

1.5: Solutions of Systems of Linear Equations

$$\text{apple} + \text{apple} + \text{apple} = 18$$

$$\text{apple} + \text{bananas} + \text{bananas} = 14$$

$$\text{bananas} - \text{cherries} = 2$$

$$\text{cherries} + \text{apple} + \text{bananas} = ?$$

Definition.

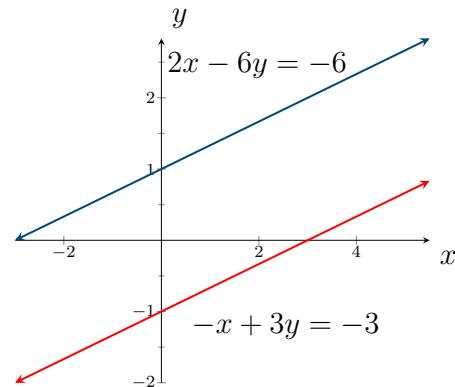
A **system of equations** is 2 (or more) equations. The ordered pairs (x, y) that satisfies *all* equations in the system are the **solutions** of the system.

When solving a system of linear equations, there are three possible outcomes:

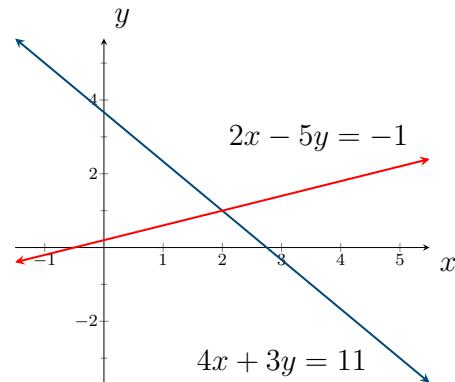
1. No solution (*Inconsistent*),
2. Exactly one solution,
3. Infinitely many solutions (*Dependent*).

Example. Use graphing to find the solutions to the following systems

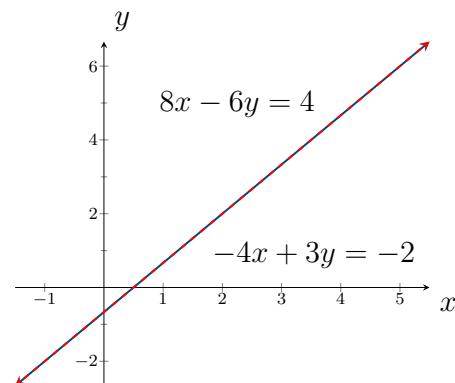
$$\begin{aligned} 2x - 6y &= -6 \\ -x + 3y &= -3 \end{aligned}$$



$$\begin{aligned} 4x + 3y &= 11 \\ 2x - 5y &= -1 \end{aligned}$$



$$\begin{aligned} -4x + 3y &= -2 \\ 8x - 6y &= 4 \end{aligned}$$



Equivalent systems result when

1. One expression is replaced by an equivalent expression.
2. Two equations are interchanged.
3. A multiple of one equation is added to another equation.
4. An equation is multiplied by a nonzero constant.

Substitution Method

Example. Solve the system $\begin{cases} 2x + 3y = 4 \\ x - 2y = 3 \end{cases}$

1. Solve one equation for either one of the variables in terms of the other.

$$x = 2y + 3$$

2. Substitute this expression into the other equation to give the equation in one unknown.

$$2(2y + 3) + 3y = 4$$

3. Solve this equation for the unknown.

$$4y + 6 + 3y = 4$$

$$7y = -2 \Rightarrow y = -\frac{2}{7}$$

4. Substitute solution into the equation in Step 1.

$$x = 2\left(-\frac{2}{7}\right) + 3 \Rightarrow x = \frac{17}{7}$$

5. Check the solution (x, y) .

$$\begin{aligned} 2\left(\frac{17}{7}\right) + 3\left(-\frac{2}{7}\right) &= 4 \\ \left(\frac{17}{7}\right) - 2\left(-\frac{2}{7}\right) &= 3 \end{aligned}$$

Example. Use the substitution method to solve the system

$$4x + 5y = 18 \quad (1)$$

$$3x - 9y = -12 \quad (2)$$

Elimination Method

Example. Solve the system $\begin{cases} 2x - 5y = 4 \\ x + 2y = 3 \end{cases}$

1. Multiply one or both equations by a nonzero number so the coefficients of one of the variables may cancel.
2. Add or subtract the equations to eliminate one of the variables.
3. Solve for the remaining variable.
4. Substitute solution in one of the original equations and solve for the other variable.
5. Check the solution (x, y)

$$\Rightarrow \begin{cases} 2x - 5y = 4 \\ -2x - 4y = -6 \end{cases}$$

$$0x - 9y = -2$$

$$\Rightarrow y = \frac{2}{9}$$

$$2x - 5\left(\frac{2}{9}\right) = 4 \Rightarrow x = \frac{23}{9}$$

$$\begin{aligned} 2\left(\frac{23}{9}\right) - 5\left(\frac{2}{9}\right) &= 4 \\ \left(\frac{23}{9}\right) + 2\left(\frac{2}{9}\right) &= 3 \end{aligned}$$

Example. Use the elimination method to solve the following systems:

$$2x - 6y = -6$$

$$-x + 3y = -3$$

$$4x + 3y = 11$$

$$2x - 5y = -1$$

$$-4x + 3y = -2$$

$$8x - 6y = 4$$

Example. A woman has \$500,000 invested in two rental properties. One yields an annual return of 10% on her investment, and the other returns 12% per year on her investment. Her total annual return from the two investments is \$53,000. Let x represent the amount of the 10% investment and y represent the amount of the 12% investment.

- Write an equation that states that the sum of investments is \$500,000.
- What is the annual return on the 10% investment? What about the 12% investment?
- Write an equation that states the sum of the annual return is \$53,000.
- Solve these two equations simultaneously to find how much is invested in each property.

Example. A nurse has two solutions that contain different concentrations of a certain medication. One is a 12.5% concentration, and the other is a 5% concentration. How many cubic centimeters of each should she mix to obtain 20 cubic centimeters of an 8% concentration?

Example. Using U.S. Bureau of Labor Statistics data for selected years from 1950 and projected to 2050, the number of men M and women W in the workforce (both in millions) can be modeled by the functions

$$M(t) = 0.591t + 37.3 \quad \text{and} \quad W(t) = 0.786t + 13.1$$

where t is the number of years after 1940. Find the year these functions predict that there will be equal numbers of men and women in the U.S. workforce.

