Relaxing Assumptions about Voter Utilities:
How the Nature of Political Preferences Shapes the Efficiency of Majority Rule Voting*

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Abstract

Spatial voting models typically assume that voters and candidates can be placed on a common policy dimension and the voters' utilities can be determined by the relative proximity of their ideal points to the respective candidates (c.f. Downs 1957). In such a framework, simple majority elections between two candidates are generally expected to lead to desirable outcomes that maximize social welfare. The goal of this paper is to examine how the underlying assumption of voter utilities based on common policy dimensions affect the expected welfare outcomes of majority voting. More specifically, we present simulational studies in order to examine the efficiency of majority elections under different scenarios. We illustrate how the assumptions underlying the ideal-point framework influence the expected social welfare outcomes of voting rules.

Keywords: Spatial Voting, Utility Assumptions, Majority Voting, Efficiency

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1 Introduction

Spatial theories of voting have a long tradition as a framework for the the study of individual political behavior in elections as well as the analyses of candidate strategies in campaigns. Pioneered in earlier work by Hotelling (1929), Downs (1957), and Black et al. (1958), many models of issue-based voting conceptualize political competition in a common multi- or unidimensional policy space in order to formalize voter preferences (for overviews, see Enelow and Hinich 1984; Merrill and Grofman 1999). Accordingly, spatial theories of voting assume that voter preferences can be described by (or approximated by) a common policy space or a single ideological dimension. Individual utilities (for example in the context of elections) are thereby formed based on the voters' ideal points in the policy space in relation to the candidates' respective positions.

However, a large body of research in political science and political psychology showed how a multitude of factors can affect voter preferences independent of pure issue positions and ideological dimensions. Such factors include, but are not limited to, the candidates' traits and personalities, their perceived competence, and the nature of the campaign (see for example Hayes 2005). For example, Todorov et al. (2005) showed that competence assessments solely based on candidate pictures successfully predicted the results in election results for the U.S. Congress (see also Mattes et al. 2010). Furthermore, the effect of candidate appearance on electoral success is not limited to the related inference about competence, but can be based on simple assessments of the beauty of candidates (Berggren, Jordahl, and Poutvaara 2010). These examples of non-issue based determinants of voter preferences indicate the underlying utilities for candidates or parties might not be reducible to a simple issue-based logic.

In the paper presented here, it will be argued, that focusing solely on policy-based utilities induces strong assumptions about the relationships between the utilities for competing candidates. Our goal is to show how relaxing such assumptions can alter our conclusions about the efficiency of voting rules. As a first step, we will focus on a simple voting scenario of two competing candidates and varying sizes of the electorate. We present simulation studies in order to examine the efficiency of majority elections under different scenarios. Based on the simulation results, we propose an experimental design in order to provide further insights as to how the assumptions underlying the ideal-point framework influence the expected social welfare outcomes of voting rules.

2 Political Preferences and the Ideal Point Framework

Spatial theories of elections and voter preferences have been very prominent in political science. The since they have been introduced by Downs (1957)

Formal models of voting behavior and political representation usually assume that the voters' utilities are

Spatial theory of voting

- common policy / ideological dimension
- utilities determined by relative *proximity*

$$U_i^{\text{cand}} = -(X_i - X^{\text{cand}})^2$$

3 Majority Voting and Social Welfare

Hastie and Kameda (2005)

4 Simulation Results

Description of simulational scenarios: - number of *voters* in each election: 2000 - number of candidates: 2 - number of simulations: 1000

Conceptualization of efficiency: does the election result *maximize the aggregate utilities* for all voters?

