Comparison of the Efficiency of Majority Election Results

Part 4: Effects of Skewness, Ideal Point Scenarios, and Analytical Solutions

May 8, 2014

1 Simulating the Effect of Skewness

1.1 Keeping the Real Mean and Variance of Skewed Normal Distribution Constant

The pdf of the skewed normal distribution is given by

$$f(x) = \frac{1}{\omega\pi} e^{-\frac{(x-\xi)^2}{2\omega^2}} \int_{-\infty}^{\alpha\left(\frac{x-\xi}{\omega}\right)} e^{-\frac{t^2}{2}} dt, \tag{1}$$

where $\alpha \in \mathbb{R}$ is the *shape* parameter (affecting skewness), $\xi \in \mathbb{R}$ is the *location* parameter, and $\omega \in \mathbb{R}^+$ is the *scale* parameter. Note that ξ is not equal to the distribution's mean μ, and ω is not it's variance σ^2. Rather, they are given by

$$\mu = \xi + \omega \frac{\alpha}{\sqrt{1+\alpha^2}} \sqrt{\frac{2}{\pi}} \tag{2}$$

$$\sigma^2 = \omega^2 \left(1 - \frac{2\left(\frac{\alpha}{\sqrt{1+\alpha^2}}\right)^2}{\pi} \right) \tag{3}$$

Accordingly, if we also want to keep the variance constant at σ^{2*}, we have to adjust the scale parameter such that:

$$\omega = \sqrt{\frac{\sigma^{2*}\pi}{\pi - 2\left(\frac{\alpha}{\sqrt{1+\alpha^2}}\right)^2}} \tag{4}$$

Furthermore, if we want to manipulate the skewness but keep the real mean of the distribution constant at μ^*, we have to adjust the location parameter such that:

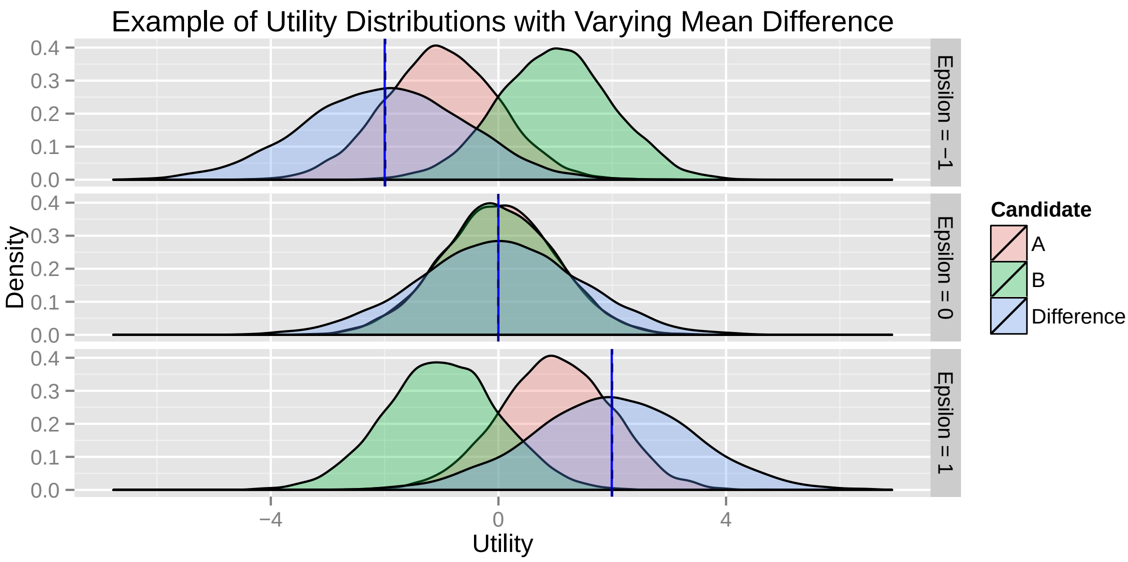
$$\xi = \mu^* - \omega \frac{\alpha}{\sqrt{1+\alpha^2}} \sqrt{\frac{2}{\pi}} \tag{5}$$

