

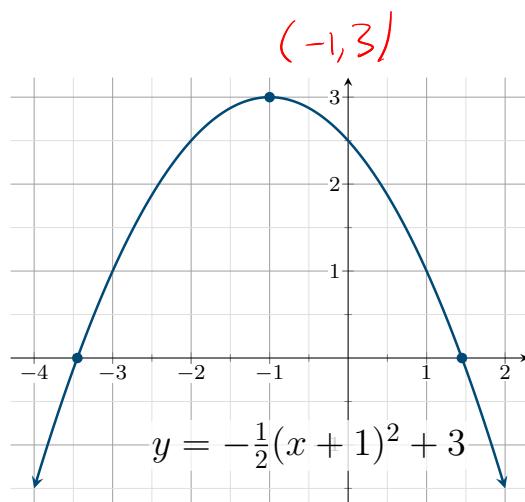
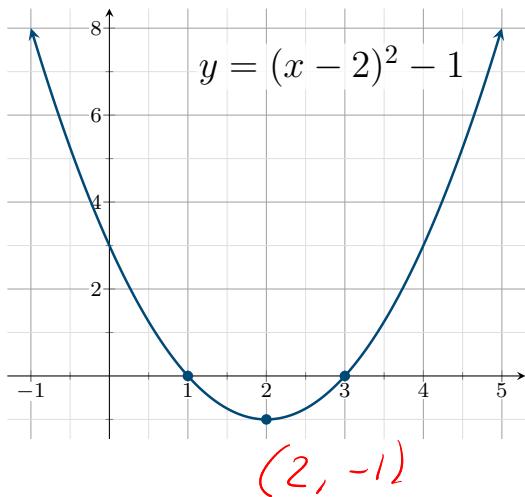
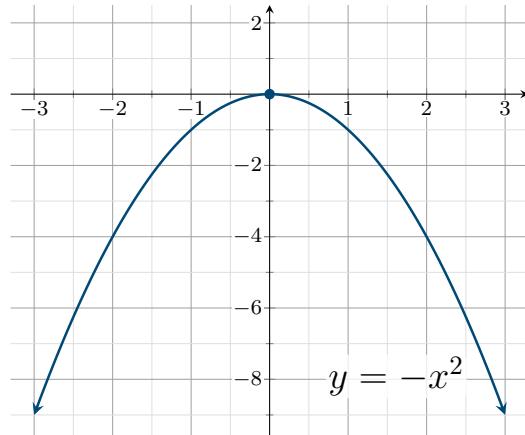
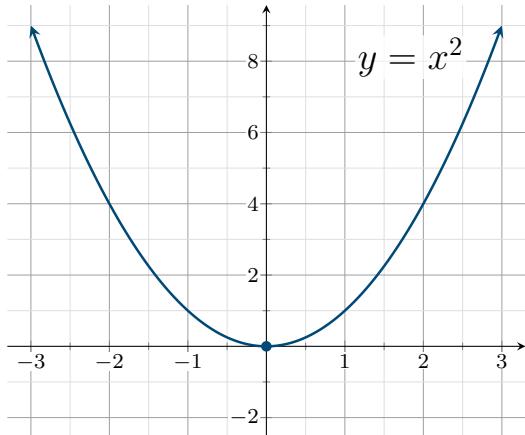
2.2: Quadratic Functions: Parabolas

Definition.

A **quadratic function** has the form

$$y = f(x) = ax^2 + bx + c \quad (a \neq 0)$$

where a , b , and c represent constants. A **parabola** is the shape of the graph of a quadratic function.



Definition.

The quadratic function $y = f(x) = ax^2 + bx + c$ has its **vertex** at

$$\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right).$$

The optimal value occurs at the vertex of a parabola:

- A maximum if $a < 0$ ↗
- A minimum if $a > 0$ ↘

Example. Consider the function

$$2x + \frac{x^2}{2}$$

Is the vertex a maximum or minimum? Locate the vertex, x -intercepts, y -intercept, and then sketch the graph.

