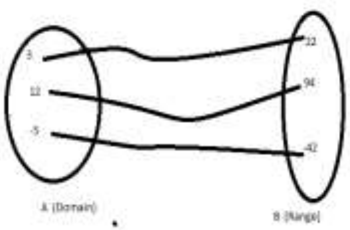
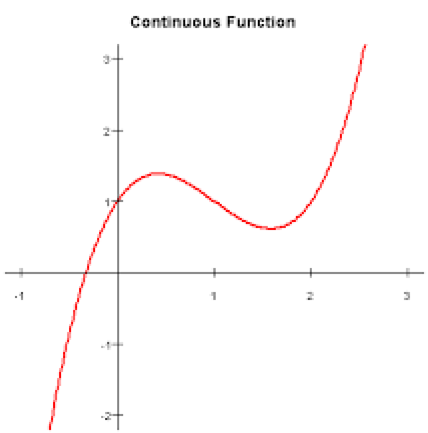
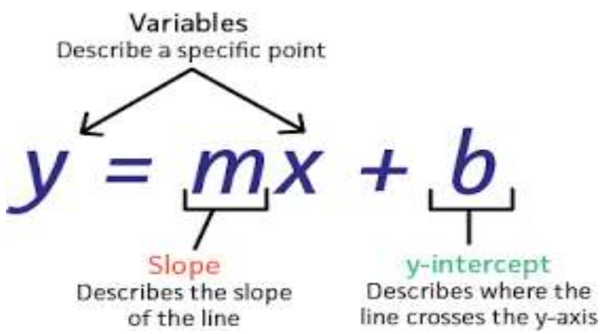
Linear Equations and Functions

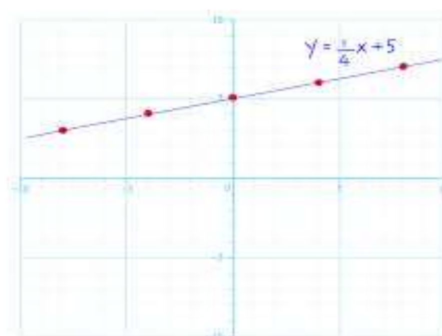
- **Function** is a relationship, a rule that connect elements of two sets. The first set is **D** (domain) and the second set is **R** (range).
- The elements of **D** are independent variable and denote by “x”. The elements of **R** are dependent variable and denote by “y”. In general, we write y = f(x), where “f” is to substitute the rule, the sentence of the rule. If we have y=4x-5 means that the value of “x” must multiply by “4” and subtract “5”. The sentence “multiply by 4 and subtract 5” is represented shortly by “f”. The graph of the function is the set of points {x,f(x)} or (x,y) on Cartesian plane



The function y = ax + b or f(x) = ax + b or y = mx + b is the linear function (its graph is a line on the Cartesian plane).



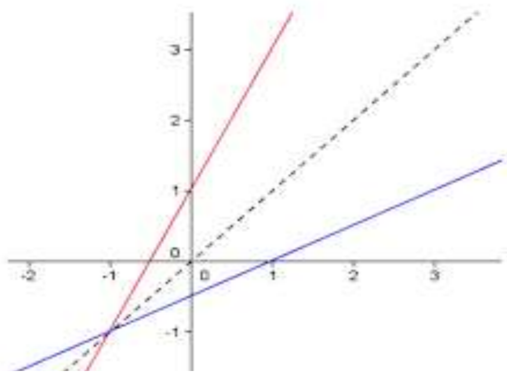




Linear Equations and Functions

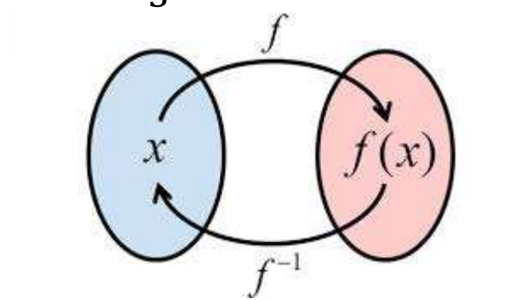
- The slope show the tangent of the angle formed by line and positive direction of “x” axe
- The intercept is the point where the line pass the “y” axe(intercept the axe “y”) The expression in the form $ax + by + c = 0$ also represent a line and its slope is $-a/b$, intercept is $–c/b$. The expression $ax + b = 0$ we consider the equation of the first degree with one variable which has one solution $x=-b/a$ for nonzero values of a. The solution is intercept of the line with “x” axe.
- The expressions $ax + b < 0$ or $ax + b>0$ are inequalities. The line divide the plane into two parts, one of which is the solution of the inequalities. Example. Sketch the lines $y = 2x + 2$ and $y = 3x – 6$

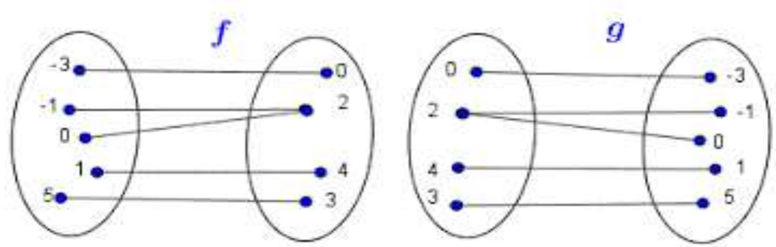
Linear Equations and Function

- If in a function y = f(x) we consider the set R(range) as domain and the set D(domain) as a range (change the role elements of the sets), then we get the **inverse** function. The graph of the inverse function is obtained by reflecting the graph of function by the line **y = x.** Let's have the function y = 3x + 2 (x independent variable and y dependent variable).If we write x = g(y) then we have "y" as independent variable and the "x" as a dependent variable. This is the inverse function g(y) = $\frac{y-2}{3}$
- 

$$f(x)=3x+2$$

$$f^{-1}(x)=\frac{x-2}{3}$$





Linear Equations and Functions

This is the way how we **find** the **inverse function**. The graph of function g(y) is symmetrical about the line y = x of the function y = f(x). A function may be given by a formula or by two or more formulas. In this case we say that the function is given by parts or in(piece-wise form).This way, **by the formula** is best way to study the function and its properties

A function may be given **by the graph**(which is the best way to see or image it). The graph in general is a curve on the Cartesian plane.

The third way is to present a function **by a table**. It has two columns or two rows.

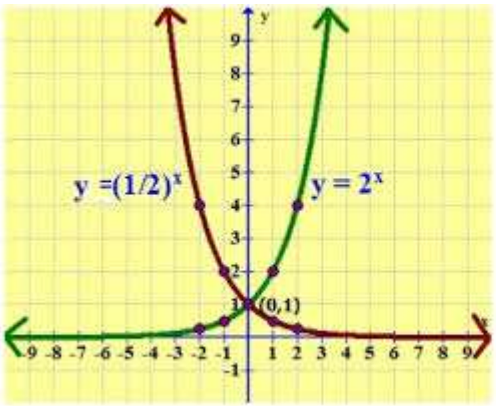
In one are the value of the independent variable and to the other are the corresponding values of the dependent variable

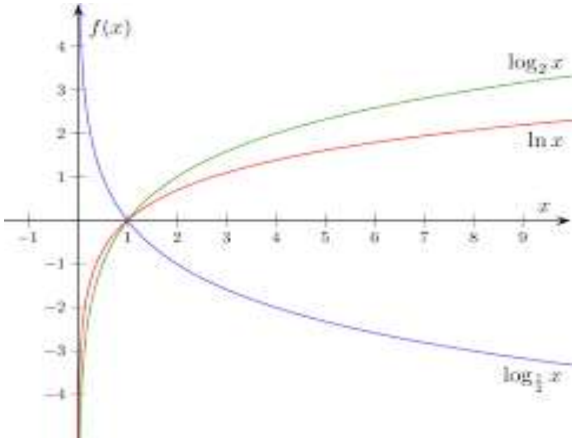
Linear equations and Functions

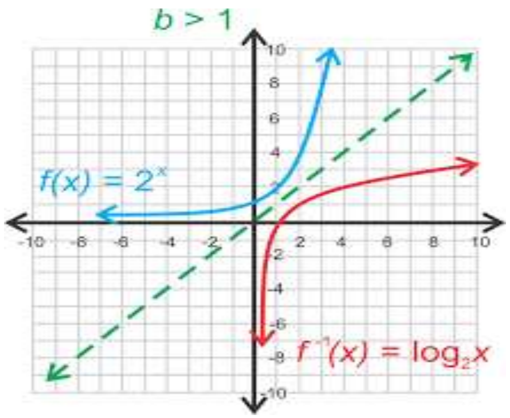
- The basic features of the functions are **Domain and Range; Boundedness; Continuity**; **Rate of Change**; **Local and Absolute extreme**; (increasing, decreasing); **Inflection Points** (concavity, concave, convex,i lugët); **Asymptotes.** Functions **Linear; Quadratic; Exponential; Logarithmic; Polynomial**; **Rationa**l; **Trigonometric functions** and their compositions of a finite number of algebraic operations are called **elementary functions .**
- Linear function $f(x) = ax + b$ $y = 3x - 11$
- Quadratic function $f(x) = ax^2 + bx + c$ $y = 3x^2 + 4x - 1$
- Exponential $f(x) = a^x$ $y = 2^x$ $y = \left(\frac{1}{3}\right)^x$
- Logarithmic function $f(x) = \log_{10} x = \log x$ $f(x) = \log_e x = \ln x$ $y = \log(2x+1)$
- Polynomial function $f(x) = \frac{p(x)}{q(x)}$ $p(x), q(x)$ $f(x) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \dots + a_{n-1} x + a_n$

