Control of Aircraft
Lateral Autopilot

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CONTROL OF AIRCRAFT

YAW AUTOPILOT (WITHOUT COUPLING)

For a symetric airframe
$CY_\beta = -CN_\alpha$
$CY_{\delta_n} = CN_{\delta_m}$
$Cn_\beta = -Cm_\alpha$
$Cn_{\delta_n} = Cm_{\delta_m}$
The study of a yaw controller is rather the same as the study of the pitch controller.

Roll autopilot (case without coupling)

Stabilization and control is achieved by a gyrometric feedback with p associated with proportional integral corrector (gain k_p and k_i), and a roll angle feedback φ associated with a proportional controller (gain k_φ).
By simplifying $p = s\varphi$ and by neglecting the gyroscopic coupling, we have:

$$I_{XX} s^2 \varphi = QS_{ref}\ell_{ref} Cl_{\delta_l}\left(((k_\varphi(\varphi_c - \varphi) - s\varphi)\frac{k_i}{s} - s\varphi)k_p\right) + QS_{ref}\ell_{ref} Cl_p \frac{\ell_{ref}}{V_a} s\varphi$$

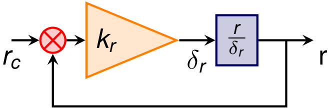
so

$$\frac{\varphi}{\varphi_c} = \frac{1}{(1+\tau s)(\frac{s^2}{\omega^2} + 2\frac{\xi}{\omega} + 1)}$$

LATERAL AUTOPILOT WITH COUPLING

A first possibility to design a coupled lateral autopilot able to deal with a coordinated turn is to command a roll linked to the wished horizontal turn rate and the new wished yaw angle, a null yaw rate in transient phase, but a constant yaw rate in steady state, to command an increase of speed and an increase of incidence.

We begin by adding a yaw damper.

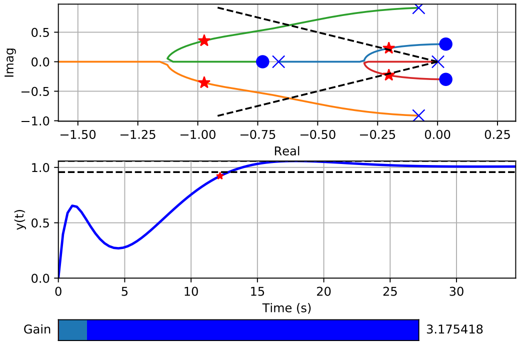


For the setting of the gain, we choose a negative gain $k_r = -3.17$, which permits to obtain for the closed loop

- a pole with a damping ratio of 0.93 and a proper pulsation of 1.04 rad/s
- a pole with a damping ration of 0.66 and a proper pulsation 0.31 rad/s

The obtained gain margin is 6.5 dB and the phase margin of 78.5°.

ROOT LOCUS AND STEP RESPONSE



-0.97315+j0.35562 xi=0.93925 w=1.03609 rad/s
-0.97315-j0.35562 xi=0.93925 w=1.03609 rad/s
-0.20313+j0.22828 xi=0.66475 w=0.30557 rad/s
-0.20313-j0.22828 xi=0.66475 w=0.30557 rad/s

OS=4.939127944822208 % tr5%=12.183001941322734 s
GM=6.5495527738059085 dB PM=78.48948043483256 deg

The aircraft can make a turn at constant altitude.
What is needed is that when the aircraft takes roll, the lift keeps on equilibrating the action of gravity.
Let N be the normalized load factor defined as the ratio of the lift R_{za} divided by weight P while the aircraft is in turn.
For null roll, we have

$$N = \frac{R_{za}}{mg}$$

While in turn, the weight is projected in the aircraft longitudinal plane.
So:

$$N = \frac{1}{\cos\varphi} = \sqrt{1 + \tan^2\varphi}$$

$$\tan\varphi = \sqrt{N^2 - 1}$$

The turn radius is, with Γ_{lat} the lateral acceleration in the horizontal plane

$$R = \frac{V_{éq}^2}{\Gamma_{lat}} = \frac{V_{éq}^2}{\frac{R_{za}}{m}\sin\varphi}$$

$$R = \frac{V_{éq}^2}{\frac{g}{\cos\varphi}\sin\varphi}$$

$$R = \frac{V_{éq}^2}{g\tan\varphi} = \frac{V_{éq}^2}{g\sqrt{N^2-1}}$$

If the turn is achieved at constant incidence, then the lift coefficient is constant (the right term is lift coefficient in turn)

$$Cz = \frac{R_{za}}{\frac{1}{2}\rho V^2 S} = \frac{mg}{\frac{1}{2}\rho V^2 S} = \frac{Nmg}{\frac{1}{2}\rho V_v^2 S}$$

