1.6: Applications of Functions in Business and Economics

Definition.
Profit is the difference between the revenue and total cost:

$$P(x) = R(x) - C(x)$$

where

$$\begin{aligned} P(x) &= \text{profit from sale of } x \text{ units,} \\ R(x) &= \text{total revenue from sale of } x \text{ units,} \\ C(x) &= \text{total cost from production and sale of } x \text{ units.} \end{aligned}$$

Note: In general, the symbols used in economics are π, TR and TC respectively.

In general, **total revenue** is

$$\text{Revenue} = (\text{price per unit})(\text{number of units})$$

The **total cost** is composed of fixed cost and variable cost:

- **Fixed costs** (FC) remain constant regardless of the number of units produced.
- **Variable costs** (VC) are directly related to the number of units produced.

The total cost is given by

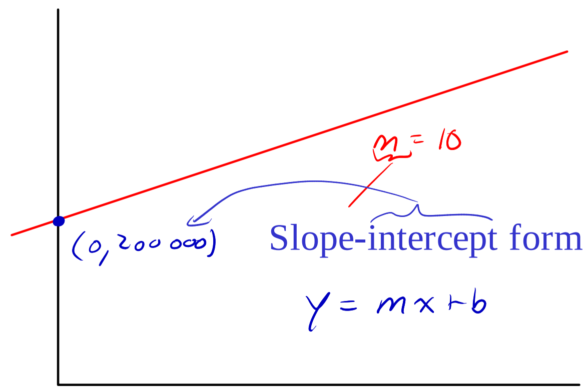
$$\text{Cost} = \text{variable costs} + \text{fixed costs}$$

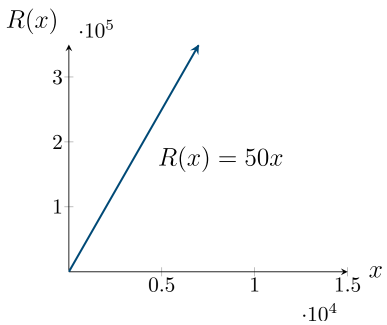
Example. Suppose a firm manufactures MP3 players and sells them for \$50 each. The costs incurred in the production and sale of the MP3 players are \$200,000 plus \$10 for each player produced and sold. Write the profit function for the production and sale of x players.

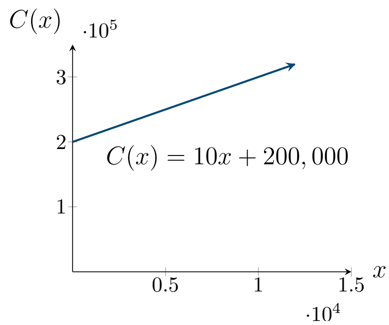
$R(x) = 50x$

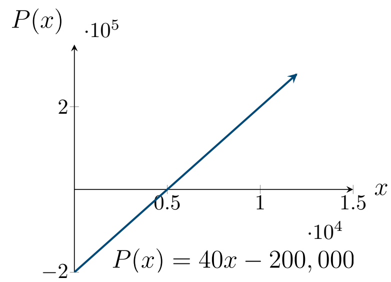
$C(x) = 10x + 200{,}000$

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= 50x - (10x + 200{,}000) \\ &= \boxed{40x - 200{,}000} \end{aligned}$$









Example. The ABC company produces widgets which sell at \$25 each. ABC can produce 30 widgets at a total cost of \$2,050, and 180 widgets at a total cost of \$4,300. Find the revenue, cost, and profit functions.

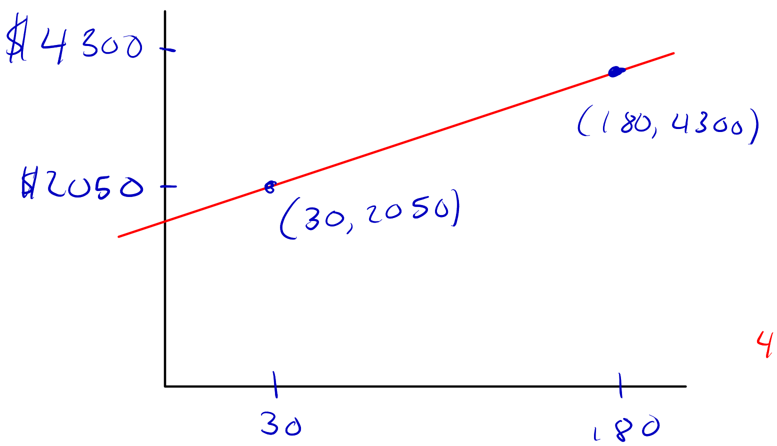
$R(x) = 25x$

$C(x) = 15x + 1600$

$P(x) = R(x) - C(x)$

$= 25x - (15x + 1600)$

$= \boxed{10x - 1600}$



$m = \dfrac{4300 - 2050}{180 - 30} = \dfrac{2250}{150} = 15$

$y - 4300 = 15(x - 180)$

$4300 + y - 4300 = 15x - 2700 + 4300$

$\boxed{y = 15x + 1600}$

Using two-points, compute the slope then use point-slope formula:

$y - y_1 = m(x - x_1)$

Definition. (Marginals)
The

- **marginal profit** ($\overline{MP}$) is the rate of change in profit...
- **marginal cost** ($\overline{MC}$) is the rate of change in costs...
- **marginal revenue** ($\overline{MR}$) is the rate of change in revenue...

with respect to the number of units produced and sold. When these functions are linear, the marginals are given by the slope of their respective function.

