Math 123 Class notes
Fall 2025

To accompany
Applied Calculus
by *Tan*

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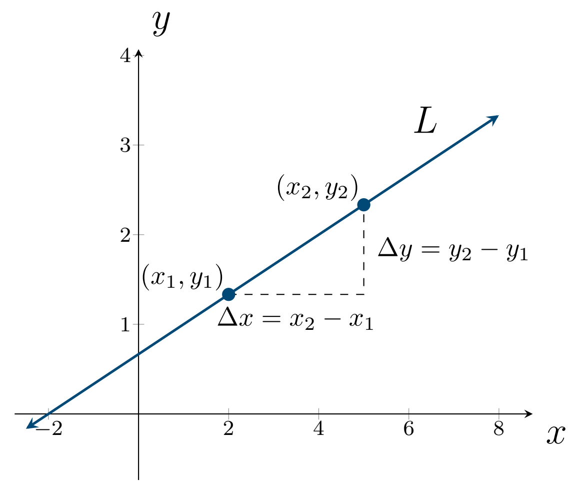
1.4: Straight Lines

Definition. (Slope of a Nonvertical Line)
If (x_1, y_1) and (x_2, y_2) are any two distinct points on a nonvertical line L, then the slope m of L is given by

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Example. Compute the slope of the line passing through the points



$(x_1, y_1) = (1, 1)$ and $(x_2, y_2) = (4, 2)$

$(x_1, y_1) = (3, 2)$ and $(x_2, y_2) = (-1, 2)$

$(x_1, y_1) = (4, 1)$ and $(x_2, y_2) = (4, 4)$

Definition. (Point-Slope Form of an Equation of a Line)
An equation of the line that has slope m and passes through the point (x_1, y_1) is given by

$$y - y_1 = m(x - x_1)$$

Example. Find the equation of the line going through the points

$(x_1, y_1) = (-2, 1)$ and $(x_2, y_2) = (3, -2)$

$(x_1, y_1) = (3, 4)$ and $(x_2, y_2) = (-1, 4)$

$(x_1, y_1) = (2, 0)$ and $(x_2, y_2) = (2, 1)$

Definition. (Slope-Intercept Form of an Equation of a Line)
An equation of the line that has slope m and intersects the y-axis at the point $(0, b)$ is given by

$$y = mx + b$$

Example. Rewrite the equations in the previous example in slope-intercept form.

Definition. (Parallel and Perpendicular lines)
Let L_1 and L_2 be lines with slopes m_1 and m_2 respectively. If L_1 and L_2 are *parallel*, then

$$m_1 = m_2.$$

If L_1 and L_2 are *perpendicular*, then

$$m_1 = -\frac{1}{m_2}.$$

Example.

Find the line *parallel* to $y = \frac{3}{2}x + 1$ that passes through the point $(-4, 10)$.

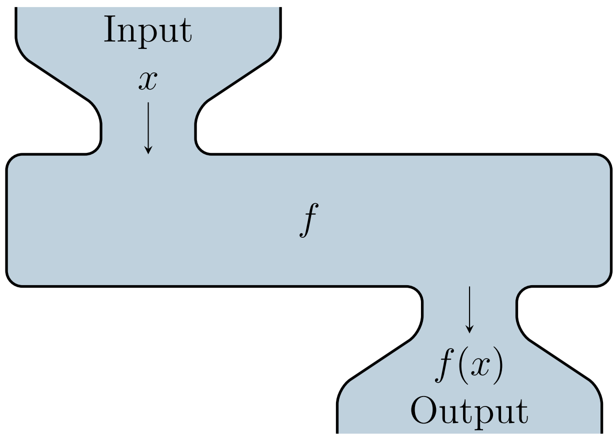
Find the line *perpendicular* to $y = \frac{3}{2}x + 1$ that passes through the point $(-3, 4)$.

