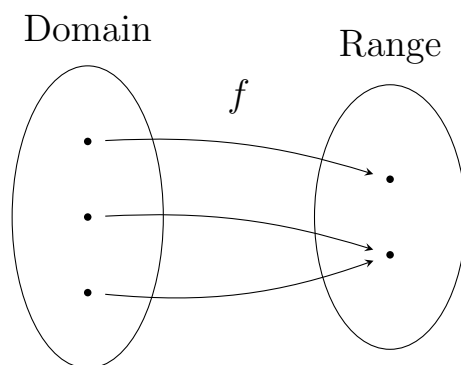
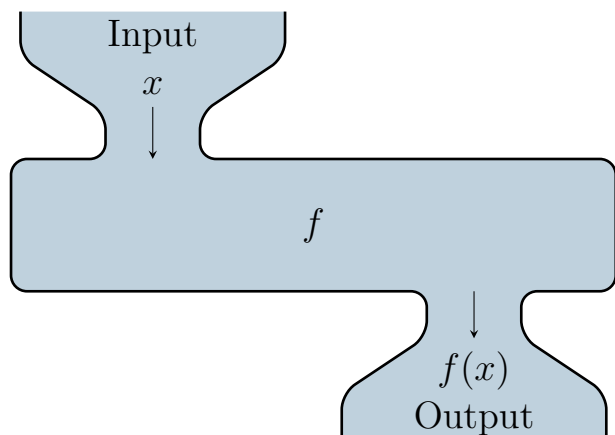


## 2.1: Functions and Their Graphs

### Definition.

A **function** is a rule that assigns to each element in a set  $A$  one and only one element in a set  $B$ .

In the context above, the set  $A$  is called the **domain**, and the set  $B$  is called the **range**.



**Example.** Let  $f(x) = 2x^2 - 2x + 1$ . Evaluate the following

$$\begin{aligned} f(1) &= 2(1)^2 - 2(1) + 1 \\ &= 2 - 2 + 1 \\ &= \boxed{1} \end{aligned}$$

$$\begin{aligned} f(-2) &= 2(-2)^2 - 2(-2) + 1 \\ &= 8 + 4 + 1 \\ &= \boxed{13} \end{aligned}$$

$$\begin{aligned} f(a) &= 2(a)^2 - 2(a) + 1 \\ &= \boxed{2a^2 - 2a + 1} \end{aligned}$$

$$\begin{aligned} f(a+h) &= 2(a+h)^2 - 2(a+h) + 1 \\ &= 2(a^2 + 2ah + h^2) - 2(a+h) + 1 \\ &= \boxed{2a^2 + 4ah + 2h^2 - 2a - 2h + 1} \end{aligned}$$