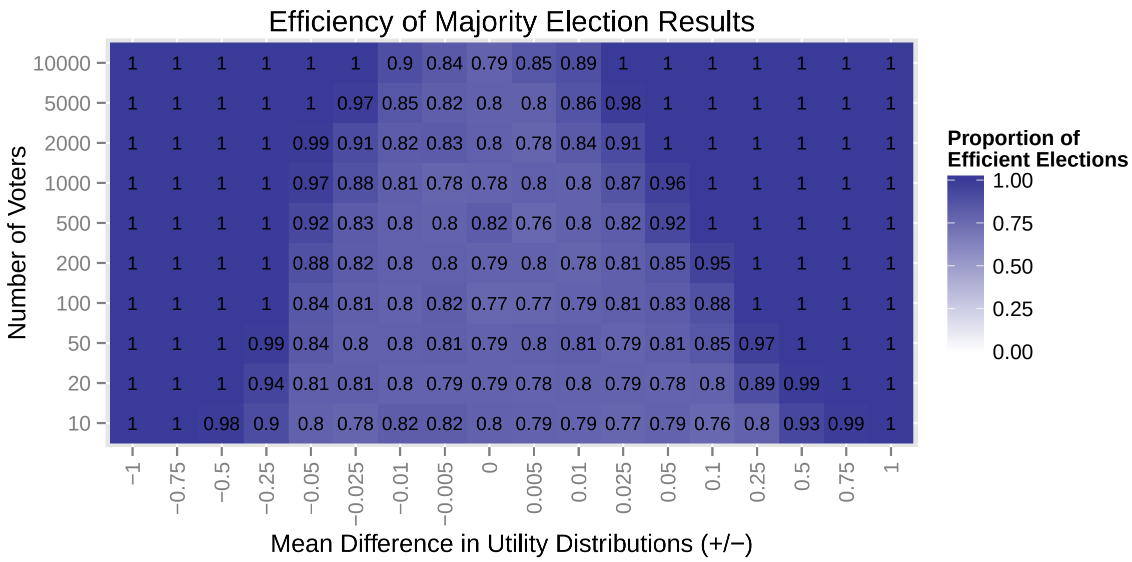
1.2 Overview of Simulation Parameters

- Number of simulations for each scenario: 1000
- Numbers of voters: 10, 20, 50, 100, 200, 500, 1000, 2000, 5000, 10000
- Utility distributions for each voter (candidates A and B):
  $U_A \sim \mathcal{N}(\mu = 0 + \epsilon, \sigma^2 = 1)$
  $$U_B \sim \mathcal{N}_{skew}\left(\xi = -\epsilon - \omega \frac{\alpha}{\sqrt{1+\alpha^2}}\sqrt{\frac{2}{\pi}}, \omega = \sqrt{\frac{\pi}{\pi - 2\left(\frac{\alpha}{\sqrt{1+\alpha^2}}\right)^2}}\right)$$
- Differences in distribution means (ϵ): -1, -0.75, -0.5, -0.25, -0.05, -0.025, -0.01, -0.005, 0, 0.005, 0.01, 0.025, 0.05, 0.1, 0.25, 0.5, 0.75, 1
- Skewness of distribution (α): 0, 1, 2, 5, 10
- For now: no correlation between utilities

