

4.5: Optimization II

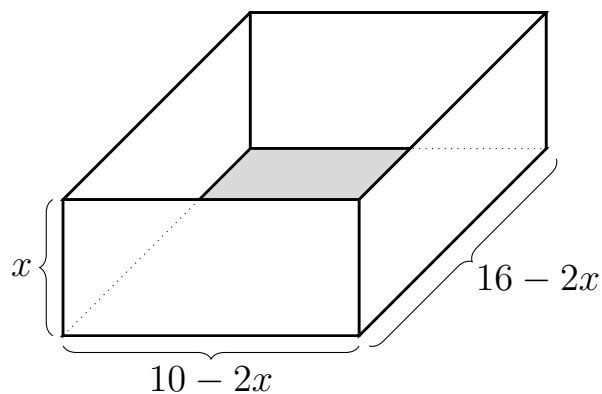
Guidelines for Solving Optimization Problems

1. Assign a letter to each variable mentioned in the problem. If appropriate, draw and label a figure.
2. Find an expression for the quantity to be optimized.
3. Use the conditions given in the problem to write the quantity to be optimized as a function f of *one* variable. Note any restrictions to be placed on the domain of f from physical considerations of the problem.
4. Optimize the function f over its domain.

Example. Show among all rectangles with an 8 meter perimeter, the one with the *largest* area is a square.

Example. A man wishes to have a rectangular-shaped garden in his backyard. He has 50 feet of fencing with which to enclose his garden. Find the dimensions for the largest garden he can have if he uses all of his fencing.

Example. By cutting away identical squares from each corner of a rectangular piece of cardboard and folding up the resulting flaps, the cardboard may be turned into an open box. If the cardboard is 16" long and 10" wide, find the dimensions of the box that will yield the maximum volume.



Example. A cylindrical can is to be made to hold 1 L (1000 cm^3) of oil. Find the dimensions of the can that will minimize the cost of the metal to manufacture the can.

Example. Of all boxes with a square base and a volume of 8 m^3 , which one has the minimum surface area?

Example. Find the point on the line $y = 2x + 3$ that is closest to the origin.

Example. A pencil cup with a capacity of 36 in^3 is to be constructed in the shape of a rectangular box with a square base and an open top. If the material for the sides cost $\$0.15/\text{in}^2$ and the material for the base costs $\$0.40/\text{in}^2$, what should the dimensions of the cup be to minimize the construction cost?