Gopinath-Observer for Flux Estimation of an Induction Machine Drive System

Oleg Buchholz, Joachim Böcker
Power Electronics and Electrical Drives
Paderborn University, 33098 Paderborn, Germany
buchholz@lea.upb.de, boecker@lea.upb.de

***Abstract*—The focus of this paper is the estimation of the rotor flux linkage of an induction motor, which is fundamental to set up a field-oriented control. In first instance, different well-known approaches for rotor flux estimation are described to get a comprehensive overview of the topic. Thereafter, an alternative observer is derived step by step. The proposed rotor flux observer can be defined as a reduced-order Gopinath-observer with a compensator in the feedback. The performance is analyzed through simulative studies in steady-state and dynamic operation. Furthermore, to identify the benefits of the proposed observer, an extensive sensitivity analysis on the impact of parameter mismatch is presented. For comparison, the classical current model-based rotor flux estimator is used.**

***Keywords*—rotor flux estimation, Gopinath-observer, induction machine**

I. INTRODUCTION

For the control of three-phase drives, such as the permanent magnet synchronous machine (PMSM) or the induction machine (IM), the field-oriented control (FOC) has become state of the art since its introduction in the 1970s [1]. It is well known that the treatment of the space vectors in a rotating reference coordinate system has several advantages from the control point of view. The main one is that the controller design can be realized quite similar to that of DC-motor drives. However, the precondition to achieve the full potential of the FOC approach is the precise knowledge of the flux vector in terms of magnitude and, even more important, orientation. The latter is used to determine the direction of the reference coordinate system. In this context, however, there is a difference between an PMSM and an IM drive. For a PMSM, the permanent magnet flux ψ_P is used to determine the orientation while for an IM typically the rotor flux linkage $\underline{\psi}_r$ is chosen. The magnitude of the permanent magnet flux is ideally constant and it can be determined *a priori*. By means of a high-resolution rotatory encoder, the magnetization direction of ψ_P (which is directly linked to the rotor) can also be determined very accurately. In contrast, the rotor flux linkage of the IM cannot be determined *a priori*, neither its magnitude nor its position. Accurate measurement of the rotor flux is, in principle, possible but only with an extremely high effort, robustness reduction and additional cost. Consequently, a common practice is the estimation of the rotor flux linkage through model-based estimators using measured electrical variables which are, in any case, needed for control purposes.

In literature, a variety of different model-based approaches to estimate the rotor flux linkage of an IM can be found [2], [3]. All of them are based on the assumption of fundamental wave distribution in the stator and rotor and can be roughly grouped into the following categories:

1) Linear flux estimators
   a. Open-loop
   b. Closed-loop
      i. Full-order
      ii. Reduced-order
2) Non-linear flux estimators

Fig. 1 shows an overview of different known model-based estimators sorted by the specified group categories. The input variables are directly measured or reconstructed from measured ones in combination with known system parameters. Hence, the dynamic behavior of the IM can be described in different reference coordinate systems (Fig. 2). Likewise, there are different alternatives for the implementation of the observers. It is only important, however, that all the information required to perform the coordinate transformation is available. Using different reference frames, it is possible to utilize potential advantages, e.g. the dependency on a variable can be eliminated. Taking this into account, the flux observer topology presented in this paper is also designed using different reference frames. The resulting benefits are explored in the following sections.

The main reason to strive for an improved rotor flux estimation (in the whole operating range) is the improvement of torque accuracy, especially in torque controlled applications. Otherwise, full utilization of the drive system cannot be achieved. In fact, even small misalignments of the rotor flux-oriented dq reference frame can lead to a significant mismatch between the measured and estimated torque. Ideally, the achievable torque accuracy can be increased to a range similar to the one achieved with PMSM drive systems, i.e., in the range of $\pm$ 5% of the nominal torque.

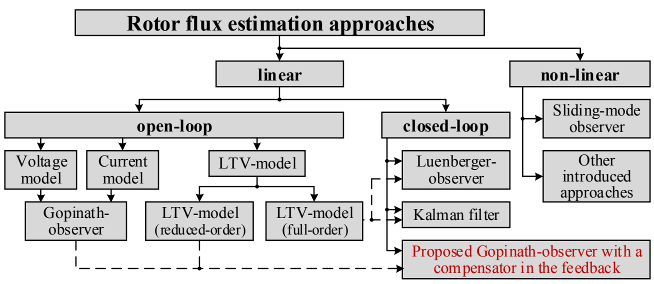


Fig. 1. Overview of flux estimation approaches for the IM

This work is funded by the German Federal Ministry for Economic Affairs and Energy (BMWi)

II. Rotor Flux Estimation Approaches

The choice of an appropriate flux estimation approach to set up the control of an IM drive depends on the application in hand: the requirements on the control performance within the expected operating range and the available hardware (computing platform, sensors, etc.). It is also clear that a certain estimation error will always remain. The challenge is, if necessary, to reduce it to a minimal level. The effort to achieve this goal can be quite high due to the variety of possible influences within the drive system, e.g. measurement noise, magnetic saturation, inverter nonlinearities, temperature dependency of the machine parameters, iron losses, skin effect, influences through harmonic components, failures through discretization, etc. Different known approaches, which were previously roughly classified, will be described shortly in this section. Therefore, different reference frames are used always considering the most common one. To distinguish between the reference frames, appropriate superscripts are chosen for the variables, e.g. $\underline{x}^s$ for the stator-oriented, $\underline{x}^r$ for the rotor-oriented, and $\underline{x}^k$ for the arbitrary-oriented reference coordinate system. For field-orientation, the arbitrary-oriented reference frame is determined in direction of the rotor flux vector ψ_r. In consequence, ψ_{rq} is zero by definition. In Fig. 2, the relations are graphically depicted for clarification of the notation.

