Exploiting Uncertified Controllers via Uniting Feedback

Paul K. Wintz

PhD Defense – October 3, 2025

UC SANTA CRUZ | Baskin Engineering

Hybrid Systems Laboratory

Introduction – Cyber-Physical Systems

Cyber-physical systems are electromechanical systems that include digital electronics (e.g., sensors and computers) that interact with physical components or processes.

Heating and Air Conditioning

Spacecraft

Walking Robots

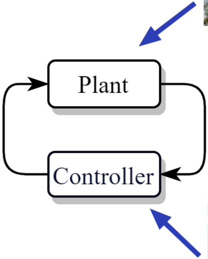
Chemical Plants

Quadcopters

Autonomous Cars

Introduction – Cyber-Physical Systems Control Problems

Physical Dynamics



Digital Processor

Fundamental Problems of Control Theory

- Don't Crash
- Get Where You're Going
- Minimize cost/Maximize rewards

Complicating Factors

- Complex dynamics
- Uncertainty
- Dynamic environments
- Computational limitations
- ...

Introduction – Spectrum of Control Methods

Experiments	Statistical Verification	Barrier Functions
Reinforcement Learning	Conformal prediction	Lyapunov Functions
Simulations	Model Predictive Control	Reachability Analysis

$\xleftarrow{\text{Easier to apply to complex systems}}$

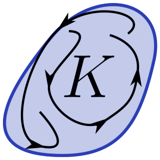
$\xrightarrow[\text{Stronger verification guarantees}]{}$

$\Longrightarrow$ How can we combine these approaches?

Control Objectives

Forward Invariance

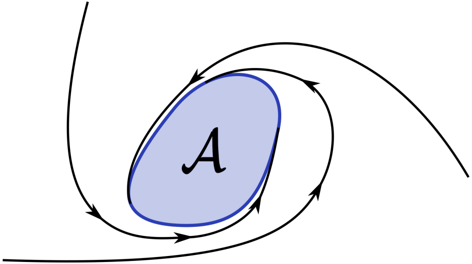
("Don't Crash")



Every trajectory remains in $\mathcal{A}$.

Global Asymptotic Stability

("Get Where You're Going")

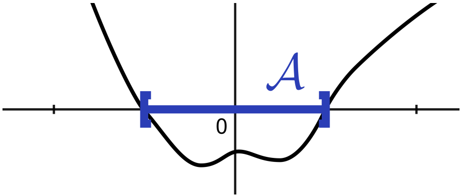


Every trajectory moves toward $\mathcal{A}$.

Certificate Functions

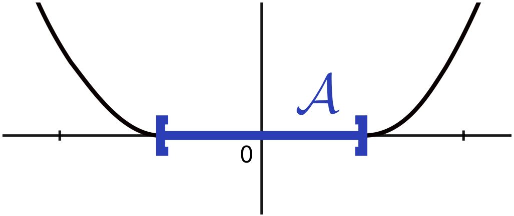
To prove forward invariance and asymptotic stability, we use *certificate functions*.

Forward Invariance



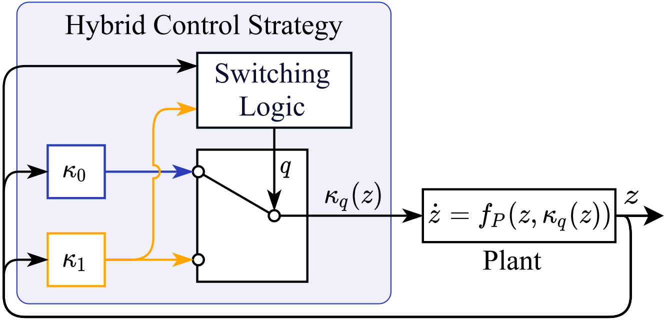
Certificate: Barrier Function

Asymptotic Stability



Certificate: Lyapunov Function

Uniting Feedback Control



κ_0 : certified controller, guaranteed to achieve control objective

κ_1 : any controller

Why Use an Uncertified Controller?

Certificates are hard to construct.

We may have an "advanced" controller that is difficult/impossible to certify but is

- Uses less energy
- Produces faster convergence
- Requires less computation
- Explores a region, collecting measurements.

Examples:

- Linear quadratic regulator (LQR) for the linearization of a system.
- Model predictive control (MPC) with computational delays.
- Black box controllers (e.g., neural network controllers).

Introduction – Previous Uses of Switched Controllers

Sometimes, a single continuous controller cannot satisfy design requirements.

Switching has been used to...

- Achieve robust global asymptotic stability around obstructions.[1]
- Unite multiple Lyapunov-certified controllers (such as local and global controllers) to achieve global asymptotic stability.[2]

[1] Mayhew, Ricardo G. Sanfelice, and Teel (2011), "Quaternion-Based Hybrid Control for Robust Global Attitude Tracking".
Ricardo G. Sanfelice et al. (2006), "Robust Hybrid Controllers for Continuous-Time Systems with Applications to Obstacle Avoidance and Regulation to Disconnected Set of Points".

[2] Prieur (2001), "Uniting Local and Global Controllers with Robustness to Vanishing Noise".
Teel and Kapoor (1997), "Uniting Local and Global Controllers".
El-Farra, Mhaskar, and Christofides (2005), "Output Feedback Control of Switched Nonlinear Systems Using Multiple Lyapunov Functions".

Simplex architecture

The *Simplex architecture* is an approach for switching between an "advanced," unverified controller and a "simple," easy-to-verify controller.[3]

Barrier functions have been used with the Simplex architecture to guarantee safety for hybrid systems while using an unverified controller.

Prior approaches have drawbacks:

- Requires costly reachability analysis and only defines "one way" switching.[4]
- Only rectangular constraints are considered, and the switching criteria depends on the extremal values of the vector field over the entire admissible set.[5]

[3]Rivera et al. (1996), *An Architectural Description of the Simplex Architecture.*
Seto et al. (1998), "The Simplex Architecture for Safe Online Control System Upgrades".
[4]Yang et al. (2017), "A Simplex Architecture for Hybrid Systems Using Barrier Certificates".
[5]Damare et al. (2022), "A Barrier Certificate-Based Simplex Architecture with Application to Microgrids".

Outline

Outline

Uniting Feedback for Safety

Uniting Feedback for Global Asymptotic Stability

Relaxed Lyapunov Conditions

Uniting Feedback with Hybrid Controllers and Hybrid Plants

Software Tools

Conclusion

Uniting Feedback for Forward Invariance

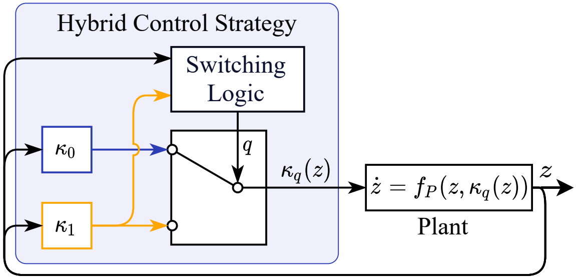
Given: A nonlinear plant

$$\dot{z} = f_{\mathrm{P}}(z, u), \quad z \in \mathbb{R}^{n_{\mathrm{P}}},\ u \in \mathbb{R}^{m_{\mathrm{P}}}.$$

and controllers

κ_0: barrier-certified to render K forward invariant

κ_1: any controller

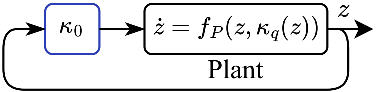


Goal: Design switching logic for $q \in \{0, 1\}$ such that

- K is forward invariant.
- κ_1 is preferred over κ_0.
- Switching does not chatter.

