Relaxing Assumptions about Voter Utilities:

How the Nature of Political Preferences Shapes the Efficiency of Majority Rule Voting\*

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Abstract

Spatial voting models typically assume that voters and candidates can be placed on a common policy

dimension and the voters' utilities can be determined by the relative proximity of their ideal points to

the respective candidates (c.f. Downs 1957). In such a framework, simple majority elections between

two candidates are generally expected to lead to desirable outcomes that maximize social welfare. The

goal of this paper is to examine how the underlying assumption of voter utilities based on common

policy dimensions affect the expected welfare outcomes of majority voting. More specifically, we present

simulational studies in order to examine the efficiency of majority elections under different scenarios. We

illustrate how the assumptions underlying the ideal-point framework influence the expected social welfare

outcomes of majority voting rules.

Keywords: Spatial Voting, Utility Assumptions, Majority Voting, Efficiency

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#### 1 Introduction

Spatial theories have a long tradition as a general framework for the the study of voting behavior as well as the analyses of candidate strategies in election campaigns. Pioneered in earlier work by Hotelling (1929), Downs (1957), and Black et al. (1958), many models of issue-based voting conceptualize political competition in a common multi- or unidimensional policy space in order to formalize voter preferences (for overviews, see Enelow and Hinich 1984; Merrill and Grofman 1999). Accordingly, spatial theories of voting assume that voter preferences can be described by (or approximated by) a common (multidimensional) policy space or a single ideological dimension. Individual utilities (for example in the context of elections) are thereby based on the voters' ideal points in the policy space in relation to the candidates' respective positions. While most applications of spatial voting models focus on the relative *proximity* between voters and candidates in order to deduce individual utilities, other scholars proposed alternative ways of mapping candidate and voter positions on actual utilities (see for example Rabinowitz and Macdonald 1989).

However, notwihstanding the debate between specific conceptualization of spatial voting models, a large body of research in political science and political psychology showed how a multitude of factors can affect voter preferences independent of pure issue positions and ideological dimensions. Classic studies such as *The American Voter* (Campbell et al. 1960) as well as subsequent work emphasized importance of party identification as a general psychological attachement in voter decision making. Other factors that have been under consideration in more recent studies include, but are not limited to, the candidates' traits and personalities, their perceived competence, and the nature of campaigns (see for example Hayes 2005). For example, Todorov et al. (2005) showed that competence assessments solely based on candidate pictures successfully predicted the results in election results for the U.S. Congress (see also Mattes et al. 2010). Furthermore, the effect of candidate appearance on electoral success is not limited to the related inference about competence, but can be based on simple assessments of the beauty of candidates (Berggren, Jordahl, and Poutvaara 2010). These examples of non-issue based determinants of voter preferences indicate the underlying utilities for candidates or parties might not be reducible to a simple issue-based logic.

In the paper presented here, it will be argued, that focusing solely on policy-based utilities induces strong assumptions about the relationships between the utilities for competing candidates. Our goal is to show how relaxing such assumptions can alter our conclusions about the efficiency of voting rules. As an initial step, we will focus on a simple voting scenario of two competing candidates and varying sizes of the electorate. We present simulation studies in order to examine the efficiency of majority elections under different scenarios. Based on the simulation results, we propose an experimental design in order to provide further insights as to

how the assumptions underlying the ideal-point framework influence the expected social welfare outcomes of voting rules.

#### 2 Political Preferences and the Ideal Point Framework

Spatial theories of voting provide a comprehensive framework to translate issue positions of candidates and members of the electorate into individual voter preferences as well as resulting candidate strategies. One of the fundamental assumptions underlying spatial models is, that individual positions regarding each policy issue or ideological dimensions can be described as an *ideal point* on a continuous scale that encompasses all possible configurations. For example, looking at general health care policies, the end points of the policy space could be described as complete privatization of health care without government regulation on the one hand, and state-controlled complementary health insurance on the other end. Each point within this space represents a unique policy configuration between both extreme points (however, see Rabinowitz and Macdonald 1989 for an alternative interpretation focusing on the relative *emphasis* on a specific issue position).