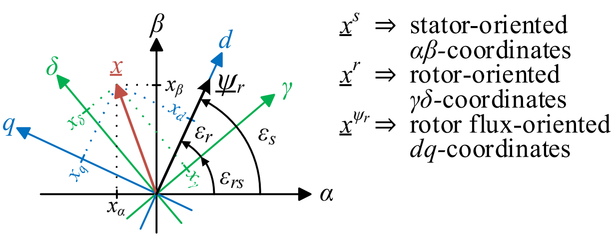


Fig. 2. Commonly used reference coordinate systems for an IM

A. *Flux estimators – open-loop types*

The main advantage of the open-loop flux estimators is their simple structure. However, the performance of these estimators is very sensitive to mismatches of the machine parameters. The most widely known open-loop flux estimator approaches are described shortly in the following.

1) Voltage or Stator model ($\underline{u}_s$-$\underline{i}_s$-model): The estimated rotor flux $\underline{\hat{\psi}}_r^s$ ($\hat{x}$ refers to estimated quantities) is calculated using the stator voltage differential equation (1) together with the relations between the currents and fluxes (2). To stabilize the open-loop integrator, the output of the integrator is fed back in a close-loop fashion by means of a P-T_1-element with an adjustable time constant τ_{FB} [2].

$$\frac{d\underline{\hat{\psi}}_s^s}{dt} = \underline{u}_s^s - R_s\underline{i}_s^s \;\Rightarrow\; \frac{d\underline{\hat{\psi}}_s^s}{dt} = \underline{u}_s^s - R_s\underline{i}_s^s - \frac{1}{\tau_{FB}}\underline{\hat{\psi}}_s^s \tag{1}$$

$$\underline{\hat{\psi}}_r^s = f\left(\underline{\hat{\psi}}_s^s, \underline{i}_s^s\right) = \frac{L_r}{L_m}\left(\underline{\hat{\psi}}_s^s - \sigma L_s \underline{i}_s^s\right) \tag{2}$$

The major disadvantages of the voltage model are the temperature dependency of the ohmic stator resistance R_s and inverter nonlinearities. Due to the feedback, the operating area is reduced to $\omega_s > 1/\tau_{FB}$ (with ω_s as the stator frequency).

2) Current or Rotor model ($\underline{i}_s$-ω_{rs}-model): Because of its simplicity and achievable performance, the current model in rotor flux-oriented reference frame is very popular and often-used for rotor flux estimation. The rotor flux is estimated utilizing a simple P-T_1-element (3). To estimate the angle $\hat{\varepsilon}_s$ required for transformation into the rotor flux-oriented reference frame, the stator frequency $\omega_s = \omega_{rs} + \hat{\omega}_r$ (with the measured rotor speed ω_{rs} and the estimated slip speed $\hat{\omega}_r$) is integrated over time (4) to obtain the transformation angle $\hat{\varepsilon}_s$.

$$\frac{d\hat{\psi}_{rd}}{dt} = \frac{L_m}{\tau_r} i_{sd} - \frac{1}{\tau_r}\hat{\psi}_{rd} \quad \text{with} \quad \tau_r = \frac{L_r}{R_r} \tag{3}$$

$$\hat{\varepsilon}_s = \int_{t_0}^{t_0+t}\left(\omega_{rs} + \frac{L_m\, i_{sq}}{\tau_r\, \hat{\psi}_{rd}}\right)dt \quad \text{with} \quad \hat{\omega}_r = \frac{L_m\, i_{sq}}{\tau_r\, \hat{\psi}_{rd}} \tag{4}$$

The performance of the current model is mainly sensitive to the temperature dependent ohmic rotor resistance R_r. An advantage is, however, that the operational range is in principle not restricted.

3) Full-order LTV-model of the IM ($\underline{u}_s$-ω_{rs}-model): It is well known that the dynamic behavior of the IM can be described in state-space representation by a full-order linear time-variable (LTV) model [4], [5]. In this case, the stator voltage $\underline{u}_s$ is input while the stator current $\underline{i}_s$ is the output variable of the model. The system matrix $\underline{A}(\omega_{rs})$ depends on the rotor speed. As the state vector, either the stator current or the stator flux vector together with the rotor flux vector can be considered resulting in the $\underline{i}_s$-$\underline{\psi}_r$- or the $\underline{\psi}_s$-$\underline{\psi}_r$-LTV-model of the IM. Normally, the stator-oriented reference frame is used for rotor flux estimation. Here, the $\underline{\psi}_s$-$\underline{\psi}_r$-LTV-model is presented.

$$\begin{gathered}\frac{d}{dt}\begin{bmatrix}\underline{\hat{\psi}}_s^s \\ \underline{\hat{\psi}}_r^s\end{bmatrix} = \begin{bmatrix}-\frac{1}{\sigma\tau_s} & \frac{L_m}{\sigma\tau_s L_r} \\ \frac{L_m}{\sigma\tau_r L_s} & -\frac{1}{\sigma\tau_r} + j\omega_{rs}\end{bmatrix}\begin{bmatrix}\underline{\hat{\psi}}_s^s \\ \underline{\hat{\psi}}_r^s\end{bmatrix} + \begin{bmatrix}1 \\ 0\end{bmatrix}\underline{u}_s^s \\ \underline{i}_s^s = \begin{bmatrix}\frac{1}{\sigma L_s} & -\frac{L_m}{\sigma L_s L_r}\end{bmatrix}\begin{bmatrix}\underline{\hat{\psi}}_s^s \\ \underline{\hat{\psi}}_r^s\end{bmatrix}\end{gathered} \tag{5}$$

It can be shown that the state-space model is fully observable, which is the precondition for state estimation. However, the achievable performance can be quite parameter-sensitive since all the machine parameters necessarily are used. If required, it should be checked which parameter deviations have the strongest effect on the estimation results.

4) Reduced-order LTV-model of the IM ($\underline{u}_s$-$\underline{i}_s$-ω_{rs}-model): The idea behind the reduced-order LTV-model is that not all states are estimated. Thus, only one state, the rotor flux, is estimated while all other quantities are measured. To derive the equation for the flux estimator (5) is converted in appropriate way so that (6) results to calculate the estimated rotor flux.

$$\underline{\hat{\psi}}_r^s = \frac{L_r}{L_m\left(j\omega_{rs} - \frac{1}{\sigma\tau_r}\right)}\left(\underline{u}_s^s - \left(R_s + R_r\frac{L_m^2}{L_r^2}\right)\underline{i}_s^s - \sigma L_s\frac{d\underline{i}_s^s}{dt}\right) \tag{6}$$

The main disadvantage of this open-loop estimator is that the derivative of the stator current is used. Consequently, this approach is very sensitive to measurement noise and rather not commonly used.