$$\sum_i U_i^W > \sum_i U_i^L$$

4.1 Comparing ideal points and independent normal utilities

$$X_i, X_a, X_b \sim \mathcal{N}(\mu = 0, \sigma^2 = 1)$$

$$U_i^a = -(X_i - X_a)^2 \qquad U_i^b = -(X_i - X_b)^2$$

$$U_i^a, U_i^b \sim \mathcal{N}(\mu = 0, \sigma^2 = 1)$$

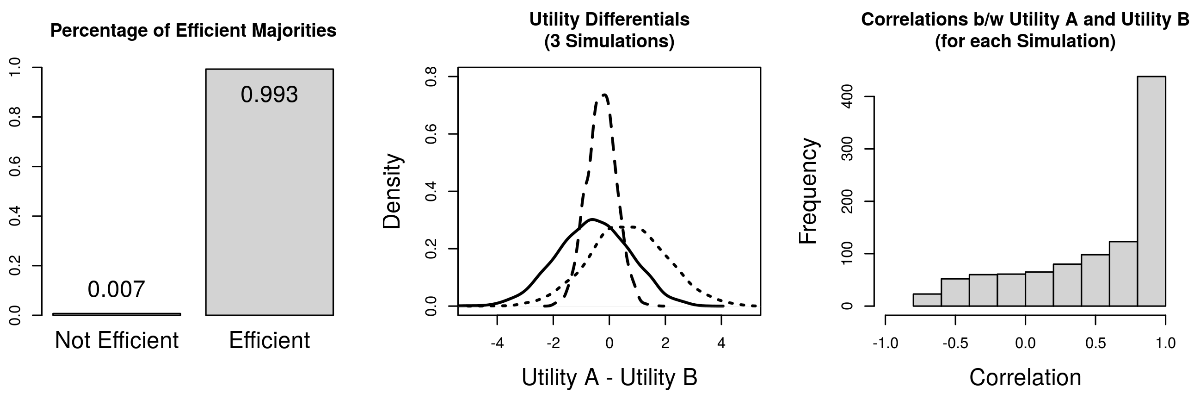

Figure 1: Normally distributed ideal points.

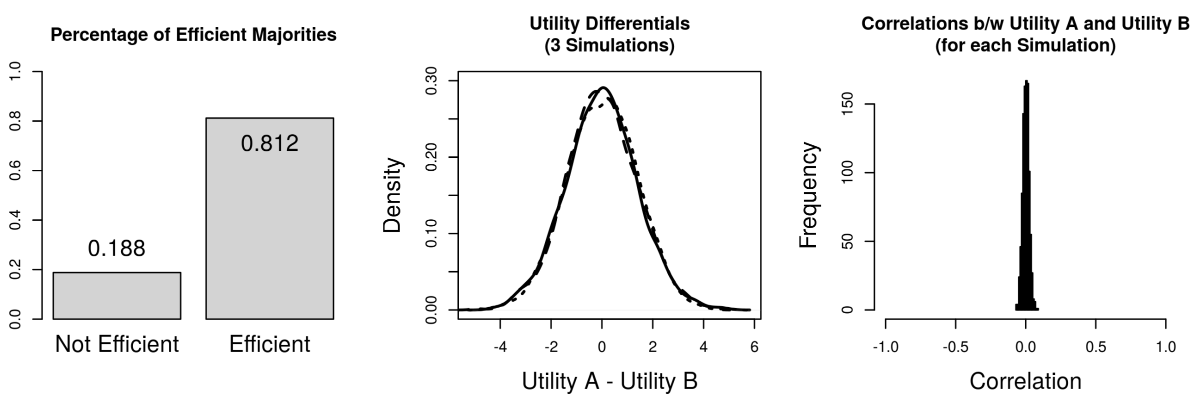

Figure 2: Independent normal utilities.

4.2 Investigating the effect of correlated utilities

$$U_a, U_b \sim \mathcal{N}\left(\mu = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{\Sigma} = \begin{pmatrix} 1 & 0.9 \\ 0.9 & 1 \end{pmatrix}\right)$$

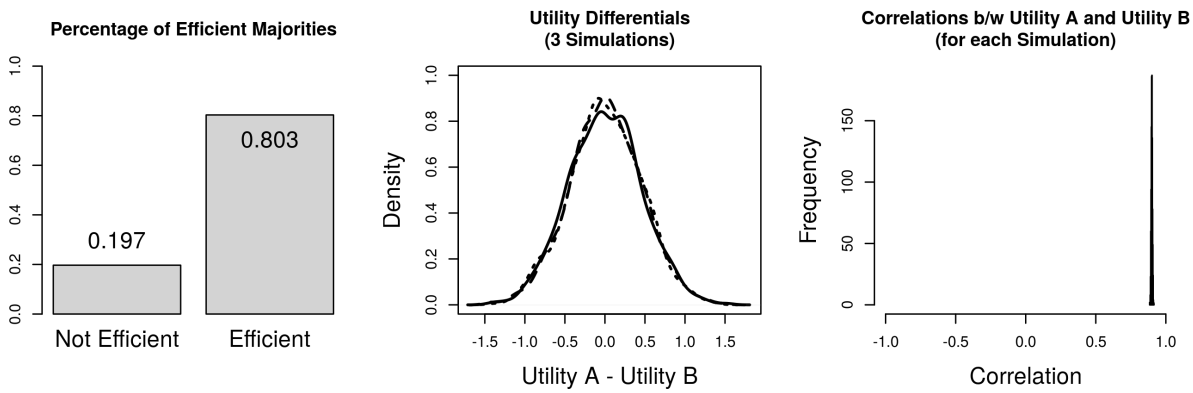

Figure 3: Positively correlated normal utilities.

