

# Welcome

- **MATH BUS 101** is the code of the mathematic program. The name of the program is: Mathematics for Economics and Business I .
- Assistant Lecturer: MSc. Vasil Lino
- Office: E- 406
- Email: [vlino@epoka.edu.al](mailto:vlino@epoka.edu.al)
- Assistant: : Email: [@epoka.edu.al](mailto:@epoka.edu.al)
- Office hours: *Monday-Friday 8:30-16:30*

## Objectives of the Course:

- To provide students with solid training in fundamental theories in both mathematics and economics.
- To equip students with quantitative reasoning skills, conceptual understanding, and the ability to effectively communicate in mathematics and in the language of economics and social science.
- Finally, to use mathematic to solve real economics' problems

# Welcome

- Important topics
- Application of mathematics in economics and business
- Studying number patterns
- Manipulate numbers
- Calculate Production Costs
- Determine Pricing
- Measure and Optimize Profits
- Analyze Finances
- Evaluate investments
- Computing compound interests

# Welcome

- The main target of every economic activity are
- The **increasing** revenue and **decreasing** the cost
- The material in this course ranges from a revision of high-school mathematics to applications of calculus (single-variate, multivariate and integral) to economics and finance. For example:
- Linear and quadratic functions are introduced in the context of demand and supply analysis.
- Geometric sequences, exponential and logarithmic functions are introduced in the context of finance for calculations of loans and saves.
- Single-variate calculus is used on the course for explaining and solution of the optimization problem, like maximization of the firm's profit, or minimization the costs.
- A consumer's utility maximization is used to motivate introducing multivariate calculus.
- Integral calculus is explained in the context of calculating the deadweight loss of taxation, consumer surplus and producer surplus.
- What you will learn during the first semester on three chapters of the book.

# Welcome

- **Chapter 1: mathematics of linear equations**
- To apply the rules of arithmetic and algebra, how to manipulate mathematical expressions
- How to solve simultaneous linear equations, systems of two equations in two unknowns using different methods.
- Applications in microeconomics and macroeconomics; how to calculate equilibrium prices and quantities, national income determination
- **Chapter 2 mathematics of non-linear equations**
- Simplest non-linear equation, known as a quadratic, solutions and graphs
- Quadratic supply and demand functions, quadratic revenue and profit functions
- Indices(powers) and logarithms

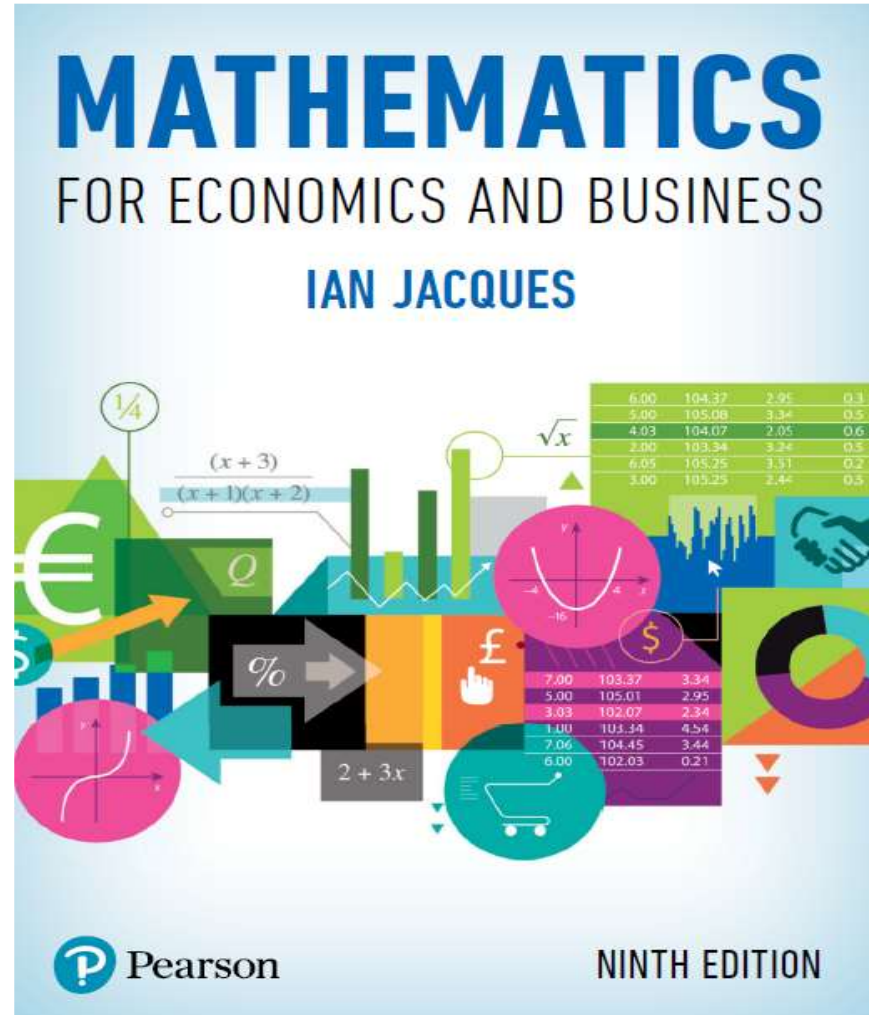
# Welcome

- Exponential and natural logarithm functions
- **Chapter 3 mathematics of finance**
- Percentages, Interest, Sequences, Present and future values, appraisal investments
- The book is MATHEMATICS FOR ECONOMICS AND BUSINESS
- IAN JACQUE NINTHH EDITION. 2018
- The book is ESSENTIAL MATHEMATIC FOR ECONOMIC ANALYSIS by Knut Sydsaeter
- Peter Hammond FIFTH EDITION 2016
- VASIL LINO MATEMATIKA PËR EKONOMINË DHE BIZNESIN 1
- Leksione për fakultetet ekonomike Tiranë, 2022

