

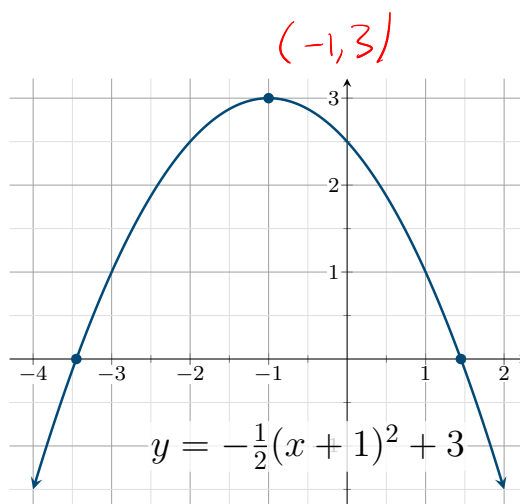
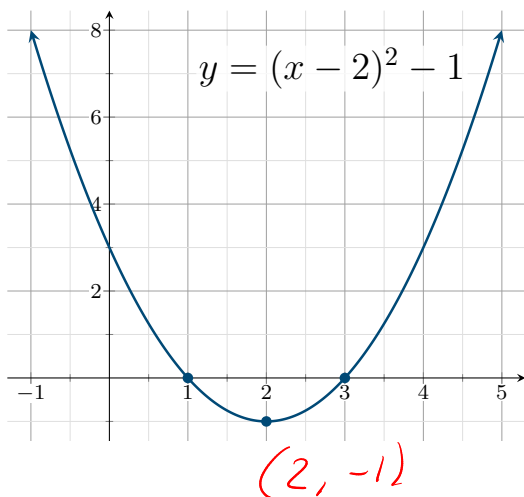
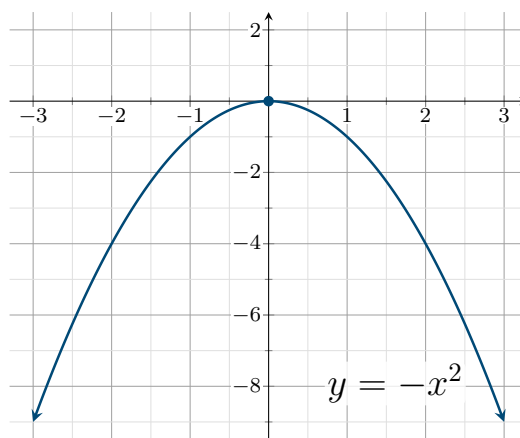
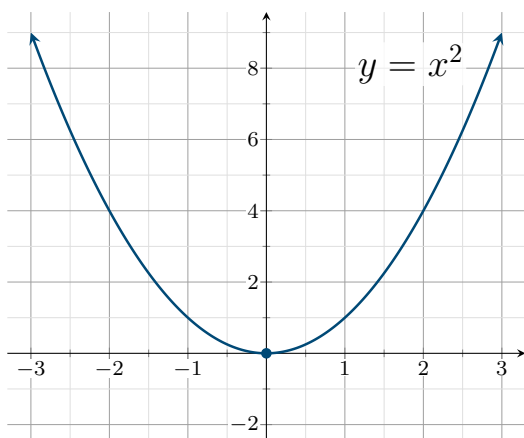
## 2.2: Quadratic Functions: Parabolas

### Definition.

A **quadratic function** has the form

$$y = f(x) = ax^2 + bx + c \quad (a \neq 0)$$

where  $a$ ,  $b$ , and  $c$  represent constants. A **parabola** is the shape of the graph of a quadratic function.



**Definition.**

The quadratic function  $y = f(x) = ax^2 + bx + c$  has its **vertex** at

$$\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right).$$

The optimal value occurs at the vertex of a parabola:

- A maximum if  $a < 0$  ↩
- A minimum if  $a > 0$  ↩

**Example.** Consider the function

$$2x + \frac{x^2}{2}$$

Is the vertex a maximum or minimum? Locate the vertex,  $x$ -intercepts,  $y$ -intercept, and then sketch the graph.

$$a = \frac{1}{2} > 0 \Rightarrow \text{Vertex is a minimum}$$

$$\text{Vertex: } \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right) = \left(-\frac{2}{2(1/2)}, f\left(-\frac{2}{2(1/2)}\right)\right) = (-2, -2)$$

