1.3: Linear Functions

Definition.
A **linear function** is a function of the form

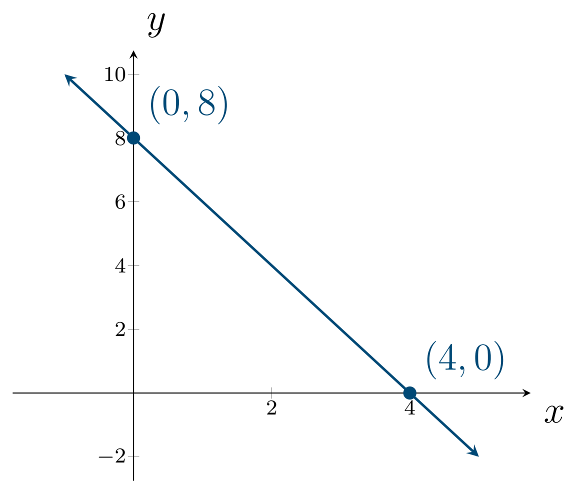
$$y = f(x) = mx + b$$

where m and b are constants.

Example. $y = -2x + 8$

$x = 0 \Rightarrow y = -2(0) + 8$
$= 8 \longrightarrow \boxed{(0,8)}$

$y = 0 \Rightarrow -8 + 0 = -2x + 8 - 8$
$\frac{-8}{-2} = \frac{-2x}{-2}$
$4 = x \longrightarrow \boxed{(4,0)}$



A linear function can be uniquely determined using only *two* distinct points.

Definition.
The point(s) where a graph intersects the axes are called intercepts. The x-coordinate of the point where the function intersects the x-axis is called the **x-intercepts**. The y-coordinate of the point where the function intersects the y-axis is called the **y-intercepts**.

- To solve for the y-intercept:
  - Set $x = 0$,
  - Solve for y.
- To solve for the x-intercept:
  - Set $y = 0$,
  - Solve for x.

Example. Find the intercepts and graph the following lines:

$3x + 2y = 12$

x-intercept:

set $y=0$, find x.

$3x + 2(0) = 12$

$\frac{3x}{3} = \frac{12}{3}$

$\boxed{x = 4} \rightarrow (4,0)$

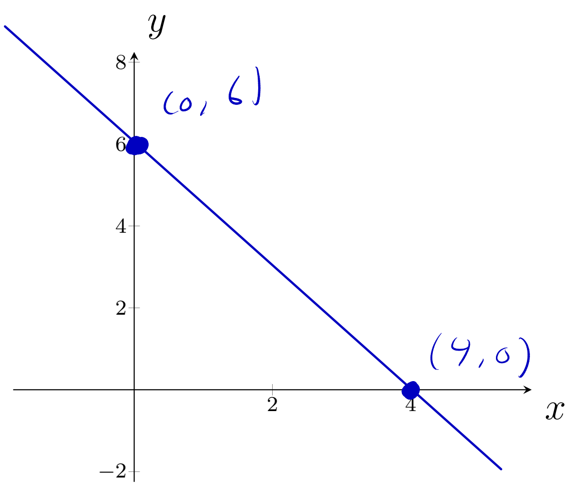
y-intercept:

set $x=0$, find y.

$3(0) + 2y = 12$

$2y = 12$

$\boxed{y = 6} \rightarrow (0,6)$



$x = 4y$

x-intercept:

set $y=0$, find x.

$x = 4(0)$

$\boxed{x = 0} \rightarrow (0,0)$

y-intercept:

set $x=0$, find y.

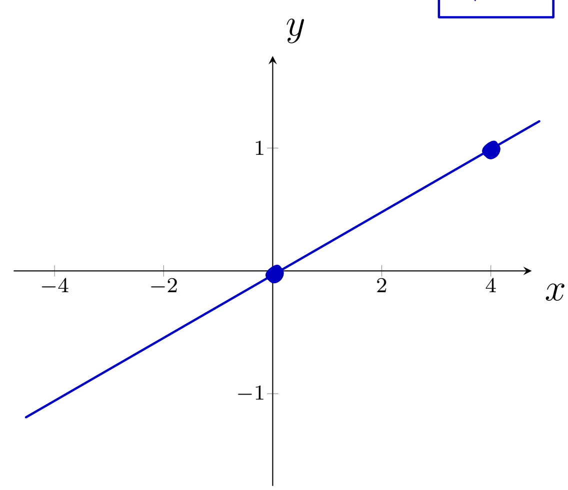
$0 = 4y$

$\boxed{y = 0} \rightarrow (0,0)$

To graph this, choose another point:

$\underline{y=1} \rightarrow$ $x = 4(1)$

$\boxed{x = 4} \rightarrow (4,1)$



Definition.
If a nonvertical line passes through the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$, its **slope**, denoted by m, is found using

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$

Δy is "delta y", and represents the change in y
Δx is "delta x", and represents the change in x

Note: The slope of a vertical line is undefined.