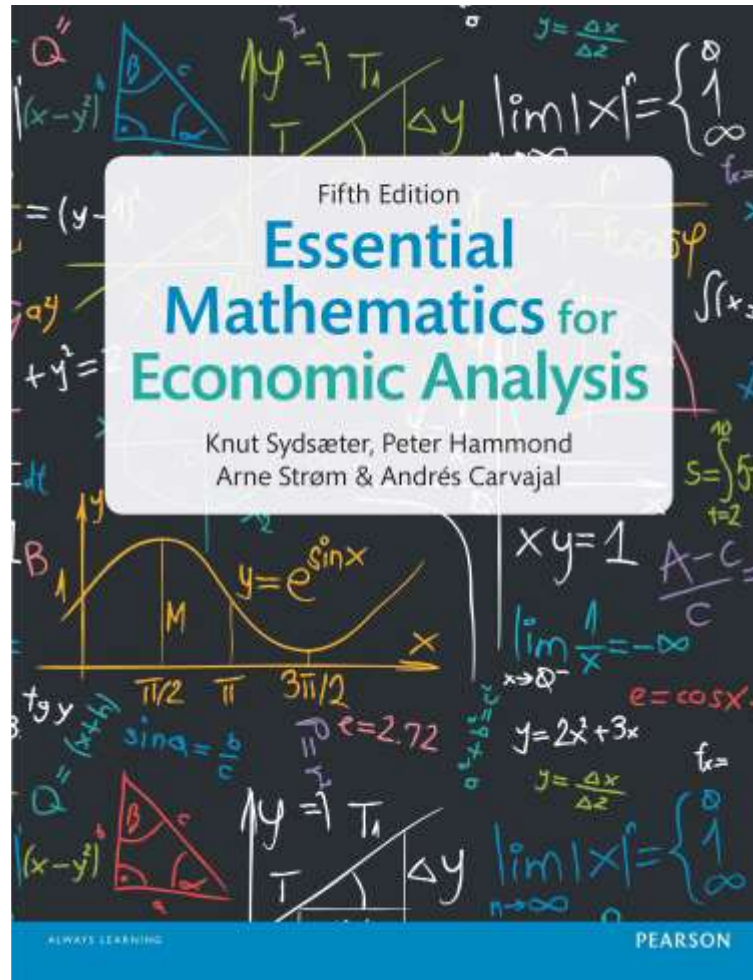
# How to use the book

- The text has nine chapters. Every chapter has sections. At the end of every
- section are key terms, exercises and starred exercises.
- Every section has its theoretical part and applications on examples, which are
- solved exercises. You can test yourself solving the practice problems.
- The non-starred exercises are designed to consolidate basic principles. The
- starred exercises are for the students with greater experience in mathematics.
- At the end of every chapter there are two types of tests. A multiple choice tests
- and some examination questions.
- If you read attentionally the theoretical material, solve the practice problems
- using solved examples and solve non-starred exercises you will get the solid
- base of the program MATH BUS 101.

# Front page



# Front page





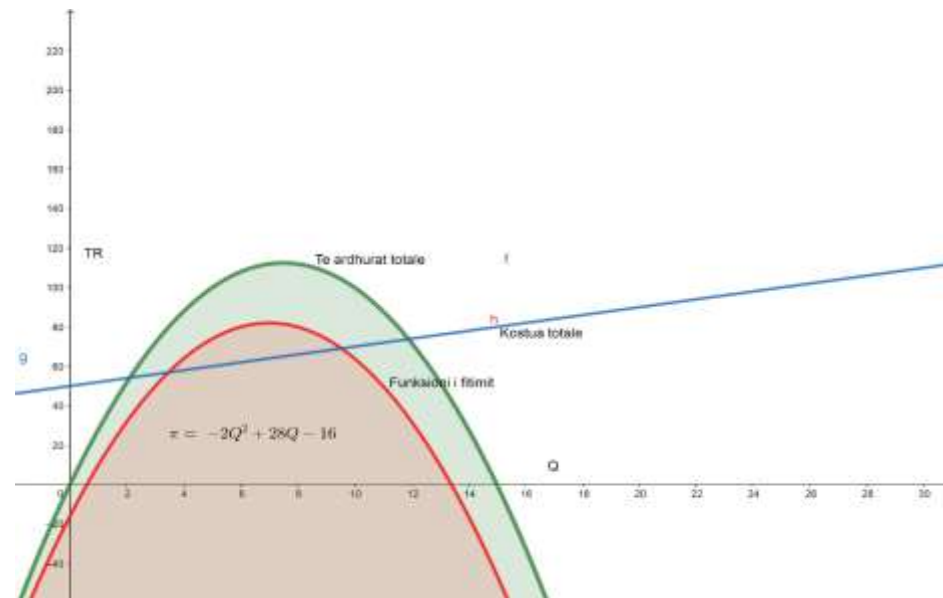
# Front page

VASIL LINO

MATEMATIKA PËR EKONOMINË

DHE BIZNESIN 1

LEKSIONE PËR FAKULTETET EKONOMIKE



Tiranë, 2022

# Syllabus

Week	Content
• 1.	Introduction to algebra, graphs of linear equations
• 2.	Solution of simultaneous linear equations. Transposition formula
• 3.	Supply and demand analysis. National income
• 4.	Quadratic functions
• 5.	Revenue cost and profit
• 6.	Indices and logarithms
• 7.	The exponential and natural logarithm functions
• 8.	<b>Midterm Exam</b>

# Syllabus

Week	Content
• 9.	Percentages
• 10.	Compound Interests
• 11.	Geometric series
• 12.	Investment appraisal
• 13.	Exercises
• 14.	Review
• 15.	<b>Final exam</b>

# Welcome

- **Grading**
- **1   Two TESTS: 5% each one**
- **2   Midterm Exam: 35%**
- **3   Final Exam: 45%**
- **4   Attendance & class performance :10%**

# Welcome

Letter grade	GPA value	Points	Description	Albanian grade system
AA	4.00	90-100	EXCELLENT	10
BA	3.50	85-89	EXCELLENT	9
BB	3.00	80-84	SUCCESS	8
CB	2.50	75-79	SUCCESS	8
CC	2.00	70-74	SUCCESS	7
DC	1.50	65-69	ON PROBITION	6
DD	1.00	60-64	ON PROBITION	5
FD	0.50	50-59	FAIL	4
FF	0.00	0-49	FAIL	0

# Introduction to algebra

SET is the **basic** concept of the mathematic

**The SET** is a group, a collection of well-defined objects or a collections of objects with a unique property , feature, characteristic.