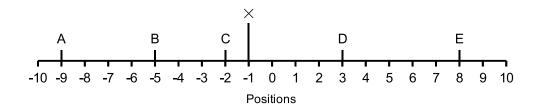


Figure 1: Illustration of Unidimensional Policy Space.

For example, Figure 1 describes policy positions of five candidates (A through E) as well as the ideal point of an individual voter (X). Using the health care example introduced above, the end points (-10 and 10) represent both extremes of complete privatization vs. government-run health care. Voter X prefers some mixture between private and state controlled health care problems with a slight preference for private health care (-1). How can the utilities that voter X could draw from each of the candidates be deduced in this framework?

Most spatial voting models assume that utilities are based on the relative *proximity* between the voter's ideal point and the respective candidate position. This conceptualization has been introduced by Downs (1957) and further developed in subsequent work (e.g. Hinich and Pollard 1981; Enelow and Hinich 1984). Most proximity voting models are conceptualized as the Euclidian distance between the voter's ideal point

and each of the candidates under consideration (c.f. Lewis and King 1999). Assuming a unidimensional framework, the utility of candidate i for each party j can be described as follows:

$$U_{i,j} = -(X_i - X_j)^2 (1)$$

Where U denotes the utility,  $X_i$  denotes the voter's ideal point, and  $X_j$  denotes candidate j's position on the policy or ideological dimension.

However, other scholars proposed a directional conceptualization... but see westholm

But it's hard to differentiate between the two (see lewis)

other approaches combine different considerations (merrill)

In the following, we will focus on the proximity conceptualization of spatial voting and political preferences.

## 3 Alternative Factors Influencing Political Preferences

Start with campbell, american voter, party identification

# 4 Majority Voting and Social Welfare

Hastie and Kameda (2005)

majority voting seems to work quite well in experiments and simulations etc.

## 5 Simulation Results

In order to investigate how different assumptions about voter utilities affect our evaluations of the efficiency of voting models, we

Description of simulational scenarios: - number of voters in each election: 2000 - number of candidates: 2 - number of simulations: 1000

<sup>&</sup>lt;sup>1</sup>Alternative conceptualizations include the absolute distance. However, we will focus on the Euclidian distance in the remainder of the paper since it represents the most common approach in proximity models.

Conceptualization of efficiency: does the election result maximize the aggregate utilities for all voters?

$$\sum_i U_i^W > \sum_i U_i^L$$

### 5.1 Comparing ideal points and independent normal utilities

$$X_i, X_a, X_b \sim \mathcal{N}(\mu = 0, \sigma^2 = 1)$$
 
$$U_i^a = -(X_i - X_a)^2 \qquad U_i^b = -(X_i - X_b)^2$$

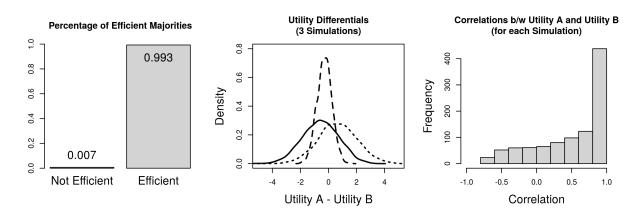


Figure 2: Normally distributed ideal points.

$$U_i^a, U_i^b \sim \mathcal{N}(\mu = 0, \sigma^2 = 1)$$

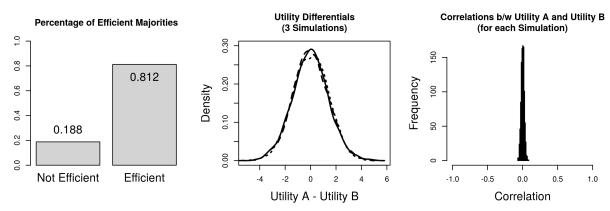


Figure 3: Independent normal utilities.

