Sequence and Series

Chapter 11
Review
MATH182A

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Example

$$\lim_{x\to 0}\frac{1-\cos x}{1+x-e^x}$$

$$\lim_{x\to 0}\frac{1-[1-\frac{x^2}{2!}+\frac{x^4}{4!}-\ldots]}{1+x-[1+x+\frac{x^2}{2!}+\ldots]}$$

$$\lim_{x\to 0}[\frac{\frac{x^2}{2}}{\frac{-x^2}{2}}][\frac{1-\frac{x^2}{12}-\ldots}{1+\frac{x}{3}+\ldots}]$$

$$\frac{\frac{1}{2}}{\frac{-1}{2}}=-1$$

Example

$$\lim_{x\to 0}\frac{\sin x-x-x^{\frac{3}{6}}}{x^3}$$

$$\lim_{x\to 0}\frac{[x-\frac{x^3}{3!}+\frac{x^5}{5!}-\ldots]-x-x^{\frac{3}{6}}}{x^3}$$

$$\lim_{x\to 0}\frac{-1}{3}+\frac{x^2}{3!}+\ldots$$

$$\frac{-1}{3}$$

Example

$$y=e^x ln(1-x)$$

$$(1+x+\frac{x^2}{2!}+\ldots)(-x-\frac{x^2}{2}-\ldots)$$

Example

$$\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{6^{2n}(2n)!}$$

$$\sum_{n=1}^{\infty} (-1)^n \frac{(\frac{\pi}{6})^{2n}}{(2n)!} = 1 - \frac{(\frac{\pi}{6})^2}{2!} + \frac{(\frac{\pi}{6})^4}{4!} = \cos(\frac{\pi}{6}) = \frac{\sqrt{3}}{2}$$

Example

$$\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n-1}}{4^{2n+1}(2n+1)!}$$

This is the sin expansion written with n.

Example

$$1 - ln2 + \frac{(ln2)^2}{2!} - \frac{(ln2)^3}{3!} + \dots$$

$$e^{-ln2} = e^{ln2^{-1}} = 2^{-1} = \frac{1}{2}$$

Example

$$\sum_{n=1}^{\infty} \frac{n^2 2^{n-1}}{(-5)^n}$$

Apply root test.

$$a_n = \frac{n^2 2^{n-1}}{(-5)^n}, a_{n+1} = \frac{(n+1)^2 2^n}{(-5)^{n+1}}$$

$$\frac{a_{n+1}}{a_n} = \frac{\frac{(n+1)^2 2^n}{(-5)^{n+1}}}{\frac{n^2 2^{n-1}}{(-5)^n}}$$

$$\frac{a_{n+1}}{a_n} = \frac{(n+1)^2 2^n 5^n}{5^{n+1} n^2 2^{n-1}} = \frac{2}{5}(\frac{n+1}{n})^2$$

$$\frac{a_{n+1}}{a_n} = \frac{2}{5}(1 + \frac{1}{n})^2 = \frac{2}{5} = L$$

$L < 1,$ Series is absolutely convergent.

Example

$$\sum_{n=2}^{\infty} \frac{1}{n\sqrt{ln(n)}}$$

Apply intergral test.

$$\int_{2}^{b} \frac{1}{n\sqrt{ln(n)}}$$

$$\int_{2}^{b} \frac{dt}{\sqrt{t}},\ t = ln(n)$$

$$\lim_{b \to \infty} 2[\sqrt{ln(b)} - \sqrt{ln(2)}]$$

$$\lim_{b \to \infty} = \infty$$