Barrier Function Certificate

$$\dot{z} = f_0(z) := f_{\mathrm{P}}(z, \kappa_0(z)).$$



Has a *barrier function* B that certifies

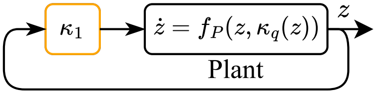
$$K = \{z \in \mathbb{R}^n \mid B(z) \leq 0\}$$

is forward invariant.
On some neighborhood U of K,

$$\dot{B}_0(z) := \langle \nabla B(z), f_0(z) \rangle \leq 0$$

for all $z \in U \setminus K$.

$$\dot{z} = f_1(z) := f_{\mathrm{P}}(z, \kappa_1(z)).$$



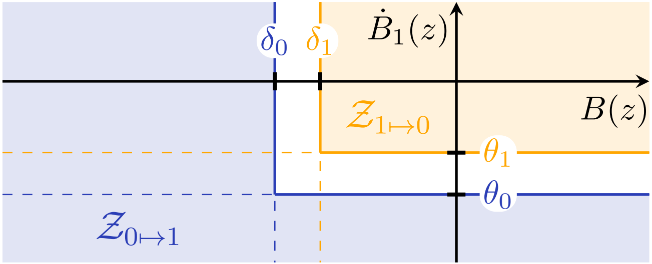
Rate of change of B when using κ_1:

$$\dot{B}_1(z) := \langle \nabla B(z), f_1(z) \rangle = ??.$$

Switching Criteria and Hold Criteria

Pick four *thresholds* δ_0, δ_1, θ_0, θ_1, such that

$$\delta_0 < \delta_1 \leq 0 \quad \text{and} \quad \theta_0 < \theta_1 \leq 0.$$



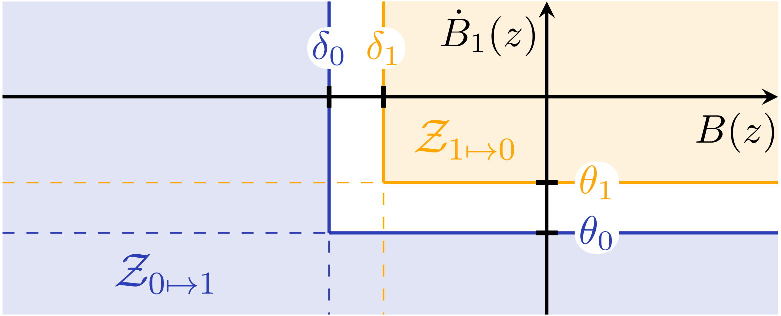
For $q = 0$ (certified controller):

- Hold $q = 0$ if $z \in \mathcal{Z}_0$.
- Switch to $q = 1$ if $z \in \mathcal{Z}_{0\mapsto 1}$.

For $q = 1$ (uncertified controller):

- Hold $q = 1$ if $z \in \mathcal{Z}_1$.
- Switch to $q = 0$ if $z \in \mathcal{Z}_{1\mapsto 0}$.

Switching Criteria and Hold Criteria



$$\mathcal{Z}_{1\mapsto 0} := \{z \in \mathbb{R}^n \mid B(z) \geq \delta_1,\ \dot{B}_1(z) \geq \theta_1\}$$

$$\mathcal{Z}_{0\mapsto 1} := \{z \in \mathbb{R}^n \mid B(z) \leq \delta_0 \text{ or } \dot{B}_1(z) \leq \theta_0\}.$$

Dynamics of Closed-Loop System

Between switches:

- z evolves according to $\dot{z} = f_{\mathrm{P}}(z,\ \kappa_q\ (z))$
- q is constant

At each switch:

- z is unchanged
- q is toggled to the opposite value in $\{0, 1\}$

Example: Linear and Affine Feedbacks

Consider the double integrator

$$\dot{z} = f_{\mathrm{P}}(z,u) := \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} z + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u.$$

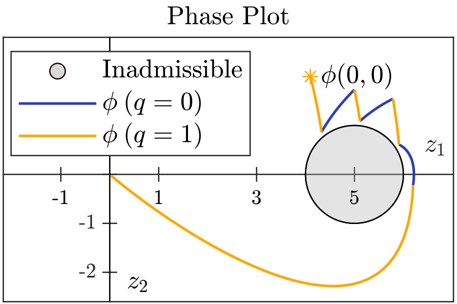
Admissible set:

$$K := \left\{ z \in \mathbb{R}^2 : |z - (5,0)| \geq 1 \right\}.$$

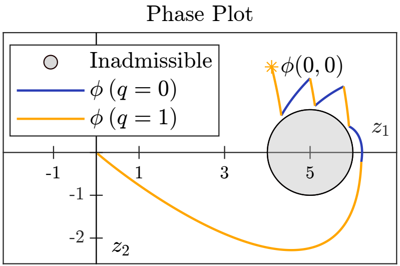
Controllers:

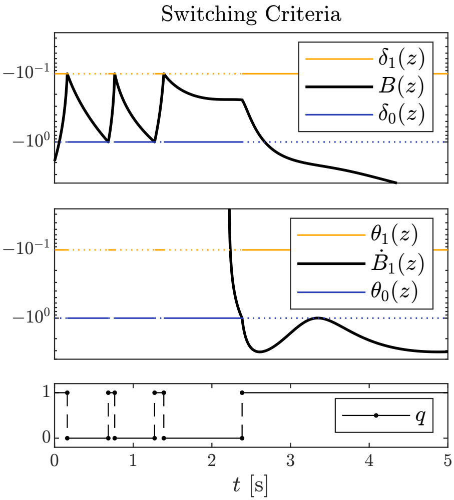
$$\kappa_0(z) = \begin{bmatrix} -1 & 1 \end{bmatrix} (z - c) \quad \text{(certified)}$$

$$\kappa_1(z) = \begin{bmatrix} -1 & -2 \end{bmatrix} z \quad \text{(uncertified)}$$

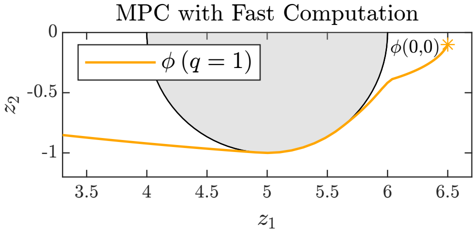


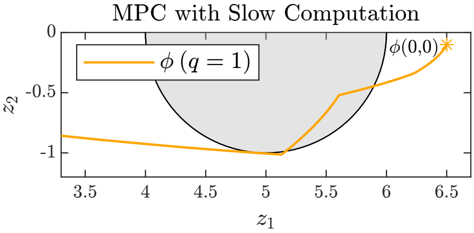
Example: Linear and Affine Feedbacks

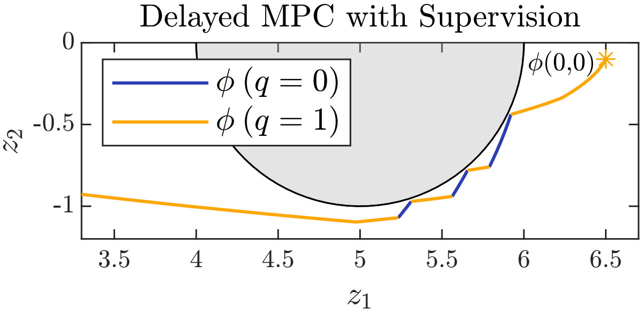




Example: MPC with Computational Delays







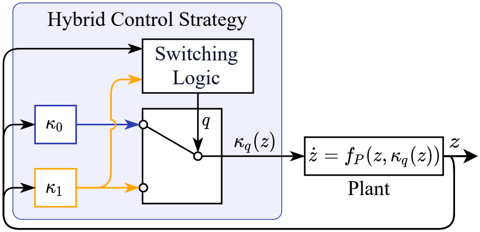
Theorem: Forward Invariance

Suppose that

- B is a continuously differentiable barrier function of K for $\dot{z} = f_0(z)$.
- f_0 and f_1 are continuous.