### 5.2 Investigating the effect of correlated utilities

$$U_a, U_b \sim \mathcal{N} \left( \mu = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{\Sigma} = \begin{pmatrix} 1 & 0.9 \\ 0.9 & 1 \end{pmatrix} \right)$$

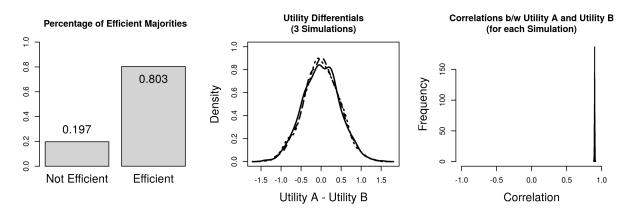


Figure 4: Positively correlated normal utilities.

$$U_a, U_b \sim \mathcal{N} \left( \mu = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{\Sigma} = \begin{pmatrix} 1 & -0.9 \\ -0.9 & 1 \end{pmatrix} \right)$$

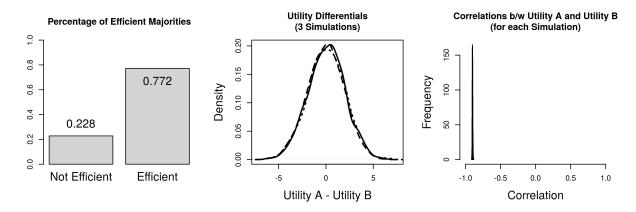


Figure 5: Negatively correlated normal utilities.

#### 5.3 Inefficiencies for varying mean differences in utilities

$$U_i^a \sim \mathcal{N}(\mu = 0, \sigma^2 = 1)$$
  $U_i^b \sim \mathcal{N}(\mu = 0 + \epsilon, \sigma^2 = 1)$ 

$$U_i^a \sim \mathcal{N}(\mu = 0, \sigma^2 = 1)$$
  $U_i^b \sim \mathcal{N}(\mu = 0 + \epsilon, \sigma^2 = 1)$ 

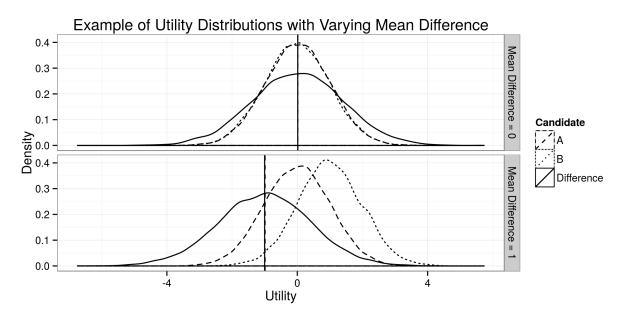


Figure 6: Inefficiencies for varying mean differences in utilities I.

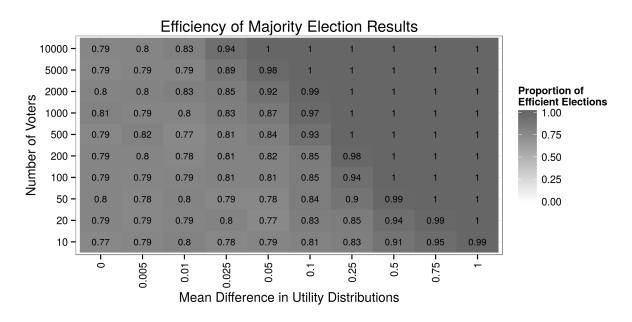


Figure 7: Inefficiencies for varying mean differences in utilities II.

# 5.4 Investigating the effect of skewed utility distributions

$$U_i^a \sim \mathcal{N}(\mu = 0 + \epsilon, \sigma^2 = 1)$$
  $U_i^b \sim \mathcal{N}_{\text{skew}}(\mu = 0 - \epsilon, \sigma^2 = 1)$ 