$$U_a, U_b \sim \mathcal{N}\left(\mu = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{\Sigma} = \begin{pmatrix} 1 & -0.9 \\ -0.9 & 1 \end{pmatrix}\right)$$

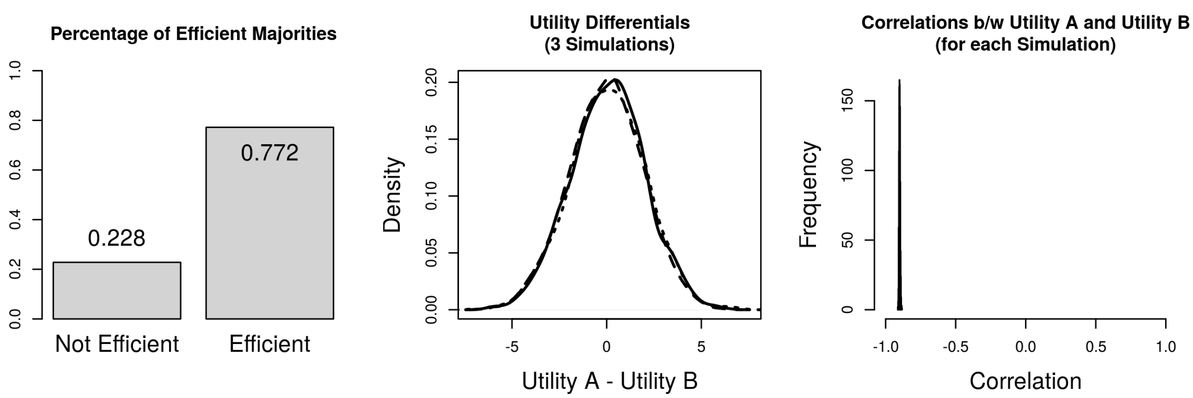

Figure 4: Negatively correlated normal utilities.

4.3 Inefficiencies for varying mean differences in utilities

$$U_i^a \sim \mathcal{N}(\mu = 0, \sigma^2 = 1) \qquad U_i^b \sim \mathcal{N}(\mu = 0 + \epsilon, \sigma^2 = 1)$$

$$U_i^a \sim \mathcal{N}(\mu = 0, \sigma^2 = 1) \qquad U_i^b \sim \mathcal{N}(\mu = 0 + \epsilon, \sigma^2 = 1)$$

