Sequence and Series

Chapter 11
Section 3
MATH182A

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Alternating Series

If you have a series in the general style of

$$\sum_{n=1}^{\infty} b_n(-1^{n-1})$$

with the following condition:

$$b_{n+1} \leq b_n$$

for all n, then b_n converges.

Example 1

$$\sum_{n=1}^{\infty} \frac{-1^{n-1}}{2n+1}$$

$$b_n = \frac{1}{2n+1}$$

$$\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{1}{2n+1} = 0$$

Therefore, $\sum_{n=1}^{\infty} \frac{-1^{n-1}}{2n+1}$ converges.

Example 2

$$\sum \frac{-1^{n-1}}{ln(n+4)}$$

$$b_n = \frac{1}{ln(n+4)}$$

Clearly, $b_{n+1} \leq b_n$ for all n.

$$\lim_{n\to\infty} \frac{1}{ln(n+4)} = 0 \text{ and } \sum b_n \text{ converges.}$$

Example 5

$$\sum_{n=1}^{\infty} -1^{n-1}(\frac{3n-1}{2n+1})$$

$$\lim_{n\to\infty} \frac{3n-1}{2n+1} = \frac{3}{2}$$

Since the limit does not equal zero, the convergence of

$$\sum_{n=1}^{\infty} -1^{n-1}(\frac{3n-1}{2n+1})$$

is not definitive based on this test.

Example 11

$$\sum -1^{n-1}\frac{n^2}{n^3+4}$$

$$b_n = \frac{n^2}{n^3+4}$$

$$b_n = \frac{1}{1+\frac{4}{n^2}}$$

$$\lim_{n\to\infty} b_n = 0$$

Therefore, $\sum -1^{n-1}\frac{n^2}{n^3+4}$ converges.

Limit Convergence Test

$$\lim_{n\to\infty} \frac{a_n}{b_n} = 0$$
$$\infty \leq c \leq 0$$

If these conditions are true, both a_n and b_n either converge or diverge.

Absolutely Convergent Series of the Ratio and the Root Tests

Definition: $\sum a_n$ is absolutely convergent if $\sum |a_n|$ converges.

Remark: if $\sum |a_n|$ diverges but $\sum a_n$ still converges, $\sum a_n$ is conditionally convergent.

Remark: if $\sum |a_n|$ converges, $\sum a_n$ must converge.

$$\text{Let } \sum a_n \text{ be a given series.}$$
$$\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = L$$
$$\text{If } L < 1, \text{ then } \sum a_n \text{ converges absolutely.}$$
$$\text{If } L > 1, \text{ then } \sum a_n \text{ diverges.}$$
$$\text{If } L = 1, \text{ then the test fails and another must be applied.}$$

Example

$$\sum \frac{n^2}{2^n}, a_n = \frac{n^2}{2^n}, a_{n+1} = \frac{(n+1)^2}{2^{n+1}}$$
$$\frac{|a_{n+1}|}{|a_n|} = (\frac{2^n}{2^{n+1}})(\frac{n+1}{n})^2$$
$$\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = (\frac{1}{2})(1+\frac{1}{n})^2 = \frac{1}{2} = L$$
$$\frac{1}{2} < 1, \text{ therefore } \sum a_n \text{ converges.}$$