Indices and Logarithms

- In the expression $M = b^n$ we understand the multiplication of "b" "n" times, or we say that b^n is the exponential form of M to base "b".
- "n" may be positive or negative number, may be integer or fraction.
- $b^n = b\times b\times b\times b\times\ldots\times b$ n times (total "n" b-s multiplied together) $b^4 = b\times b\times b\times b$
- $b^{-n} = 1/b^n$ $b^0 = 1$ $b^{1/n}$ is "n" th root of "b"
- $b^{1/n}$ n-th root of b $\sqrt[n]{b}$
- $9^{1/2}$ square root of 9 is 3 or minus 3 because
- $8^{1/3}$ cube root of 8 is 2 because
- $625^{1/4}$ fourth root of 625 is 5 or minus five because
- $b^{m/n} = \sqrt[n]{b^m}$
- Usually we find first "n" th root of "b", then the power "m"
- Rules of indices (multiplication, division, power of power, power of product, ratio)

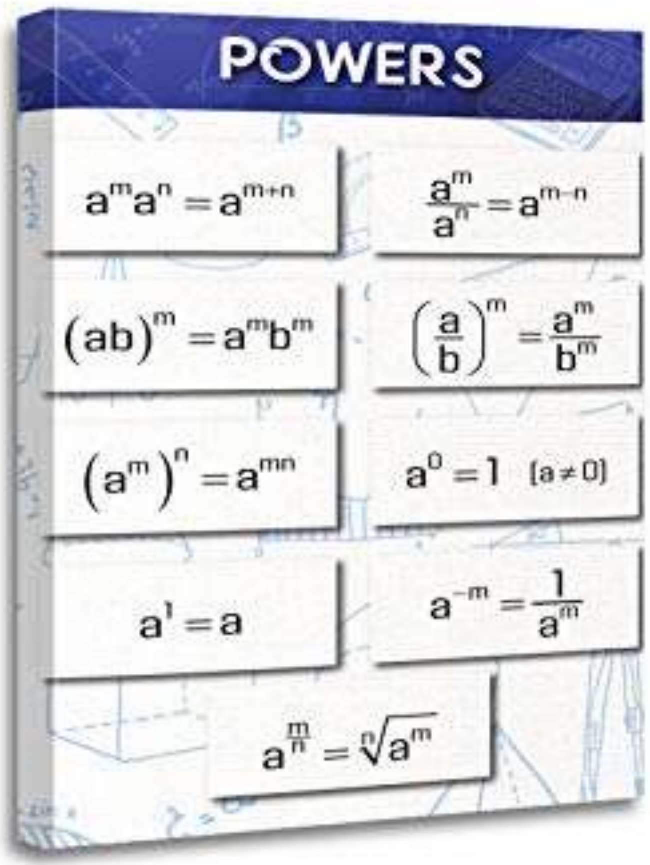
Indices and Logarithms

- There are economic problems which are solved used indices (powers) and logarithms. There are many factors influenced on the production of a good. They are known as **factors of production.** One of them is **Capital, K**, denotes all man-made aids to production such as buildings, tools and plant machinery. The other is **Labour, L**, denotes all paid work in the production process. The dependence of Q on K and L may be written as Q = f(K, L). This function is called a **production function**. For example, if
- Q = $100K^{1/3}L^{1/2}$then the inputs K = 27 and L = 100 lead to an output
- Q = $100(27)^{1/3}(100)^{1/2}$= 100(3)10 = 3000.
- These are with indices or powers. If capital and labor both double, what's happen with production level. To answer this question, we must compute Q based on the power's operation properties.
- Q = $100(2K)^{1/3}(2L)^{1/2}$= $2^{5/6}$($100K^{1/3}L^{1/2}$)= 1.78($100K^{1/3}L^{1/2}$). The capital and labor are double, but output is only 1.78 and not double.

Indices and Logarithms

- In general, we can write the formula as Q = $100(\lambda K)^{1/3}(\lambda L)^{1/2} = \lambda^{5/6}(100K^{1/3}L^{1/2})$ where λ is a scalar.
- If we can write the production function in form $f(\lambda K, \lambda L) = \lambda^{n} f(K, L)$ is said that this function is **homogeneous,** and the power "n" is called the **degree of homogeneity**.
- If $n < 1$, the function is said to display **decreasing returns to scale**.
- If $n = 1$, the function is said to display **constant returns to scale**.
- If $n > 1$, the function is said to display **increasing returns to scale.**
- Let's see now the powers formulas