5) Gopinath-observer ($\underline{u}_s$-$\underline{i}_s$-ω_{rs}-model): The Gopinath-observer consists of the two open-loop estimators, the voltage and current model. The idea is that at low speeds the flux estimation of the current model is used for the FOC and when the speed rises then the estimated flux is provided by the voltage model. The smooth transition between both open-loop estimators is realized by a *PI*-controller. In this context, the bandwidth of the controller determines the transition frequency and can be adjusted to meet the application demands [2]. The Gopinath-observer is listed here under open-loop flux estimators, due to the reason that just a switch-over between two open-loop estimators is realized. There is no feedback of the observer error and hence no correction of the state observation.

B. Flux estimators – closed-loop types

To this group belong all flux estimators that include a feedback of the observation error $\underline{e}_i^s = \underline{i}_s^s - \hat{\underline{i}}_s^s$ to correct the state estimation. The model representation is based on the full order LTV-model described in (5) extended by the observer error feedback through the observer matrix $\underline{K}$ (7). For the design of the observer matrix, different assumptions regarding the system description are made [6]. This results in two different approaches: Luenberger observer or Kalman filter.

$$\frac{d}{dt}\begin{bmatrix}\hat{\underline{\psi}}_s^s\\ \hat{\underline{\psi}}_r^s\end{bmatrix} = \begin{bmatrix}-\frac{1}{\sigma\tau_s} & \frac{L_m}{\sigma\tau_s L_r}\\ \frac{L_m}{\sigma\tau_r L_s} & -\frac{1}{\sigma\tau_r}+j\omega_{rs}\end{bmatrix}\begin{bmatrix}\hat{\underline{\psi}}_s^s\\ \hat{\underline{\psi}}_r^s\end{bmatrix} + \begin{bmatrix}1\\0\end{bmatrix}\underline{u}_s^s + \underline{K}\,\underline{e}_i^s$$
$$\underline{i}_s^s = \begin{bmatrix}\frac{1}{\sigma L_s} & -\frac{L_m}{\sigma L_s L_r}\end{bmatrix}\begin{bmatrix}\hat{\underline{\psi}}_s^s\\ \hat{\underline{\psi}}_r^s\end{bmatrix} \tag{7}$$

1) Luenberger observer: For the Luenberger observer the observer matrix is designed in such a way that the eigenvalues of the matrix $\underline{F}=\left(\underline{A}(\omega_{rs}) - \underline{K}\,\underline{C}\right)$ are placed left compared to the eigenvalues of the system matrix $\underline{A}(\omega_{rs})$. This way, the observer error is decreasing over time, ideally reaching zero steady-state error $\lim_{t\to\infty} \underline{e}_i^s = 0$. The matrix entries can be chosen either constant or speed-dependent. With the choice of the matrix entries, the dynamic of the observer is determined. Therefore, the design of $\underline{K}$ is carried out comparing the dynamical behavior of the observer to the one of the system described in state-space representation.

2) Kalman filter: It is well known that the Kalman filter takes the stochastical characteristics of the observed system into account. For this purpose, the LTV-model (5) is extended by $\underline{q}(t)$ and $\underline{r}(t)$ considering this way the system and measurement noise. The result is the stochastical system description (8).

$$\frac{d\underline{x}(t)}{dt} = \underline{A}\,\underline{x}(t) + \underline{B}\,\underline{u}(t) + \underline{q}(t)$$
$$\underline{y}(t) = \underline{C}\,\mathrm{x}(t) + \underline{r}(t) \tag{8}$$

To design the observer matrix $\underline{K}$, the cost function J, which considers the square of the observation error, is minimized $\min J = \min\sum_{i=1}^{n}\mathrm{E}\{\tilde{x}_i^2\}$. The optimal solution for $\underline{K}$ (10) is obtained by solving the Riccati differential equation (9). The stochastically characteristics of the system are thereby considered within the noise covariance matrices $\underline{Q}$ and $\underline{R}$.

$$\dot{\underline{P}} = \underline{A}\,\underline{P} + \underline{P}\,\underline{A}^T - \underline{P}\,\underline{C}^T\,\underline{R}^{-1}\,\underline{C}\,\underline{P} + \underline{Q} \tag{9}$$
$$\underline{K} = \underline{P}\,\underline{C}^T\underline{R}^{-1} \tag{10}$$

The Kalman filter is a very powerful approach and used for different applications. However, the biggest disadvantage is that (9) must be solved for every time step, which makes the state estimation with a Kalman filter computationally very intensive. An approach to reduce the computational effort is to consider the steady-state solution of (9), this way the matrix entries in the noise covariance matrices are assumed to be constant. The advantage is, that the observer matrix can be computed *a priori* and stored in a look-up table (LUT) for online flux estimation. The computational effort decreases drastically and the achievable results are quite similar to the ones achieved by the computational intensive online solution.

C. Non-linear flux estimators

Usually non-linear flux estimators are designed using sliding-mode theory [7] - [12]. Typically, the fundamental wave model of the IM is used as basis for the observer synthesis combined or supplemented by varying sliding-mode techniques. In addition, different combinations of non-linear concepts [13] for flux estimation are investigated always aiming at a robust performance.

III. GOPINATH-OBSERVER WITH A COMPENSATOR IN THE FEEDBACK

In this section, an alternative approach for flux estimation is elaborated which, according to authors best knowledge, is not presented in any prior publication. Basically, this paper presents one variation of a reduced-order observer in Gopinath-style. In [14], an observer approach consisting of the voltage model, a slightly modified current model, and a feedback of the observer error $\underline{e}_i^s$ in a sliding-mode manner is presented. As opposed to [14], an alternative way for the feedback of the observer error is derived in this paper. For this purpose, a compensating element is used, which is chosen as a *PD-T*$_1$-element. In section III.B, in which the design of the feedback is presented, the need for the *PD-T*$_1$-element is justified. The discrete observer model is derived in section III.C.