So, we can speak for the set of the {Boys of our classroom} or the set of the {Students over 20 years old} , the set of odd numbers smaller than ten

$$A = \{1, 3, 5, 7, 9\}$$

A set is given if for every element we can show that **is member, belongs, is element** to the set or no using the notation  $x \in A$ . For our set above we can say that 7( $\in$ ) is element to the set and 4 ( $\notin$ ) is not element, member of this set.

There are two ways to represent the set: **describing** (saying the characteristic feature) and **specifying** (showing all the members of the set)

. We denote the sets with upper letters A, B, R, etc

$A = \{x \in \mathbb{N} / 7 \leq x \leq 12\}$   $B = \{7, 8, 9, 10, 11, 12\}$   $A = B$  .A set may be **finite, infinite(endless) or empty  $\phi$**

A part of a set is called **subset**

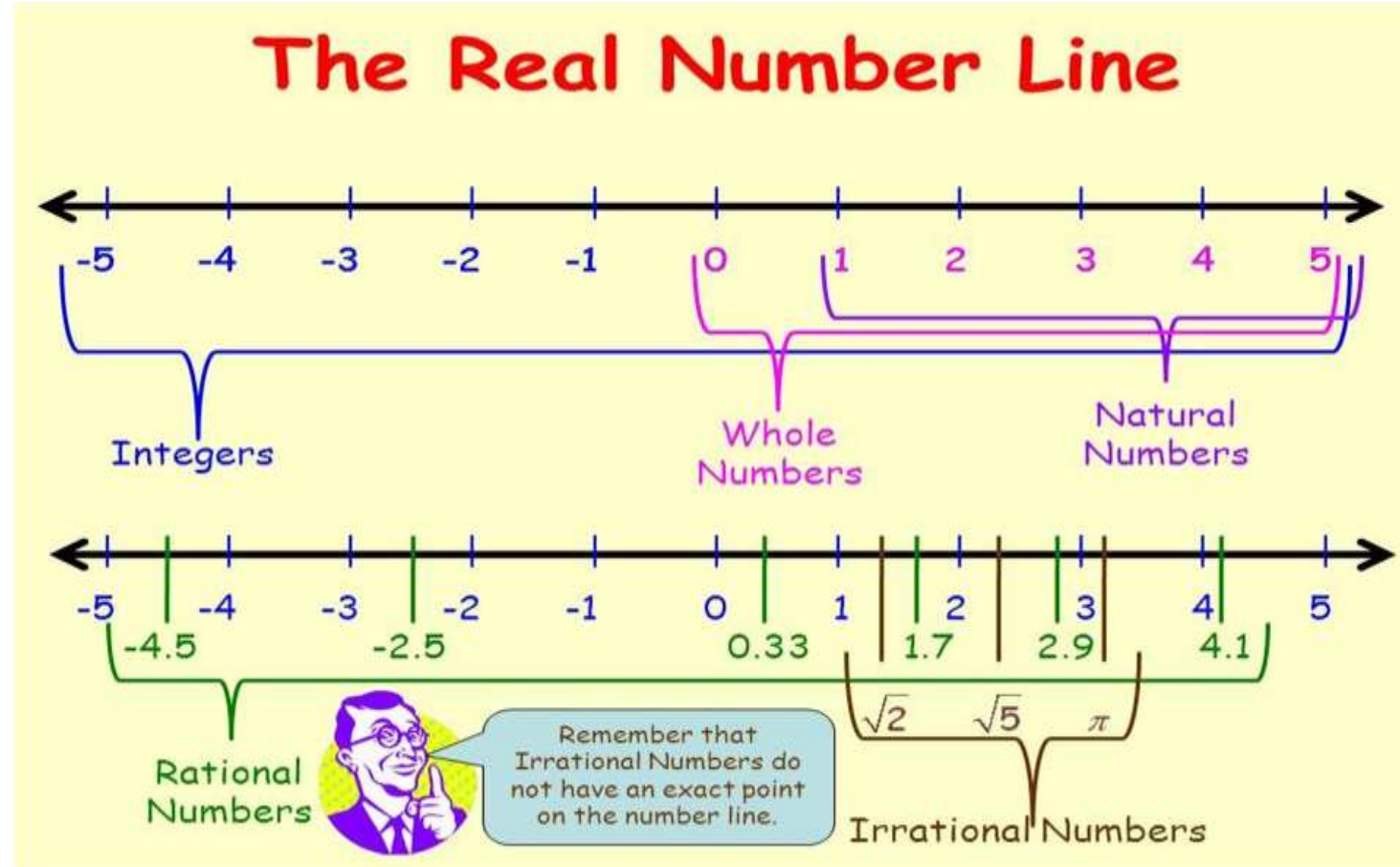
**Numerical sets** are

# Introduction to algebra

- **Natural numbers**  $\mathbf{N} = \{1, 2, 3, \dots\}$
- **Whole numbers**  $\mathbf{W} = \{0, 1, 2, 3, \dots\}$
- **Integers numbers**  $\mathbf{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$     **Rational numbers**  $\mathbf{Q} = \{Z; m/n \mid m, n \in Z; \text{decimal; decimal periodic (repeated its values at regular intervals) endless numbers} \}$
- **Irrational numbers**  $\mathbf{I} = \{\text{decimal non periodic endless numbers as } \pi, e, \sqrt{15}\}$
- **Real numbers**  $\mathbf{R} = \mathbf{Q} \cup \mathbf{I}$
- We can present all real numbers in a line called **real number line** or **axis** ( $x'x$ )