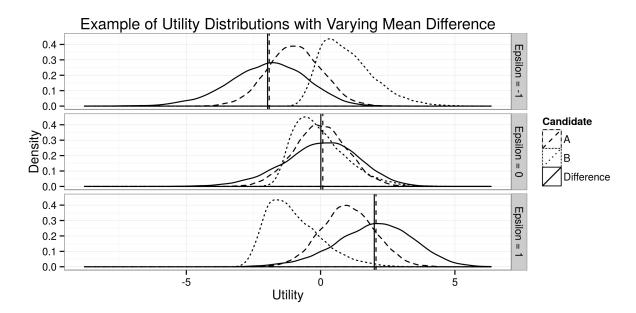


Figure 8: Investigating the effect of skewed utility distributions I.

$$U_i^a \sim \mathcal{N}(\mu = 0 + \epsilon, \sigma^2 = 1)$$
  $U_i^b \sim \mathcal{N}_{\text{skew}}(\mu = 0 - \epsilon, \sigma^2 = 1)$ 

#### 5.5 Inducing inefficiencies with ideal point utilities

$$X_i \sim \mathcal{N}_{\text{skew}}(\mu=0,\sigma^2=1) \qquad \quad X_a, X_b \sim \mathcal{N}(\mu=0,\sigma^2=1)$$
 
$$U_i^a = -(X_i - X_a)^2 \qquad \quad U_i^b = -(X_i - X_b)^2$$

$$X_i, X_a \sim \mathcal{N}(\mu = 0, \sigma^2 = 1)$$
  $X_b = -1 * X_a$  
$$U_i^a = -(X_i - X_a)^2 \qquad U_i^b = -(X_i - X_b)^2$$

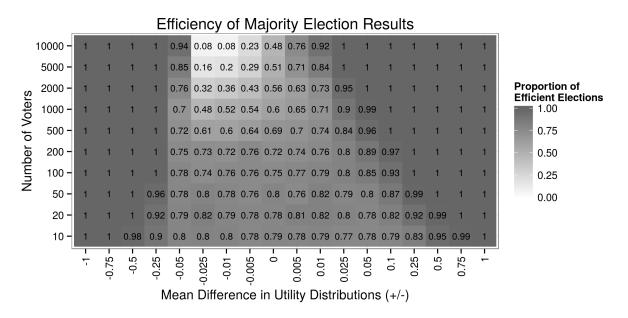


Figure 9: Investigating the effect of skewed utility distributions II.

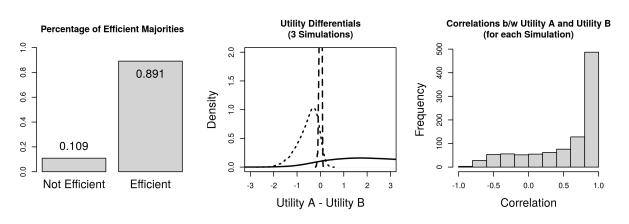


Figure 10: Skewed ideal points.

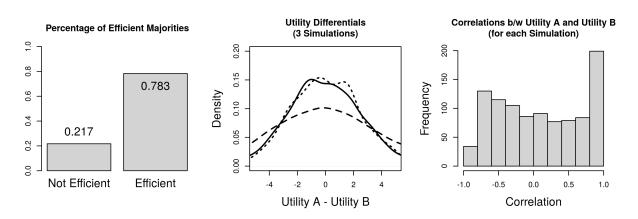


Figure 11: Aggregate indifference between ideal points.

#### 5.6 Further simulational scenarios

# 6 Possible Experimental Designs and Further Developments

- Performance of compensation elections / bidding mechanisms in the context of binary choices Oprea, Smith, and Winn (2007)
- Effect of (endogenous) electoral abstention on election efficiency
- Multi-candidate elections

Oprea, Smith, and Winn (2007)
comparing auction mechanism to voting
uncertainty about issue positions

## 7 Conclusion

- relaxing assumptions about ideal-point based preferences can reduce the likelihood that election results are efficient
- mean difference and skewness of the distributions of individual utilities for each candidate affects the likelihood of inefficiencies
- under some scenarios, increasing the *size of the electorate* actually reduces the efficiency of majority voting!
- Question: conceptualization of utility reasonable? These results would not hold if preferences were purely ordinal (and utilities not comparable across individuals)

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