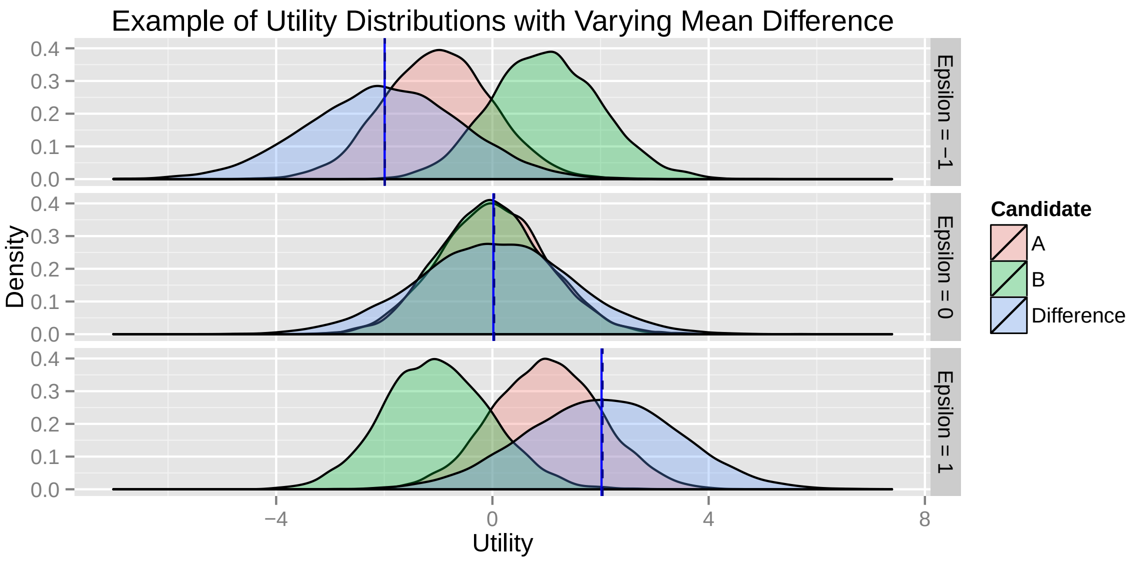
1.3 Simulations

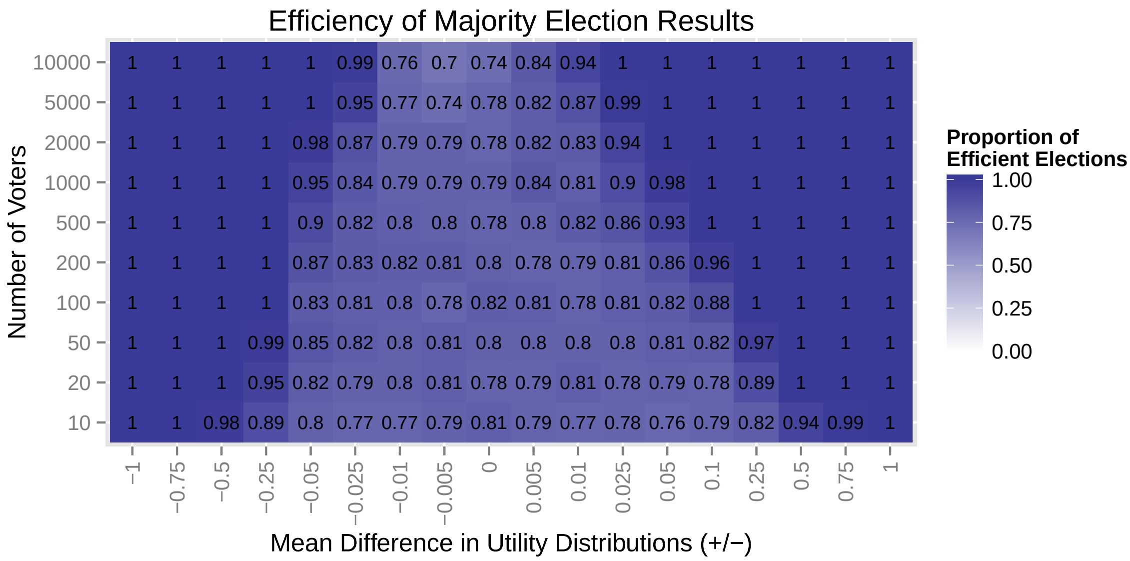
1.3.1 Skewness: alpha = 0



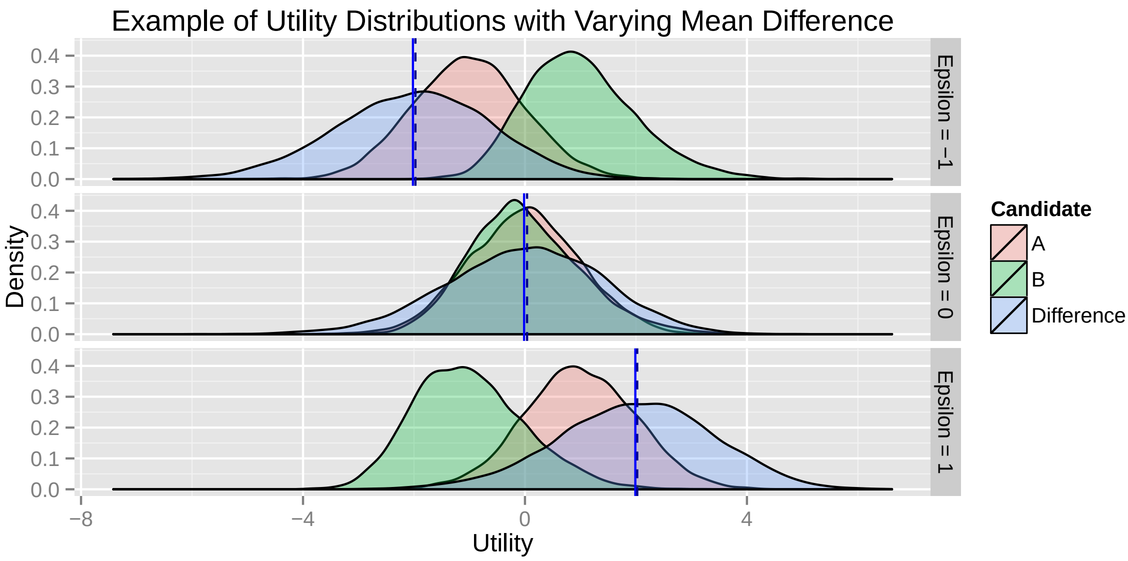


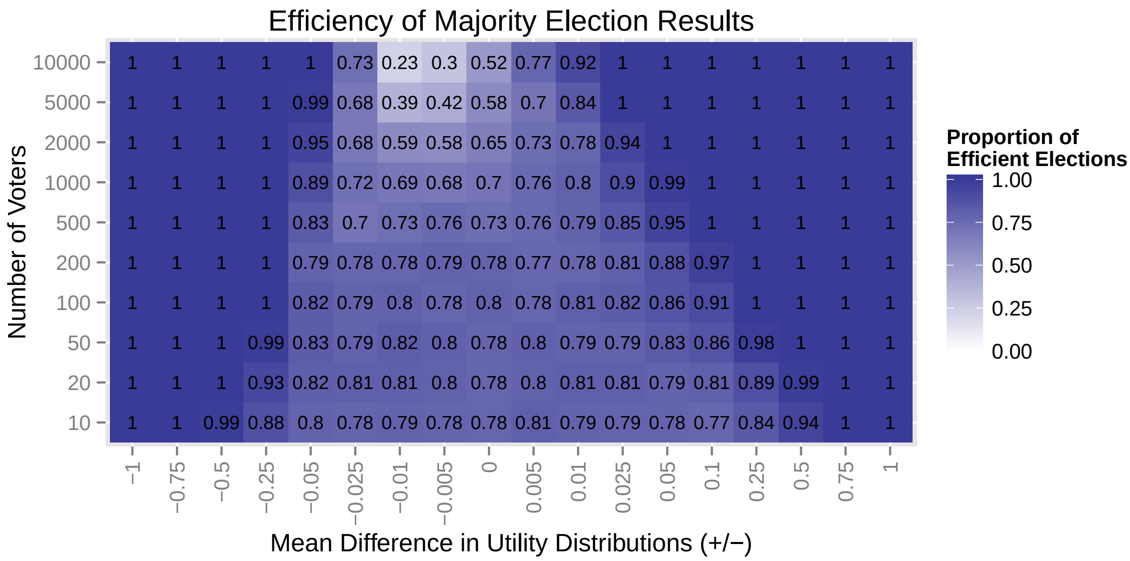
1.3.2 Skewness: alpha = 1



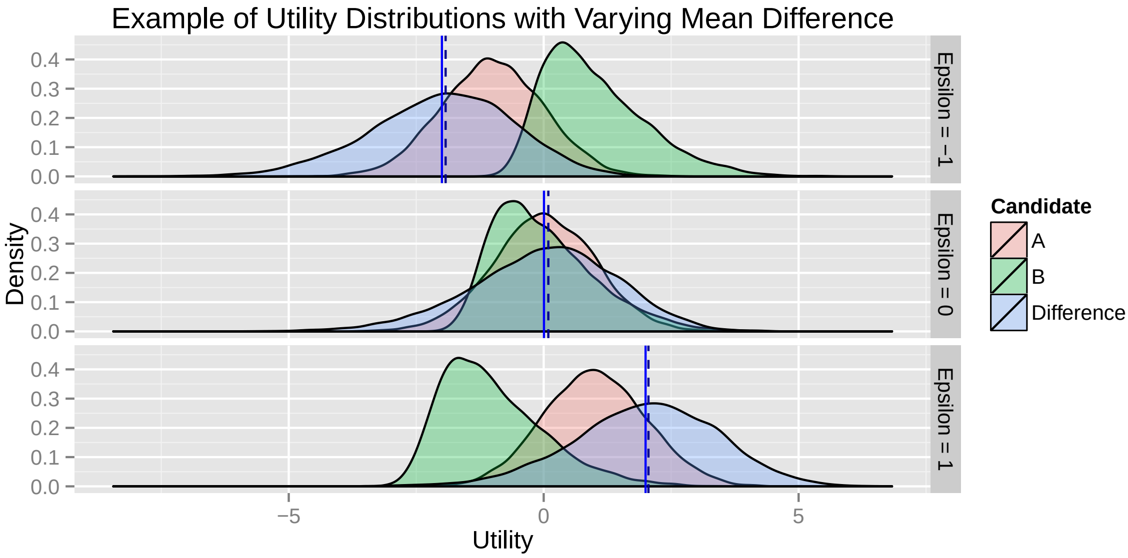


