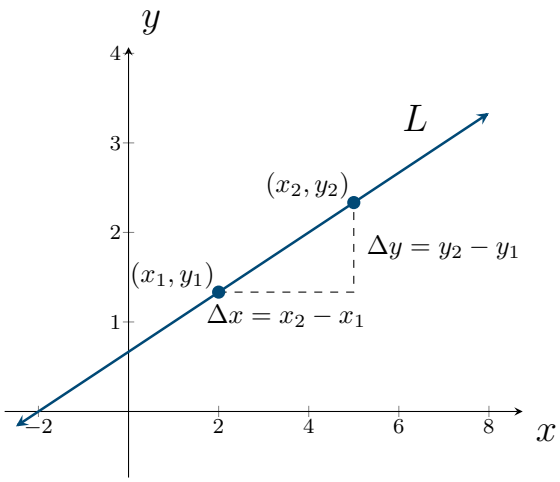
1.4: Straight Lines

Definition. (Slope of a Nonvertical Line)
If (x_1, y_1) and (x_2, y_2) are any two distinct points on a nonvertical line L, then the slope m of L is given by

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Example. Compute the slope of the line passing through the points



$(x_1, y_1) = (1, 1)$ and $(x_2, y_2) = (4, 2)$

$m = \frac{2-1}{4-1} = \boxed{\frac{1}{3}}$

$(x_1, y_1) = (3, 2)$ and $(x_2, y_2) = (-1, 2)$

$m = \frac{2-2}{-1-3} = \frac{0}{-4} = \boxed{0}$

$(x_1, y_1) = (4, 1)$ and $(x_2, y_2) = (4, 4)$

$m = \frac{4-1}{4-4} = \frac{3}{0}$ undefined

Definition. (Point-Slope Form of an Equation of a Line)
An equation of the line that has slope m and passes through the point (x_1, y_1) is given by

$$y - y_1 = m(x - x_1)$$

Example. Find the equation of the line going through the points

$(x_1, y_1) = (-2, 1)$ and $(x_2, y_2) = (3, -2)$

$m = \frac{-2-1}{3-(-2)} = \frac{-3}{5}$

$y - 1 = -\frac{3}{5}(x - (-2))$

$(x_1, y_1) = (3, 4)$ and $(x_2, y_2) = (-1, 4)$

$m = \frac{4-4}{-1-3} = 0$

$y - 4 = 0\,(x - 3)$

$(x_1, y_1) = (2, 0)$ and $(x_2, y_2) = (2, 1)$

$m = \frac{1-0}{2-2} = \frac{1}{0}$ undefined

Definition. (Slope-Intercept Form of an Equation of a Line)
An equation of the line that has slope m and intersects the y-axis at the point $(0, b)$ is given by

$$y = mx + b$$

Example. Rewrite the equations in the previous example in slope-intercept form.

$+1 \quad +y - 1 = -\frac{3}{5}(x - (-2)) \quad +1$

$y = -\frac{3}{5}x - \frac{6}{5} + 1\left(\frac{5}{5}\right)$

$\boxed{y = -\frac{3}{5}x - \frac{1}{5}}$

$+4 \quad +y - 4 = 0 \quad (x - 3) \quad +4$

$\boxed{y = 4}$

Horizontal lines are of the form $y = a$

$\boxed{x = 2}$

Vertical lines are of the form $x = b$

- undefined slope
- no y-intercept

Definition. (Parallel and Perpendicular lines)
Let L_1 and L_2 be lines with slopes m_1 and m_2 respectively. If L_1 and L_2 are *parallel*, then

$$m_1 = m_2.$$

If L_1 and L_2 are *perpendicular*, then

$$m_1 = -\frac{1}{m_2}.$$

Example.

Find the line *parallel* to $y = \frac{3}{2}x + 1$ that passes through the point $(-4, 10)$.

$\Rightarrow m = \frac{3}{2}$ $\quad y - 10 = \frac{3}{2}(x - (-4))$

$$\boxed{y = \frac{3}{2}x + 16}$$

Find the line *perpendicular* to $y = \frac{3}{2}x + 1$ that passes through the point $(-3, 4)$.

$\Rightarrow m = -\frac{1}{3/2} = -\frac{2}{3}$ $\quad y - 4 = -\frac{2}{3}(x - (-3))$

$$\boxed{y = -\frac{2}{3}x + 2}$$

Forms of Linear Equations

$$\begin{aligned}
\text{General form: } & Ax + By = C \\
\text{Point-slope form: } & y - y_1 = m(x - x_1) \\
\text{Slope-intercept form: } & y = mx + b \\
\text{Vertical line: } & x = a \\
\text{Horizontal line: } & y = b
\end{aligned}$$