## 3.2: The Product and Quotient Rules

**Rule 5: The Product Rule**

$$\frac{d}{dx}[f(x)\cdot g(x)] = f'(x)\cdot g(x) + f(x)\cdot g'(x)$$

*Note:*

$$\frac{d}{dx}[f(x)\cdot g(x)] \neq f'(x)\cdot g'(x)$$

**Example.** Find the derivative of the following functions

- by expanding (← omitted)
- by using the product rule

$f(x) = (2x^2 - 1)(x+3)$

$$f'(x) = \frac{d}{dx}[2x^2-1](x+3) + (2x^2-1)\frac{d}{dx}[x+3]$$
$$= 4x(x+3) + (2x^2-1)(1)$$
$$= 4x^2 + 12x + 2x^2 - 1$$
$$= \boxed{6x^2 + 12x - 1}$$

$g(x) = x^3(\overbrace{\sqrt{x}+1}^{x^{1/2}+1})$

$$g'(x) = \frac{d}{dx}[x^3](\sqrt{x}+1) + x^3\frac{d}{dx}[\sqrt{x}+1]$$
$$= 3x^2(\sqrt{x}+1) + x^3\frac{1}{2\sqrt{x}}$$
$$= 3x^{5/2} + 3x^2 + \frac{1}{2}x^{5/2}$$
$$= \boxed{\frac{7}{2}x^{5/2} + 3x^2}$$

*Note:*

$$\frac{d}{dx}[fghj] = f'ghj + fg'hj + fgh'j + fghj'$$

**Rule 6: The Quotient Rule**

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{\left[g(x)\right]^2}$$

"Lo De Hi, minus Hi De Lo, over the square of what's below"

**Example.** Find the derivative of the following functions

$$f(x) = \frac{3x^2 - 4x + 7}{x}$$

$$f'(x) = \frac{x \frac{d}{dx}[3x^2-4x+7] - (3x^2-4x+7)\frac{d}{dx}[x]}{x^2} = \frac{x(6x-4) - (3x^2-4x+7)1}{x^2}$$

$$= \frac{6x^2 \cancel{-4x} - 3x^2 \cancel{+4x} - 7}{x^2} = \frac{3x^2-7}{x^2} = \boxed{3 - \frac{7}{x^2}}$$

$$g(x) = \frac{x}{2x-4}$$

$$g'(x) = \frac{(2x-4)\frac{d}{dx}[x] - x\frac{d}{dx}[2x-4]}{(2x-4)^2} = \frac{(2x-4)1 - x\cdot 2}{(2x-4)^2}$$

$$= \frac{\cancel{2x} - 4 \cancel{-2x}}{(2x-4)^2} = \boxed{\frac{-4}{(2x-4)^2}}$$

$$h(x) = \frac{x^2+1}{x^2-1}$$

$$h'(x) = \frac{(x^2-1)\frac{d}{dx}[x^2+1] - (x^2+1)\frac{d}{dx}[x^2-1]}{(x^2-1)^2} = \frac{(x^2-1)2x - (x^2+1)2x}{(x^2-1)^2}$$

$$= \frac{\cancel{2x^3} - 2x - \cancel{2x^3} - 2x}{(x^2-1)^2}$$

$$= \boxed{\frac{-4x}{(x^2-1)^2}}$$

$$j(x) = \frac{\sqrt{x}}{x^2+1}$$

$$j'(x) = \frac{(x^2+1)\frac{d}{dx}[x^{1/2}] - x^{1/2}\frac{d}{dx}[x^2+1]}{(x^2+1)^2} = \frac{(x^2+1)\frac{1}{2}x^{-1/2} - x^{1/2}\cdot 2x}{(x^2+1)^2}$$

$$= \frac{\frac{1}{2}x^{3/2} + \frac{1}{2}x^{-1/2} - 2x^{3/2}}{(x^2+1)^2}$$

$$= \frac{-\frac{3}{2}x^{3/2} + \frac{1}{2}x^{-1/2}}{(x^2+1)^2}$$

$$= \boxed{\frac{-3x^{3/2} + x^{-1/2}}{2(x^2+1)^2}}$$

$$k(x) = \frac{3x(x^2+1)}{x^2-1}$$

$$k'(x) = \frac{(x^2-1)\overbrace{\frac{d}{dx}\left[3x\cdot(x^2+1)\right]}^{\text{Use the product rule here}} - 3x(x^2+1)\frac{d}{dx}\left[x^2-1\right]}{(x^2-1)^2}$$

$$= \frac{(x^2-1)\overbrace{\left(3x\frac{d}{dx}\left[(x^2+1)\right] + \frac{d}{dx}\left[3x\right](x^2+1)\right)}^{\text{This is the product rule}} - 3x(x^2+1)2x}{(x^2-1)^2}$$

$$= \frac{(x^2-1)\left(3x\cdot 2x + 3(x^2+1)\right) - 3x(x^2+1)2x}{(x^2-1)^2}$$

$$= \frac{(x^2-1)\left(6x^2+3x^2+3\right) - 6x^2(x^2+1)}{(x^2-1)^2}$$

$$= \frac{x^2(9x^2+3) - 1(9x^2+3) - 6x^4 - 6x^2}{(x^2-1)^2}$$

$$= \frac{9x^4+3x^2-9x^2-3-6x^4-6x^2}{(x^2-1)^2}$$

$$= \boxed{\frac{3x^4-12x^2-3}{(x^2-1)^2}}$$