Example. A manufacturer sells widgets for \$10 per unit. The manufacturer's variable costs are \$2.50 per unit, and the total cost of 100 units is \$1,450.

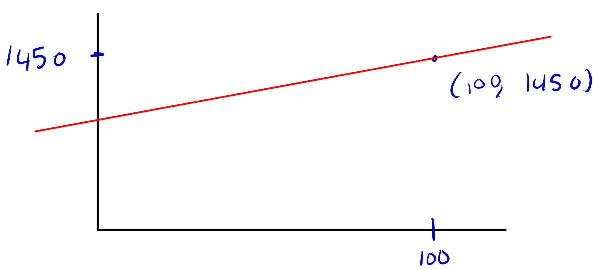
- Find the profit function. What are the marginal revenue, cost and profit?

$R(x) = 10x$

$C(x) = 2.50x + 1200$

$P(x) = R(x) - C(x)$

$= \boxed{7.5x - 1200}$



$y - y_1 = m(x - x_1)$

$y - 1450 = 2.5(x - 100)$

$+1450 + y - 1450 = 2.5x - 250 + 1450$

$\boxed{y = 2.5x + 1200}$

- Find the **break-even point** (where $R(x) = C(x)$ and $P(x) = 0$). What happens if we sell more or less than the break-even point?

Solve $R(x) = C(x)$ $\longrightarrow$ Equivalently $P(x) = 0$

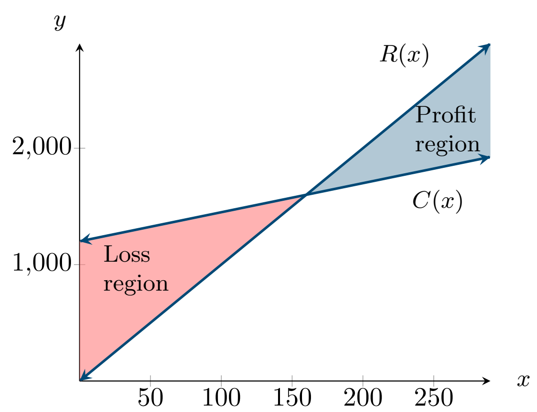
$10x = 2.5x + 1200$

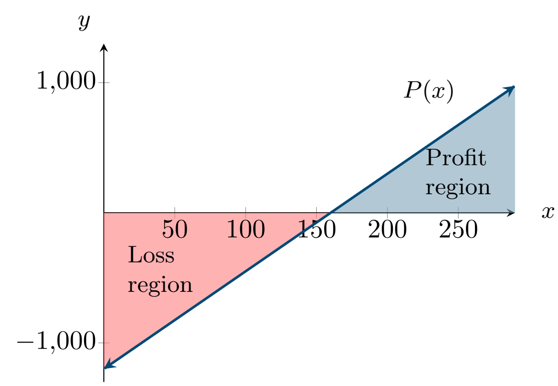
$7.5x = 1200$

$\boxed{x = 160}$

$x > 160 \Rightarrow P(x) > 0 \Rightarrow$ profit!

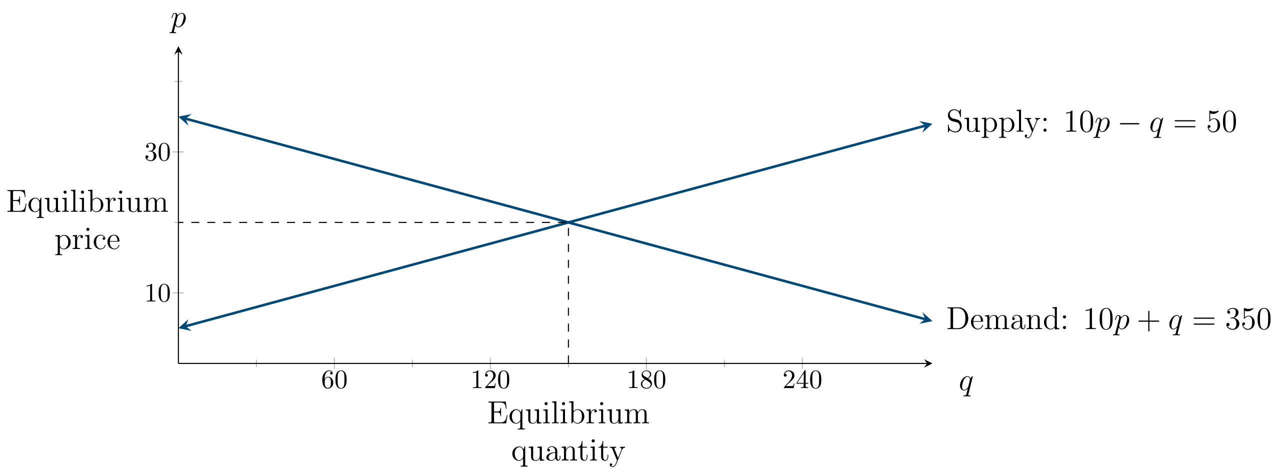
$x < 160 \Rightarrow P(x) < 0 \Rightarrow$ loss !!