  $y = 4x^3 + 3x^2 - 6x + 13$
- Rational function are polynomials $f(x) = \frac{p(x)}{q(x)}$ $y = \frac{2x^3 + 5x - 7}{7x^2 - x + 1}$
- Trigonometric functions f(x) = sinx; f(x) = cosx; f(x) = tanx; f(x) = ctanx

Linear equations and Functions

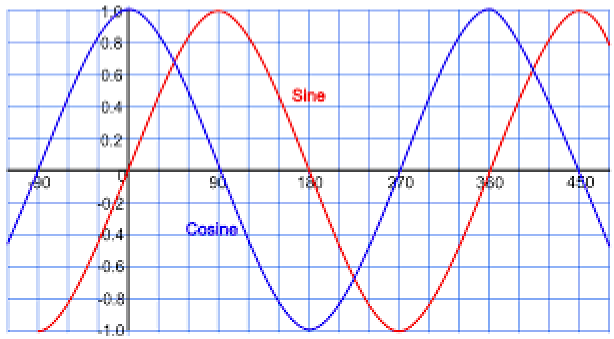


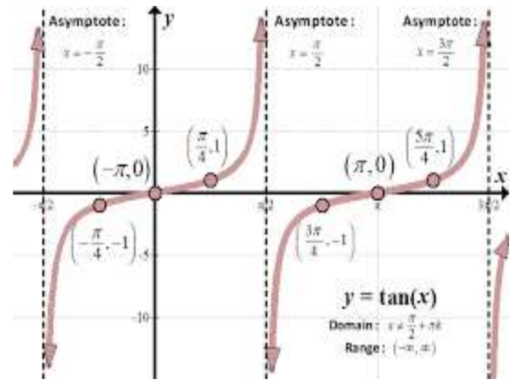




Linear Equations and Functions

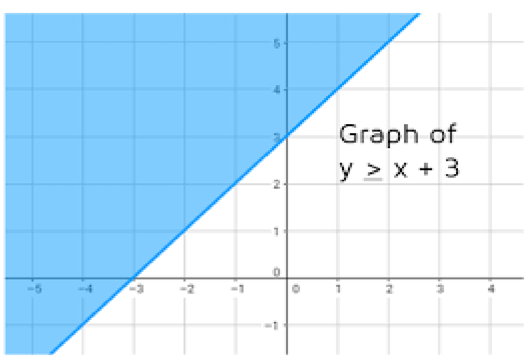
- The graphs of the trigonometric functions.

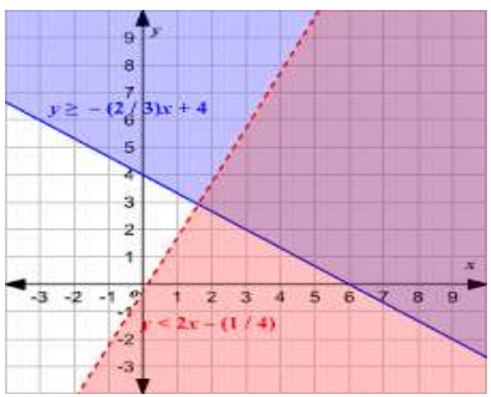




- How we find the domain and the range for the elementary functions?
- Functions $y_1 = a_1x + b_1$ and $y_2 = a_2x + b_2$ present two lines on the Cartesian plane
- If $a_1 = a_2$ the lines are parallel. If $a_1 = -\frac{1}{a_2}$ the lines are orthogonal. If $a_1 \neq a_2$ the lines intersect.
- If we consider as system of linear equations, it has exactly one solution if $a_1 \neq a_2$ and it
- has not solution if $a_1 = a_2$ .If in stead of equality "=" we have inequality greater ">"
- or smaller"<" then we have an inequality or a system of inequalities. The solution is a part of the Cartesian plane.

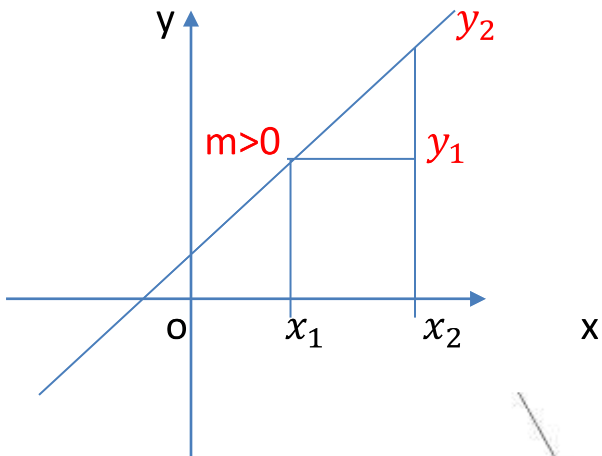
$$y_1 = a_1x + b_1$$
$$y_2 = a_2x + b_2$$

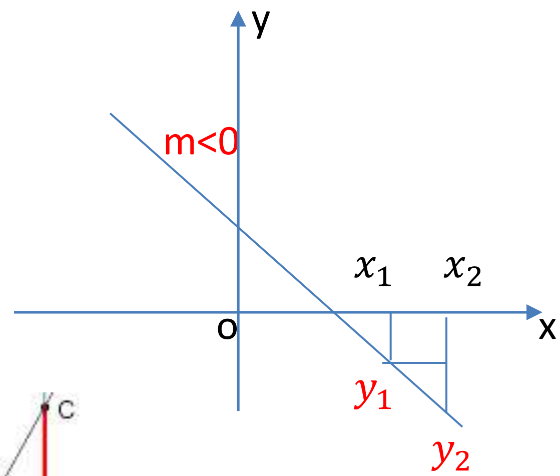




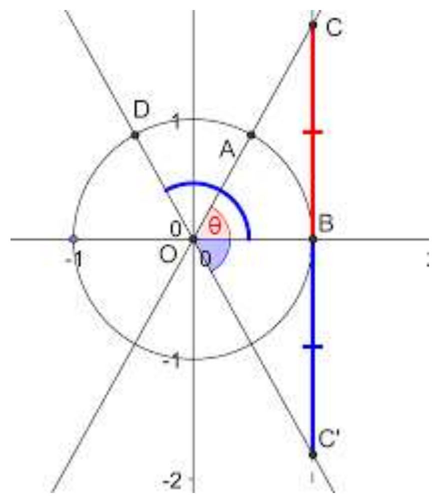
Linear Equations and Functions

- Positive slope x increases and y increases
- Negative slope x increases and y decreases



