

5.4: Differentiation of Exponential Functions

Rule 1: Derivative of the Exponential Function

$$\frac{d}{dx}[e^x] = e^x$$

Example. Find the derivative of the following functions

$$f(x) = x^2 e^x$$

$$f'(x) = \frac{d}{dx} [x^2] e^x + x^2 \frac{d}{dx} [e^x]$$

$$= 2x e^x + x^2 e^x$$

$$= (2x + x^2) e^x$$

$$g(t) = (e^t + 2)^{3/2}$$

$$g'(t) = \frac{3}{2} (e^t + 2)^{1/2} \underbrace{e^t}_{\nearrow}$$

Rule 2: The Chain Rule for Exponential Functions

If $f(x)$ is a differentiable function, then

$$\frac{d}{dx} \left[e^{f(x)} \right] = e^{f(x)} f'(x)$$

Example. Find the derivative of the following functions

$$f(x) = e^{2x}$$

$$f'(x) = e^{2x} \frac{d}{dx}[2x] = 2e^{2x}$$

$$y = e^{-3x}$$

$$y' = e^{-3x} \frac{d}{dx}[-3x] = -3e^{-3x}$$

$$g(t) = e^{2t^2+t}$$

$$g'(t) = e^{2t^2+t} \frac{d}{dt} [2t^2+t] = (4t+1) e^{2t^2+t}$$

$$y = xe^{-2x}$$

$$y' = \frac{d}{dx} [x] e^{-2x} + x \frac{d}{dx} [e^{-2x}]$$

$$= 1 e^{-2x} + x e^{-2x} (-2)$$

$$= (1 - 2x) e^{-2x}$$

Example. Find the inflection points of the function $f(x) = e^{-x^2}$.

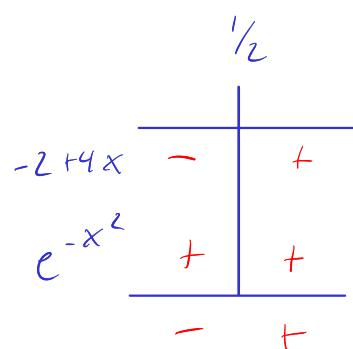
$$f'(x) = e^{-x^2} \cdot (-2x) = -2x e^{-x^2}$$

$$f''(x) = -2e^{-x^2} - 2x e^{-x^2}(-2x) = (-2 + 4x^2)e^{-x^2}$$

Solve $f''(x)=0$

$$\underbrace{(-2+4x)}_{=0} e^{-x^2} = 0$$

$$-2+4x=0 \\ x=\frac{1}{2}$$



↗ ↘

\Rightarrow Inflection point at $(\frac{1}{2}, f(\frac{1}{2})) \rightarrow (\frac{1}{2}, e^{-\frac{1}{4}})$