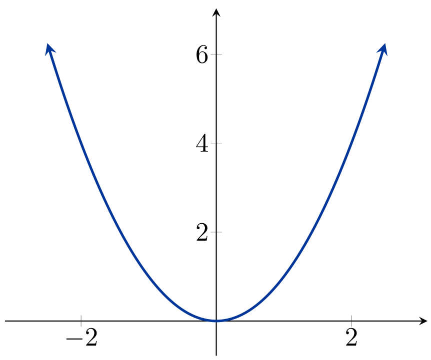
4.4: Optimization I

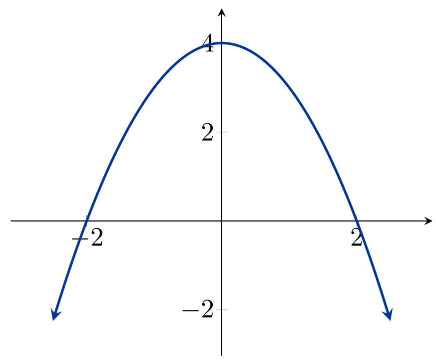
Definition. (Absolute Extrema)
Let f be defined on a set D containing c. If

- $f(c) \geq f(x)$ for every x in D, then $f(c)$ is an **absolute maximum** value of f
- $f(c) \leq f(x)$ for every x in D, then $f(c)$ is an **absolute minimum** value of f

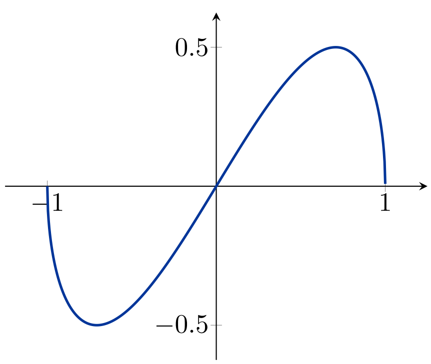
$f(0) = 0$ is the absolute minimum;
No absolute maximum



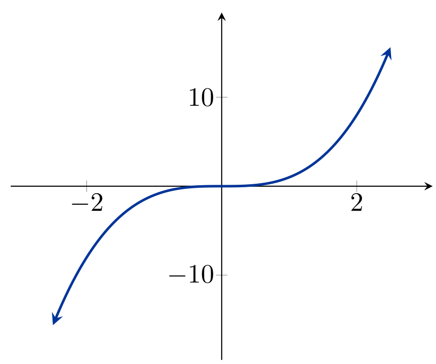
No absolute minimum;
$f(0) = 4$ is the absolute maximum



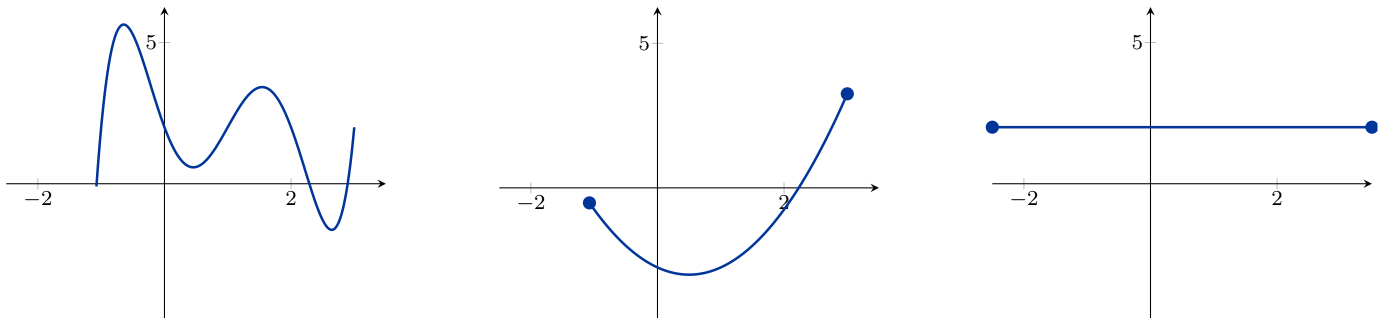
$f(-\sqrt{2}) = -\frac{1}{2}$ is the absolute minimum;
$f(\sqrt{2}) = \frac{1}{2}$ is the absolute maximum



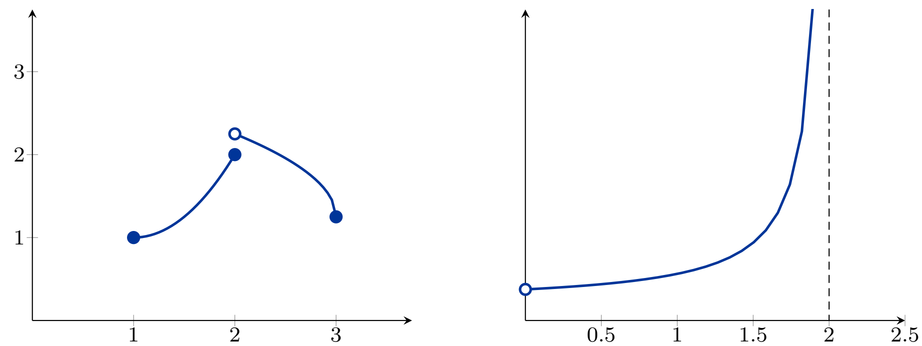
No absolute minimum;
No absolute maximum



> **Theorem 3**
> A function that is continuous on a closed interval $[a, b]$ has an absolute maximum value and an absolute minimum value on that interval.



Note: It is important that the function is both continuous *and* the interval is closed:



Finding the Absolute Extrema of f on a Closed Interval

1. Find the critical points of f within the interval (a, b).
2. Compute $f(x)$ at $x = a$, $x = b$, and at each of the critical points found above.
3. The absolute maximum and absolute minimum will correspond to the largest and smallest values found above.

Example. Find the absolute extrema of the following functions on the intervals indicated

$f(x) = x^2$ on $[-1, 2]$ Graphs

$g(x) = x^3 - 2x^2 - 4x + 4$ on $[0, 3]$

$h(x) = x^{2/3}$ on $[-1, 8]$

Example. The daily average cost function (in dollars per unit) of Elektra Electronics is given by

$$\overline{C}(x) = 0.0001x^2 - 0.08x + 40 + \frac{5000}{x} \qquad (x > 0)$$

where x stands for the number of graphing calculators that Elektra produces. Show that a production level of 500 units per day results in a minimum average cost for the company.

Example. The altitude (in feet) of a rocket t seconds into flight is given by

$$s = f(t) = -t^3 + 96t^2 + 5 \qquad (t \geq 0)$$

Find the maximum altitude attained by the rocket.

Find the maximum velocity attained by the rocket.