Sequence and Series

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Example 17

$$a_n = 1 - (0.2)^n$$

$$\begin{aligned}
\lim_{n\to\infty} a_n &= \lim_{n\to\infty} 1 - (0.2)^n \\
&= 1 - \lim_{n\to\infty} (0.2)^n \\
&= 1
\end{aligned}$$

Example 18

$$a_n = \frac{n^3}{n^3 + 1}$$

$$\begin{aligned}
\lim_{n\to\infty} a_n &= \lim_{n\to\infty} \frac{n^3}{n^3 + 1} \\
&= \lim_{n\to\infty} \frac{n^3}{(n^3)(1 + \frac{1}{n^3})} \\
&= \lim_{n\to\infty} \frac{1}{1 + \frac{1}{n^3}} \\
&= 1
\end{aligned}$$

Example 20

$$a_n = e^{\frac{1}{n}}$$

$$\begin{aligned}
\lim_{n\to\infty} a_n &= \lim_{n\to\infty} e^{\frac{1}{n}} \\
&= \lim_{n\to\infty} \frac{1}{n} = 0 \\
&= \lim_{n\to\infty} e^0 \\
&= 1
\end{aligned}$$

Example 22

$$a_n = \tan \frac{2n\pi}{1+8n}$$

Consider the insides of the tangent function:

$$\begin{aligned}
\lim_{n\to\infty} a_n &= \lim_{n\to\infty} \frac{2n\pi}{1+8n} \\
&= \lim_{n\to\infty} \frac{(n)2\pi}{(n)\frac{1}{n}+8}
\end{aligned}$$

As n appraoches infinity, $\frac{1}{n}$ approaches zero.

$$\begin{aligned}
&= \lim_{n\to\infty} \frac{2\pi}{8} \\
&= \lim_{n\to\infty} \frac{\pi}{4}
\end{aligned}$$

Insert back into original limit...

$$\begin{aligned}
&= \lim_{n\to\infty} \tan \frac{\pi}{4} \\
&= 1
\end{aligned}$$

Example 25

$$a_n = \frac{(-1)^n}{n^2+1}$$

$$\begin{aligned}
|a_n| &= \frac{n}{n^2+1} \\
&= \frac{(n)}{(n^2)(\frac{1}{n^2}+1)} \\
&= \frac{\frac{1}{n}}{\frac{1}{n^2}+1}
\end{aligned}$$

As n approaches infinity, $\frac{1}{n}$ and $\frac{1}{n^2}$ approach 0.

Therefore, since $\lim_{n\to\infty} |a_n|$ is equal to $\lim_{n\to\infty} a_n$, $\lim_{n\to\infty} a_n = 0$.

Example 29

$$a_n = \frac{(2n-1)!}{(2n+1)!}$$

$$\begin{aligned}
\lim_{n\to\infty} a_n &= \lim_{n\to\infty} \frac{(2n-1)!}{(2n+1)(2n+1-1)!} \\
&= \lim_{n\to\infty} \frac{(2n-1)!}{(2n+1)(2n)(2n-1)!} \\
&= \lim_{n\to\infty} \frac{1}{(2n+1)(2n)} \\
&= 0
\end{aligned}$$

Example 31

$$a_n = \frac{e^n + e^(-n)}{e^(2n) - 1}$$

$$\begin{aligned}
\lim_{n\to\infty} a_n &= \lim_{n\to\infty} \frac{e^n + e^(-n)}{e^n(e^n - e^(-n))} \\
&= \lim_{n\to\infty} \frac{e^n(e^(2n) + 1)}{e^n(e^n - e^(-n))} \\
&= 0
\end{aligned}$$

General Theorem

$$a_n = \lim_{n\to\infty} (1 + \frac{2}{n})^n$$

$$\begin{aligned}
y = y(x) &= (1 + \frac{2}{x})^x \\
lny &= xln(1 + \frac{2}{x}) \\
\lim_{x\to\infty} y &= e^{\lim_{x\to\infty} y} \\
&= e^2
\end{aligned}$$

In general, the theorem for this set of circumstances is:

$$\lim_{n\to\infty} (1 + \frac{1}{x})^x = e^x$$

If you find anything wrong in these notes, please email me at pxe1833@g.rit.edu with the subject line "Notes Error".