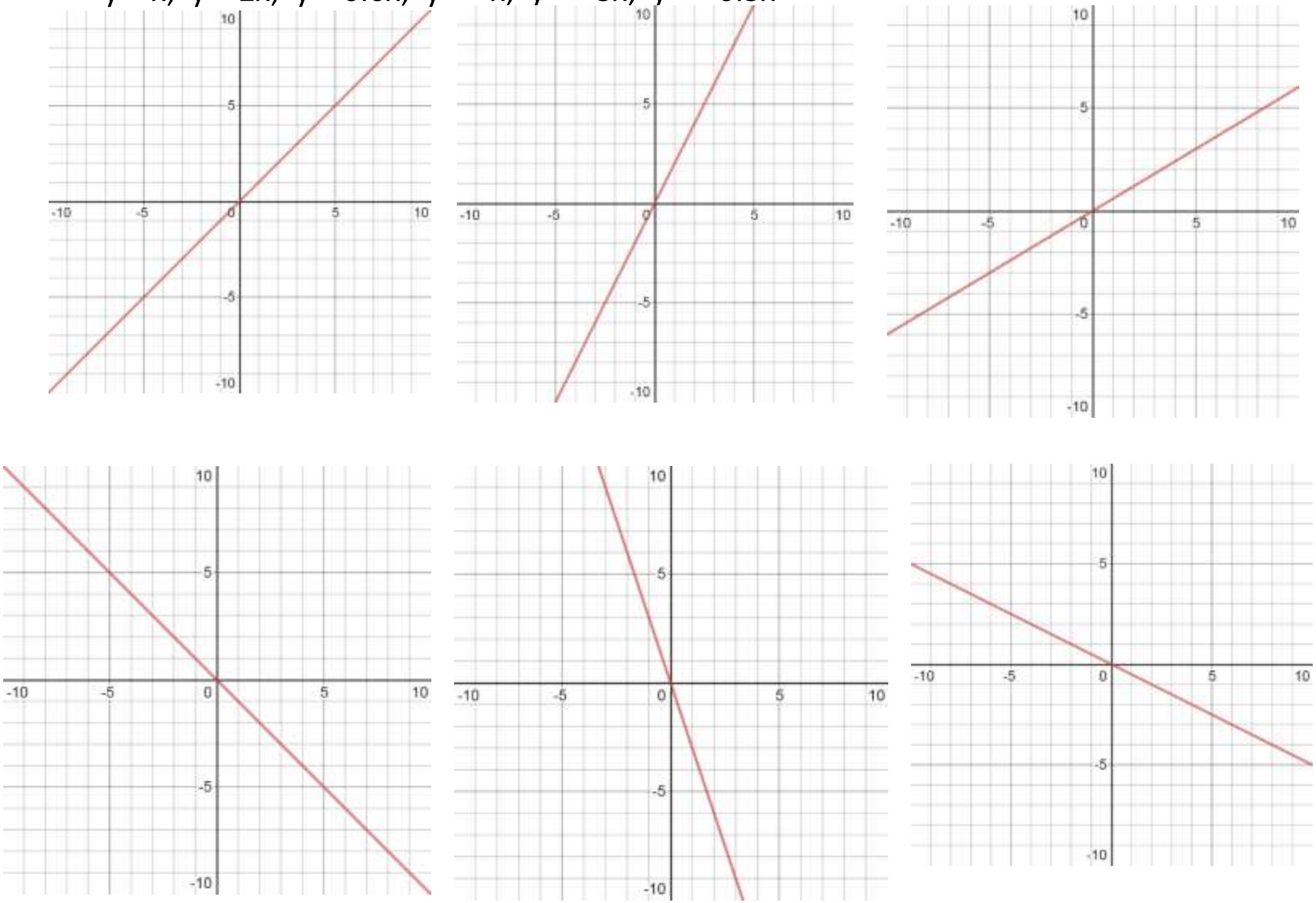


- Trigonometric circle



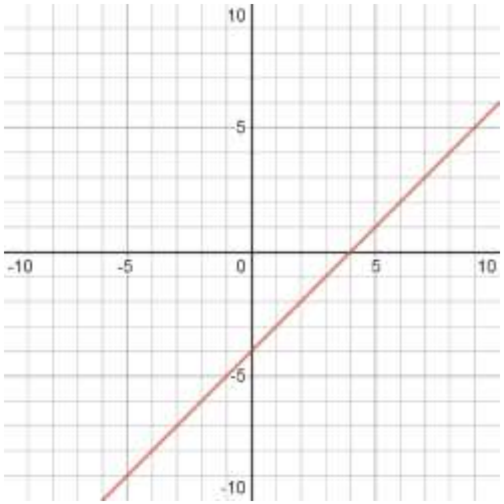
Linear equations and Functions

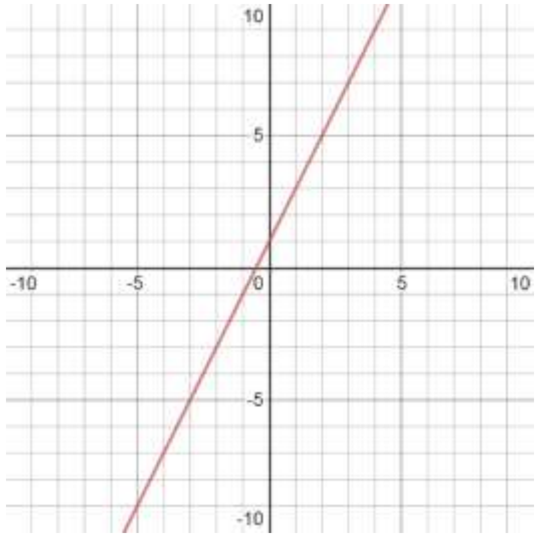
- y = x; y = 2x; y = 0.6x; y = - x; y = - 3x; y = - 0.5x

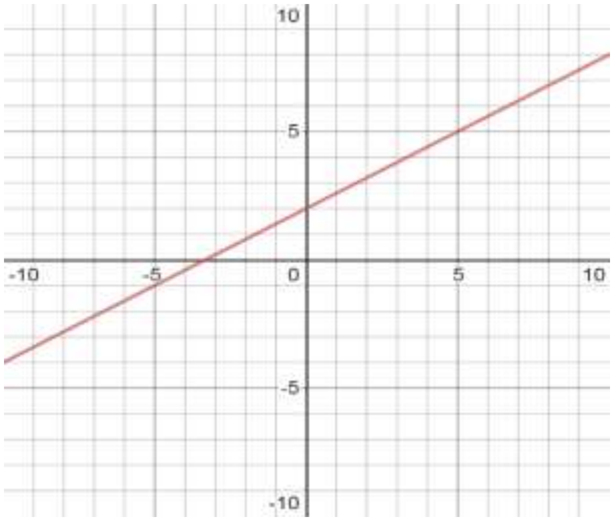


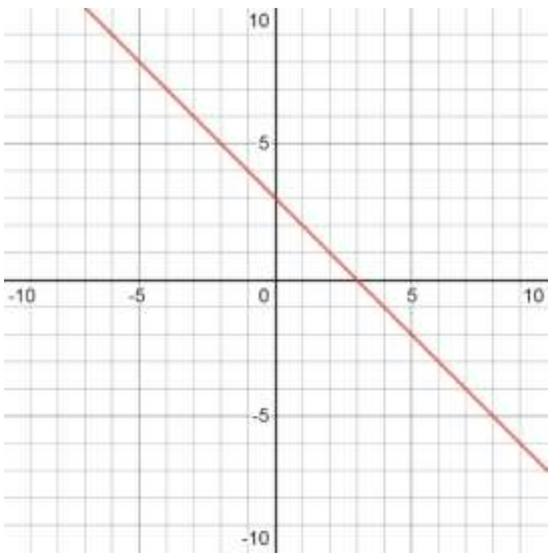
Linear Equations and Functions

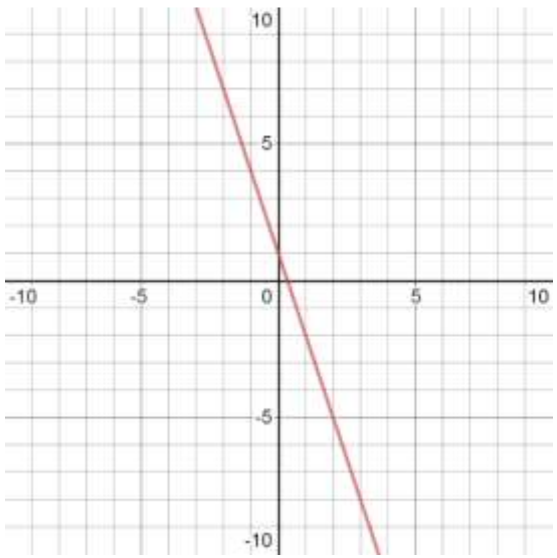
- y = x – 4 ;   y = 2x + 1;   y = 0.6x + 2;   y = - x + 3;   y = - 3x + 1;   y = - 0.5x – 2

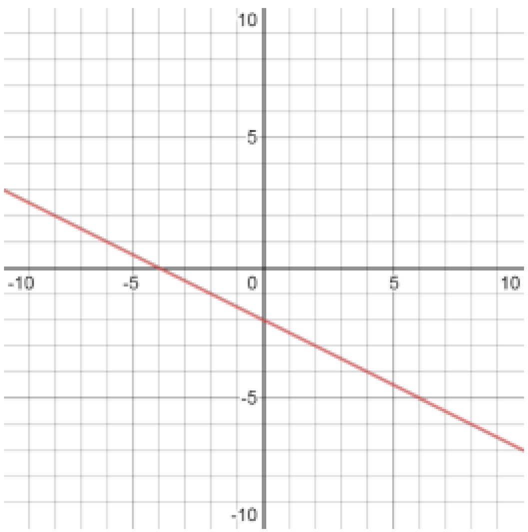












Linear Equations and Functions

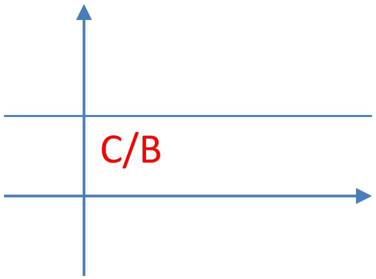
- If A is not equal to zero and B is not equal to zero, then $Ax + By = C$ can be written as

  $$y = -\frac{A}{B}x + \frac{C}{B}$$

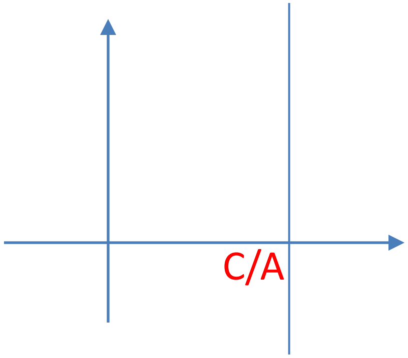
  - $-\frac{A}{B}$ is the slope and $\frac{C}{B}$ is the intercept

