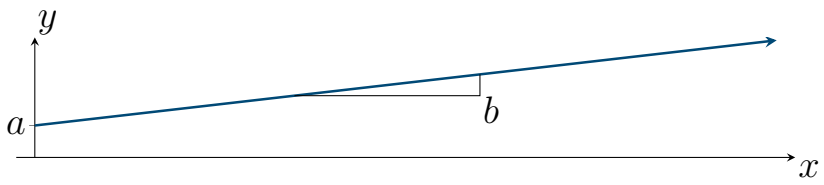
4.3: Modeling Linear Trends

Definition.
The **regression line** is a model used for making predictions about *future* observed values. The equation of the regression line is

$$y = a + bx$$

(when x=0)

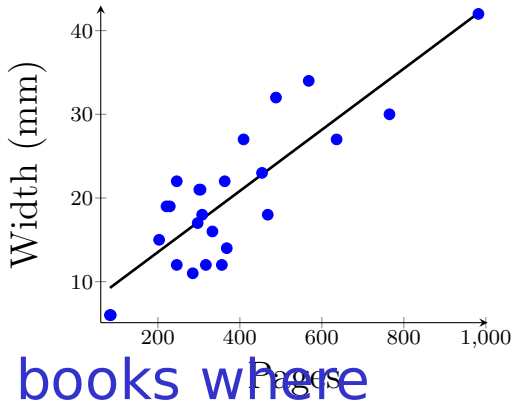
where a is the y**-intercept** and b is the **slope**.



- The input variable x is also know as the
  - Independent variable
  - Predictor variable
  - Explanatory variable
- The output variable y is known as the
  - Dependent variable
  - Predicted variable
  - Response variable

Example. Below is a scatterplot comparing number of pages a book has against the width of the book. Interpret the intercept and the slope of the regression line.

Predicted Width=6.22+0.0366 Pages

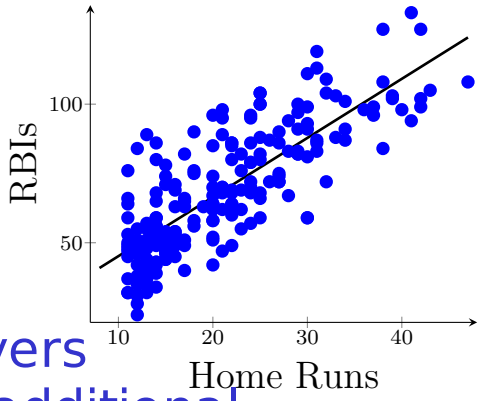


Intercept: 6.22mm
Width of a book with zero pages

Slope: 0.0366 mm per page
When comparing two books where one book has an additional page, the other is 0.0366mm thicker

Example. Below is a scatterplot comparing the number of home runs and RBIs in the 2016 season. Interpret the intercept and slope of the regression line.

Predicted RBI=23.84+2.13 HR



Intercept: 23.84 RBI
Could make sense, but is outside the data range

Slope: 2.13 RBI/HR
When comparing two players where one player hits an additional HR, their predicted RBI is expected to be 2.13 RBI higher

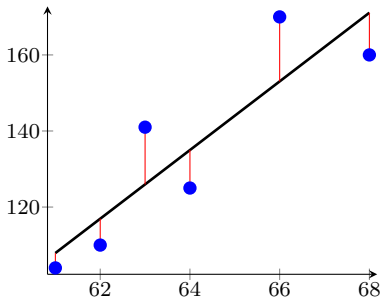
Definition.
Now we define the formula of the regression line:

$$y = a + bx$$

Where

$$b = r\frac{s_y}{s_x} \quad \text{and} \quad a = \overline{y} - b\overline{x}.$$

These formulae minimize the residual error: Try this!



Example. Below are the heights and weights of six women:

Heights	61	62	63	64	66	68
Weights	104	110	141	125	170	160

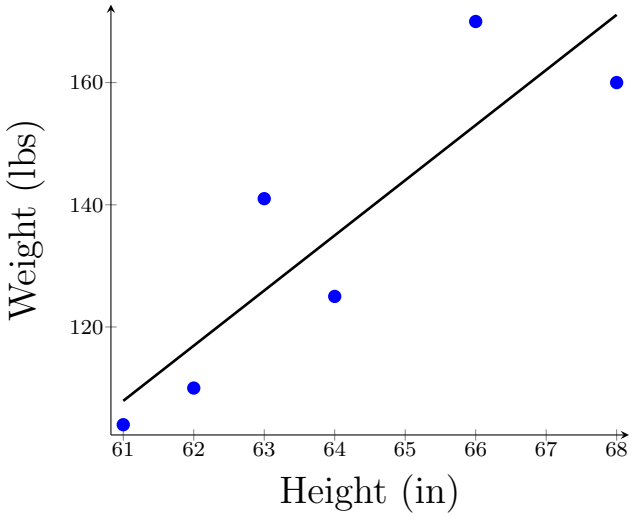
From this we get

$$\overline{x} = 64 \qquad s_x = 2.608$$
$$\overline{y} = 135 \qquad s_y = 26.728$$
$$r = 0.881$$

Find the equation of the regression line.

