How the Nature of Political Preferences Shapes the Efficiency of Majority Rule Voting

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Questions

- Is the majority rule efficient?
- How does its efficiency depend on how we conceptualize individual preferences and utilities?

Background

- Condorcet Jury Theorem
  - majority rule is efficient
  - information aggregation
- What about conflicting preferences?

Inefficient Majorities

Table : Example for Inefficient Majorities

Voter	Candidate A	Candidate B
Alice	\$0	\$100
Betty	\$20	\$10
Carol	\$10	\$0

Adding a third candidate candidate

Table : Example for Cyclic Preferences

Voter	Candidate A	Candidate B	Candidate C
Alice	$0	$100	$10
Betty	$20	$10	$0
Carol	$10	$0	$20

Voting, Ideal Points, and Utilities I

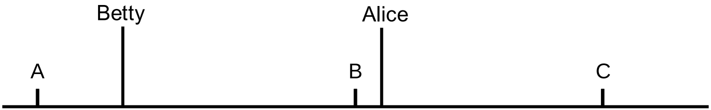
- Spatial theory of voting (e.g. Downs, 1957; Westholm, 1997):
  - common policy / ideological dimension
  - utilities determined by relative proximity

$$U_i^{\text{cand}} = -(X_i - X^{\text{cand}})^2$$

Voting, Ideal Points, and Utilities II

Table : Example for Cyclic Preferences

Voter	Candidate A	Candidate B	Candidate C
Alice	\$0	\$100	\$10
Betty	\$20	\$10	\$0
Carol	\$10	\$0	\$20



Back to the initial question

- Formal voting models: common policy space
- Other factors influence preferences and utilities, e.g. candidate traits and appearance (e.g. Hayes, 2005; Todorov et al., 2005)
- Question: How can relaxing assumptions of issue-based utilities alter our conclusions about the efficiency of voting rules?

Simulation Scenarios

- Overview:
  - Number of voters in each election: 2000
  - Number of candidates in each election: 2
  - Number of simulations for each scenario: 1000
  - Individual utilities based on ideal points or directly simulated from distributions; voters vote for the candidate that maximizes their utility
  - Goal: investigate the efficiency of majority voting under varying assumptions about voter preferences
- Conceptualization of efficiency:
  - Does the election result maximize the aggregated utilities for all voters?
  - $\sum_i U_i^W > \sum_i U_i^L$

