Custom Math Notes00 ¡00

Sequence and Series

Chapter 11
Taylor's Series
MATH182A

Patrick Ehrenreich

December 6, 2016

Taylor Series

We know the following:

$$f(x) = \sum_{n=0}^{\infty} C_n (x-a)^n$$

$$|x-a| < R$$

$$C_0 + C_1(x-a) + C_2(x-a)^2 + \dots$$

But, we don't know much about the coefficients (C_0, C_1, etc.) To calculate these coefficients, use the following:

$$C_n = \frac{\frac{d^n y}{dx^n}(a)}{n!}$$

$$\text{For example, } C_1 = \frac{f'(a)}{1!}, C_2 = \frac{f''(a)}{2!}, \dots$$

This is a Taylor series, where a need not be centered at any specific value. In the case where we have $a = 0$, the series becomes a Maclauren series.

Example

Find Maclauren expansion for e^x.

$$f(x) = e^x, e^x = \sum_{n=0}^{\infty} \frac{f^n(a)}{n!}(x^n)$$

$$\text{Maclauren Expansion: } e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

Example

Find Maclauren expansion for $sin(x)$.

$$\text{Formulaic expansion: } f(0)(x^0) + \frac{f'(0)}{1!}(x^1) + \frac{f''(0)}{2!}(x^2) + \dots$$

$$\text{Substitute in... } 0 + \frac{x}{1!} + 0 - \frac{x^3}{3!} - 0 + \frac{x^5}{5!} + \dots$$

Binomial Series

$$\text{Suppose } |x| < 1 \text{ and } (1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n}.$$

$$\sum_{n=0}^{\infty} \binom{k}{n} = 1 + \frac{k(x)}{1!} + \frac{k(k-1)(x^2)}{2!} + \frac{k(k-1)(k-2)(x^3)}{3!} + \dots$$

$$\text{For the } nth \text{ term... } \sum_{n=0}^{\infty} \binom{k}{n} = \frac{k(k-1)\dots(k-(n+1))(x^n)}{n!}$$

Example

$$f(x) = x^4 - 3x^2 + 1, a = 1$$

n	f^n	C_n
0	$f(1)$	-1
1	$f'(1)$	-2
2	$f''(1)$	6
3	$f'''(1)$	24
4	$f''''(1)$	24
5	$f'''''(1)$	0

Based off this table, the Taylor expansion at $a = 1$ is:

$$f(x) = -1 + (-2)(x-a) + 6(x-a)^2 + 24(x-a)^3 + 24(x-a)^4 + 0 + 0 + \dots$$