

1.3 Linear Functions

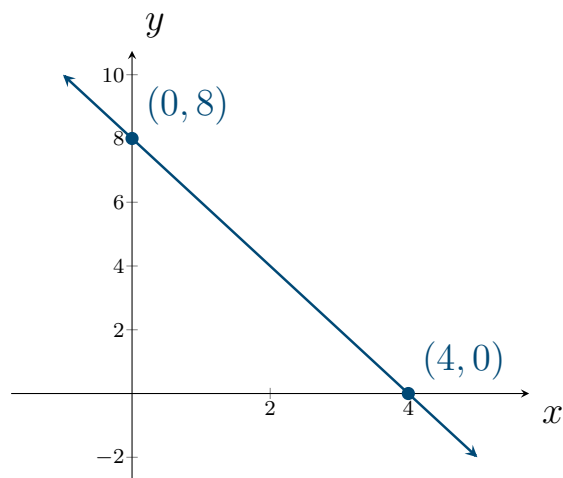
Definition.

A **linear function** is a function of the form

$$y = f(x) = ax + b$$

where a and b are constants.

Example. $y = -2x + 8$



A linear function can be uniquely determined using only *two* distinct points.

Definition.

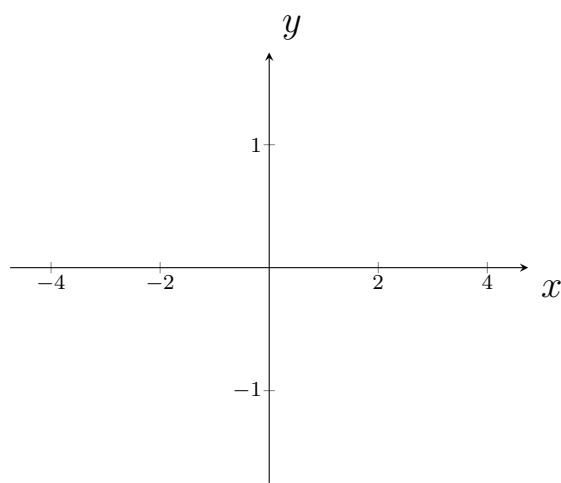
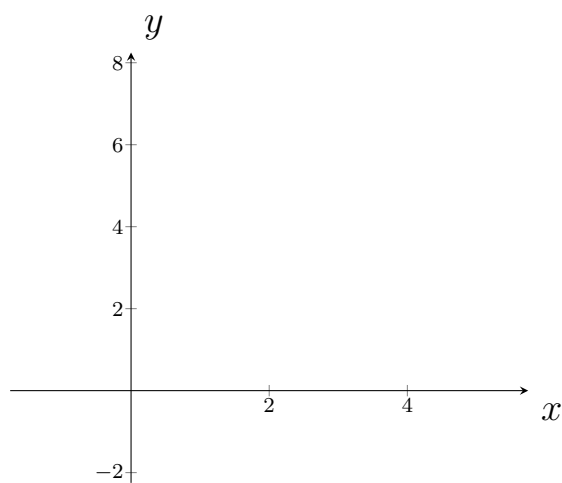
The point(s) where a graph intersects the axes are called intercepts. The x -coordinate of the point where the function intersects the x -axis is called the **x -intercepts**. The y -coordinate of the point where the function intersects the x -axis is called the **y -intercepts**.

- To solve for the y -intercept:
 - Set $x = 0$,
 - Solve for y .
- To solve for the x -intercept:
 - Set $y = 0$,
 - Solve for x .

Example. Find the intercepts and graph the following lines:

$$3x + 2y = 12$$

$$x = 4y$$



Definition.

If a nonvertical line passes through the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$, its **slope**, denoted by m , is found using

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$

Δy is “delta y ”, and represents the change in y

Δx is “delta x ”, and represents the change in x

Note: The slope of a vertical line is undefined.

Example. Find the slope of the line passing through the points $(-2, 1)$ and $(5, 3)$.

Note:

- Two distinct nonvertical lines are *parallel* if and only if their slopes are *equal*.
- Two distinct nonvertical lines are *perpendicular* if and only if their slopes are *negative reciprocals*:
e.g. If ℓ_1 has a nonzero slope m , then ℓ_2 is perpendicular if its slope is $-1/m$.

Point-slope form

Definition.

The equation of the line passing through the point (x_1, y_1) with slope m can be written in the point-slope form:

$$y - y_1 = m(x - x_1)$$

Example. Find the equation of each line that passes through the point $(-3, 4)$ and has

a slope of $m = \frac{1}{4}$

the point $(-2, 1)$ on the line

a slope of zero (horizontal)

an undefined slope (vertical)

Slope-intercept form

Definition.

The slope-intercept form of the equation of a line with slope m and y -intercept b is

$$y = mx + b$$

Example (Example 7, p.82). The population of U.S. males, y (in thousands), projected from 2015 to 2060 can be modeled by

$$y = 1125.9x + 142,960$$

where x is the number of years after 2000.

- Find the slope and y -intercept of the graph of this function.
- What does the y -intercept tell us about the population of U.S. males?
- Interpret the slope as a rate of change.

Forms of Linear Equations

General form: $ax + by + c = 0$

Point-slope form: $y - y_1 = m(x - x_1)$

Slope-intercept form: $y = mx + b$

Vertical line: $x = a$

Horizontal line: $y = b$