9.4: Hypothesis Testing for Means

Similar to hypothesis testing for proportions, we have the following four steps:

1. **Hypothesize:** formulate your hypotheses
2. **Check conditions:**
   - **Random and Independent:** The sample must be randomly collected from the population, and observations are independent of each other.
   - **Large Sample:** Either the population is Normal, *or* the sample size is large ($n \geq 25$).
   - **Large Population:** If the sample is collected without replacement, the population of size N must be at least 10 times bigger than the sample: $N \geq 10n$

   If these conditions are met, we compute the test statistic for the One-Sample t-Test which follows a t-distribution with $n-1$ degrees of freedom:

$$t = \frac{\overline{x} - \mu_0}{SE_{\text{est}}}, \qquad \text{where} \qquad SE_{\text{est}} = \frac{s}{\sqrt{n}}$$

3. **Compute:** Stating a significance level, compute the observed test statistic t and/or p-value.
4. **Interpret:** Decide whether to reject or fail to reject the null hypothesis.

Two-Sided	One-Sided (Left)	One-Sided (Right)
$H_0 : \mu = \mu_0$	$H_0 : \mu = \mu_0$	$H_0 : \mu = \mu_0$
$H_a : \mu \neq \mu_0$	$H_a : \mu < \mu_0$	$H_a : \mu > \mu_0$

Example. McDonald's advertises that its ice cream cones have a mean weight of 3.2 ounces. To test this, we find the weights of a sample of 5 cones:

$$4.2, 3.6, 3.9, 3.4, 3.3$$

Formulate the null and alternative hypotheses

Check the conditions required to perform a hypothesis test.

Find the test statistic and p-value

Using a significance level of $\alpha = 0.05$, decide whether to reject or fail to reject the null hypothesis.

Example. In the 2011-2012 academic year, the mean cost of attending two-year colleges in the United States was $\$3,831$. Has this increase over time? A random sample of 35 two-year colleges in 2014-2015 had a mean tuition of $\$4,173$, with a standard deviation of $\$2,590$.

Formulate the null and alternative hypotheses

Check the conditions required to perform a hypothesis test.

Find the test statistic and p-value

Using a significance level of $\alpha = 0.05$, decide whether to reject or fail to reject the null hypothesis.

Repeat this hypothesis test with a sample size of $n = 175$. What happens to the standard error when the sample size increases?