  This is known as **slope-intercept form**

- If $A = 0$ and B is not equal to zero, then the graph is a **horizontal line**

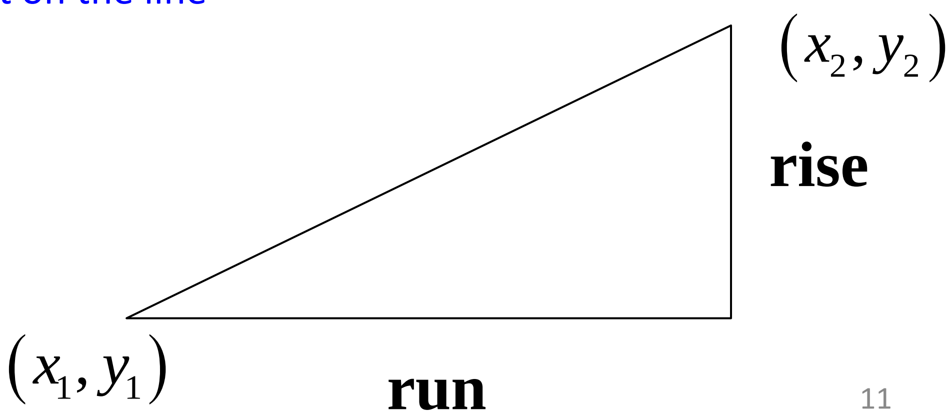


-
- If A is not equal to zero and $B = 0$, then the graph is a **vertical line**



- The **slope** of a straight line is simply the change in the value of y brought about by a 1 unit increase in the value of x.
- The slope of a line is constant at every point on the line

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{rise}{run}$$



Linear Equations and Functions

- The point-slope form of the equation of a line is

$$y - y_1 = m(x - x_1)$$

where m is the slope and (x_1, y_1) is a given point. For two points

(x_1, y_1) and (x_2, y_2) the equation of the line and the slope are as below

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1} \qquad \frac{y_2 - y_1}{x_2 - x_1} = m$$

Find the equation of the lines which, forms the angle 135°, degree, 60° degree and pass to the point (2; -1) (-1; 2) (remember trigonometric circle)

Find the equation of the line which pass through the points (1; 4) and (-2; 0) (2; -3) and (-3; 5)

Linear Equations and Functions

Monthly sales revenue, S (in \$), and monthly advertising expenditure, A (in \$), are modelled by the linear relation, $S = 9000 + 12A$.

- a) If the firm does not spend any money on advertising, what is the expected sales revenue that month?
- b) If the firm spends \$800 on advertising one month, what is the expected sales revenue?
- c) How much does the firm need to spend on advertising to achieve monthly sales revenue of \$15000?
- d) If the firm increases monthly expenditure on advertising by \$1, what is the corresponding increase in sales revenue?
- a) $A = 0$ so $S = 9000$
- b) $A = 800$ so $S = 9000 + 12\cdot 800$ $\quad S = 9000 + 9600 = 18600$
- c) $S = 15000$ so $15000 = 9000 + 12A$ $\quad 12A = 6000$ $\quad A = 500$
- d) $S' = 9000 + 12(A+1) = 9000 + 12A + 12$ so $S' = S + 12$. Increasing is \$ 12. This increasing is equal to the value of the slope. (Sometimes we call it rate of change)

Linear Equations and Functions

- Three companies can supply a university with some mathematical software. Each company has a different pricing structure:
  - Company 1 provides a site licence which costs \$130 000 and can be used by anyone at the university;
  - Company 2 charges \$1000 per user;
  - Company 3 charges a fixed amount of \$40 000 for the first 60 users and \$500 for each additional user.

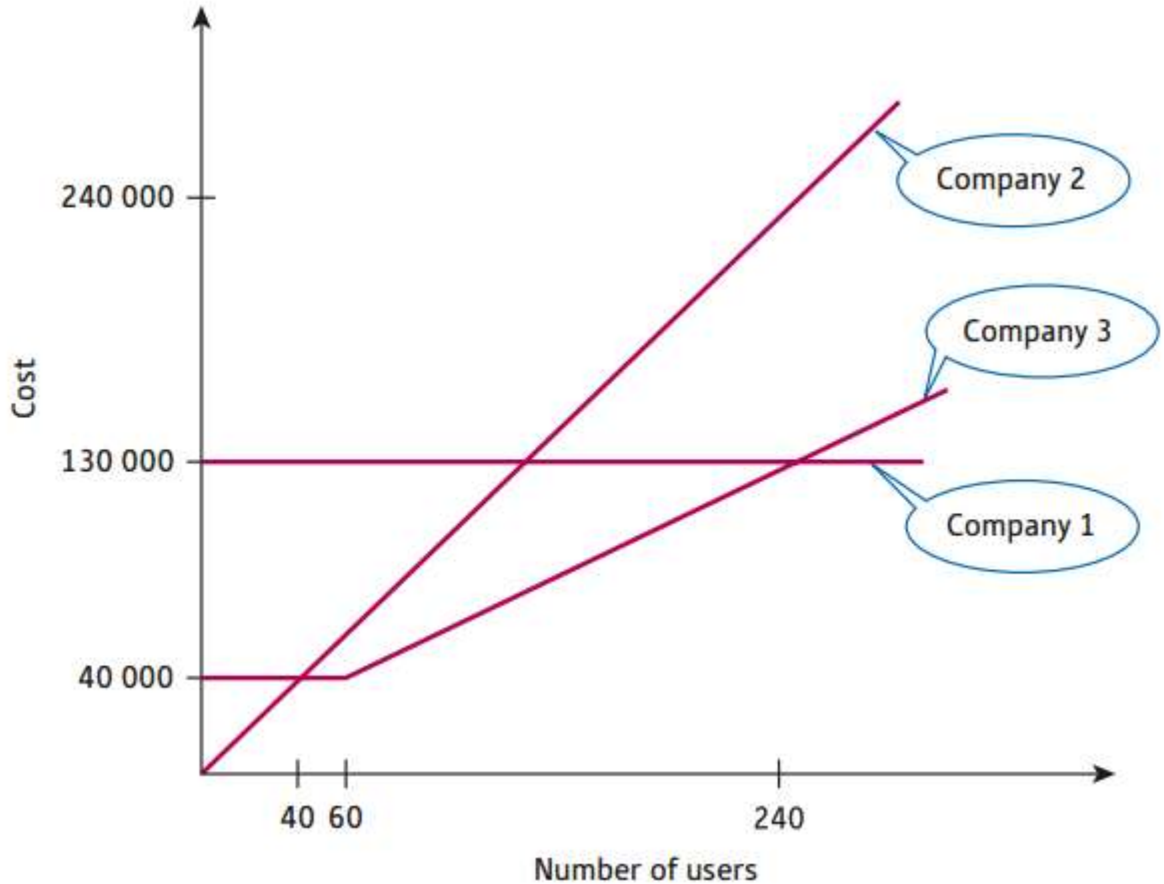
(a) Draw a graph of each cost function on the same set of axes.

(b) What advice can you give the university about which company to use?

The first company has fixed cost \$ 130000 for 240 students or more or less

The second company has \$ 240000 for 240 students, \$ 180000, for 180 students and \$ 300000, for 300 students.

Linear Equations and Functions



Linear Equations and Functions

The third company has \$ 40000 plus 180·500 = 90000. Total \$ 130000 for 240 students. Total cost for “n” students is C = 40000 + (n - 60)500 = 500n + 40000.

Advice: If the number of the students is less than 40 students, second company is low cost.