# Introduction to algebra

- Real number line

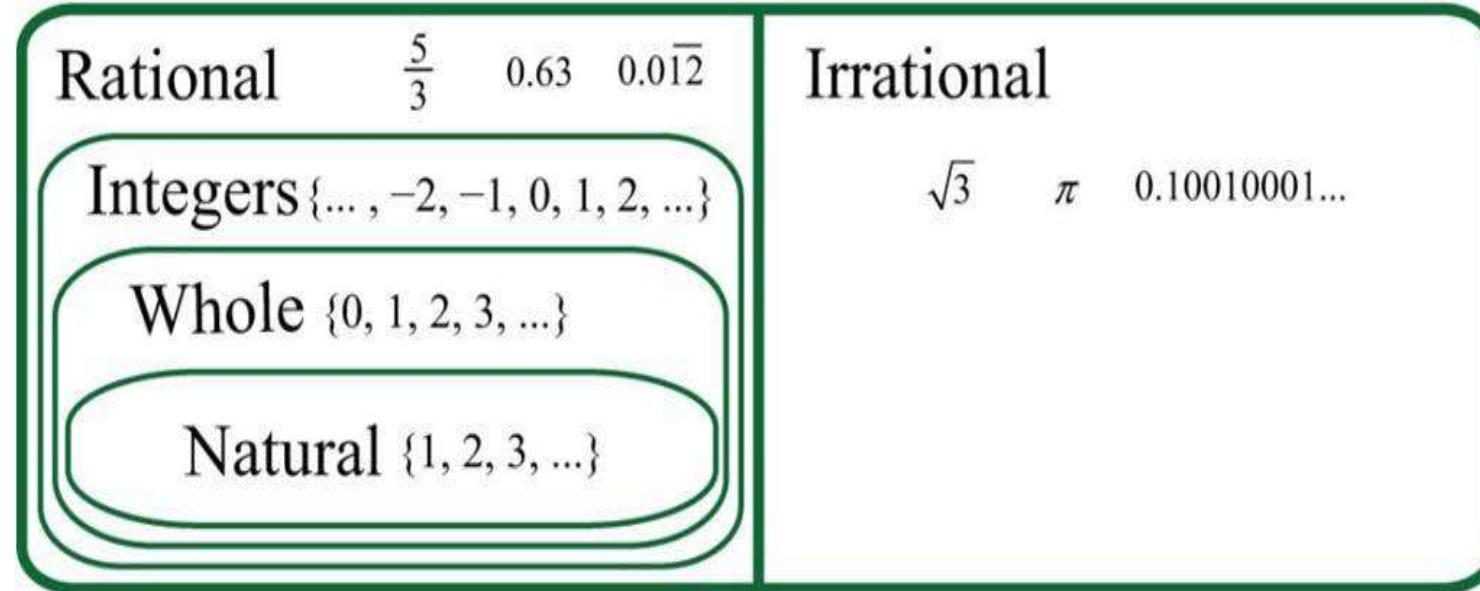




# Introduction to algebra

- Real numbers and its subsets

## Real Numbers



# Introduction to algebra

- Set operations and their symbols

## Set Theory Symbols

Symbol	Name	Example	Explanation
$\{ \}$	Set	$A = \{1, 3\}$ $B = \{2, 3, 9\}$ $C = \{3, 9\}$	Collection of objects
$\cap$	Intersect	$A \cap B = \{3\}$	Belong to both set A and set B
$\cup$	Union	$A \cup B = \{1, 2, 3, 9\}$	Belong to set A or set B
$\subset$	Proper Subset	$\{1\} \subset A$ $C \subset B$	A set that is contained in another set
$\subseteq$	Subset	$\{1\} \subseteq A$ $\{1, 3\} \subseteq A$	A set that is contained in or equal to another set
$\not\subset$	Not a Proper Subset	$\{1, 3\} \not\subset A$	A set that is not contained in another set
$\supset$	Superset	$B \supset C$	Set B includes set C
$\in$	Is a member	$3 \in A$	3 is an element in set A
$\notin$	Is not a member	$4 \notin A$	4 is not an element in set A

Using this symbols, we can write  $N \cap Z = N$ ,  $Q \subset R$ ,  $Z \cup Q = Z$   $\frac{3}{4} \in Z$

Notations for union and intersection of the real numbers sets

$A \cap B = \{x \in R / x \in A \wedge x \in B\}$   $A \cup B = \{x \in R / x \in A \vee x \in B\}$  symbols

“and” “or”

# Introduction to algebra

- Basic operations

Operations with numbers. Integer numbers, fractions, decimal numbers.

Numbers	Addition	Subtraction	Multiplication	Division
Integer	Same sign, no change the sign Different sign, the sign of the greater	Change the sign of the second term and apply addition	Put "+" for numbers with the same sign and "-" for numbers with different sign	Put "+" for numbers with the same sign and "-" for numbers with different sign
Fractions	Turn fractions with the common denominator, add the numerators	Turn fractions with the common denominator, subtract the numerators	Multiply the numerators, Multiply the denominators	Multiply the first fraction with inverse of the second fraction
Decimals	Apply the well known rules	Apply the well known rules	The sum of digits of decimal parts	Turn the first digit to whole number

Operations with roots.

We can apply the addition and subtraction for similar roots (the same index and the same expression under the root).

We can multiply and divide the expressions under the root if they have the same index.

# Introduction to algebra

- Operations with numbers.
- A set is **closed** under an operation if the result of the operation between two elements of the set is **also** element of the set
- The set N is closed under addition but not under subtraction. The set Z is closed under multiplication but not under division
- The set of real numbers is **closed** under basic operations. Addition, subtraction, multiplication and division.
- The fundamental properties of operations on real numbers set are:
- Commutative property; Associative property; Distributive property.
- Rules of operations with real numbers (numbers with sign, positive or negative; fractions)

$$\begin{array}{c} \text{numerator} \\ \text{(number of parts we have)} \quad 2 \\ \hline 5 \quad \text{denominator} \\ \text{(total parts in whole)} \end{array}$$

We can add or subtract two fractions only if they have the same denominator

So called common denominator. It is the common lower factor

To divide two fractions, we multiply the first one with the inverse of the second

When multiply or divide two real numbers with different sign the result is negative number.

The operation of multiplication “n” times of number “a” is called exponentiation (power) “a” is the base and “n” is the exponent (indicia)

# Introduction to algebra



Powers properties

POWERS	
$a^m a^n = a^{m+n}$	$\frac{a^m}{a^n} = a^{m-n}$
$(ab)^m = a^m b^m$	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
$(a^m)^n = a^{mn}$	$a^0 = 1 \quad (a \neq 0)$
$a^1 = a$	$a^{-m} = \frac{1}{a^m}$
$a^{\frac{m}{n}} = \sqrt[n]{a^m}$	

The expression  $2x+3y-5xy$  is called Algebraic expression

This expressions combines the algebraic operations numbers and letters (unknown or variable )