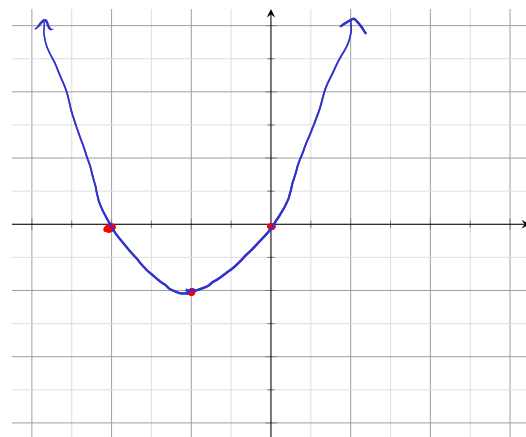
$x$ -intercepts ( $y=0$ )

$$0 = 2x + \frac{x^2}{2} = \frac{1}{2}x(4+x)$$

$$\begin{array}{c} \downarrow \quad \quad \downarrow \\ x=0 \quad \quad x=-4 \\ \boxed{(0,0)} \quad \boxed{(-4,0)} \end{array}$$

$y$ -intercept ( $x=0$ )

$$y = 2(0) + \frac{0^2}{2} = 0 \Rightarrow \boxed{(0,0)}$$



**Example.** Consider the function

$$x^2 + 5 - 4x$$

Is the vertex a maximum or minimum? Locate the vertex,  $x$ -intercepts,  $y$ -intercept, and then sketch the graph.

$$a = 1 > 0 \Rightarrow \text{Vertex is a minimum}$$

$$\text{Vertex: } \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right) = \left(-\frac{-4}{2(1)}, f\left(-\frac{-4}{2(1)}\right)\right) = (2, 1)$$

$x$ -intercepts ( $y=0$ )

$$0 = \underbrace{x^2}_{a} - \underbrace{4x}_{b} + \underbrace{5}_{c}$$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)}$$

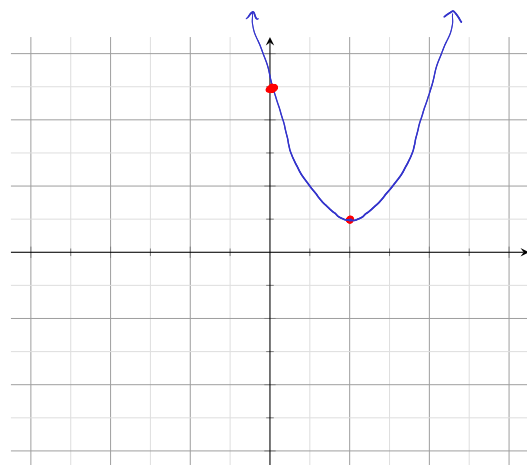
Determinant:

$$(-4)^2 - 4(1)(5) = -4 < 0 \Rightarrow \text{No solutions!}$$

$\Rightarrow$  No  $x$ -intercepts

$y$ -intercept ( $x=0$ )

$$y = 0^2 - 4(0) + 5 = 5 \rightarrow (0, 5)$$



**Example.** Ace Cruises offers a sunset cruise to a group of 50 people for a price of \$30 per person, but it reduces the price per person by \$0.50 for each additional person above 50. Find the revenue function. What price maximizes the revenue? What is this maximal value?

Number of people	Price per person	Total Revenue
50	\$ 30.00	\$ 1500.00
51	\$ 29.50	\$ 1504.50
52	\$ 29.00	\$ 1508.00
⋮		
50+x	30-0.5x	$R(x) = (50+x)(30-0.5x)$

$$R(x) = (50+x)(30-0.5x)$$

$$= \underbrace{1500}_c + \underbrace{5x}_b - \underbrace{0.5x^2}_a$$

$a < 0 \Rightarrow$  Vertex is a maximum.

$$\text{Vertex } \left( -\frac{b}{2a}, R\left(-\frac{b}{2a}\right) \right)$$

$$-\frac{b}{2a} = -\frac{5}{2(-0.5)} = 5$$

$$R(5) = 1500 + 5(5) - 0.5(5)^2$$

$$= 1500 + 25 - \frac{25}{2}$$

$$= \boxed{\$1512.50}$$