Test

November 15, 2024

[1]:
```python
# Add the Model Scripts folder to the path
import sys
sys.path.append("Scripts")
sys.path.append("Weights")


# Import the necessary libraries
import torch
import random
import torch.nn as nn
import torch.optim as optim
from torch.utils.data import Dataset
from torch.utils.data import DataLoader
import matplotlib.pyplot as plt
```

```
Intel MKL WARNING: Support of Intel(R) Streaming SIMD Extensions 4.2 (Intel(R)
SSE4.2) enabled only processors has been deprecated. Intel oneAPI Math Kernel
Library 2025.0 will require Intel(R) Advanced Vector Extensions (Intel(R) AVX)
instructions.
Intel MKL WARNING: Support of Intel(R) Streaming SIMD Extensions 4.2 (Intel(R)
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instructions.
```

[2]:
```python
# Import the function to create the model
from Scripts.ArithmeticTransformer import create_arithmetic_transformer

# Create a model with default parameters
model = create_arithmetic_transformer()

# Or create a model with custom parameters
model = create_arithmetic_transformer(
    vocab_size=14,
    embed_size=128,
    num_heads=4,
    ff_dim=512,
    num_layers=3,
    max_length=64
```

```
)
```

[10]:

```
import torch

def load_model(model_path, device=None):
    # Determine device
    if device is None:
        device = torch.device('cuda' if torch.cuda.is_available() else 'cpu')

    # Load checkpoint
    print(f"Loading model on {device}")
    checkpoint = torch.load(model_path, map_location=device)

    # Extract config
    config = checkpoint['model_config']
    print("Model configuration:", config)

    # Create model
    model = create_arithmetic_transformer(
        vocab_size=config['vocab_size'],
        embed_size=config['embed_size'],
        num_heads=config['num_heads'],
        ff_dim=config['ff_dim'],
        num_layers=config['num_layers'],
        max_length=config['max_length'],
        dropout=config['dropout']
    )

    # Load state dict
    model.load_state_dict(checkpoint['model_state_dict'])

    # Move model to device and set to eval mode
    model = model.to(device)
    model.eval()

    # Get vocab if available
    vocab = checkpoint.get('vocab')
    inv_vocab = checkpoint.get('inv_vocab')

    print(f"Model loaded successfully! Best accuracy: {checkpoint['accuracy']:.
 ↪4f}")

    return model, vocab, inv_vocab, device, config

# Usage:
model_path = './Weights/large_addition_model.pth'
model, vocab, inv_vocab, device, config = load_model(model_path)
```

```
Loading model on cpu
Model configuration: {'vocab_size': 14, 'embed_size': 512, 'num_heads': 8,
'ff_dim': 2048, 'num_layers': 8, 'max_length': 42, 'dropout': 0.15}
Model loaded successfully! Best accuracy: 1.0000
```

[38]:
```
# If you need to use the model for inference, you'll want these helper␣
↪functions:
def preprocess_input(input_str, max_length, vocab):
    # Reverse the input string
    input_str = input_str[::-1]
    # Tokenize
    tokens = [vocab[c] for c in input_str if c in vocab]
    # Pad
    padded = tokens + [vocab['<PAD>']] * (max_length - len(tokens))
    return torch.tensor(padded).unsqueeze(0)  # Add batch dimension

def decode_output(output_tensor, inv_vocab):
    _, predicted = output_tensor.max(2)
    decoded = []
    for token in predicted[0]:
        token_val = token.item()
        if token_val == vocab['<EOS>']:
            break
        if token_val != vocab['<PAD>']:
            decoded.append(inv_vocab[token_val])
    return ''.join(decoded)[::-1]  # Reverse at the end

# Example usage:
def test_addition(num1, num2, model, vocab, inv_vocab, max_length):
    input_str = f"{num1}+{num2}="
    input_tensor = preprocess_input(input_str, max_length, vocab)
    with torch.no_grad():
        output = model(input_tensor)
        result = decode_output(output, inv_vocab)
    print(f"{num1} + {num2} = {result}")
    print(f"Correct result: {num1 + num2}")
    print(f"Model's prediction is {'correct' if int(result) == num1 + num2 else␣
↪'incorrect'}")
```

[39]:
```
# Test a simple addition
test_addition(123, 456, model, vocab, inv_vocab, config['max_length'])

# or test multiple additions in a loop
test_cases = [
    (5, 7),
    (42, 58),
    (123, 456),
```

```
    (1234, 5678)
]

for num1, num2 in test_cases:
    test_addition(num1, num2, model, vocab, inv_vocab, config['max_length'])
```

```
123 + 456 = 579
Correct result: 579
Model's prediction is correct
5 + 7 = 12
Correct result: 12
Model's prediction is correct
42 + 58 = 100
Correct result: 100
Model's prediction is correct
123 + 456 = 579
Correct result: 579
Model's prediction is correct
1234 + 5678 = 6912
Correct result: 6912
Model's prediction is correct
```

[40]:

```
  # Test a simple addition
test_addition(123, 456, model, vocab, inv_vocab, config['max_length'])

# or test multiple additions in a loop
test_cases = [
    (5, 7),
    (42, 58),
    (123, 456),
    (1234, 5678),
    (10304923, 123123123),
    (123123123, 10304923)

]

for num1, num2 in test_cases:
    test_addition(num1, num2, model, vocab, inv_vocab, config['max_length'])
```

```
123 + 456 = 579
Correct result: 579
Model's prediction is correct
5 + 7 = 12
Correct result: 12
Model's prediction is correct
42 + 58 = 100
Correct result: 100
Model's prediction is correct
```

```
123 + 456 = 579
Correct result: 579
Model's prediction is correct
1234 + 5678 = 6912
Correct result: 6912
Model's prediction is correct
10304923 + 123123123 = 133428046
Correct result: 133428046
Model's prediction is correct
123123123 + 10304923 = 133428046
Correct result: 133428046
Model's prediction is correct
```

[41]:

```
import torch
import numpy as np
from torch.utils.data import DataLoader
import matplotlib.pyplot as plt
from collections import defaultdict
import seaborn as sns
import random

class ArithmeticModelTester:
    def __init__(self, model, vocab, inv_vocab, max_seq_length):
        self.model = model
        self.vocab = vocab
        self.inv_vocab = inv_vocab
        self.max_seq_length = max_seq_length
        self.model.eval()
        self.results = defaultdict(dict)

    def preprocess_input(self, input_str):
        """Preprocess input string for model"""
        input_str = input_str[::-1]  # Reverse string
        tokens = [self.vocab[c] for c in input_str if c in self.vocab]
        padded = tokens + [self.vocab['<PAD>']] * (self.max_seq_length -␣
↪len(tokens))
        return torch.tensor(padded).unsqueeze(0)

    def decode_output(self, output_tensor):
        """Decode model output"""
        _, predicted = output_tensor.max(2)
        decoded = []
        for token in predicted[0]:
            token_val = token.item()
            if token_val == self.vocab['<EOS>']:
                break
            if token_val != self.vocab['<PAD>']:
```

```
            decoded.append(self.inv_vocab[token_val])
    return ''.join(decoded)[::-1]

def test_single_digits(self, num_trials=100):
    """Test single digit additions"""
    correct = 0
    for _ in range(num_trials):
        n1 = random.randint(0, 9)
        n2 = random.randint(0, 9)
        result = self.predict_addition(n1, n2)
        if result == n1 + n2:
            correct += 1
    self.results['single_digits'] = {'accuracy': correct/num_trials}
    return correct/num_trials

def test_commutative_property(self, max_digit=999, num_trials=100):
    """Test if a + b == b + a"""
    correct = 0
    for _ in range(num_trials):
        n1 = random.randint(0, max_digit)
        n2 = random.randint(0, max_digit)
        result1 = self.predict_addition(n1, n2)
        result2 = self.predict_addition(n2, n1)
        if result1 == result2 == (n1 + n2):
            correct += 1
    self.results['commutative'] = {'accuracy': correct/num_trials}
    return correct/num_trials

def test_zero_property(self, max_digit=999, num_trials=100):
    """Test additions with zero"""
    correct = 0
    for _ in range(num_trials):
        n = random.randint(0, max_digit)
        result1 = self.predict_addition(n, 0)
        result2 = self.predict_addition(0, n)
        if result1 == result2 == n:
            correct += 1
    self.results['zero_property'] = {'accuracy': correct/num_trials}
    return correct/num_trials

def test_by_length(self, max_length=5):
    """Test additions with different number lengths"""
    results = {}
    for length in range(1, max_length + 1):
        correct = 0
        trials = 100
        for _ in range(trials):
```

```
                n1 = random.randint(10**(length-1), 10**length - 1)
                n2 = random.randint(10**(length-1), 10**length - 1)
                result = self.predict_addition(n1, n2)
                if result == n1 + n2:
                    correct += 1
            results[length] = correct/trials
        self.results['length_wise'] = results
        return results

    def test_carries(self):
        """Test additions requiring different numbers of carries"""
        test_cases = [
            (9, 1),       # Single carry
            (99, 1),      # Double carry
            (999, 1),     # Triple carry
            (19, 81),     # Multiple carries
            (999999, 1)   # Many carries
        ]
        results = {}
        for n1, n2 in test_cases:
            result = self.predict_addition(n1, n2)
            results[f"{n1}+{n2}"] = result == n1 + n2
        self.results['carries'] = results
        return results

    def predict_addition(self, n1, n2):
        """Make a prediction for n1 + n2"""
        input_str = f"{n1}+{n2}="
        input_tensor = self.preprocess_input(input_str)
        with torch.no_grad():
            output = self.model(input_tensor)
            result_str = self.decode_output(output)
            try:
                return int(result_str)
            except ValueError:
                return None

    def visualize_results(self):
        """Visualize test results"""
        plt.figure(figsize=(15, 10))

        # Plot accuracy by number length
        if 'length_wise' in self.results:
            plt.subplot(2, 2, 1)
            lengths = list(self.results['length_wise'].keys())
            accuracies = list(self.results['length_wise'].values())
            plt.plot(lengths, accuracies, marker='o')
```

```python
            plt.title('Accuracy by Number Length')
            plt.xlabel('Number Length (digits)')
            plt.ylabel('Accuracy')

        # Plot bar chart of different properties
        properties = ['single_digits', 'commutative', 'zero_property']
        accuracies = [self.results[prop]['accuracy'] for prop in properties if␣
↪prop in self.results]
        if accuracies:
            plt.subplot(2, 2, 2)
            plt.bar(properties, accuracies)
            plt.title('Accuracy by Property')
            plt.xticks(rotation=45)
            plt.ylabel('Accuracy')

        plt.tight_layout()
        plt.show()

# Example usage:
"""
# Load your model and necessary components
model = load_model('path_to_model.pth')
vocab = {...}  # Your vocabulary
inv_vocab = {...}  # Inverse vocabulary
max_seq_length = 42  # Your max sequence length

# Create tester instance
tester = ArithmeticModelTester(model, vocab, inv_vocab, max_seq_length)

# Run tests
tester.test_single_digits()
tester.test_commutative_property()
tester.test_zero_property()
tester.test_by_length()
tester.test_carries()

# Visualize results
tester.visualize_results()
"""
```

[41]:
```
"\n# Load your model and necessary components\nmodel =
load_model('path_to_model.pth')\nvocab = {…}  # Your vocabulary\ninv_vocab =
{…}  # Inverse vocabulary\nmax_seq_length = 42  # Your max sequence
length\n\n# Create tester instance\ntester = ArithmeticModelTester(model, vocab,
inv_vocab, max_seq_length)\n\n# Run tests\ntester.test_single_digits()\ntester.t
est_commutative_property()\ntester.test_zero_property()\ntester.test_by_length()
\ntester.test_carries()\n\n# Visualize results\ntester.visualize_results()\n"
```

