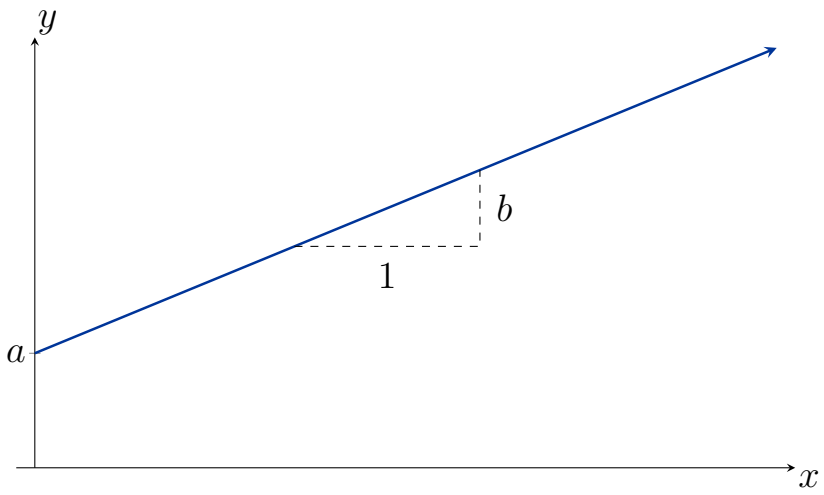
4.3: Modeling Linear Trends

Definition.
The **regression line** is a model used for making predictions about *future* observed values. The equation of the regression line is

$$y = a + bx$$

(when x=0)

where a is the **y-intercept** and b is the **slope**.



The input variable x is known as the

- Independent variable
- Predictor variable
- Explanatory variable

The output variable y is known as the

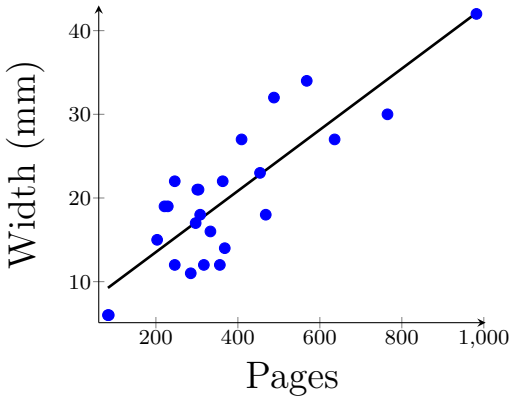
- Dependent variable
- Predicted variable
- Response variable

Example. Below is a scatterplot comparing number of pages a book has against the width of the book. Interpret the intercept and the slope of the regression line.

Predicted Width=6.22+0.0366 Pages

Intercept: Book with no pages (cover only) is 6.22mm wide

Slope: When we compare to a book with one more page, that book will be 0.0366mm wider

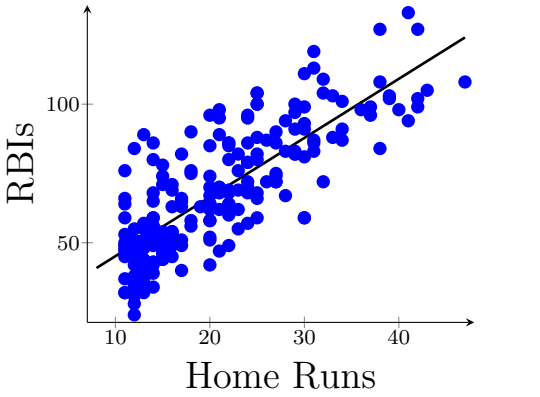


Example. Below is a scatterplot comparing the number of home runs and RBIs in the 2016 season. Interpret the intercept and slope of the regression line.

Predicted RBI=23.84+2.13 HR

Intercept: 23.84 RBI when number of HR=0

Slope: When comparing one player to another with one additional HR, the other player's RBI is expected to be 2.13 higher



Definition.

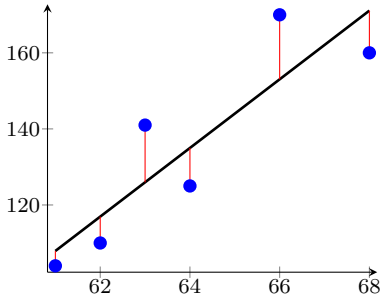
Now we define the formula of the regression line:

$$y = a + bx$$

Where

$$b = r\frac{s_y}{s_x} \quad \text{and} \quad a = \overline{y} - b\overline{x}.$$

These formulae minimize the residual error: Try this!



