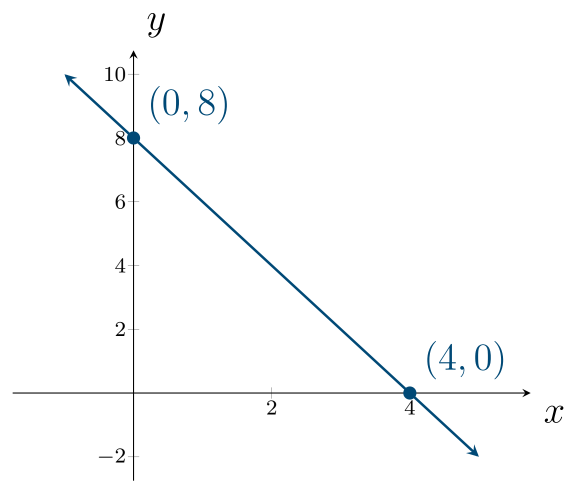
1.3: Linear Functions

Definition.
A **linear function** is a function of the form

$$y = f(x) = mx + b$$

where m and b are constants.

Example. $y = -2x + 8$



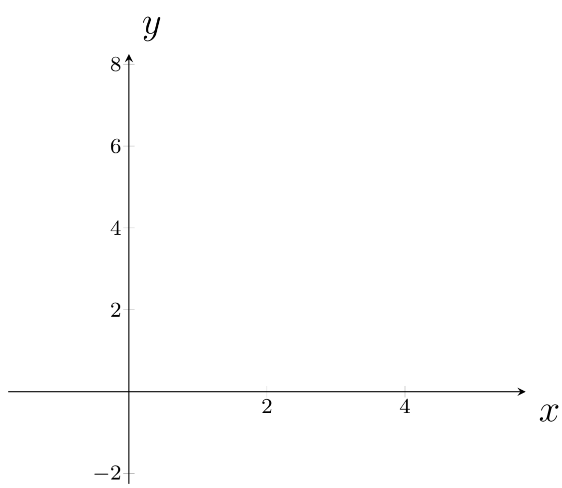
A linear function can be uniquely determined using only *two* distinct points.

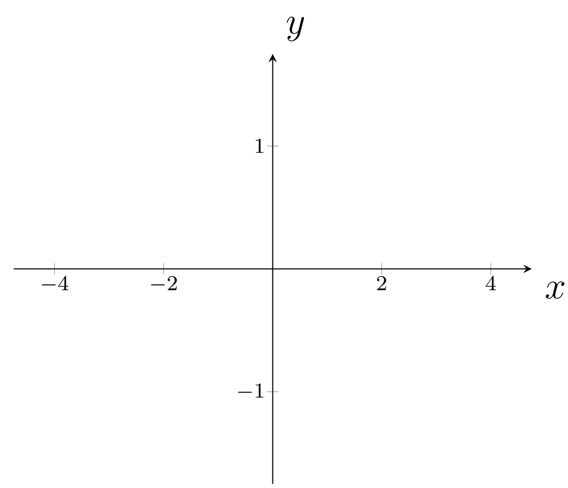
Definition.
The point(s) where a graph intersects the axes are called intercepts. The x-coordinate of the point where the function intersects the x-axis is called the **x-intercepts**. The y-coordinate of the point where the function intersects the y-axis is called the **y-intercepts**.

- To solve for the y-intercept:
  - Set $x = 0$,
  - Solve for y.
- To solve for the x-intercept:
  - Set $y = 0$,
  - Solve for x.

Example. Find the intercepts and graph the following lines:

$3x + 2y = 12$ $\qquad\qquad$ $x = 4y$





Definition.
If a nonvertical line passes through the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$, its **slope**, denoted by m, is found using

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$

Δy is "delta y", and represents the change in y
Δx is "delta x", and represents the change in x

Note: The slope of a vertical line is undefined.

Example. Find the slope of the line passing through the points $(-2, 1)$ and $(5, 3)$.

Note:

- Two distinct nonvertical lines are *parallel* if and only if their slopes are *equal*.
- Two distinct nonvertical lines are *perpendicular* if and only if their slopes are *negative reciprocals*:
  e.g. If ℓ_1 has a nonzero slope m, then ℓ_2 is perpendicular if its slope is $-1/m$.

Point-slope form

Definition.
The equation of the line passing through the point (x_1, y_1) with slope m can be written in the point-slope form:

$$y - y_1 = m(x - x_1)$$

Example. Find the equation of each line that passes through the point $(-3, 4)$ and has

a slope of $m = \dfrac{1}{4}$

the point $(-2, 1)$ on the line

a slope of zero (horizontal)

an undefined slope (vertical)

Slope-intercept form

Definition.
The slope-intercept form of the equation of a line with slope m and y-intercept b is

$$y = mx + b$$

Example (Example 7, p.82)**.** The population of U.S. males, y (in thousands), projected from 2015 to 2060 can be modeled by

$$y = 1125.9x + 142,960$$

where x is the number of years after 2000.

- Find the slope and y-intercept of the graph of this function.

- What does the y-intercept tell us about the population of U.S. males?

- Interpret the slope as a rate of change.

Example. Each day, a young person should sleep 8 hours plus $\frac{1}{4}$ hour for each year the person is under 18 years of age. Assuming that the relation is linear, write the equation relating hours of sleep y and age x

Forms of Linear Equations

$$\begin{aligned} \text{General form: } & Ax + By = C \\ \text{Point-slope form: } & y - y_1 = m(x - x_1) \\ \text{Slope-intercept form: } & y = mx + b \\ \text{Vertical line: } & x = a \\ \text{Horizontal line: } & y = b \end{aligned}$$

1.4: Graphs and Graphing Utilities

As graphing calculators are *not* required for this course, we will use Desmos:

desmos.com/calculator

Example. For a certain city, the cost C of obtaining drinking water with p percent impurities (by volume) is given by

$$C = \frac{120,000}{p} - 1200$$

The equation for C requires that $p \neq 0$, and because p is the percent impurities, we know $0 < p \leq 100$. Use the restriction on p and a graphing calculator to obtain an accurate graph of the equation.