9.4: Hypothesis Testing for Means

Similar to hypothesis testing for proportions, we have the following four steps:

1. **Hypothesize:** formulate your hypotheses
2. **Check conditions:**
   - **Random and Independent:** The sample must be randomly collected from the population, and observations are independent of each other.
   - **Large Sample:** Either the population is Normal, *or* the sample size is large ($n \geq 25$).
   - **Large Population:** If the sample is collected without replacement, the population of size N must be at least 10 times bigger than the sample: $N \geq 10n$

   If these conditions are met, we compute the test statistic for the One-Sample t-Test which follows a t-distribution with $n-1$ degrees of freedom:

   $$t = \frac{\overline{x} - \mu_0}{SE_{\text{est}}}, \qquad \text{where} \qquad SE_{\text{est}} = \frac{s}{\sqrt{n}}$$
3. **Compute:** Stating a significance level, compute the observed test statistic t and/or p-value.
4. **Interpret:** Decide whether to reject or fail to reject the null hypothesis.

Two-Sided	One-Sided (Left)	One-Sided (Right)
$H_0 : \mu = \mu_0$	$H_0 : \mu = \mu_0$	$H_0 : \mu = \mu_0$
$H_a : \mu \neq \mu_0$	$H_a : \mu < \mu_0$	$H_a : \mu > \mu_0$

Example. McDonald's advertises that its ice cream cones have a mean weight of 3.2 ounces. To test this, we find the weights of a sample of 5 cones:

$$4.2, 3.6, 3.9, 3.4, 3.3$$

Formulate the null and alternative hypotheses

$H_0 : \mu = 3.2$
$H_A : \mu \neq 3.2$

Check the conditions required to perform a hypothesis test.

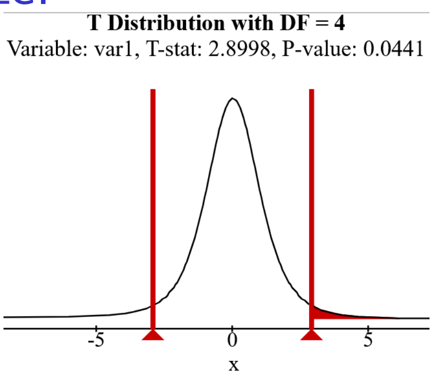
1. The sample is random and independent
2. It's safe to assume that the distribution of cone weights is Norma
3. Since there are more than 50 cones, the population is at least 10 times larger than the sample size.

Find the test statistic and p-value

One sample T hypothesis test:
μ : Mean of variable
$H_0 : \mu = 3.2$
$H_A : \mu \neq 3.2$

Hypothesis test results:

Variable	Sample Mean	Std. Err.	DF	T-Stat	Critical t	P-value
var1	3.68	0.16552945	4	2.899786	2.776445	0.0441



Using a significance level of $\alpha = 0.05$, decide whether to reject or fail to reject the null hypothesis.

Since the p-value=0.0441 is less than our significance level, we reject the n hypothesis, and assert that the true mean weight is significanly different fro 3.2 ounces.

Similarly, since the t-statistic is greater than our critical t (in absolute value we reject the null hypothesis.

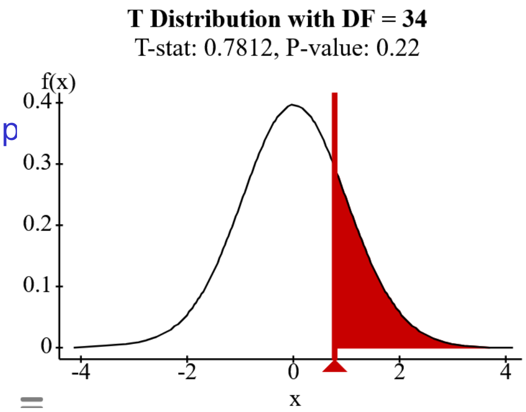
Example. In the 2011-2012 academic year, the mean cost of attending two-year colleges in the United States was $\$3,831$. Has this increase over time? A random sample of 35 two-year colleges in 2014-2015 had a mean tuition of $\$4,173$, with a standard deviation of $\$2,590$.

Formulate the null and alternative hypotheses

$$H_0 : \mu = 3831$$
$$H_A : \mu > 3831$$

Check the conditions required to perform a hypothesis test.

1. The sample is random and independent
2. Our sample size is larger enough: n=35
3. Since there are more than 350 two-year colleges,
   the population is at least 10 times larger than the samp



Find the test statistic and p-value

One sample T summary hypothesis test:
μ : Mean of population
$H_0 : \mu = 3831$
$H_A : \mu > 3831$

Hypothesis test results:

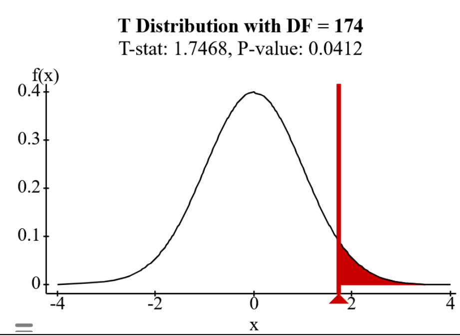
Mean	Sample Mean	Std. Err.	DF	T-Stat	Critical t	P-value
μ	4173	437.7899	34	0.78119664	1.6909243	0.22

Using a significance level of $\alpha = 0.05$, decide whether to reject or fail to reject the null hypothesis.

Since the p-value=0.22 is greater than our significance level, we fail to reject the null hypothesis, m
there is insufficient evidence to suggest the true mean tuition cost is significantly greater than 3831

Similarly, since the t-statistic is less than our critical t (in absolute value), we fail to reject the null hy

Repeat this hypothesis test with a sample size of $n = 175$. What happens to the standard error when the sample size increases?



One sample T summary hypothesis test:
μ : Mean of population
$H_0 : \mu = 3831$
$H_A : \mu > 3831$

Hypothesis test results:

Mean	Sample Mean	Std. Err.	DF	T-Stat	Critical t	P-value
μ	4173	195.7856	174	1.7468088	1.653658	0.0412

When the sample size increases, the standard error decreases.
Since the p-value is less than the significance level, we would reject the null hypothesis.