Then, K is forward invariant w.r.t. z for the closed-loop system.

Closed-loop System



Remark. We also give conditions to ensure that

- Solutions exist for all $t \geq 0$.
- The time between switches is not too short.

Outline

Problem Setting

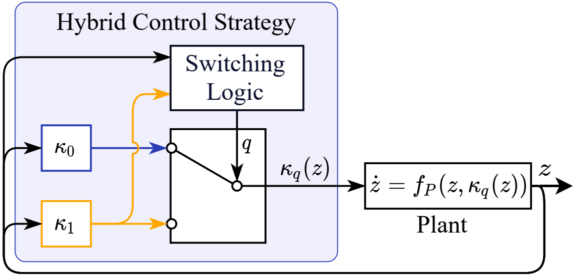
Given: A control system

$$\dot{z} = f_{\mathrm{P}}(z, u), \quad z \in \mathbb{R}^{n_{\mathrm{P}}}, u \in \mathbb{R}^{m_{\mathrm{P}}}.$$

and controllers

κ_0: Lyapunov-certified to render $\mathcal{A}$ globally asymptotically stable

κ_1: any controller



Goal: Design switching logic for $q \in \{0, 1\}$ such that

- $\mathcal{A}$ is globally asymptotically stable
- κ_1 is preferred over κ_0
- switching does not chatter.

Problem Setting – Lyapunov-certified Controller

For the *Lyapunov-certified* controller κ_0, there exists a Lyapunov function

$$V_{\mathrm{P}} : \mathbb{R}^{n_{\mathrm{P}}} \to [0, \infty)$$

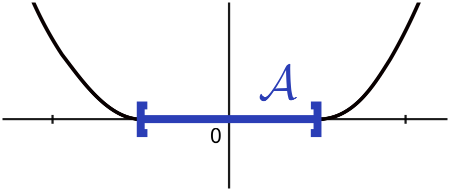
for

$$\dot{z} = f_0(z) := f_{\mathrm{P}}(z, \kappa_0(z)),$$

that guarantees $\mathcal{A}$ is globally asymptotically stable.

Value of V_{P} decreases outside $\mathcal{A}$:

$$\dot{V}_0 := \langle \nabla V_{\mathrm{P}}, f_0(z) \rangle \le -\sigma_0(z).$$



For the *uncertified* controller κ_1, no assumptions on the rate of change of V_{P},

$$\dot{V}_1 := \langle \nabla V_{\mathrm{P}}, f_1(z) \rangle = ??$$

for

$$\dot{z} = f_1(z) := f_{\mathrm{P}}(z, \kappa_1(z)).$$

Example: Model Predictive Controller with Slow Computation

Consider a nonlinear plant

$$\dot{z} = f_{\mathrm{P}}(z, u)$$

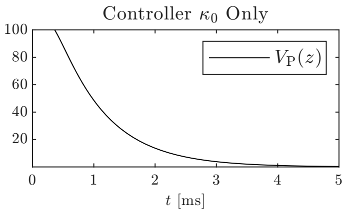
and two controllers:

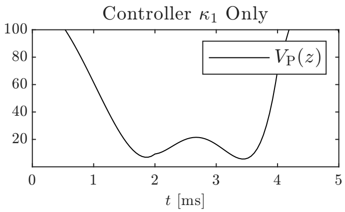
κ_0: Lyapunov-certified controller

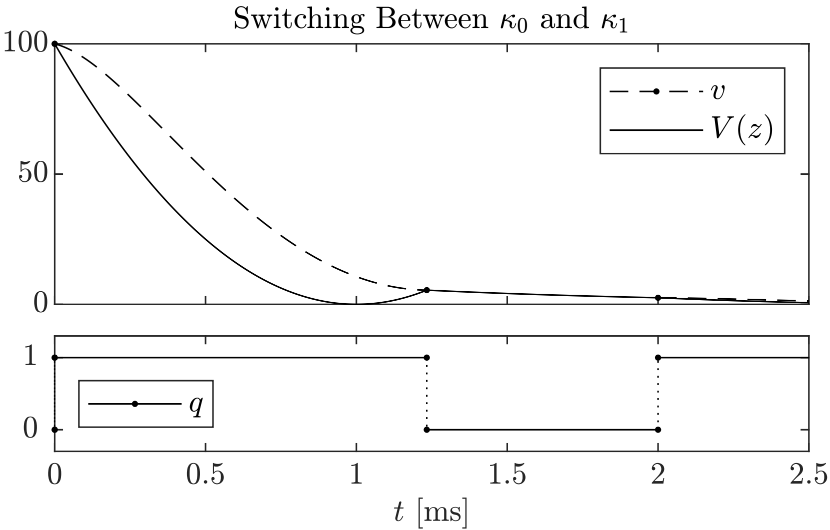
κ_1: Model predictive controller (MPC) with a sampling period of $1\,\mathrm{ms}$

Suppose new MPC feedback value is not available at $1\,\mathrm{ms}$.

When should we switch?







- The dynamics of v are described later.

Hybrid Control Strategy – Switching Logic

Buffer function: Pick a continuous, positive function $x \mapsto \delta(x) > 0$

For $q = 0$ (certified controller):

V_{P} is "small enough to switch to $q = 1$" if

$$V_{\mathrm{P}}(z) + \delta(z) \leq v$$

V_{P} is "large enough to hold $q = 0$" if

$$V_{\mathrm{P}}(z) + \delta(z) \geq v$$

For $q = 1$ (uncertified controller):