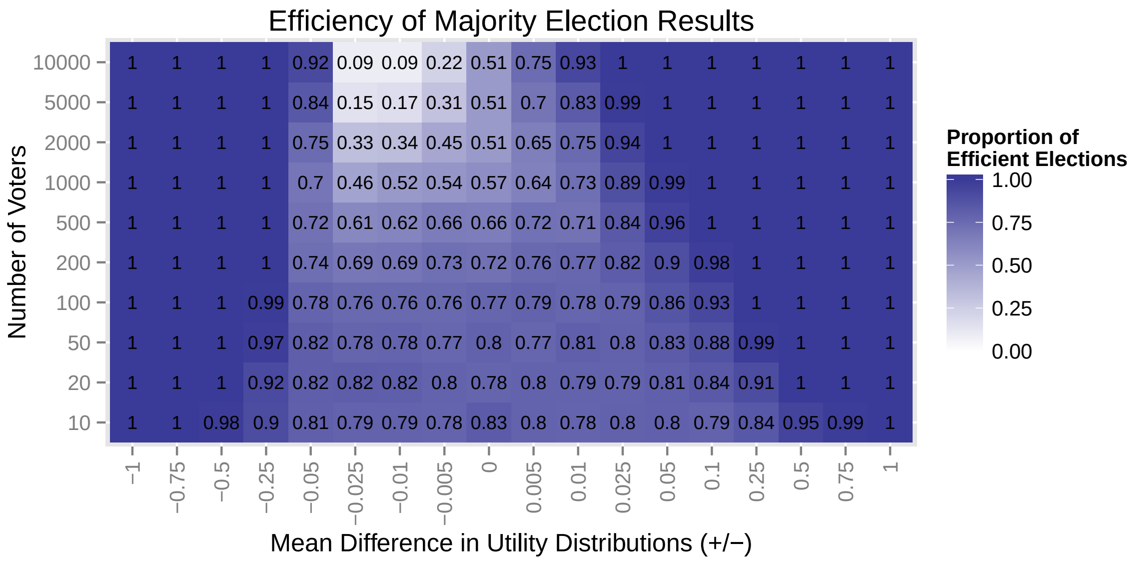
1.3.3 Skewness: alpha = 2



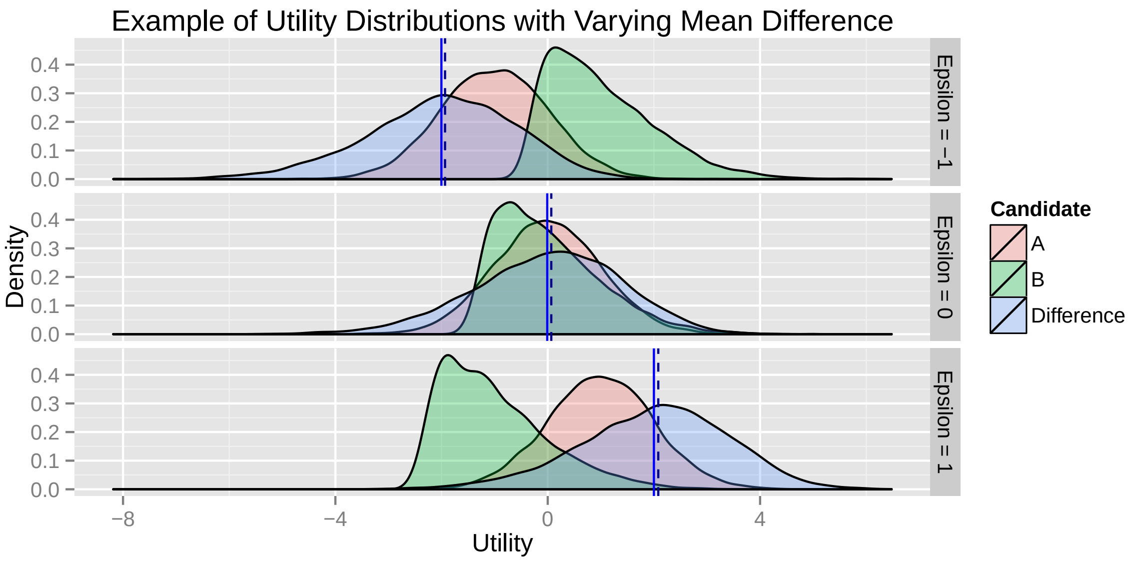


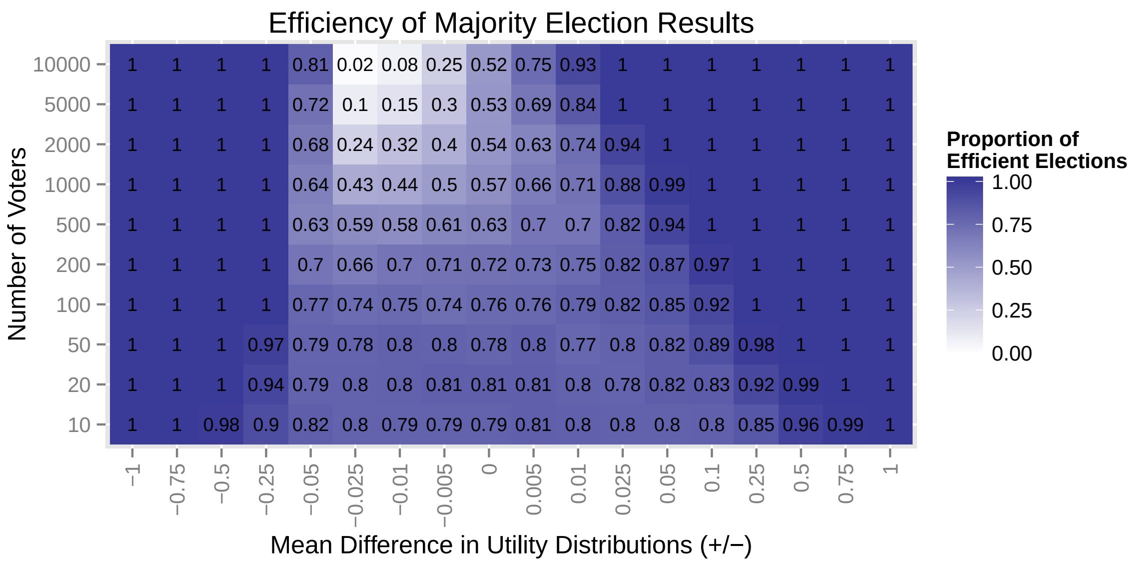
1.3.4 Skewness: alpha = 5





1.3.5 Skewness: alpha = 10



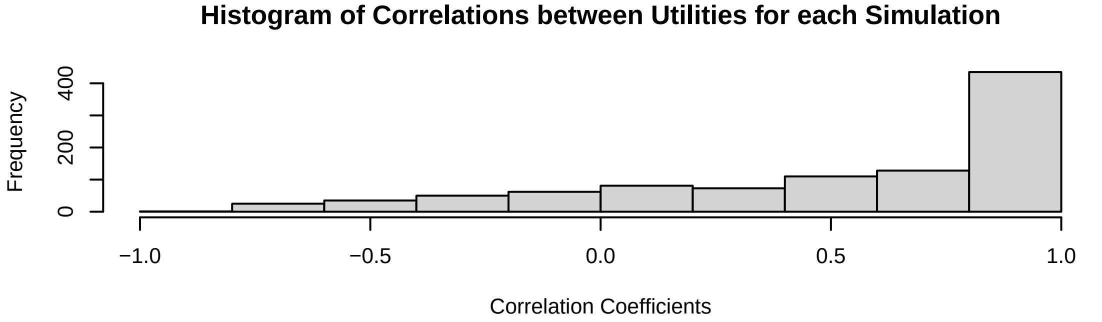


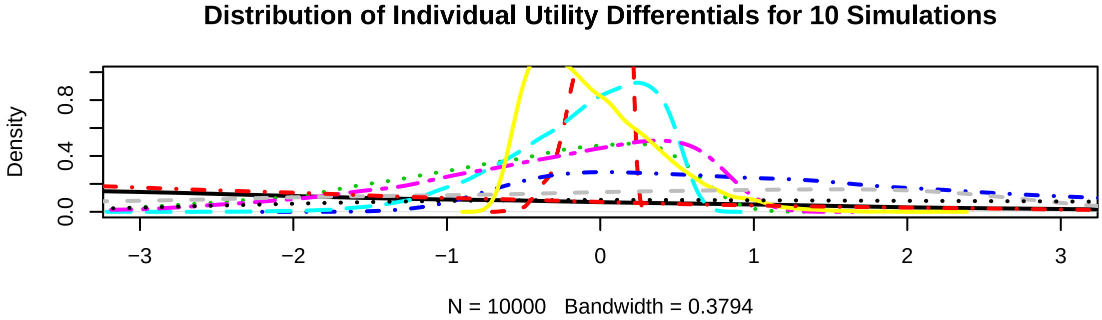
