Calculus II In Review

Formulas and Important Information by Topic

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Substitution Rule for Integration

Proceudre for Substitution in Integration:

1. Choose a portion of the integral to substitute with a new variable, such that, when substituted, the original variable will be eliminated from the equation.
2. Set this portion equal to the variable and take the derivative of both sides.
3. Substitute back into the original integral.
4. Integrate.
5. Change the substituted variable back to the portion of the formula which was originally subbed out.

Partial Fractions

Begin with an integration problem such as this:

$$\frac{x-1}{x^2+3x+2}$$

Break apart the bottom:

$$\frac{x-1}{(x+1)(x+2)}$$

Split the fraction:

$$\frac{x-1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$$

Multiply out all demoninators:

$$x - 1 = A(x+2) + B(x+1)$$

Choose a value for x which will eliminate one variable, and solve for the other (in this case, I use $x = -1$):

$$-1 - 1 = A(-1 + 2) + B(-1 + 1)$$

$$-2 = A(1) + 0$$
$$-2 = A$$

Substitute back in with a different x value for the second variable:

$$-2 - 1 = A(-2 + 2) + B(-2 + 1)$$

$$-3 = 0 + B(-1)$$
$$3 = B$$

With A and B found, integrate as follows:

$$\int \frac{x - 1}{(x + 1)(x + 2)} dx = \int \frac{A}{x + 1} dx + \int \frac{B}{x + 2}$$
$$= A(ln(x + 1)) + B(ln(x + 2))$$
$$= -2(ln(x + 1)) + 3(ln(x + 2))$$

Integration by Parts

Begin with an integration problem such as this, which can be theoretically split into two functions:

$$\int f'(x)g(x)dx$$

Whichever function is easier to integrated should be taken as integrable, and the other as differentiable, displayed above as $f'(x)$ (integrable) and $g(x)$ (differentiable). Once this information is found, substitute into the following formula:

$$\int f'(x)g(x)dx = f(x)g(x) - \int f(x)g'(x)dx$$

Area Under a Curve

To find the are under a curve, set up an integral as such, where $f(x)$ is the curve in question and (a, b) are the limits between which you are interested in the area.

$$A = \int_a^b f(x)dx$$

Volume by Integration, Approach 1 (Discs or Washers)

Finding the volume between two curves, $f(x)$ and $g(x)$, plays off the formula $A = \pi r^2$, again using some (a, b) as bounds of integration.

$$V = \pi \int_a^b (f(x))^2 - (g(x))^2 dx$$

Volume by Integration, Approach 2 (Cylindrical Shells)

This formula uses only one curve, $f(x)$, and bounds (a, b), with the following general formula:

$$V = 2\pi \int_a^b x(f(x))dx$$

Arc Length

The arc length formula uses the derivative of a given curve, $f(x)$, and bounds (a, b), with the following general formula:

$$\int_a^b \sqrt{1 + (\frac{dy}{dx})^2}\ dx$$

Polar Coordinates

Polar coordinates are given as follows:

$$(r, \theta)$$

This can be converted as such:

$$x = r\cos(\theta)$$

$$y = r\sin(\theta)$$

Arc Length in Polar Form

The arc length formula changes significantly using polar coordinates, but can be logically found using the base arc length formula:

$$\int_a^b \sqrt{r^2 + (\frac{dr}{d\theta})^2}\ dx$$