Percentages

- The percentage is "**a part**" of "**a unit**". Percentages means per cent, per hundred or for every hundred. This may be greater than unit (increased) or smaller than unit (decreased).
- When we want to find the percentage of a number (considering as a unit) we just multiply the percentage with this
- number. Example. Find 45% of 74; 68% of 36; 108% of 82; 168% of 324
- Solution 45% = $\frac{45}{100}$ 45% of 74 is $\frac{45}{100} \times 74 = \frac{3330}{100}$ = 33.3
- 108% = $\frac{108}{100}$ 108% of 82 is $\frac{108}{100} \times 82 = \frac{8856}{100}$ = 88.56
- In general, when we speak about **"n percent**" of something **we simply mean the fraction** $\boldsymbol{\frac{n}{100}}$
- Whatever any numerical quantity increased, or decreased economics usually refer to this change in percentages terms.
- Example. The investments of a company raised from \$4768 to \$5364. Express the increase as a percentage
- Find at first the difference 5364 – 4768 = 596. As a fraction of the original, it is $\frac{596}{4768}$ = 0.125 or 12.5%. So the
- percentage of rise is 12.5%
- The price of a product is reduced 18%. Find the new price if the previous price was \$4326. Find at first 18% of 4326 which
- is $\frac{18}{100} \times 4326 = 0.18 \times 4326 = 778.68$ so the new price is 4326 – 778.68 = 3547.32

Percentages

- Problem. A firm's annual sales rise from 50000 to 55000 from one year to the next. Express the rise as a percentage
- of the original. Solution. 55000 – 50000 = 5000 $\frac{5000}{50000}$ = 0.10 or 10%
- The government imposes a 15% tax on price of a good. How much does the consumer pay for a good priced by firm at
- $1360. Solution. Find 15% of the price which is 1360$\times$ 0.15 = 204 the new price is 1360 + 204 = 1564
- If we consider the price before tax as 100% the new price is 100% + 15% = 115% so the new price is 1360$\times$ 115%
- or 1360 $\times \frac{115}{100}$ = 1564. we have the same result multiplying with 1.15 the previous price. This number
- 1.15 = 1 + 0.15 = $\frac{100}{100} + \frac{15}{100}$ = 1 + $\frac{15}{100}$ = 1.15 is called **scale factor.**
- Investments fall during the course of a year by 7%. Find the value of an investment at the end of the year if it was
- worth $9500 at the beginning of the year
- Find at first 7% of 9500 9500$\times$0.07 = 665 then 9500 – 665 = $8835
- If we consider the investment at the beginning of the year as 100% and falls are 7% that means the falls are 93%.
- so 9500$\times$93% = 9500$\times \frac{93}{100}$ = 8835. Is the same result.
- Number 0.93 = 1 – 0.07 = $\frac{100}{100} - \frac{7}{100}$ = 1 - $\frac{7}{100}$ = 0.93 is called **scale factor.**

Percentages

- In general, we can say that **the scale factor for increasing in r% is** $\mathbf{1 + \frac{r}{100}}$
- In general, we can say that **the scale factor for decreasing in r% is** $\mathbf{1 - \frac{r}{100}}$
- If we know the previous data, we can find the after data (going **forward** in time) just **multiply by scale factor**
- If we know the after data, we can find the previous data (going **backward** in time) just **divide by scale factor**
- **Previous data scale factor After data Previous data scale factor After data**
- **multiply scale factor ? ? scale factor divide**

- Problem. a) The value of a good rises by 13% in a year. If it was worth \$6.5million at the beginning of the year,
- find its value at the end of the year.
- Solution. Scale factor is $1 + 0.13 = 1.13$ so $1.13 \times 6.5 = 7.345$ million
- b) The GNP of a country has increased by 63% over the past five years and is now \$124 billion. What was the GNP five years ago?
- Solution. Scale factor is $1 + 0.63 = 1.63$ so (going backward) $124 \div 1.63 = 76.07$ billion.
- c) sales rise from 115000 to 123050 in a year. Find the annual percentage rise.
- Solution. The annual rise is $123050 - 115000 = 8050$ as percentage is $(8050 \times 100) \div 115000 = 805000 \div 115000 = 7$
- the rise was 7%