First comparison of ideal points and normal utilities I

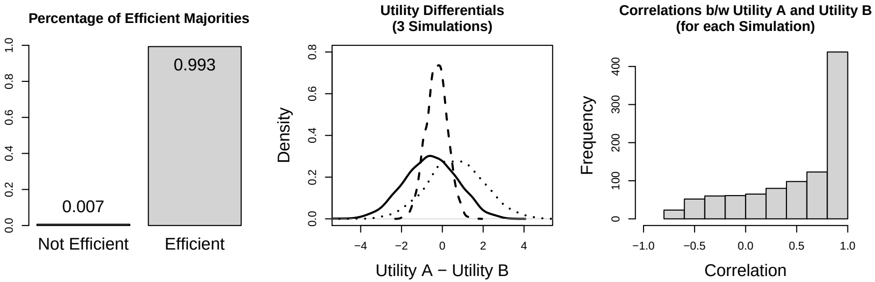


Figure : Normally distributed ideal points

$$X_i, X_a, X_b \sim \mathcal{N}(\mu = 0, \sigma^2 = 1)$$

$$U_i^a = -(X_i - X_a)^2 \qquad U_i^b = -(X_i - X_b)^2$$

First comparison of ideal points and normal utilities II

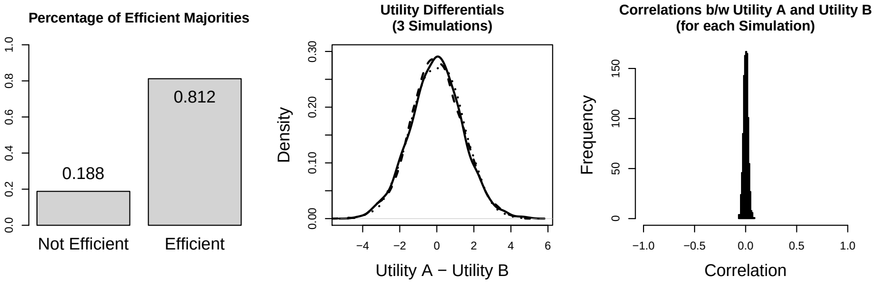


Figure : Independent normal utilities

$$U_i^a, U_i^b \sim \mathcal{N}(\mu = 0, \sigma^2 = 1)$$

Investigating the effect of correlated utilities I

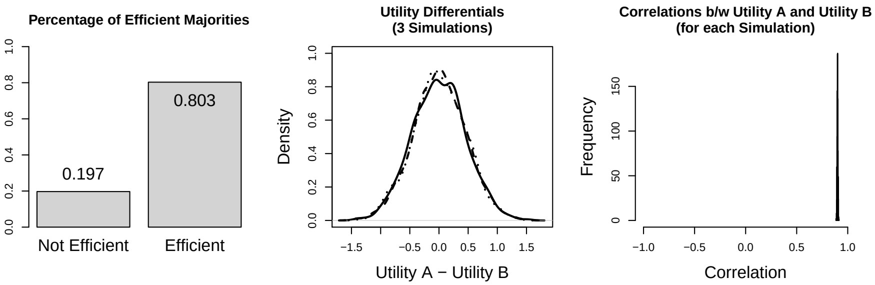


Figure : Positively correlated normal utilities

$$U_a, U_b \sim \mathcal{N}\left(\mu = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \boldsymbol{\Sigma} = \begin{pmatrix} 1 & 0.9 \\ 0.9 & 1 \end{pmatrix}\right)$$

Investigating the effect of correlated utilities II

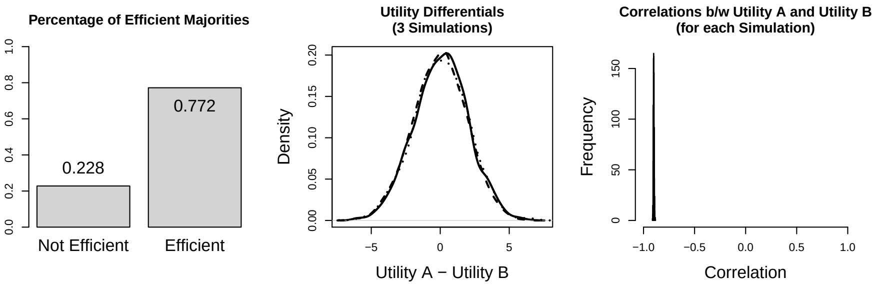
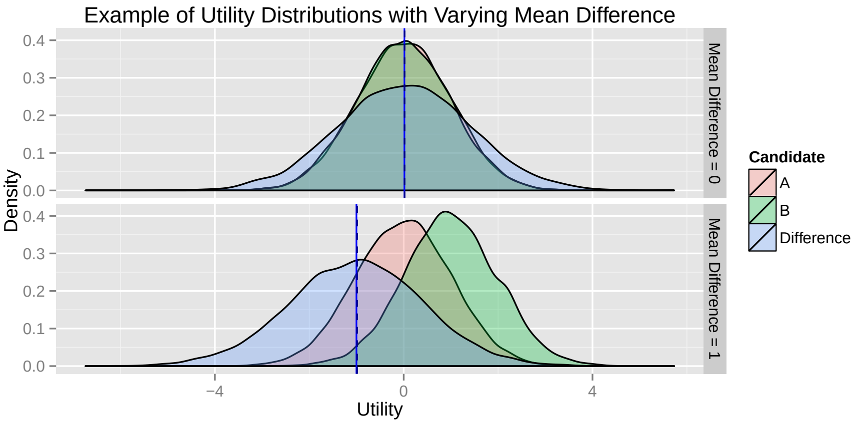