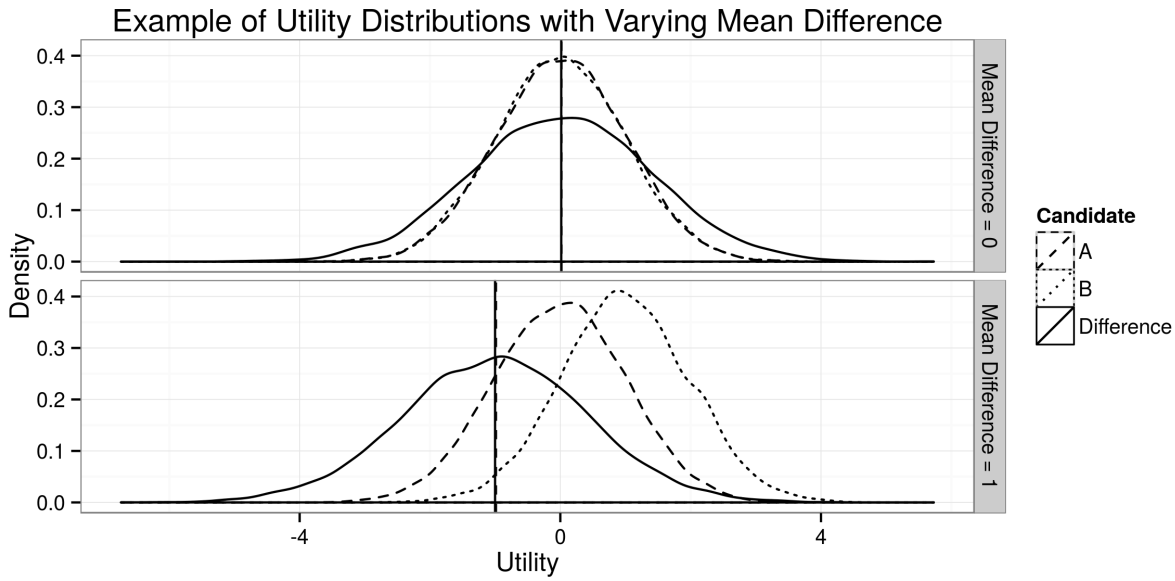

Figure 5: Inefficiencies for varying mean differences in utilities I.

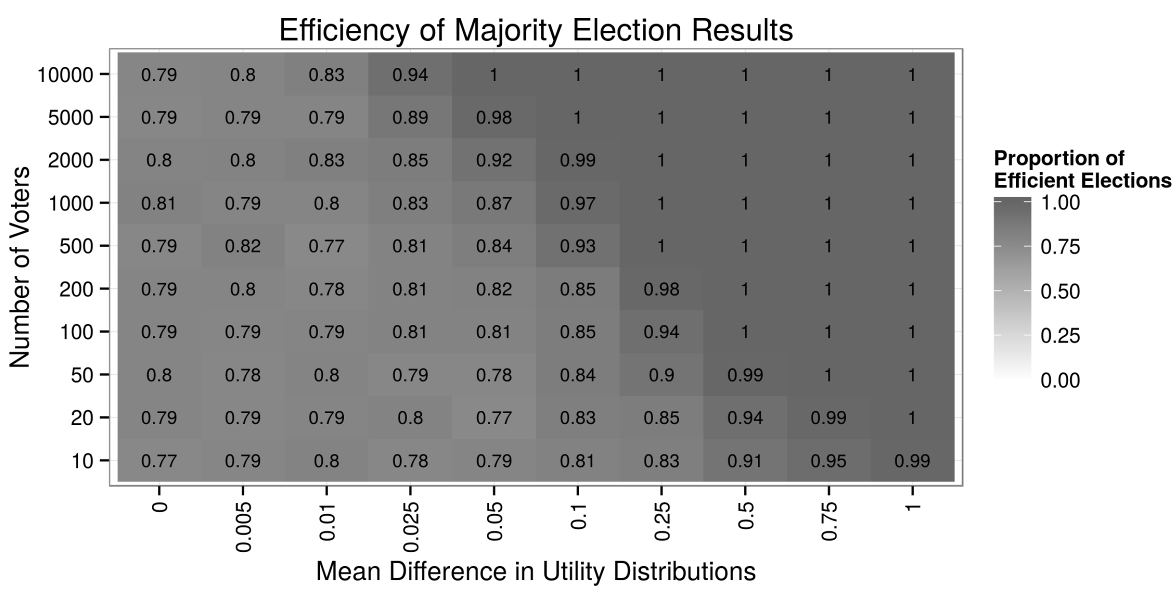

Figure 6: Inefficiencies for varying mean differences in utilities II.

4.4 Investigating the effect of skewed utility distributions

$$U_i^a \sim \mathcal{N}(\mu = 0 + \epsilon, \sigma^2 = 1) \qquad U_i^b \sim \mathcal{N}_{\text{skew}}(\mu = 0 - \epsilon, \sigma^2 = 1)$$