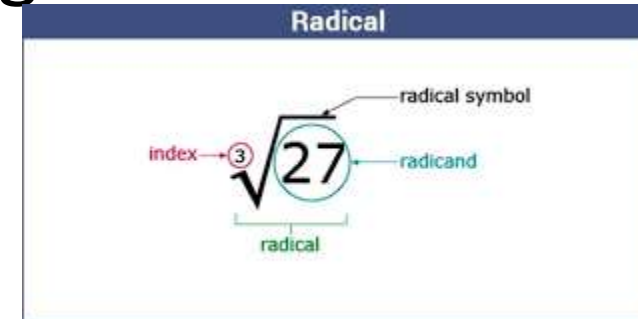
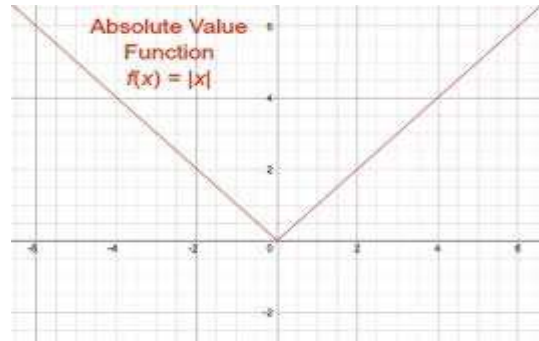
The operations with algebraic expressions are like operations with numbers

# Introduction to algebra

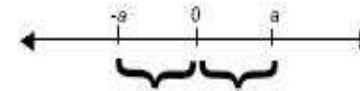
- An algebraic expression which has two operations multiplication and exponentiation is called **monomial**. It is consisting of a single term. **Polynomial** is the expression with two or more monomials.
- The order of operations is **BIDMAS** Brackets, Indices(powers), Division, Multiplication, Addition, Subtraction.
- Operations with brackets multiplication, simplification, factorization. Examples.
- $(a + b)(c + d) = a(c + d) + b(c + d)$ . We apply simplifications for similar terms, **like terms** which are multiples of the same letter (or letters) in the same power.
- Operations with powers and roots. Examples. We can change the roots on powers with rational exponent.
- Operations with fractions. Simplifications. Equivalent fractions. Examples.  $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$ . We can simplify the same factor in numerator and in the denominator. To divide two fractions, multiply the first with the inverse of the second (the divisor is turned upside down).
- The common errors with algebraic operations. Examples.
- The absolute value of a number presents the distance of the number from the origin 0 on the axis.

# Introduction to algebra

$$|x| = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$$



## Absolute Value



Distance from 0 to  $-a = |-a| = a$       Distance from 0 to  $a = |a| = a$

The real problems are expressed by word. To solve them we must transform to the algebraic expressions connected with suitable operations.  
The **unknown, variable** transform, write as **letters**  
The **information** transform in **operations**

# Introduction to algebra

**Set Operations**

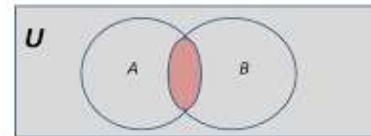
- **Union**
  - Let A and B be sets.
  - The union of two sets A and B is the set that contains all elements in A, B, or both.
  - It is denoted by  $A \cup B$ .
- **Example**
  - $A = \{1, 2\}$
  - $B = \{2, 3\}$
  - $A \cup B = \{1, 2, 3\}$

$A \cup B = \{x \mid x \in A \vee x \in B\}$

## Set Operators: Intersection

- **Definition:** The **intersection** of two sets A and B is the set that contains all elements that are element of both A and B. We write:

$$A \cap B = \{x \mid (a \in A) \wedge (b \in B)\}$$



$$(a, b) = ]a, b[ = \{x \in \mathbb{R} \mid a < x < b\},$$

$$[a, b) = [a, b[ = \{x \in \mathbb{R} \mid a \leq x < b\},$$

$$(a, b] = ]a, b] = \{x \in \mathbb{R} \mid a < x \leq b\},$$

$$[a, b] = [a, b] = \{x \in \mathbb{R} \mid a \leq x \leq b\}.$$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\} \quad A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

$$\wedge \text{ and symbol} \quad \vee \text{ or symbol}$$

**Intervals**

Name of interval	Notation	Inequality description	Number line representation
Finite and closed	$[a, b]$	$a \leq x \leq b$	
Finite and open	$(a, b)$	$a < x < b$	
Finite and half-open	$[a, b)$	$a \leq x < b$	
Infinite and closed	$[-\infty, b]$ $[a, +\infty)$	$-\infty \leq x \leq b$ $a \leq x < +\infty$	
Infinite and open	$(-\infty, b)$ $[a, +\infty)$	$-\infty < x < b$ $a < x < +\infty$	
Infinite and open	$(-\infty, +\infty)$	$-\infty < x < +\infty$	

Let's be sets  $A = \{-3, -1, 7, 14, 21, \}$   $B = \{2, x, 7, 12, \}$

$C = \{-1, 4, 12, 14, \}$

$-1 \in A$ ,  $x \in B$ ,  $12 \in C$ ,  $-1 \in C$ ,  $4 \notin A$ .

Find  $A \cap B$ ,  $A \cap C$ ,  $B \cap C$ ,  $A \cup B$ ,  $B \cup C$ ,  $A \cup C$ ,  $n(A \cup B)$ ,

$n(B \cup C)$   $n(A \cup C)$ ,  $n(A \cap B)$ ,  $n(B \cap C)$

Show the sets below on the real number line

$(-4, 3)$ ;  $(-1, 8)$ ;  $[-2, 4]$ ;  $[2, 5]$ ;  $(-5, 6]$ ;  $[1, 9)$ ;  $(-\infty, -7]$ ;  $[7, +\infty)$



# Introduction to algebra

- The real problems are expressed by **words and sentences**. To solve the problems, we must **transform** words and sentences to the algebraic expressions connected with suitable operations.
- The **unknown, variable** **transform (write as)** **letters**
- The **information** **transform** **into operations**
- Example.

