Sequence and Series

Chapter 11
Section 2
MATH182A

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Example 23

$$\sum_{n=2}^{\infty} \frac{n^2}{n^2-1}$$

$$\text{if } a_n \to 0 \text{ as } n \to \infty, \text{ series is finite.}$$

Else, the series goes to infinity (diverges).

$$\text{Therefore, since } a_n \to 1 \text{ as } n \to \infty, \sum a_n \to \ \infty, \text{ and } a_n \text{ diverges.}$$

Example 25

$$\sum \frac{1+2^n}{3^n} = \sum \frac{1}{3^n} + \sum \frac{2^n}{3^n}$$

$$= \sum (\frac{1}{3})^n + \sum (\frac{2}{3})^n$$

$$\sum (\frac{1}{3})^n = \frac{a}{1-r} \text{ since } r = \frac{a_n+1}{a_n} = \frac{1}{3}, \text{ meaning } |r| < 1. \text{Therefore } \frac{a}{1-r} = \frac{1}{2}.$$

$$\sum (\frac{2}{3})^n = \frac{a}{1-r} \text{ as well, but in this case } \frac{a}{1-r} = 2.$$

$$\text{Therefore, } \sum \frac{1+2^n}{3^n} = \frac{1}{2} + 2 = \frac{5}{2}.$$

Example 29

$$\sum_{n=1}^{\infty} ln(\frac{n^2+1}{2n^2+1})$$

$$a_n = ln(\frac{n^2+1}{2n^2+1})$$

$$\frac{n^2+1}{2n^2+1} = \frac{n^2(1+\frac{1}{n^2})}{n^2(2+\frac{1}{n^2})} \; = \frac{1+\frac{1}{n^2}}{2+\frac{2}{n^2}}$$

$$\frac{1}{n^2} \& \frac{2}{n^2} \to 0 \text{ as } n \to \infty, \text{ so } \; a_n = ln(\frac{n^2+1}{2n^2+1}) \to ln(\frac{1}{2}) \text{ as } n \to \infty.$$

Therefore, a_n diverges.

Example 33

$$\sum_{n=1}^{\infty}[\frac{1}{e^n} + \frac{1}{n(n+1)}] = \sum_{n=1}^{\infty}\frac{1}{e^n} \; + \sum_{n=1}^{\infty}\frac{1}{n(n+1)}$$

$$\sum_{n=1}^{\infty}\frac{1}{e^n} = \frac{1}{e} + \frac{1}{e^2} + \dots$$

$$\sum_{n=1}^{\infty}\frac{1}{e^n} = \frac{a}{1-r} = \frac{\frac{1}{e}}{1-\frac{1}{e}} \; = \frac{\frac{1}{e}}{\frac{(e-1)}{e}} = \frac{1}{e-1}$$

$$\sum_{n=1}^{\infty}\frac{1}{n(n+1)} = [1-\frac{1}{2}] + [\frac{1}{2} - \frac{1}{3}] + \dots$$

$\sum_{n=1}^{\infty}\frac{1}{n(n+1)} = 1$ because each term consistently cancels out the last term added...

Therefore, $\sum_{n=1}^{\infty}[\frac{1}{e^n} + \frac{1}{n(n+1)}] = 1 + \frac{1}{e-1}$

Example 35

$$\sum_{n=2}^{\infty} \frac{2}{n^2-1} = \frac{2}{(n+1)(n-1)} = \frac{1}{n-1} - \frac{1}{n+1}$$

$$\sum_{n=2}^{\infty} a_n = (1-\frac{1}{3}) + (\frac{1}{2}-\frac{1}{4}) + (\frac{1}{3}-\frac{1}{5}) + \dots$$

$$\sum_{n=2}^{\infty} a_n = 1 + \frac{1}{2} - \frac{1}{n-1} - \frac{1}{n+1}$$

$$\text{Take limit: } \lim_{n\to\infty} 1 + \frac{1}{2} - \frac{1}{n-1} - \frac{1}{n+1} = \frac{3}{2}$$

because the last two terms go to zero as n goes to infinity.

$$\text{Therefore, } \sum_{n=2}^{\infty} \frac{2}{n^2-1} = \frac{3}{2}$$