3.5: Higher–Order Derivatives

Definition.
The **second derivative** of f is

$$f''(x) = \frac{d}{dx}[f'(x)] = \lim_{h\to 0}\frac{f'(x+h)-f'(x)}{h}$$

We can repeatedly take the derivative of $f(x)$:

$$f'(x), f''(x), f'''(x), \ldots, f^{(n)}(x)$$

Example. Find all derivatives of

$$f(x) = x^5 - 7x^4 - 5x^3 - 2x^2 + 6x - 6$$

$f'(x) = 5x^4 - 28x^3 - 15x^2 - 4x + 6$

$f''(x) = 20x^3 - 84x^2 - 30x - 4$

$f'''(x) = 60x^2 - 168x - 30$

$f^{(4)}(x) = 120x - 168$

$f^{(5)}(x) = 120$

$f^{(n)}(x) = 0$ for $n \geq 6$

Example. Let $f(x) = x^{2/3}$. Find $f'''(x)$.

$$f'(x) = \frac{2}{3} x^{-1/3}$$

$$f''(x) = -\frac{2}{9} x^{-4/3}$$

$$f'''(x) = \frac{8}{27} x^{-7/3}$$

Example. Find the second derivative of $y = \left(2x^2 + 3\right)^{3/2}$

$$y' = \frac{3}{2}(2x^2+3)^{1/2} \cdot 4x = 6x(2x^2+3)^{1/2}$$

$$y'' = 6(2x^2+3)^{1/2} + 6x\frac{1}{2}(2x^2+3)^{-1/2} \cdot 4x$$

$$= 6(2x^2+3)^{1/2} + 12x^2(2x^2+3)^{-1/2}$$

Example. The position function of a maglev train (in feet) is given by

$$s(t) = 4t^2, \qquad (0 \le t \le 30).$$

Find the velocity and the acceleration of the maglev train at time t

$v(t) = s'(t) = 8t$

$a(t) = v'(t) = s''(t) = 8$

Example. Find $\dfrac{d^2y}{dx^2}$ if $x^2 + y^2 = 4$.

$$2x + 2y\,\frac{dy}{dx} = 0$$

$$2y\,\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-x}{y}$$

$$\frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{d}{dx}\left[-\frac{x}{y}\right]$$

$$\frac{d^2y}{dx^2} = \frac{y(-1) - (-x)\frac{dy}{dx}}{y^2}$$

$$= \frac{-y + x\frac{-x}{y}}{y^2}$$

$$= -\frac{\left(\frac{y}{y}\right)y + \frac{x^2}{y}}{y}$$

$$\boxed{\frac{d^2y}{dx^2} = -\frac{y^2 + x^2}{y^2}}$$