[4]:

```
import torch
import numpy as np
from torch.utils.data import DataLoader
import matplotlib.pyplot as plt
import seaborn as sns
from tqdm import tqdm
import random
import pandas as pd
from datetime import datetime

class ModelEvaluator:
    def __init__(self, model, vocab, inv_vocab, max_seq_length, random_seed=42):
        """
        Initialize evaluator with model and set random seed for reproducibility
        """
        self.model = model
        self.vocab = vocab
        self.inv_vocab = inv_vocab
        self.max_seq_length = max_seq_length
        self.model.eval()

        # Set random seeds for reproducibility
        torch.manual_seed(random_seed)
        np.random.seed(random_seed)
        random.seed(random_seed)

        # Store test configuration
        self.test_config = {
            'random_seed': random_seed,
            'timestamp': datetime.now().strftime('%Y-%m-%d_%H-%M-%S'),
            'max_seq_length': max_seq_length
        }

    def generate_test_case(self, min_digits=1, max_digits=20):
        """Generate a random test case with specified digit length range"""
        len1 = random.randint(min_digits, max_digits)
        len2 = random.randint(min_digits, max_digits)

        num1 = random.randint(10**(len1-1), 10**len1 - 1)
        num2 = random.randint(10**(len2-1), 10**len2 - 1)

        return num1, num2, len1, len2

    def preprocess_input(self, input_str):
        """Preprocess input string for model"""
        input_str = input_str[::-1]  # Reverse string
        tokens = [self.vocab[c] for c in input_str if c in self.vocab]
```

```
        padded = tokens + [self.vocab['<PAD>']] * (self.max_seq_length -
↪len(tokens))
        return torch.tensor(padded).unsqueeze(0)

    def decode_output(self, output_tensor):
        """Decode model output"""
        _, predicted = output_tensor.max(2)
        decoded = []
        for token in predicted[0]:
            token_val = token.item()
            if token_val == self.vocab['<EOS>']:
                break
            if token_val != self.vocab['<PAD>']:
                decoded.append(self.inv_vocab[token_val])
        return ''.join(decoded)[::-1]

    def evaluate_model(self, num_samples=10000):
        """
        Evaluate model on random test cases and collect detailed metrics
        """
        results = []

        for _ in tqdm(range(num_samples), desc="Evaluating Model"):
            # Generate test case
            num1, num2, len1, len2 = self.generate_test_case()
            true_result = num1 + num2

            # Get model prediction
            input_str = f"{num1}+{num2}="
            input_tensor = self.preprocess_input(input_str)

            with torch.no_grad():
                output = self.model(input_tensor)
                pred_str = self.decode_output(output)

            try:
                pred_result = int(pred_str)
                is_correct = pred_result == true_result
            except ValueError:
                pred_result = None
                is_correct = False

            # Collect detailed information about this test case
            results.append({
                'num1': num1,
                'num2': num2,
                'len1': len1,
```

```
                'len2': len2,
                'max_len': max(len1, len2),
                'total_len': len1 + len2,
                'true_result': true_result,
                'predicted_result': pred_result,
                'is_correct': is_correct,
                'carries': self.count_carries(num1, num2)
            })

        return pd.DataFrame(results)

    def count_carries(self, num1, num2):
        """Count the number of carry operations in addition"""
        carry = 0
        carries = 0

        while num1 > 0 or num2 > 0 or carry:
            digit1 = num1 % 10
            digit2 = num2 % 10

            if digit1 + digit2 + carry >= 10:
                carries += 1
                carry = 1
            else:
                carry = 0

            num1 //= 10
            num2 //= 10

        return carries

    def analyze_results(self, df):
        """
        Analyze results and generate comprehensive metrics
        """
        metrics = {
            'test_config': self.test_config,
            'overall_accuracy': df['is_correct'].mean(),
            'total_samples': len(df),
            'metrics_by_length': {},
            'metrics_by_carries': {}
        }

        # Analyze by maximum length of operands
        for length in range(1, 21):
            length_df = df[df['max_len'] == length]
            if len(length_df) > 0:
```

```
            metrics['metrics_by_length'][length] = {
                'accuracy': length_df['is_correct'].mean(),
                'samples': len(length_df)
            }

    # Analyze by number of carries
    for carries in df['carries'].unique():
        carries_df = df[df['carries'] == carries]
        metrics['metrics_by_carries'][int(carries)] = {
            'accuracy': carries_df['is_correct'].mean(),
            'samples': len(carries_df)
        }

    return metrics

def visualize_results(self, df, metrics):
    """
    Create visualizations of model performance
    """
    plt.figure(figsize=(20, 10))

    # Plot 1: Accuracy by maximum length
    plt.subplot(2, 2, 1)
    lengths = list(metrics['metrics_by_length'].keys())
    accuracies = [metrics['metrics_by_length'][l]['accuracy'] for l in␣
↪lengths]
    plt.plot(lengths, accuracies, marker='o')
    plt.title('Accuracy by Maximum Length of Operands')
    plt.xlabel('Maximum Length (digits)')
    plt.ylabel('Accuracy')

    # Plot 2: Accuracy by number of carries
    plt.subplot(2, 2, 2)
    carries = list(metrics['metrics_by_carries'].keys())
    carry_accuracies = [metrics['metrics_by_carries'][c]['accuracy'] for c␣
↪in carries]
    plt.plot(carries, carry_accuracies, marker='o', color='orange')
    plt.title('Accuracy by Number of Carries')
    plt.xlabel('Number of Carries')
    plt.ylabel('Accuracy')

    # Plot 3: Distribution of test cases
    plt.subplot(2, 2, 3)
    sns.histplot(data=df, x='max_len', bins=20)
    plt.title('Distribution of Test Cases by Maximum Length')
    plt.xlabel('Maximum Length (digits)')
```

```python
    # Plot 4: Heatmap of length combinations
    plt.subplot(2, 2, 4)
    heatmap_data = pd.crosstab(df['len1'], df['len2'])
    sns.heatmap(heatmap_data, cmap='YlOrRd')
    plt.title('Distribution of Length Combinations')
    plt.xlabel('Length of Second Number')
    plt.ylabel('Length of First Number')

    plt.tight_layout()
    plt.show()

    # Print summary statistics
    print("\nSummary Statistics:")
    print(f"Overall Accuracy: {metrics['overall_accuracy']:.4f}")
    print(f"Total Samples: {metrics['total_samples']}")
```

[6]:

```python
import torch
from torch.utils.data import DataLoader
import matplotlib.pyplot as plt
import seaborn as sns
from tqdm import tqdm
import random
import pandas as pd
from datetime import datetime

# Recreate the model architecture (you'll need to import your model class)
from Scripts.ArithmeticTransformer import *


# First, let's load the model correctly
def load_model_and_config(model_path):
    """Load the model and its configuration"""
    # Load checkpoint
    print(f"Loading model on {device}")
    checkpoint = torch.load(model_path, map_location=device)

    # Extract config
    config = checkpoint['model_config']
    print("Model configuration:", config)

    # Create model
    model = create_arithmetic_transformer(
        vocab_size=config['vocab_size'],
        embed_size=config['embed_size'],
        num_heads=config['num_heads'],
        ff_dim=config['ff_dim'],
        num_layers=config['num_layers'],
```

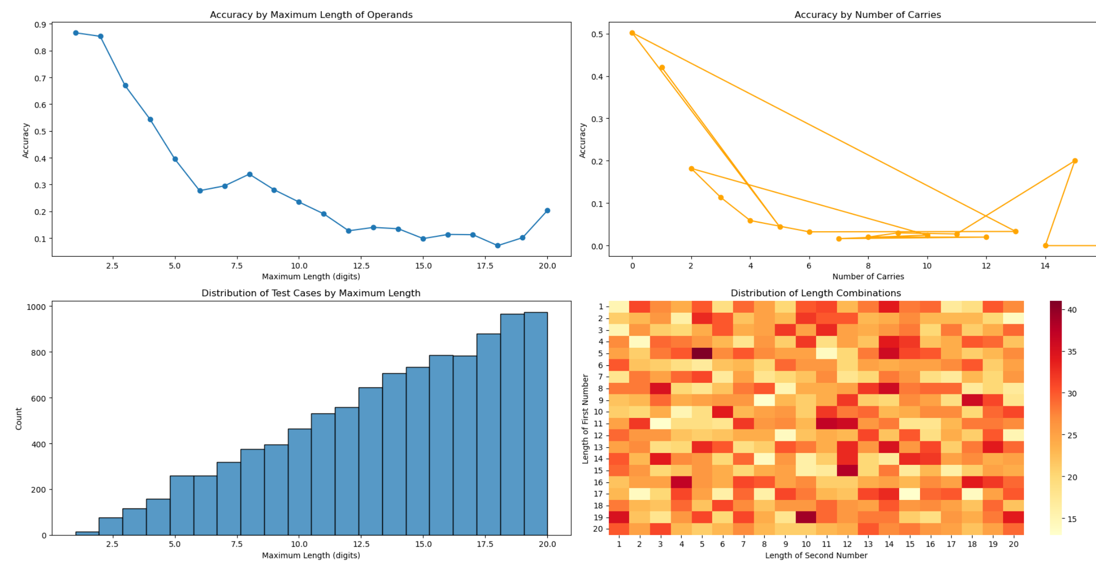
```
        max_length=config['max_length'],
        dropout=config['dropout']
    )

    # Load state dict
    model.load_state_dict(checkpoint['model_state_dict'])

    # Move model to device and set to eval mode
    model = model.to(device)
    model.eval()

    # Get vocab if available
    vocab = checkpoint.get('vocab')
    inv_vocab = checkpoint.get('inv_vocab')

    print(f"Model loaded successfully! Best accuracy: {checkpoint['accuracy']:.
↪4f}")

    return model, vocab, inv_vocab, config['max_length']

# [Previous ModelEvaluator class code remains the same]
```

[15]:

```
# SMALL MODEL EVALUATION
# Load model and components
model_path = './Weights/small_addition_model.pth'  # Update with your model path
model, vocab, inv_vocab, max_seq_length = load_model_and_config(model_path)

# Create evaluator with specific random seed
evaluator = ModelEvaluator(model, vocab, inv_vocab, max_seq_length,␣
↪random_seed=42)

# Run evaluation
results_df = evaluator.evaluate_model(num_samples=10000)
metrics = evaluator.analyze_results(results_df)

# Visualize results
evaluator.visualize_results(results_df, metrics)

# Save results
timestamp = datetime.now().strftime('%Y-%m-%d_%H-%M-%S')
results_df.to_csv(f'model_evaluation_{timestamp}.csv')

# Print some interesting statistics
print("\nDetailed Statistics:")
print(f"Number of correct predictions: {results_df['is_correct'].sum()}")
print(f"Average number of carries: {results_df['carries'].mean():.2f}")
```

```python
# Print performance by length ranges
print("\nPerformance by length ranges:")
length_ranges = [(1,5), (6,10), (11,15), (16,20)]
for start, end in length_ranges:
    mask = (results_df['max_len'] >= start) & (results_df['max_len'] <= end)
    acc = results_df[mask]['is_correct'].mean()
    print(f"{start}-{end} digits: {acc:.4f}")
```

```
Loading model on cpu
Model configuration: {'vocab_size': 14, 'embed_size': 64, 'num_heads': 2,
'ff_dim': 256, 'num_layers': 2, 'max_length': 42, 'dropout': 0.1}
Model loaded successfully! Best accuracy: 0.7937

Evaluating Model: 100%|          | 10000/10000 [00:23<00:00, 430.51it/s]
```

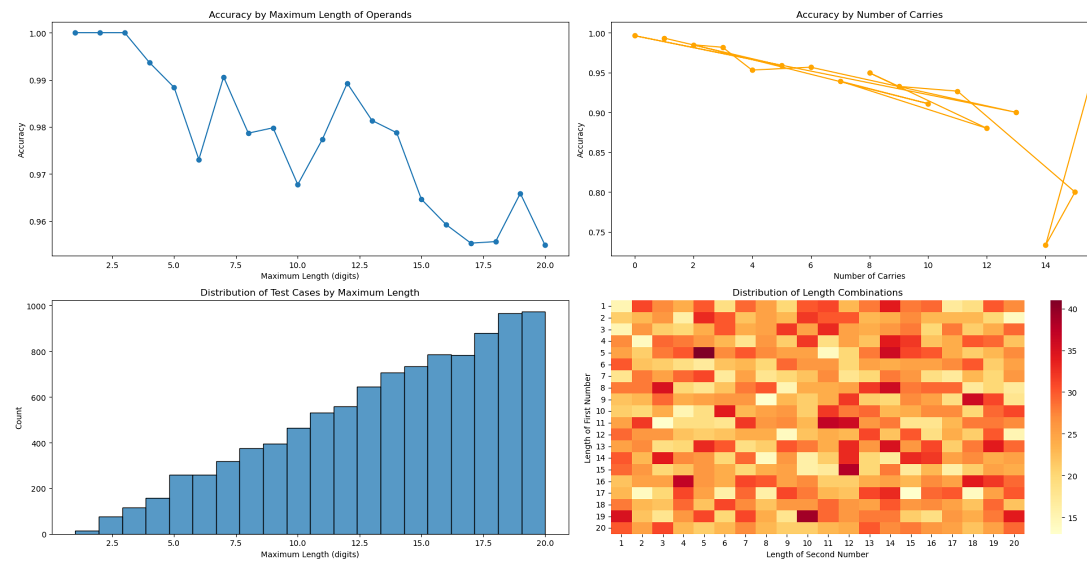



```
Summary Statistics:
Overall Accuracy: 0.1820
Total Samples: 10000

Detailed Statistics:
Number of correct predictions: 1820
Average number of carries: 3.65

Performance by length ranges:
1-5 digits: 0.5507
6-10 digits: 0.2826
11-15 digits: 0.1351
16-20 digits: 0.1221
```

[51]:

```
# MEDUIM MODEL EVALUATION
# Load model and components
model_path = './Weights/medium_addition_model.pth'  # Update with your model␣
↪path
model, vocab, inv_vocab, max_seq_length = load_model_and_config(model_path)

# Create evaluator with specific random seed
evaluator = ModelEvaluator(model, vocab, inv_vocab, max_seq_length,␣
↪random_seed=42)

# Run evaluation
results_df = evaluator.evaluate_model(num_samples=10000)
metrics = evaluator.analyze_results(results_df)

# Visualize results
evaluator.visualize_results(results_df, metrics)

# Save results
timestamp = datetime.now().strftime('%Y-%m-%d_%H-%M-%S')
results_df.to_csv(f'model_evaluation_{timestamp}.csv')

# Print some interesting statistics
print("\nDetailed Statistics:")
print(f"Number of correct predictions: {results_df['is_correct'].sum()}")
print(f"Average number of carries: {results_df['carries'].mean():.2f}")

# Print performance by length ranges
print("\nPerformance by length ranges:")
length_ranges = [(1,5), (6,10), (11,15), (16,20)]
for start, end in length_ranges:
    mask = (results_df['max_len'] >= start) & (results_df['max_len'] <= end)
    acc = results_df[mask]['is_correct'].mean()
    print(f"{start}-{end} digits: {acc:.4f}")
```

```
Loading model on cpu
Model configuration: {'vocab_size': 14, 'embed_size': 256, 'num_heads': 4,
'ff_dim': 1024, 'num_layers': 4, 'max_length': 42, 'dropout': 0.1}
Model loaded successfully! Best accuracy: 0.9987

Evaluating Model: 100%|     | 10000/10000 [01:44<00:00, 96.03it/s]
```

