

6.5: Loans and Amortization

Definition.

Amortization is the process of repaying a loan. When we have a debt of A_n , with an interest rate of i per period, amortized by n equal periodic payments (at the end of each period), the size of each payment is

$$R = A_n \left[\frac{i}{1 - (1 + i)^{-n}} \right]$$

Example. A company modernizes its production by buying a new piece of equipment for \$200,000. It makes a \$50,000 down payment and agrees to amortize the rest of the debt with quarterly payments over the next 10 years. If the interest on the debt is 12%,

find the size of the quarterly payments

R ← Find

$$A_n = 200,000 - 50,000 = 150,000$$

$$i = \frac{r}{m} = \frac{0.12}{4} = 0.03$$

$$n = m \cdot t = 4 \cdot 10 = 40$$

$$R = 150,000 \left[\frac{0.03}{1 - (1.03)^{-40}} \right] \approx \$6,489.36$$

find the total amount of the payments

$$40 \cdot R = \$259,574.27$$

find the total amount of interest paid

$$I_{\text{int cost}} = \$259,574.27 - \$150,000 = \$109,574.27$$

Example. For the following, calculate the regular payments

\$100,000 for 3 years at 9% compounded annually

R ← Find

$$A_n = 100,000$$

$$i = \frac{r}{m} = \frac{0.09}{1} = 0.09$$

$$n = m \cdot t = 1 \cdot 3 = 3$$

$$R = \frac{100,000}{\left[\frac{0.09}{1 - (1.09)^{-3}} \right]} \approx \$39,505.48$$

\$30,000 for 5 years at 7% compounded annually

R ← Find

$$A_n = 30,000$$

$$i = \frac{r}{m} = \frac{0.07}{1} = 0.07$$

$$n = m \cdot t = 1 \cdot 5 = 5$$

$$R = \frac{30,000}{\left[\frac{0.07}{1 - (1.07)^{-5}} \right]} \approx \$7,316.72$$

\$20,000 for 1 year at 9% compounded quarterly

R ← Find

$$A_n = 20,000$$

$$i = \frac{r}{m} = \frac{0.09}{4} = 0.0225$$

$$n = m \cdot t = 4 \cdot 1 = 4$$

$$R = 20,000 \left[\frac{\frac{0.09}{4}}{1 - (1 + \frac{0.09}{4})^{-4}} \right] \approx \$5284.38$$

\$50,000 for $2\frac{1}{2}$ years at 9% compounded semiannually

R ← Find

$$A_n = 20,000$$

$$i = \frac{r}{m} = \frac{0.09}{2} = 0.045$$

$$n = m \cdot t = 2 \cdot 2.5 = 5$$

$$R = 20,000 \left[\frac{0.045}{1 - (1.045)^{-5}} \right] \approx \$11,389.58$$

Example. Chuckie and Angelica have \$30,000 for a down payment, and their budget can accommodate a monthly mortgage of \$1,200. What is the most expensive home they can buy if they borrow money for 30 years at 7.8% compounded monthly?

$$A_n + 30,000$$

$$R = 1200$$

$$A_n = \leftarrow \text{Find}$$

$$i = \frac{r}{m} = \frac{0.078}{12} = 0.0065$$

$$n = m \cdot t = 12 \cdot 30 = 360$$

$$1200 = A_n \left[\frac{0.0065}{1 - (1.0065)^{-360}} \right]$$

≈ 0.0071987

$$\frac{1200}{0.0071987} = A_n = \$166,696.65$$

$$A_n + \text{Down payment} = \$196,696.65$$

Use the exact value!
Don't round!!

Example. A loan of \$10,000 is to be amortized with 10 equal quarterly payments. If the interest rate is 6% compounded quarterly, what is the periodic payment?

$$R = \leftarrow \text{Find}$$

$$A_n = 10,000$$

$$i = \frac{r}{m} = \frac{0.06}{4} = 0.015$$

$$n = m \cdot t = 4 \cdot 10 = 40$$

$$R = 10,000 \left[\frac{0.015}{1 - (1.015)^{-40}} \right]$$

$= \$334.27$