Figure : Negatively correlated normal utilities

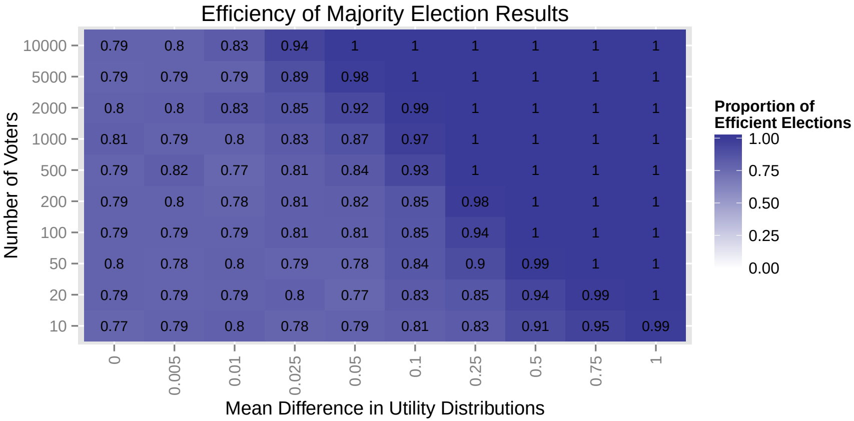
$$U_a, U_b \sim \mathcal{N}\left(\mu = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{\Sigma} = \begin{pmatrix} 1 & -0.9 \\ -0.9 & 1 \end{pmatrix}\right)$$

Inefficiencies for varying mean differences in utilities I



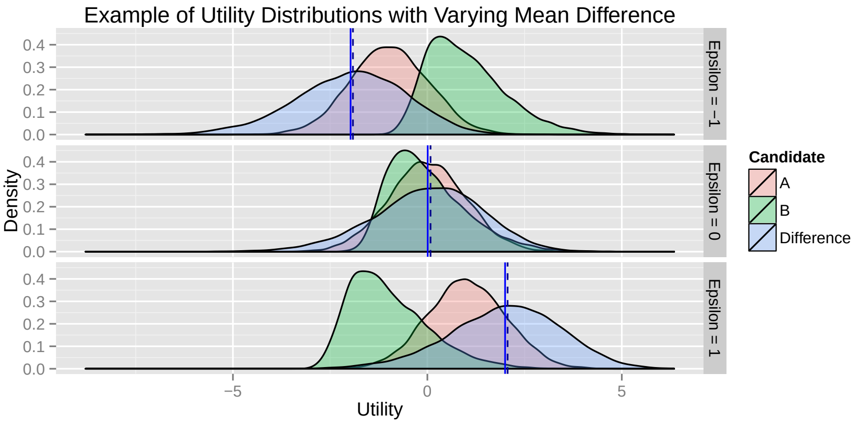
$$U_i^a \sim \mathcal{N}(\mu = 0, \sigma^2 = 1) \qquad U_i^b \sim \mathcal{N}(\mu = 0 + \epsilon, \sigma^2 = 1)$$

Inefficiencies for varying mean differences in utilities II



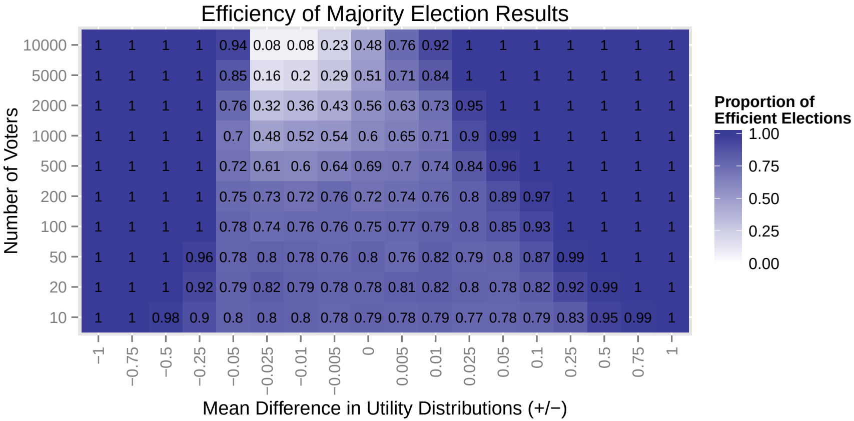
$$U_i^a \sim \mathcal{N}(\mu = 0, \sigma^2 = 1) \qquad U_i^b \sim \mathcal{N}(\mu = 0 + \epsilon, \sigma^2 = 1)$$

Investigating the effect of skewed utility distributions I



$$U_i^a \sim \mathcal{N}(\mu = 0 + \epsilon, \sigma^2 = 1) \qquad U_i^b \sim \mathcal{N}_{\text{skew}}(\mu = 0 - \epsilon, \sigma^2 = 1)$$

