

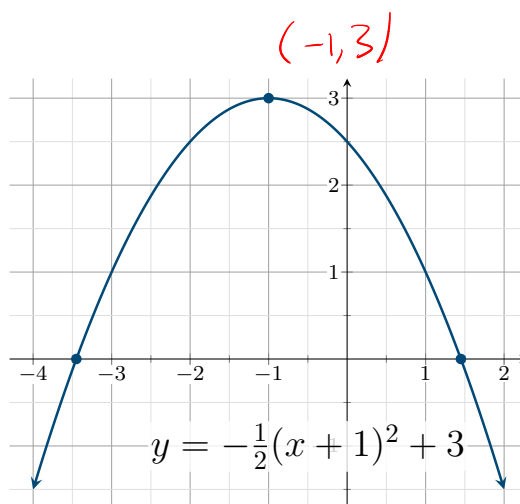
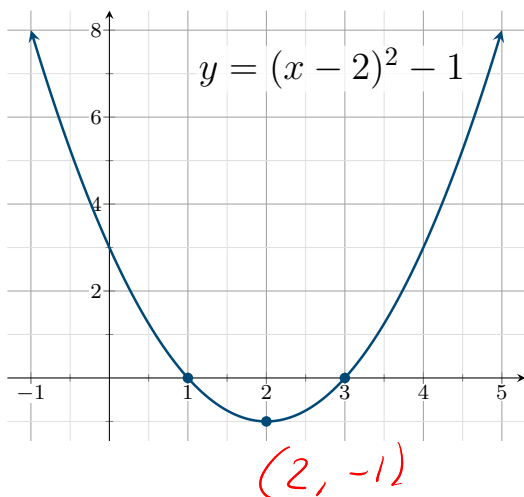
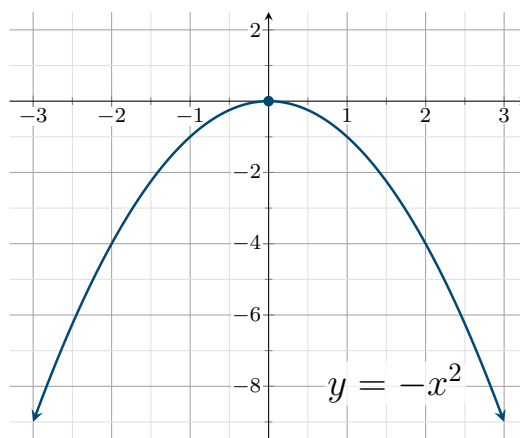
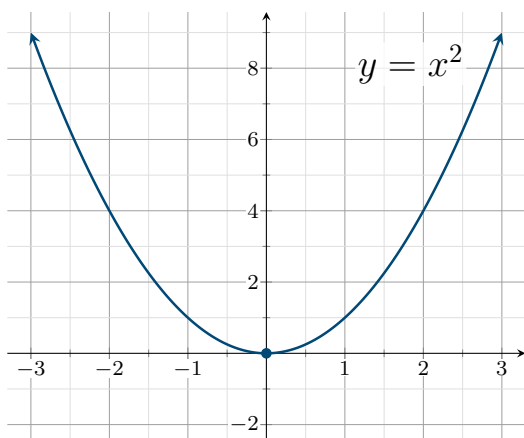
2.2: Quadratic Functions: Parabolas

Definition.

A **quadratic function** has the form

$$y = f(x) = ax^2 + bx + c \quad (a \neq 0)$$

where a , b , and c represent constants. A **parabola** is the shape of the graph of a quadratic function.



Definition.

The quadratic function $y = f(x) = ax^2 + bx + c$ has its **vertex** at

$$\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right).$$

The optimal value occurs at the vertex of a parabola:

- A maximum if $a < 0$
- A minimum if $a > 0$

Example. Consider the function

$$2x + \frac{x^2}{2}$$

Is the vertex a maximum or minimum? Locate the vertex, x -intercepts, y -intercept, and then sketch the graph.

$$a = \frac{1}{2} > 0 \Rightarrow \text{Vertex is a minimum}$$

$$\text{Vertex: } \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right) = \left(-\frac{2}{2(1/2)}, f\left(-\frac{2}{2(1/2)}\right)\right) = (-2, -2)$$