Example. Find the slope of the line passing through the points $(-2, 1)$ and $(5, 3)$.

(x_1, y_1) (x_2, y_2)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 1}{5 - (-2)} = \boxed{\frac{2}{7}}$$

Note:

- Two distinct nonvertical lines are *parallel* if and only if their slopes are *equal*.
- Two distinct nonvertical lines are *perpendicular* if and only if their slopes are *negative reciprocals*:
  e.g. If ℓ_1 has a nonzero slope m, then ℓ_2 is perpendicular if its slope is $-1/m$.

Point-slope form

Definition.
The equation of the line passing through the point (x_1, y_1) with slope m can be written in the point-slope form:

$$y - y_1 = m(x - x_1)$$

Example. Find the equation of each line that passes through the point $(-3, 4)$ and has

a slope of $m = \dfrac{1}{4}$

$y - 4 = \frac{1}{4}(x - (-3))$

$+4 + y - 4 = \frac{x}{4} + \frac{3}{4}$ $+4$

$y = \frac{x}{4} + \frac{3}{4} + 4\left(\frac{4}{4}\right)$

$y = \frac{x}{4} + \frac{3}{4} + \frac{16}{4} \Rightarrow \boxed{y = \frac{x}{4} + \frac{19}{4}}$

the point $(-2, 1)$ on the line

(x_2, y_2) (x_1, y_1)

$m = \dfrac{4 - 1}{-3 - (-2)} = \dfrac{3}{-1} = -3$

$y - 4 = -3(x - (-3))$

$y = -3x - 9 + 4$

$\boxed{y = -3x - 5}$

$y - 1 = -3(x - (-2))$

$y = -3x - 6 + 1$

$\boxed{y = -3x - 5}$

a slope of zero (horizontal)

$y - 4 = 0\,(x - (-3))$

$\boxed{y = 4}$

an undefined slope (vertical)

$\boxed{x = -3}$

Slope-intercept form

Definition.
The slope-intercept form of the equation of a line with slope m and y-intercept b is

$$y = mx + b$$

Example (Example 7, p.82)**.** The population of U.S. males, y (in thousands), projected from 2015 to 2060 can be modeled by

$$y = 1125.9x + 142,960$$

where x is the number of years after 2000.

- Find the slope and y-intercept of the graph of this function.

set $x=0$ $\longrightarrow$ $y = 1125.9(0) + 142,960$

$$\boxed{y = 142,960}$$

- What does the y-intercept tell us about the population of U.S. males?

Zero years after 2000, the population of U.S. males is 142,960.

- Interpret the slope as a rate of change.

Each year, the population of U.S. males is expected to increase by 1125.9.

Example. Each day, a young person should sleep 8 hours plus $\frac{1}{4}$ hour for each year the person is under 18 years of age. Assuming that the relation is linear, write the equation relating hours of sleep y and age x

x	y
18	8
17	$8 + 1/4$
16	$8 + 2(1/4)$
15	$8 + 3(1/4)$
14	9

$\Rightarrow$ Use Point-slope formula

$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{8 - 9}{18 - 14} = \boxed{-\frac{1}{4}}$$

$$y - y_1 = m(x - x_1)$$

$$y - 8 = -\frac{1}{4}(x - 18)$$

$$y = 8 - \frac{x}{4} + \frac{\cancel{18}^{\,9}}{\cancel{4}_{\,2}}$$

$$\boxed{y = \frac{25}{2} - \frac{x}{4}}$$

Forms of Linear Equations

$$\begin{aligned}
\text{General form: } & Ax + By = C \\
\text{Point-slope form: } & y - y_1 = m(x - x_1) \\
\text{Slope-intercept form: } & y = mx + b \\
\text{Vertical line: } & x = a \\
\text{Horizontal line: } & y = b
\end{aligned}$$