Indices and Logarithms



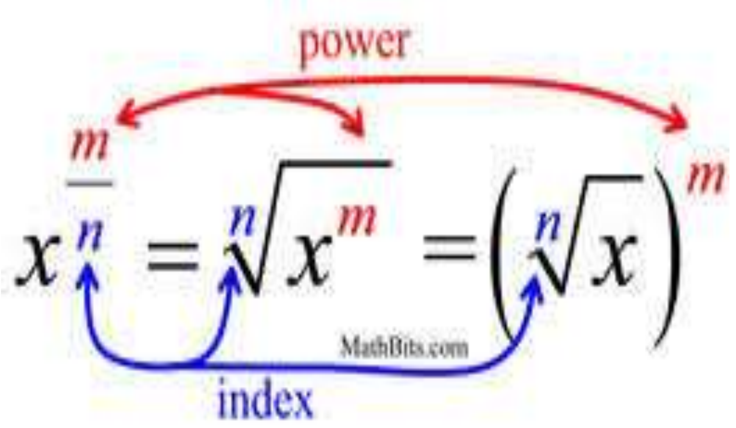
If a^m = a^n then m = n

if x^n = b^n then x = b

These formula are used to solve equations

Indices and Logarithms

- The index may be integer number or fraction.
- The formula below show how we compute the powers which have as index a fraction.
- Example $125^{\frac{2}{3}} = \sqrt[3]{125}^{2} = 5^2 = 25$ (at first, we find the root and then the power.
- For the production function Q = $200K^{1/4}L^{2/3}$find the output when
- K = 16; L = 27 and K = 10000; L = 1000
- Substituting the values get Q = 200· $16^{1/4} \cdot 27^{2/3}$
- Q = 200·2·9 = 3600



Indices and Logarithms

- Which of the following production functions are homogeneous? For those functions which are homogeneous write down their degrees of homogeneity and comment their return to scale .
- Q = $500K^{\frac{1}{3}}L^{\frac{1}{4}}$ Q = 3LK + L^2 Q = L + $5L^2K^3$
- f($\lambda K, \lambda L$) = $500(\lambda K)^{\frac{1}{3}}(\lambda L)^{\frac{1}{4}}$ = $\lambda^{\frac{7}{12}}$ $500K^{\frac{1}{3}}L^{\frac{1}{4}}$ = $\lambda^{\frac{7}{12}}$ f(K,L) degree homogeneity is $\frac{7}{12}\frac{7}{12}$< 1
- decreasing return to scale
- f($\lambda K, \lambda L$) = 3λLλK + λL^2 = λ^2 (3LK + L^2) = λ^2 f(KL) degree homogeneity is 2 2 > 1
- increasing return to scale
- f($\lambda K, \lambda L$) = λL + 5 $(\lambda K)^3(\lambda L)^2$ = λL + 5λ^5 $(K)^3(L)^2 \neq \lambda^n$f(K,L). This function is not homogeneous

Indices and Logarithms

- To solve the equation 3^x = 81 we must express 81 as power of 3 so $3^x = 3^4$ and x = 4.
- To solve the equation 3^x = 243 we must express 243 as power of 3 so $3^x = 3^5$ and x = 5.
- To find the unknown exponent "n" on expression a^n = b we use the logarithm and write n = log_ab. So we want to find how must be the power of "a" to give us "b". This is invers process
- of exponentiation. In exponentiation we know "a" ; "n" and find "b".
- In logarithms we know "a" ; "b" and find "n"
- Example $log_3 81$ = 4 because 3^4 = 81
- log_2(3x - 5) = log_2(2x - 2) 3x – 5 = 2x – 2 or x = 3
- To solve the equation 5^x = 16 we take logarithms of both sides
- (the same base, usually we use as base number 10 and
- log_{10}a denote simple as loga).
- log5^x = log16 or xlog5 = log16 or x = $\frac{log16}{log5}$ = $log_5 16$
- Exercises page 169.

Logarithmic Properties	
Product Rule	$\log_a(xy) = \log_a x + \log_a y$
Quotient Rule	$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$
Power Rule	$\log_a x^p = p\log_a x$
Change of Base Rule	$\log_a x = \frac{\log_b x}{\log_b a}$
Equality Rule	If $\log_a x = \log_a y$ then $x = y$

Indices and Logarithms

- Exponential function is
- y =a^x It is decreased function for 0 < a <1 and increased for a > 1. The value of “y” is positive number and never zero.
- Inverse of this function is logarithmic function which is determined for positive values of “x”. It may be positive or negative number, but never zero
- Exercise nr 3 page 168
- nr 6 ; 7 ; 8; 9; 10 ; 11 ; 12 page 169
- nr 8 ; 12 page 171. Find values
- using your calculator.
- 3^3 $2^{0.4}$ $5^{0.8}$ $7^{3.6}$ $4^{3,12}$
- log 10, log 10000, log23, log 738
- log114.26