Compound Interest

- Every type of business is based on investments (on money). Someone, to increase, or developed the business needs money, needs financial support. To obtain this support they use **loans** (money borrow from the bank with interest).
- The interest may be annually, monthly, weekly or daily.
- In finance, a **loan** is the lending of money by one or more individuals, organizations, or other entities to other individuals, organizations etc. The recipient (i.e., the borrower) incurs a debt, and is usually liable to pay interest on that debt until it is repaid, and to repay the principal amount borrowed.
- If you need money, someone gives you money under conditions and interest options. Options are **time** of repay and **percentages of interests**.
- The interest may be **simple interest** or **compound interest or continuous compound interest**
- Simple interest is when the amount of interest is the same for all time.
- **Compound interest** or “interest over interest” is when the interest of the first period is added on principal and the interest of the second period (being the same as percentage) is computed over the new principal (the first principal plus interest of the first period). This process continuous until the last period.
- Period of time may be daily, weekly, monthly, quarterly or annually. In general, the period is annually

Compound Interest

A principal of \$ 9000 is invested at 8% simple interest annually for 5 years. Find the value of the interest after the period of time. $9000\times0.08 = \$720$ which is the interest of the first period (one year). For 5 years it is $720\times5 = \$3600$

A principal of \$ 9000 is invested at 8% interest compounded annually for 5 years. Find the value of the interest after the period.

Solution. After one year the interest is $9000\times0.08 = \$720$. Now the new principal is $9000 + 720 = \$ 9720$.

After the second year the interest is $9720\times0.08 = 777.6$ Now the new principal is $9720 + 777.6 = \$10497.6$

After the third year the interest is $10497.6\times0.08 = 839.8$ Now the new principal is $10497.6 + 839.8 = \$11337.4$

After the fourth year the interest is $11337.4\times0.08 = 906.9$ Now the new principal is $11337.4 + 906.9 = \$12244.3$

After the fifth year(last period) the interest is $12244.3\times0.08 = 979.5$ Now the new principal is $12244.3 + 979.5 = \$13223.8$

Let's denote the principal as P. We know that scale factor associated with an increase of 8% is $1 + \frac{8}{100} = 1.08$. So, at the end of one year the total amount invested is $P\times1.08 = 9000\times1.08 = 9720$. after two years we get

$(P \times 1.08) \times 1.08 = P \times(1.08)^2 = 9000 \times 1.1664 = 10497.6$. After three years we get

$P \times(1.08)^2 \times 1.08 = P \times(1.08)^3 = 9000 \times 1.259 = 11337.408$. we can continue at the same way and finally we get

$P \times(1.08)^4 \times 1.08 = P \times(1.08)^5 = 9000 \times 1.46932 = 13223.8$

Compound Interest

- In general, if the interest rate is r% compound annually, then the scale factor is $1 + \frac{r}{100}$. If the original sum of
- money invested is P (principal) and the period is "n" years then the total sum of investment
- (principal + compound interest) is given by the formula S = P $(1 + \frac{r}{100})^n$ this formula involves four variables. If you
- know three of them you can find the fourth. The variables are S; P; r; n
- Exercise. Use this formula to find values in (find the total sum)
- a) Ten years time of $ 1000 invested at 8% compound annually
- b) Seven years time of $ 2000 invested at 6% compound annually
- c) Three years time of $ 15000 invested at 4.5% compound annually
- Use this formula to find
- a) After how many years a principal of $25000 invested at 12% compound interest annually
- will exceed $ 250000 (find the time "n")
- b) After how many years a principal of $12000 invested at 7% compound interest annually
- will exceed $ 24000 (find the time "n")

Compound Interest

- Use this formula to find
- a) How must be the principal to give us total sum \$14000 after three years invested at 5% compound interest
- The formula is P = S $(1 + \frac{r}{100})^{-n}$
- b) How must be the principal to give us total sum \$18000 after five years invested at 4% compound interest
- use this formula to find
- a) How must be the rate of compound interest to give us total sum \$10000 after four years investing \$7500
- b) How must be the rate of compound interest to give us total sum \$12000 after six years investing \$8500
- Solutions S = P $(1 + \frac{r}{100})^{n}$ S = 1000 $(1 + \frac{8}{100})^{10}$= 1000($1.08^{)10}$ = 1000· 2.1589 = 2158.9
- 24000 = 12000 $(1 + \frac{7}{100})^{n}$ 2 = $(1.07)^{n}$ log 2 = nlog (1.07) 0.301 = n 0.029 n = $\frac{0.301}{0.029}$ = 10.37

Compound Interest

- We observe that compound interest rises as the time increased. In a same period increasing the frequency
- of compounding and interest increased. But as the frequency increased, also the interest increased, and it approaches
- a fixed value.
- Example. A principal of $10 is invested for one year. Determine the future value if the interest 12% is compounded
- a) annually b) semi-annually c) quarterly d) monthly e) weekly
- using formula S = P $(1 + \frac{r}{100})^n$ we have a) n = 1 S = $\$10(1.12)^1$ = $11.20
- b) n = 2 r = $\frac{12}{2}$ = 6% S = $\$10(1.06)^2$ = 11.236 c) n = 4 r = $\frac{12}{4}$ = 3% S = $\$10(1.03)^4$ = 11.255
- d) n = 12 r = $\frac{12}{12}$ = 1% S = $\$10(1.01)^{12}$ = 11.2682 e) n = 52 r = $\frac{12}{52}$ = 0.23% S = $\$10(1.0023)^{52}$ = 11.2689
- from this example we se that increasing the frequency (period divide by a number) the total sum approach to a
- fix value.
- The type of compounding in which the interest is added on increasing frequency is called **continuous compound**
- There is a formula by which we can find the future value, S, of a principal, P, compounded continuously for “t” years
- at an annual rate of “r%” S = P$e^{\frac{rt}{100}}$ where “e” is the number e = 2.718281828… applying this formula find for us
- exercise. The principal P = $10, r = 12%, t = 1. S = $\$10e^{\frac{12\times1}{100}}$ = $\$10e^{0.12}$ = 11.2749. Compound S = 10×1.12 = 11.20

Compound Interest Exercises

- 1. Find the future value of $20000 in two years' time if compounded quarterly at 8% interest.
- 2. The value of an asset, currently priced at $ 100000, is expected to increase by 20% a year. A) Find its value in 10 years' time. B) After how many years will it be worth $ 1 million?
- 3. How long will it take for a sum of money to double if it is invested at 5% interest compound annually
- 4. A principal, $ 7000, is invested at 9% interest for 8 years. Determine its future value if the interest is compounded a) annually, b) semi-annually, c) monthly, d) continuously.
- 5. How long will it take for a sum of money to triple in value if invested at an annual rate of 3% compounded continuously

Compound Interest Exercises

- Find the missing values of the table below if the interest rate is a)annually compounded b) annually continuous compounded.

	S	P	r (compounded, continuous compounded)	n
1		12000	3.6%	8
2	42000		4.2%	6
3	20800	13000		5
4	21000	15000	0.6%	
5		6000	0.2%	10

Compound Interest

- Key words
- Loans
- Simple interest
- Compound interest
- Continuous compound interest
- Principal, present value
- Future value, total sum
- Formulas S = P $(1 + \frac{r}{100})^{n}$ P = S $(1 + \frac{r}{100})^{-n}$ S = P$e^{\frac{rt}{100}}$

  Note
- For every key word ask the question "what is" and try to answer it