V_{P} is "small enough to hold $q = 1$" if

$$V_{\mathrm{P}}(z) \leq v$$

V_{P} is "large enough to switch to $q = 0$" if

$$V_{\mathrm{P}}(z) \geq v$$

Example: Switching Logic

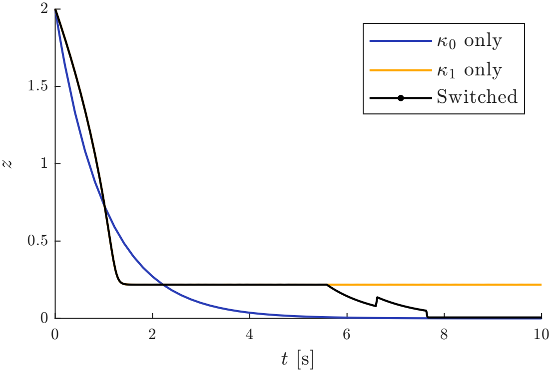
Consider the plant

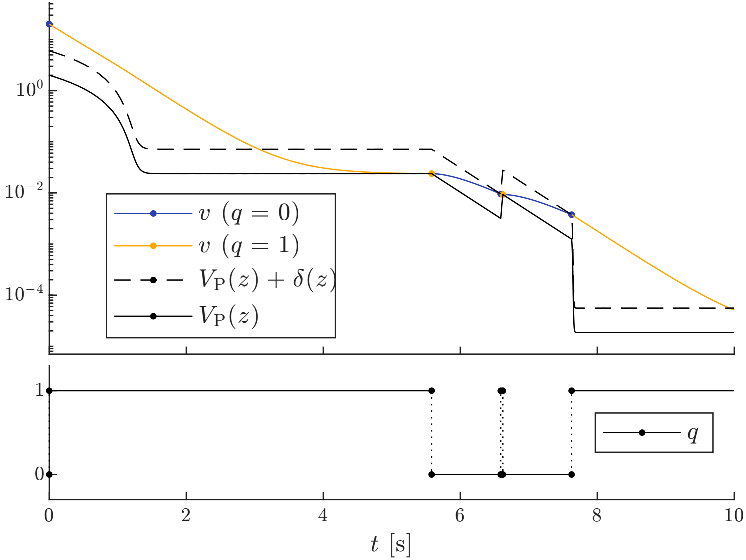
$$\dot{z} = u$$

with $z > 0$ and $u \in \mathbb{R}$ and controllers

$$\kappa_0(z) := -z$$

$$\kappa_1(z) := -2\sin\left(\frac{1}{z+0.1}\right)$$





10^0
10^{-2}
10^{-4}
$v\ (q=0)$
$v\ (q=1)$
$V_{\mathrm{P}}(z)+\delta(z)$
$V_{\mathrm{P}}(z)$
1
0
q
0
2
4
6
8
10
t [s]

Dynamics of Auxiliary v Variable

At each switch:

- v is set to $\max\{V_{\mathrm{P}}(z), v\}$

Between switches:

- v evolves according to

$$\dot{v} := -\gamma \tanh(v)\sigma_0(z) - \mu\big(v - V_{\mathrm{P}}(z)\big),$$

where $\gamma > 0$, $\mu > 0$, and σ_0 is continuous and positive definite.

- v converges to 0.
- If $q = 0$ and $v < V_{\mathrm{P}}(z)$, then v can increase gradually.

Example: Linear Quadratic Regulator of Linearized System

Consider the nonlinear plant

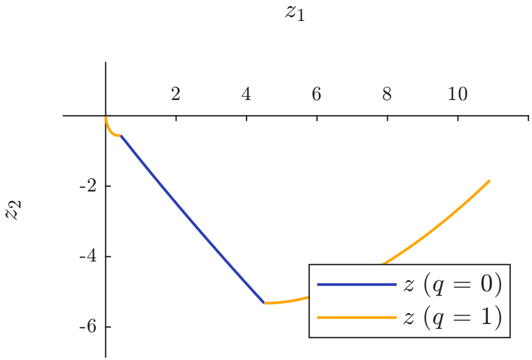
$$\dot{z} = Az + Bu + \underbrace{f(z,u)}_{\text{Nonlinear component}} .$$

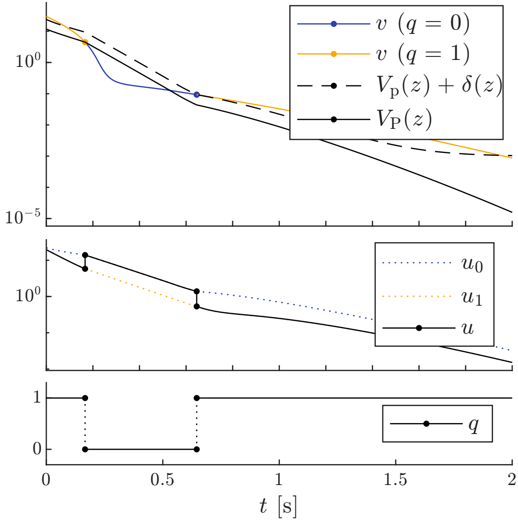
Let κ_0 be an (inefficient) Lyapunov-certified controller.

Let κ_1 be the LQR feedback that solves the following LQR problem:

$$\begin{aligned} \underset{u(\cdot)}{\text{minimize}} \quad & \int_0^\infty |z(t)|^2 + |u(t)|^2 \, dt \\ \text{subject to} \quad & \dot{z} = Az + Bu. \end{aligned}$$

Example: LQR of Linearized System





Theorem:[6] Global Asymptotic Stability

Suppose that

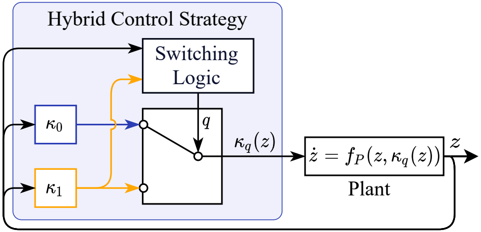
- $\mathcal{A}$ is compact;
- f_0 and f_1 are continuous;
- V_{P} is a Lyapunov function for $\dot{z} = f_0(z)$.

Then,

$$\widetilde{\mathcal{A}} := \{(z, v, q) \mid z \in \mathcal{A}, v = 0\}$$

is (uniformly) globally asymptotically stable for the closed-loop system.

Closed-loop System



Remark. The asymptotic stability of $\widetilde{\mathcal{A}}$ is robust to small perturbations.

[6] Paul K. Wintz, Ricardo G. Sanfelice, and Hespanha (2022), "Global Asymptotic Stability of Nonlinear Systems While Exploiting Properties of Uncertified Feedback Controllers via Opportunistic Switching".

Summary

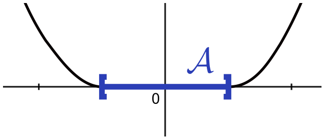
- Lyapunov-certified controller acts as a backup to ensure convergence while using an uncertified controller.
- Exploit useful properties of **any** uncertified continuous controller without losing the convergence guarantee.

Next steps

- We consider more general systems later.

Outline

Lyapunov Theorems



Given: A dynamical system and a set $\mathcal{A} \in \mathbb{R}^n$.

Goal: Prove that $\mathcal{A}$ is asymptotically stable.

Method: Construct $V : \mathbb{R}^n \to [0, \infty)$ such that

1. V is positive definite with respect to $\mathcal{A}$.
2. $t \mapsto V(x(t))$ is decreasing for each solution $t \mapsto x(t)$ while $x(t) \notin \mathcal{A}$.

