Topic 4

Representation and Reasoning with Uncertainty

Contents

4.1 Probabilistic methods (Bayesian)

Interpretations of the meaning of "probability"

1. **Subjective:** the probabilities characterize the beliefs that the observer has about the problem.
2. **Frequentalist:** the values of the probabilities only can come from repetitions of experiments.

$$p(a) \equiv lím_{\#cases \to \infty} \left(\frac{\#cases\ of\ a}{\#cases} \right)$$

Practically, we consider:

$$p(a) \approx \frac{\#observed\ cases\ of\ a}{\#observed\ cases}$$

3. **Objective:** the probabilities are real aspects of the Universe, not just beliefs of the observer, or observation.

4.1 Probabilistic methods (Bayesian)

Interpretation of conditional probability under the frecuency interpretation:

$$p(a \mid b) = \frac{p(a \wedge b)}{p(b)} =$$

$$= \left(\frac{\#cases\ of\ \ a \wedge b}{\#cases} \right) / \left(\frac{\#cases\ of\ \ b}{\#cases} \right) = \left(\frac{\#cases\ of\ \ a \wedge b}{\#cases\ of\ \ \ b} \right)$$

In other words, of the cases in which *b* ocurs, the percentage where *a* also occurs.

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4.1 Probabilistic methods (Bayesian)

Example:

$$p(a \mid b) = \frac{p(a \wedge b)}{p(b)} = \quad = \left(\frac{\#cases\ of\ \ a \wedge b}{\#cases} \right) / \left(\frac{\#cases\ of\ \ b}{\#cases} \right) = \left(\frac{\#cases\ of\ \ a \wedge b}{\#cases\ of\ \ \ b} \right)$$

	has_cold	¬ has_cold
sneezing	15	7
¬ sneezing	5	60

p(sneezing|has_cold) = p(sneezing&has_cold) / p(has_cold)
= 15/92 / (15+5)/92
= 15 / 20 = 0.75

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Conditional Probabilities as Rules

- Conditional Probabilities can be represented as rules:

- p(a|b) = 0.8 — if *b* occurs then *a* has a 0.8 chance of occuring

- (if (b) then (a)) (0.8)
  if b is true then a is true
  (in 80% of the cases)

4.1 Probabilistic methods (Bayesian)

Bayes rule in KBSs

p (patient sneezes | patient has cold) **=** 0.75

Is the same as saying:

IF patient has cold **THEN** patient sneezes (0.75)

4.1 Probabilistic methods (Bayesian)

Combining probabilies of premises with rule probabilities

- We have seen that facts can have associated probabilities:
  - p(Pedro has cold) = 0.5
- We have also seen that rules can have associated probabilities (the conditional probability):

  IF X has cold **THEN** X sneezes (0.75)

- How do we combine these probabilities?

- E.g., if our premises have only partial probability, how probable is our conclusion?

4.1 Probabilistic methods (Bayesian)

Combining probabilies of premises with rule probabilities

- How do we combine these probabilities?
- The solution is to convert the rule back to a conditional probability format:

  IF X has cold **THEN** X sneezes (0.75)
  = P(X sneezes | X has cold)=0.75

- We can then produce the probability of the conclusion as follows:
  - We saw earlier that:

$$p(s_1 \wedge s_2) = p(s_2) \cdot p(s_1 \mid s_2)$$

  - Thus
    P(P sneezes & P has cold) = P(P has cold) * P(P sneezes | P has cold)
    = 0.5 * 0.75
    = 0.375

4.1 Probabilistic methods (Bayesian)

Another use of the rule: **Abduction**

IF X has cold **THEN** X sneezes (0.75)

If the doctor sees Pedro sneezing (absolute certainty).
Which is the probability that he has a cold?

We are thus asking:

p (X has a cold | X is sneezing)

when we know…

p (X is sneezing | X has a cold) **=** 0.75

How do we calculate $p(a \mid b)$ from $p(b \mid a)$?

-> Bayes Rule

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4.1 Probabilistic methods (Bayesian)

Another use of the rule: **Abduction**

Bayes rules:

$$p(x \mid y) = p(y \mid x) * p(x) / p(y)$$

We have:

IF X has cold **THEN** X sneezing (0.75)

or : p(X sneezing|X has cold) = 0.75

We can calculate:

$$p(\text{X has cold}|\text{X sneezing}) = \frac{p(\text{X sneezing}|\text{X has cold}) * p(\text{X has cold})}{p(\text{X sneezing})}$$

We thus would need to know:

- p(X has cold) (proportion of the population that has a cold)
- p(X sneezing) (proportion of the population that is sneezing)

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Example:

p (sneezing) = 0.4

p (cold) = 0.3

p (sneezing | has cold) = 0.75

What is the probability of having a cold if sneezing?

p(cold|sneezing) = p(sneezing|cold)*p(cold)/p(sneezing)

= 0.75 * 0.3 / 0.4

= 0.56

Example:

p (sneezing) = 0.1

p (cold) = 0.3

p (sneezing | has cold) = 0.75

What is the probability of having a cold if sneezing?

p(cold|sneezing) = p(sneezing|cold)*p(cold)/p(sneezing)

= ??

Example:

p (sneezing) = 0.1

p (cold) = 0.3

p (sneezing | has cold) = 0.75

What is the probability of having a cold if sneezing?

p(cold|sneezing) = p(sneezing|cold)*p(cold)/p(sneezing)
= 0.75 * 0.3 / 0.1
= 2.25

BUT probabilities have to be between 0 and 1!!!!

Explanation?

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- **Example**:

p (sneezing) = 0.1

p (cold) = 0.3

p (sneezing | has cold) = 0.75

What is the probability of having a cold if sneezing?

p(cold|sneezing) = p(sneezing|cold)*p(cold)/p(sneezing)
= 0.75 * 0.3 / 0.1
= 2.25

BUT probabilities have to be between 0 and 1!!!!

It is necessary: $p(x) \geq p(x \,\&\, y)$

We know: $p(x \mid y) = p(x \,\&\, y) / p(y)$

And thus: $p(x \,\&\, y) = p(x \mid y) * p(y)$

Substituting: p(sneeze & cold) = p(sneeze|cold) * p(cold)
= 0.75 * 0.3
= 0.225

THUS: p(sneeze) ≥ 0.225

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