A. Continuous-time model of the Gopinath-observer

In order to derive the complete observer model, the flux differential equations (11) in stator-oriented and (12) in rotor flux-oriented reference frame are used. Thereby, the rotor current in (12) is eliminated by utilizing the relations with the flux linkages $\underline{i}_r = f(\underline{\psi}_s,\underline{\psi}_r) = \frac{1}{\sigma L_r}(\underline{\psi}_r - \frac{L_m}{L_s}\underline{\psi}_s)$.

$$\frac{d\underline{\psi}_s^s}{dt} = \underline{u}_s^s - R_s\underline{i}_s^s \tag{11}$$

$$\frac{d\underline{\psi}_r^{\psi_r}}{dt} = \frac{L_m}{\sigma L_s\tau_r}\hat{\underline{\psi}}_s^{\psi_r} - \left(\frac{1}{\sigma\tau_r} + j\hat{\omega}_r\right)\hat{\underline{\psi}}_r^{\psi_r} \tag{12}$$

The dependency of the slip speed $\hat{\omega}_r$ can be eliminated in (12) if only the real part is considered since the imaginary part of the rotor flux is by definition zero ($\psi_{rq} \overset{\text{def}}{=} 0$). From the evaluation of the real part, the simple *P-T*$_1$ relation (13) results.

$$\frac{d\hat{\psi}_{rd}}{dt} = \frac{L_m}{\sigma L_s \tau_r}\hat{\psi}_{sd} - \frac{1}{\sigma \tau_r}\hat{\psi}_{rd} \;\Rightarrow\; \hat{\psi}_{rd} = \frac{\frac{L_m}{L_s}}{\sigma \tau_r\, s+1}\hat{\psi}_{sd} \tag{13}$$

To perform the transformation into the rotor flux-oriented reference frame, the transformation angle $\hat{\varepsilon}_s = \tan^{-1}(\hat{\psi}_{r\beta}/\hat{\psi}_{r\alpha})$ is estimated by calculating the rotor flux vector $\underline{\hat{\psi}}_r^s = f(\underline{i}_s^s, \underline{\hat{\psi}}_s^s) = \frac{L_r}{L_m}(\underline{\hat{\psi}}_s^s - \sigma L_s \underline{i}_s^s)$. With the knowledge of $\hat{\varepsilon}_s$, the stator flux computed by the voltage model (11) can be transformed into rotor flux-oriented coordinates using the rotation operator (14). From the knowledge of $\hat{\psi}_{sd}$, the rotor flux $\hat{\psi}_{rd}$ follows directly from (13) and can be transformed back in stator-oriented coordinates $\underline{\hat{\psi}}_r^s$ using (14).

$$\underline{\hat{\psi}}_s^{\psi_r} = e^{-j\hat{\varepsilon}_s}\underline{\hat{\psi}}_s^s \quad \text{or} \quad \underline{\hat{\psi}}_r^s = e^{j\hat{\varepsilon}_s}\underline{\hat{\psi}}_s^{\psi_r} \tag{14}$$

With the derived stator and rotor flux linkages described in stator-oriented coordinates, it is possible to calculate the estimated stator current $\underline{\hat{i}}_s^s = f(\underline{\hat{\psi}}_s^s, \underline{\hat{\psi}}_r^s) = \frac{1}{\sigma L_s}(\underline{\hat{\psi}}_s^s - \frac{L_m}{L_r}\underline{\hat{\psi}}_r^s)$. The resulting structure for the Gopinath-observer is shown in Fig. 3. Up to this point, just the open-loop observer structure is derived whose main disadvantage is the open-loop integrator within the voltage model. In the next section, the feedback of the observer error with a compensating element will be described. The objective is, on the one hand, to stabilize the integrator and, on the other, to correct the state observation.

B. Design of the compensator in the feedback

The first step is to identify the overall system transfer function. This is done in the rotor flux-oriented reference frame. Therefore, the voltage model is expressed as in (15).

$$\frac{d\underline{\hat{\psi}}_s^{\psi_r}}{dt} = \underline{u}_s^{\psi_r} - R_s \underline{i}_s^{\psi_r} - j\omega_s \underline{\hat{\psi}}_s^{\psi_r} \tag{15}$$

The conversion of (15) into the Laplace domain considering $\Delta\underline{u}_s^{\psi_r} = \underline{u}_s^{\psi_r} - R_s \underline{i}_s^{\psi_r}$ as the stator voltage reduced by the ohmic voltage drop and the cross-coupling voltage term $\underline{u}_{cc}^{\psi_r} = -j\omega_s \underline{\hat{\psi}}_s^{\psi_r} = -j\omega_s(\frac{L_m}{L_r}\underline{\hat{\psi}}_r^{\psi_r} + \sigma L_s \underline{i}_s^{\psi_r})$, introduced by the coordinate transformation, leads to (16).

$$s\underline{\hat{\psi}}_s^{\psi_r} = \Delta\underline{u}_s^{\psi_r} + \underline{u}_{cc}^{\psi_r} \tag{16}$$

For further analysis, only the real part of (16) is used as the modified current model also only consists of the real part.

$$s\hat{\psi}_{sd} = u_{sd} - R_s i_{sd} + \omega_s \sigma L_s i_{sq} = \Delta u_{sd} + u_{cc,d} \tag{17}$$

By assuming that the cross-coupling voltage $u_{cc,d}$ can be eliminated by a feedforward compensation, a simple transfer function $G_{u,d}(s)$ results for the voltage model in d-axis.

$$G_{u,d}(s) = \frac{\hat{\psi}_{sd}}{\Delta u_{sd}} = \frac{1}{s} \tag{18}$$

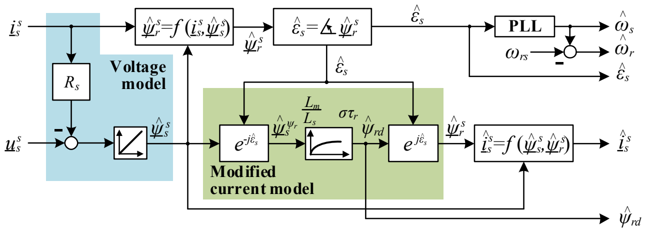


Fig. 4. Structure of the proposed Gopinath-observer

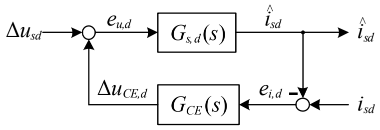


Fig. 3. Block diagram to design the compensating element $G_{CE}(s)$

To consider the dynamic behavior of the remaining part of the observer, the modified current model (13) and the subsequent computation of the estimated stator current are summarized by the transfer function $G_{i,d}(s)$.