Investigating the effect of skewed utility distributions II



$$U_i^a \sim \mathcal{N}(\mu = 0 + \epsilon, \sigma^2 = 1) \qquad U_i^b \sim \mathcal{N}_{\text{skew}}(\mu = 0 - \epsilon, \sigma^2 = 1)$$

Discussion of Results

- Relaxing assumptions about ideal-point based preferences can reduce the likelihood that election results are efficient
  - mean difference and skewness of the distributions of individual utilities for each candidate affects the likelihood of inefficiencies
  - under some scenarios, increasing the size of the electorate actually reduces the efficiency of majority voting!
- Question: conceptualization of utility reasonable? These results would not hold if preferences were purely ordinal (and utilities not comparable across individuals)

Possible Experimental Designs and Further Developments

- Performance of compensation elections / bidding mechanisms in the context of binary choices (Oprea et al., 2007)
- Effect of (endogenous) electoral abstention on election efficiency
- Multi-candidate elections

References

Downs, Anthony. 1957. *An economic theory of democracy*. New York.

Hayes, Danny. 2005. "Candidate qualities through a partisan lens: A theory of trait ownership." *American Journal of Political Science* 49(4):908–923.

Oprea, Ryan D, Vernon L Smith and Abel M Winn. 2007. "A compensation election for binary social choice." *Proceedings of the National Academy of Sciences* 104(3):1093–1096.

Todorov, Alexander, Anesu N Mandisodza, Amir Goren and Crystal C Hall. 2005. "Inferences of competence from faces predict election outcomes." *Science* 308(5728):1623–1626.

Westholm, Anders. 1997. "Distance versus Direction: The Illusory Defeat of the Proximity Theory of Electoral Choice." *The American Political Science Review* 91(4):865–883.

Inducing inefficiencies with ideal point utilities I

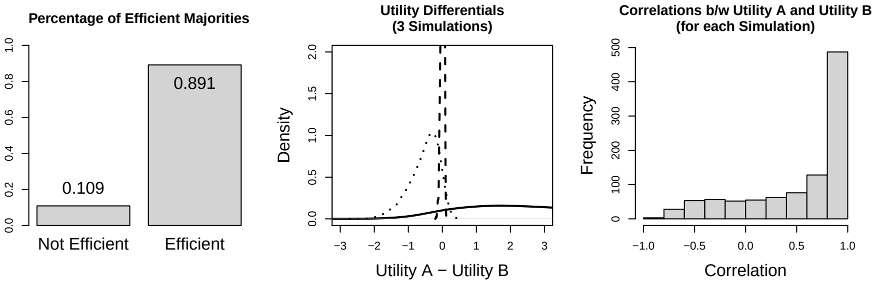


Figure : Skewed ideal points

$$X_i \sim \mathcal{N}_{\text{skew}}(\mu = 0, \sigma^2 = 1) \qquad X_a, X_b \sim \mathcal{N}(\mu = 0, \sigma^2 = 1)$$

$$U_i^a = -(X_i - X_a)^2 \qquad U_i^b = -(X_i - X_b)^2$$

Inducing inefficiencies with ideal point utilities II

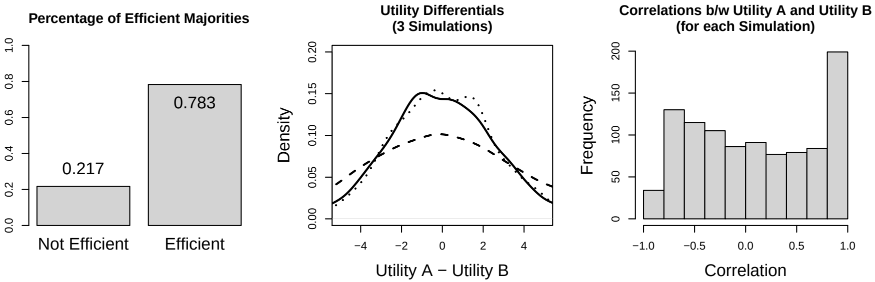


Figure : Aggregate indifference between ideal points

$$X_i, X_a \sim \mathcal{N}(\mu = 0, \sigma^2 = 1) \qquad X_b = -1 * X_a$$

$$U_i^a = -(X_i - X_a)^2 \qquad U_i^b = -(X_i - X_b)^2$$