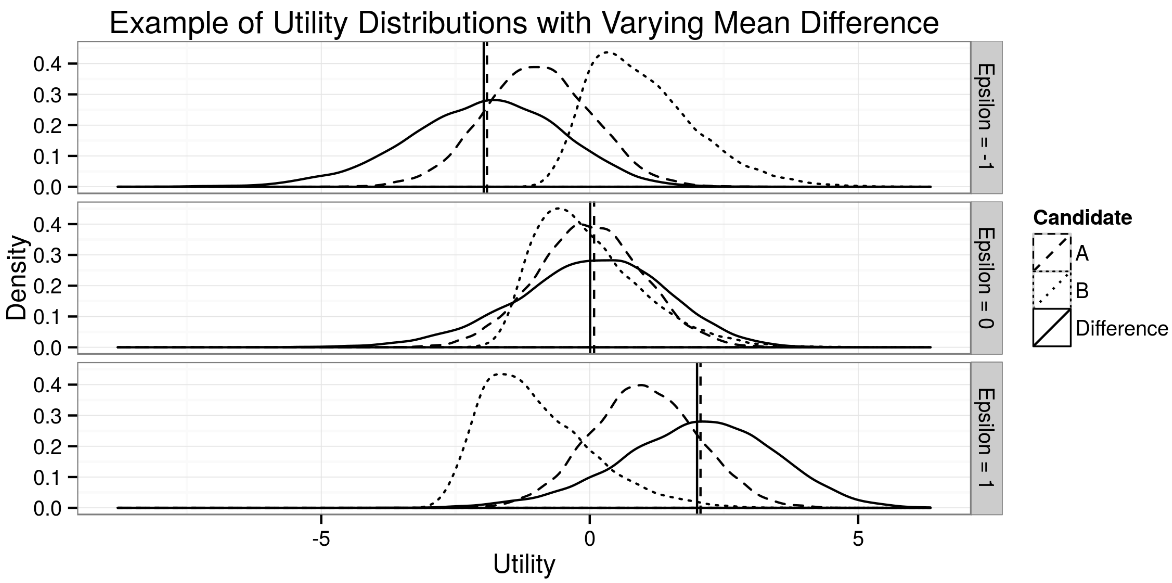

Figure 7: Investigating the effect of skewed utility distributions I.

$$U_i^a \sim \mathcal{N}(\mu = 0 + \epsilon, \sigma^2 = 1) \qquad U_i^b \sim \mathcal{N}_{\text{skew}}(\mu = 0 - \epsilon, \sigma^2 = 1)$$

4.5 Inducing inefficiencies with ideal point utilities

$$X_i \sim \mathcal{N}_{\text{skew}}(\mu = 0, \sigma^2 = 1) \qquad X_a, X_b \sim \mathcal{N}(\mu = 0, \sigma^2 = 1)$$

$$U_i^a = -(X_i - X_a)^2 \qquad U_i^b = -(X_i - X_b)^2$$

$$X_i, X_a \sim \mathcal{N}(\mu = 0, \sigma^2 = 1) \qquad X_b = -1 * X_a$$

$$U_i^a = -(X_i - X_a)^2 \qquad U_i^b = -(X_i - X_b)^2$$

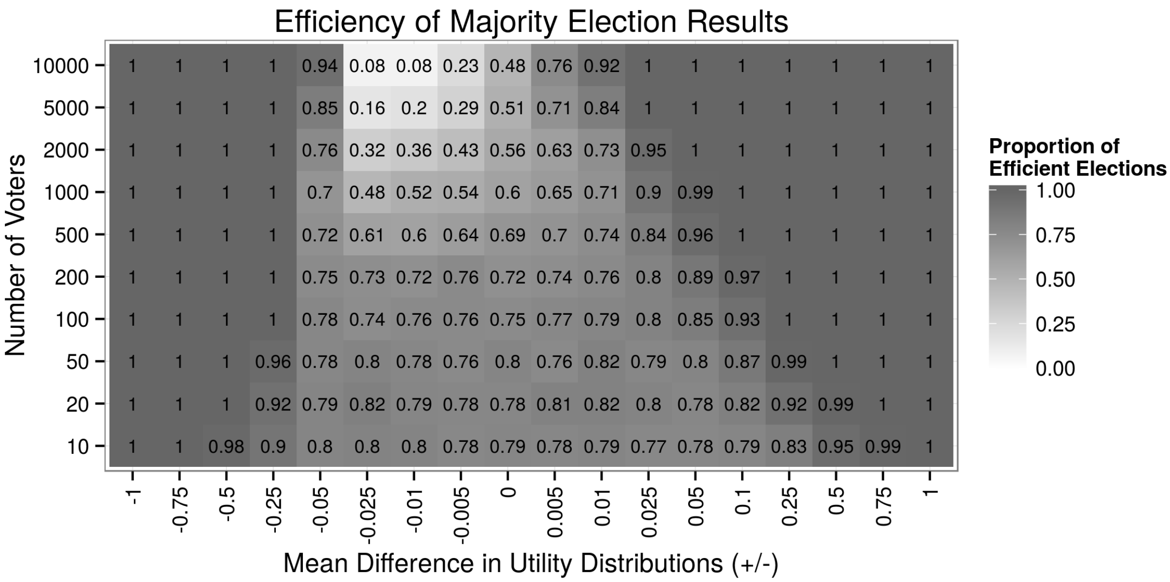

Figure 8: Investigating the effect of skewed utility distributions II.