$$G_{i,d}(s) = \frac{\hat{i}_{sd}(s)}{\hat{\psi}_{sd}(s)} = \frac{1}{L_s}\frac{(\tau_r s+1)}{(\sigma\tau_r s+1)} = \frac{K_s(\tau_r s+1)}{(\sigma\tau_r s+1)} \quad \text{with } K_s = \frac{1}{L_s} \tag{19}$$

Considering (18) and (19), the open-loop transfer function in d-axis results in (20) for the observer shown in Fig. 3.

$$G_{s,d}(s) = \frac{\hat{i}_{sd}(s)}{\Delta u_{s,d}(s)} = G_{u,d}(s)G_{i,d}(s) = \frac{K_s(\tau_r s+1)}{s(\sigma\tau_r s+1)} \tag{20}$$

After identifying the open-loop transfer function, the feedback of the observer error $\underline{e}_i^s = \underline{i}_s^s - \underline{\hat{i}}_s^s$ in front of the integrator (within the voltage model) can be realized. Once again, only the real part $e_{i,d}$ must be considered. The transfer function for the compensating element (CE) is $G_{CE}(s)$, like shown in Fig. 4 for the resulting block diagram of the closed-loop observer. The transfer behavior in closed-loop can be derived utilizing this block diagram. Hereafter, the abbreviations $a(s)$ and $b(s)$ are chosen to describe the numerator and denominator polynomials of a transfer function, e.g. $G(s) = \frac{a(s)}{b(s)}$.

$$\begin{aligned}
\hat{i}_{sd} &= G_{s,d}(s)e_{u,d} = G_{s,d}(s)(\Delta u_{sd} + \Delta u_{CE,d}) \\
&= G_{s,d}(s)(\Delta u_{sd} + G_{CE}(s)e_{i,d}) \\
&= G_{s,d}(s)\big(\Delta u_{sd} + G_{CE}(s)(i_{sd} - \hat{i}_{sd})\big) \\
&= \frac{G_{s,d}(s)}{1+G_{s,d}(s)G_{CE}(s)}\Delta u_{sd} + \frac{G_{s,d}(s)G_{CE}(s)}{1+G_{s,d}(s)G_{CE}(s)}i_{sd} \\
&= \frac{a_s b_{CE}}{b_s b_{CE} + a_s a_{CE}}\Delta u_{sd} + \frac{a_s a_{CE}}{b_s b_{CE} + a_s a_{CE}}i_{sd} \\
&= \frac{K_s(\tau_r s+1)b_{CE}}{s(\sigma\tau_r s+1)b_{CE} + K_s(\tau_r s+1)a_{CE}}\Delta u_{sd} \\
&\quad + \frac{K_s(\tau_r s+1)a_{CE}}{s(\sigma\tau_r s+1)b_{CE} + K_s(\tau_r s+1)a_{CE}}i_{sd}
\end{aligned} \tag{21}$$

To determine the transfer behavior of the closed-loop observer a *PD-T_1*-element is chosen. The derivative time T_D and the time constant T_1 are determined to $T_1 = \tau_r > T_D = \sigma\tau_r$ while the proportional gain K_p provides a degree of freedom (22) and is determined as follows.

$$G_{CE}(s) = \frac{a_{CE}}{b_{CE}} = K_p\frac{(1+T_D s)}{(1+T_1 s)} = K_p\frac{(1+\sigma\tau_r s)}{(1+\tau_r s)} \tag{22}$$

Replacing (22) in (21) leads to (23):

$$\hat{i}_{sd} = \frac{K_s(\tau_r s+1)}{(s+K_s K_p)(1+\sigma\tau_r s)}\Delta u_{sd} + \frac{K_s K_p}{(s+K_s K_p)}i_{sd} \tag{23}$$

From (23), and considering that $K_s = 1/L_s$ (as given in (19)), the proportional gain can be chosen in an appropriate way to adjust the pole $s = -K_s K_p$. Thus, for example, an integer multiple m of the stator inductance L_s can be chosen for the proportional gain (24). In consequence, the resulting pole $s = -m$ is determined by the choice of m. The overall structure of the derived Gopinath-observer with a compensator in the feedback is shown in Fig. 5.

$$K_p = m \cdot L_s \tag{24}$$

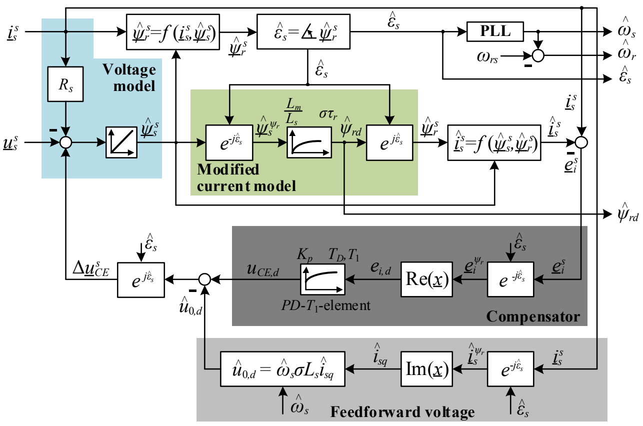


Fig. 5. Structure of the proposed Gopinath-observer in closed-loop

C. Discrete-time model

For the discretization of the proposed Gopinath-observer the well-known Tustin's approximation with

$$s \approx \frac{2}{T_s}\frac{z-1}{z+1} = \frac{2}{T_s}\frac{1-z^{-1}}{1+z^{-1}} \tag{25}$$

is chosen. The resulting small steady-state discretization error favors a higher accuracy for the correct alignment of the rotor flux-oriented reference frame. The discretized voltage model in (26) results from applying (25) to (11), directly.