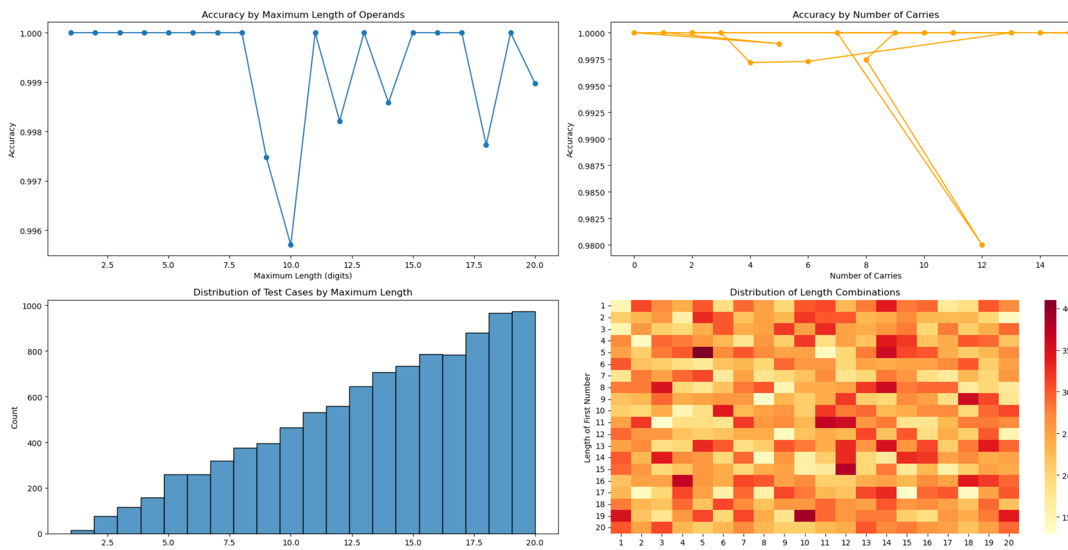



```
Summary Statistics:
Overall Accuracy: 0.9701
Total Samples: 10000

Detailed Statistics:
Number of correct predictions: 9701
Average number of carries: 3.65

Performance by length ranges:
1-5 digits: 0.9936
6-10 digits: 0.9774
11-15 digits: 0.9776
16-20 digits: 0.9583
```

[53]:
```python
# LARGE MODEL EVALUATION
# Load model and components
model_path = './Weights/large_addition_model.pth'  # Update with your model path
model, vocab, inv_vocab, max_seq_length = load_model_and_config(model_path)

# Create evaluator with specific random seed
evaluator = ModelEvaluator(model, vocab, inv_vocab, max_seq_length, random_seed=42)

# Run evaluation
results_df = evaluator.evaluate_model(num_samples=10000)
metrics = evaluator.analyze_results(results_df)

# Visualize results
```

```python
evaluator.visualize_results(results_df, metrics)

# Save results
timestamp = datetime.now().strftime('%Y-%m-%d_%H-%M-%S')
results_df.to_csv(f'model_evaluation_{timestamp}.csv')

# Print some interesting statistics
print("\nDetailed Statistics:")
print(f"Number of correct predictions: {results_df['is_correct'].sum()}")
print(f"Average number of carries: {results_df['carries'].mean():.2f}")

# Print performance by length ranges
print("\nPerformance by length ranges:")
length_ranges = [(1,5), (6,10), (11,15), (16,20)]
for start, end in length_ranges:
    mask = (results_df['max_len'] >= start) & (results_df['max_len'] <= end)
    acc = results_df[mask]['is_correct'].mean()
    print(f"{start}-{end} digits: {acc:.4f}")
```

```
Loading model on cpu
Model configuration: {'vocab_size': 14, 'embed_size': 512, 'num_heads': 8,
'ff_dim': 2048, 'num_layers': 8, 'max_length': 42, 'dropout': 0.15}
Model loaded successfully! Best accuracy: 1.0000

Evaluating Model: 100%|          | 10000/10000 [13:21<00:00, 12.47it/s]
```




```
Summary Statistics:
Overall Accuracy: 0.9992
Total Samples: 10000
```

```
Detailed Statistics:
Number of correct predictions: 9992
Average number of carries: 3.65

Performance by length ranges:
1-5 digits: 1.0000
6-10 digits: 0.9983
11-15 digits: 0.9994
16-20 digits: 0.9993
```

[7]:

```
# Helper functions for extended range testing
def generate_extended_test_case(min_digits=21, max_digits=25):
    """Generate a test case with specific digit length"""
    len1 = random.randint(min_digits, max_digits)
    len2 = random.randint(min_digits, max_digits)
    num1 = random.randint(10**(len1-1), (10**len1) - 1)
    num2 = random.randint(10**(len2-1), (10**len2) - 1)
    return num1, num2, len1, len2

class ExtendedRangeEvaluator(ModelEvaluator):
    def evaluate_model(self, samples_per_length=200):
        """
        Evaluate model with uniform distribution across 21-25 digits
        """
        results = []
        for target_length in range(21, 26):
            for _ in tqdm(range(samples_per_length),
                          desc=f"Evaluating {target_length} digits"):
                # Generate numbers
                num1, num2, len1, len2 = generate_extended_test_case(
                    target_length, target_length)
                true_result = num1 + num2

                # Get model prediction
                input_str = f"{num1}+{num2}="
                input_tensor = self.preprocess_input(input_str)

                with torch.no_grad():
                    output = self.model(input_tensor)
                    pred_str = self.decode_output(output)

                try:
                    pred_result = int(pred_str)
                    is_correct = pred_result == true_result
                except ValueError:
                    pred_result = None
```

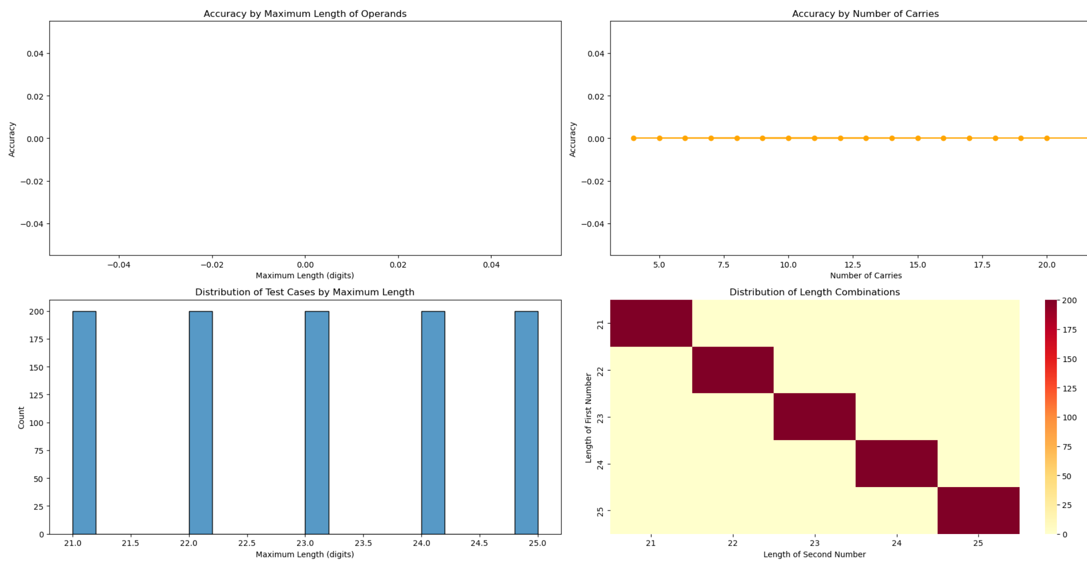
```
                    is_correct = False

                results.append({
                    'num1': num1,
                    'num2': num2,
                    'len1': len1,
                    'len2': len2,
                    'max_len': max(len1, len2),
                    'true_result': true_result,
                    'predicted_result': pred_result,
                    'is_correct': is_correct,
                    'carries': self.count_carries(num1, num2)
                })

        return pd.DataFrame(results)
```

[11]:

```
# First cell: SMALL MODEL EXTENDED RANGE EVALUATION
print("# SMALL MODEL EXTENDED RANGE EVALUATION")
# Load model and components
model_path = './Weights/small_addition_model.pth'
model, vocab, inv_vocab, max_seq_length = load_model_and_config(model_path)

# Create evaluator
evaluator = ExtendedRangeEvaluator(model, vocab, inv_vocab, max_seq_length,␣
 ↪random_seed=42)

# Run evaluation
results_df = evaluator.evaluate_model(samples_per_length=200)
metrics = evaluator.analyze_results(results_df)

# Visualize results
evaluator.visualize_results(results_df, metrics)

# Save results
timestamp = datetime.now().strftime('%Y-%m-%d_%H-%M-%S')
results_df.to_csv(f'extended_small_model_evaluation_{timestamp}.csv')

# Print statistics
print("\nDetailed Statistics:")
print(f"Number of correct predictions: {results_df['is_correct'].sum()}")
print(f"Average number of carries: {results_df['carries'].mean():.2f}")

print("\nPerformance by length:")
for length in range(21, 26):
    mask = (results_df['max_len'] == length)
    acc = results_df[mask]['is_correct'].mean()
    samples = sum(mask)
```

```python
    print(f"{length} digits: {acc:.4f} ({samples} samples)")
```

```
# SMALL MODEL EXTENDED RANGE EVALUATION
Loading model on cpu
Model configuration: {'vocab_size': 14, 'embed_size': 64, 'num_heads': 2,
'ff_dim': 256, 'num_layers': 2, 'max_length': 42, 'dropout': 0.1}
Model loaded successfully! Best accuracy: 0.7937

Evaluating 21 digits: 100%|      | 200/200 [00:00<00:00, 323.16it/s]
Evaluating 22 digits: 100%|      | 200/200 [00:00<00:00, 290.10it/s]
Evaluating 23 digits: 100%|      | 200/200 [00:00<00:00, 257.09it/s]
Evaluating 24 digits: 100%|      | 200/200 [00:00<00:00, 253.23it/s]
Evaluating 25 digits: 100%|      | 200/200 [00:01<00:00, 169.77it/s]
```

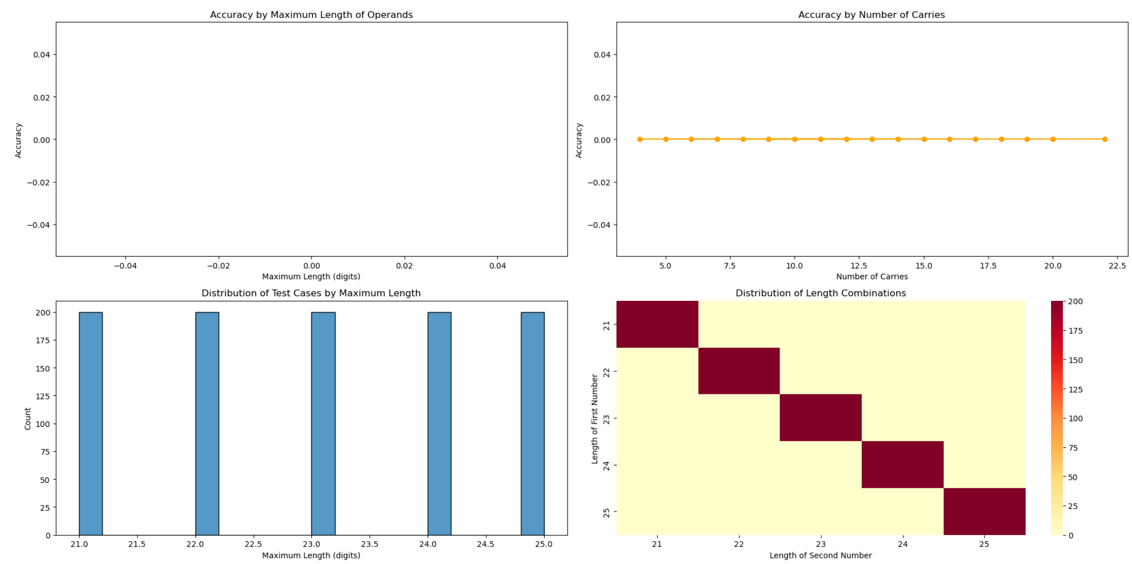



```
Summary Statistics:
Overall Accuracy: 0.0000
Total Samples: 1000

Detailed Statistics:
Number of correct predictions: 0
Average number of carries: 11.56

Performance by length:
21 digits: 0.0000 (200 samples)
22 digits: 0.0000 (200 samples)
23 digits: 0.0000 (200 samples)
24 digits: 0.0000 (200 samples)
25 digits: 0.0000 (200 samples)
```

