

2.1: Quadratic Equations

Definition.

A **quadratic equation** in one variable is an equation of second degree that can be written in the *general form* as

$$ax^2 + bx + c = 0 \quad (a \neq 0)$$

where a , b , and c represent constants.

The **zero product property** states that for real numbers a and b , $ab = 0$ if and only if $a = 0$ or $b = 0$ or both.

Example. Solve the following for x :

$$x(x + 3) = 0$$

$$(x - 4)(3x + 1) = 0$$

Solving quadratic equations via factoring:

Example. Solve $2x^2 + x = 3x + 12$

1. Rewrite the equation in the general form: $2x^2 - 2x - 12 = 0$

2. Rewrite bx using factors of ac : $2x^2 - 6x + 4x - 12 = 0$

3. Factor out like terms: $2x(x - 3) + 4(x - 3) = 0$

4. Factor by grouping: $(x - 3)(2x + 4) = 0$

5. Solve for the roots: $x = 3$ and $x = -2$

Example. Solve the following for x via factoring:

$$(x + 3)(x - 1) = 5$$

$$-4x^2 + 8x - 3 = 0$$

Solutions to $x^2 = C$ are $x = \pm\sqrt{C}$

Example. Solve the following:

$$(x - 1)^2 = 9$$

$$4x^2 - 1 = 0$$

Definition.

The **quadratic formula**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

gives the solutions to $ax^2 + bx + c = 0$.

Quadratic equations can have one, two, or no solutions. The **discriminant** is $b^2 - 4ac$:

- $b^2 - 4ac > 0$: The equation has *exactly* two distinct real solutions.
- $b^2 - 4ac = 0$: The equation has *exactly* one real solution.
- $b^2 - 4ac < 0$: The equation has no real solutions.

Example. Suppose some hooligans kick a ball up in the air off the roof of the library. Assuming the height, in ft , of the ball t seconds after kicking it is given by

$$h(t) = -32t^2 + 64t + 40$$

Solve for t when

the ball is 80 feet off of the ground

the ball is 72 feet off of the ground

the ball is 40 feet off of the ground

the ball hits the ground

Example. The Social Security Trust Fund balance B , in billions of dollars, can be described by the function $B = -7.97t^2 + 312t - 356$ where t is the number of years past the year 1995. For planning purposes, it is important to know when the trust fund balance will be 0. Solve

$$0 = -7.97t^2 + 312t - 356.$$