Example. Below are the heights and weights of six women:

Heights	61	62	63	64	66	68
Weights	104	110	141	125	170	160

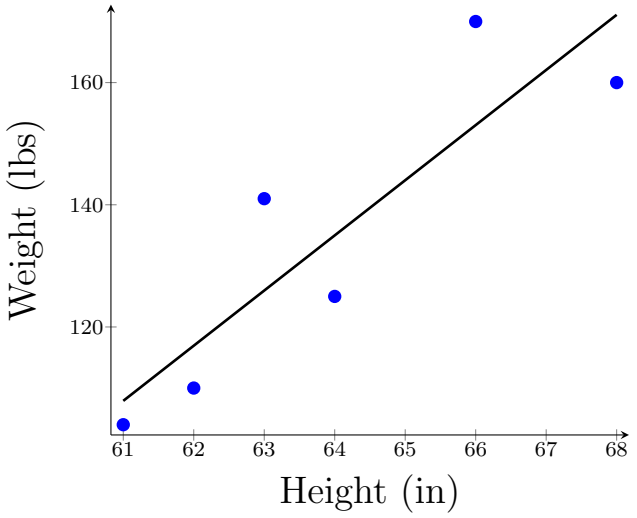
From this we get

$$\overline{x} = 64 \qquad s_x = 2.608$$
$$\overline{y} = 135 \qquad s_y = 26.728$$
$$r = 0.881$$

Find the equation of the regression line.

$$b = 0.881 \cdot \left(\frac{26.728}{2.608}\right) = 9.03$$

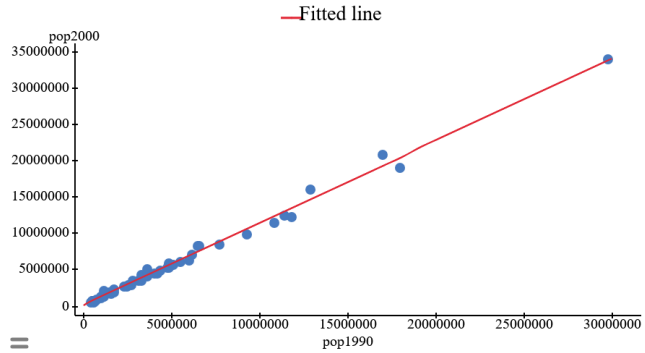
$$a = 135 - 9.03(64) = -442.92$$



Example. Open the `popdensity_and_crime` dataset in StatCrunch, use the "Simple Linear" tool under the `Stat›Regression` menu to find the regression line for the following columns. Interpret the slope and intercept where appropriate.

the `pop1990` and `pop2000` columns,

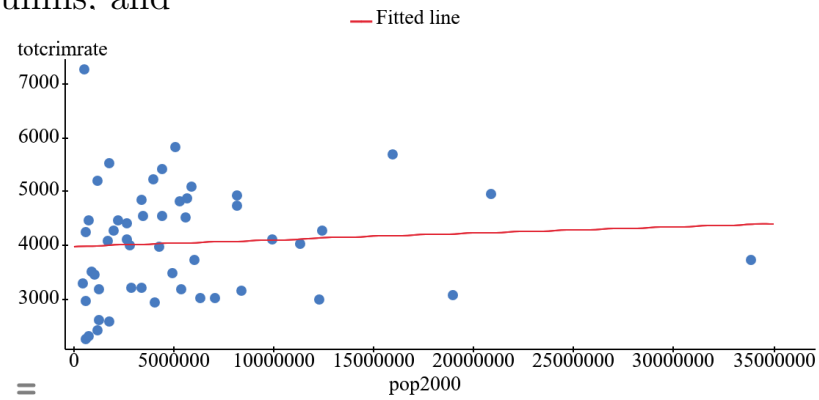
pop2000 = 12266.759 + 1.1295246 pop1990
R (correlation coefficient) = 0.99649554



Intercept: If pop=0 in 1990, it's 12,267 in 2000
(Not appropriate since data is 454k-30mil)

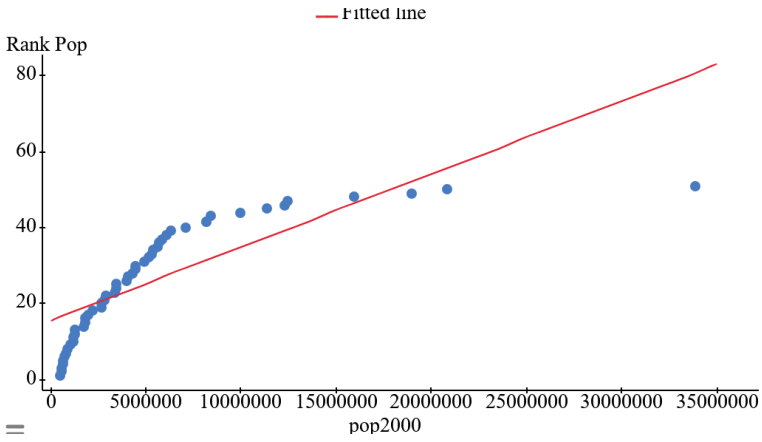
Slope: When comparing pop in 1990 where one group has 1 more person, that group is expected to have 1.13 more people in 2000

the `pop2000` and `totcrimerate` columns, and



Low correlation coefficient (~0.07)
Looks nonlinear
Linear regession is not appropriate here

the `pop2000` and `Rank Pop` columns.



Large correlation coefficient (~0.8),
but a nonlinear trend

Linear regression is not appropriate

4.4: Evaluating the Linear Model

Guidelines:

- Don't fit linear models to nonlinear associations!

- Correlation is not causation

- Beware of outliers (a.k.a. **influential points**)

- Don't extrapolate (make predictions beyond the range of the data)

Definition.
The **coefficient of determination** is the correlation coefficient coefficient squared:

$$r^2$$

This is sometimes also called r**-squared**.