If f is Lipschitz continuous, then the "decreasing" condition for $\dot{x} = f(x)$ is

$$\dot{V}(x) := \langle \nabla V(x),\, f(x) \rangle < 0.$$

Lyapunov-like Theorems (Non-smooth Systems)

We also consider *non-smooth* systems:

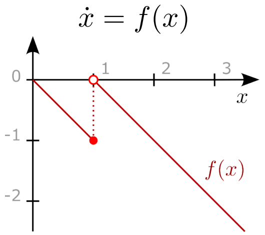
1. $\dot{x} = f(x)$ with f discontinuous (Non-smooth ODE)
2. $\dot{x} \in F(x)$ (differential inclusion)
3. $x^+ \in G(x)$ (difference inclusion)
4. $\mathcal{H} : \begin{cases} \dot{x} \in F(x) & \forall x \in C \\ x^+ \in G(x) & \forall x \in D. \end{cases}$ (hybrid system)

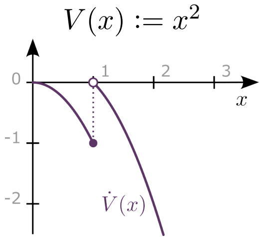
We can also have V non-differentiable.

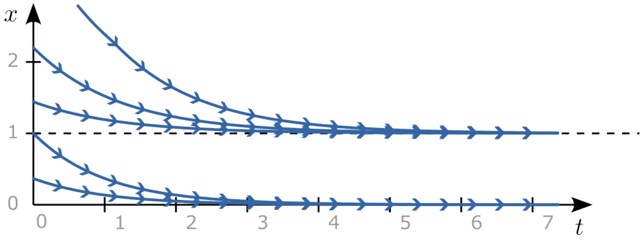
For non-smooth systems,

$$\left(\dot{V}(x) < 0 \text{ for all } x \notin \mathcal{A}\right) \not\Rightarrow \left(\mathcal{A} \text{ is globally asymptotically stable}\right)$$

Example: $\dot{V} < 0$ without convergence to 0





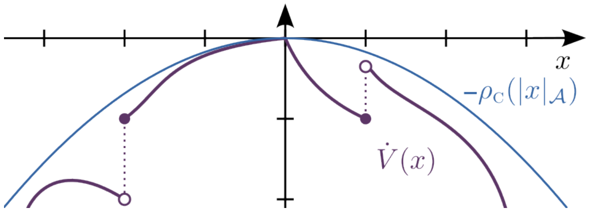


Lyapunov-like Theorems (Nonsmooth Systems)

For nonsmooth systems, prior results replace "$\dot{V}(x) < 0$" with

$$\dot{V}(x) \leq -\rho_{\mathrm{C}}(|x|_{\mathcal{A}}) \quad \forall x \in \mathbb{R}^n, \tag{1}$$

where $\rho_{\mathrm{C}} : [0, \infty) \to [0, \infty)$ is continuous and positive definite.



Lyapunov-like theorems – Relaxed Lyapunov Condition

It is often difficult to construct ρ_{C}.

- Must be continuous.
- Must be a function of the *distance* from $\mathcal{A}$.

When $\mathcal{A}$ is compact, we found a relaxation of the Lyapunov conditions.[7]

If there exists $\sigma_{\mathrm{LSC}} : \mathbb{R}^n \to [0, \infty)$ such that

- σ_{LSC} is a function of x (instead of $|x|_{\mathcal{A}}$),
- σ_{LSC} is lower semicontinuous (instead of continuous),
- σ_{LSC} is positive definite with respect to $\mathcal{A}$, and
- $\dot{V}(x) \leq -\sigma_{\mathrm{LSC}}(x)$...

...then there exists ρ_{C} (continuous and positive definite) such that

$$\dot{V}(x) \leq -\rho_{\mathrm{C}}(|x|_{\mathcal{A}}).$$

[7] Paul K Wintz and Ricardo G Sanfelice (2025), "Relaxed Lyapunov Conditions".

Problem Statement: Construction of ρ_{C}

Given: $\mathcal{A}$ (compact) and σ_{LSC}

Goal: Construct ρ_{C} such that

$$\sigma_{\mathrm{LSC}}(x) \geq \quad \rho_{\mathrm{C}}(|x|_{\mathcal{A}}) \qquad \forall x \in \mathbb{R}^n.$$

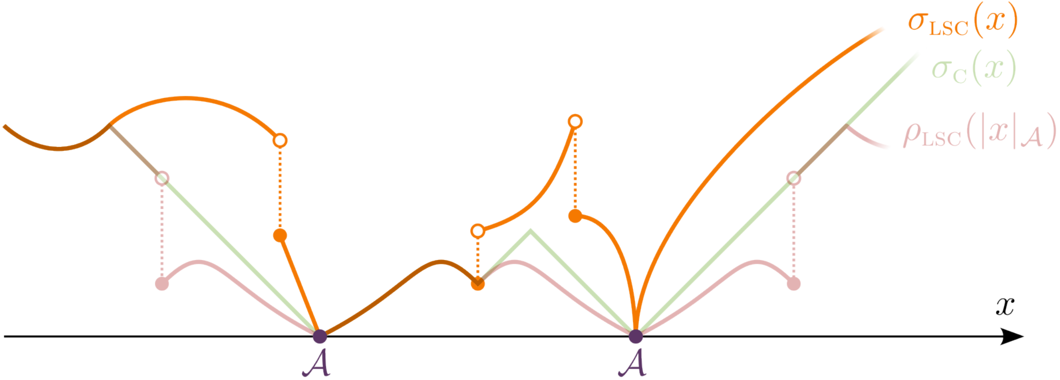
$$-\sigma_{\mathrm{LSC}}(x) \leq -\rho_{\mathrm{C}}(|x|_{\mathcal{A}}) \qquad \forall x \in \mathbb{R}^n.$$

Then,

$$\left(\dot{V}(x) \leq -\sigma_{\mathrm{LSC}}(x)\right) \implies \left(\dot{V}(x) \leq -\sigma_{\mathrm{LSC}}(x) \leq -\rho_{\mathrm{C}}(|x|_{\mathcal{A}})\right).$$

Construction of ρ_{C} Outline

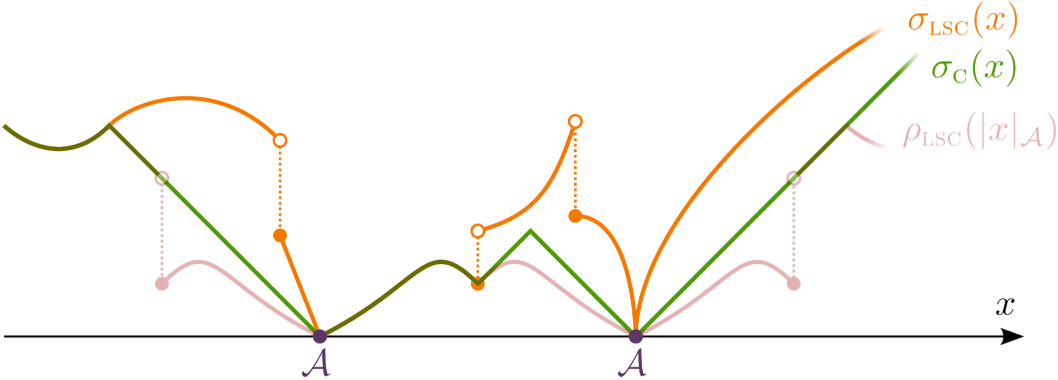
$\sigma_{\mathrm{LSC}}(x)$ $> 0 \qquad \forall x \notin \mathcal{A}.$



