Assignment #1

Due date: October 11th, 2022 at 11:59pm

- Please write your name here ....
- Please write your UID here ....
- Please write your solutions in the free space after problem statements. Hand in the hard copy of your solutions before the class begins.
- Your grades depend on the correctness and clarity of your answers.
- Write your answers with enough detail about your approach and concepts used, so that the grader will be able to understand it easily. You should ALWAYS prove the correctness of your algorithms either directly or by referring to a proof in the book.
- Write your answers in the spaces provided. If needed, attach other pages.
- You should scan and submit your solutions one week before the deadline to show your progress. Send your .pdf file to XXX@gmail.com with title "HW1-so-far".

(1)

1. A standard 52-card deck includes thirteen *ranks* of {2, 3, ... ,10, *J, Q, K, A*} in each of the four *suits* {♦, ♣, ♥, ♠}. In Poker, any subset of 5 cards is called a *hand*. A hand is called a *full house* if it includes exactly three cards of the same rank and two cards of another rank.

(a) Suppose that we draw 5 cards out of the 52 cards randomly. What is the probability that these 5 cards form a full house?

(b) Suppose that we draw 7 cards out of the 52 cards randomly. What is the probability that there exists at least one full-house hand within these 7 cards?

(2)

2. Suppose that you play the following game against a house in Las Vegas. You pick a number between one and six, and then the house rolls three dices. The house pays you \$1,500 if your number comes up on one dice, \$2,000 if your number comes up on two dices, and \$2,500 if your number comes up on all three dices. However, *you must pay the house \$1,000* if your number does not show up at all. How much can you expect to win (or lose)?

(3)

3. You are looking for your hat in one of six drawers. There is a 10% chance that it is not in the drawers at all, but if it is in a drawer, it is equally likely to be in each. Suppose that we have opened the first two drawers and noticed that the hat is not in them.

(a) What is the probability that the hat is in the third drawer?

(b) What is the probability that the hat is not in any of the drawers?

(4)

4. Let S be the set of all sequences of three rolls of a dice.

- Let X be the sum of the number of dots on the three rolls. What is $E(X)$?
- Let Y be the product of the number of dots on the three rolls. What is $E(Y)$?

(5)

5. Find x, y, and z such that

- $x \leq 10$
- $x + y \leq 17$
- $2x + 3z \leq 25$
- $y + z \geq 11$
- $15x + 2y + z$ is maximized

Your solution:

- x : [ ]
- y : [ ]
- z : [ ]
- $15x + 2y + z$: [ ]

(6)

6. We are given a graph G with vertex set $V(G)$ and edge set $E(G)$. Every edge e has a weight w_e. Write an LP whose optimal solution is equal to the length of the shortest path from a vertex s to a vertex t in this graph. You may write some constraints over the edge set or vertex set of the graph. For instance, you can write

$$\forall (u,v) \in E(G) \qquad\qquad w_u f_u + w_v f_v \leq 10.$$

In the above example, f_i's are the variables of your LP.

(7)

7. Prove that the expected value of the binomial distribution with n draws and probability p is equal to np and its variance is $np(1 - p)$.

(8)

8. Assume there is a set J of jobs and a set C of CPU cores such that $|C| = |J|$. The time required for a job $j \in J$ to be processed on CPU $c \in C$ is $t_{j,c}$.

(a) Write an integer program to assign each job to a CPU which minimizes the total amount of time required to finish all the jobs. Note that each CPU must receive exactly one job.

(b) Give an example with two jobs and two CPU cores for which the LP relaxation of your program has a lower objective value than the optimal integer one.

(9)

9. The bisection of a graph of a graph is defined as the smallest number of crossing edges when dividing the vertices of the graph into two sets of equal size (there is no connectivity requirements for the sets). Write an integer program that computes the bisection of a graph with even number of vertices.