Problem 3 $\theta = (\theta_a, \theta_{0b}, \theta_{1b})$ $d\alpha = d\theta_a \times d\theta_{0b} \times d\theta_{1b}$ assuming the parameters are indep.

we can use ML separately for parameters:

(a) $P(D|G,\theta_a) = \binom{4}{2}\theta_a^2(1-\theta_a)^2 \Rightarrow \hat{\theta}_a^{ML} = \frac{1}{2}$

$P(D|G,\theta_{1b}) = \binom{2}{2}\theta_{1b}^2 \Rightarrow \hat{\theta}_{1b}^{ML} = 1$, $P(D|G,\theta_{0b}) = \binom{0}{2}(1-\theta_{0b})^2 \Rightarrow \hat{\theta}_{0b}^{ML} = 0$

(b) $P(D|G) = \int P(D|\theta,G)P(\theta|G)d\theta = \left(\int P(D|\theta_a,G)P(\theta_a|G)d\theta_a\right)\left(\int \dots d\theta_{1b}\right)\left(\int \dots d\theta_{0b}\right)$

each factor has generic form $\int \binom{k+\ell}{k}\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}x^{k+a-1}(1-x)^{\ell+b-1}dx = \binom{k+\ell}{k}\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}\cdot\frac{\Gamma(a+b+k+\ell)}{\Gamma(a+k)\Gamma(b+\ell)}$

I don't feel like crunching the numbers...

(c) the posterior of binomial-beta is beta:

$P(\theta_a|G,D) \propto \theta^2(1-\theta)^2 \cdot \theta^2(1-\theta)^0 \Rightarrow P(\theta_a|GD) = B(5,5)$

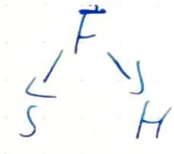
$P(\theta_{0b}|G,D) \propto \theta^4(1-\theta)^2 \Rightarrow B(5,3)$, $P(\theta_{1b}|G,D) = B(3,5)$

Problem 4 (a) $P(B=1|A=0,\theta_0) = \binom{10}{8}\theta_0^8(1-\theta_0)^2 \Rightarrow \theta_0^{ML} = 0.8$

(b) $P(\theta_0|D) = B(8+1, 2+1) = B(9,3)$

Problem 2 (a) $P(F,S,H) = P(S|F)P(H|F)P(F)$

$S \perp H \mid F$



(b) $P(F=1) = 0.1 \Rightarrow P(F=0) = 0.9$

(c) $P(F=0|S=1) = \frac{P(S=1|F=0)P(F=0)}{\sum_{i=0,1}P(S=1|F=i)P(F=i)} = \frac{10^{-3}\cdot 0.9}{10^{-3}\cdot 0.9 + 0.09} = 0.0099$

(d) $P(S,H|F) = P(S|F)P(H|F)$

$P(S,H) = \sum_{i=0,1}P(S,H|F=i)P(F=i) \approx 0.089$

$P(F=1|S,H=1) = P(S|F=1)P(F=1)P(H=1|F=1)/P(S,H=1) \approx 0.999$