To maintain the altitude, we need to increase the lift by increasing the speed and attain a new higher speed in the turn $V_v = \sqrt{N}V$. It is obtained by setting a higher throttle control.
If we increase the incidence α, we need to increase throttle to compensate the added drag created by an increase of α.
While in turn, we have on the aircraft y axis:

$$mV_{éq}\dot{\psi}\cos\varphi = mg\sin\varphi$$

$$\tan\varphi = \frac{V_{éq}\dot{\psi}}{g}$$

For $\varphi \ll 1$

$$\tan\varphi \approx \varphi$$

We want that ψ acts as a first order system, such as its response is not instantaneous (it can also remove the high frequency noise in the commanded φ).

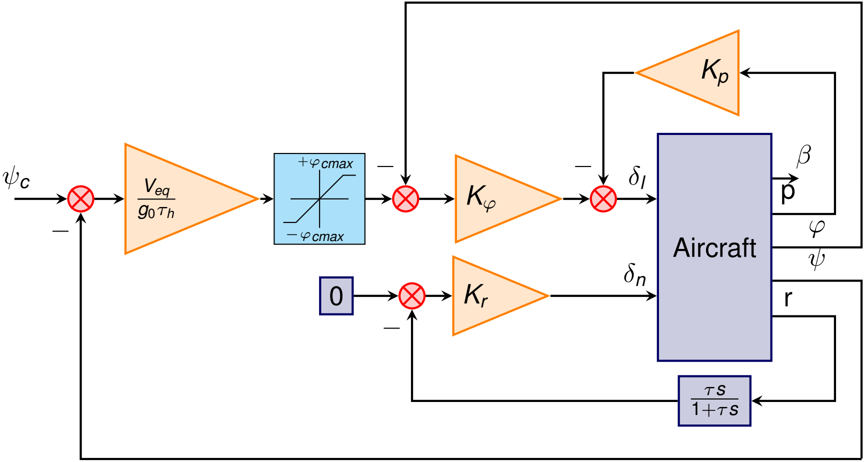
$$\frac{\psi}{\psi_{co}} = \frac{1}{\tau_h s + 1}$$

Then we have

$$\varphi_{co} = \frac{V_{éq}}{g}\dot{\psi} = \frac{V_{éq}}{\tau_h g}(\psi_{co} - \psi)$$

On the following figures, we have the lateral autopilot architecture (yaw+roll), with the trajectory and the angles φ and ψ as a function of time.
On yaw axis, a null rotation speed r is commanded. We notice a washout filter ($\frac{\tau s}{\tau s+1}$) on the gyrometric feedback r, so that the autopilot allows a steady state phase with non null rotation speed r.

LATERAL AUTOPILOT



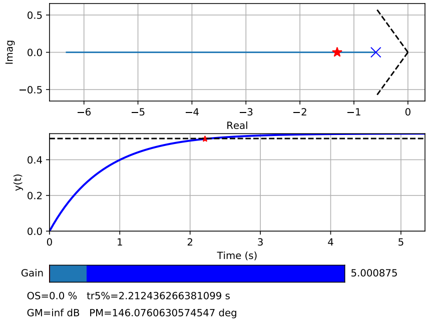
Gains of the roll control loop

The transfer function between the aileron deflection angle and the roll rotation speed can be approximated by

$$\frac{p}{\delta_l} = \frac{l_{\delta_l}}{1 - l_p}$$

First, the gain of the roll rotation speed feedback is chosen positive, and a gain $k_p = 5$ allows to have a settling time of about 2 s. Note that we could also have dealt with PD corrector (p and φ) at the same time.

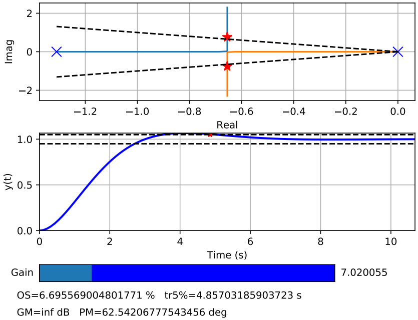
```
pdeltal=tf(l_deltal,[1,-l_p])
sisotool(pdeltal)
```



Gains of the roll control loop

We then calculate the closed loop transfer function with the p feedback loop. We can then find the gain for the φ feedback loop. The gain is chosen to have a damping ratio for double complex conjugate pole of about 0.65. We obtain a gain of $k_\varphi = 7$.

```
olrolldamper=series(feedback(\
pdeltal,tf(dxdt.kp,1)),tf(1,[1,0]))
sisotool(olrolldamper)
```



Then τ_h is chosen between 10 s and 20 s. τ_h determines the ψ rate during a coordinated turn.