> **Definition.**
>
> - **Market equilibrium** occurs when the quantity of a commodity demanded is equal to the quantity supplied.
> - The **law of demand** states that the quantity demanded will decrease as the price increases
> - The **law of supply** states that the quantity supplied will increase as the price increases

Example. Below is a graph containing a supply and demand curve. Find the market equilibrium.



Solve this as a system of equations via elimination method:

$$
\begin{array}{r}
10p - q = 50 \\
+\ 10p + q = 350 \\
\hline
20p \quad = 400
\end{array}
$$

$$\frac{20p}{20} = \frac{400}{20}$$

$$\boxed{p = 20}$$

$$10(20) - q = 50$$

$$200 - q = 50$$

$$\boxed{150 = q}$$

Example. Find the market equilibrium point for the following demand and supply functions:

$$\text{Supply:} \quad p = 2q + 170$$
$$\text{Demand:} \quad p = -5q + 450$$

Solve this as a system of equations via substitution method:

$$+5q + 2q + 170 = -5q + 450 \quad +5q$$
$$-170 + 7q + 170 = 450 \quad -170$$
$$\frac{7q}{7} = \frac{280}{7}$$
$$\boxed{q = 40} \longrightarrow p = -5(40) + 450$$
$$= -200 + 450$$
$$\boxed{p = 250}$$

Example. Using the supply and demand functions above, modify the supply function to include a \$14 tax per unit sold, then find the new market equilibrium point.

$$2q + 170 + 14 = -5q + 450$$
$$-184 + 7q + 184 = 450 \quad -184$$
$$\frac{7q}{7} = \frac{266}{7}$$
$$\boxed{q = 38}$$

$$p = -5(38) + 450$$
$$= -190 + 450$$
$$\boxed{p = 260}$$

A \$14 tax will shift the curves. The new equilibrium will be at (q,p)=(38,260) demand

Example. Retailers will buy 45 Wi-Fi routers from a wholesaler if the price is \$10 each but only 20 if the price is \$60. The wholesaler will supply 56 routers at \$42 each and 70 at \$50 each. Assuming that the supply and demand functions are linear, find the market equilibrium point.

Retailers

Compute slope → use point-slope formula

$(45, \$10)$
$(20, \$60)$

$$m = \frac{60-10}{20-45} = \frac{50}{-25}$$

$$m = -2$$

$$y - y_1 = m(x - x_1)$$
$$p - 60 = -2(q - 20)$$
$$p - 60 = -2q + 40$$
$$\boxed{p = -2q + 100}$$

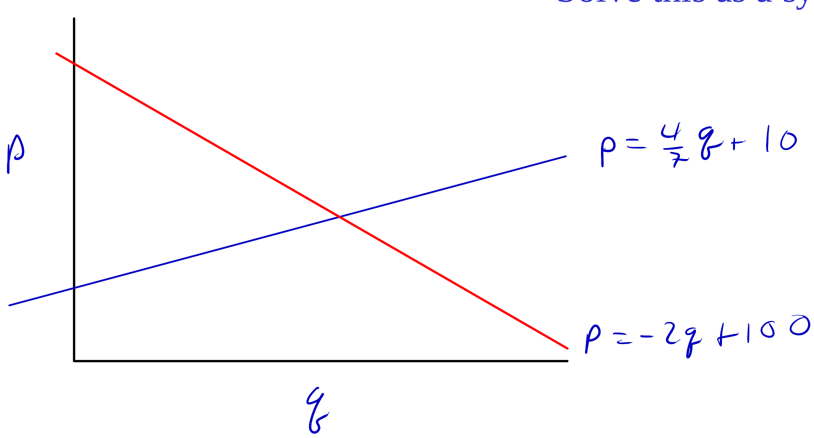
Wholesaler

$(56, \$42)$
$(70, \$50)$

$$m = \frac{50-42}{70-56} = \frac{8}{14} = \frac{4}{7}$$

$$y - y_1 = m(x - x_1)$$
$$p - 42 = \frac{4}{7}(q - 56)$$
$$p - 42 = \frac{4}{7}q - 32$$
$$\boxed{p = \frac{4}{7}q + 10}$$

Solve this as a system of equations via substitution method:



$$7\left(\frac{4}{7}q + 10\right) = (-2q + 100)\,7$$
$$4q + 70 = -14q + 700$$
$$18q = 630$$
$$\boxed{q = 35}$$
$$p = \frac{4}{7}(35) + 10$$
$$= 20 + 10$$
$$\boxed{p = 30}$$