Spatial Filtering

Fundamentals of Spatial Filtering:

- The name filter is borrowed from frequency domain processing, where 'filtering' refers to accepting (passing) or rejecting certain frequency components.
- Eg: A filter that passes low frequencies is called a lowpass filter. The net effect produced by a lowpass filter is to blur (smooth) an image.
- We can accomplish a similar smoothing directly on the image itself by using spatial filters.

Spatial filtering

- Operating on the neighborhood with a sub-image of size same as that of the neighborhood
- The sub-image is called **kernel, window, template, filter or mask**
- Can be used to smooth, blur, sharpen and to find edges of an image
- The values in the sub-image are called "coefficients" and not pixels
- Spatial filtering term is the filtering operations that are performed directly on the pixels of an image

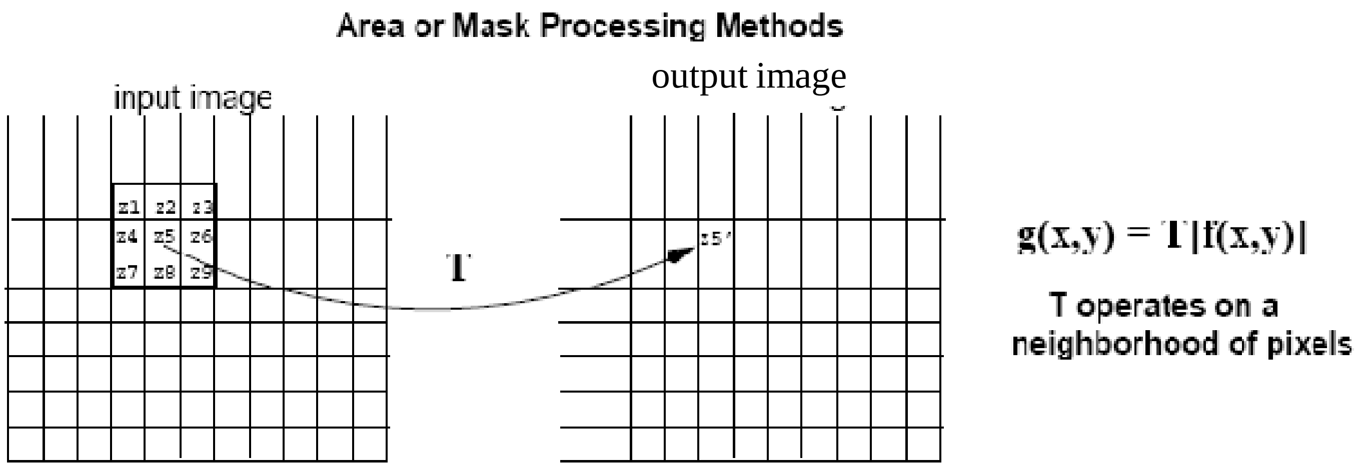
Spatial filtering – Operation

<u>Example</u>: weighted sum of input pixels.

$$z5' = R = w1z1 + w2z2 + \ldots + z9w9$$

mask weights:

w1	w2	w3
w4	w5	w6
w7	w8	w9



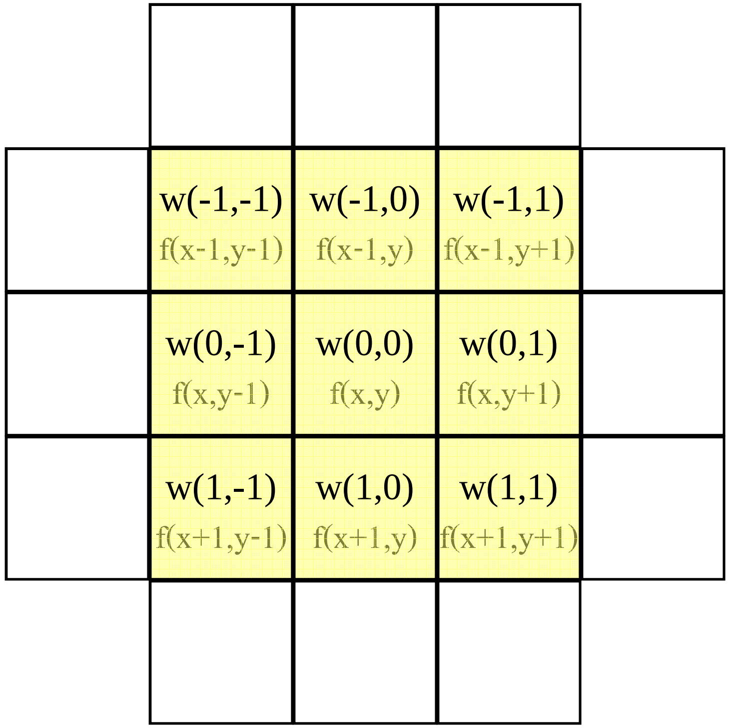
A filtered image is generated as the center of the mask moves to every pixel in the input image.