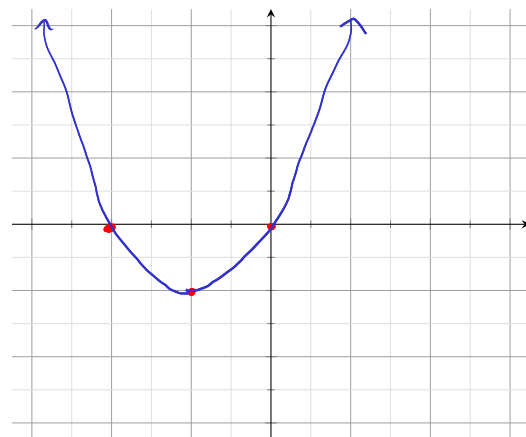
x -intercepts ($y=0$)

$$0 = 2x + \frac{x^2}{2} = \frac{1}{2}x(4+x)$$

$$\begin{array}{ccc} \downarrow & & \downarrow \\ x=0 & & x=-4 \\ \boxed{(0,0)} & & \boxed{(-4,0)} \end{array}$$

y -intercept ($x=0$)

$$y = 2(0) + \frac{0^2}{2} = 0 \Rightarrow \boxed{(0,0)}$$



Example. Consider the function

$$x^2 + 5 - 4x$$

Is the vertex a maximum or minimum? Locate the vertex, x -intercepts, y -intercept, and then sketch the graph.

$$a = 1 > 0 \Rightarrow \text{Vertex is a minimum}$$

$$\text{Vertex: } \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right) = \left(-\frac{-4}{2(1)}, f\left(-\frac{-4}{2(1)}\right)\right) = (2, 1)$$

x -intercepts ($y=0$)

$$0 = \underbrace{x^2}_{a} - \underbrace{4x}_{b} + \underbrace{5}_{c}$$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)}$$

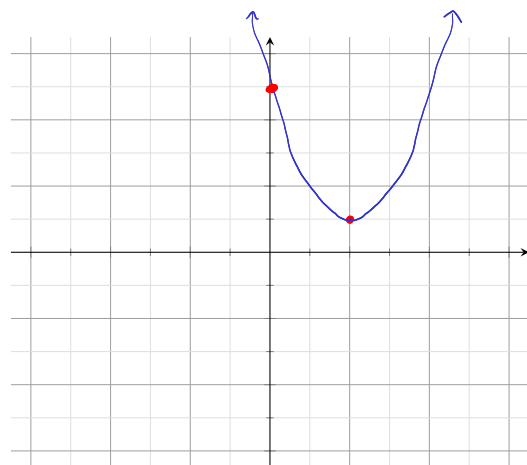
Determinant:

$$(-4)^2 - 4(1)(5) = -4 < 0 \Rightarrow \text{No solutions!}$$

\Rightarrow No x -intercepts

y -intercept ($x=0$)

$$y = 0^2 - 4(0) + 5 = 5 \rightarrow \boxed{(0, 5)}$$



Example. Ace Cruises offers a sunset cruise to a group of 50 people for a price of \$30 per person, but it reduces the price per person by \$0.50 for each additional person above 50. Find the revenue function. What price maximizes the revenue? What is this maximal value?

Number of people	Price per person	Total Revenue
50	\$ 30.00	\$ 1500.00
51	\$ 29.50	\$ 1504.50
52	\$ 29.00	\$ 1508.00
⋮		
50+x	$30 - 0.5x$	$R(x) = (50+x)(30-0.5x)$

$$\begin{aligned}
 R(x) &= (50+x)(30-0.5x) \\
 &= \underbrace{1500}_c + \underbrace{5x}_b - \underbrace{0.5x^2}_a
 \end{aligned}$$

$a < 0 \Rightarrow$ Vertex is a maximum.

$$\text{Vertex } \left(-\frac{b}{2a}, R\left(-\frac{b}{2a}\right) \right)$$

$$-\frac{b}{2a} = -\frac{5}{2(-0.5)} = 5$$

$$\begin{aligned}
 R(5) &= 1500 + 5(5) - 0.5(5)^2 \\
 &= 1500 + 25 - \frac{25}{2} \\
 &= \boxed{\$1512.50}
 \end{aligned}$$