# Introduction to algebra

- A law firm seeks to recruit top-quality experienced lawyers. The total package offered is the sum of three separate components: a basic salary which is 1.2 times the candidate's current salary together with an additional \$3000 for each year work as a qualified lawyer and an extra \$1000 for every year that they are over the age of 21.
- Work out a formula that could be used to calculate the total salary,  $S$ , offered to someone who is  $A$  years of age, has  $E$  years of relevant experience and who currently earns  $\$N$ . Hence work out the salary offered who is 30 years old with five years' experience and who currently earns \$150000.
- Key words
  - 1. Basic salary (connected with current salary  $N$ ).
  - 2. Experience (\$3000 for every year experience related with  $E$ ).
  - 3. Extra for years over 21 years old (\$1000 every year over 21 related with  $A$ ).
- We transform the variables on letters. Now transform the information in operations.
  - 1. The first component is basic salary which is 1.2 times current salary means  $1.2N$
  - 2. The second component is additional money \$ 3000 for every year means  $3000E$ .
  - 3. The third component is an extra for difference of age 21  $A - 21$  means  $1000(A - 21)$ .
- Final formula is the sum of three components "S" means  $S = 1.2N + 3000E + 1000(A - 21)$ .
- Knowing the values for  $N$ ;  $E$ ; and the years of age  $A$  we can find easily the total salary.
-

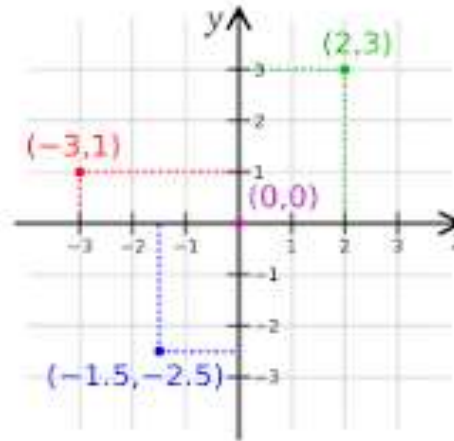
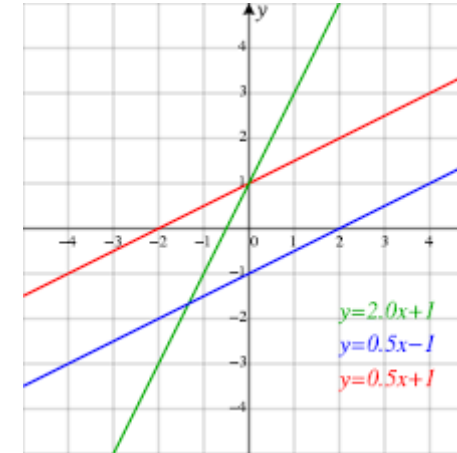
# Introduction to algebra

Words corresponding to operations

- Bigger, Higher, More, Greater, Longer, etc. are **for addition**.
- Less, Smaller, Lower, Shorter, etc. are **for subtraction**.
- Times more, times greater, etc. are **for multiplication**.
- Times lower, times smaller, etc. are **for division**.
- An algebraic expression is **identity** if it is true for **every value** of the variable.
- $z(x \pm y) = zx \pm zy$  true for every value  $x, y, z$
- An algebraic expression is **equation** if it is true for **some values** of the variable.
- $3x - 5 = x + 3$  true for  $x = 4$
- An algebraic expression is **inequality** if it is true for **a set values** of the variable.
- $x - 1 > 0$  true for the set  $A = \{x \in \mathbb{R} / x > 1\}$
- The fundamental property of equation. You can apply whatever mathematical operation over equations, but you must be sure that you are doing the same thing to the both sides of the equation.
- If the both sides of an equality are multiplied or divided by a negative number, then the sense of the inequality is reversed

# Introduction to algebra

- Cartesian plane. Line on a plane. Slope and intercept.  $Ax + By = C$  general form of the line
- $y = ax + b$  simple form or function of the first-degree. "a" is the slope "b" is intercept  
 $y = ax + b$  as a function represent a line on the Cartesian plane
- $ax + b = 0$  is the linear equation
- or equation of the first-degree
- $ax + b < 0$  is the linear inequality of the first.
- Each line divide the plane in two part,
- one of which is the solution set
- of the inequality.
- All **rules** and **properties** mentioned here are well known for all of you.
- We **remind** them because you
- must
- **not forget** and **use them correctly**



# Lesson 1

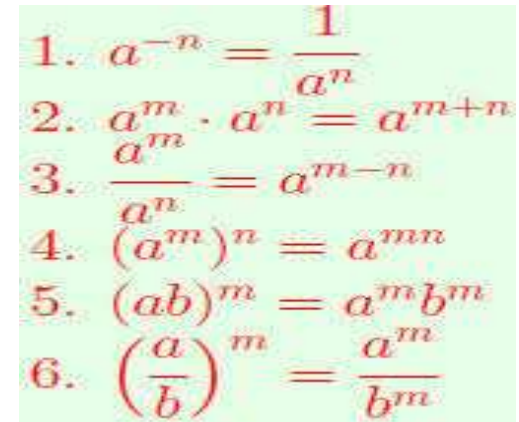
- Important notation
- $P \times Q = P \cdot Q = PQ$ ;  $m \times 7 = 7m$ ;  $r \times t \times 5 = 5rt$ ;  $1 \times y = y \cdot 1 = y$ ;  $b \times b = b^2$
- Write three finite numerical sets given by describing, specifying
- The sets below are subsets of N, W, Z, Q, I, R?
- $A = \{7, 14, 26, 36\}$   $B = \{0, 2, 5, 12, 25\}$   $C = \{0, 1, 8, -7, -12, -21\}$   $D = \{-4, -3/5, -6/15, 2/7, 9, 0.33\ldots\}$   $E = \{\sqrt{3}, \pi, \sqrt{7}, e, \}$
- For the set below find subsets of natural number, integer, rational, irrational.
- $A = \{-7, -4, -3/4, 0, \sqrt{5}, \sqrt{11}, 15, 31, 42.615615615\ldots\}$
- Present on the real line the set of numbers  $A = \{-6, -1, -3/4, 1/8, 0.33\ldots=1/3, \sqrt{2} \approx 1.41 \ldots\}$
- Closed under an operation  $a, b \in A$ ; a "operation"  $b = c$  **if and only if**  $c \in A$
- Properties of the operations **commutative, associative, distributive.**
- Order of operations BIDMAS.

# Lesson 1

- Indices, Powers. Properties
- Equations. If  $a^m = a^n$  then  $m = n$   $3^{2x} = 9^{2-x}$ ,  $25^{3x-1} = \sqrt{5}^{4x-3}$
- Properties of radicals.