$$\underline{\psi}_s^s[k+1]=\underline{\psi}_s^s[k]+\frac{T_s}{2}\left(\underline{u}_s^s[k+1]+\underline{u}_s^s[k]\text{-}R_s\left(\underline{i}_s^s[k+1]+\underline{i}_s^s[k]\right)\right) \tag{26}$$

According to [4], to derive the discrete modified current model, the discretization in rotor-oriented coordinates is executed as first step, followed by the coordinate transformations (27) into the rotor flux-oriented reference frame.

$$\hat{\underline{\psi}}_r^{\psi_r}[k]=e^{-j\hat{\varepsilon}_r[k]}\hat{\underline{\psi}}_r^r[k] \text{ and } \hat{\underline{\psi}}_r^{\psi_r}[k+1]=e^{-j\hat{\varepsilon}_r[k+1]}\hat{\underline{\psi}}_r^r[k+1] \tag{27}$$

By doing so, the discretized modified current model results in (28) with the difference angle $\Delta\varepsilon_r[k] = \varepsilon_r[k+1]\text{-}\varepsilon_r[k]$.

$$\begin{aligned}\underline{\psi}_r^{\psi_r}[k+1] &= \underline{\psi}_r^{\psi_r}[k]e^{-j\Delta\varepsilon_r[k]}\frac{2-\frac{T_s}{\sigma\tau_r}}{2+\frac{T_s}{\sigma\tau_r}} \\ &+\frac{\frac{T_s}{\sigma\tau_r}\frac{L_m}{L_s}}{2+\frac{T_s}{\sigma\tau_r}}\left(\underline{\psi}_s^{\psi_r}[k+1]+\underline{\psi}_s^{\psi_r}[k]e^{-j\Delta\varepsilon_r[k]}\right)\end{aligned} \tag{28}$$

On the right side of (26) and (28), for one sample time step T_s predicted values $\underline{x}[k+1]$ are needed. These are unknown and therefore replaced by the steady-state approximation $\underline{x}[k+1]=\underline{x}[k]e^{j\Delta\varepsilon[k]}$. Finally, (26) and (28) simplifies to (29) and (30) for the discrete-time model of the observer.

$$\begin{aligned}&\underline{\psi}_s^s[k+1] \\ &= \underline{\psi}_s^s[k]+\frac{T_s}{2}\left(\underline{u}_s^s[k](1+e^{j\Delta\varepsilon_s[k]})\text{-}R_s\left(\underline{i}_s^s[k](1+e^{j\Delta\varepsilon_s[k]})\right)\right)\end{aligned} \tag{29}$$

$$\begin{aligned}&\underline{\psi}_r^{\psi_r}[k+1] \\ &= \underline{\psi}_r^{\psi_r}[k]e^{-j\Delta\varepsilon_r[k]}\frac{2-\frac{T_s}{\sigma\tau_r}}{2+\frac{T_s}{\sigma\tau_r}}+\frac{\frac{T_s}{\sigma\tau_r}\frac{L_m}{L_s}}{2+\frac{T_s}{\sigma\tau_r}}\underline{\psi}_s^{\psi_r}[k](2\cos\Delta\varepsilon_r[k])\end{aligned} \tag{30}$$

IV. Simulative Performance Analysis

To analyze the performance of the proposed Gopinath-observer (within a field-oriented controlled drive system) simulation results for different operating points are presented in section IV.A. Furthermore, in section IV.B, the results of an extensive sensitivity analysis on parameter mismatch are shown. To identify the benefits in comparison to another flux estimation approach, the classic current model presented in section II.A.2) was used. For the sensitivity analysis, the detuning of the ohmic resistances R_s, R_r and the main inductance L_m were considered.

The simulated drive system (power rating of $P_{me}\approx 150$ kW) runs in torque controlled mode. It was developed as an axial drive for traction drive applications of purely electrically driven vehicles. Characteristically for traction motor drives is a noticeable magnetic saturation of the main inductance for increasing magnetizing current magnitudes $|i_\mu|$ which is considered for the simulation model by means of a look-up-table $L_m = f(|i_\mu|)$ derived from measurements.

A. Simulation results

The performance of the proposed Gopinath-observer in steady-state and dynamic operation was simulated for two different constant speeds with varying load conditions. The results are shown in Fig. 6. As it can be seen, the torque dynamics are restricted due to reduced rotor flux magnitude in partial-load range operation. The reference rotor flux is adjusted depending on the reference torque value $\psi_{r,d}^* = f(T^*)$ for improved efficiency of the traction drive system. In addition, relevant error curves for the absolute $\Delta x=x_{MM}\text{-}\hat{x}$ or relative errors $\Delta x/x_N$ (with x_N as the nominal value) compared with variables of the motor model x_{MM} are presented Fig. 6 a) and b).

B. Sensitivity analysis on parameter mismatch

To assess the parameter sensitivity of the proposed Gopinath-observer, extensive simulation studies for a field-oriented controlled drive system at a constant speed were performed. Aside from the evaluation of the resulting steady-state errors between three selected estimated variables $\hat{x}$ (rotor flux magnitude $\hat{\psi}_r$, rotor flux vector orientation $\hat{\varepsilon}_s$ and torque $\hat{T}$) and the equivalent variables of the motor model x_{MM}, a comparison with the current model approach for rotor flux estimation was performed. Both observer models were simulated under same steady-state operating conditions (constant speed, different load torques and parameter detuning levels) for this comparison. The results are illustrated in Fig. 7 using different colors for both rotor flux observers. In doing so, the pros and cons of the Gopinath-observer approach compared to one of the most used methods for rotor flux estimation were identified.

