

6.5: Evaluating Definite Integrals

The Fundamental Theorem of Calculus

Let f be continuous on $[a, b]$. Then,

$$\int_a^b f(x) dx = F(x) \Big|_a^b = F(b) - F(a)$$

where F is any antiderivative of f ; that is, $F'(x) = f(x)$.

Example. Let R be the region under the graph of $f(x) = x$ on the interval $[1, 3]$. Find the area of R

[Graph](#)

using geometry

using the Fundamental Theorem of Calculus

Properties of the Definite Integral

Let f and g be integrable functions; then,

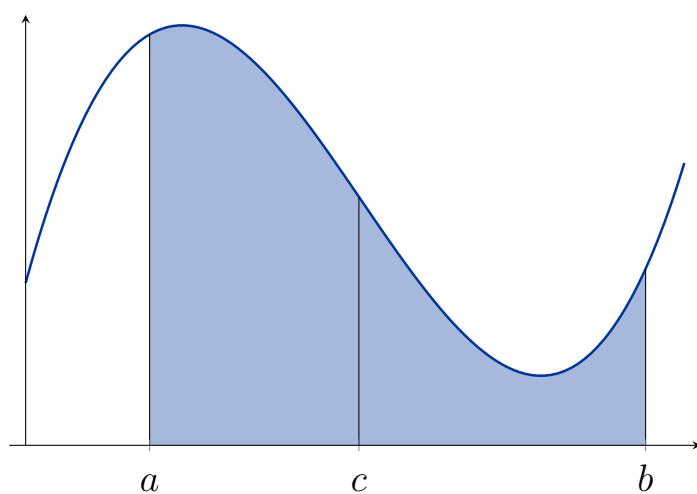
$$1. \int_a^a f(x) dx = 0$$

$$2. \int_a^b f(x) dx = - \int_b^a f(x) dx$$

$$3. \int_a^b cf(x) dx = c \int_a^b f(x) dx \quad (c \text{ constant})$$

$$4. \int_a^b [f(x) \pm g(x)] dx = \int_a^b f(x) dx \pm \int_a^b g(x) dx$$

$$5. \int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx \quad (a < c < b)$$



Example. Evaluate the following definite integrals

$$\int_0^4 x \sqrt{9 + x^2} dx$$

$$\int_0^2 x e^{2x^2} dx$$

Example. Find the area of each region R described below:

[Graphs](#)

Under $f(x) = \sqrt{x}$ from $x = 1$ to $x = 4$

Under $f(x) = e^{x/2}$ from $x = -1$ to $x = 1$