

1.3: Linear Functions

Definition.

A **linear function** is a function of the form

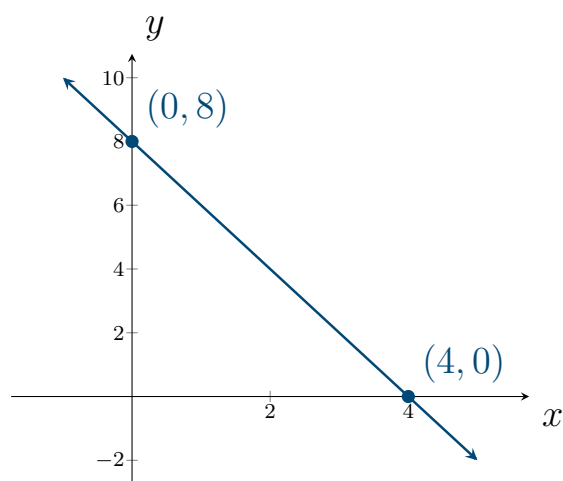
$$y = f(x) = mx + b$$

where m and b are constants.

Example. $y = -2x + 8$

$$\begin{aligned} x=0 &\Rightarrow y = -2(0) + 8 \\ &= 8 \quad \rightarrow \quad \boxed{(0, 8)} \end{aligned}$$

$$\begin{aligned} y=0 &\Rightarrow -8 + 0 = -2x + 8 - 8 \\ &\quad \underline{-8} \quad \underline{-2x} \\ &\quad \underline{-2} \quad \underline{-2} \\ &4 = x \quad \rightarrow \quad \boxed{(4, 0)} \end{aligned}$$



A linear function can be uniquely determined using only *two* distinct points.

Definition.

The point(s) where a graph intersects the axes are called intercepts. The x -coordinate of the point where the function intersects the x -axis is called the **x -intercepts**. The y -coordinate of the point where the function intersects the y -axis is called the **y -intercepts**.

- To solve for the y -intercept:
 - Set $x = 0$,
 - Solve for y .
- To solve for the x -intercept:
 - Set $y = 0$,
 - Solve for x .

Example. Find the intercepts and graph the following lines:

$$3x + 2y = 12$$

$$x = 4y$$

x-intercept:

set $y=0$, find x .

$$3x + 2(0) = 12$$

$$3x = 12$$

$$\frac{3}{3}x = \frac{12}{3}$$

$$x = 4 \rightarrow (4, 0)$$

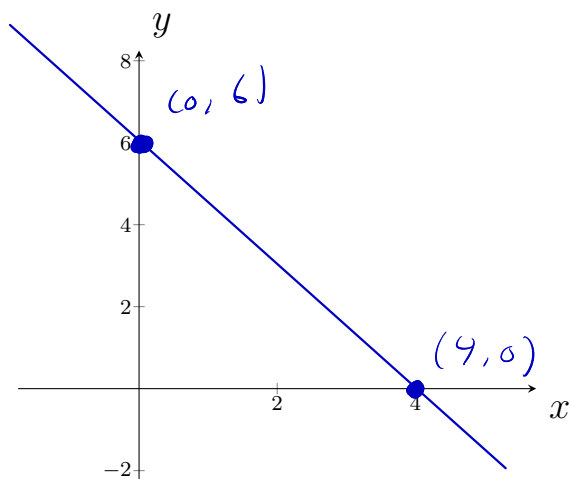
y-intercept:

set $x=0$, find y .

$$3(0) + 2y = 12$$

$$2y = 12$$

$$y = 6 \rightarrow (0, 6)$$



x-intercept:

set $y=0$, find x .

$$x = 4(0)$$

$$x = 0 \rightarrow (0, 0)$$

y-intercept:

set $x=0$, find y .

$$0 = 4y$$

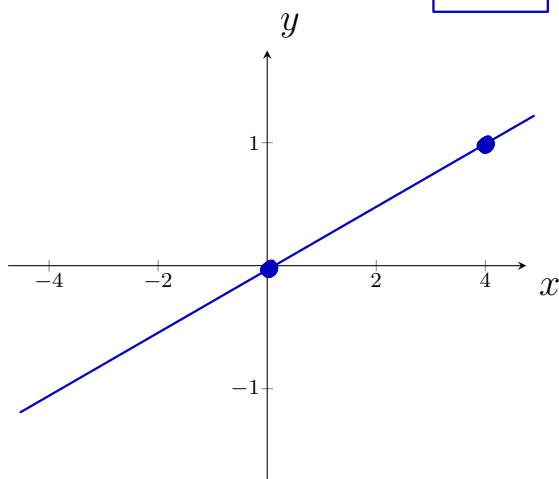
$$y = 0 \rightarrow (0, 0)$$

To graph this, choose another point:

$$y = 1 \rightarrow$$

$$x = 4(1)$$

$$x = 4 \rightarrow (4, 1)$$



Definition.

