

6.2: Integration by Substitution

Example. Find the derivative of the following functions:

$$f(x) = \frac{(2x+1)^4}{4}$$

$$g(x) = \frac{1}{x+3}$$

Let $u = g(x)$, where g is differentiable on an interval, and let f be continuous on the corresponding range of g . On that interval,

$$\int f'(g(x)) \cdot g'(x) dx = \int f(u) du$$

Procedure: Substitution Rule (Change of Variables)

1. Let $u = g(x)$, where $g(x)$ is part of the integrand, usually the “inside function” of a composite function $f(g(x))$.
2. Find $du = g'(x) dx$.
3. Use the substitution $u = g(x)$ and $du = g'(x) dx$ to convert the *entire* integral into one involving only u .
4. Find the resulting integral
5. Replace u by $g(x)$ to obtain the final solution as a function of x .

Example. Evaluate the following integrals:

$$\int 2x(x^2 + 3)^4 dx$$

$$\int (2x + 1)^3 dx$$

$$\int x^2 \sqrt{x^3 + 1} \, dx$$

$$\int t \sqrt[4]{1 - t^2} \, dt$$

$$\int \sqrt{4 - t} \, dt$$

$$\int (2 - x)^6 \, dx$$

$$\int e^{-3x} dx$$

$$\int \frac{t}{3t^2 + 1} dt$$

$$\int \frac{(\ln(x))^2}{2x} dx$$

$$\int u^3(u^2 + 1)^{3/2} du$$