[12]:

```python
# Second cell: MEDIUM MODEL EXTENDED RANGE EVALUATION
print("\n# MEDIUM MODEL EXTENDED RANGE EVALUATION")
# Load model and components
model_path = './Weights/medium_addition_model.pth'
model, vocab, inv_vocab, max_seq_length = load_model_and_config(model_path)

# Create evaluator
evaluator = ExtendedRangeEvaluator(model, vocab, inv_vocab, max_seq_length,␣
 ↪random_seed=42)

# Run evaluation
results_df = evaluator.evaluate_model(samples_per_length=200)
metrics = evaluator.analyze_results(results_df)

# Visualize results
evaluator.visualize_results(results_df, metrics)

# Save results
timestamp = datetime.now().strftime('%Y-%m-%d_%H-%M-%S')
results_df.to_csv(f'extended_medium_model_evaluation_{timestamp}.csv')

# Print statistics
print("\nDetailed Statistics:")
print(f"Number of correct predictions: {results_df['is_correct'].sum()}")
print(f"Average number of carries: {results_df['carries'].mean():.2f}")

print("\nPerformance by length:")
for length in range(21, 26):
    mask = (results_df['max_len'] == length)
    acc = results_df[mask]['is_correct'].mean()
    samples = sum(mask)
    print(f"{length} digits: {acc:.4f} ({samples} samples)")
```

```
# MEDIUM MODEL EXTENDED RANGE EVALUATION
Loading model on cpu
Model configuration: {'vocab_size': 14, 'embed_size': 256, 'num_heads': 4,
'ff_dim': 1024, 'num_layers': 4, 'max_length': 42, 'dropout': 0.1}
Model loaded successfully! Best accuracy: 0.9987

Evaluating 21 digits: 100%|      | 200/200 [00:03<00:00, 57.93it/s]
Evaluating 22 digits: 100%|      | 200/200 [00:03<00:00, 54.02it/s]
Evaluating 23 digits: 100%|      | 200/200 [00:03<00:00, 52.04it/s]
Evaluating 24 digits: 100%|      | 200/200 [00:03<00:00, 51.76it/s]
Evaluating 25 digits: 100%|      | 200/200 [00:03<00:00, 59.23it/s]
```

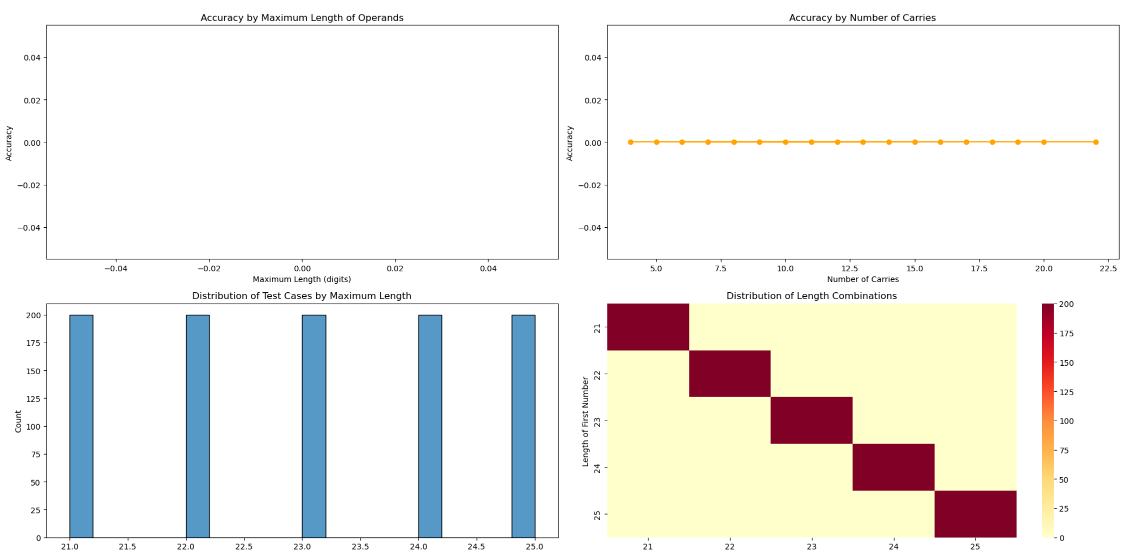



```
Summary Statistics:
Overall Accuracy: 0.0000
Total Samples: 1000

Detailed Statistics:
Number of correct predictions: 0
Average number of carries: 11.56

Performance by length:
21 digits: 0.0000 (200 samples)
22 digits: 0.0000 (200 samples)
23 digits: 0.0000 (200 samples)
24 digits: 0.0000 (200 samples)
25 digits: 0.0000 (200 samples)
```

[13]:
```python
# Third cell: LARGE MODEL EXTENDED RANGE EVALUATION
print("\n# LARGE MODEL EXTENDED RANGE EVALUATION")
# Load model and components
model_path = './Weights/large_addition_model.pth'
model, vocab, inv_vocab, max_seq_length = load_model_and_config(model_path)

# Create evaluator
evaluator = ExtendedRangeEvaluator(model, vocab, inv_vocab, max_seq_length, random_seed=42)

# Run evaluation
results_df = evaluator.evaluate_model(samples_per_length=200)
metrics = evaluator.analyze_results(results_df)
```

```python
# Visualize results
evaluator.visualize_results(results_df, metrics)

# Save results
timestamp = datetime.now().strftime('%Y-%m-%d_%H-%M-%S')
results_df.to_csv(f'extended_large_model_evaluation_{timestamp}.csv')

# Print statistics
print("\nDetailed Statistics:")
print(f"Number of correct predictions: {results_df['is_correct'].sum()}")
print(f"Average number of carries: {results_df['carries'].mean():.2f}")

print("\nPerformance by length:")
for length in range(21, 26):
    mask = (results_df['max_len'] == length)
    acc = results_df[mask]['is_correct'].mean()
    samples = sum(mask)
    print(f"{length} digits: {acc:.4f} ({samples} samples)")
```

```
# LARGE MODEL EXTENDED RANGE EVALUATION
Loading model on cpu
Model configuration: {'vocab_size': 14, 'embed_size': 512, 'num_heads': 8,
'ff_dim': 2048, 'num_layers': 8, 'max_length': 42, 'dropout': 0.15}
Model loaded successfully! Best accuracy: 1.0000

Evaluating 21 digits: 100%|      | 200/200 [00:15<00:00, 13.04it/s]
Evaluating 22 digits: 100%|      | 200/200 [00:14<00:00, 13.69it/s]
Evaluating 23 digits: 100%|      | 200/200 [00:15<00:00, 13.03it/s]
Evaluating 24 digits: 100%|      | 200/200 [00:16<00:00, 12.05it/s]
Evaluating 25 digits: 100%|      | 200/200 [00:14<00:00, 13.62it/s]
```



```
Summary Statistics:
Overall Accuracy: 0.0000
Total Samples: 1000

Detailed Statistics:
Number of correct predictions: 0
Average number of carries: 11.56

Performance by length:
21 digits: 0.0000 (200 samples)
22 digits: 0.0000 (200 samples)
23 digits: 0.0000 (200 samples)
24 digits: 0.0000 (200 samples)
25 digits: 0.0000 (200 samples)
```

[ ]:

```
[20]:
```

```python
# Import necessary libraries
import torch
import torch.nn.functional as F
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
from tqdm import tqdm
import random
from datetime import datetime

class ComprehensiveEvaluator(ModelEvaluator):
    def evaluate_model(self, samples_per_length=100):
        """
        Evaluate model with multiple metrics per Schaeffer et al.
        - Traditional accuracy (discontinuous)
        - Token edit distance (linear)
        - Per-token probability (continuous)
        - Brier score for digit prediction (continuous)
        """
        results = []
        digit_lengths = range(1, 21)  # Testing on 1-20 digit numbers

        for target_length in digit_lengths:
            for _ in tqdm(range(samples_per_length),
                          desc=f"Evaluating {target_length} digits"):
                # Generate numbers with exact target length
```

```
                num1 = random.randint(10**(target_length-1), 10**target_length␣
↪- 1)
                num2 = random.randint(10**(target_length-1), 10**target_length␣
↪- 1)
                true_result = num1 + num2

                # Get model prediction
                input_str = f"{num1}+{num2}="
                input_tensor = self.preprocess_input(input_str)

                with torch.no_grad():
                    output = self.model(input_tensor)
                    logits = F.softmax(output, dim=-1)
                    pred_str = self.decode_output(output)

                try:
                    pred_result = int(pred_str)
                    is_correct = pred_result == true_result
                except ValueError:
                    pred_result = None
                    is_correct = False

                # Calculate additional metrics
                target_tokens = str(true_result)
                pred_tokens = str(pred_result) if pred_result is not None else␣
↪""
                edit_distance = self.
↪calculate_token_edit_distance(target_tokens, pred_tokens)
                target_token_probs = self.calculate_target_token_probs(logits,␣
↪str(true_result))
                brier_score = self.calculate_brier_score(logits,␣
↪str(true_result))

                results.append({
                    'num1': num1,
                    'num2': num2,
                    'len1': len(str(num1)),
                    'len2': len(str(num2)),
                    'max_len': max(len(str(num1)), len(str(num2))),
                    'true_result': true_result,
                    'predicted_result': pred_result,
                    'is_correct': is_correct,
                    'carries': self.count_carries(num1, num2),
                    'edit_distance': edit_distance,
                    'mean_token_prob': np.mean(target_token_probs),
                    'brier_score': brier_score
```

```
            })

        return pd.DataFrame(results)

    def calculate_token_edit_distance(self, target, pred):
        """Calculate minimum number of token operations to transform pred into␣
↪target"""
        m, n = len(target), len(pred)
        dp = [[0] * (n + 1) for _ in range(m + 1)]

        for i in range(m + 1):
            dp[i][0] = i
        for j in range(n + 1):
            dp[0][j] = j

        for i in range(1, m + 1):
            for j in range(1, n + 1):
                if target[i-1] == pred[j-1]:
                    dp[i][j] = dp[i-1][j-1]
                else:
                    dp[i][j] = min(dp[i-1][j], dp[i][j-1], dp[i-1][j-1]) + 1

        return dp[m][n]

    def calculate_target_token_probs(self, logits, target):
        """Calculate probability model assigned to each correct target token"""
        probs = []
        for i, digit in enumerate(target):
            if i < logits.size(1):  # Ensure we don't exceed sequence length
                digit_idx = self.vocab[digit]
                prob = logits[0, i, digit_idx].item()
                probs.append(prob)
        return probs

    def calculate_brier_score(self, logits, target):
        """Calculate Brier score for digit prediction"""
        brier_scores = []
        for i, digit in enumerate(target):
            if i < logits.size(1):
                target_dist = torch.zeros_like(logits[0, i])
                target_dist[self.vocab[digit]] = 1.0
                brier_scores.append(torch.mean((logits[0, i] - target_dist)**2).
↪item())
        return np.mean(brier_scores)

def analyze_and_visualize_results(results_df, model_name):
```

```
    """Generate visualizations and save reports for a model's evaluation␣
↪results"""
    # Save raw results
    timestamp = datetime.now().strftime('%Y-%m-%d_%H-%M-%S')
    results_df.to_csv(f'{model_name}_raw_results_{timestamp}.csv')

    # Calculate aggregated statistics
    agg_stats = {}

    # Overall metrics
    agg_stats['overall'] = {
        'accuracy': results_df['is_correct'].mean(),
        'edit_distance': results_df['edit_distance'].mean(),
        'token_probability': results_df['mean_token_prob'].mean(),
        'brier_score': results_df['brier_score'].mean()
    }

    # Metrics by length range
    length_ranges = [(1,5), (6,10), (11,15), (16,20)]
    for start, end in length_ranges:
        mask = (results_df['max_len'] >= start) & (results_df['max_len'] <= end)
        range_name = f'length_{start}_to_{end}'
        agg_stats[range_name] = {
            'accuracy': results_df[mask]['is_correct'].mean(),
            'edit_distance': results_df[mask]['edit_distance'].mean(),
            'token_probability': results_df[mask]['mean_token_prob'].mean(),
            'brier_score': results_df[mask]['brier_score'].mean()
        }

    # Save aggregated statistics
    pd.DataFrame(agg_stats).to_csv(f'{model_name}_aggregated_stats_{timestamp}.
↪csv')

    # Visualization
    plt.figure(figsize=(20, 10))

    # Plot 1: Traditional Accuracy (Discontinuous)
    plt.subplot(2, 2, 1)
    accuracy_by_length = results_df.groupby('max_len')['is_correct'].mean()
    plt.plot(accuracy_by_length.index, accuracy_by_length.values, marker='o')
    plt.title(f'{model_name}: Traditional Accuracy by Length\n(Discontinuous␣
↪Metric)')
    plt.xlabel('Number Length (digits)')
    plt.ylabel('Accuracy')

    # Plot 2: Token Edit Distance (Linear)
    plt.subplot(2, 2, 2)
```

