8.3: Hypothesis Tests in Detail

Definition. (Type I and type II errors)

- A **type I error** is rejecting the null hypothesis, H_0, when it is actually true.
- A **type II error** is failing to reject the null hypothesis, H_0, when it is false.

The probability of committing a type I error is the level of significance: α

	Null Hypothesis is true	Null Hypothesis is false
Reject null hypothesis	Type I error	True positive
Fail to reject null hypothesis	True negative	Type II error

Example. For the following scenarios, identify the type I and type II errors:

"The Boy Who Cried Wolf"

H_0 : There is no wolf
H_a : There is a wolf

Type I: Villagers believe there IS A wolf when there isn't a wolf
Type II: Villagers believe there IS NO wolf when there is a wolf

In a court of law, a person is considered innocent until proven guilty.

H_0 : Not guilty
H_a : Guilty

Type I: An innocent person is convicted
Type II: A guilty person is NOT convicted

Testing someone for a disease (e.g. Covid)

H_0 : Not sick/infected
H_a : Sick/infected

Type I: Someone tests positive but is not sick or infected
Type II: The test fails to detect when someone is sick or infected

Pregnancy test

H_0 : Not pregnant
H_a : Pregnant

Type I: Pregnancy test is positive but person is not pregnant
Type II: Pregnancy test fails to detect when a person is pregnant

- When we "fail to reject H_0", we are **not proving** the null hypothesis
- Don't change your hypothesis after you gather your results
- Statistically significant means something likely did not occur by chance
- Confidence intervals vs. Hypothesis testing

Confidence Intervals	Hypothesis Tests
Estimates parameters	Test parameters
Range of values	Is data consistent?

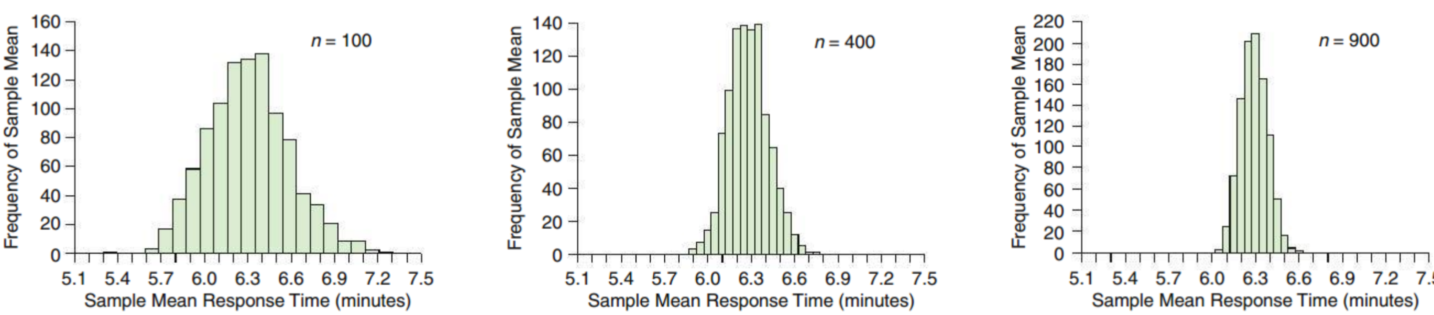
9.1: Sample Means of Random Samples

Statistics		Parameters	
Sample mean	$\overline{x}$	Population mean	μ
Sample standard deviation	s	Population standard deviation	σ
Sample proportion	$\hat{p}$	Population proportion	p

Definition.

- The **sampling distribution** is the distribution of the sample means $\bar{x}$.
- The mean of the sampling distribution is μ so the statistic $\bar{x}$ is an **unbiased estimator**.
- The standard deviation of the sampling distribution is the **standard error**:

$$SE = \frac{\sigma}{\sqrt{n}}$$



9.2: The Central Limit Theorem for Sample Means

Definition. (Central Limit Theorem (CLT))

When estimating a population mean, μ, if

1. *Random and Independent*: Each observation is collected randomly from the population, and observations are independent of each other.
2. *Large Sample*: Either the population distribution is Normal, or the sample size is large ($n \geq 25$).
3. *Big population*: If the sample is collected without replacement (e.g. SRS), then the population size must be at least 10 times bigger than the sample size.

