Complex Systems - Vilhelm part - Assignment 4

(2) (A) let $T, S \in \mathbb{R}^{n\times n}$ be stochastic and let $x \in \mathbb{R}^n_+$ be a distribution;

$\|x\|_1 = \sum x_i = 1$, let $\alpha \in [0,1] \Rightarrow \|Tx\|_1 = \|Sx\|_1 = 1$

then: $\|(\alpha T + (1-\alpha)S)x\|_1 \underset{\uparrow}{=} \alpha\|T\|_1 + (1-\alpha)\|S\|_1 = \alpha + (1-\alpha) = 1$

because $\alpha, (1-\alpha), x, T, S \geq 0$

(B) $Q \stackrel{\Delta}{=} \frac{1}{N}E$

$$p_{n+1} = [\alpha Q + (1-\alpha)T]p_n = \dots = [\alpha Q + (1-\alpha)T]^n p_0 \qquad q \stackrel{\Delta}{=} Qp_0 = \frac{1}{N}\mathbb{1} \stackrel{\Delta}{=} p_0$$

given that the limit exists: $\lim_{n\to\infty} p_n = p = \lim_{n\to\infty}[\alpha Q + (1-\alpha)T]p_n = [\alpha Q + (1-\alpha)T]p$

it must follow that $p = Ip = [\alpha Q + (1-\alpha)T]p = \alpha Qp + (1-\alpha)Tp = \alpha q + (1-\alpha)Tp$

$\Leftrightarrow \alpha q = (I - (1-\alpha)T)p$

$\Leftrightarrow p = \alpha(I - (1-\alpha)T)^{-1}q$

<u>remark</u> for any $v \in \mathbb{R}^n_+$ s.t $\|v\|_1 = 1$ we have $Qv = q = (\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n})$

so p_0 doesn't matter as long as we pick a distribution.