CS421 HW11
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Problem 1.

By α-conversion, substitute y into z:
  (λy.xy)(λx.λy.yx) -- α → (λz.xz)(λx.λz.zx)
Since α-conversion implies α-equivalence:
  (λy.xy)(λx.λy.yx) ~ α ~ (λz.xz)(λx.λz.zx)   (1)
By α-conversion, substitute x into y:
  (λx.λz.zx) -- α → (λy.λz.zy)
Since α-conversion implies α-equivalence:
  (λx.λz.zx) ~ α ~ (λy.λz.zy)
Do the α-conversion and α-equivalence on z to x:
  (λy.λz.zy) -- α → (λy.λx.xy)
  (λy.λz.zy) ~ α ~ (λy.λx.xy)
So:
  (λx.λz.zx) ~ α ~ (λy.λx.xy)
By congruence for application, we have:
  (λz.xz)(λx.λz.zx) ~ α ~ (λz.xz)(λy.λx.xy)
From (1) we have:
  (λy.xy)(λx.λy.yx) ~ α ~ (λz.xz)(λy.λx.xy)

Problem 2.

a. eager evaluation:
  (λx.x(λy.xy))((λu.u)(λw.w))
  -- β --> (λx.x(λy.xy))(λw.w)
  -- β --> ((λw.w)(λy.(λw.w)y))
  -- β --> λy.(λw.w)y
b. lazy evaluation:
  (λx.x(λy.xy))((λu.u)(λw.w))
  -- β --> (λu.u)(λw.w)(λy.(λu.u)(λw.w)y)
  -- β --> (λw.w)(λy.(λu.u)(λw.w)y)
  -- β --> λy.(λu.u)(λw.w)y
c. unrestricted αβ-reduction:
  (λx.x(λy.xy))((λu.u)(λw.w))
  -- β --> (λx.x(λy.xy))(λw.w)
  -- β --> ((λw.w)(λy.(λw.w)y))
  -- β --> λy.(λw.w)y
  -- β --> λy.y