2.2: The Algebra of Functions

Definition.
Let f and g be functions with domains A and B, respectively. Then the **sum** $f+g$, **difference** $f-g$, and **product** fg of f and g are functions with domain $A \cap B$.

$$(f+g)(x) = f(x) + g(x)$$
$$(f-g)(x) = f(x) - g(x)$$
$$(fg)(x) = f(x)g(x)$$

The **quotient** f/g of f and g has domain $A \cap B$ excluding all numbers x such that $g(x) = 0$ and rule given by

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

Example. Let $f(x) = \sqrt{x+1}$ and $g(x) = 4 - x$. Find the domain of the following:

$f(x) + g(x) = \sqrt{x+1} + (4-x)$

$x+1 \geq 0$

$x \geq -1$

Domain $[-1, \infty)$

$f(x) - g(x) = \sqrt{x+1} - (4-x)$

$x+1 \geq 0$

$x \geq -1$

Domain $[-1, \infty)$

$f(x)g(x) = (4-x)\sqrt{x+1}$

$x+1 \geq 0$

$x \geq -1$

Domain $[-1, \infty)$

$\frac{f(x)}{g(x)} = \frac{\sqrt{x+1}}{4-x}$

$x+1 \geq 0$

$x \geq -1$

$4 - x \neq 0$

$4 \neq x$

Domain $[-1, 4) \cup (4, \infty)$

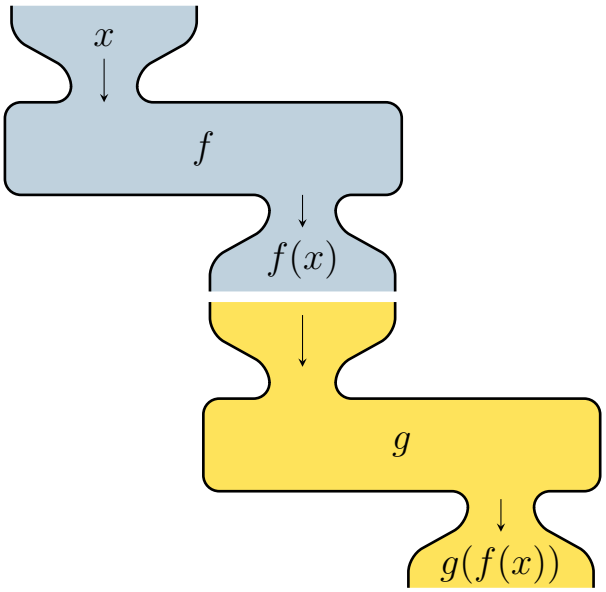
Definition. (The Composition of Two Functions)
Let f and g be functions. Then the composition of g and f is the function $g \circ f$ defined by

$$(g \circ f)(x) = g(f(x))$$

The domain of $g \circ f$ is the set of all x is the domain of f such that $f(x)$ lies in the domain of g.

Example. Let $f(x) = \sqrt{x+1}$ and $g(x) = 4 - x$. Find the domain of the following:

$g(f(x)) =$ $4 - \sqrt{x+1}$



$f(g(x)) =$ $\sqrt{(4-x)+1}$

$= \sqrt{3-x}$

$f(f(x)) =$ $\sqrt{\sqrt{x+1}+1}$