As can be seen in Fig. 7 a), the Gopinath-observer is robust with respect to parameter mismatch of the rotor resistance in motor as well as generator operation. This is a substantial advantage since the detuning is always present during normal operation and the temperature dependent rotor resistance can typically not be measured online. A dynamic tracking $\hat{R}_r=f(\hat{\vartheta}_r)$ in an adaptive manner is normally connected with

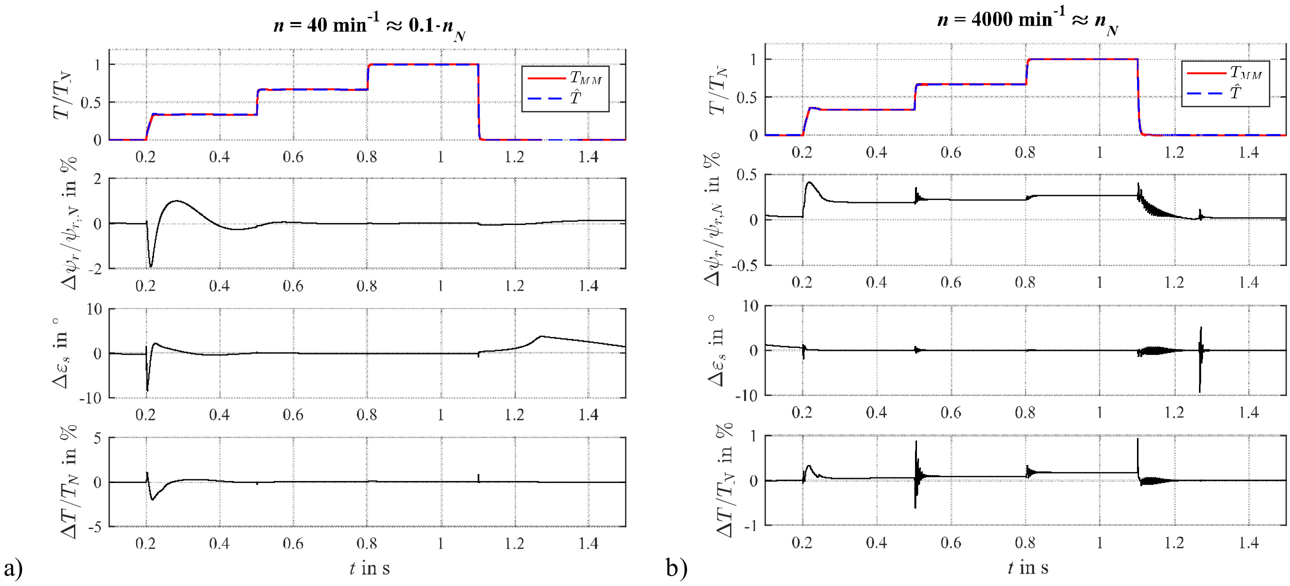
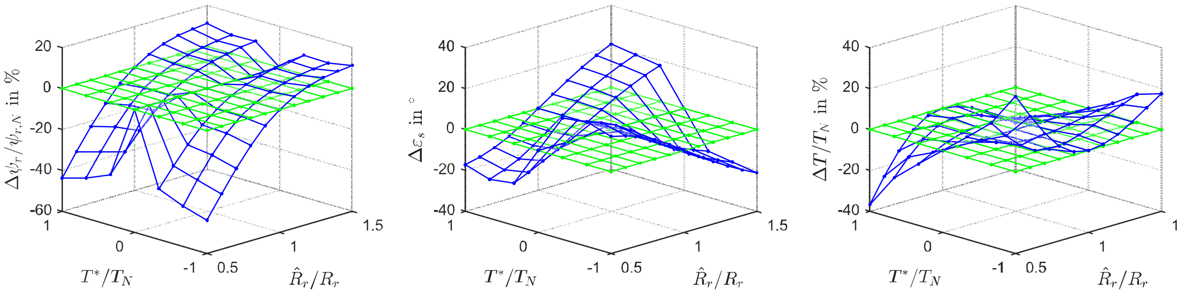
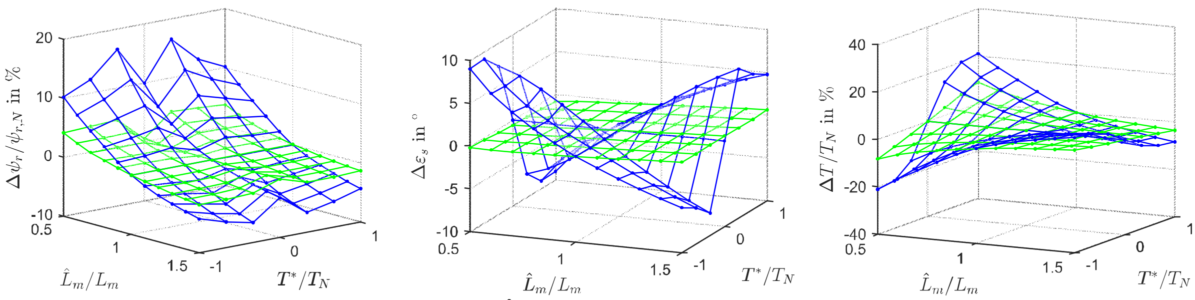


Fig. 6. Simulation results for two different speeds and varying load torque conditions.

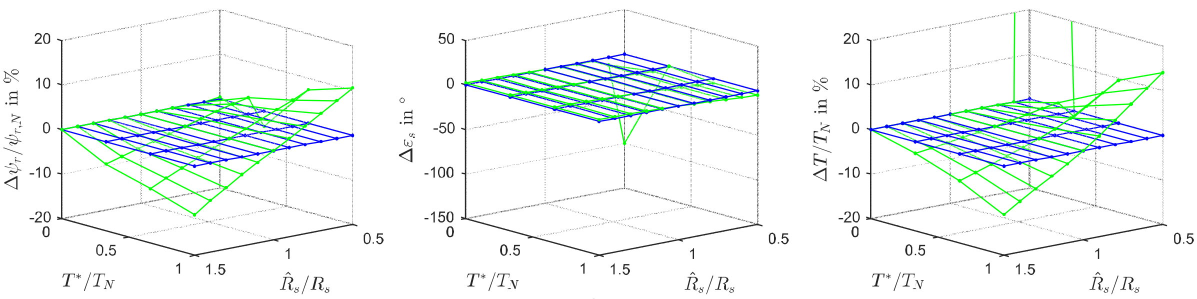
Constant speed operation at $n = 400\ \mathrm{min}^{-1} \approx 0.1 \cdot n_N$



a) Rotor resistance detuning $\hat{R}_r$=0.5·R_r ... 1.5·R_r (motor and generator operation)



b) Main inductance detuning $\hat{L}_m$=0.5·L_m ... 1.5·L_m (motor and generator operation)



c) Stator resistance detuning $\hat{R}_s$=0.5·R_s ... 1.5·R_s (motor operation)

