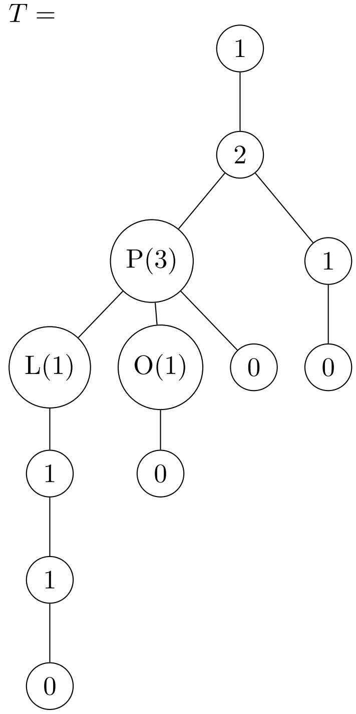
Ordered Trees

Given an ordered tree T, Let O be the first node in a preorder traversal of T that is not in the left-down path of T. Let P be O's parent, and let L be P's leftmost child (or, equivalently, O's left sibling). The tree below gives an example illustrating these three nodes.

$D = [1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 1, 1, 0, 0, 1, 0, 0, 1, 1, 0, 0, 0]$

$T =$



Let D be the dyck word corresponding to T and let k be the index of the first 01 substring in D. Note that the node O within T corresponds to D_k.
The cool-lex rule for shifts in ordered trees can be broken down into 3 cases:

- Case 1: O has at least 1 child

  Since O corresponds to D_k, O having at least one child corresponds to the case where $D_{k+1} = 1$

  Shift L to be O's first child.

- Case 2: O has no children and O is the child of the root

  This case corresponds to the case where $D_{k+1} = 0$ and the non-increasing prefix is tight (i.e., the non-increasing prefix hsa exactly as many ones as zeroes).

  Shifts in this case are the same as in case 1.

- Case 3: O has no children and O is not child of the root

This case corresponds to the case where $D_{k+1} = 0$ and the non-increasing prefix is not-tight (i.e., the non-increasing prefix has more ones than zeroes).

Shift L to be the first child of P's parent

Shift O to be the first child of the root.

Note: The order of these shifts matters. P cannot be the root, but if P's parent is the root, the O and L are both shifted to be P's first child. The shifting of O must be done second so that after both shifts are done O is the first child of the root.

Illustration of case 1 (shifting a 1):

[1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 1, 1, 0, 0, 1, 0, 0, 1, 1, 0, 0, 0] $\Longrightarrow$
[1, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 1, 1, 0, 0, 0]

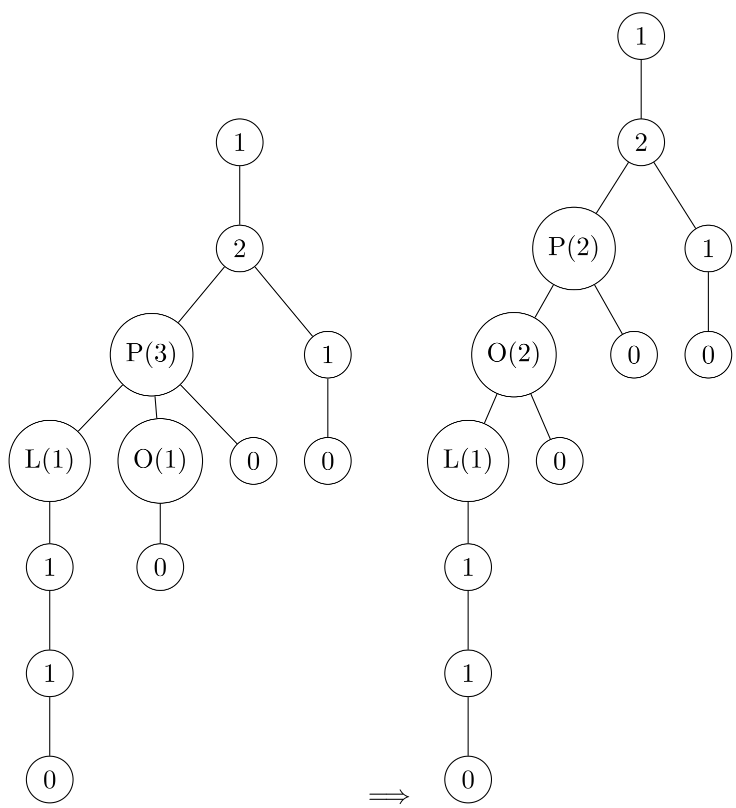


Illustration of case 3 (shifting a 0):

[1, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 1, 1, 0, 0, 0] $\Longrightarrow$
[1, 0, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 1, 0, 1, 0, 0, 1, 1, 0, 0, 0]

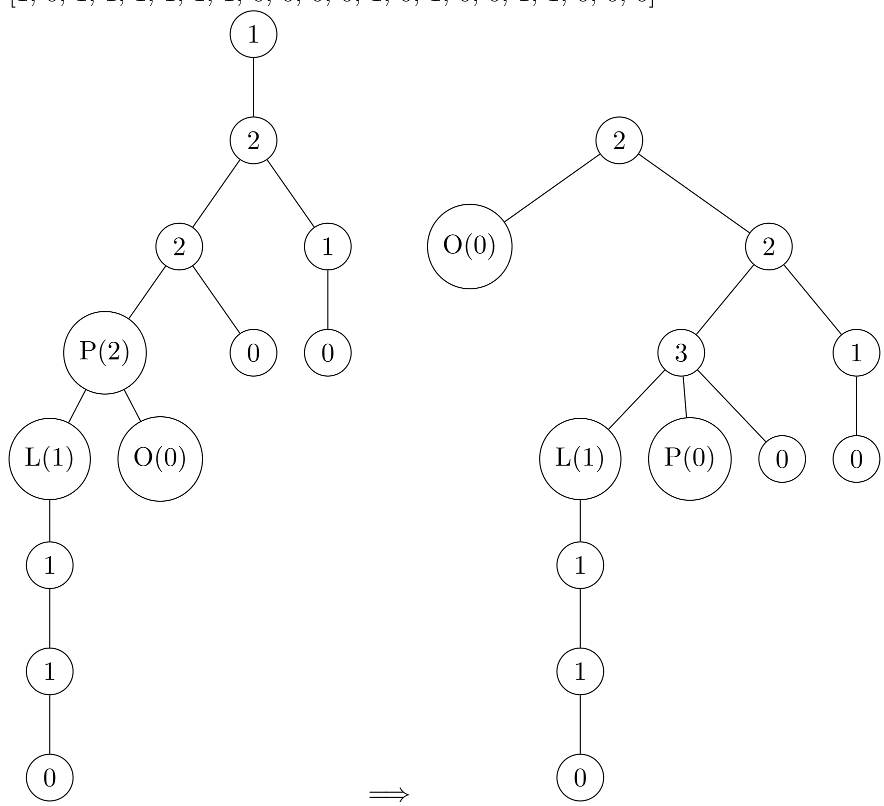


Illustration of case 2 ($D_k = 0$, prefix is tight, shift a 1)

[1, 1, 1, 0, 0, 0, 1, 0, 1, 0, 1, 0] $\Longrightarrow$
[1, 1, 1, 1, 0, 0, 0, 0, 1, 0, 1, 0]