Construction of ρ_{C} Outline

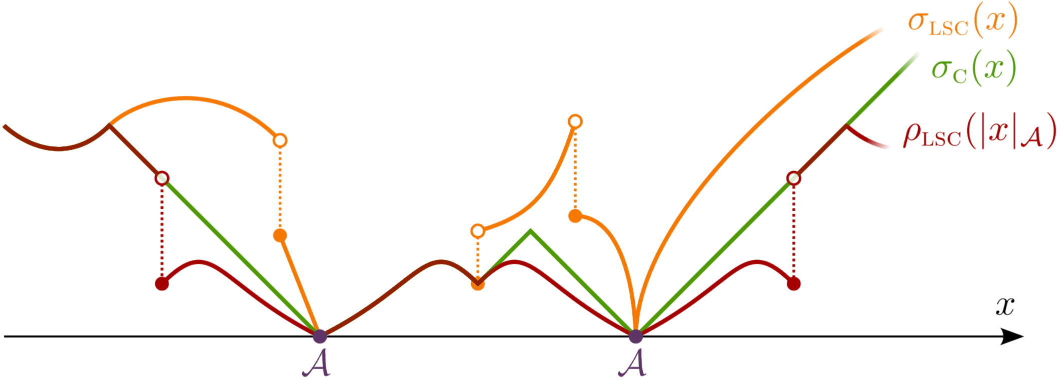
$\sigma_{\mathrm{LSC}}(x) \geq \sigma_{\mathrm{C}}(x)$ $> 0 \quad \forall x \notin \mathcal{A}.$

continuous



Construction of ρ_{C} Outline

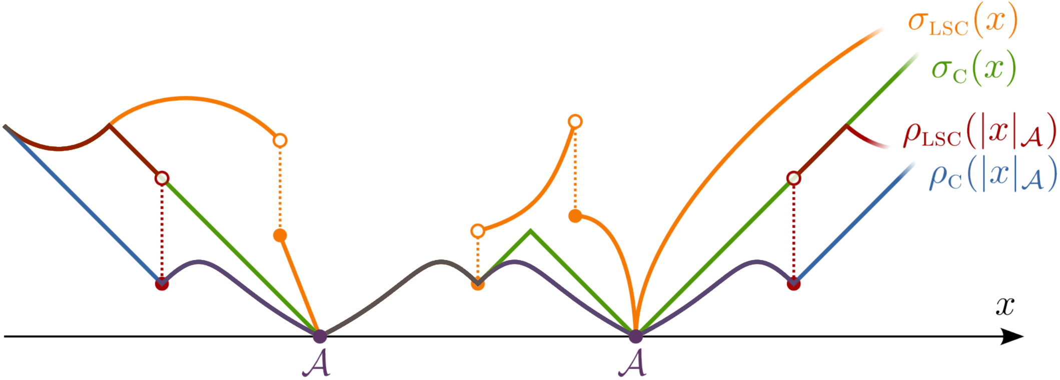
$$\sigma_{\mathrm{LSC}}(x) \geq \underbrace{\sigma_{\mathrm{C}}(x)}_{\text{continuous}} \geq \rho_{\mathrm{LSC}}(|x|_{\mathcal{A}}) \qquad > 0 \qquad \forall x \notin \mathcal{A}.$$



Construction of ρ_{C} Outline

$$\underbrace{\sigma_{\mathrm{LSC}}(x) \geq \sigma_{\mathrm{C}}(x)}_{} \geq \rho_{\mathrm{LSC}}(|x|_{\mathcal{A}}) \geq \rho_{\mathrm{C}}(|x|_{\mathcal{A}}) > 0 \qquad \forall x \notin \mathcal{A}.$$

continuous (under $\sigma_{\mathrm{C}}(x)$)   continuous (under $\rho_{\mathrm{C}}(|x|_{\mathcal{A}})$)



Given $\sigma_{\mathrm{LSC}}(x)$.

Make continuous

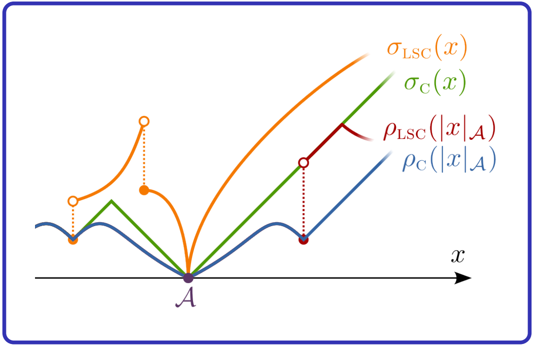
$$\sigma_{\mathrm{C}}(x) := \min_{x' \in \mathbb{R}} \big(\sigma_{\mathrm{LSC}}(x') + |x' - x| \big).$$

Make function of distance (but LSC)

$$\rho_{\mathrm{LSC}}(r) := \min_{x \in \mathbb{R}^n} \big\{ \sigma_{\mathrm{C}}(x) : |x|_{\mathcal{A}} = r \big\}.$$

Make continuous, function of distance

$$\rho_{\mathrm{C}}(r) := \min_{r' \geq 0} \big(\rho_{\mathrm{LSC}}(r) + |r' - r| \big).$$



Relaxed Lyapunov Theorem for Continuous-time Systems

Consider a hybrid system $\dot{x} \in F(x)$ on $\mathbb{R}^n$, a nonempty compact set $\mathcal{A} \subset \mathbb{R}^n$, and a Lyapunov function candidate V with respect to $\mathcal{A}$ for $\mathcal{H}$.

Suppose that

1. there exists $\alpha \in \mathcal{K}_\infty$ such that $\alpha(|x|_\mathcal{A}) \leq V(x)$ for all $x \in \mathbb{R}^n$, and
2. there exist LSC function σ_{LSC} that is positive definite w.r.t. $\mathcal{A}$ such that

$$\dot{V}(x) \leq -\sigma_{\text{LSC}}(x) \qquad\qquad \forall x \in \mathbb{R}^n.$$

Then, $\mathcal{A}$ is (uniformly) globally asymptotically stable for $\dot{x} \in F(x)$.

How pick a good choice for σ_{LSC}

For any function $f : \mathbb{R}^n \to \mathbb{R}$, the function

$$x \mapsto \liminf_{x' \to x} f(x')$$

is lower semicontinuous and

$$\liminf_{x' \to x} f(x') \leq f(x) \qquad \forall x \in \operatorname{dom} f$$

Thus, if we pick

$$x_0 \mapsto \sigma_{\mathrm{LSC}}(x_0) := \liminf_{x \to x_0} \Big(-\dot{V}(x) \Big),$$

then

$$\dot{V}(x) \leq -\sigma_{\mathrm{LSC}}(x_0).$$

Example (Continuous-time with discontinuous f)

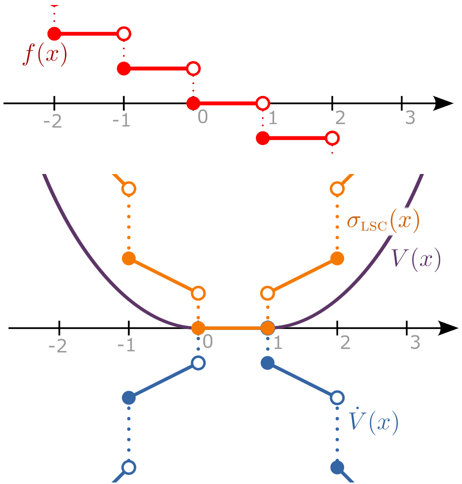
Consider the continuous-time system

$$\dot{x} = f(x) := -\lfloor x \rfloor \quad \forall x \in \mathbb{R},$$

Let $\mathcal{A} := [0, 1]$ and $x \mapsto V(x) := |x|^2_{\mathcal{A}}$.
Let

$$\sigma_{\mathrm{LSC}}(x) := \liminf_{x' \to x} -\dot{V}(x)$$

$\implies$ $\dot{V}(x) \leq -\sigma_{\mathrm{LSC}}(x)$.
$\implies$ $\mathcal{A}$ is globally asymptotically stable.



