HW 2 Solution

CS 421 – Spring 2014

Revision 1.0

Assigned January 30, 2014
Due February 9, 2014, 11:59 pm

1 Change Log

1.0 Initial Release.

2 Solutions

1. (25 pts) Below is a fragment of OCaml code, with various program points indicated by numbers with comments.

   (code inlined in solution)

   For each of program points 1, 2, and 3, please describe the environment in effect after evaluation has reached that point (8pts). Finally, show step by step how the application of `f_z y` would be evaluated (17pts). You may assume that the evaluation begins in an empty environment, and that the environment is cumulative thereafter. The program points are supposed to indicate points at which all complete preceding declarations (including local ones) have been fully evaluated.

   Solution:

   let x = 2;;

   let plus_x = fun y -> x + y;;

   let y = 2;;

   let z = plus_x y;;

   ρ_1={x $\mapsto$ 2, plus_x $\mapsto$ c_{plus}, y $\mapsto$ 2, z $\mapsto$ 4} where c_{plus}=<y $\rightarrow$ x+y, {x $\mapsto$ 2}>

   let sub_z = fun x -> y - z + x;;

   let y = sub_z y;;

   ρ_2= {sub_z $\mapsto$ c_{sub}, y $\mapsto$ 0} + ρ_1 = {x $\mapsto$ 2, plus_x $\mapsto$ c_{plus}, y $\mapsto$ 2, z $\mapsto$ 4, sub_z $\mapsto$ c_{sub}}
   where c_{plus}=<y $\rightarrow$ x+y, {x $\mapsto$ 2}> and c_{sub}=<x $\rightarrow$ y-z+x,ρ_1>

let f_z x = if plus_x x $<$ z then plus_x z else sub_z x;;

$$\begin{aligned}
\rho_3 &= \{\text{f\_z} \mapsto c_f\} + \rho_2 \\
&= \{\text{f\_z} \mapsto c_f,\ \text{sub\_z} \mapsto c_{sub},\ y \mapsto 0\} + \rho_1 \\
&= \{\text{x} \mapsto 2,\ \text{plus\_x} \mapsto c_{plus},\ \text{y} \mapsto 0,\ \text{z} \mapsto 4,\ \text{sub\_z} \mapsto c_{sub},\ \text{f\_z} \mapsto c_f\} \\
&\ \text{where } c_{plus} =< \text{y} \to \text{x} + \text{y},\ \{\text{x} \mapsto 2\} > \\
&\ \text{and } c_{sub} =< \text{x} \to \text{y} - \text{z}, \rho_1 > \\
&\ \text{and } c_f =< x \to \text{if plus\_x x} < \text{z then plus\_x z else sub\_z x}, \rho_2 >
\end{aligned}$$

f_z y;;

Eval (f_z y, $\{$x $\mapsto$ 2, plus_x $\mapsto$ c_{plus}, y $\mapsto$ 0, z $\mapsto$ 4, sub_z $\mapsto$ c_{sub}, f_z $\mapsto$ $c_f\}$)

$\Rightarrow$ Eval (f_z 0, $\{$x $\mapsto$ 2, plus_x $\mapsto$ c_{plus}, y $\mapsto$ 0, z $\mapsto$ 4, sub_z $\mapsto$ c_{sub}, f_z $\mapsto$ $c_f\}$)

$\Rightarrow$ Eval (app <x$\to$ if plus_x x $<$ z then plus_x z else sub_z x,$\rho_2 >$ 0,$\{$x $\mapsto$ 2, plus_x $\mapsto$ c_{plus}, y $\mapsto$ 0, z $\mapsto$ 4, sub_z $\mapsto$ c_{sub}, f_z $\mapsto$ $c_f\}$)

$\Rightarrow$ Eval (if plus_x x $<$ z then plus_x z else sub_z x,$\{$x$\mapsto$ 0$\}$+ρ_2)

$\Rightarrow$ Eval (if plus_x x $<$ 4 then plus_x z else sub_z x,$\{$x$\mapsto$ 0$\}$+ρ_2)

$\Rightarrow$ Eval (if plus_x 0 $<$ 4 then plus_x z else sub_z x,$\{$x$\mapsto$ 0$\}$+ρ_2)

$\Rightarrow$ Eval (if (app <y $\to$ x+y, $\{$x $\mapsto$ 2$\}$> 0) $<$ 4 then plus_x z else sub_z x,$\{$x$\mapsto$ 0$\}$+ρ_2)

$\Rightarrow$ Eval (if (Eval(x+y, $\{$y $\mapsto$ 0$\}$ + $\{$x $\mapsto$ 2$\}$> 0)) $<$ 4 then plus_x z else sub_z x,$\{$x$\mapsto$ 0$\}$+ρ_2)

$\Rightarrow$ Eval (if (Eval(x+0, $\{$y $\mapsto$ 0, x $\mapsto$ 2$\}$> 0)) $<$ 4 then plus_x z else sub_z x,$\{$x$\mapsto$ 0$\}$+ρ_2)

$\Rightarrow$ Eval (if (Eval(2+0, $\{$y $\mapsto$ 0, x $\mapsto$ 2$\}$> 0)) $<$ 4 then plus_x z else sub_z x,$\{$x$\mapsto$ 0$\}$+ρ_2)

$\Rightarrow$ Eval (if 2 $<$ 4 then plus_x z else sub_z x,$\{$x$\mapsto$ 0$\}$+ρ_2)

$\Rightarrow$ Eval (if true then plus_x z else sub_z x,$\{$x$\mapsto$ 0$\}$+ρ_2)

$\Rightarrow$ Eval (plus_x z $\{$x$\mapsto$ 0$\}$+ρ_2)

$\Rightarrow$ Eval (plus_x 4,$\{$x $\mapsto$ 0, plus_x $\mapsto$ c_{plus}, y $\mapsto$ 0, z $\mapsto$ 4, sub_z $\mapsto$ $c_{sub}\}$) where c_{plus}=$<$ y $\to$ x $+$ y, $\{$x $\mapsto$ 2$\}$ $>$

$\Rightarrow$ Eval (app <y $\mapsto$ x+y, $\{$x $\mapsto$ 2$\}$> 4,$\{$x $\mapsto$ 0, plus_x $\mapsto$ c_{plus}, y $\mapsto$ 0, z $\mapsto$ 4, sub_z $\mapsto$ $c_{sub}\}$)

$\Rightarrow$ Eval (x+y,$\{$y$\mapsto$ 4$\}$ + $\{$x $\mapsto$ 2$\}$)

$\Rightarrow$ Eval (x+4,$\{$x $\mapsto$ 2,y $\mapsto$ 4$\}$)

$\Rightarrow$ Eval (2+4,$\{$x $\mapsto$ 2,y $\mapsto$ 4$\}$) = 6