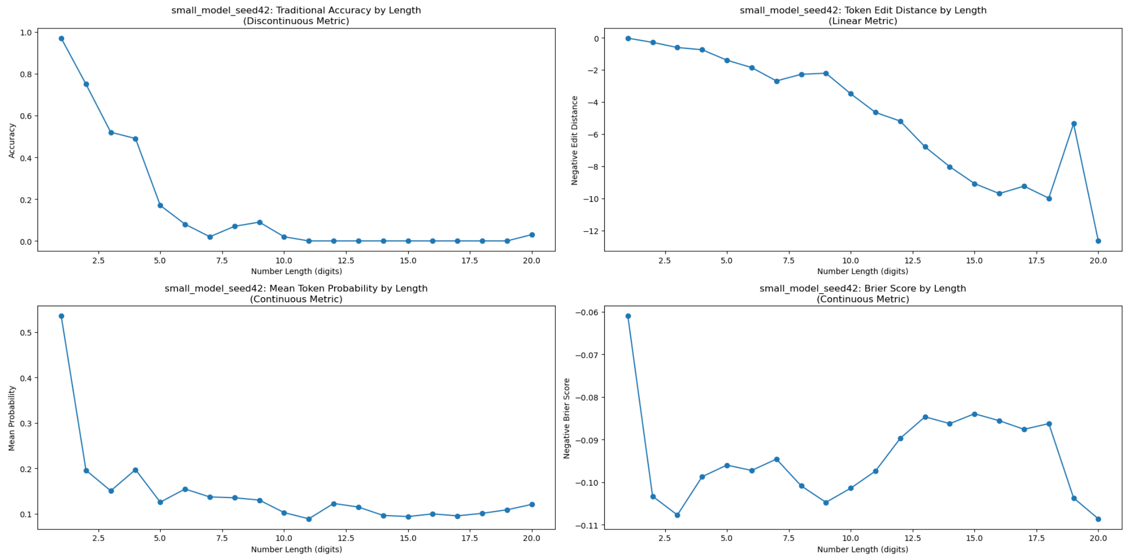
```
    edit_distance_by_length = -results_df.groupby('max_len')['edit_distance'].
↪mean()
    plt.plot(edit_distance_by_length.index, edit_distance_by_length.values,␣
↪marker='o')
    plt.title(f'{model_name}: Token Edit Distance by Length\n(Linear Metric)')
    plt.xlabel('Number Length (digits)')
    plt.ylabel('Negative Edit Distance')

    # Plot 3: Mean Token Probability (Continuous)
    plt.subplot(2, 2, 3)
    prob_by_length = results_df.groupby('max_len')['mean_token_prob'].mean()
    plt.plot(prob_by_length.index, prob_by_length.values, marker='o')
    plt.title(f'{model_name}: Mean Token Probability by Length\n(Continuous␣
↪Metric)')
    plt.xlabel('Number Length (digits)')
    plt.ylabel('Mean Probability')

    # Plot 4: Brier Score (Continuous)
    plt.subplot(2, 2, 4)
    brier_by_length = -results_df.groupby('max_len')['brier_score'].mean()
    plt.plot(brier_by_length.index, brier_by_length.values, marker='o')
    plt.title(f'{model_name}: Brier Score by Length\n(Continuous Metric)')
    plt.xlabel('Number Length (digits)')
    plt.ylabel('Negative Brier Score')

    plt.tight_layout()
    plt.savefig(f'{model_name}_analysis_plots_{timestamp}.png')
    plt.show()

    # Print summary statistics
    print(f"\nDetailed Statistics for {model_name}:")
    print(f"Number of samples: {len(results_df)}")
    print("\nPerformance by metric type:")
    print("\nDiscontinuous Metrics:")
    print(f"Overall Accuracy: {agg_stats['overall']['accuracy']:.4f}")

    print("\nLinear Metrics:")
    print(f"Mean Edit Distance: {agg_stats['overall']['edit_distance']:.4f}")

    print("\nContinuous Metrics:")
    print(f"Mean Token Probability: {agg_stats['overall']['token_probability']:.
↪4f}")
    print(f"Mean Brier Score: {agg_stats['overall']['brier_score']:.4f}")

    print("\nPerformance by length ranges:")
    for start, end in length_ranges:
        range_name = f'length_{start}_to_{end}'
```

```
        print(f"\n{start}-{end} digits:")
        print(f"  Accuracy: {agg_stats[range_name]['accuracy']:.4f}")
        print(f"  Edit Distance: {agg_stats[range_name]['edit_distance']:.4f}")
        print(f"  Token Probability: 
 ↪{agg_stats[range_name]['token_probability']:.4f}")
        print(f"  Brier Score: {agg_stats[range_name]['brier_score']:.4f}")
```

[22]:
```
# SMALL MODEL COMPREHENSIVE ANALYSIS
print("\n# SMALL MODEL COMPREHENSIVE ANALYSIS")

# Load model and components
model_path = './Weights/small_addition_model.pth'
model, vocab, inv_vocab, max_seq_length = load_model_and_config(model_path)

# Create evaluator with specific random seed for test generation
evaluator = ComprehensiveEvaluator(model, vocab, inv_vocab, max_seq_length, 
 ↪random_seed=42)

# Run evaluation
results_df = evaluator.evaluate_model(samples_per_length=100)

# Generate visualizations and save reports
analyze_and_visualize_results(results_df, "small_model_seed42")
```

```
# SMALL MODEL COMPREHENSIVE ANALYSIS
Loading model on cpu
Model configuration: {'vocab_size': 14, 'embed_size': 64, 'num_heads': 2,
'ff_dim': 256, 'num_layers': 2, 'max_length': 42, 'dropout': 0.1}
Model loaded successfully! Best accuracy: 0.7937

Evaluating 1 digits: 100%|      | 100/100 [00:00<00:00, 172.18it/s]
Evaluating 2 digits: 100%|      | 100/100 [00:00<00:00, 216.54it/s]
Evaluating 3 digits: 100%|      | 100/100 [00:00<00:00, 155.49it/s]
Evaluating 4 digits: 100%|      | 100/100 [00:00<00:00, 126.37it/s]
Evaluating 5 digits: 100%|      | 100/100 [00:00<00:00, 178.02it/s]
Evaluating 6 digits: 100%|      | 100/100 [00:00<00:00, 172.43it/s]
Evaluating 7 digits: 100%|      | 100/100 [00:00<00:00, 121.08it/s]
Evaluating 8 digits: 100%|      | 100/100 [00:00<00:00, 196.02it/s]
Evaluating 9 digits: 100%|      | 100/100 [00:00<00:00, 191.95it/s]
Evaluating 10 digits: 100%|      | 100/100 [00:00<00:00, 192.51it/s]
Evaluating 11 digits: 100%|      | 100/100 [00:00<00:00, 142.94it/s]
Evaluating 12 digits: 100%|      | 100/100 [00:00<00:00, 150.06it/s]
Evaluating 13 digits: 100%|      | 100/100 [00:00<00:00, 151.69it/s]
Evaluating 14 digits: 100%|      | 100/100 [00:00<00:00, 102.24it/s]
Evaluating 15 digits: 100%|      | 100/100 [00:00<00:00, 141.71it/s]
Evaluating 16 digits: 100%|      | 100/100 [00:00<00:00, 128.76it/s]
Evaluating 17 digits: 100%|      | 100/100 [00:00<00:00, 105.92it/s]
```

```
Evaluating 18 digits: 100%|          | 100/100 [00:00<00:00, 115.92it/s]
Evaluating 19 digits: 100%|          | 100/100 [00:00<00:00, 113.37it/s]
Evaluating 20 digits: 100%|          | 100/100 [00:00<00:00, 104.53it/s]
```




```
Detailed Statistics for small_model_seed42:
Number of samples: 2000

Performance by metric type:

Discontinuous Metrics:
Overall Accuracy: 0.1605

Linear Metrics:
Mean Edit Distance: 4.8140

Continuous Metrics:
Mean Token Probability: 0.1451
Mean Brier Score: 0.0940

Performance by length ranges:

1-5 digits:
  Accuracy: 0.5800
  Edit Distance: 0.6160
  Token Probability: 0.2406
  Brier Score: 0.0934

6-10 digits:
```

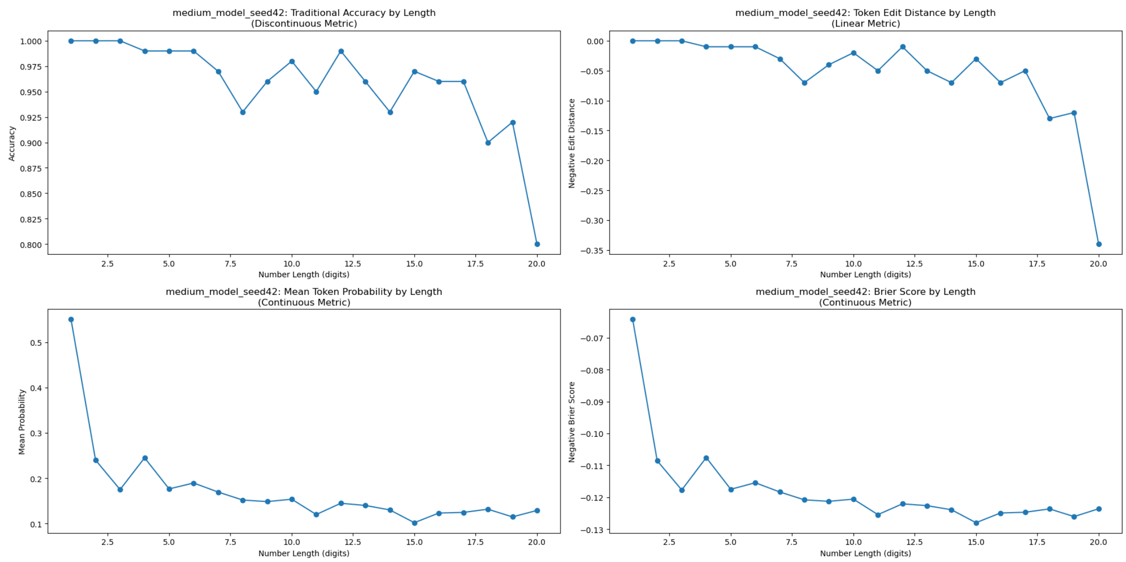
```
  Accuracy: 0.0560
  Edit Distance: 2.5060
  Token Probability: 0.1317
  Brier Score: 0.0998

11-15 digits:
  Accuracy: 0.0000
  Edit Distance: 6.7480
  Token Probability: 0.1031
  Brier Score: 0.0884

16-20 digits:
  Accuracy: 0.0060
  Edit Distance: 9.3860
  Token Probability: 0.1049
  Brier Score: 0.0943
```

[23]:

```
# MEDIUM MODEL COMPREHENSIVE ANALYSIS
print("\n# MEDIUM MODEL COMPREHENSIVE ANALYSIS")

# Load model and components
model_path = './Weights/medium_addition_model.pth'
model, vocab, inv_vocab, max_seq_length = load_model_and_config(model_path)

# Create evaluator with different random seed
evaluator = ComprehensiveEvaluator(model, vocab, inv_vocab, max_seq_length,␣
 ↪random_seed=42)

# Run evaluation
results_df = evaluator.evaluate_model(samples_per_length=100)

# Generate visualizations and save reports
analyze_and_visualize_results(results_df, "medium_model_seed42")
```

```
# MEDIUM MODEL COMPREHENSIVE ANALYSIS
Loading model on cpu
Model configuration: {'vocab_size': 14, 'embed_size': 256, 'num_heads': 4,
'ff_dim': 1024, 'num_layers': 4, 'max_length': 42, 'dropout': 0.1}
Model loaded successfully! Best accuracy: 0.9987

Evaluating 1 digits: 100%|      | 100/100 [00:05<00:00, 17.70it/s]
Evaluating 2 digits: 100%|      | 100/100 [00:03<00:00, 30.96it/s]
Evaluating 3 digits: 100%|      | 100/100 [00:03<00:00, 31.89it/s]
Evaluating 4 digits: 100%|      | 100/100 [00:02<00:00, 37.50it/s]
Evaluating 5 digits: 100%|      | 100/100 [00:03<00:00, 27.69it/s]
Evaluating 6 digits: 100%|      | 100/100 [00:02<00:00, 33.61it/s]
Evaluating 7 digits: 100%|      | 100/100 [00:02<00:00, 33.95it/s]
```

```
Evaluating 8 digits: 100%|          | 100/100 [00:02<00:00, 35.91it/s]
Evaluating 9 digits: 100%|          | 100/100 [00:02<00:00, 36.67it/s]
Evaluating 10 digits: 100%|          | 100/100 [00:03<00:00, 32.60it/s]
Evaluating 11 digits: 100%|          | 100/100 [00:04<00:00, 20.86it/s]
Evaluating 12 digits: 100%|          | 100/100 [00:03<00:00, 27.21it/s]
Evaluating 13 digits: 100%|          | 100/100 [00:04<00:00, 21.47it/s]
Evaluating 14 digits: 100%|          | 100/100 [00:05<00:00, 19.64it/s]
Evaluating 15 digits: 100%|          | 100/100 [00:15<00:00,  6.56it/s]
Evaluating 16 digits: 100%|          | 100/100 [00:04<00:00, 24.57it/s]
Evaluating 17 digits: 100%|          | 100/100 [00:02<00:00, 37.49it/s]
Evaluating 18 digits: 100%|          | 100/100 [00:03<00:00, 27.08it/s]
Evaluating 19 digits: 100%|          | 100/100 [00:05<00:00, 19.55it/s]
Evaluating 20 digits: 100%|          | 100/100 [00:03<00:00, 27.48it/s]
```




```
Detailed Statistics for medium_model_seed42:
Number of samples: 2000

Performance by metric type:

Discontinuous Metrics:
Overall Accuracy: 0.9575

Linear Metrics:
Mean Edit Distance: 0.0555

Continuous Metrics:
Mean Token Probability: 0.1730
Mean Brier Score: 0.1179
```