$$N \geq 10n$$

then the sampling distribution for $\overline{x}$ is approximately Normal, with mean μ and standard deviation

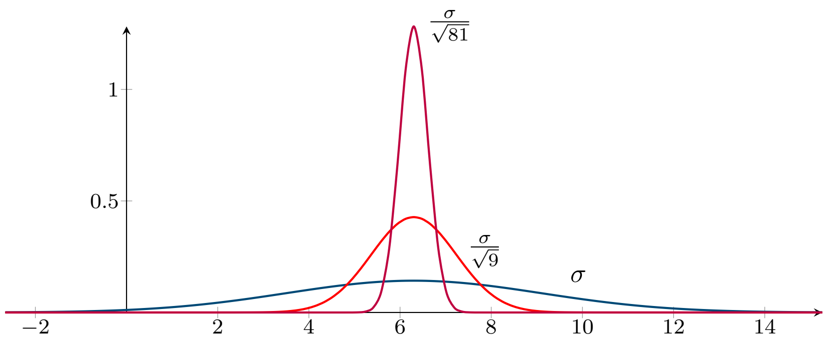
$$SE = \frac{\sigma}{\sqrt{n}}.$$

This distribution is denoted as

$$N\left(\mu, \frac{\sigma}{\sqrt{n}}\right).$$

Example. The population distribution of *all* emergency response times from the LA Fire Department is right-skewed. Suppose we repeatedly take random samples of a certain size from this population and calculate the mean response time. We know that the population has mean $\mu = 6.3$ and standard deviation $\sigma = 2.8$ minutes.

Describe the sampling distribution if the sample size is $n = 9$, and again when $n = 81$.



Note: Even if the population distribution has an unusual shape, the sampling distribution is fairly symmetric and unimodal.

Example. According to one very large study done in the US, the mean resting pulse rate of adult women is about $\mu = 74$ BPM, with standard deviation $\sigma = 13$ BPM, where the distribution is known to be skewed right. Suppose we take a random sample of 36 women from this population.

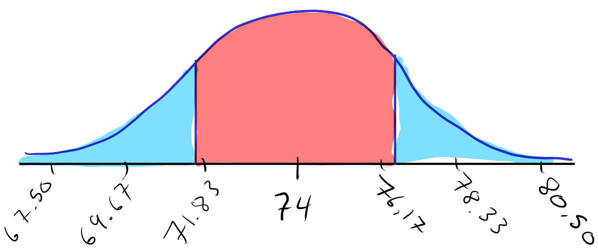
What is the approximate probability that the average pulse rate of this sample will be below 71 or above 77?

$SE = \frac{\sigma}{\sqrt{n}} = 2.167 \longrightarrow N(74, 2.167)$

$\mu = 74$

$\sigma = 13$

$n = 36$



$N(74, 2.167)$

$N(0,1)$ -3 -2 -1 0 1 2 3

CLT conditions

1. Random & independent ✓

2. $n \geq 25$ $\quad n = 36 \geq 25$ ✓

3. $N \geq 10n$ $\quad 10(36) = 360$ ✓

There are at least 360 adult women in the US

$P(\bar{x} < 71 \text{ or } \bar{x} > 77)$

$= 1 - P(71 \leq \bar{x} \leq 77)$

$= 1 - 0.8338$

$= 0.1662$

Found via Rossman/Chance or Stat Crunch

Can we find the probability that a single adult woman, randomly selected from this population, will have a resting pulse rate more than 3 BPM away from the mean value, $\mu = 74$?

No. The CLT only applies to sample means and not individual observations.

Definition. (The t-Distribution)
The hypothesis tests and confidence intervals we will use for estimating and testing the mean are based on the **t-statistic**:

$$t = \frac{\overline{x} - \mu}{SE_{est}}$$

$$SE_{est} = \frac{s}{\sqrt{n}}$$

The t-statistic follows the **t-distribution**. With the t-distribution, we do not need to check conditions for the CLT, but the distribution's shape is dependent on the **degrees of freedom (df)**.

If we know the population standard deviation σ, then we have the familiar z-statistic:

$$z = \frac{\overline{x} - \mu}{\left(\frac{\sigma}{\sqrt{n}}\right)}$$