2 Investigating Ideal Point Scenarios that Lead to Skewed Utility Differentials

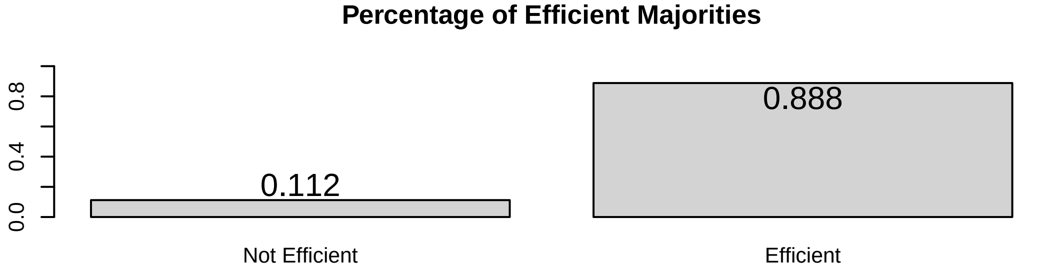
$$X_{cand1}, X_{cand2} \sim \mathcal{N}(\mu = 0, \sigma^2 = 1)$$

$$X_i \sim \mathcal{N}_{skew}\left(\xi = 0 - \omega \frac{\alpha}{\sqrt{1+\alpha^2}}\sqrt{\frac{2}{\pi}}, \omega = \sqrt{\frac{\pi}{\pi - 2\left(\frac{\alpha}{\sqrt{1+\alpha^2}}\right)^2}}\right)$$

$$U_{i1,i2} = -(X_{cand1,cand2} - X_i)^2$$







3 Analytical Solution for Efficiency Probability (for non-skewed utilities)

Coming next: assume that true mean difference is zero, and median falls on one side of the zero point or the other. What is the probability that the individuals on the opposite side outweigh the median side (in order to pull the mean on their side). This probability decreases with increasing sample size (relative size of groups on either side etc...)