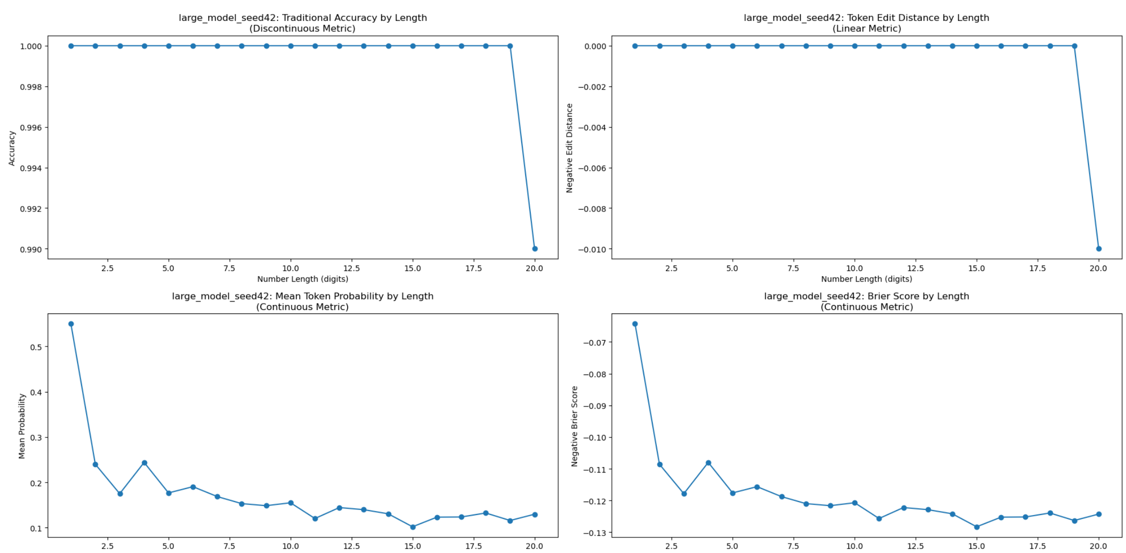
```
Performance by length ranges:

1-5 digits:
  Accuracy: 0.9960
  Edit Distance: 0.0040
  Token Probability: 0.2773
  Brier Score: 0.1031

6-10 digits:
  Accuracy: 0.9660
  Edit Distance: 0.0340
  Token Probability: 0.1626
  Brier Score: 0.1193

11-15 digits:
  Accuracy: 0.9600
  Edit Distance: 0.0420
  Token Probability: 0.1274
  Brier Score: 0.1244

16-20 digits:
  Accuracy: 0.9080
  Edit Distance: 0.1420
  Token Probability: 0.1247
  Brier Score: 0.1246
```

[24]:

```python
# LARGE MODEL COMPREHENSIVE ANALYSIS
print("\n# LARGE MODEL COMPREHENSIVE ANALYSIS")

# Load model and components
model_path = './Weights/large_addition_model.pth'
model, vocab, inv_vocab, max_seq_length = load_model_and_config(model_path)

# Create evaluator with another different random seed
evaluator = ComprehensiveEvaluator(model, vocab, inv_vocab, max_seq_length,␣
 ↪random_seed=42)

# Run evaluation
results_df = evaluator.evaluate_model(samples_per_length=100)

# Generate visualizations and save reports
analyze_and_visualize_results(results_df, "large_model_seed42")
```

```
# LARGE MODEL COMPREHENSIVE ANALYSIS
Loading model on cpu
Model configuration: {'vocab_size': 14, 'embed_size': 512, 'num_heads': 8,
```

```
'ff_dim': 2048, 'num_layers': 8, 'max_length': 42, 'dropout': 0.15}
Model loaded successfully! Best accuracy: 1.0000

Evaluating 1 digits: 100%|      | 100/100 [00:09<00:00, 10.70it/s]
Evaluating 2 digits: 100%|      | 100/100 [00:08<00:00, 11.67it/s]
Evaluating 3 digits: 100%|      | 100/100 [00:08<00:00, 11.85it/s]
Evaluating 4 digits: 100%|      | 100/100 [00:08<00:00, 12.24it/s]
Evaluating 5 digits: 100%|      | 100/100 [00:08<00:00, 11.86it/s]
Evaluating 6 digits: 100%|      | 100/100 [00:08<00:00, 12.08it/s]
Evaluating 7 digits: 100%|      | 100/100 [00:08<00:00, 11.77it/s]
Evaluating 8 digits: 100%|      | 100/100 [00:08<00:00, 11.39it/s]
Evaluating 9 digits: 100%|      | 100/100 [00:08<00:00, 11.49it/s]
Evaluating 10 digits: 100%|      | 100/100 [00:09<00:00, 10.58it/s]
Evaluating 11 digits: 100%|      | 100/100 [00:08<00:00, 11.62it/s]
Evaluating 12 digits: 100%|      | 100/100 [00:09<00:00, 11.09it/s]
Evaluating 13 digits: 100%|      | 100/100 [00:08<00:00, 11.68it/s]
Evaluating 14 digits: 100%|      | 100/100 [00:09<00:00, 10.01it/s]
Evaluating 15 digits: 100%|      | 100/100 [00:08<00:00, 11.51it/s]
Evaluating 16 digits: 100%|      | 100/100 [00:09<00:00, 10.64it/s]
Evaluating 17 digits: 100%|      | 100/100 [00:09<00:00, 10.71it/s]
Evaluating 18 digits: 100%|      | 100/100 [00:08<00:00, 11.37it/s]
Evaluating 19 digits: 100%|      | 100/100 [00:08<00:00, 11.17it/s]
Evaluating 20 digits: 100%|      | 100/100 [00:08<00:00, 11.30it/s]
```




```
Detailed Statistics for large_model_seed42:
Number of samples: 2000

Performance by metric type:
```

```
Discontinuous Metrics:
Overall Accuracy: 0.9995

Linear Metrics:
Mean Edit Distance: 0.0005

Continuous Metrics:
Mean Token Probability: 0.1731
Mean Brier Score: 0.1181

Performance by length ranges:

1-5 digits:
  Accuracy: 1.0000
  Edit Distance: 0.0000
  Token Probability: 0.2771
  Brier Score: 0.1033

6-10 digits:
  Accuracy: 1.0000
  Edit Distance: 0.0000
  Token Probability: 0.1631
  Brier Score: 0.1196

11-15 digits:
  Accuracy: 1.0000
  Edit Distance: 0.0000
  Token Probability: 0.1273
  Brier Score: 0.1247

16-20 digits:
  Accuracy: 0.9980
  Edit Distance: 0.0020
  Token Probability: 0.1249
  Brier Score: 0.1250
```

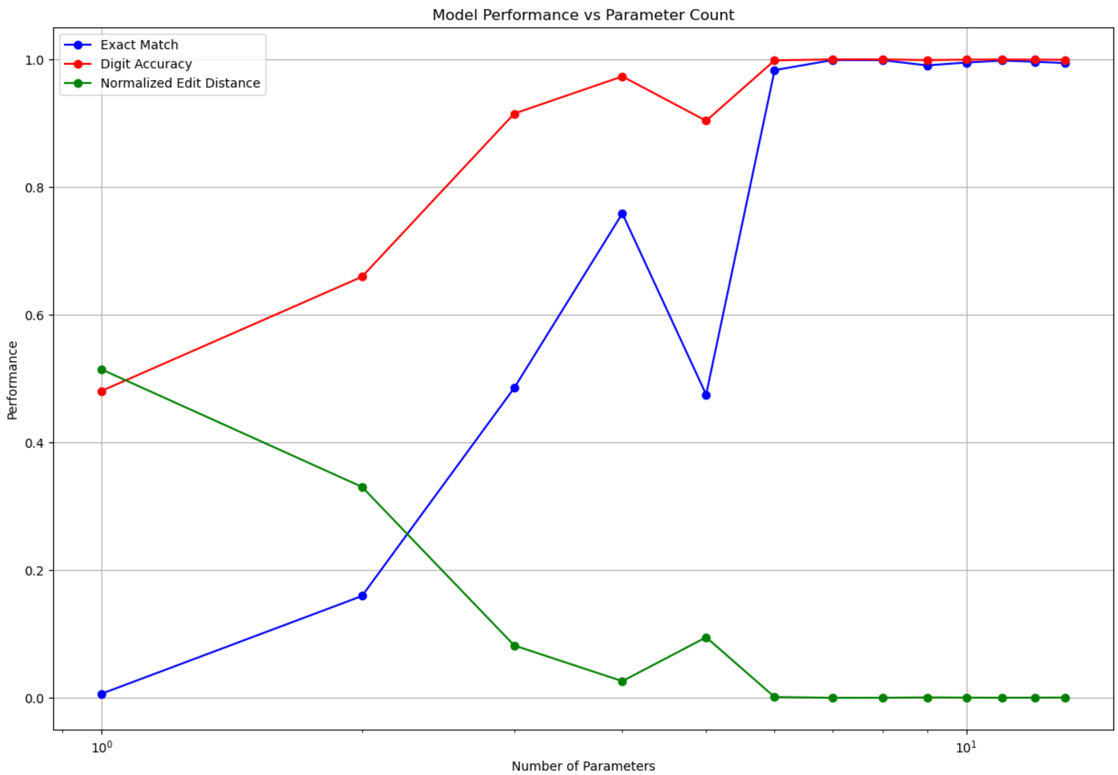
[25]:

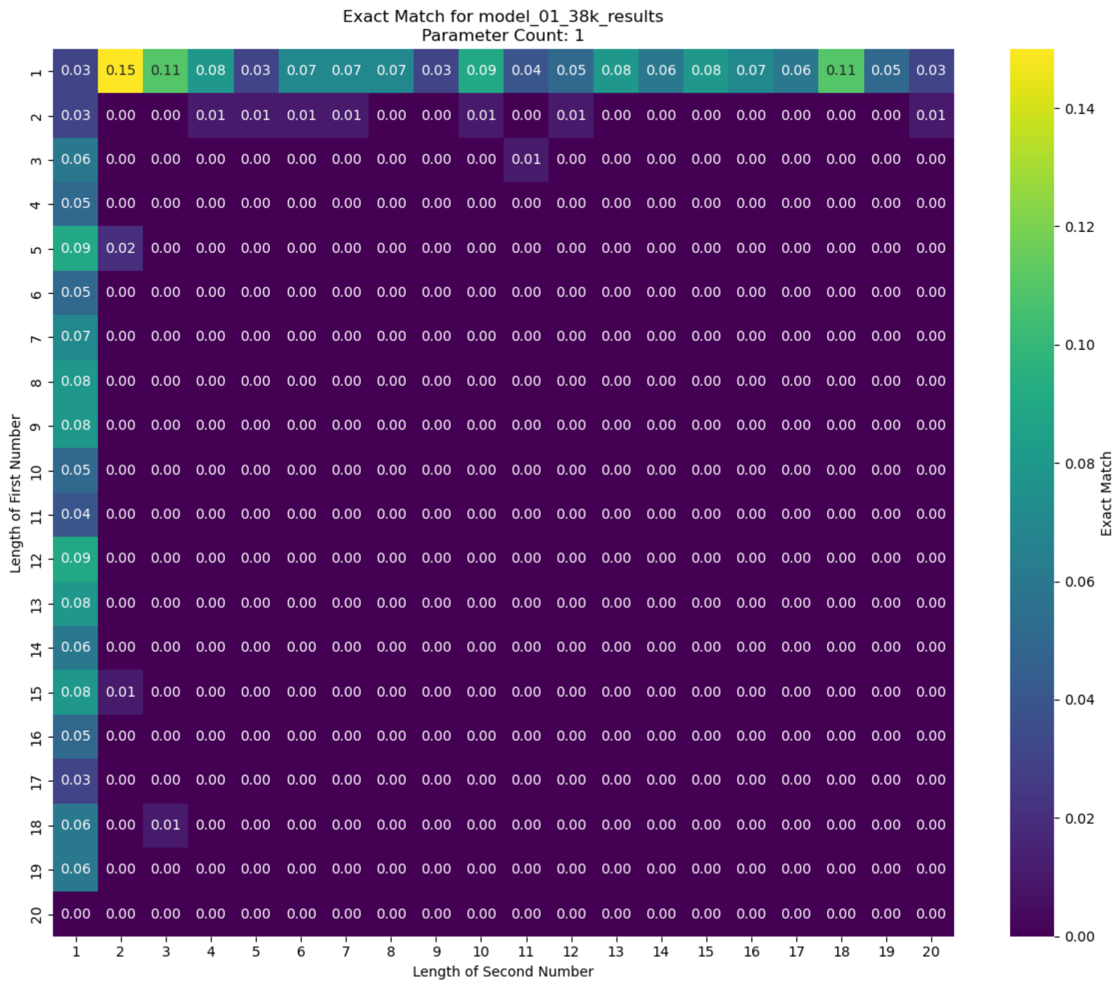
```
# Import cell
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
from pathlib import Path
import re

# Cell to load and prepare data
def extract_param_count(model_name):
    """Extract parameter count from model name and convert to numeric"""
```