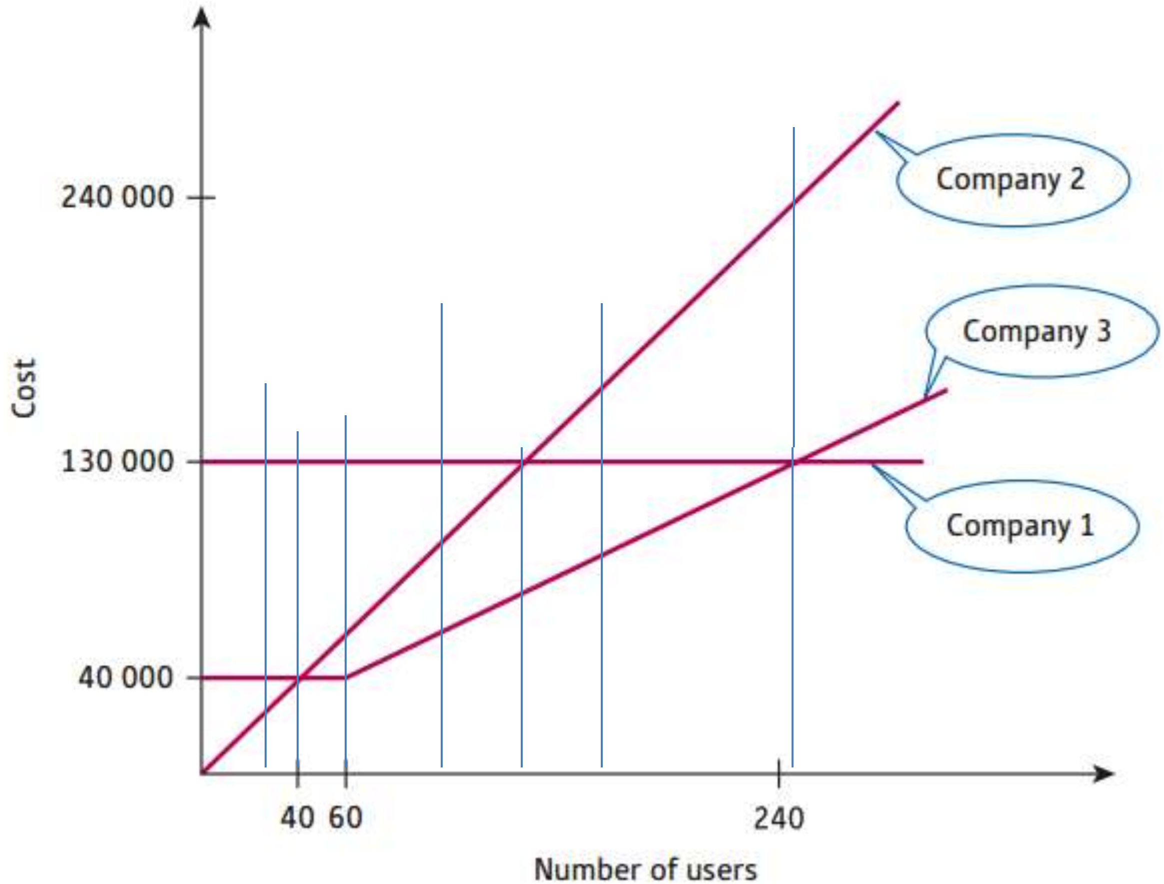
If the number of the students is more than 40, but less than 60, third company is low cost.

If the number of the students is more than 60, but less than 240, third company is low cost.

If the number of the students is more than 240, the first company is low cost.

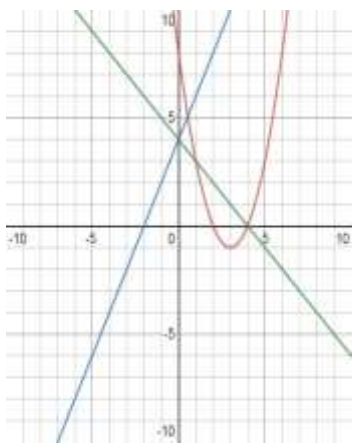
- Company third is a function given by parts
-
- f(x) = $\begin{cases} 40000 & for \quad 0 < x < 60 \\ 500x + 40000 & for \quad x > 60 \end{cases}$

Linear Equations and Functions



Linear Equations and Functions

- Intersections of lines and curves. Intersections of the red curve with axes are points (2;0) (4;0) and (0;8). For the green line are (4;0) and (0;4). For the blue line are (0;4) and (-2;0). Green red intersections are points (4;0)and (1;3). Blue red intersections are (0.5;5)and another.
- The linear function y = ax +b we can see as equation ax + b = 0 ,for a $\neq$ 0 has one
- solution or as inequality
- ax + b > 0 or ax + b < 0 and
- studding the sign, we get the
- solution. The solution is a set
- of numbers (a part of x'x axe)
- The inequality in general form
- ax + by < 0 or ax + by > 0 we
- can see as a line which divide the
- Cartesian plane into two parts. One
- is the solution of the inequality. Using
- the graphs of the lines we solve the
- system of the inequalities.



Linear Equations and Functions

- Is given the expression 2x + 4y – 12z = 16. let's express one variable in terms of the others
- 2x = - 4y + 12z + 16 or x = -2y + 6z + 8
- 4y = -2x + 12z + 16 or y = -0.5x +3z + 4
- 12z = 2x + 4y – 16 or z = 1/6x + 1/3y – 4/3
- These are transposition formulas
- S = P $(1+\frac{r}{100})^{n}$ express S in terms of the others
- P = S$(1+\frac{r}{100})^{-n}$ $(1+\frac{r}{100})^{n}$ = $\frac{S}{P}$ $1+\frac{r}{100}$ = $\sqrt[n]{\frac{S}{P}}$
- $\frac{r}{100}$ = $\sqrt[n]{\frac{S}{P}}$ - 1 or r = 100 $\sqrt[n]{\frac{S}{P}}$ - 100

Linear Equations and Functions

- To transpose a given formula you must procced as follow
- Step 1 Remove fractions
- Step 2 Multiply out the brackets
- Step 3 Collect all terms with the selected variable to the
- left-hand side
- Step 4 Take out the factor of the variable
- Step 5 Divide by the coefficient of the variable

Solution of simultaneous linear equations

- Simultaneous linear equations is a set linear equations which have (usually) the same number of equations and variables. We know these equations as system of the linear equations. The solution consists of finding the values of the variables which satisfies all equations at the same time.
- There are three methods of finding the solutions of the systems (if it exists).
- Substitution method.

Step 1. From one equation express a variable in terms of the other variable and substitute to the other variable, getting an equation of one variable.

Step 2. Solving this obtain a value for one variable.

Step 3. Substitute this value to the first equation and get the value for the second variable.

Solution of simultaneous linear equations

- Elimination method.
- Step 1. Multiply by a constant one or two equations to equate the constants of one variable.
- Step 2. Add/Subtract one equation from the other eliminating one variable and getting a first-degree linear equation of a variable
- Step 3. Substitute this value to whatever equation and get the value for the second variable.
- The elimination method is based in so called elementary **row operations**. These operations are : 1. Interchange the position of two rows.
- 2. Scaling. Multiply all entries in a row by a nonzero constant.
- 3. Replace one row by the sum of itself and multiple of another row.
- For systems of two equations with two variables the first operation has not sense. At this case we can summarize the following steps.
- Step 1 Add a multiple of one equation to a multiple of the other to eliminate one variable or subtract a multiple of one equation to a multiple of the other to eliminate one variable.

  Step 2 Solve the resulting equation for one variable.

  Step 3 Substitute the value of the variable you found into one of the original equations to find the value of the other variable.

Solution of simultaneous linear equations

The substitution method we can summarize in following steps.

Step 1. Express one variable in terms the other variable from one of the equations.

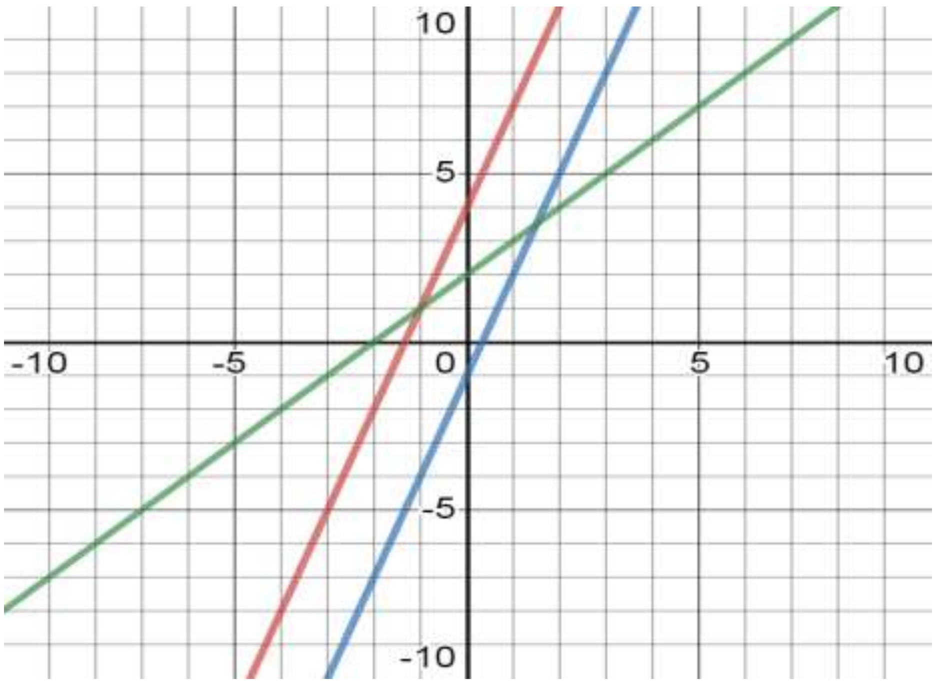
Step 2. Substitute the founded variable into the next equation and find the value of one variable

Step 3. Go to the result of step one and apply back substitution to the value of the next variable

- Graphical method.
- Step 1. Sketch the graphs of the two lines at the same Cartesian plane.
- Step 2. By the graph write the coordinates of the intersection point which is the solution of the system

Solution of simultaneous linear equations

- Solve the system using three methods
- $3x + 2y = 12$ $2x – y = 5$ $2x + 3y = - 18$ $3x + 4y = 1$
- $x + y = 5$ $3x + 2y = 4$ $5x – 6y = 9$ $-5x – 9y = 3$



