3.1: Basic Rules of Differentiation

Rule 1: Derivative of a Constant

$$\frac{d}{dx}[c] = 0$$

Rule 2: The Power Rule

If n is any real number, then

$$\frac{d}{dx}[x^n] = nx^{n-1}$$

Example. Find the derivative of the following functions

$f(x) = x$

$f'(x) = 1$

$g(x) = x^8$

$g'(x) = 8x^{8-1} = 8x^7$

$h(x) = x^{\frac{5}{2}}$

$h'(x) = \frac{5}{2}x^{5/2-1} = \frac{5}{2}x^{3/2}$

$j(x) = \sqrt{x} = x^{1/2}$

$j'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

$k(x) = \frac{1}{\sqrt[3]{x}} = x^{-1/3}$

$k'(x) = -\frac{1}{3}x^{-4/3} = \frac{-1}{3\sqrt[3]{x^4}}$

$\ell(x) = \pi^4$ ← Constant function

$\ell'(x) = 0$

Rule 3: Derivative of a Constant Multiple of a Function

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)]$$

Rule 4: The Sum Rule

$$\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$$

Example. Find the derivative of the following functions

$f(x) = 5x^3$

$f'(x) = 5 \cdot 3x^{3-1}$

$= 15x^2$

$g(x) = \frac{3}{\sqrt{x}} = 3x^{-1/2}$

$g'(x) = -\frac{3}{2}x^{-3/2} = \frac{-3}{2\sqrt{x^3}}$

$h(x) = 4x^5 + 3x^4 - 8x^2 + x + 3$

$h'(x) = 20x^4 + 12x^3 - 16x + 1$

$j(t) = \frac{t^2}{5} + \frac{5}{t^2} + \pi = \frac{1}{5}t^2 + 5t^{-2} + \pi$

$j'(t) = \frac{2}{5}t - 10t^{-3}$

$= \frac{2}{5}t - \frac{10}{t^3}$

Example. Find the line tangent to the curve

$$f(x) = 2x + \frac{1}{\sqrt{x}} = 2x + x^{-1/2}$$

at the point $(1, 3)$ [Graph]

$$f'(x) = 2 - \frac{1}{2}x^{-3/2} = 2 - \frac{1}{2\sqrt{x^3}}$$

Equation of a line:

$$y - y_1 = m(x - x_1) \longrightarrow y - f(1) = f'(1)(x - 1)$$

$f(1) = 3$

$f'(1) = \frac{3}{2}$

1

$$y - 3 = \frac{3}{2}(x - 1)$$

$$y = \frac{3}{2}x - \frac{3}{2} + 3$$

$$\boxed{y = \frac{3}{2}x + \frac{3}{2}}$$

Example. An experimental rocket lifts off vertically. Its altitude (in feet) t seconds into flight is given by

$$s = f(t) = -t^3 + 96t^2 + 5, \qquad (t \geq 0)$$

Find an expression v for the rocket's velocity at any time t.

$v = f'(t) = -3t^2 + 192t$

Compute the rocket's velocity when $t = 0, 30, 50, 64$, and 70. Interpret your results.

t	$v(t)$
0	0
30	3060
50	2100
64	0
70	−1260

At time t=50 seconds, the rocket is moving at a rate of 2100 ft/sec.

Using the results from above and the observation that at the highest point in its trajectory the rocket's velocity is zero, find the maximum altitude attained by the rocket.

$f(64) = 131{,}077$ ft.