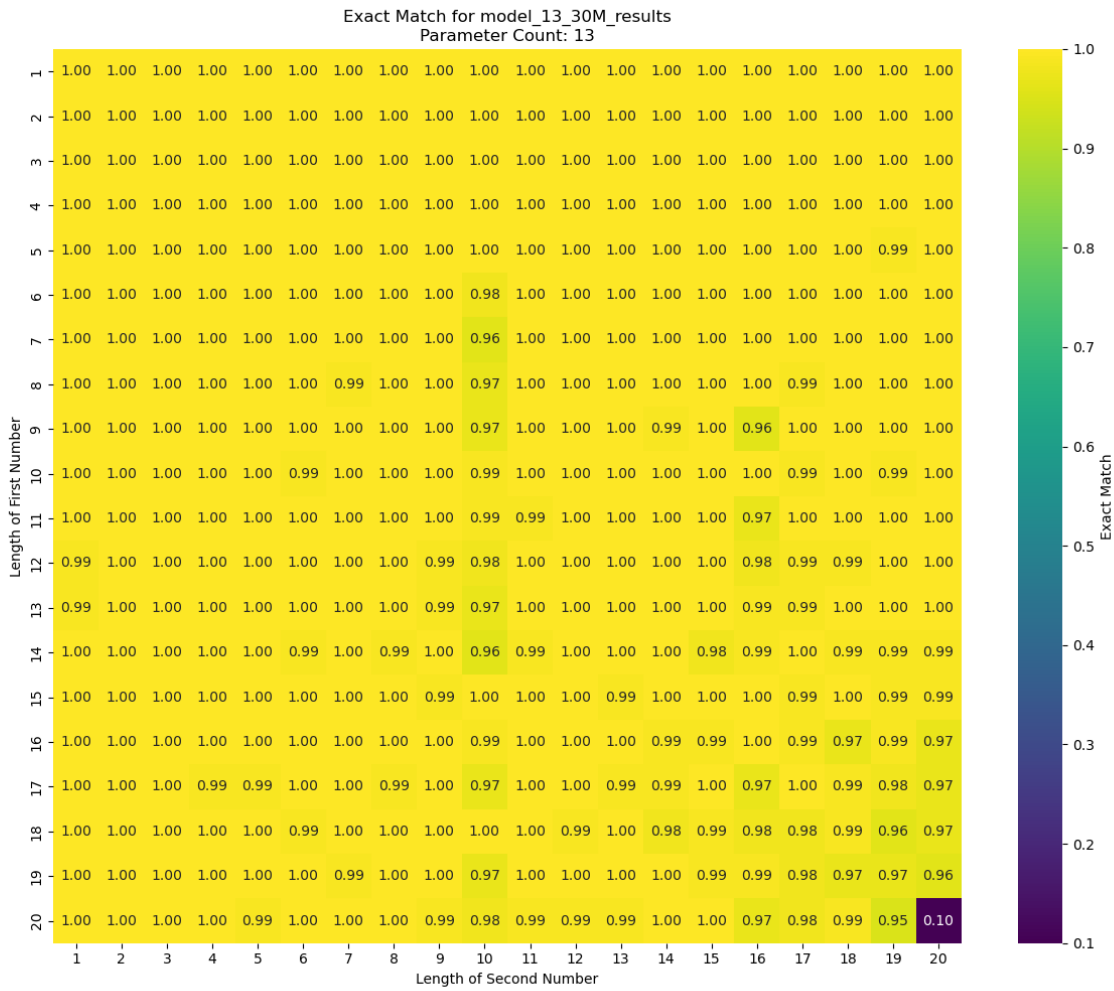
```
    param_str = model_name.split('_')[1]
    if 'k' in param_str:
        return float(param_str.replace('k', '')) * 1000
    elif 'M' in param_str:
        return float(param_str.replace('M', '')) * 1_000_000
    return float(param_str)

# Load all results
results_dir = Path('Emergence_Testing/Results')
model_results = {}

# Load individual model results
for csv_file in results_dir.glob('model_*.csv'):
    model_name = csv_file.stem
    df = pd.read_csv(csv_file)
    df['param_count'] = extract_param_count(model_name)
    model_results[model_name] = df

# Load aggregate results if it exists
aggregate_path = results_dir / 'aggregate_results.csv'
if aggregate_path.exists():
    aggregate_df = pd.read_csv(aggregate_path)
    # Sort by parameter count
    aggregate_df['parameters'] = pd.to_numeric(aggregate_df['parameters'])
    aggregate_df = aggregate_df.sort_values('parameters')
```

[27]:
```
# Cell for emergence curve plots
plt.figure(figsize=(15, 10))

# Plot different metrics from aggregate results
metrics = ['avg_exact_match', 'avg_digit_accuracy',␣
 ↪'avg_normalized_edit_distance']
colors = ['blue', 'red', 'green']

for metric, color in zip(metrics, colors):
    plt.semilogx(aggregate_df['parameters'],
                 aggregate_df[metric],
                 marker='o',
                 linestyle='-',
                 color=color,
                 label=metric.replace('avg_', '').replace('_', ' ').title())

plt.grid(True)
plt.xlabel('Number of Parameters')
plt.ylabel('Performance')
plt.title('Model Performance vs Parameter Count')
plt.legend()
```

```
plt.show()

# Cell for heatmap visualization
def create_length_heatmap(model_name, metric='exact_match'):
    """Create heatmap showing performance across different input lengths"""
    df = model_results[model_name]
    pivot_data = df.pivot(index='length1', columns='length2', values=metric)

    plt.figure(figsize=(12, 10))
    sns.heatmap(pivot_data, cmap='viridis', annot=True, fmt='.2f', cbar_kws={'label': metric.replace('_', ' ').title()})
    plt.title(f'{metric.replace("_", " ").title()} for {model_name}\nParameter Count: {int(df["param_count"].iloc[0]):,}')
    plt.xlabel('Length of Second Number')
    plt.ylabel('Length of First Number')
    plt.tight_layout()
    plt.show()

# Create heatmaps for largest and smallest models
model_names = sorted(model_results.keys(),
                    key=lambda x: model_results[x]['param_count'].iloc[0])
smallest_model = model_names[0]
largest_model = model_names[-1]

create_length_heatmap(smallest_model, 'exact_match')
create_length_heatmap(largest_model, 'exact_match')
```

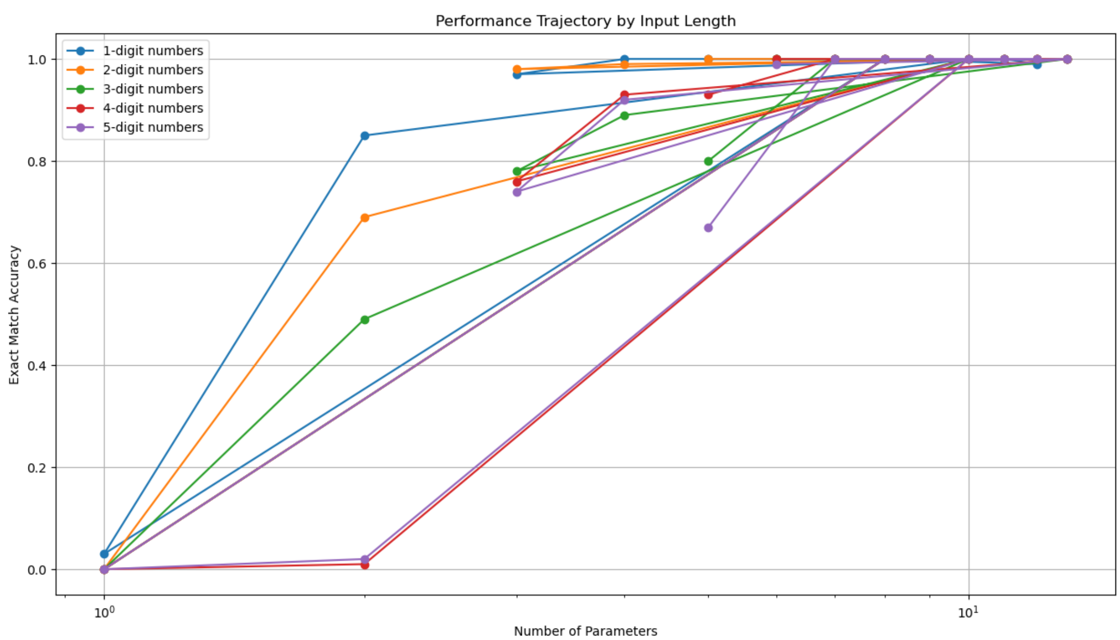

Model Performance vs Parameter Count
Exact Match
Digit Accuracy
Normalized Edit Distance
Performance
1.0
0.8
0.6
0.4
0.2
0.0
10^0
10^1
Number of Parameters


Exact Match for model_01_38k_results
Parameter Count: 1
Length of First Number
Length of Second Number
Exact Match



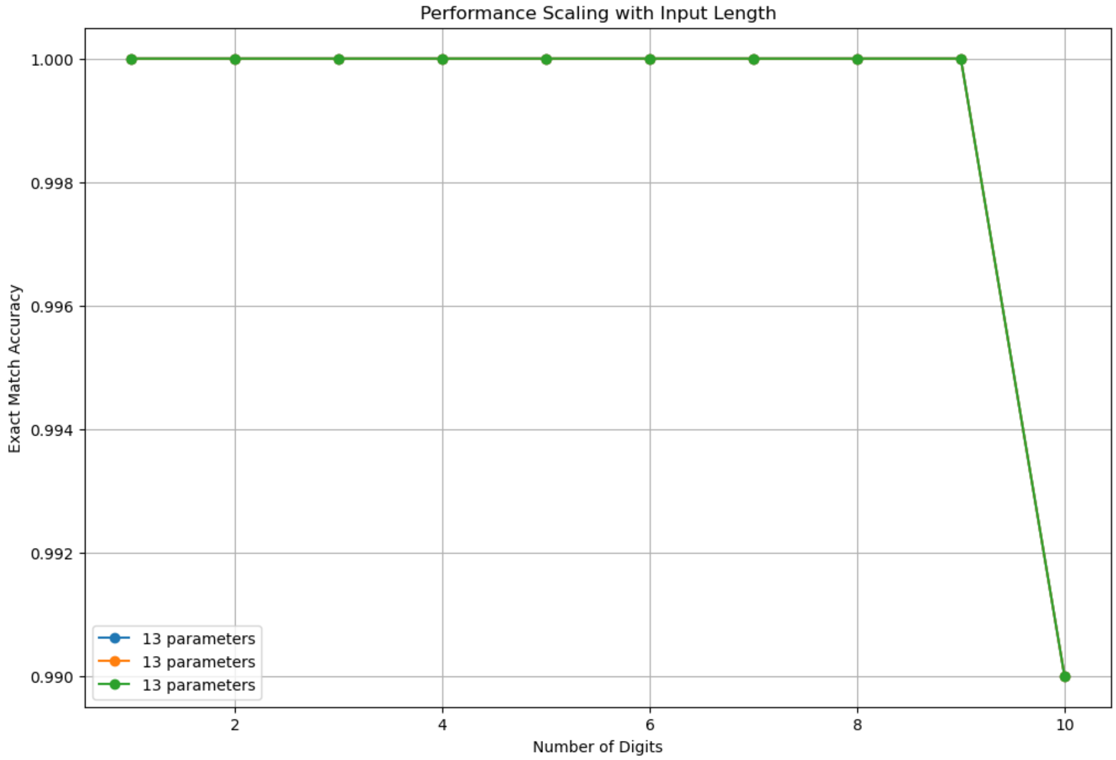

```
[28]: # Cell for performance trajectory analysis
def plot_performance_by_length(max_length=5):
    """Plot how performance changes with parameter count for different input␣
↪lengths"""
    plt.figure(figsize=(15, 8))

    for length in range(1, max_length + 1):
        performances = []
        param_counts = []

        for model_name, df in model_results.items():
            # Get results where both numbers have the same length
            length_results = df[(df['length1'] == length) & (df['length2'] ==␣
↪length)]
            if not length_results.empty:
                performances.append(length_results['exact_match'].iloc[0])
```

```
                param_counts.append(df['param_count'].iloc[0])

        plt.semilogx(param_counts, performances,
                     marker='o',
                     linestyle='-',
                     label=f'{length}-digit numbers')

    plt.grid(True)
    plt.xlabel('Number of Parameters')
    plt.ylabel('Exact Match Accuracy')
    plt.title('Performance Trajectory by Input Length')
    plt.legend()
    plt.show()

plot_performance_by_length()
```



[29]:
```
# Cell for performance drop-off analysis
def plot_length_scaling(param_counts=[100000, 1000000, 10000000]):
    """Plot how performance scales with input length for different model␣
↪sizes"""
    plt.figure(figsize=(12, 8))

    for param_count in param_counts:
        # Find model closest to target parameter count
        closest_model = min(model_results.keys(),
```

```
                              key=lambda x: abs(model_results[x]['param_count'].
↪iloc[0] - param_count))
        df = model_results[closest_model]

        # Get diagonal performance (where length1 == length2)
        diagonal_perf = []
        lengths = []
        for length in range(1, 11):  # Up to 10-digit numbers
            perf = df[(df['length1'] == length) & (df['length2'] ==␣
↪length)]['exact_match']
            if not perf.empty:
                diagonal_perf.append(perf.iloc[0])
                lengths.append(length)

        plt.plot(lengths, diagonal_perf,
                marker='o',
                label=f'{int(df["param_count"].iloc[0]):,} parameters')

    plt.grid(True)
    plt.xlabel('Number of Digits')
    plt.ylabel('Exact Match Accuracy')
    plt.title('Performance Scaling with Input Length')
    plt.legend()
    plt.show()

plot_length_scaling()
```