Position of the lines on the Cartesian plane

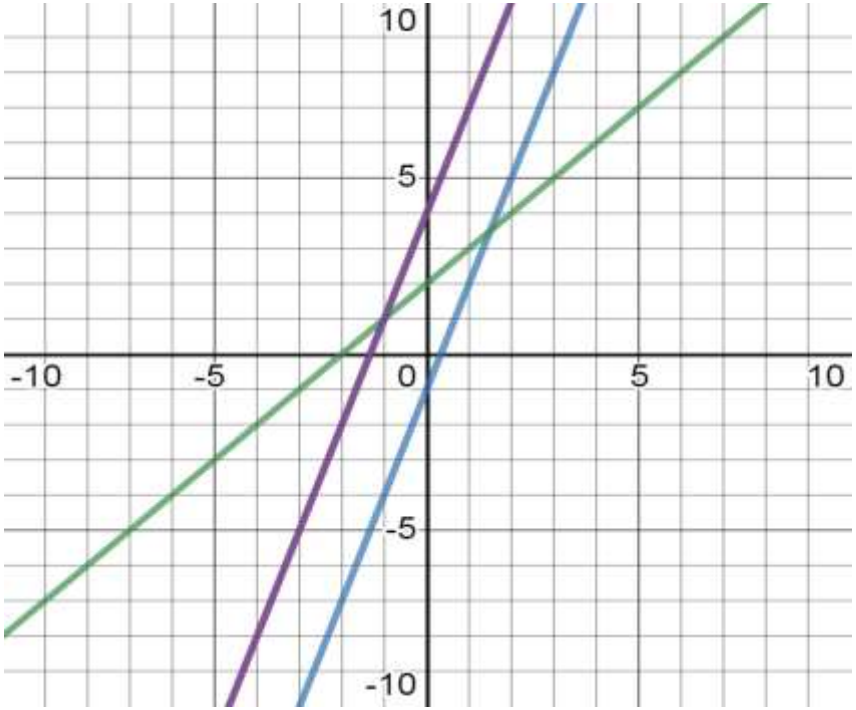
$y = 3x + 4$

$y = 3x – 1$

$y = x + 2$

Solution of simultaneous linear equations

- Graphs of the lines. Position of lines on Cartesian plane
- Intersect lines one solution. Parallel lines no solution



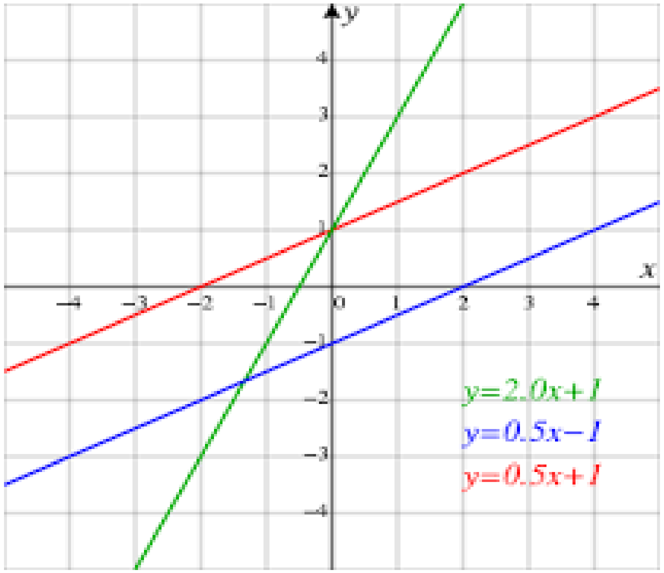
- Coincides lines
- infinite solutions
- $6x - 2y + 8 = 0$
- $y = 3x + 4$

Solution of simultaneous linear equations

- Applying substitution method, we can solve the system of
- three equations and three variables.
- Practice problem on the book page 64
- $2x + 2y - 5z = -5$ From one equation express one variable
- $x - y + z = 3$ in terms of the other variables.
- $-3x + y + 2z = -2$ Substitute this expression to the other
- two equations, getting a system of two equations with two
- variables. Finding the values of the two variables, do back
- substitution to the first transposition getting the solution of the system

Graphs of Linear Functions

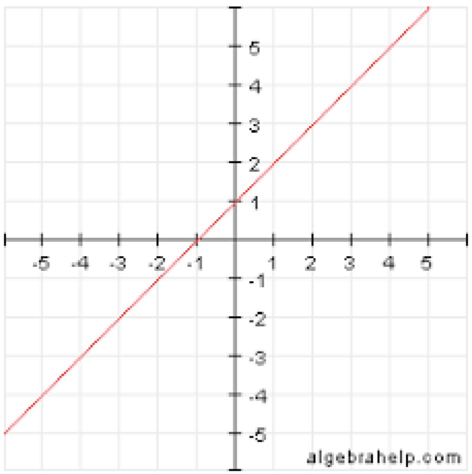
- The expression f(x) = ax + b or y = ax + b are formulas of the linear function where a; b; are real numbers. The graph of this function is an infinite (endless)set of points. The graph of this function is an infinite (endless) set of points M(x, y) on the Cartesian plane. It is a straight-line L lays on the Cartesian plane. If the point M(x, y) is element of the line L y = ax + b, or lies on the line, then the values x (abscise) and y (ordinate) turn the equation y = ax +b into an identity. The equations y = ax + b, y = mx + b are equations of the line in slope intersect form. The linear function in economic science we can find in forms like g(Q) = aQ + b or P(Q) = -2Q + 7. We can find the value of g(Q), P(Q) (depended variable) for a given value of Q (independent variable). The equation of the line we can find in forms ax + by + c = 0 or ax + by = c . The general equation of a straight line takes the form
- **a multiple of x + b multiple of y = c (a number).** In general, to sketch a line from its mathematical equation, is sufficient to calculate the coordinates of any two distinct

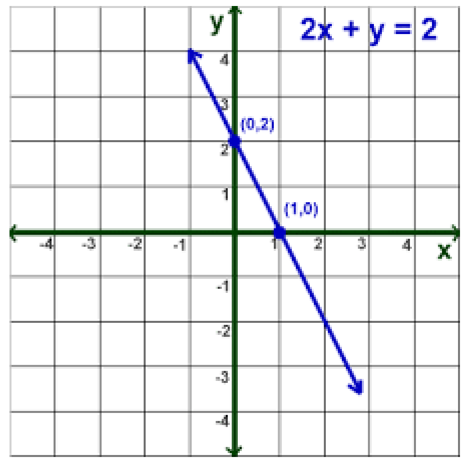


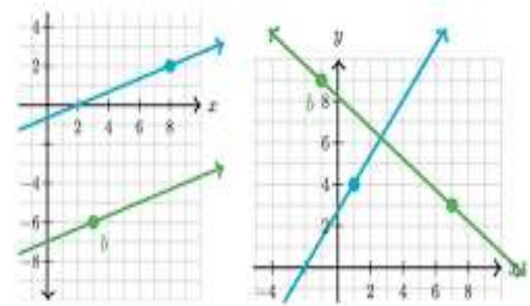
points lying on it. These two points can be plotted on graph paper and a ruler used to draw the line passing through them. One way of finding the coordinates of a point on a line is simply to choose a numerical value for x and to substitute it into the equation to find the corresponding value of y. The same process can be repeated to find the coordinates of the second point by choosing another value for x.

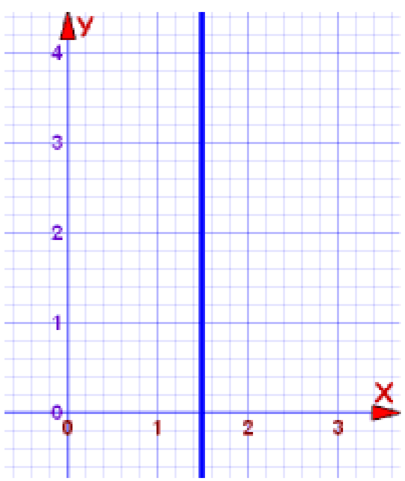
Graphs of Linear Functions

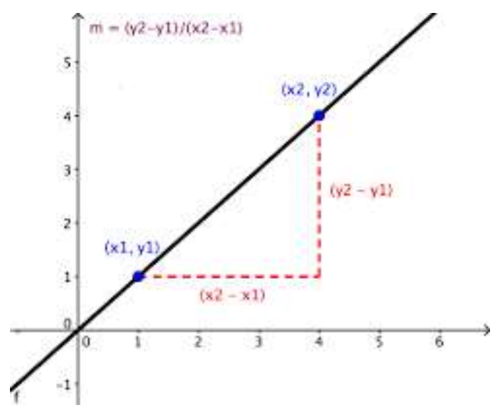
- Different positions of the line on the Cartesian plane