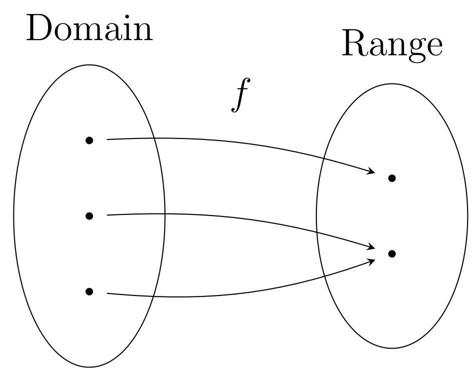
Forms of Linear Equations

$$
\begin{aligned}
\text{General form: } & Ax + By = C \\
\text{Point-slope form: } & y - y_1 = m(x - x_1) \\
\text{Slope-intercept form: } & y = mx + b \\
\text{Vertical line: } & x = a \\
\text{Horizontal line: } & y = b
\end{aligned}
$$

2.1: Functions and Their Graphs

Definition.
A **function** is a rule that assigns to each element in a set A one and only one element in a set B.
In the context above, the set A is called the **domain**, and the set B is called the **range**.





Example. Let $f(x) = 2x^2 - 2x + 1$. Evaluate the following

$f(1)$

$f(-2)$

$f(a)$

$f(a+h)$

Example. Find the domain and range of the following functions:

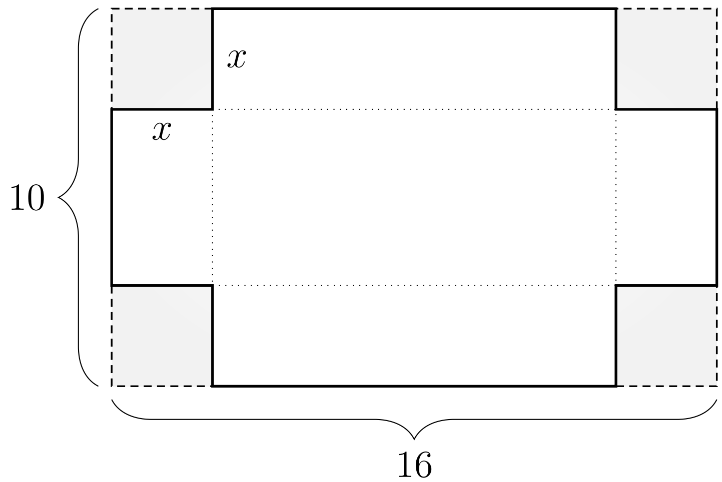
$f(x) = x$

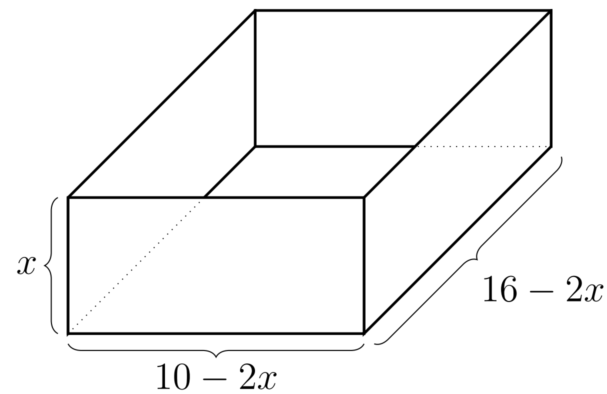
$A = \pi r^2$

$y = \sqrt{x-1}$

$y = \dfrac{1}{x^2-4}$

Example. An open box is to be made from a rectangular piece of cardboard 16 inches long and 10 inches wide by cutting away identical squares (x inches by x inches) from each corner and folding up the resulting flaps. Find an expression that gives the volume V of the box as a function of x. What is the domain of the function?





Definition.
A **piecewise** function is a function with different definitions for different portions of the domain.

Example. Rewrite the following as piecewise functions:

$|x| =$

$\dfrac{x}{|x|} =$

$|x-1| + |4-x| =$

> **Definition. (Vertical Line Test)**
> A curve in the xy-plane is the graph of a function $y = f(x)$ (an explicit function) if and only if each vertical line intersects it in at most one point

Example. Use the vertical line test on the following graphs to determine which graphs may represent an explicit function: