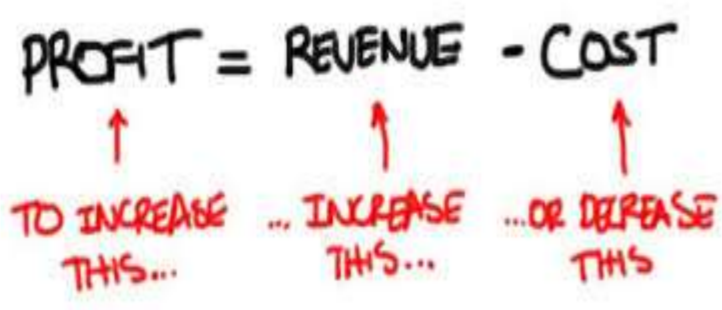
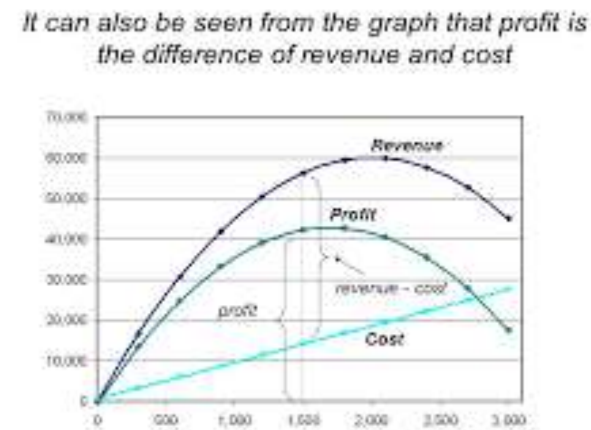
Revenue Cost and profit

- Economists denotes by **P** the price and with **Q** the quantity
- **TR** total revenue (is the amount of money received by the firm from the sale of its goods)
- **TC** total cost (is the amount of money that the firm must spend to produce these goods)
- **FC** fixed cost (include the cost of land, equipment, rent and possibly skilled labour)
- **VC** variable cost (include raw materials, components, energy and unskilled labour)
- **TVC** total variable cost.
- **AC** average cost.
- **π** Profit (must maximize) $\pi = TR - TC$. This is the profit function. We can maximize this function in two ways. 1. **Maximizing** the **TR** 2. **Minimizing** the **TC**.
- We know that $TR = PQ$ (price times quantity) and demand function $P = aQ + b$
- where $a < 0$ and $b > 0$
- So, we can write $TR = PQ = (aQ + b)Q = aQ^2 + bQ$. This is quadratic function with variable Q and has an inverted U shape. $TVC = (VC)Q$ and $TC = FC + (VC)Q$. This is an important economic function

Revenue Cost and profit

- 



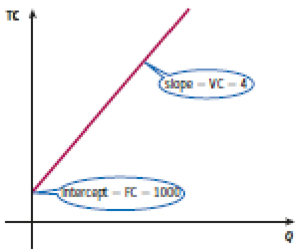
- There is another element important on
- studies of the economic problems known as average cost function
- **AC** average cost (it is obtained by dividing the total cost by output).
- AC = TC/Q . This formula is like y = a/x which is hyperbola (rectangular hyperbola) and sometimes economists named as **L-shaped**

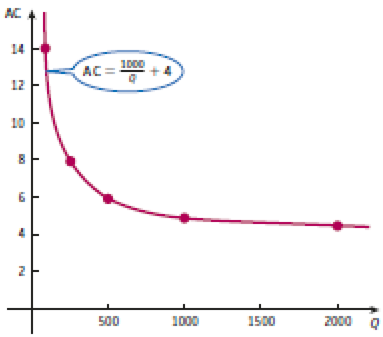
Revenue Cost and profit

- TR (total revenue) is P times Q TR = PQ. It is received from the sale of Q goods at price P.
- If price is given by demand function, then TR = f(Q)Q where f(Q) is demand function.
- Example. Demand function is P = - 3Q + 42 then TR = (- 3Q + 42)Q. We can sketch the graph
- of this function with independent variable Q.
- TC (total cost) is the sum of fixed costs and variable costs TC = FC +T(VC)
- VC (variable cost) is a function of product. If denote as VC the variable cost per unit of output, then the total variable cost is TVC = (VC)Q.
- Finally, TC = FC + (VC)Q.
- AC (average cost) is obtained by dividing the total cost by output AC = $\frac{TC}{Q} = \frac{FC+(VC)Q}{Q}$
- TC is linear function of Q
- TR is quadratic function of Q.
- AC is an L shape hyperbola.

Revenue Cost and profit

- Total cost graph is a line
- Average cost graph is a rectangular hyperbola (y = $\frac{a}{x}$)



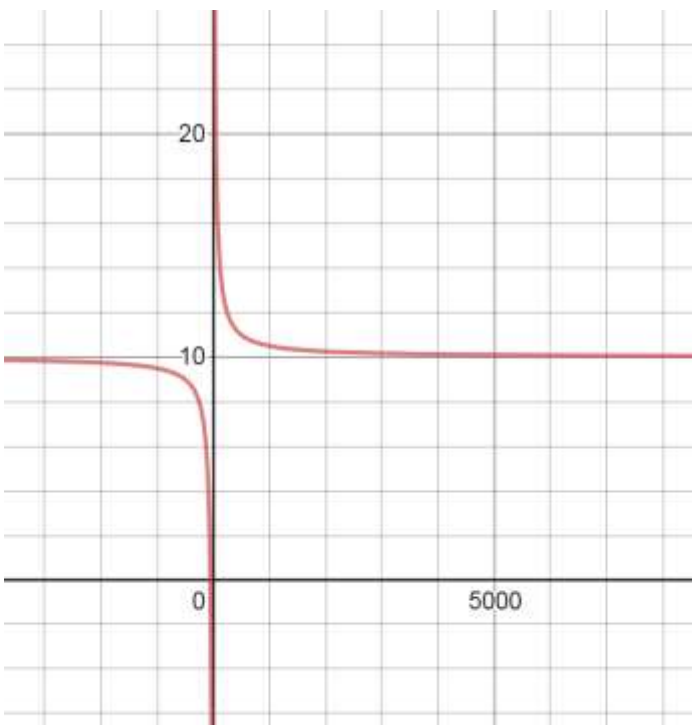


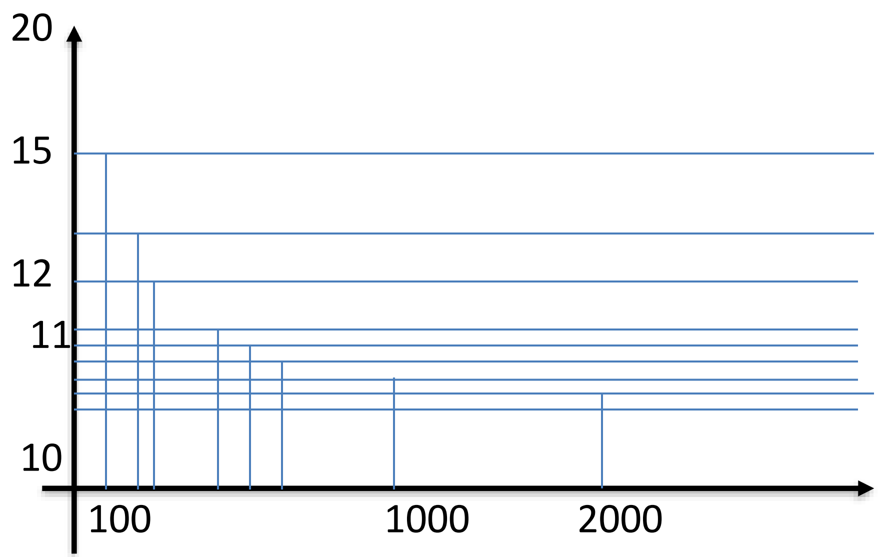
- Total cost function TC = FC + (VC)Q on this case FC is constant. Intercept is FC and slope VC
-
- Average cost AC = $\frac{TC}{Q}$ = $\frac{FC+(VC)Q}{Q}$ = $\frac{FC}{Q}$ + VC
- Total cost function TC is an important economic function, but it can not used to compare two individual firms. For this we can use the average cost function. The graph of this function is hyperbola (L shape). From the graph we see that AC decreased as the quantity (the number of units produced) increased, but it is not linear. To sketch this graph, we use the data from the table, plotted them on the graph and then join the points together.

Revenue Cost and profit

- Example. Given that fixed costs are 500 and that variable costs are 10 per unit, express TC and AC as functions of Q. Sketch their graphs.
- TC = FC + (VC)Q   TC = 500 + 10Q   AC = $\frac{TC}{Q} = \frac{FC+(VC)Q}{Q} = \frac{FC}{Q}$ + VC   AC = $\frac{500}{Q}$ + 10

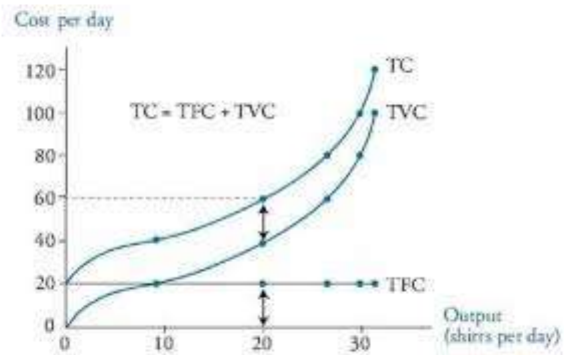
Q	100	200	250	400	500	800	1000	2000
AC	15	12.5	12	11.25	11	10.625	10.5	10.25

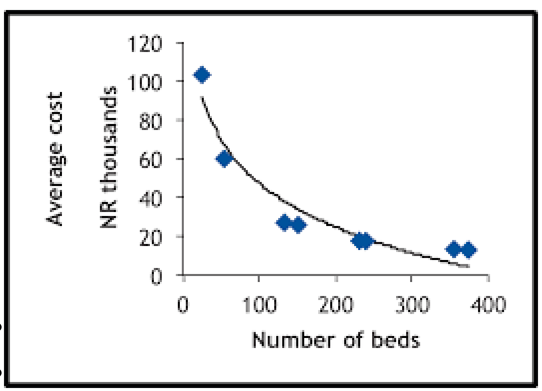


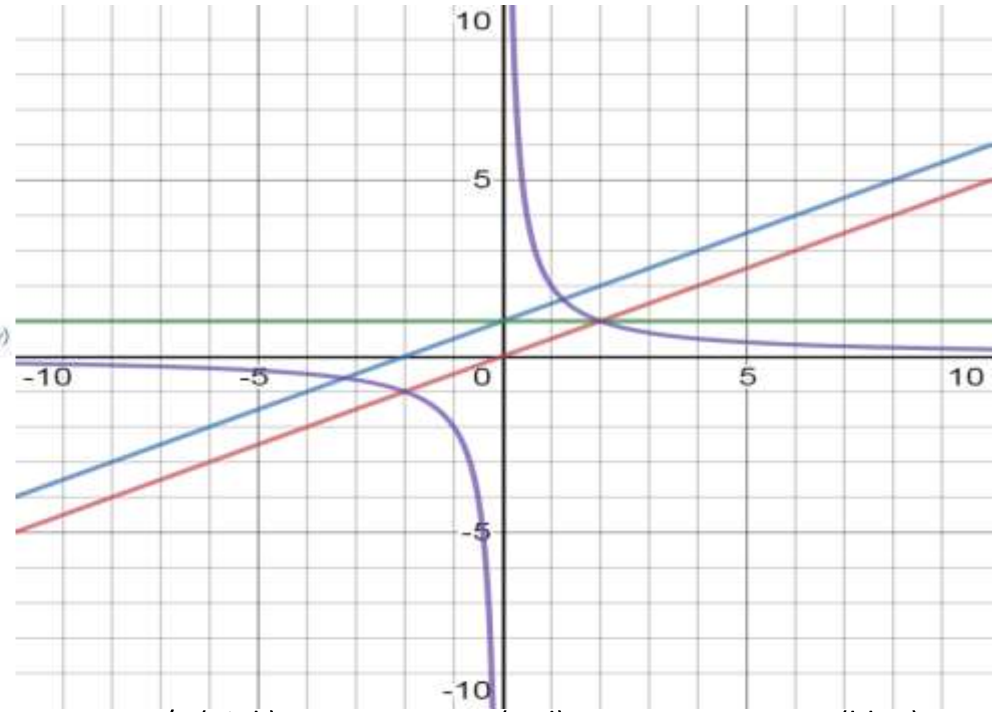


Revenue Cost and Profit

- TC = FC + (VC)Q AC = (TC/Q) = [FC + (VC)Q]/Q = (FC/Q) + VC.
- Total cost is linear function with variable Q. Average cost is a hyperbola (L-shaped) with variable Q.
- Total cost line TC = FC + (VC)Q has intercept FC and slope VC.
- The hyperbola curve of the average cost decreased as quantity increased.







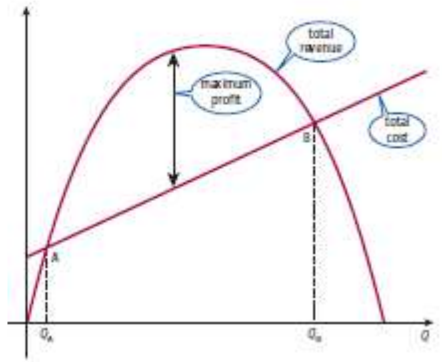
y = 2/x (pink) y = 0.5x (red) y = 0.5x + 1(blue)

average cost function total cost function

Revenue Cost and profit

- Total revenue TR = PQ P = aQ + b (demand function) so TR = aQ^2 + bQ a < 0 (quadratic function)
- Total revenue has "a" negative, so the graph has an inverted U shape.
- Total revenue has the constant term zero (quadratic function), so the graph crosses the TR axis at the origin.
- For the two functions TR and TC, the horizontal axis represents quantity, Q.
- As for the revenue function, Q denotes the **quantity** of goods sold, for the cost function it denotes the **quantity produced**. Sketching them at the same diagram we suppose that the firm **sells all**, the goods that it produces.

On this graph we see two intersection points cost and revenue are equal and the breaks even.



if the quantity is smaller than Q_A or greater than Q_B TC curves lies above TR curve, so cost exceed revenue and the firm make a loss. If Q_A< Q < Q_B TC curves lies below, cost is smaller than revenue. The firm makes a profit. Maximum profit is when the distance between two curves (for the same Q) is the largest.

Revenue Cost and Profit

- 1. a) If the demand function of a good is given by $P = 80 - 3Q$ find the price when $Q = 10$, and the total revenue.
- Solution. For $Q = 10$ $P = 80 - 3 \cdot 10 = 50$. $TR = PQ = 50 \cdot 10 = 500$.
- b) If fixed costs are 100 and variable costs are 5 per unit, find the total cost when Q=10
- $TC = FC + (VC)Q$ $(VC)Q = 5 \cdot 10 = 50$ $TC = 100 + 50 = 150$
- c) Find the corresponding profit $\pi = TR - TC = 500 - 150 = 350$.
- 2. Express TR as a function of Q and sketch their graphs if $P = 4$; $P = 7/Q$; $P = 10 - 4Q$
- Solution. $TR = P \cdot Q$ $TR = 4 \cdot Q = 4Q$ $TR = P \cdot Q = \frac{7}{Q} Q = 7$ $TR = (10 - 4Q)Q = 10Q - 4Q^2$
- Analogy $y = 4x$, $y = \frac{7}{x} x = 7$ and $y = -4x^2 + 10x$

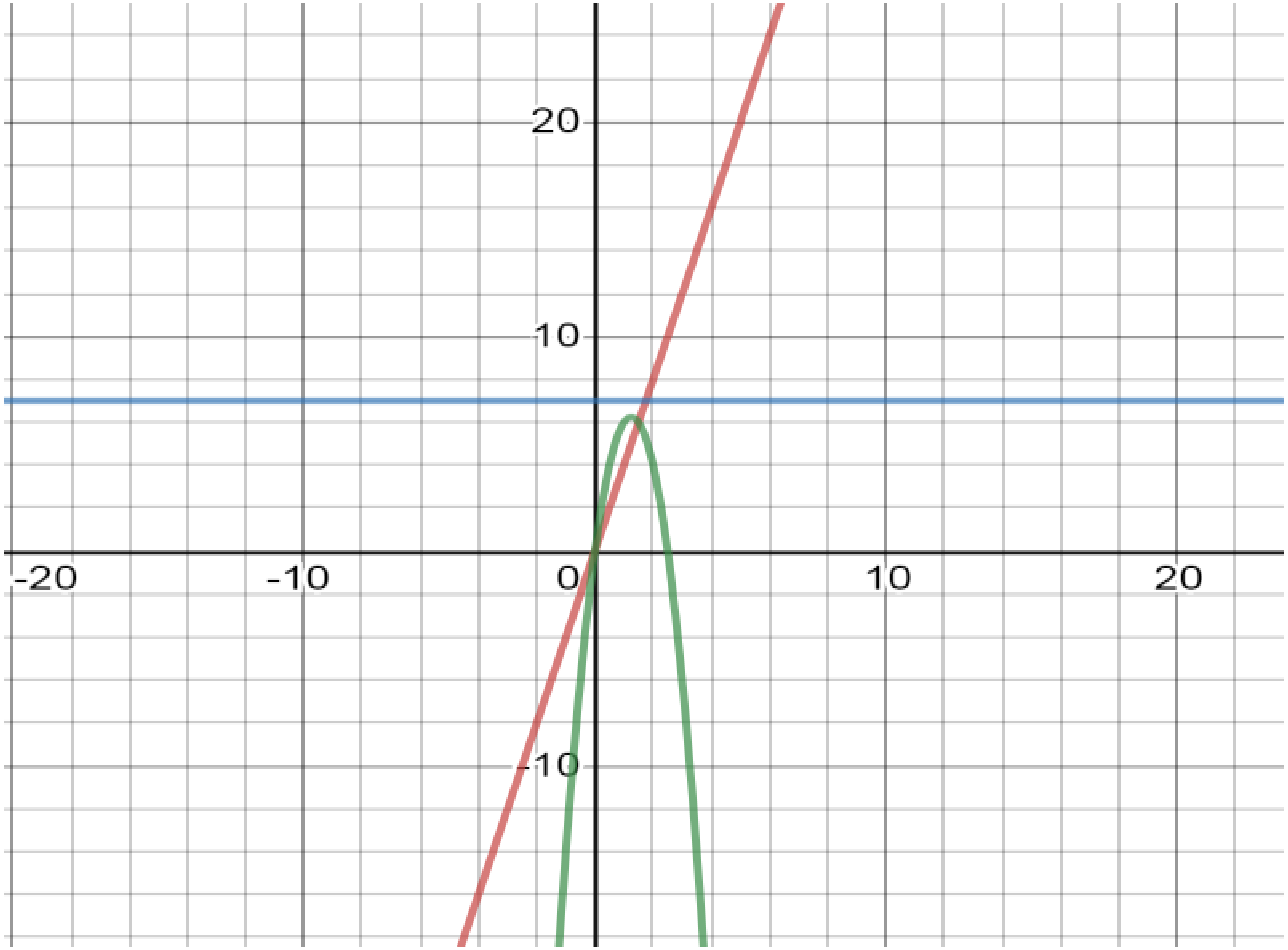
3. Sketch , on the same diagram, graphs of the total revenue and total cost functions.

$TR = -2Q^2 + 14Q$ $TC = 2Q + 10$

Use the graphs to estimate the values of for which the firm a) breaks even points b) maximize profit. TR = TC $-2Q + 14 = 2Q + 10$ or $-2Q^2 + 12Q - 10 = 0$ $Q = 1$; $Q = 5$

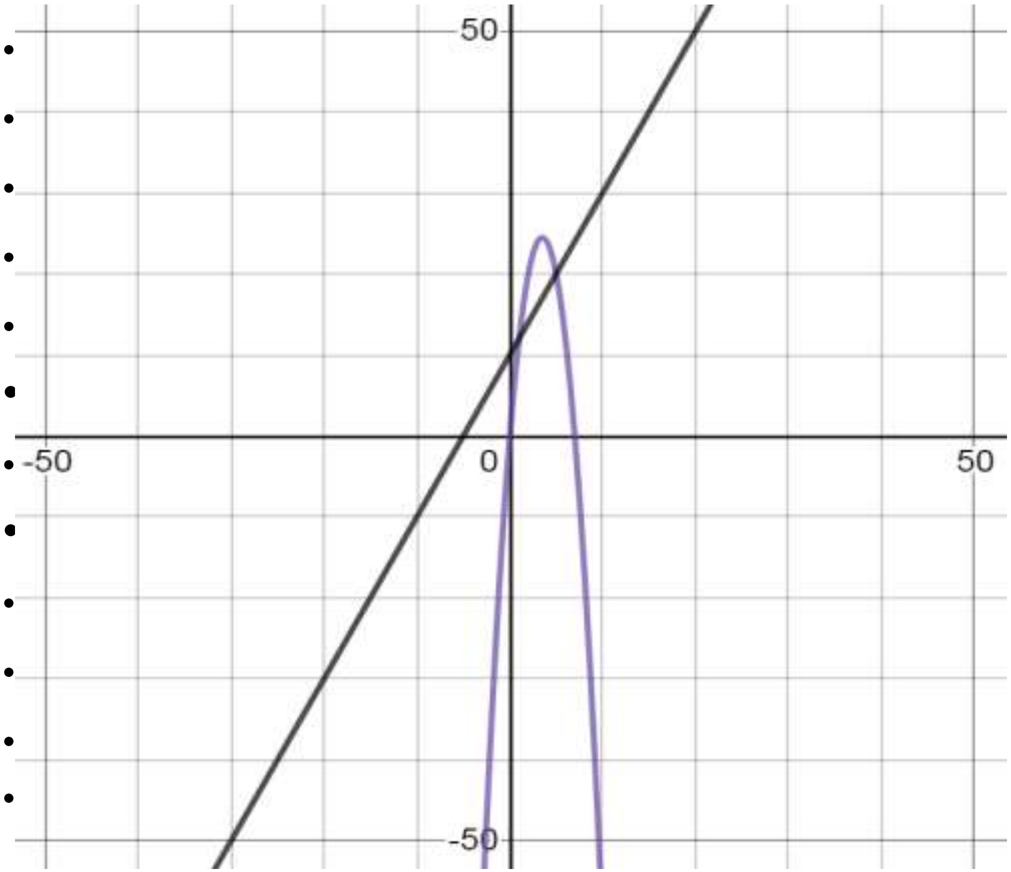
- $\pi = TR - TC = -2Q^2 + 14Q - 2Q - 10$ $\pi = -2Q^2 + 12Q - 10$
- $\pi' = -4Q + 12$ $Q = 3$ $\pi_{max} = -2(3)^2 + 12(3) - 10 = -18 + 36 - 10 = 8$
- $D = 12^2 - 4(-2)(-10) = 144 - 80 = 64$ $\pi_{max} = \frac{-D}{-4a} = \frac{-64}{4(-2)} = 8$

Revenue Cost and Profit

- 

Revenue Cost and Profit

- $y = -2x^2 + 14x$ $y = 2x + 10$ $-2x^2 + 14x = 0$ $x(-2x + 14) = 0$ $x = 0$ and $x = 7$



intersection with axis are points

(0;0) (7;0) (-5;0) (0;10)

intersections line and curve are

(1;12) and (5;20) which are

breaks even points.

maximum point of the curve is

(3.5;24.5)

maximum profit is for

$x = 3 = (1+5)/2 = 6/2$

TR for Q = 3 is $-2\cdot3^2 + 14\cdot3 = 24$

TC for Q = 3 is $2\cdot3 + 10 = 16$

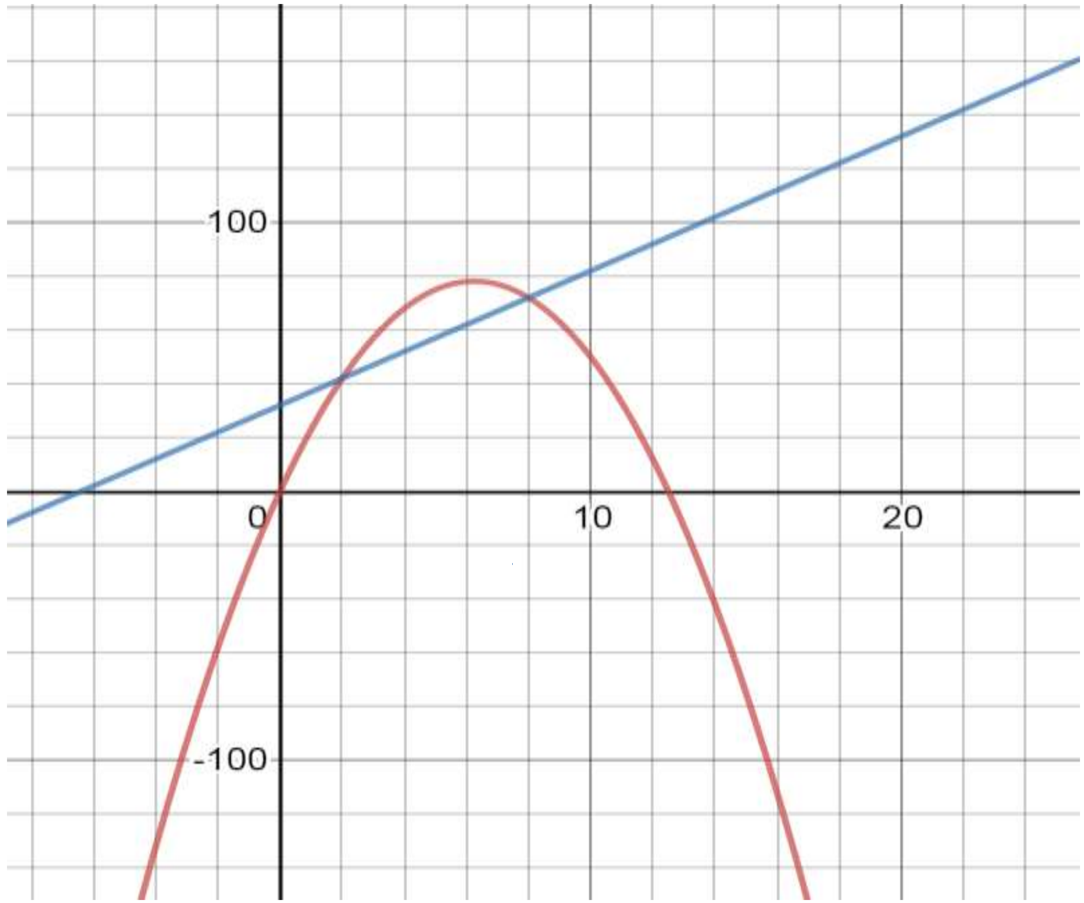
Maximum Profit is $24 - 16 = 8$

Revenue Cost and Profit

- Exercise nr 9 page 149 (ninth edition)
- Find an expression for the profit function given the demand function 2Q + P = 25 and the
- average cost function AC = $\frac{32}{Q}$ + 5.
- Sketch the graph of profit function.
- Find the values of Q for which the firm breaks even.
- Find the values of Q for which the firm makes a loss of 432 units.
- Find the values of Q for which the firm maximizes profit.
- Solution . π = TR – TC TR = PQ = (-2Q + 25)Q TC = FC + (VC)Q = 5Q + 32
- π =(- $2Q^2$ + 25Q) – (5Q + 32) = - $2Q^2$+ 20Q – 32
- For profit zero (breaks even points) Q has values 2 and 8.
- For profit - 432 find Q solving the equation - $2Q^2$+ 20Q + 400 = 0. The values of Q are
- -10 and 20
- Maximum of the profit is when π' = 0 or – 4Q + 20 = 0 or Q = 5

Revenue Cost and Profit

- The graphs of functions
- TR = (- $2Q^2$ + 25Q) and
- TC = 5Q + 32



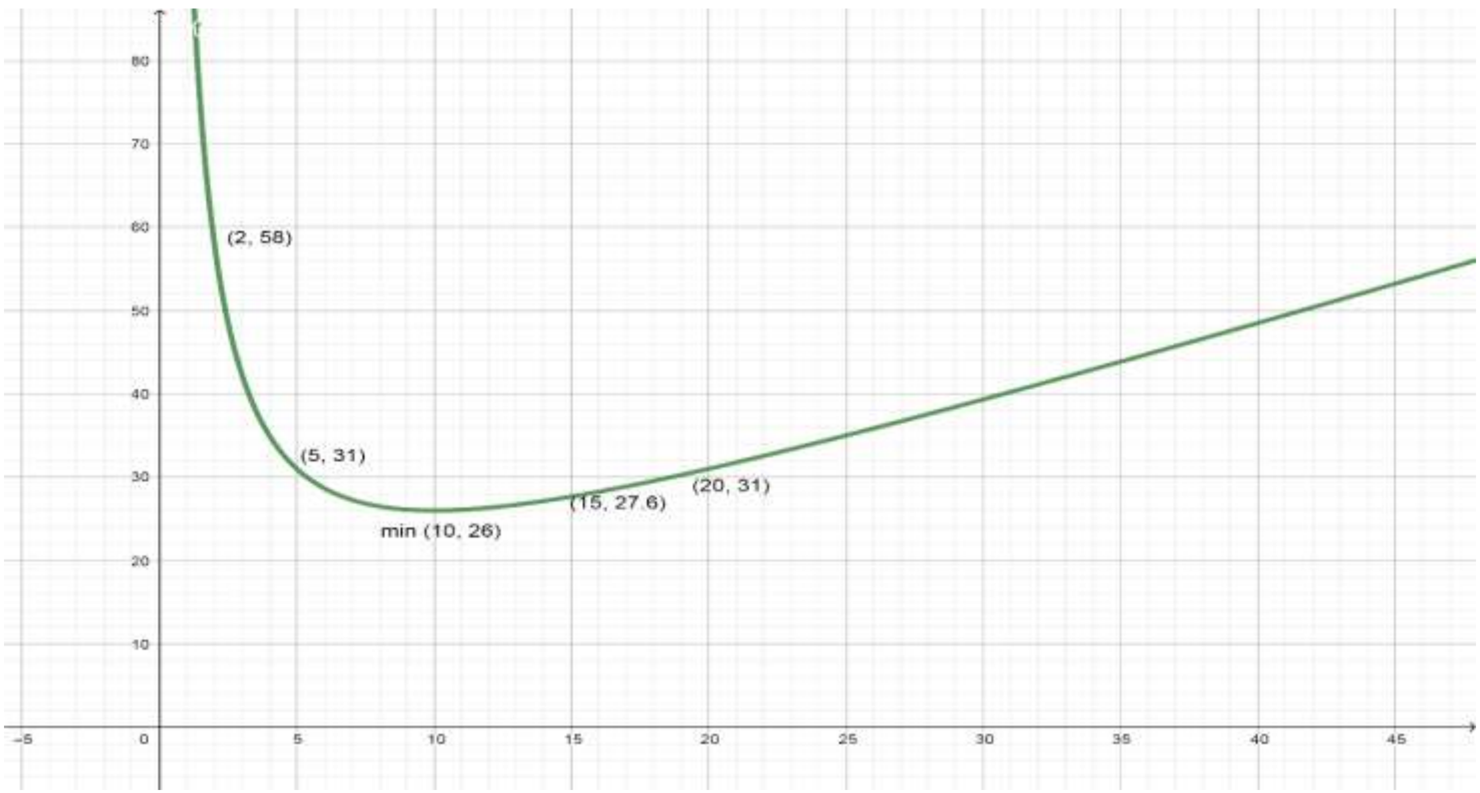
Revenue Cost and Profit

- A company has observed that when it produces 5 units the total cost is \$42, and when it produces 10 units the total cost is \$52. This company sells 10 units of its products at price \$5, and 6 units at price \$13.
- a) Find the total cost function (linear function).
- b) Find the demand function (linear function).
- c) Write the profit function.
- d) Sketch at the same diagram all these functions.
- e) Find the breaks even points.
- f) Find the maximum profit.