Summary

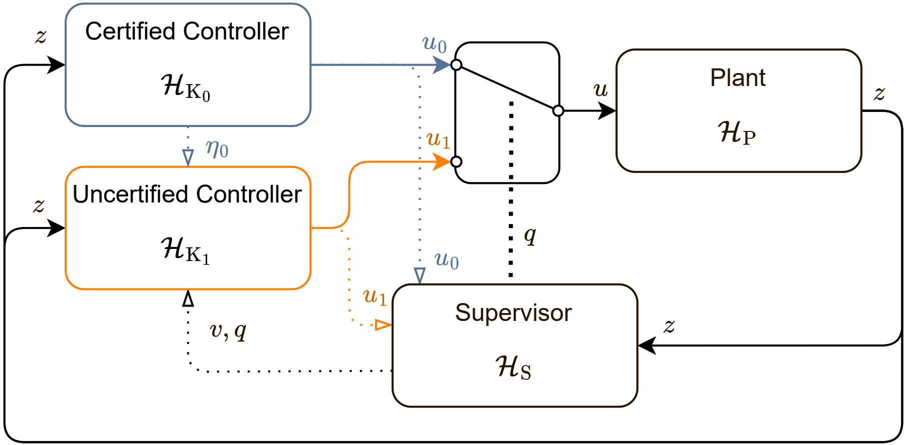
We presented

- relaxation of Lyapunov conditions
- several insertion theorems for positive definite functions.

Future Work

- Generalize relaxed Lyapunov conditions:
  - consider $\mathcal{A}$ non-compact
  - Other types of Lyapunov functions, e.g., ISS Lyapunov functions.

Outline


z
Certified Controller
$\mathcal{H}_{\mathrm{K}_0}$
u_0
Plant
$\mathcal{H}_{\mathrm{P}}$
u
z
η_0
u_1
z
Uncertified Controller
$\mathcal{H}_{\mathrm{K}_1}$
q
u_0
u_1
Supervisor
$\mathcal{H}_{\mathrm{S}}$
v, q
z

Hybrid Systems as Models of Cyber-Physical Systems

Hybrid dynamical systems are a type of mathematical model of dynamical systems that combine continuous flows and discrete jumps.

Hybrid dynamical systems are a natural choice for modeling cyber-physical systems.

flows
- continuous evolution of physical state.

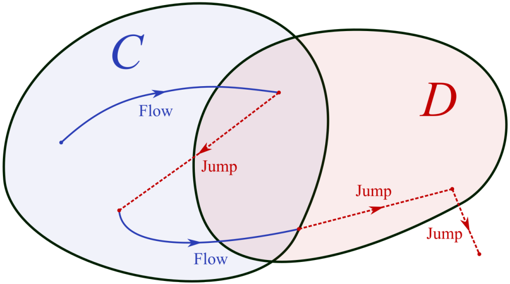
jumps
- evolution of digital components
- impacts in physical states

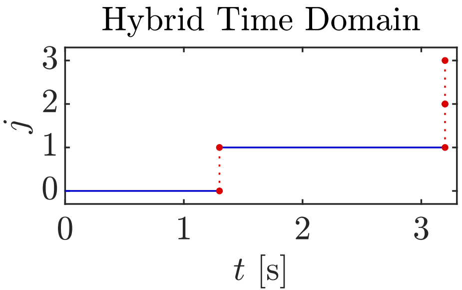
Hybrid Dynamical Systems

$$\mathcal{H} : \begin{cases} \dot{x} = f(x) & x \in C \\ x^+ = g(x) & x \in D \end{cases}$$

- flow set $C \subset \mathbb{R}^n$
- flow map $f : C \to \mathbb{R}^n$

- jump set $D \subset \mathbb{R}^n$
- jump map $g : D \to \mathbb{R}^n$





Hybrid Dynamical Systems with Set-valued Dynamics

$$\mathcal{H} : \begin{cases} \dot{x} \in F(x) & x \in C \\ x^+ \in G(x) & x \in D \end{cases}$$

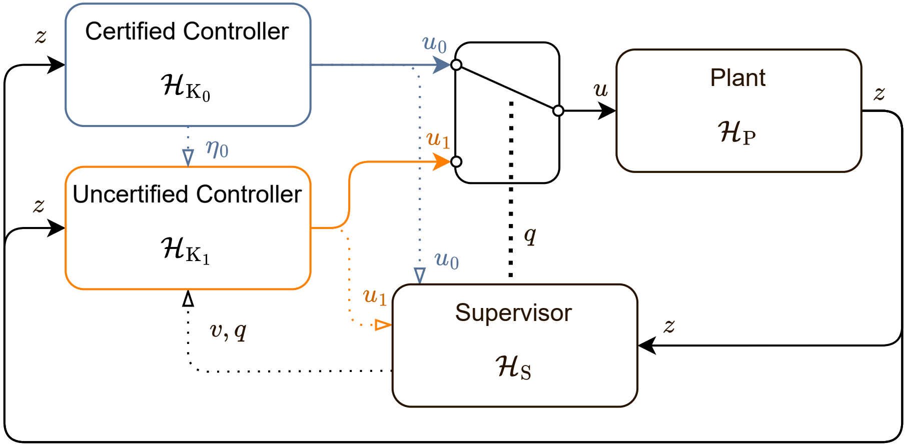
Hybrid Solutions

A *solution* ϕ to $\mathcal{H}$ is defined on a hybrid time domain $\operatorname{dom}\phi \subset [0, \infty) \times \mathbb{N}$:

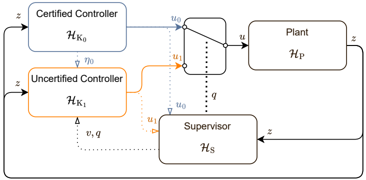
$$\operatorname{dom}\phi = ([t_0, t_1] \times \{0\}) \cup ([t_1, t_2] \times \{1\}) \cup \cdots .$$
$$0 = t_0 \leq t_1 \leq t_2 \leq \cdots .$$

A hybrid arc $(t, j) \mapsto \phi(t, j)$ is a solution to $\mathcal{H}$ if

- $\dot{\phi}(t, j) \in F(\phi(t, j))$ for almost all t in each intervals of flow $[t_j, t_{j+1}]$
- $\phi(t_j, j + 1) \in G(\phi(t_j, j))$ for each jump time t_j in $\operatorname{dom}(\phi)$.