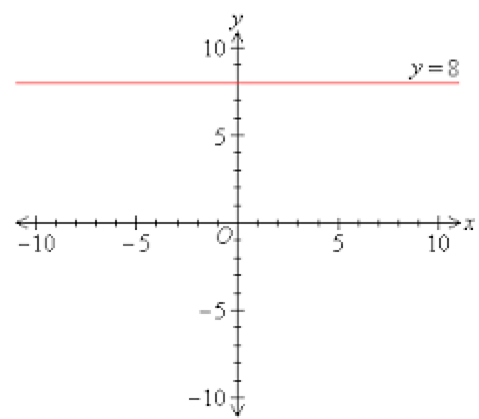










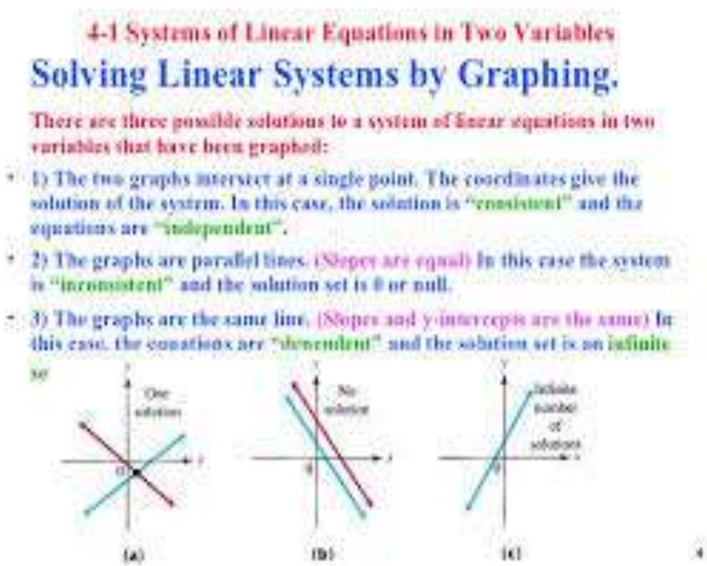


Graphs of Linear Functions

- Two lines may be intersecting in a single point
- may be parallel (not intersection)
- may be coincides and hence intersect at every point of the line.
- In economics it is sometimes necessary to handle more than one equation at the same time.
- For example, in supply and demand analysis we are interested in two equations, the supply equation and demand equation. Both involve the same variables Q and P, so it makes sense to sketch them on the same diagram. This enables the market equilibrium quantity and price to be determined by finding the point of intersection of the two lines. There are many occasions in economics and business studies when it is necessary to determine the coordinates of points of intersections.
- Find intersection of lines $x + y = 30$
- $2x - y = 30$
- From the general form of the equation of the line we can find the slope intercept form after some algebraic operations.
- Two equations of two lines forms the system of two equations with two variables.
- The system may have no solution, zero solutions (parallel lines)
- exactly one solution (lines intersect in a single point).
- many solutions, infinite solutions (coincident lines, the two
- equations represent the same line.)

Graphs of Linear Functions

- The system of two linear equations and two variables. System of two simultaneous linear equations
- Intersect in a single point (consistent, linearly independent). One solution.
- Parallel line (inconsistent, linearly dependent) No solution, or zero solution.
- The same line, or coincides lines (inconsistent, linearly dependent)
- The solution is an infinite set, every point of the lines(line, because is the same line) is solution for the system.



Graphs of Linear Functions

$M_1(x_1;y_1);M_2(x_2,y_2)$ Are two points of the line. Equation of this line is $\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$ And the slope is

$\frac{y_2-y_1}{x_2-x_1}$ If a>0 the angel α is smaller than 90° if a<0 the angel α is greater than 90°.For two lines if $a_{1.}=a_2$ the lines are parallel. If $a_{1.}\cdot a_2=-1$ The lines are perpendicular, orthogonal

Examples of Systems of Linear Equations

- $8a - b = 9$
  $4a + 9b = 7$
- $3a - b + 14c = 7$
  $2a + 2b + 3c = 0$
  $a - 12b - 18c = 33$
- $3r + s - 7t = 15$
  $r - 12s + t = 0$
  $5s - 4t = 8$
- $x + 7y = 9$
  $2x - y = 18$

There are two methods for solutions the systems of linear equations with two, three, or more variables. The **elimination** method and **substitution** method

For systems of two linear equations and two variables we can find the solution using the **graphical** method. The solution is the point of intersections of two lines(it's coordinates). Example

4x - 6y = -4 -8x + 12 y = 8
8x + 2y = 48 8x + 2y = 48 after addition have 14y = 56 y = 4
The value y=4 substitute in one of the equations and find x = 5

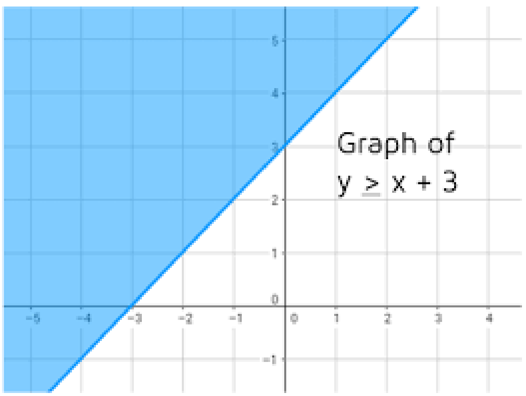
Simultaneous linear equations (usually) is a set of linear equations in which the number of variables is equal with number of equations.

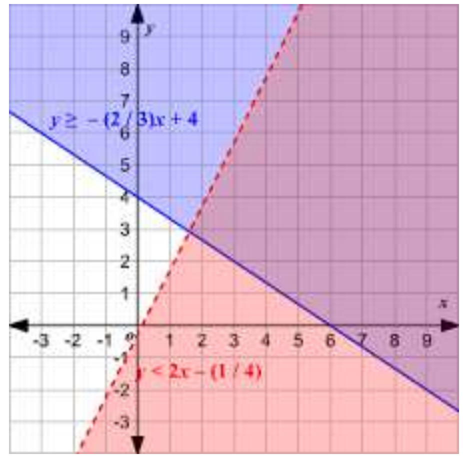
The expressions ax + b>0 or ax + b<o are inequalities of the first degree with one variable. The solution is a subset of R . 2x + 6 > 0 2x > -6 x > -3 Solution set is A={xϵR/ x > -3}

4x - 6y > -4
8x + 2y < 48 is a system of two inequalities with two variables. The solution is a subset of the plane.

Graphs of Linear Functions

- Solutions of linear inequalities and system of two linear inequalities with two variables. The solution





is a part of the plane xoy.
For y=ax + b and “a” nonzero number we can see the sign of the ”y” as on the table below.
The signs are different on the left and on the right of the value of the root -b/a. (root of the equation ax + b=0 for nonzero “a”)

Y=ax+b x=-b/a

Value of x		
Value of y	Sign of - a	0 sign of a

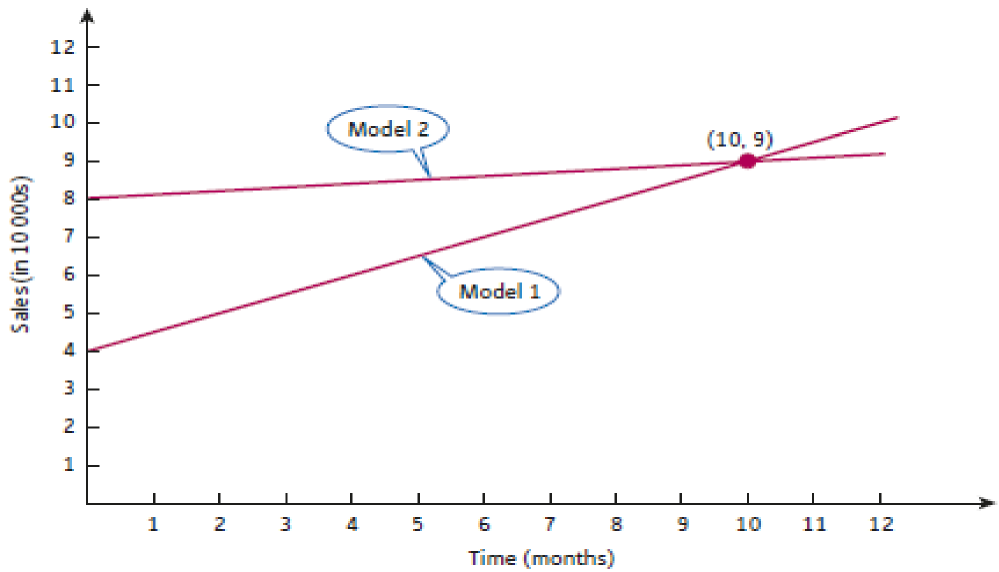
Graphs of linear equations

- Two models' phone are launched on 1 January 2018. Predictions of sales are given by the models
- Model 1: $S_1 = 4 + 0.5n$ Model 2: $S_2 = 8 + 0.1n$ where S_i (in tens of thousands) denotes the monthly sales of model “i” after “n” months.
- State the values of the slope and intercept of each line and give an interpretation.
- Illustrate the sales of both models during the first year by drawing graphs on the same axes
- Use the graph to find the month when sales of Model 1 overtake those of Model 2

Graphs of Linear Functions

- The intercept for Model 1 is 4. There are 40000 sales of this phone when the product is launched. The slope is 0.5, so each month, sales are increased by 5000. The corresponding figures for Model 2 are 8 (80000 sales), and 0.1 (1000 items increased per month)
- The intercept for Model 1 is 4, so the line passes through (0, 4). For every-one unit increase in "n" the of S_1 increases by 0.5. So, for example, a two-unit increases in "n" results in a one-unit increase in S_1. The line passes through (2, 5), (4, 6) and so on.
- For the Model 2, the line passes through (0, 8), and since the slope is 0.1, it passes through (10, 9).
- The graph intersects at (10, 9), so sales of Model 1 overtake of Model 2 after 10 months.