Operations with fractions. Simplify the expression  $\frac{3a-4b}{2b} - \frac{4a-3b}{3a} + \frac{20}{6}$

$$\frac{2x-4}{\sqrt{x-1}} - \frac{\sqrt{x-1}}{x}$$



1.  $a^{-n} = \frac{1}{a^n}$
2.  $a^m \cdot a^n = a^{m+n}$
3.  $\frac{a^m}{a^n} = a^{m-n}$
4.  $(a^m)^n = a^{mn}$
5.  $(ab)^m = a^m b^m$
6.  $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Root of a product:  $\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

Root of a quotient:  $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Fractional exponent:  $\sqrt[n]{a^m} = a^{m/n}$

# Lesson 1

- Absolute value. Sketch the graph of lines.  $y = |x - 2|$   $y = |x + 1|$
- A plumber has a fixed call-out charge of \$80 and has an hourly rate of \$60. Work out the total charge,  $C$ , for a job that takes  $L$  hours in which the cost of materials and parts is \$  $K$
- We must find the total cost (the sum) of the costs. “information cost transform on operation sum, addition”. There are three elements
  - a) call-out charge,
  - b) hourly rate
  - c) cost of materials and parts
- The first is fixed equal \$ 80. The second is variable(depended on the numbers of hours), so it is  $60L$ .
- The third is unknown  $K$

# Lesson 1

- We must find the total cost (the sum) of the costs. “information cost transform on operation sum addition”. There are three elements a) call-out charge, b) hourly rate and c) cost of materials and parts
- The first is fixed equal \$ 80.
- The second is variable (depended on the numbers of hours), so it is  $60L$ .
- The third also is variable because we don't know the prices (transform on letters). It is a simple  $K$ .
- Finally, the total cost is the sum  $C = 80 + 60L + K$
- An airport currency exchange shop charges a fixed fee of \$10 on all transactions and offers an exchange rate of 1 dollar to 0.8 euros. Find the total charge,  $C$ , (in dollars) for buying “ $x$ ” euros.



# Lesson 1

- Again, total charge (the sum of all charges).
- The first is fixed fee \$10.
- The second is exchange rate which is \$1 for 0.8 euros or \$1.25 for one euro
- ( $\frac{1}{0.8} = 1.25$ ) and for  $x$  euros is  $1.25x$ .
- Total charge is the sum  $C = 10 + 1.25x$
- A car hire company charges \$C a day together with an additional \$c per mile. Work out the total charge, \$X, for hiring a car for “d” days and travelling “m” miles during that time.
- Total charge (addition). The first \$C in one day. For “d” days is C times d (Cd).
- The second is \$c for one mile. For “m” miles is c times m (cm).
- So, the total charge is the sum  $T = Cd + cm$

# Lesson 1

- The design costs of an advertisement in a glossy magazine are \$ 9000 and the cost per  $cm^2$  of print is \$ 50
- A) write down an expression for the total cost of publishing an advert which covers " $x$ "  $cm^2$
- B) the advertising budget is between \$10800 and \$ 12500.
- Write down and solve an inequality to work out the minimum and maximum area that could be used.
- Total cost is fixing cost plus cost of print  $C = 9000 + 50x$ .
- Total cost must be greater or equal 10800 and smaller or equal 12500.
- Values of the cost must be inside the closed interval  $[10800; 12500]$ .
- $10800 \leq 9000 + 50x \leq 12500$

# Lesson 1

- An amount \$ P is placed in a saving account. The interest rate is r% compounded annually so that after n years the savings, S , will be  $S = P\left(1 + \frac{r}{100}\right)^n$
- Find S when P = 2000, n = 5 and r = 10% (apply the formula)
- Find P when S = 6556.362, n = 3 and r = 3% (transpose the formula)  $P = S\left(1 + \frac{r}{100}\right)^{-n}$ . In a formula the process to express one variable in terms of the others is called transposition of a formula.
- Find r when S = 7500, P = 5000 and n = 4
- Find n when S = 7200, P = 6000 and r = 2.8
- $1 + \frac{10}{100} = 1 + 0.1 = 1.1$      $1.1^5 = 1.61051$      $2000 \cdot 1.61051 = 3221.02$      $S = 3221.02$  dollars
- $1 + \frac{3}{100} = \frac{103}{100} = 1.03$      $1.03^3 = 1.092727$      $P = 6556.62 \div 1.092727 = 6000.236$  dollars
- $7500 = 5000\left(1 + \frac{r}{100}\right)^4$  ;  $\left(1 + \frac{r}{100}\right)^4 = 1.5$  or  $\left(1 + \frac{r}{100}\right) = \sqrt[4]{1.5} = 1.1066$      $\frac{r}{100} = 0.1066$      $r = 10.66\%$   
 $7200 = 6000\left(1 + \frac{2.8}{100}\right)^n$  ;  $\log(1 + 0.028)^n = \log 1.2$      $n = \frac{\log 1.2}{\log 1.028} = \frac{0.0791}{0.01199} = 7.6$  years

# Lesson 1

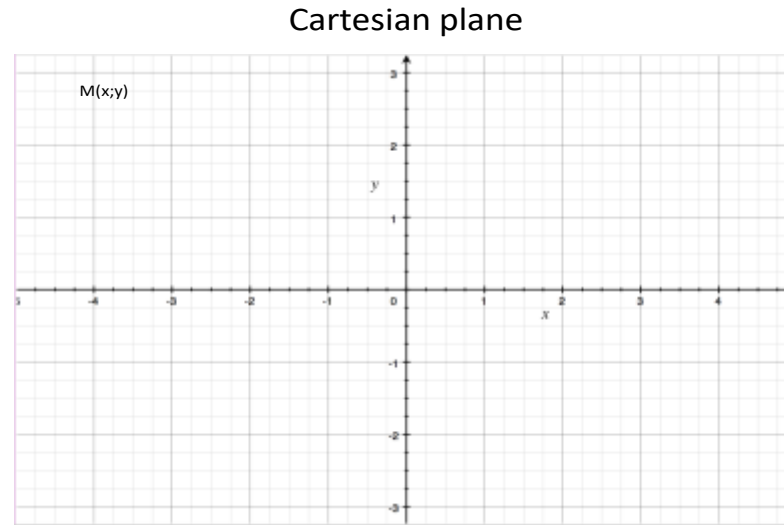
- Multiply out the brackets
- $(4 + a)(3 - 5b + x)$   $(M + 5N)(6N - 7M + 2)$   $(C + 2P - Q)(4 - 2Q + 5P)$
- Factorize  $q^2 - 49$   $32p^2 - 50q^2$   $49C^2 - 36R^2$
- Evaluate without calculator  $20572^2 - 19428^2$ ;  $12,51^2 - 11,49^2$
- Factorize  $2PQ^2 - 6P^2Q$   $7K^3L^2 + 21KL^3 - (KL)^3$   $M^2 - 5M + 6$
- Solve the equation:
- $3^{x+3} - 2 \cdot 3^{x+2} - 3^{x+1} = 54$
- Simplify the expressions:
- $\frac{2}{\sqrt{7}-\sqrt{5}}$ ;  $\frac{5}{\sqrt{5}} + \frac{7}{\sqrt{7}}$ ;  $\frac{2x^3-8x^2+6x}{x^2-5x+4}$