$$a = \frac{1}{2} > 0 \Rightarrow \text{Vertex is a minimum}$$

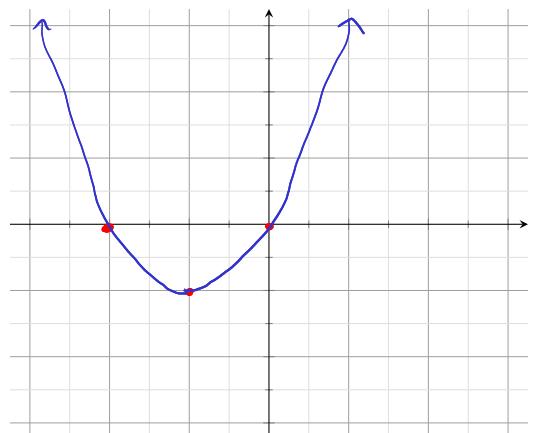
$$\text{Vertex: } \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right) = \left(-\frac{2}{2(\frac{1}{2})}, f\left(-\frac{2}{2(\frac{1}{2})}\right)\right) = (-2, -2)$$

x -intercepts ($y=0$)

$$0 = 2x + \frac{x^2}{2} = \frac{1}{2} \times (4 + x)$$
$$\downarrow \qquad \qquad \qquad \downarrow$$
$$x=0 \qquad \qquad x=-4$$
$$(0, 0) \qquad (4, 0)$$

y -intercept ($x=0$)

$$y = 2(0) + \frac{0^2}{2} = 0 \Rightarrow (0, 0)$$



Example. Consider the function

$$x^2 + 5 - 4x$$

Is the vertex a maximum or minimum? Locate the vertex, x -intercepts, y -intercept, and then sketch the graph.

$a = 1 > 0 \Rightarrow$ Vertex is a minimum

$$\text{Vertex: } \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right) \right) = \left(-\frac{-4}{2(1)}, f\left(-\frac{-4}{2(1)}\right) \right) = (2, 1)$$

x -intercepts ($y=0$)

$$0 = \frac{x^2}{a} - \frac{4x}{b} + \frac{5}{c}$$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)}$$

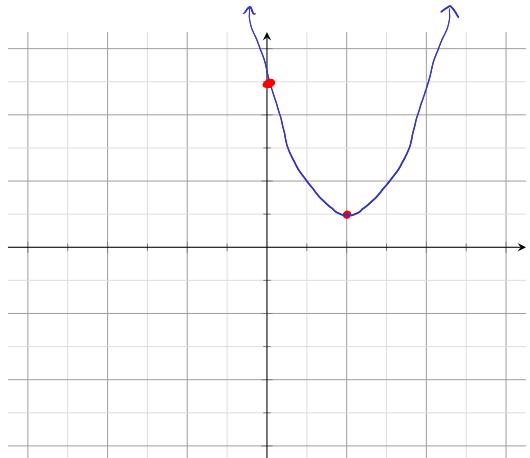
Determinant:

$$(-4)^2 - 4(1)(5) = -4 < 0 \Rightarrow \text{No solutions!}$$

\Rightarrow No x -intercepts

y -intercept ($x=0$)

$$y = 0^2 - 4(0) + 5 = 5 \rightarrow (0, 5)$$



Example. Ace Cruises offers a sunset cruise to a group of 50 people for a price of \$30 per person, but it reduces the price per person by \$0.50 for each additional person above 50. Find the revenue function. What price maximizes the revenue? What is this maximal value?

| Number of people | Price per person | Total Revenue |
|------------------|------------------|----------------------------|
| 50 | \$30.00 | \$1500.00 |
| 51 | \$29.50 | \$1504.50 |
| 52 | \$29.00 | \$1508.00 |
| ⋮ | ⋮ | ⋮ |
| 50+x | $30 - 0.5x$ | $R(x) = (50+x)(30 - 0.5x)$ |

$$R(x) = (50+x)(30 - 0.5x)$$

$$= \frac{1500}{c} + \frac{5x}{b} - \frac{0.5x^2}{a}$$

$a < 0 \Rightarrow$ vertex is a maximum.

$$\text{Vertex } \left(-\frac{b}{2a}, R\left(-\frac{b}{2a}\right) \right) \quad -\frac{b}{2a} = -\frac{5}{2(-0.5)} = 5$$

$$R(5) = 1500 + 5(5) - 0.5(5)^2$$

$$= 1500 + 25 - \frac{25}{2}$$

$$= \boxed{\$1512.50}$$