

## 4.1: Linear Inequalities in Two Variables

### Properties of Inequalities

**Substitution Property:** The inequality formed by substituting one expression for an equal expression is equivalent to the original inequality:

$$5x - 4x + 2 < 6$$

$$x < 4 \Rightarrow \text{The solution set is } \{x : x < 6\}$$

**Addition Property:** The inequality formed by adding the same quantity to both sides of an inequality is equivalent to the original inequality:

$$x - 4 < 6$$

$$x - 4 + 4 < 6 + 4$$

$$x < 10$$

$$x + 5 \geq 12$$

$$x + 5 + (-5) \geq 12 + (-5)$$

$$x \geq 7$$

**Multiplication Property** The inequality formed by multiplying both sides of an inequality by the same *positive* quantity is equivalent to the original inequality. The direction of the inequality is flipped when multiplying by a *negative* quantity:

$$\frac{1}{3}x > 6$$

$$3\left(\frac{1}{3}x\right) > 3(6)$$

$$x > 18$$

$$5x - 5 + 5 \leq 6x + 20 + 5$$

$$\frac{-x}{-1} \leq \frac{25}{-1}$$

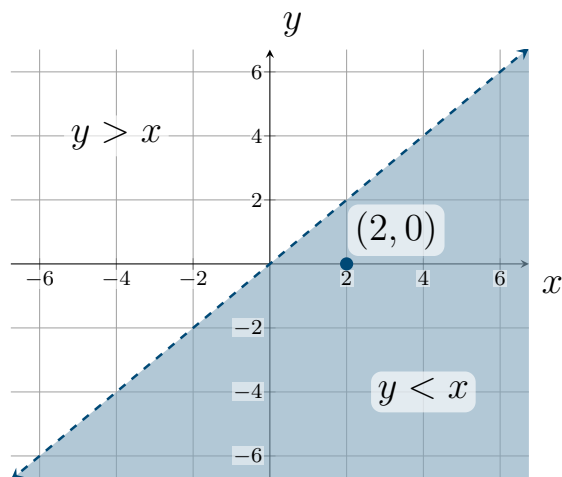
$$x \geq -25$$

## One Linear Inequality in Two Variables

### Definition.

Consider the inequality  $y < x$ :

The line created by this inequality divides the  $xy$ -plane into two **half-planes**. We can determine which half-plane is the solution region by selecting any point *not on the line* as a **test point**.



**Example.** Graph the inequality  $3x - 2y \leq 6$

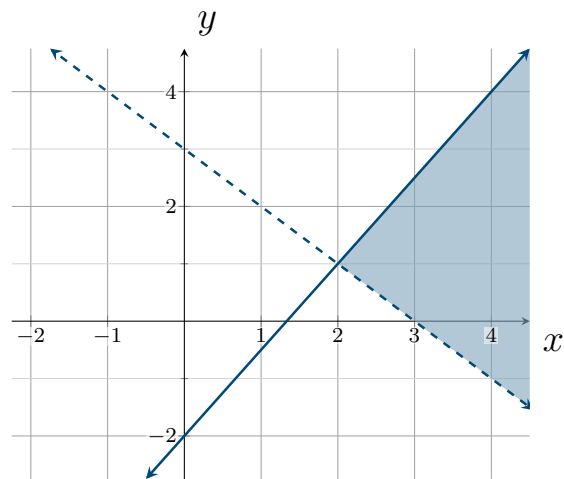


**Definition.**

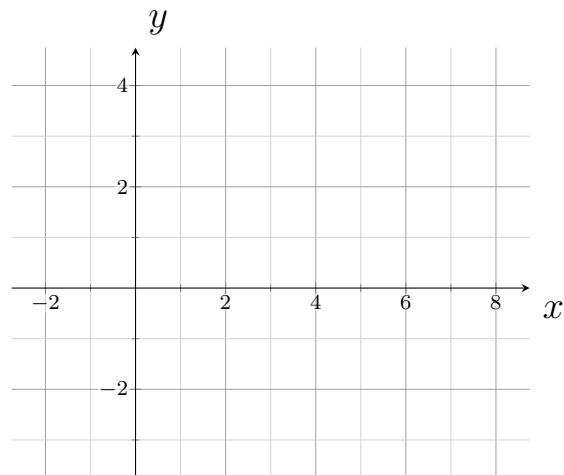
A **system of inequalities** has two or more inequalities in two or more variables. The solution of the system is the intersection of the individual solution sets.

**Example.**

$$\begin{aligned}3x - 2y &\geq 4 \\ x + y - 3 &> 0\end{aligned}$$

**Example.** Graph the solution of the system

$$\begin{aligned}3x - 4y &\leq 12 \\ 2x + 5y &> 10 \\ x - 8y &\geq -16\end{aligned}$$



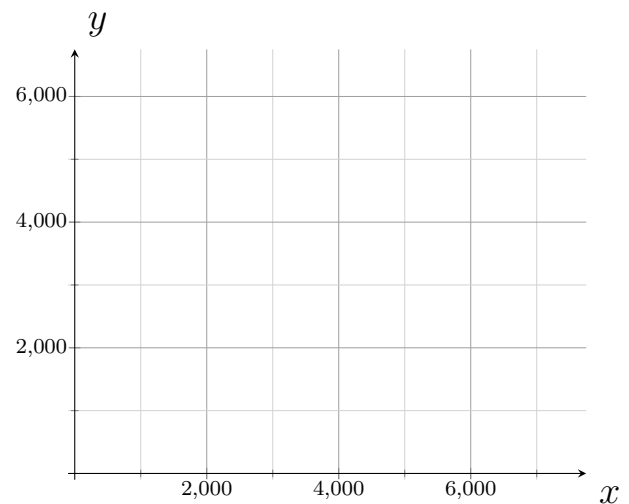
**Example.** CDF Appliances has assembly plants in Atlanta and Fort Worth, where the company produces a variety of kitchen appliances, including a 12-cup coffee maker and a cappuccino machine. The following table shows each factory’s assembly capabilities for the two products and the numbers needed to fill orders.

	Atlanta	Fort Worth	Needed
Coffee maker	160/hr	800/hr	At least 64,000
Cappucino machine	200/hr	200/hr	At least 40,000

Write the system of inequalities that describes the number of assembly hours needed at each plant to fill the orders and graph the solution region for the system



**Example.** A farm co-op has 6000 acres available to plant with corn and soybeans. Each acre of corn requires 9 gallons of fertilizer/herbicide and  $\frac{3}{4}$  hour of labor to harvest. Each acre of soybeans requires 3 gallons of fertilizer/herbicide and 1 hour of labor to harvest. The co-op has available at most 40,500 gallons of fertilizer/herbicide and at most 5250 hours of labor for harvesting. The number of acres of each crop is limited (constrained) by the available resources: land, fertilizer/herbicide, and labor for harvesting. Write the system of inequalities that describes the constraints and graph the solution region for the system.



**Example.** Graph the solution region for the system

$$5x+2y \leq 54$$

$$2x+4y \leq 60$$

$$x \geq 0, y \geq 0$$