On modern fly by wire controlled aircrafts, the yaw damper (r feedback loop) is switched on before take-off, and switched off after landing, to damp the Dutch roll mode.

We have $V_x = V_v cos(\psi + \beta)$ and $V_y = -V_v sin(\psi + \beta)$.

By integration we can find the aircraft center of gravity coordinates. Note that in order to simulate this autopilot with a non linearity (saturation) the time response tools of the control toolbox can no longer be used. A simulation with an ODE (ordinary differential equations) integrator must be used instead.

Lateral autopilot simulation

Finally, the used gains and time constant are

$$k_p = 5$$

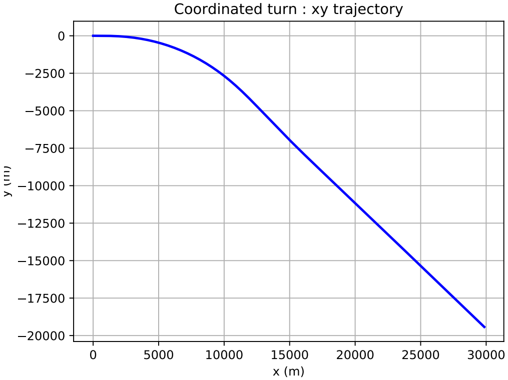
$$k_\varphi = 7$$

$$k_r = -3.17$$

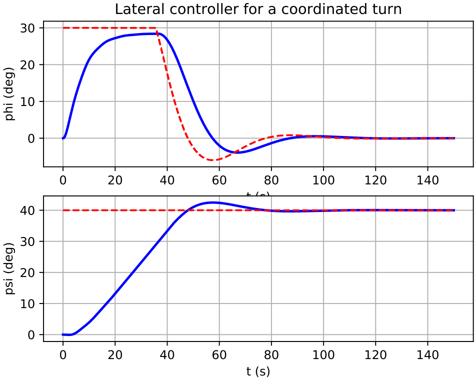
$$\tau_h = 10$$

and $\tau = 0.5$ s for the high-pass filter.
A rudder and aileron actuators are introduced. They have a first order transfer function with a time constant of 0.1 s.
The autopilot receives a commanded heading ψ_{co} of 40°.
The commanded roll is saturated to stay within ±30°

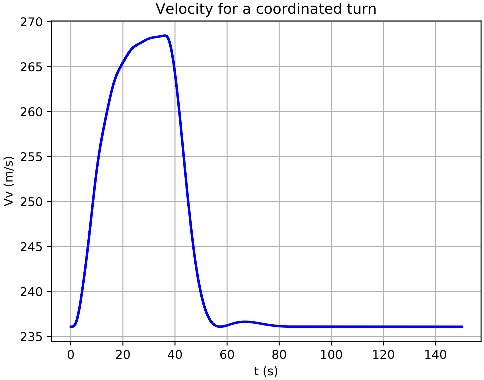
Trajectory in the XY plane



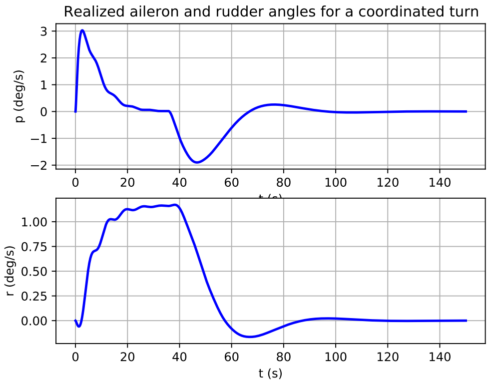
φ AND ψ



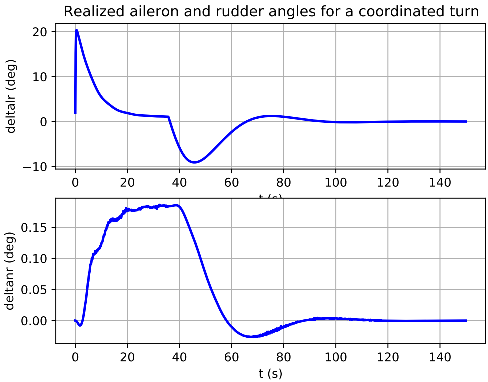
VELOCITY


Velocity for a coordinated turn
Vv (m/s)
t (s)
270
265
260
255
250
245
240
235
0
20
40
60
80
100
120
140

