CS421 HW07
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Problem 1.

1. L_1 is a regular language.
   Regular Expression: L_1 = {0(0*)1((0v1)*)}
   Regular Grammar:
   - S -> XY
   - X -> 0A
   - Y -> 1B
   - A -> 0A | ε
   - B -> B0 | B1 | ε

2. L_2 is not a regular language.
   Prove by pumping lemma:
   Let p be the pumping length. let w = (0^p)(1^p), and w belongs to L_2. By pumping lemma, w= xyz where |xy| <= p, |y| > 0, and x(y^i)z should still belongs to L_2 .
   Becasue |xy| < p, so y only contains 0s. Thus, x = (0^(p-j)), y = (0^j), z = (1^p), j > 0.
   When we have w' = x(y^2)z = (0^(p-j+2j))(1^p) = (0^(p+j))(1^p). This w' is clearly not belonging L_2. Therefore, L_2 is not regular.

3. L_3 is a regular language.
   Regular Expression: L_3 = {(avb)(((avb)(avb))*)}
   Regular Grammar:
   - S -> AB
   - B -> AA | ε
   - A -> a | b

4. L_4 is a regular language.
   Regular Expression: L_4 = {(((((a*)c)*) ((bvc)*)) v (a*))*}
   Regular Grammar:
   - S -> CXS | AS | ε
   - C -> aAc | ε
   - A -> Aa | ε
   - X -> Xb | Xc | ε

5. L_5 is not a regular language.
   Prove by pumping lemma:
   Let p be the pumping length. Let w = a^(p^3).
   Then w = xyz, where |xy| <= p, |y| > 0, and x(y^i)z should still belongs to L_5 .
   Choosing i = 2, then w' = x(y^2)z,
   Because 0 < |y| <= p, So, |w'| = p^3 + |y| <= p^3 + p < (p+1)^3.
   Then p^3 < |w'| < (p+1)^3. Therefore, w' is not belonging to L_5. L_5 is not regular.