9.3: Answering Questions about the Mean of a Population

Definition.
Suppose that we wish to estimate a population mean μ based on a sample mean $\overline{x}$.

- A **confidence interval** is an interval about the point estimate $\overline{x}$ that we can be confident contains the true population mean μ:

$$\overline{x} \pm m$$

- The **margin of error (ME)** is half the width of the confidence interval. When estimating a population proportion, the margin of error is

$$m = t^* SE$$

  where

$$SE_{est} = \frac{s}{\sqrt{n}}$$

- The **confidence level** measures how often the estimation method is successful. A larger confidence level results in a larger margin of error.

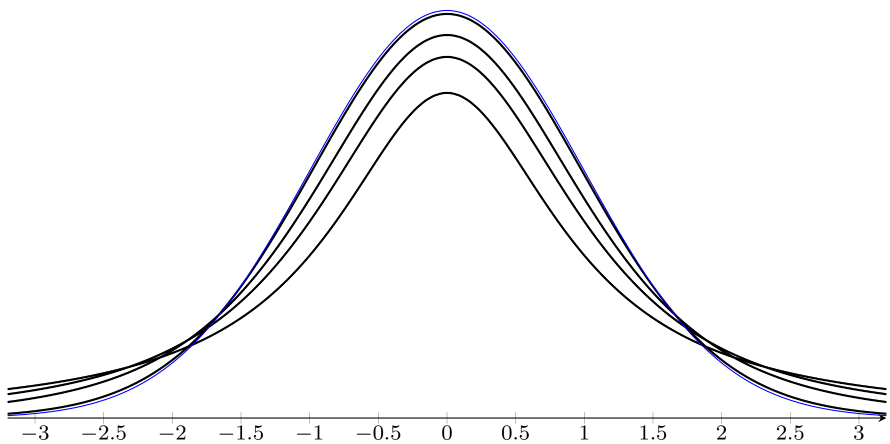
The multiplier t^* is found using the t-distribution and $n - 1$ degrees of freedom.

The t-distribution is used when

- the sample size is small
- the population standard deviation is unknown

As the sample size n (and degrees of freedom, $n-1$) increases, the t-distribution gets closer to the Standard Normal Distribution.

The graph below shows the t-distribution in black where the degrees of freedom are 1, 2, 4, and 30, and the Standard Normal Distribution in blue.



	Confidence Level									
df	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
1	1.000	1.376	1.963	3.078	6.314	12.710	31.820	63.660	318.310	636.620
2	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
z	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291

Example. A random sample of 35 two-year colleges in 2014-2015 had a mean tuition of \$4,173, with a standard deviation of \$2,590. Find the 90% and 95% confidence intervals and interpret their results.

We are 90% confident that the true population mean lies within the interval (3432.7304, 4913.2696)

We are 95% confident that the true population mean lies within the interval (3283.3039, 5062.6961)

Example. A study to test the life of iPad batteries reports that in a random sample of 30 iPads, the mean battery life was 9.7 hours, and the standard deviation was 1.2 hours.

Compute a 95% confidence interval and interpret the results.

We are 95% confident that the true mean battery life in hours lies within the interval (9.2519,10.1481).

Suppose Apple claims the iPad has a mean battery life of 10 hours. Based on this confidence interval, do we reject or fail to reject this claim?

Fail to reject; 10 hours is in this confidence interval, so it is reasonable to assert that the true mean battery life is 10 hours.

Example. Suppose I am interested in estimating my mean commute time. Over the course of a week, I record my commute times:

18.96	20.65	17.47	19.38	17.11

Compute a 95% confidence interval for my actual commute times.

We are 95% confident that the true mean commute time lies within the interval (16.9176,20.5104)