Rotation speeds

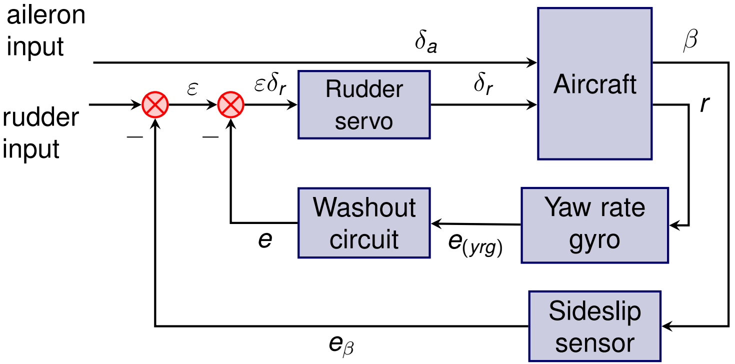


FIN DEVIATION ANGLES



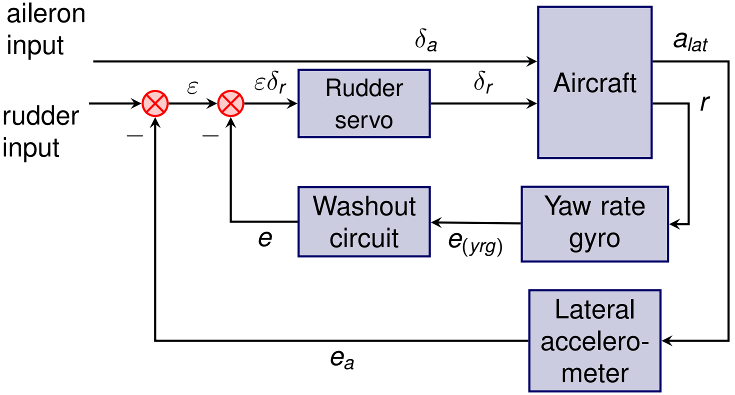
ALTERNATIVE LATERAL AUTOPILOT

In order to obtain a coordinated turn, a possibility is to command the sideslip to 0.



Alternative lateral autopilot

In order to obtain a coordinated turn, another possibility is to command the lateral acceleration to 0.



CONTROL OF AIRCRAFT

1 YAW-ROLL CONTROLLER SYNTHESIS

2 OTHER SYNTHESIS METHODS

NASLIN CRITERION

Let $F(s)$ be be a transfer function under the form

$$F(s) = \frac{1}{a_n s^n + ... + a_{k+1} x^{k+1} + a_k x^k + a_{k-1} x^{k-1} + ... + a_0}$$

For this transfer function being stable, with a damping ratio of 0.7 it is sufficient that all fractions $\frac{a_i^2}{a_{i+1} a_{i-1}}$ be equal to 2 $\frac{a_i^2}{a_{i+1} a_{i-1}} = 2$ for $i \in [1, n-1]$.

POLE PLACEMENT

The controller synthesis methods by pole placement consists in specifying the closed loop poles of the system, the real part of imaginary poles allowing to specify a proper pulsation and the argument allowing to specify a damping ratio.
The used controller is generally state feedback controller.

LQ CONTROLLER

Optimal control by state feedback, this method allows to compute the command u minimizing a quadratic criterion using weights on state (generally the final state) and command.
By fixing a criterion on the final state, the method tries to obtain a null static error on variable to be controlled (such as the acceleration for example).
Minimizing a quadratic criterion on the command is equivalent to searching to minimize the demand of energy of the system needed to achieve the servo control.
This LQ method (Linear Quadratic) can be applied to multivariable system.
When this LQ controller is coupled to a state observer (such as a Kalman filter for example), used to filter noisy measurements and to estimate the non measured states), we talk about LQG controller (Linear Quadratic Gaussian).

$\mathcal{H}_\infty$ CONTROLLER

Optimal control with output feedback, this method allows to compute the command u minimizing the norm $\mathcal{H}_\infty$ of the transfer function of the system, the system being possibly the process to be controlled to which syntheses filers have added.
The idea is that bounded inputs and in spite of uncertainties on transfer functions of the system, the outputs must remain bounded, whatever the frequency could be.
This is a linear robust controller, which could be applied to multivariable system.