6.4: Present Values of Annuities

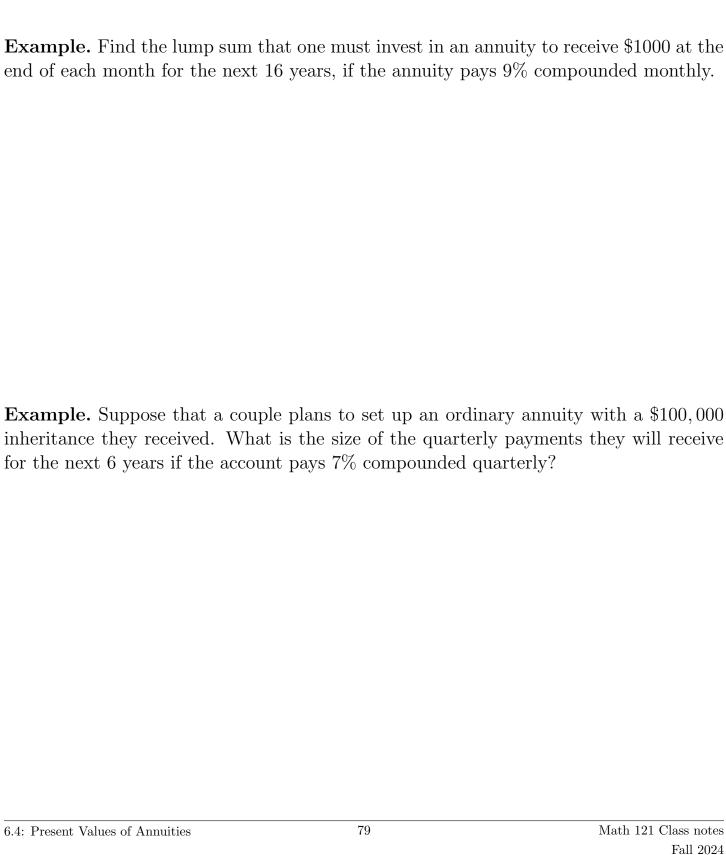
Example. Suppose we wish to invest a lump sum of money, A_n , into an annuity that earns interest at a rate of 10% per year, so that we may receive payments of \$100 for 5 years. What is the amount of the lump sum?

Definition.

If a payment of R is to be withdrawn at the *end of each period* for n periods from an account that earns interest at a rate of i per period, then the account is an **ordinary annuity** and the **present value** is

$$A_n = R \cdot a_{\overline{n}|i} = R \left\lceil \frac{1 - (1+i)^{-n}}{i} \right\rceil$$

The notation $a_{\overline{n}|i}$ represents the present value of an ordinary annuity of \$1 per period for n periods with an interest rate of i per period.





Definition.

If a payment of R is to be withdrawn at the *beginning of each period* for n periods from an account that earns interest at a rate of i per period, then the account is an **annuity due** and the **present value** is

$$A_{(n,\text{due})} = R \cdot a_{\overline{n}|i}(1+i) = R \left[\frac{1 - (1+i)^{-n}}{i} \right] (1+i)$$

The notation $a_{\overline{n}|i}$ represents the present value of an ordinary annuity of \$1 per period for n periods with an interest rate of i per period.

Example. Suppose that a court settlement results in a \$750,000 award. If this is invested at 9% compounded semiannually, how much will it provide at the *beginning* of each half-year for a period of 7 years?

Fall 2024

Comparing annuity calculations:

	Ordinary annuities	Annuity due	
Future Value	S	$S_{ m due}$	Regular payments Total $after n$ periods
Present Value	A_n	$A_{(n,\mathrm{due})}$	Regular withdrawals Total $before \ n$ periods
	end of each period	beginning of each period	