6.5: Loans and Amortization

Definition.
Amortization is the process of repaying a loan. When we have a debt of A_n, with an interest rate of i per period, amortized by n equal periodic payments (at the end of each period), the size of each payment is

$$R = A_n \left[\frac{i}{1 - (1+i)^{-n}} \right]$$

Example. A company modernizes its production by buying a new piece of equipment for $200,000. It makes a $50,000 down payment and agrees to amortize the rest of the debt with quarterly payments over the next 10 years. If the interest on the debt is 12%,

find the size of the quarterly payments

$R \longleftarrow$ Find

$A_n = 200{,}000 - 50{,}000 = 150{,}000$

$i = \frac{r}{m} = \frac{0.12}{4} = 0.03$

$n = m \cdot t = 4 \cdot 10 = 40$

$$R = 150{,}000 \left[\frac{0.03}{1-(1.03)^{-40}} \right] \approx \$6{,}489.36$$

find the total amount of the payments

$40 \cdot R = \boxed{\$259{,}574.27}$

find the total amount of interest paid

Interest $= \$259{,}574.27 - \$150{,}000 = \boxed{\$109{,}574.27}$

Example. For the following, calculate the regular payments

$100,000 for 3 years at 9% compounded annually

$R \longleftarrow$ Find

$A_n = 100{,}000$

$i = \frac{r}{m} = \frac{0.09}{1} = 0.09$

$n = m \cdot t = 1 \cdot 3 = 3$

$$R = 100{,}000\left[\frac{0.09}{1-(1.09)^{-3}}\right] \approx \boxed{\$39{,}505.48}$$

$30,000 for 5 years at 7% compounded annually

$R \longleftarrow$ Find

$A_n = 30{,}000$

$i = \frac{r}{m} = \frac{0.07}{1} = 0.07$

$n = m \cdot t = 1 \cdot 5 = 5$

$$R = 30{,}000\left[\frac{0.07}{1-(1.07)^{-5}}\right] \approx \boxed{\$7{,}316.72}$$

\$20,000 for 1 year at 9% compounded quarterly

$R \longleftarrow$ Find

$A_n = 20{,}000$

$i = \frac{r}{m} = \frac{0.09}{4} = 0.0225$

$n = m \cdot t = 4 \cdot 1 = 4$

$$R = 20{,}000\left[\frac{\frac{0.09}{4}}{1-\left(1+\frac{0.09}{4}\right)^{-4}}\right] \approx \boxed{\$5284.38}$$

\$50,000 for 2 1/2 years at 9% compounded semiannually

$R \longleftarrow$ Find

$A_n = 20{,}000$

$i = \frac{r}{m} = \frac{0.09}{2} = 0.045$

$n = m \cdot t = 2 \cdot 2.5 = 5$

$$R = 20{,}000\left[\frac{0.045}{1-(1.045)^{-5}}\right] \approx \boxed{\$11{,}389.58}$$

Example. Chuckie and Angelica have $\$30,000$ for a down payment, and their budget can accommodate a monthly mortgage of $\$1,200$. What is the $\underbrace{\text{most expensive home}}_{A_n + 30,000}$ they can buy if they borrow money for 30 years at 7.8% compounded monthly?

$R = 1200$

$A_n = \longleftarrow$ Find

$i = \frac{r}{m} = \frac{0.078}{12} = 0.0065$

$n = m \cdot t = 12 \cdot 30 = 360$

$$1200 = A_n \left[\underbrace{\frac{0.0065}{1-(1.0065)^{-360}}}_{\approx 0.0071987} \right]$$

$$\frac{1200}{0.0071987} = A_n = \$166,696.65$$

Use the exact value!
Don't round!!

$A_n +$ Down payment $= \boxed{\$196,696.65}$

Example. A loan of $\$10,000$ is to be amortized with 10 equal quarterly payments. If the interest rate is 6% compounded quarterly, what is the periodic payment?

$R = \longleftarrow$ Find

$A_n = 10,000$

$i = \frac{r}{m} = \frac{0.06}{4} = 0.015$

$n = m \cdot t = 4 \cdot t = 10$

$$R = 10,000 \left[\frac{0.015}{1-(1.015)^{-10}} \right]$$

$$= \boxed{\$1,084.34}$$