

## 1.1: Variables

### Definition.

A **variable** is a placeholder for something which may or may not be unknown.

**Example.** Is there a number with the following property: doubling it and adding 3 gives the same result as squaring it?

- Is there a number  $x$  with the property that  $2x + 3 = x^2$ ?
- Is there a number  $\square$  with the property that  $2 \cdot \square + 3 = \square^2$ ?

**Example.** No matter what number might be chosen, if it is greater than 2, then its square is greater than 4.

- No matter what number  $n$  might be chosen, if  $n$  is greater than 2,  
then  $n^2$  is greater than 4.

**Example.** Use variables to rewrite the following sentences:

Are there numbers with the property that the sum of their squares equals the square of their sum?

Do there exist numbers  $a$  &  $b$  such that  $(a+b)^2 = a^2 + b^2$ ?

Given any real number, its square is nonnegative.

For any real number  $r$ ,  $r^2 \geq 0$

### Definition.

- A **universal statement** says that a certain property is true for all elements in a set.
- A **conditional statement** says that if one thing is true, then some other thing also has to be true.
- Given a property that may or may not be true, an **existential statement** says that there is at least one thing for which the property is true.

## Definition.

A **universal conditional statement** is both universal and conditional:

For every animal  $a$ , if  $a$  is a dog, then  $a$  is a mammal.

Conditional statements can be rewritten in ways that make them appear more to be purely universal or purely conditional:

If  $a$  is a dog, then  $a$  is a mammal.

All dogs are mammals

**Example.** Rewrite the following universal condition statement:

For ever real number  $x$ , if  $x$  is nonzero then  $x^2$  is positive.

If a real number is nonzero, then its square is positive.

For every nonzero real number  $x$ ,  $x^2$  is positive.

If  $x$  is a nonzero real number, then  $x^2$  is positive.

The square of any nonzero real number is positive.

All nonzero real numbers have positive squares.

### Definition.

A **universal existence statement** is a statement that is universal because its first part says that a certain property is true for all objects of a given type, and it is existential because its second part asserts the existence of something:

Every real number has an additive inverse.

In the above example, note that the particular additive inverse depends on the given real number:

For every real number  $r$ , there is an additive inverse for  $r$ .

**Example.** Rewrite the following universal existence statement:

Every pot has a lid

Not every lid will fit every pot

All pots have lids.

For ever pot  $P$ , there is a lid.

For every pot  $P$ , there is a lid  $L$  such that  $L$  is a lid for  $P$ .

## Definition.

An **existential universal statement** is a statement that is existential because its first part asserts that a certain object exists and is universal because its second part says that the object satisfies a certain property for all things of a certain kind:

There is a positive integer that is less than or equal to every positive integer.

The number one satisfies the above statement, which can also be rewritten:

There is a positive integer  $m$  that is less than or equal to every positive integer.

**Example.** Rewrite the following existence universal statement:

There is a person in my class who is at least as old as every person in my class.

Some person in my class is at least as old as every person in my class.

There is a person  $p$  in my class such that  $p$  is at least as old as every person in my class.

There is a person  $p$  in my class with the property that for every person  $q$  in my class,  $p$  is at least as old as  $q$ .