

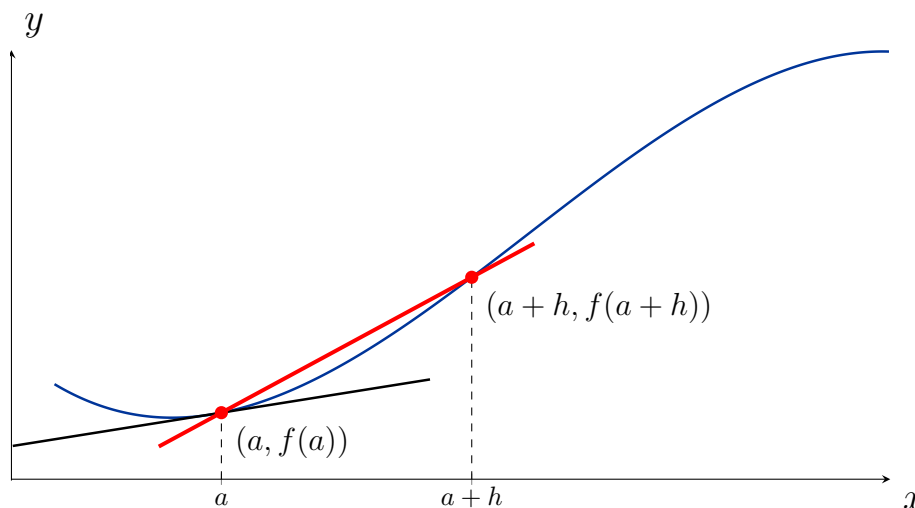
## 2.6: The Derivative

### Definition.

Given a function  $f(x)$ :

- the **secant line** is the line that passes through two *distinct* points lying on the graph of  $f(x)$ ,
- the **tangent line** is the line that intersects  $f(x)$  in exactly one place (locally) and matches the slope of the graph at that point.

[Graph](#)



### Definition. (Slope of a Tangent Line)

The slope of the tangent line to the graph of  $f$  at the point  $P(x, f(x))$  is given by

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

if it exists.

**Definition. (Average and Instantaneous Rates of Change)**

The **average rate of change** of  $f$  over the interval  $[x, x+h]$  or **slope of the secant line** to the graph of  $f$  through the points  $(x, f(x))$  and  $(x+h, f(x+h))$  is

$$\frac{f(x+h) - f(x)}{h}$$

The above fraction is referred to as the **difference quotient**.

The **instantaneous rate of change** of  $f$  at  $x$  or **slope of the tangent line** to the graph of  $f$  at  $(x, f(x))$  is

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

**Definition. (Derivative of a Function)**

The derivative of a function  $f$  with respect to  $x$  is the function  $f'$  (read “ $f$  prime”),

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

The domain of  $f'$  is the set of all  $x$  for which the limit exists.

Some other notations for the derivative are

$$D_x f(x) \qquad \frac{dy}{dx} \qquad y'$$

**Example.** Find the slope of the line tangent to the graph  $f(x) = 3x + 5$  at any point  $(x, f(x))$

**Example.** Let  $f(x) = x^2$ .

- Find  $f'(x)$ .
- Compute  $f'(2)$  and interpret your result.

**Example.** Let  $f(x) = x^2 - 4x$ . Find the point on the graph where the tangent line is horizontal.

**Example.** Let  $f(x) = \frac{1}{x}$ . Find the equation of the tangent line at  $x = 1$ .

## Differentiability and Continuity

If a function is differentiable at  $x = a$ , then it is continuous at  $x = a$ .

**Example.** For the graph below, identify each point where the derivative is undefined.