**Example.** Find the domain and range of the following functions:

$$f(x) = x$$

$$\text{Domain: } (-\infty, \infty)$$

$$\text{Range: } (-\infty, \infty)$$

$$A = \pi r^2$$

$$\text{Domain: } (-\infty, \infty)$$

$$\text{Range: } [0, \infty)$$

$$y = \sqrt{x-1}$$

$$x-1 \geq 0$$

$$x \geq 1$$

$$\text{Domain: } [1, \infty)$$

$$\text{Range: } [0, \infty)$$

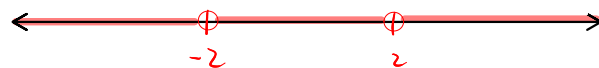
$$y = \frac{1}{x^2 - 4}$$

$$x^2 - 4 \neq 0$$

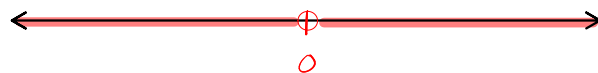
$$x^2 \neq 4$$

$$x \neq \pm 2$$

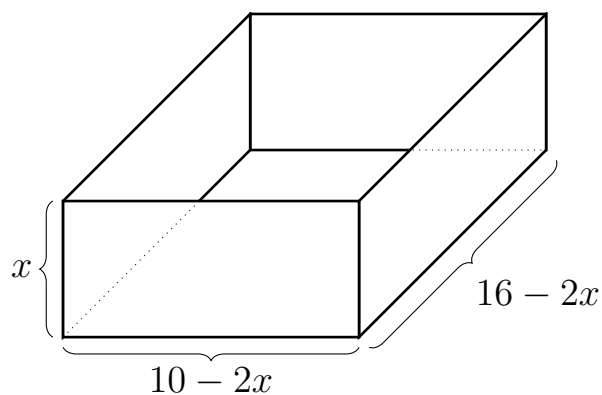
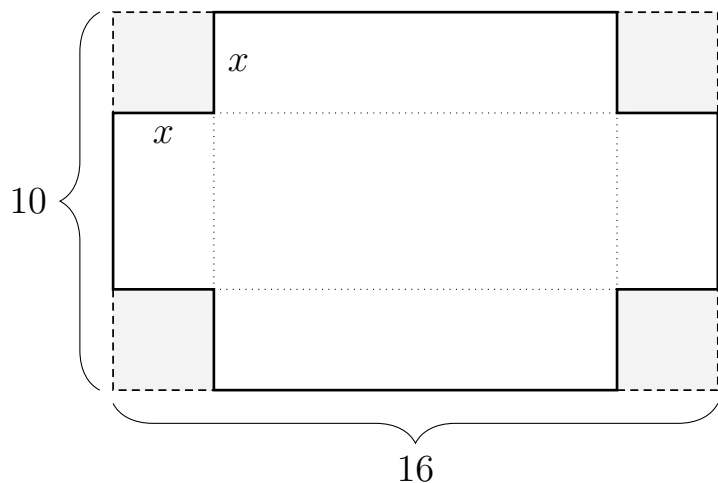
$$\text{Domain: } (-\infty, -2) \cup (-2, 2) \cup (2, \infty)$$



$$\text{Range: } (-\infty, 0) \cup (0, \infty)$$



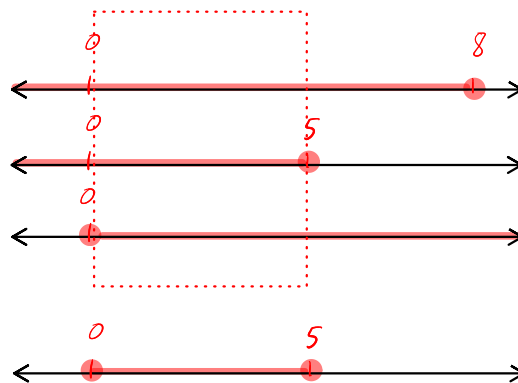
**Example.** An open box is to be made from a rectangular piece of cardboard 16 inches long and 10 inches wide by cutting away identical squares ( $x$  inches by  $x$  inches) from each corner and folding up the resulting flaps. Find an expression that gives the volume  $V$  of the box as a function of  $x$ . What is the domain of the function?



$$\begin{aligned}
 V &= (\text{length})(\text{width})(\text{height}) \\
 &= (16-2x)(10-2x)x \\
 &= x(160 - 52x + 4x^2) \\
 &= 4x^3 - 52x^2 + 160x
 \end{aligned}$$

$$\begin{aligned}
 16-2x &\geq 0 \\
 10-2x &\geq 0 \\
 x &\geq 0
 \end{aligned}
 \Rightarrow \begin{cases} x \leq 8 \\ x \leq 5 \\ x \geq 0 \end{cases}$$

$$\text{Domain: } [0, 5]$$



**Definition.**

A **piecewise** function is a function with different definitions for different portions of the domain.

**Example.** Rewrite the following as piecewise functions:

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$\frac{x}{|x|} = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases}$$

$$|x-1| + |4-x| = \begin{cases} -(x-1) + (4-x), & x \leq 1 \\ (x-1) + (4-x), & 1 < x \leq 4 \\ (x-1) - (4-x), & 4 \leq x \end{cases} = \begin{cases} -2x+5, & x \leq 1 \\ 3, & 1 < x \leq 4 \\ 2x-5, & 4 \leq x \end{cases}$$

**Definition. (Vertical Line Test)**

A curve in the  $xy$ -plane is the graph of a function  $y = f(x)$  (an explicit function) if and only if each vertical line intersects it in at most one point

**Example.** Use the vertical line test on the following graphs to determine which graphs may represent an explicit function:

