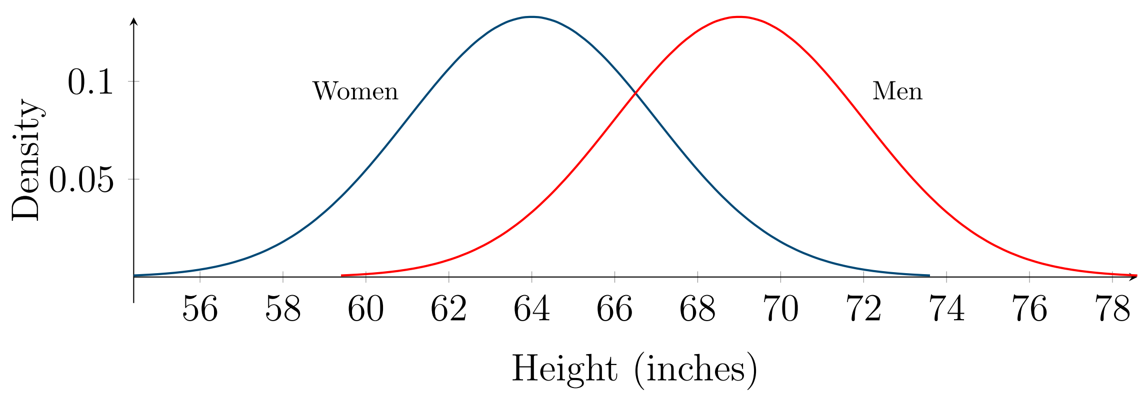




Example. Below is the graph of two Normal Distributions with equal means, but different standard deviations.



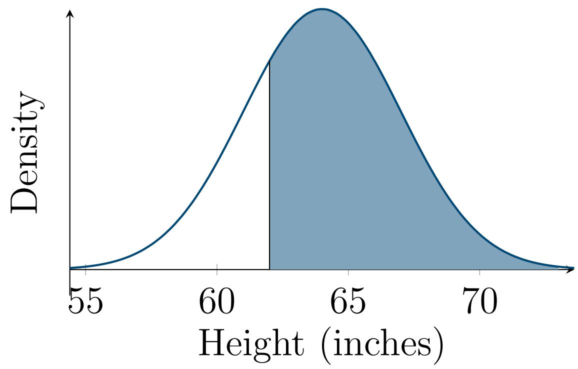
Example. Below is the graph of two Normal Distributions with equal standard deviations, but different means.



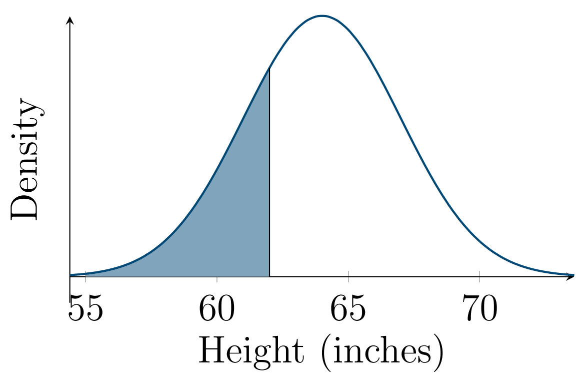
What is the area under each of the curves above?

Example. Suppose that the Normal model $N(64, 3)$ gives a good approximation to the distribution of adult women's height in the United States. If a women is chosen at random, what is the probability that

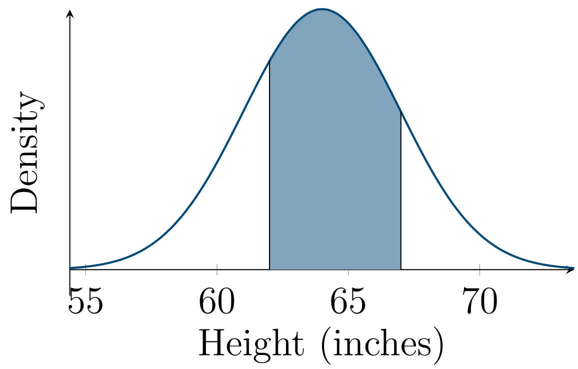
she is taller than 62"?



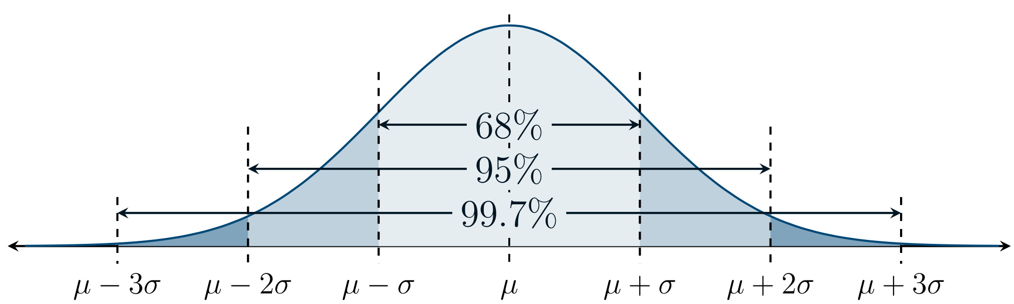
she is shorter than 62"?



her height is between 62” and 67”?



Example. “Verify” the emperical rule by using technology to find the probability that an observation lies within 1, 2, and 3 standard deviations.



Definition.
The **Standard Normal Distribution** is a $N(0,1)$:

$$\frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}$$

We use the Standard Normal Distribution in conjunction with z-scores to compute probabilities:

$$z = \frac{X-\mu}{\sigma}$$

Example. Suppose the length of a newborn seal pup follows a Normal Distribution with a mean length of 29.5, and standard deviation 1.2. Solve the following by finding the z-score and then using a z-score table to compute the probability that a seal pup's length is

shorter than 28",

longer than 31", and

is between 28" and 31".

Example. Assume that women's heights follow a Normal distribution with mean 64" and standard deviation 3". Find the 25th and 75th percentile using

technology and

by hand.