

Problem 1.

(1)

1. $\rho_1 = \{z \rightarrow 4, y \rightarrow 2, \text{plus_x} \rightarrow \langle y \rightarrow x+y, \{x \rightarrow 2\} \rangle, x \rightarrow 2\}$
 (* So, plus_x n = n+2 *)
2. $\rho_2 = \{y \rightarrow 0, \text{sub_z} \rightarrow \langle x \rightarrow y-z+x, \rho_1 \rangle, z \rightarrow 4, \text{plus_x} \rightarrow \langle y \rightarrow x+y, \{x \rightarrow 2\} \rangle, x \rightarrow 2\}$
 (* So, sub_z n = n-2 *)
3. $\rho_3 = \{f_z \rightarrow \langle x \rightarrow \text{if}(\text{plus_x } x < z) \text{ then plus_x } z \text{ else sub_z } x, \rho_2 \rangle, \rho_2\}$
 (* So, f_z n = if(n+2 < 4) then 6, else n-2 *)

(2) First, $\rho_3 = \{f_z \rightarrow \langle x \rightarrow \text{if}(\text{plus_x } x < z) \text{ then plus_z else sub_z } x, \rho_2 \rangle, \rho_2\}$ Eval (f_z y, ρ_3) =Eval ((app $\langle x \rightarrow \text{if}(\text{plus_x } x < z) \text{ then plus_z else sub_z } x, \rho_2 \rangle$) y, ρ_2) =Eval ((app $\langle x \rightarrow \text{if}(\text{plus_x } x < z) \text{ then plus_z else sub_z } x, \rho_2 \rangle$) 0, ρ_2) =Eval (if(plus_x x < z) then plus_x z else sub_z x, {y → 0} + ρ_2) =Eval (if(plus_x 0 < z) then plus_x z else sub_z 0, ρ_2) =Eval (if($\langle y \rightarrow x+y, \{x \rightarrow 2\} \rangle > 0 < z$) then $\langle y \rightarrow x+y, \{x \rightarrow 2\} \rangle z$ else $\langle x \rightarrow y-z+x, \rho_1 \rangle 0$, ρ_2) =Eval (if (2 < 4) then $\langle y \rightarrow x+y, \{x \rightarrow 2\} \rangle > 4$ else $\langle x \rightarrow y-z+x, \rho_1 \rangle 0$, ρ_2) =Eval $\langle y \rightarrow x+y, \{x \rightarrow 2\} \rangle > 4 = 6$