Fig. 7. Results for the sensitivity analysis on parameter mismatch: Resulting steady-state errors simulated for the Gopinath-Observer (in green) and the current model rotor flux estimation approach (in blue)

relatively high effort, e.g. by utilizing rotor temperature ($\hat{\vartheta}_r$) observer strategies. From Fig. 7 b), it is evident that a detuning regarding the main inductance has an impact on the performance of the Gopinath-observer in rotor and generator operation. However, it can be seen the achievable results are much better than with the current model approach. In Fig. 7 c), the results for a detuning of the stator resistance are shown. As expected, the current model approach is insensitive regarding stator resistance detuning. The Gopinath-observer approach, however, is strongly influenced by a stator resistance detuning. Only the results for motor operation are shown, since the results for generator operation are not acceptable and the performance is intolerable. However, the stator resistance is suitable for online tracking with relatively small effort in comparison with the other machine parameters. This basically makes it possible to reduce the detuning regarding the stator resistance to an acceptable level.

V. Conclusions and Outlook

In this paper, a rotor flux estimator was proposed based on the known Gopinath-observer approach. To reduce the observer error, the design of a compensator in the feedback, identified to a *PD*-T_1-element, was presented. Moreover, its performance was proved through simulation results for steady-state operation as well as for dynamic changes of the operating point. The main conclusions are:

- The *PD*-T_1-element in the feedback is well suited to stabilize the performance of the proposed observer and ensures a rotor flux estimation with a small remaining steady-state observer error.
- Using different reference frames to design the observer model, it is possible to design the compensator only for the *d*-component of the rotor flux-oriented reference frame. However, due to the transformation back in stator-oriented coordinates, both open-loop integrators in the voltage model are stabilized and are, therefore, utilized to reduce the observer error. Thus, no integral part in the compensator is required.

Outlook for further investigations:

- Evaluation of the Gopinath-observer testbench performance and enhancement of the observer performance with regard to stator resistance detuning.

Acknowledgement

Supported by:
Federal Ministry for Economic Affairs and Energy
on the basis of a decision by the German Bundestag

This work is supported by the German Federal Ministry for Economic Affairs and Energy (BMWi) identifiable under the acronym ”SichElAn” or rather the funding code ”01MY15003A”. The full project name can be translated as ”Increase of Functional Safety and Fault Tolerance of Electrical Drive Systems”. The author is responsible for the contents of this publication.

List of Symbols

$\underline{u}_s, \underline{i}_s$	stator voltage and current
$\underline{\psi}_s, \underline{\psi}_r$	stator and rotor flux linkages
R_s, R_r	stator and rotor ohmic resistance
L_m, L_s, L_r	main, stator, and rotor inductance
$\sigma = 1 - \frac{L_m^2}{L_r L_s}$	leakage coefficient
$\omega_s, \omega_{rs}, \omega_r$	stator, rotor, and slip angular frequencies
$\tau_s = \frac{L_s}{R_s}, \tau_r = \frac{L_r}{R_r}$	stator, or rotor time constant
ε	electrical angle

References

[1] F. Blaschke, “A new method for the structural decoupling of ac induction machines,” in Conf.Rec.IFAC, Duesseldorf, Germany, pp. 1-15, October 1971.

[2] J. Böcker, and S. Mathapati, “State of the art of induction motor control,” IEEE International Electrical Machines & Drives Conference (IEMDC), Antalya, Turkey, 2007.

[3] W. Leonhard, “Control of electrical drives,” Springer, 3rd edition, Berlin, 2001.

[4] J. Böcker, “Discrete-time model of an induction motor,” ETEP, Vol. 1, No. 2, pp. 65-71, March/April 1991.

[5] J. Böcker, and J. Janning, “Discrete-time flux observer for PWM inverter fed induction motors,” Fifth European Conference on Power Electronics and Applications, 1993.

[6] F. L. Lewis, “Applied optimal control and estimation,” Prentice Hall and Texas Instruments, USA, 1992.

[7] A. B. Proca, and A. Keyhani, “Sliding-mode flux observer with online rotor parameter estimation for induction motors,” IEEE Transactions on inductrial electronics, vol. 54, no.2, April 2007.

[8] M. Comanescu, “A nonlinear full order observer for rotor flux position estimation of induction motors,” Industrial Electronics Society, IECON - 36th Annual Conference of the IEEE, Glendale, AZ, USA, 2010.

[9] M.Comanescu, “A sensorless sliding mode observer for the flux magnitude of the induction motor based on the synchronous reference frame model,” Industrial Electronics Society, IECON - 39th Annual Conference of the IEEE, Vienna, Austria, 2013.

[10] Z. Dehong, and Z. Jin, “A sliding mode flux observer for online rotor and stator resistance estimation in predictive torque controlled induction motor drive,” 34th Chinese Control Conference (CCC), Hangzhou, China, 2015.

[11] E. Jiménez, O. Jaramillo, J. D. Sánchez-Torres, and H. Botero, “A second order sliding mode state and parameter estimator for induction motors,” 12th International Conference on Electrival Engineering, Computing Science and Automatic Control (CCE), Mexico City, Mexico, 2015.

[12] X. Huihui, Z. Feng, C. Wei, and P. Wei, “Study of a new rotor flux estimator for induction machine based on sliding mode control,” IEEE Vehicle Power and Propulsion Conference (VPPC), Hangzhou, China, 2016.

[13] Z. Kandoussi, Z. Boulghasoul, A. Elbacha, and A. Tajer, “Fuzzy sliding mode observer based sensorless indirect FOC for IM drives,” Third World Conference on Complex Systems (WCCS), Marrakech, Morocco, 2015.

[14] C. Lascu, I. Boldea, and F. Blaabjerg, “Very-low-speed variable-structure control of sensorless induction machine drives without signal injection,” IEEE Transactions on industry applications, vol. 41, no. 2, March/April 2005.