# Lesson 1

- Equations of the line  $Ax + By + C = 0$ ;  $y = ax + b$   $y - y_0 = a(x - x_0)$  line pass through one-point;  
 $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$  line pass through two-points.
- $y = ax + b$  “a” is called slope or gradient. May be positive or negative number
- b is called intercept (which is the intersection of the y’y axis
- Intersections of the two lines we find solving the system of the two equations and two variables.  
 $y = a_1x + b_1$  and  $y = a_2x + b_2$
- To sketch the graph of a line on the cartesian system are enough (needed) only two points and these points are intercepts or (intersections with axes)
- Example. Sketch the graphs of the lines;  $y = 3x - 9$  ;  $y = -2x + 8$
- $y = 0$   $x = 3$   $x = 0$   $y = 9$
- $y = 0$   $x = -4$   $x = 0$   $y = 8$

# Lesson one

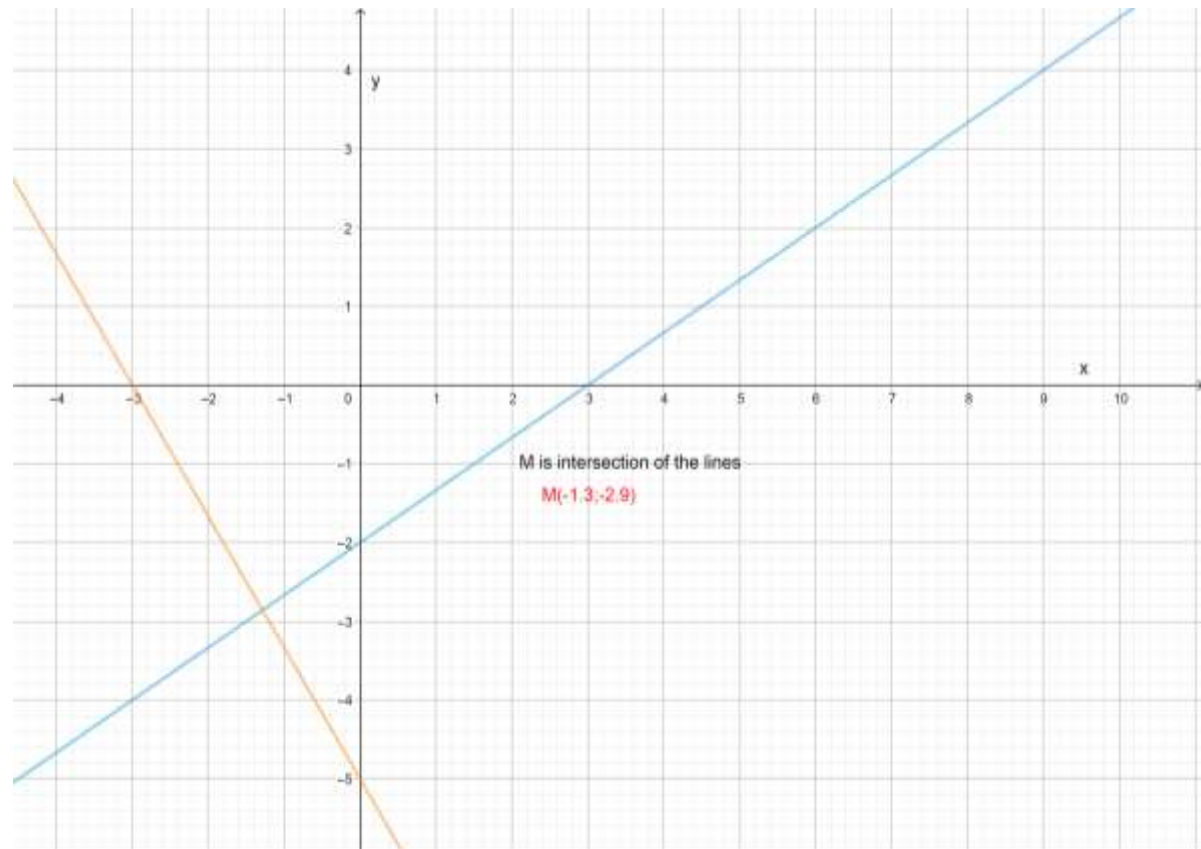
- POINTS ON THE PLANE  $M(x, y)$



- POSITION OF LINES ON THE PLANE
- Equations of the line  $Ax + By + C = 0$ ;  $y = ax + b$
- $y - y_0 = a(x - x_0)$  line pass through one-point;  $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$  line pass
- through two-points

# Lesson 1

- Are given lines;  $2x - 3y - 6 = 0$  and  $5x + 3y + 15 = 0$ . write the slope and intercept. Sketch the graphs of these lines and find the coordinates of the intersection.
- $x = 0 \quad y = -2 \quad y = 0 \quad x = 3;$   
 $x = 0 \quad y = -5 \quad y = 0 \quad x = -3$



# Lesson one

- $y = ax + b$  “a” is called slope or gradient. “a” may be positive or negative
- “b” is called intercept (which is the intersection of the oy axis)



# Introduction to algebra

- **Key words.**
  - Set; Subset; Member; Element.
  - N Natural number; Z Integer number; Q Rational number; R Real number
  - Set symbols  $\cap$  intersect;  $\subset$  subset of;  $\cup$  union. Not element  $\notin$  Element  $\in$
  - Closed under an operation. Power properties. Absolute value.
  - Slope; Intersect; Intersection of two curves or two lines. Properties of the intersection point.
  - Transform variables on letters and information in operations
  - Identity; Equation; Inequality.
  - Like terms.
  - Factorization
  - Transposition
- 
- **Note.**
  - **On every key word add the question “what is” and try to answer it.**
  - Thank you