If a nonvertical line passes through the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$, its **slope**, denoted by m , is found using

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$

Δy is “delta y ”, and represents the change in y

Δx is “delta x ”, and represents the change in x

Note: The slope of a vertical line is undefined.

Example. Find the slope of the line passing through the points $(-2, 1)$ and $(5, 3)$.

(x_1, y_1) (x_2, y_2)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 1}{5 - (-2)} = \boxed{\frac{2}{7}}$$

Note:

- Two distinct nonvertical lines are *parallel* if and only if their slopes are *equal*.
- Two distinct nonvertical lines are *perpendicular* if and only if their slopes are *negative reciprocals*:
e.g. If ℓ_1 has a nonzero slope m , then ℓ_2 is perpendicular if its slope is $-1/m$.

Point-slope form

Definition.

The equation of the line passing through the point (x_1, y_1) with slope m can be written in the point-slope form:

$$y - y_1 = m(x - x_1)$$

Example. Find the equation of each line that passes through the point $(-3, 4)$ and has

a slope of $m = \frac{1}{4}$

$$y - 4 = \frac{1}{4}(x - (-3))$$
$$+4 + y - 4 = \frac{x}{4} + \frac{3}{4} +4$$

$$y = \frac{x}{4} + \frac{3}{4} + 4\left(\frac{4}{4}\right)$$

$$y = \frac{x}{4} + \frac{3}{4} + \frac{16}{4} \Rightarrow y = \frac{x}{4} + \frac{19}{4}$$

a slope of zero (horizontal)

$$y - 4 = 0(x - (-3))$$
$$y = 4$$

the point $(-2, 1)$ on the line

$$m = \frac{4 - 1}{-3 - (-2)} = \frac{3}{-1} = -3$$

$$y - 4 = -3(x - (-3))$$
$$y = -3x - 9 + 4$$

$$y = -3x - 5$$

$$y - 1 = -3(x - (-2))$$
$$y = -3x - 6 + 1$$

$$y = -3x - 5$$

an undefined slope (vertical)

$$x = -3$$

Slope-intercept form

Definition.

The slope-intercept form of the equation of a line with slope m and y -intercept b is

$$y = mx + b$$

Example (Example 7, p.82). The population of U.S. males, y (in thousands), projected from 2015 to 2060 can be modeled by

$$y = 1125.9x + 142,960$$

where x is the number of years after 2000.

- Find the slope and y -intercept of the graph of this function.

Set $x=0 \rightarrow y = 1125.9(0) + 142,960$

$$y = 142,960$$

- What does the y -intercept tell us about the population of U.S. males?

Zero years after 2015, the population of U.S. males is 142,960.

- Interpret the slope as a rate of change.

Each year, the population of U.S. males is expected to increase by 1125.9.

Example. Each day, a young person should sleep 8 hours plus $\frac{1}{4}$ hour for each year the person is under 18 years of age. Assuming that the relation is linear, write the equation relating hours of sleep y and age x

x	y
18	8
17	$8 + \frac{1}{4}$
16	$8 + 2(\frac{1}{4})$
15	$8 + 3(\frac{1}{4})$
14	9

\Rightarrow Use point-slope formula

$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{8 - 9}{18 - 14} = \boxed{-\frac{1}{4}}$$

$$y - y_1 = m(x - x_1)$$

$$y - 8 = -\frac{1}{4}(x - 18)$$

$$y = 8 - \frac{x}{4} + \frac{18}{2}$$

$$y = \frac{25}{2} - \frac{x}{4}$$

Forms of Linear Equations

General form: $Ax + By = C$

Point-slope form: $y - y_1 = m(x - x_1)$

Slope-intercept form: $y = mx + b$

Vertical line: $x = a$

Horizontal line: $y = b$