

6.1: Antiderivatives and the Rules of Integration

Definition. (Antiderivatives)

A function F is an **antiderivative** of f on an interval I if $F'(x) = f(x)$ for all x in I .

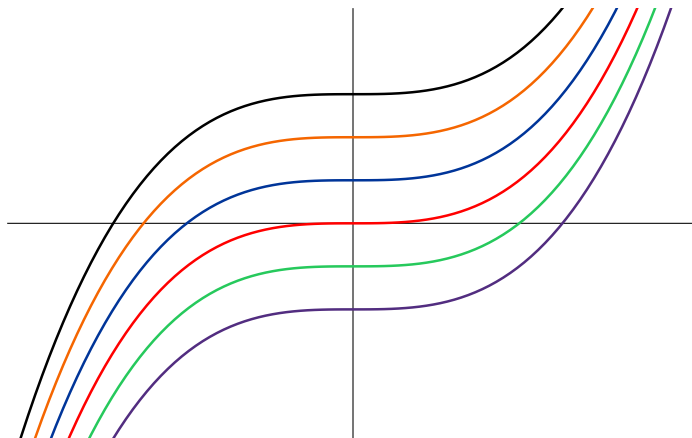
Example. Show that $F(x) = \frac{1}{3}x^3 - 2x^2 + x - 1$ is an antiderivative of $f(x) = x^2 - 4x + 1$.

Example. Let $F(x) = x$, $G(x) = x + 2$, and $H(x) = x + C$, where C is a constant. Show that F , G , and H are all antiderivatives of $f(x) = 1$.

Theorem 1

Let G be an antiderivative of a function f on an interval I . Then, every antiderivative of F of f on I must be of the form $F(x) = G(x) + C$, where C is a constant.

Example. If $f'(x) = x^2$, then $f(x) = \frac{x^3}{3} + C$ is the family of antiderivatives of $f'(x)$.

**Definition. (Integration)**

The process of finding the antiderivative is called **integration**:

$$\int f(x) dx = F(x) + C$$

The **indefinite integral** of f is the family of functions given by $F(x) + C$ where $F'(x) = f(x)$. The function to be integrated, f , is called the **integrand**. C is the **constant of integration**.

Rule 1: The Indefinite Integral of a Constant

$$\int k \, dx = kx + C \quad (k, \text{ a constant})$$

Rule 2: The Power Rule

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1} + C \quad (n \neq -1)$$

Example. Find each of the following indefinite integrals

$$\int 2 \, dx$$

$$\int \pi^2 \, dx$$

$$\int x^3 \, dx$$

$$\int \frac{1}{x^{3/2}} \, dx$$

Rule 3: The Indefinite Integral of a Constant Multiple of a Function

$$\int cf(x) dx = c \int f(x) dx \quad (c, \text{ a constant})$$

Rule 4: The Sum Rule

$$\int [f(x) \pm g(x)] dx = \int f(x) dx \pm \int g(x) dx$$

Example. Find each of the following indefinite integrals

$$\int \frac{1}{5} - \frac{2}{t^3} + 2t dt$$

$$\int 3x^5 + 4x^{3/2} - 2x^{-1/2} dx$$

Rule 5: The Indefinite Integral of the Exponential Function

$$\int e^x dx = e^x + C$$

Rule 6: The Indefinite Integral of the Function $f(x) = x^{-1}$

$$\int x^{-1} dx = \int \frac{1}{x} dx = \ln|x| + c \quad (x \neq 0)$$

Example. Find each of the following indefinite integrals

$$\int 2e^x - x^3 + x^e - e^e dx$$

$$\int 2x + \frac{3}{x} + \frac{4}{x^2} dx$$

$$\int \frac{2}{\sqrt{x}} - \frac{2}{x} dx$$

$$\int \frac{1}{4e^x} - \frac{4}{x} + e^x dx$$

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