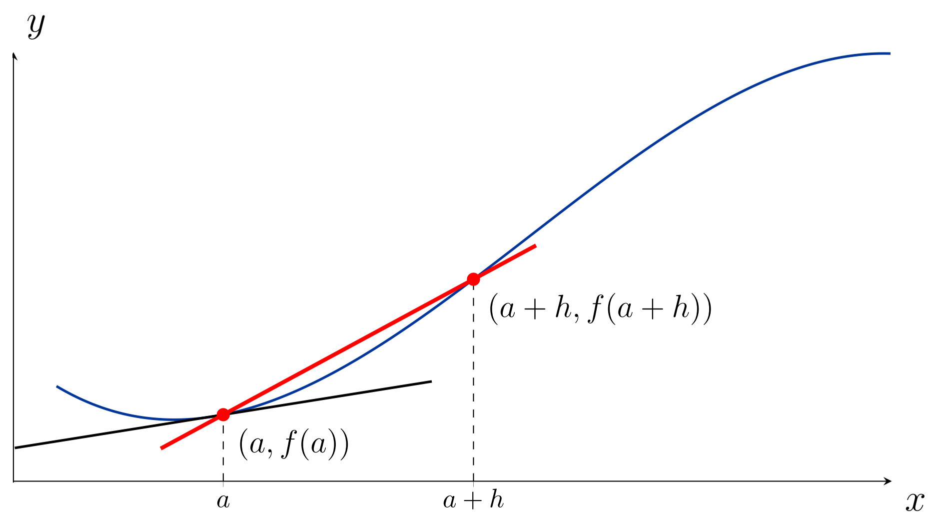
2.6: The Derivative

Definition.
Given a function $f(x)$:

- the **secant line** is the line that passes through two *distinct* points lying on the graph of $f(x)$,
- the **tangent line** is the line that intersects $f(x)$ in exactly one place (locally) and matches the slope of the graph at that point.

Graph



Definition. (Slope of a Tangent Line)
The slope of the tangent line to the graph of f at the point $P(x, f(x))$ is given by

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

if it exists.

Definition. (Average and Instantaneous Rates of Change)
The **average rate of change** of f over the interval $[x, x+h]$ or **slope of the secant line** to the graph of f through the points $(x, f(x))$ and $(x+h, f(x+h))$ is

$$\frac{f(x+h)-f(x)}{h}$$

The above fraction is referred to as the **difference quotient**.

The **instantaneous rate of change** of f at x or **slope of the tangent line** to the graph of f at $(x, f(x))$ is

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

Definition. (Derivative of a Function)
The derivative of a function f with respect to x is the function f' (read "f prime"),

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

The domain of f' is the set of all x for which the limit exists.

Some other notations for the derivative are

$$D_x f(x) \qquad\qquad \frac{dy}{dx} \qquad\qquad y'$$

Example. Find the slope of the line tangent to the graph $f(x) = 3x + 5$ at any point $(x, f(x))$

Example. Let $f(x) = x^2$.

- Find $f'(x)$.
- Compute $f'(2)$ and interpret your result.

Example. Let $f(x) = x^2 - 4x$. Find the point on the graph where the tangent line is horizontal.

Example. Let $f(x) = \dfrac{1}{x}$. Find the equation of the tangent line at $x = 2$.

Differentiability and Continuity
If a function is differentiable at $x = a$, then it is continuous at $x = a$.

Example. For the graph below, identify each point where the derivative is undefined.