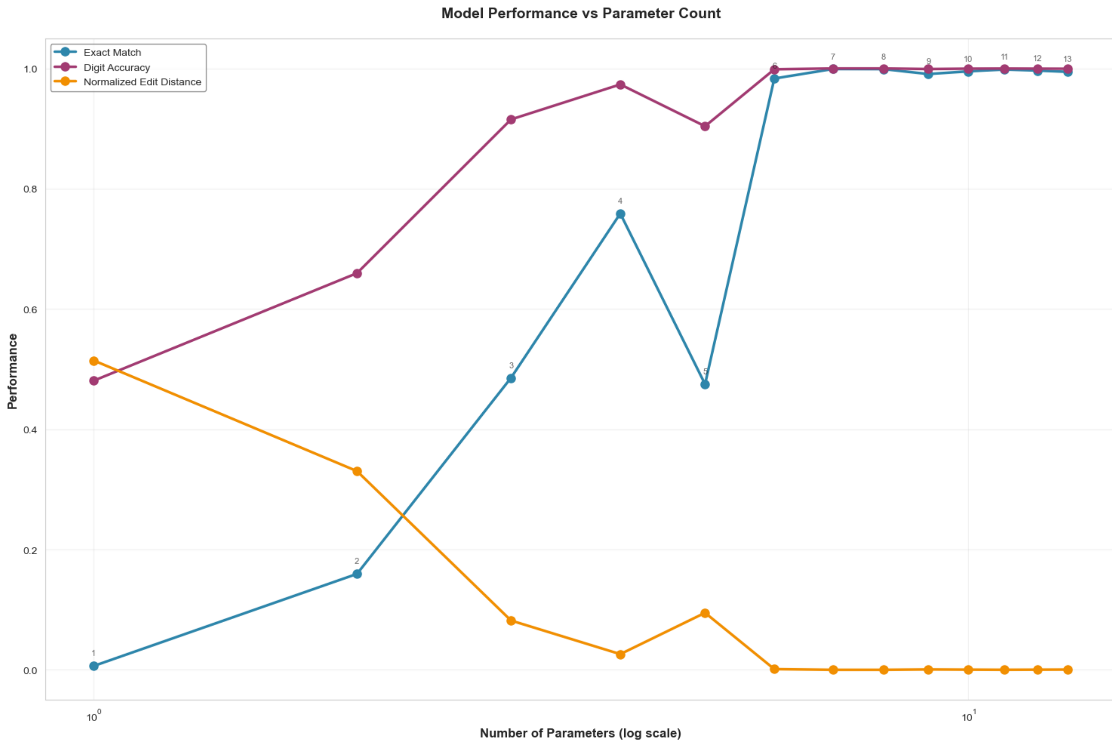
[30]:

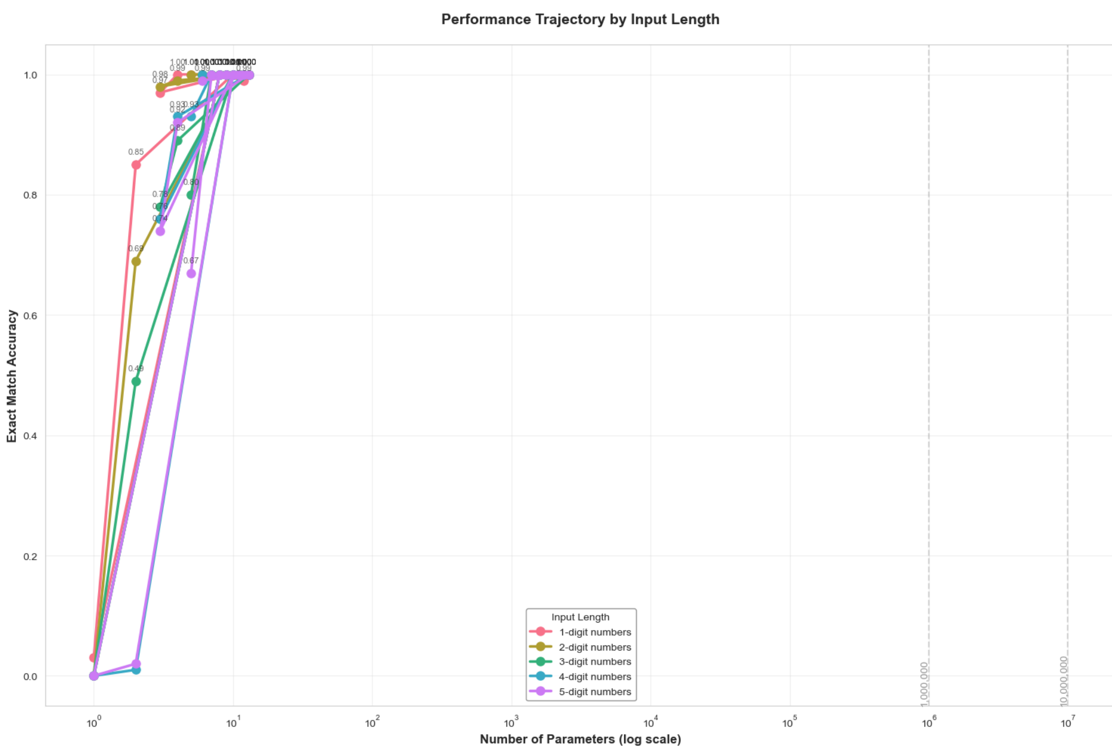
```
# Performance vs Parameter Count with improved styling
plt.figure(figsize=(15, 10))
sns.set_style("whitegrid")

# Plot different metrics with improved styling
metrics = ['avg_exact_match', 'avg_digit_accuracy',␣
↪'avg_normalized_edit_distance']
colors = ['#2E86AB', '#A23B72', '#F18F01']  # More distinctive color palette
labels = ['Exact Match', 'Digit Accuracy', 'Normalized Edit Distance']

for metric, color, label in zip(metrics, colors, labels):
    plt.semilogx(aggregate_df['parameters'],
                 aggregate_df[metric],
                 marker='o',
                 markersize=8,
                 linestyle='-',
                 linewidth=2.5,
                 color=color,
                 label=label)

plt.grid(True, alpha=0.3)
plt.xlabel('Number of Parameters (log scale)', fontsize=12, fontweight='bold')
```

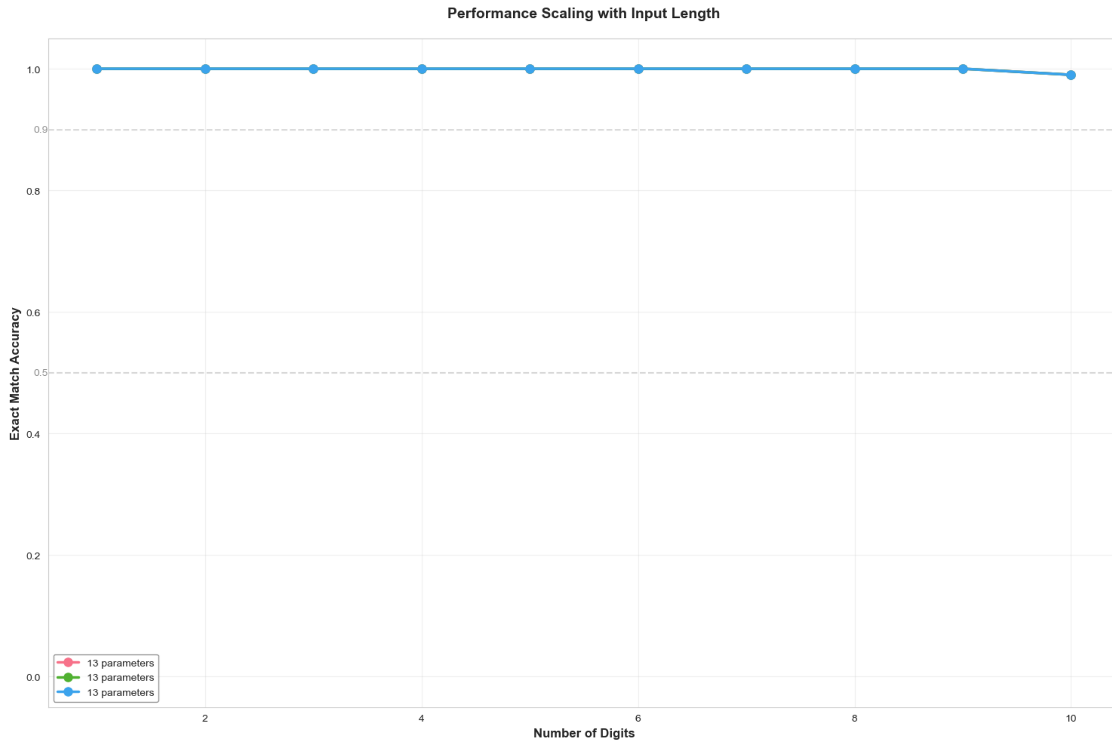
```python
plt.ylabel('Performance', fontsize=12, fontweight='bold')
plt.title('Model Performance vs Parameter Count', fontsize=14,␣
↪fontweight='bold', pad=20)
plt.legend(fontsize=10, frameon=True, facecolor='white', edgecolor='gray')
plt.tick_params(axis='both', which='major', labelsize=10)

# Add parameter count annotations
for x, y in zip(aggregate_df['parameters'], aggregate_df['avg_exact_match']):
    plt.annotate(f'{int(x):,}',
                (x, y),
                textcoords="offset points",
                xytext=(0,10),
                ha='center',
                fontsize=8,
                alpha=0.7)

plt.tight_layout()
plt.show()

# Improved Performance Trajectory Plot
plt.figure(figsize=(15, 10))
sns.set_style("whitegrid")

def plot_performance_by_length(max_length=5):
    """Plot how performance changes with parameter count for different input␣
↪lengths"""
    colors = sns.color_palette("husl", max_length)

    for length, color in zip(range(1, max_length + 1), colors):
        performances = []
        param_counts = []

        for model_name, df in model_results.items():
            # Get results where both numbers have the same length
            length_results = df[(df['length1'] == length) & (df['length2'] ==␣
↪length)]
            if not length_results.empty:
                performances.append(length_results['exact_match'].iloc[0])
                param_counts.append(df['param_count'].iloc[0])

        plt.semilogx(param_counts, performances,
                    marker='o',
                    markersize=8,
                    linestyle='-',
                    linewidth=2.5,
                    color=color,
                    label=f'{length}-digit numbers')
```

```
        # Add data point annotations
        for x, y in zip(param_counts, performances):
            if y > 0.1:  # Only annotate significant points
                plt.annotate(f'{y:.2f}',
                             (x, y),
                             textcoords="offset points",
                             xytext=(0,10),
                             ha='center',
                             fontsize=8,
                             alpha=0.7)

    plt.grid(True, alpha=0.3)
    plt.xlabel('Number of Parameters (log scale)', fontsize=12,␣
↪fontweight='bold')
    plt.ylabel('Exact Match Accuracy', fontsize=12, fontweight='bold')
    plt.title('Performance Trajectory by Input Length', fontsize=14,␣
↪fontweight='bold', pad=20)
    plt.legend(fontsize=10, frameon=True, facecolor='white', edgecolor='gray',
              title='Input Length', title_fontsize=10)
    plt.tick_params(axis='both', which='major', labelsize=10)

    # Add parameter thresholds vertical lines
    thresholds = [1e6, 1e7]  # Example thresholds
    for threshold in thresholds:
        plt.axvline(x=threshold, color='gray', linestyle='--', alpha=0.3)
        plt.text(threshold, plt.ylim()[0], f'{int(threshold):,}',
                rotation=90, va='bottom', ha='right', alpha=0.5)

    plt.ylim(-0.05, 1.05)  # Add some padding to y-axis
    plt.tight_layout()

plot_performance_by_length()
plt.show()

# Improved Length Scaling Plot
def plot_length_scaling(param_counts=[100000, 1000000, 10000000]):
    """Plot how performance scales with input length for different model␣
↪sizes"""
    plt.figure(figsize=(15, 10))
    sns.set_style("whitegrid")

    colors = sns.color_palette("husl", len(param_counts))

    for param_count, color in zip(param_counts, colors):
        closest_model = min(model_results.keys(),
```

```
                          key=lambda x: abs(model_results[x]['param_count'].
↪iloc[0] - param_count))
        df = model_results[closest_model]
        actual_params = df['param_count'].iloc[0]

        diagonal_perf = []
        lengths = []
        for length in range(1, 11):
            perf = df[(df['length1'] == length) & (df['length2'] ==␣
↪length)]['exact_match']
            if not perf.empty:
                diagonal_perf.append(perf.iloc[0])
                lengths.append(length)

        plt.plot(lengths, diagonal_perf,
                marker='o',
                markersize=8,
                linewidth=2.5,
                color=color,
                label=f'{int(actual_params):,} parameters')

    plt.grid(True, alpha=0.3)
    plt.xlabel('Number of Digits', fontsize=12, fontweight='bold')
    plt.ylabel('Exact Match Accuracy', fontsize=12, fontweight='bold')
    plt.title('Performance Scaling with Input Length', fontsize=14,␣
↪fontweight='bold', pad=20)
    plt.legend(fontsize=10, frameon=True, facecolor='white', edgecolor='gray')
    plt.tick_params(axis='both', which='major', labelsize=10)

    # Add horizontal threshold lines
    thresholds = [0.5, 0.9]
    for threshold in thresholds:
        plt.axhline(y=threshold, color='gray', linestyle='--', alpha=0.3)
        plt.text(plt.xlim()[0], threshold, f'{threshold:.1f}',
                va='center', ha='right', alpha=0.5)

    plt.ylim(-0.05, 1.05)
    plt.tight_layout()
    plt.show()

plot_length_scaling()
```


Model Performance vs Parameter Count
Exact Match
Digit Accuracy
Normalized Edit Distance
Performance
Number of Parameters (log scale)


Performance Trajectory by Input Length
Exact Match Accuracy
Number of Parameters (log scale)
Input Length
1-digit numbers
2-digit numbers
3-digit numbers
4-digit numbers
5-digit numbers
1,000,000
10,000,000


Performance Scaling with Input Length
Exact Match Accuracy
Number of Digits
1.0
0.9
0.8
0.6
0.5
0.4
0.2
0.0
2
4
6
8
10
13 parameters
13 parameters
13 parameters

```
[ ]:
```