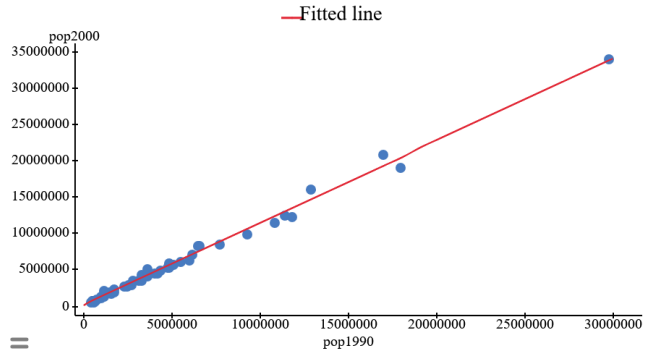
$$b = 0.881 \cdot \left(\frac{26.728}{2.608}\right) = 9.03$$

$$a = 135 - 9.03(64) = -442.92$$



Example. Open the `popdensity_and_crime` dataset in StatCrunch, use the "Simple Linear" tool under the `Stat›Regression` menu to find the regression line for the following columns. Interpret the slope and intercept where appropriate.

the `pop1990` and `pop2000` columns,



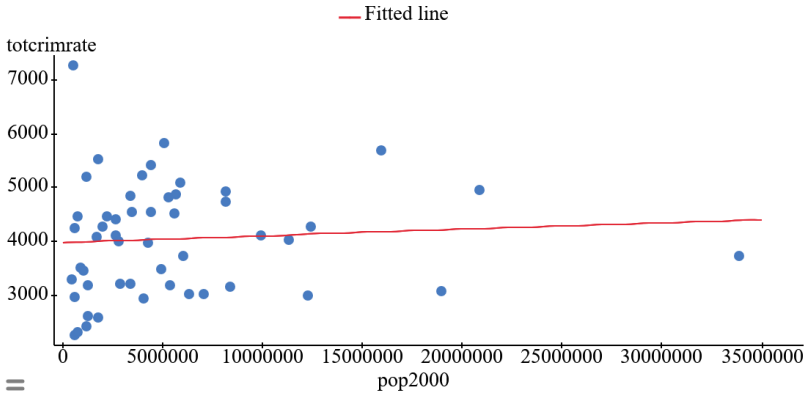
pop2000 = 12266.759 + 1.1295246 pop1990
R (correlation coefficient) = 0.99649554

Intercept: If pop=0 in 1990, it's 12,267 in 2000
(Not appropriate since data is 454k-30mil)

Slope: When comparing pop in 1990 where one group has 1 more person, that group is expected to have 1.13 more people in 2000

=

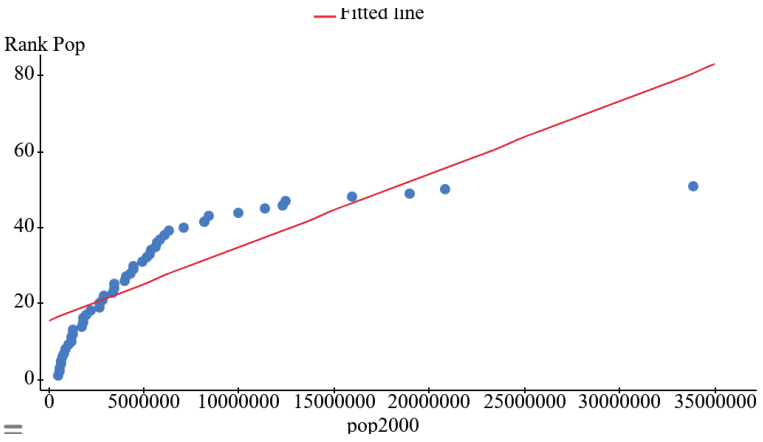
the `pop2000` and `totcrimerate` columns, and



Low correlation coefficient (~0.07)
Looks nonlinear
Linear regession is not appropriate here

=

the `pop2000` and `Rank Pop` columns.



Large correlation coefficient (~0.8),
but a nonlinear trend

Linear regression is not appropriate

=

4.4: Evaluating the Linear Model

Guidelines:

- Don't fit linear models to nonlinear associations!

- Correlation is not causation

- Beware of outliers (a.k.a. **influential points**)

- Don't extrapolate (make predictions beyond the range of the data)

Definition.
The **coefficient of determination** is the correlation coefficient coefficient squared:

$$r^2$$

This is sometimes also called r**-squared**.