Graphs of Linear Functions



Graphs of Linear Functions

- A firm's human resource department has a budget of \$25000 to spend on training and laptops. Training courses cost \$700 and new laptops are \$1200
- If the department trains "E" employees and buys "L" laptops, write down an inequality for "E" and "L".
- If 12 employees attend courses, how many laptops could be bought?
- How many employees can follow training courses if the department buy 12 laptops
- Solution
- There are 700E dollars for training and 1200L dollars for laptops. The total amount does not exceed 25000 dollars. So, $700E + 1200L \leq 25000$
- Substituting $E = 12$ into the inequality gives $8400 + 1200L \leq 25000$ or
- $1200L \leq 16600$ (subtract 8400 from both sides)
- $L \leq 13\frac{5}{6}$ (divide both sides by 1200). So, a maximum of 13 laptops could be
- $700E + 1200 \times 12 \leq 25000$ or $700E \leq 25000 - 14400$ or $700E \leq 10600$
- and dividing by 700 get $E \leq 15.1$. the department can pay the training of 15 employees.

Graphs of linear equations

- Problem
- Two-fifths of Suzan's money together with three-fourths of Emily's money are \$380
- Peter has \$500 more than Suzan and \$300 more than Emily. Find out the money which has Suzan, Emily and Peter.
- Denote x the money of Suzan. y the money of Emily and z the money of Peter
- We have three equations
- $\frac{2}{5}x + \frac{3}{4}y = 380;$ $8x + 15y = 380 \cdot 20$
- $z = x + 500$ $x + 500 = y + 300$
- $z = y + 300;$
- $x - y = -200$ $x = y - 200$ $8(y - 200) + 15y = 7600$ $23y = 9200$
- $y = 9200 \div 23$
- $y = 400$ $z = 400 + 300 = 700$ $700 = x + 500$ $x = 200$

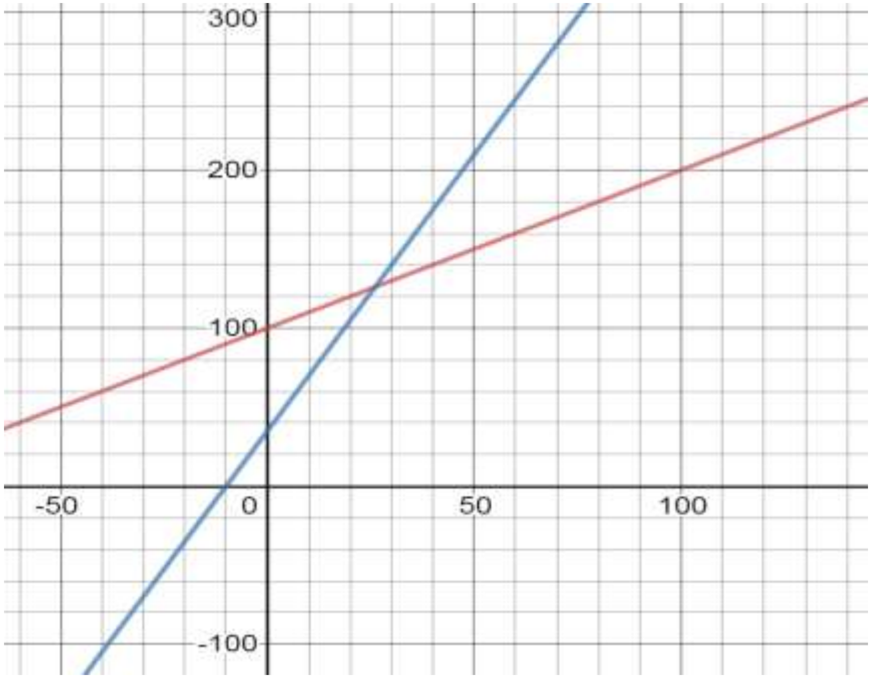
Graphs of linear functions

A)The Feelgood Gym has a monthly membership fee of $100 and it charges an additional $1 per hour. If I use the gym x hours in a month, write down an expression for the total cost in terms of "x"

B) Repeat part (a) for the Fabulous Me Gym, which charges $35 per month and $3.5 per hour.

C) By plotting the both graphs on the same axes, find the number of hours which gives the same total cost

Graphs of linear functions

- x hours plus \$100 fee. $y = x + 100$
- 3.5x plus fee \$35. $y = 3.5x + 35$
- Intersection of the lines we can find solving the system
- $x + 100 = 3.5x + 35 \quad 3.5x – x = 100 – 35 \quad 2.5x = 65 \quad x = 26$
- 

Graphs of linear functions

- 1. A bakery discovers that if it decreases the price of its birthday cakes by \$1, it sells 12 more cakes each month.

a) Assuming that monthly sales, M, are related to prices , P, by a linear model, $M = aP + b$, state the value of "a"

b) If the bakery sells 240 cakes in a month when the price of the cake is \$14, work out the value of "b"

c) Use this model to estimate monthly sales when the price is \$9

d) If the bakery can make only 168 cakes in a month, work out the price that it needs to charge to sell them all.

2. a) Show that the lines $ax + by = c$ and $dx + ey = f$ are parallel whenever $ae – bd = 0$

b) Write down the coordinates of the points where the line $ax + by = c$ intersect the axes .

c) Use the result of part "a" to comment on the solution of the following simultaneous equations.

$2x – 4y = 1$

$– 3x + 6y = 7$

Graphs of linear functions

- Suppose that the sales of the previous month were M and the prices were P. After decrease, the prices the sales were M' and the price were P'. $M' = M + 12$ and $P' = P – 1$.
- The model is $M = aP + b$ and $M' = aP' + b$, or $M' = a(P - 1) + b$ After substitution have
- $a(P -1) + b = aP + b + 12$ or $aP – a + b = aP + b + 12$ so $- a = 12$ or $a = - 12$.
- $M = 240$, $P = 14$ $a = - 12$ or $240 = - 12 \cdot 14 + b$ or $b = 240 + 168 = 408$.
- $M = - 12 \cdot 9 + 408$ $M = - 108 + 408 = 300$.
- $M = 168$, $b = 408$, $a = - 12$ so $168 = -12P + 408$ or $- 12P = 168 – 408 = - 240$ $P = 20$

Graphs of linear functions

- A company seeks to hire a contract specialist for 5 years. It pays 3% more on actual one year specialist's salary. It pays $ 150 a month for every 3 years of experience behind 35 year olds and $ 60 for every child under 18 years old.
- It has two candidate, A and B
- The candidate A is 39 years old, has two children 17 years and 11 years and its actual on one year specialist's salary is $ 36000.
- The candidate B is 37 years old, has two children 16 years and 13 years and its actual on one year specialist's salary is $ 44000.
- Write down the formula of the total cost for one year for every candidate. Denote "x" current one year salary, "y" three years' experience and z_1; z_2 children's age.
- Compute the cost of the contract for one year for each candidate.
- Compute the total cost of the contract for five years for each candidate.
-
- Formula $C = x + 0.03x + 12\frac{y-35}{3}150 + 12(18 - z_1)60 + 12(18 - z_2)60$

Graphs of linear functions

	A			B		
Year	Y	Z_1	Z_2	Y	Z_1	Z_2
1	39	17	11	37	16	13
2	40	18	12	38	17	14
3	41		13	39	18	15
4	42		14	40		16
5	43		15	41		17

Linear Equation and Functions

- **Key words**
- Function; Domain; Range; How to find it? Which is difference between Domain and Solution set
- Function given by a: Formula; Piece wise; By Parts; Graph; Table
- Linear Functions; Their Graph; Line. Positions of line on the plane. Intersection of lines.
- Inverse Function; How to find it? Properties
- Properties of the functions: Boundedness; Continuity; Rate of Change
- Increase; Decrease; Local and Absolute Extreme
- Elementary functions: Properties; Their Graphs; Sketch their graph.
- Elimination method; substitution method; graphical method.
- Linear equation and inequations. solutions
- Linear inequations of two variables. Graphical solution of system of linear inequationsof two or more inequalities.
- Intersection's lines, parallel lines, coincides lines

Note

- **On every key word add the question “what is ” and try answer it**

Thank   You