6.4: Present Values of Annuities

Example. Suppose we wish to invest a lump sum of money, A_n, into an annuity that earns interest at a rate of 10% per year, so that we may receive payments of \$100 for 5 years. What is the amount of the lump sum?

Definition.
If a payment of $\$R$ is to be withdrawn at the *end of each period* for n periods from an account that earns interest at a rate of i per period, then the account is an **ordinary annuity** and the **present value** is

$$A_n = R \cdot a_{\overline{n}|i} = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

The notation $a_{\overline{n}|i}$ represents the present value of an ordinary annuity of \$1 per period for n periods with an interest rate of i per period.

Example. Find the lump sum that one must invest in an annuity to receive \$1000 at the end of each month for the next 16 years, if the annuity pays 9% compounded monthly.

Example. Suppose that a couple plans to set up an ordinary annuity with a $\$100,000$ inheritance they received. What is the size of the quarterly payments they will receive for the next 6 years if the account pays 7% compounded quarterly?

Example. An inheritance of $\$250,000$ is invested at 9% compounded monthly. If \$2500 is withdrawn at the end of each month, how long will it be until the account balance is \$0?

Definition.
If a payment of $\$R$ is to be withdrawn at the *beginning of each period* for n periods from an account that earns interest at a rate of i per period, then the account is an **annuity due** and the **present value** is

$$A_{(n,\text{due})} = R \cdot a_{\overline{n}|i}(1+i) = R\left[\frac{1-(1+i)^{-n}}{i}\right](1+i)$$

The notation $a_{\overline{n}|i}$ represents the present value of an ordinary annuity of \$1 per period for n periods with an interest rate of i per period.

Example. Suppose that a court settlement results in a $\$750,000$ award. If this is invested at 9% compounded semiannually, how much will it provide at the *beginning* of each half-year for a period of 7 years?

Comparing annuity calculations:

	Ordinary annuities	Annuity due	
Future Value	S	S_{due}	Regular payments Total *after* n periods
Present Value	A_n	$A_{(n,\text{due})}$	Regular withdrawals Total *before* n periods
	end of each period	beginning of each period	