Percentages

- a) Current monthly output from a factory is 25000. In a recession, this is expected to fall by 65%. Estimate the new level
- output.
- Solution. Scale factor is 1 – 0.65 = 0.35 so (going forward in time) the new level is 25000 $\times$ 0.35 = 8750
- b) As a result of a modernization program, a firm can reduce the size of its workforce by 24%. If it now employs
- 570 workers, how many people did it employ before restructuring?
- Solution. Scale factor is 1 – 0.24 = 0.76 so (going backward) 570 $\div$ 0.76 = 750.
- c) Shares originally worth \$10.50 fall in a stock market crash to \$2.10. Find the percentage decrease.
- Solution. The difference of prices is 10.50 – 2.10 = 8.4 and in percentage it is 8.4 $\div$10.5 = 0.8 or 80%.
- The ratio of prices give us $\frac{new\ price}{old\ price} = \frac{2.10}{10.50}$ = 0.20 . This can consider as 1 – 0.80 = 1 - $\frac{80}{100}$. So, the fall is 80%
- The final application of scale factors is calculation of the percentage changes. On the different periods of time we
- often change the price of a good having several individual percentage changes. Is more useful to replace these by an
- equivalent single percentage for the entire period.
- This can be done **simply multiplying together successive scale factors.**

Percentages

- There are three variables, the first percentage, the second percentage and the single percentage. If we know two of them, we can find the third.
- Problem. Find the single percentage increase or decrease equivalent to
- a) an increase of 30% followed by an increase of 40%.
- solution. The scale factor of the first increasing is 1 + $\frac{30}{100}$ = 1.30 and for the second increasing is 1 + $\frac{40}{100}$ =1.40
- the single percentage is 1.30$\times$ 1.40 = 1.82 which can be thought as 1 + $\frac{82}{100}$ so increasing is 82% (different from 30% + 40%)
- b) a decrease of 30% followed by a decrease of 40%.
- Solution. At the same way find scale factors which are 1 – 0.30 = 0.70 and 1 – 0.40 = 0.60. After multiplication have
- 0.70$\times$ 0.60 = 0.42 or 1 - $\frac{58}{100}$ so we have decrease 58%
- c) an increase of 10% followed by a decrease of 50%
- Solution scale factors are 1 + 0.10 = 1.10 and 1 – 0.50 = 0.50. After multiplication we have 1.1 $\times$ 0.5 = 0.55
- If the result is **greater than one have increase** and if the result is **less than one have decrease**. In our case have
- decrease 0.55 = 1 - $\frac{45}{100}$ so decrease is 45%

Percentage

- Economic data often take the form of a time series; values of economic indicators are available on an annual, quarterly or monthly bases, and we are interested in analyzing the **rise** and the **falls** of these numbers over time. **Index numbers** enable us to identify trends and relationships in the data.
- Using the example below let's see how we calculate the index number taking a period as base and how interpret the result discovering the trends and relationships in data.

	Year				
	1	2	3	4	5
Household spending	686.9	697.2	723.7	716.6	734.5

Choose as a base year 2, so the index number of Y2 is 100. to find the index number of Y3 we use scale factor related Y2 (the base)and multiply by 100. **index number = scale factor from base year× 100**. In our case $\frac{723.7}{697.2} \times 100 = 103.8$

- For the Y4 the value is $\frac{716.6}{697.2} \times 100 = 102.8$
- During the year 3 the household spending increased 3.8%. During year 4 household spending increased 2.8% between Y2 and Y4. Comparing with Y3, is fact that spending fell slightly during Y4.

Percentage

- **Index numbers have no units**. They merely express the value of some quantity as a percentage of a base number. It enable us to compare how values of quantities change in relation to each other.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug
Share A	0.31	0.28	0.31	0.34	0.40	0.39	0.45	0.52
Share B	6.34	6.40	6.45	6.52	6.57	6.43	6.65	7.00

- if invested \$1000 on share A on January could have bought 3225 ($1000 \div 0.31 = 3225$) of them and in August their value is \$1677 ($3225 \times 0.52 = 1677$) the profit is \$677
- If invested on share B \$1000 ($1000 \div 6.34 = 157.7$) and their value on, August is \$1104.1 ($157.7 \times 7 = 1104.1$) the profit is \$104.1
- Example

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug
Index of share price A (base Apr)	91.2	82.3	91.2	100	117.6	114.7	132.4	152.9
Index of share price B (base Apr)	97.2	98.2	98.9	100	100.8	98.6	102.0	107.4

Percentages

- From the tables of the index numbers, we can find the percentage changes between any pairs of values. Compare in
- percentage prices A and B on period March- Jun.
- 114.7÷ 91.2 = 1.25 = 1 + 0.25 increase 25% 98.6÷ 98.9 = 0.99 = 1 – 0.01 decrease 1%
- Exercises page 211

Percentages

- Key words
- Percentage
- Scale factor
- Going forward in time
- Going backward in time
- Base data
- Index number

- Note
- For every key word ask the question “what is” and try to answer it