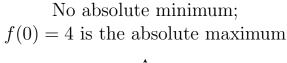
4.4: Optimization I

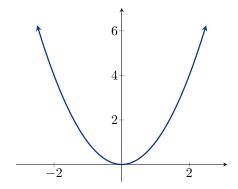
Definition. (Absolute Extrema)

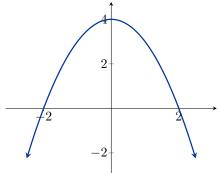
Let f be defined on a set D containing c. If

- $f(c) \ge f(x)$ for every x in D, then f(c) is an **absolute maximum** value of f
- $f(c) \leq f(x)$ for every x in D, then f(c) is an **absolute minimum** value of f

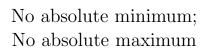
f(0) = 0 is the absolute minimum; No absolute maximum

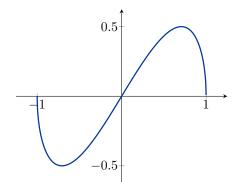


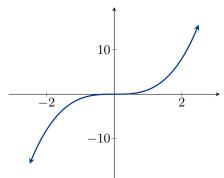




$$f(-\sqrt{2}) = -\frac{1}{2}$$
 is the absolute minimum;
 $f(\sqrt{2}) = \frac{1}{2}$ is the absolute maximum

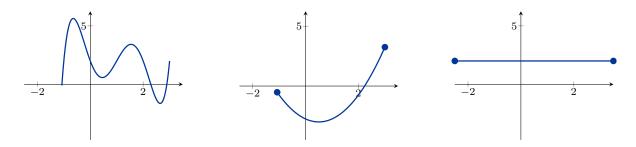




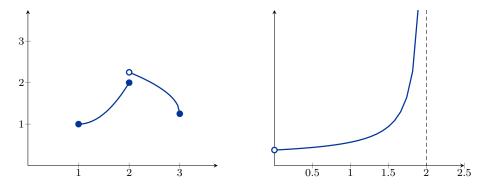


Theorem 3

A function that is continuous on a closed interval [a, b] has an absolute maximum value and an absolute minimum value on that interval.



Note: It is important that the function is both continuous and the interval is closed:



Finding the Absolute Extrema of f on a Closed Interval

- 1. Find the critical points of f within the interval (a, b).
- 2. Compute f(x) at x = a, x = b, and at each of the critical points found above.
- 3. The absolute maximum and absolute minimum will correspond to the largest and smallest values found above.

Example. Find the absolute extrema of the following functions on the intervals indicated

$$f(x) = x^2$$
 on $[-1, 2]$

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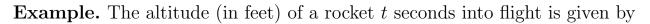
$$g(x) = x^3 - 2x^2 - 4x + 4$$
 on $[0, 3]$

$$h(x) = x^{2/3}$$
 on $[-1, 8]$

Example. The daily average cost function (in dollars per unit) of Elektra Electronics is given by

 $\overline{C}(x) = 0.0001x^2 - 0.08x + 40 + \frac{5000}{x}$ (x > 0)

where x stands for the number of graphing calculators that Elektra produces. Show that a production level of 500 units per day results in a minimum average cost for the company.



$$s = f(t) = -t^3 + 96t^2 + 5 \qquad (t \ge 0)$$

Find the maximum altitude attained by the rocket.

Find the maximum velocity attained by the rocket.