## 3.2: Predicates and Quantified Statements II

**Definition.**

- The negation of a statement of the form

$$\forall x \text{ in } D,\ Q(x)$$

is logically equivalent to a statement of the form

$$\exists x \text{ in } D \text{ such that } \sim Q(x).$$

$$\sim (\forall x \in D,\ Q(x)) \equiv \exists x \in D \text{ such that } \sim Q(x).$$

- The negation of a statement of the form

$$\exists x \text{ in } D \text{ such that } Q(x)$$

is logically equivalent to a statement of the form

$$\forall x \text{ in } D,\ \sim Q(x).$$

$$\sim (\exists x \in D \text{ such that } Q(x)) \equiv \forall x \in D,\ \sim Q(x)$$

**Example.** Negate the following statements:

$\forall$ primes $p$, $p$ is odd

$\exists$ a triangle $T$ such that the sum of the angles of $T$ equals $200°$

**Example.** Rewrite the following statements formally, then write the formal and informal negations.

No politicians are honest

The number $1,357$ is not divisible by any integer between 1 and 37.

**Example.** Write informal negations for the following statements:

All computer programs are finite.

Some computer hackers are over 40.

**Negation of a Universal Conditional Statement**

$$\sim (\forall x,\ \text{if } P(x) \text{ then } Q(x)) \equiv \exists x \text{ such that } P(x) \text{ and } \sim Q(x)$$

**Definition.**
A statement of the form

$$\forall x \text{ in } D,\ \text{if } P(x) \text{ then } Q(x)$$

is called **vacuously true** or **true by default** if, and only if, $P(x)$ is false for every $x$ in $D$.

**Example.** The following statement is vacuously true since it's negation is false:

All elephants enrolled in my class are passing.

**Definition.**
Consider a statement of the form $\forall x \in D$, if $P(x)$ then $Q(x)$.

1. Its **contrapositive** is the statement $\forall x \in D$, if $\sim Q(x)$ then $\sim P(x)$.
2. Its **converse** is the statement $\forall x \in D$, if $Q(x)$ then $P(x)$.
3. Its **inverse** is the statement $\forall x \in D$, if $\sim P(x)$ then $\sim Q(x)$.

**Example.** Write a formal and informal contrapositive, converse, and inverse for the following statement:

If a real number is greater than 2, then its square is greater than 4.

**Definition.**

- "$\forall x, r(x)$ is a **sufficient condition** for $s(x)$" $\rightarrow$ "$\forall x$, if $r(x)$ then $s(x)$"
- "$\forall x, r(x)$ is a **necessary condition** for $s(x)$" $\rightarrow$ "$\forall x$, if $\sim r(x)$ then $\sim s(x)$"
  $\rightarrow$ "$\forall x$, if $s(x)$ then $r(x)$"
- "$\forall x, r(x)$ **only if** $s(x)$" $\rightarrow$ "$\forall x$, if $\sim s(x)$, then $\sim r(x)$"
  $\rightarrow$ "$\forall x$, if $r(x)$ then $s(x)$"

**Example.** Rewrite each of the following as a universal conditional statement, quantified either explicitly or implicitly. Do not use the word *necessary* or *sufficient.*

Squareness is a sufficient condition for rectangularity.

Being at least 35 years old is a necessary condition for being president of the United States.

**Example.** Rewrite the following as a universal conditional statement:

A product of two numbers is 0 only if one of the numbers is 0.