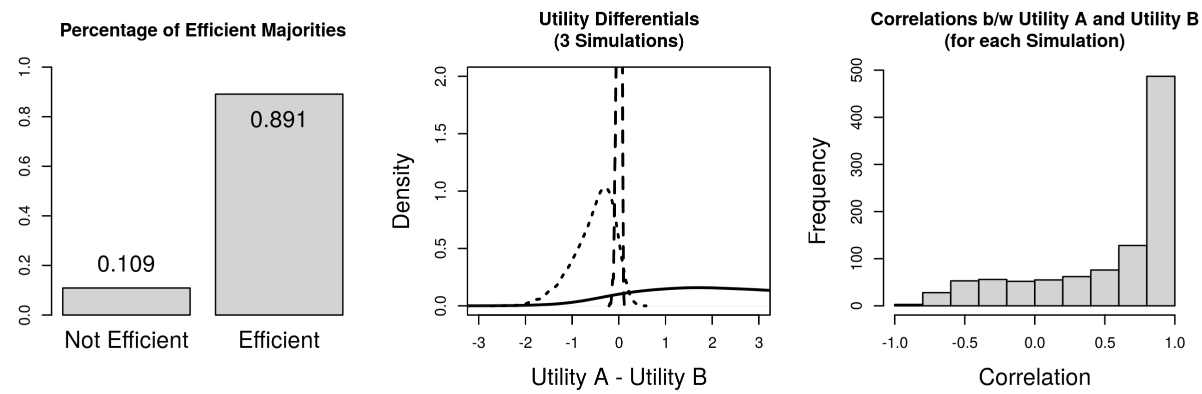

Figure 9: Skewed ideal points.

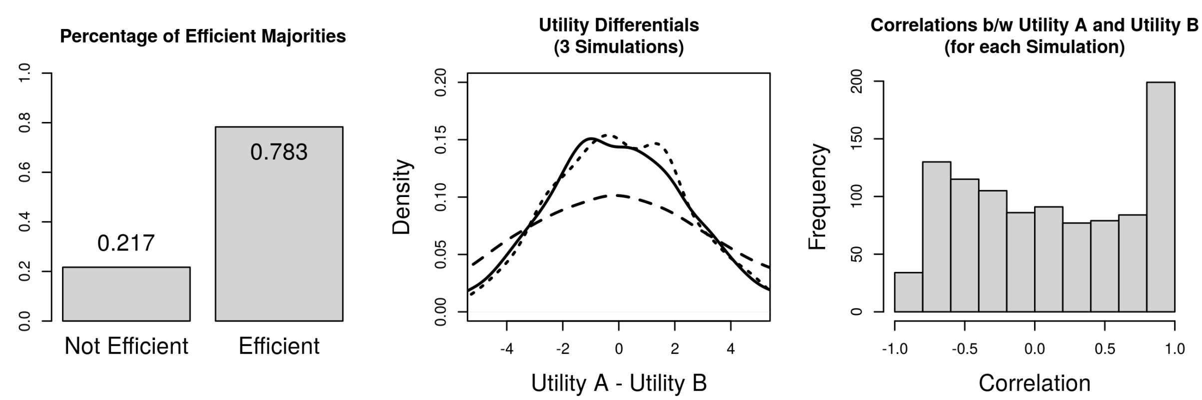

Figure 10: Aggregate indifference between ideal points.

4.6 Further simulational scenarios

5 Possible Experimental Designs and Further Developments

- Performance of *compensation elections / bidding mechanisms* in the context of binary choices Oprea, Smith, and Winn (2007)
- Effect of (endogenous) electoral *abstention* on election efficiency
- Multi-candidate elections

Oprea, Smith, and Winn (2007)

comparing auction mechanism to voting

uncertainty about issue positions

6 Conclusion

- *relaxing assumptions* about ideal-point based preferences can reduce the likelihood that election results are efficient
- *mean difference* and *skewness* of the distributions of individual utilities for each candidate affects the likelihood of inefficiencies
- under some scenarios, increasing the *size of the electorate* actually reduces the efficiency of majority voting!
- *Question*: conceptualization of utility reasonable? These results would not hold if preferences were purely ordinal (and utilities not comparable across individuals)

7 References

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