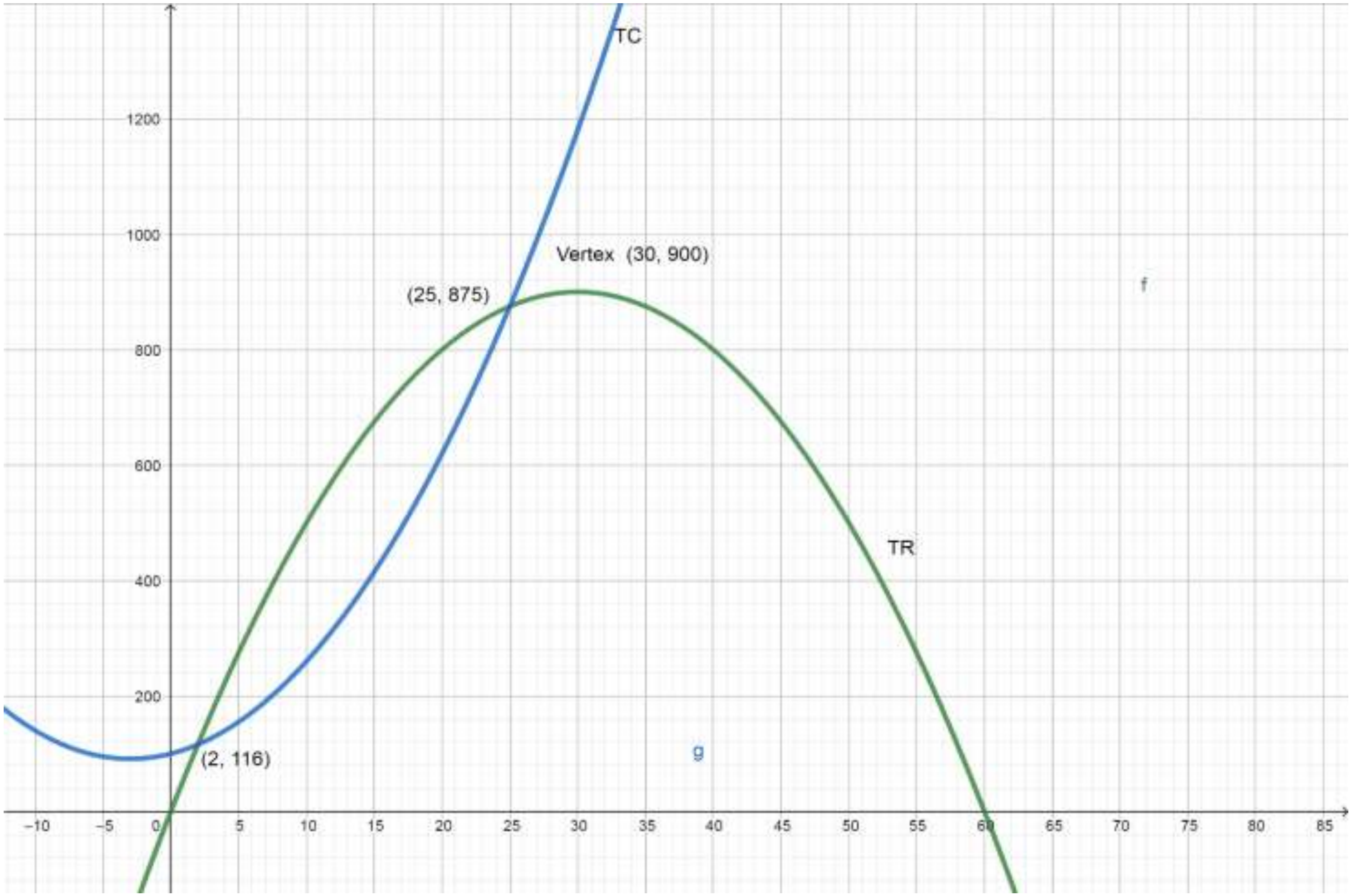
Revenue Cost and Profit

- Exercise. The demand function for a firm's product is given by P = 60 – Q
- fixed costs are 100, and the variable costs per good are Q + 6
- a) Write down an expression for total revenue, TR, and sketch the graph.
- show the intersections with coordinate axes.
- b) Write down an expression for total costs, TC, and find the average cost function. Find the value of AC if the values of Q are, 2, 5, 10, 15, 20. Plot these points on the coordinates system and sketch the graph.
- c) Write down an expression for the profit function, find break even points, determine the maximum profit, and sketch the graph of the profit function
- TR = (60 – Q)Q TC = 100 + (Q + 6)Q AC = $\frac{TC}{Q}$ = $\frac{100}{Q}$ + Q + 6
- π = TR – TC = (-Q^2 + 60Q) – (100 + Q^2 + 6Q) = - 2Q^2 + 54Q - 100

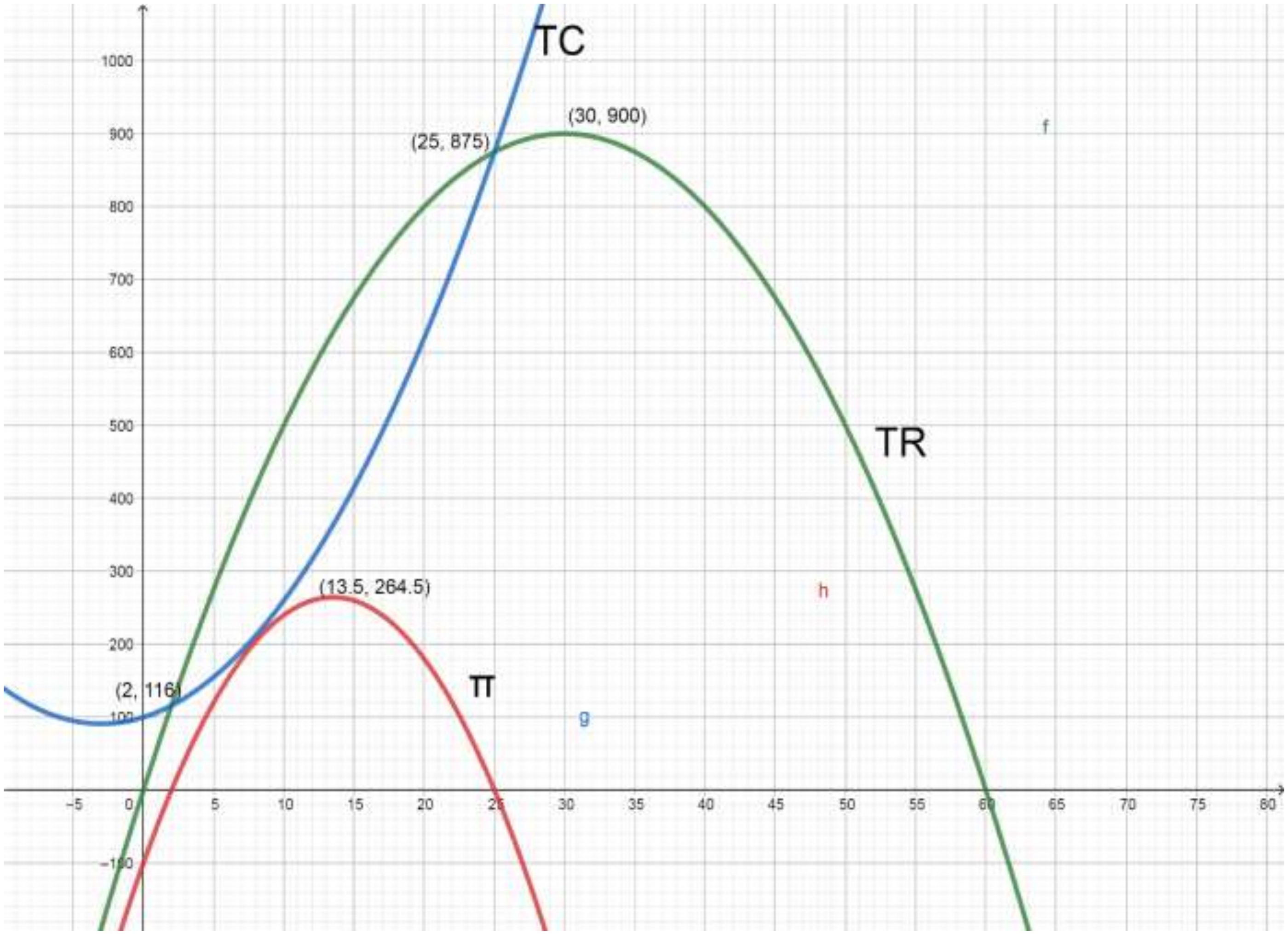
Revenue Cost and Profit



Revenue Cost and Profit



Revenue Cost and Profit



Revenue Cost and Profit

- Key words
- Profit. Formula. Ways of maximizing the profit
- Total Revenue. Notation. Formula.
- Total Cost. Formula. notation
- Fixed Cost. Notation. Examples.
- Variable Cost. Notation. Examples.
- Average Cost. Formula. Notation.
- Rectangular hyperbola. L-shaped
- Breaks even points $\pi = 0$ or $TR = TC$

- Note
- **On every key word add the question "what is" and try to answer it.**
- Thank You