Certified Controller
$\mathcal{H}_{\mathrm{K}_0}$
Uncertified Controller
$\mathcal{H}_{\mathrm{K}_1}$
Plant
$\mathcal{H}_{\mathrm{P}}$
Supervisor
$\mathcal{H}_{\mathrm{S}}$
z
z
z
z
u_0
u_0
u_1
u_1
u
η_0
q
v, q



Plant

$$\mathcal{H}_P : \begin{cases} \dot{z} \in F_P(z, u) & (z, u) \in C_P \\ z^+ \in G_P(z, u) & (z, u) \in D_P \end{cases}$$

Certified Controller

$$\mathcal{H}_{K_0} : \begin{cases} \dot{\eta}_0 \in F_{K_0}(z, \eta_0) & (z, \eta_0) \in C_{K_0} \\ \eta_0^+ \in G_{K_0}(z, \eta_0) & (z, \eta_0) \in D_{K_0} \\ u_0 = \kappa_0(z, \eta_0). \end{cases}$$

Uncertified Controller

$$\mathcal{H}_{K_1} : \begin{cases} \dot{\eta}_1 \in F_{K_1}(x) & x \in C_{K_1} \\ \eta_1^+ \in G_{K_1}(x) & x \in D_{K_1} \\ u_1 \in \kappa_1(x), \end{cases}$$

Supervisor Design for Global Asymptotic Stability

Given a Lyapunov function $(z, \eta_0) \mapsto V_{\mathrm{P}}(z, \eta)$ for $\mathcal{H}_{\mathrm{P}} \times \mathcal{H}_{\mathrm{K}_0}$ that certifies that $\mathcal{A}_{\mathrm{P}}$ is asymptotically stable for (z, η_0).

We extended the previous supervisor design as follows:

$$\mathcal{H}_{\mathrm{S}} : \begin{cases} \begin{bmatrix} \dot{v} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} f_{\mathsf{v}}(z, \eta_0, v) \\ 0 \end{bmatrix} & (z, \eta_0, \eta_1, v, q) \in C_{\mathrm{S}} \\ \begin{bmatrix} v^+ \\ q^+ \end{bmatrix} = \begin{bmatrix} \max\{V(z, \eta_0),\ v\} \\ 1 - q \end{bmatrix} & (z, \eta_0, \eta_1, v, q) \in D_{\mathrm{S}} \end{cases}$$

where

$$f_{\mathsf{v}}(z, \eta_0, v) = -\gamma \tanh(v) \sigma_0(z, \eta_0) - \mu\big(v - V_{\mathrm{P}}(z, \eta_0)\big).$$

Theorem: Global Asymptotic Stability

Suppose that

- $\mathcal{A}_{\mathrm{P}}$ is compact,
- Regularity conditions hold (outer semicontinuity, local boundedness of functions, closed sets)
- V_{P} is a Lyapunov function for $\mathcal{H}_{\mathrm{P}} \times \mathcal{H}_{\mathrm{K}0}$ with strict decrease during flows.
- The state of $\mathcal{H}_{\mathrm{K}_1}$ is constrained to compact set

Then, the set $\mathcal{A}_{\mathrm{P}} \times \{0\}$ is (uniformly) asymptotically stable for (z, η_0, z).

Proof Sketch

We introduce a new Lyapunov function

$$V(x) := \max\{V_{\mathrm{P}}(z, \eta_0), v\},$$

then show that it satisfies our relaxed Lyapunov conditions.

Let

$$\sigma_{\mathrm{LSC}}(x) := \begin{cases} \sigma_0(z, \eta_0) & \text{if } V_{\mathrm{P}}(z, \eta_0) > v \\ -f_{\mathsf{v}}(z, \eta_0, v) & \text{if } V_{\mathrm{P}}(z, \eta_0) \leq v \end{cases}$$

We show

- σ_{LSC} is LSC and positive definite with respect to $\mathcal{A}$ on C
- $\dot{V}(x) \leq -\sigma_{\mathrm{LSC}}(x)$.

Proof Sketch

At jumps

$$v^+ = \max\{V_{\mathrm{P}}(z, \eta_0), v\}.$$

Thus,

$$V(x^+) = \max\{V_{\mathrm{P}}(z, \eta_0), v^+\} \leq V(x),$$

so $V(x)$ does not increase at jumps.

We apply a hybrid version of our relaxed Lyapunov theorem to get existence of solutions.

Proof Sketch

We prove existence of solutions with unbounded domains:

1. At each point in $C \setminus D$, solutions can flow because
$$F(x) \cap T_C(x) \neq \varnothing.$$
2. The state cannot jump out of $C \cup D$ because
$$G(D) \subset C \cup D.$$

To show that $t \to \infty$ in the domain of each solution:

- For each subsystem, there is a minimum time between sequential jumps, so number of jumps is bounded in finite time.

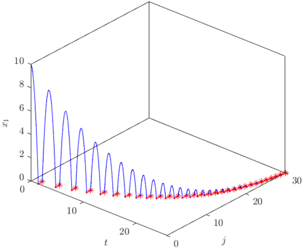
Outline

Software Tools for Hybrid Systems

SHARC

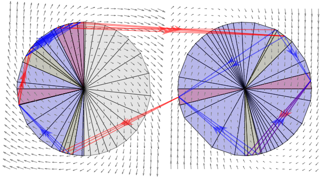
Simulate cyber-physical systems with accurate computational delays.

Hybrid Equations Toolbox



Simulation and plotting of composite hybrid systems

Conical Transition Graphs



Algorithmically check asymptotic stability in hybrid systems.

Outline

Acknowledgements

Thank you to everybody who helped me get here today!

My Family.

- Mom, Dad, Mary Alice. Love you!

My Teachers.

- Anna Parmely.
- Michael Sommermann.

My Friends

- Especially Ryan Johnson, Callie Chappell, Adam Grant – For helping with preparation.

BSOE IT support

- Ryan Meckel, David, and Sean (they saved my defense!)

My advisor,

- Ricardo Sanfelice

My committee

- Qi Gong
- Daniele Venturi
- Alessandro Pinto

Everybody in the Hybrid Systems Lab.

Funding

Publications

Wintz and Ricardo G Sanfelice. “Relaxed Lyapunov Conditions”. American Control Conference (ACC). 2025.

Wintz, Yasin Sonmez, et al. SHARC: Simulator for Hardware Architecture and Real-time Control”. ACM Conference on Hybrid Systems: Computation and Control (HSCC). 2025.

Wintz and Ricardo G. Sanfelice. “Conical Transition Graphs for Analysis of Asymptotic Stability in Hybrid Dynamical Systems”. IFAC Conference on Analysis and Design of Hybrid Systems (ADHS). 2024.

Wintz and Ricardo G. Sanfelice. “Forward Invariance-Based Hybrid Control Using Uncertified Controllers”. IEEE Conference on Decision and Control (CDC). 2023.

Wintz, Ricardo G. Sanfelice, and João P. Hespanha, “Global Asymptotic Stability of Nonlinear Systems while Exploiting Properties of Uncertified Feedback Controllers via Opportunistic Switching”, American Control Conference (ACC). 2022.

Wintz “Optimal Control of a Noncircular Wheel”. UC Santa Cruz, MS Thesis. 2020.

Under Review

Wintz and Ricardo G. Sanfelice. “Conical Transition Graphs for Analysis of Asymptotic Stability in Hybrid Dynamical Systems”.

In Preparation

Wintz and Ricardo G. Sanfelice. “Exploiting Uncertified Controllers via Uniting Feedback”.

Wintz and Ricardo G. Sanfelice. “Relaxed Lyapunov Conditions for Non-smooth Dynamical Systems”

Questions?