

3.6: Implicit Differentiation and Related Rates

Implicit Functions

$$x^2y + y - x^2 + 1 = 0$$

$$x^2 + y^2 = 4$$

$$y^3 + y^2 - xy + \frac{x^4}{4} = y$$

Explicit Functions

$$y = \frac{x^2 - 1}{x^2 + 1}$$

$$y = \pm\sqrt{4 - x^2}$$

[Graph](#)

Implicit Differentiation:

1. Differentiate both sides of the equation with respect to x , treating y as a differentiable function of x .
2. Collect the terms with dy/dx on one side of the equation.
3. Solve for dy/dx .

Example. Find the derivatives of the following by rewriting each function explicitly before taking the derivative, and by using implicit differentiation. Compare the results.

$$y^2 = x$$

$$\sqrt{x} + \sqrt{y} = 4$$

Example. Find $\frac{dy}{dx}$ given the equation

$$y^3 - y + 2x^3 - x = 8$$

Example. Consider the equation $x^2 + y^2 = 4$.

Find $\frac{dy}{dx}$ by implicit differentiation.

Find the slope of the tangent line to the graph of the function $y = f(x)$ at the point $(1, \sqrt{3})$.

Find an equation of the tangent line.

Related Rates:

Related rates are problems that use a mathematical relationship between two or more objects under specific constraints. From this, we can differentiate this relationship and examine how each variable changes with respect to time.

The volume of a cone with radius r and height h is given by

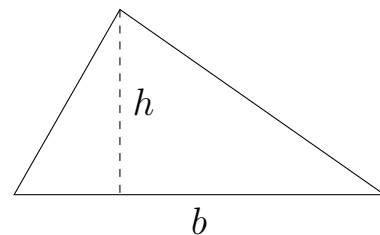
$$V = \frac{1}{3}\pi r^2 h$$

Find dV/dt when r and h are changing.

Find dV/dt when r is constant and h is changing.

Find dV/dt when r is changing and h is constant.

Example. The altitude of a triangle is increasing at a rate of $1\text{ cm}/\text{min}$ while the area of the triangle is increasing at a rate of $2\text{ cm}^2/\text{min}$. How fast is the base of the triangle changing when the altitude is 10 cm and the area is 100 cm^2 .



Example. The base of a 13-ft ladder leaning against a wall begins to slide away from the wall. At the instant of time when the base is 12 ft from the wall, the base is moving at a rate of 8 ft/sec. How fast is the top of the ladder sliding down the wall at that instant of time?

Example. At noon, ship A is 100 km west of ship B. Ship A is sailing south at 35 km/h and ship B is sailing north at 25 km/h. How fast is the distance between the ships changing at 4pm?