Mechanics of spatial filtering

- The process consists simply of moving the filter mask from point to point in an image.
- At each point (x,y) the response of the filter at that point is calculated using a predefined relationship

Linear spatial filtering

Pixels of image



The result is the sum of products of the mask coefficients with the corresponding pixels directly under the mask

Mask coefficients

w(-1,-1)	w(-1,0)	w(-1,1)
w(0,-1)	w(0,0)	w(0,1)
w(1,-1)	w(1,0)	w(1,1)

$$
\begin{aligned}
f(x,y) = \; & w(-1,-1)f(x-1,y-1)+w(-1,0)f(x-1,y)+w(-1,1)f(x-1,y+1)+ \\
& w(0,-1)f(x,y-1)+w(0,0)f(x,y)+w(0,1)f(x,y+1)+ \\
& w(1,-1)f(x+1,y-1)+w(1,0)f(x+1,y)+w(1,1)f(x+1,y+1)
\end{aligned}
$$

Linear spatial filtering

- The coefficient w(0,0) coincides with image value f(x,y), indicating that the mask is centered at (x,y) when the computation takes place.
- For a mask of size mxn, we assume that m=2a+1 and n=2b+1, where a and b are nonnegative integer. Then m and n are odd.
- In general, linear filtering of an image f of size MxN with a filter mask of size mxn is given by the expression:

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

Convolution Masks

- The process of linear filtering similar to a frequency domain concept called "*convolution*"
- Linear spatial filtering often referred to as "convolving a mask with an image". In general represented as

<u>Simplify expression</u>

$$R = w_1 z_1 + w_2 z_2 + \dots + w_{mn} z_{mn} = \sum_{i=1}^{mn} w_i z_i$$

$$R = w_1 z_1 + w_2 z_2 + \dots + w_9 z_9 = \sum_{i=1}^{9} w_i z_i$$

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

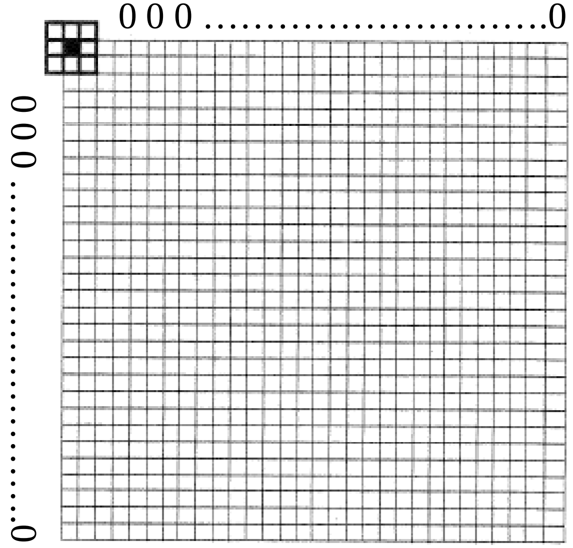
Where the w's are mask coefficients, the z's are the value of the image gray levels corresponding to those coefficients

Linear Spatial Filtering Methods

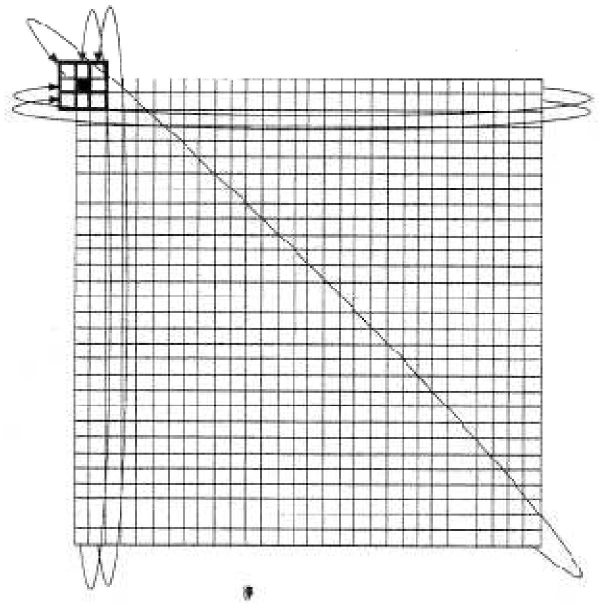
- Correlation = process of moving a filter mask over the image and computing the sum of products at each location
- Convolution = same mechanics but with a filter rotated by 180°

Handling Pixels Close to Boundaries

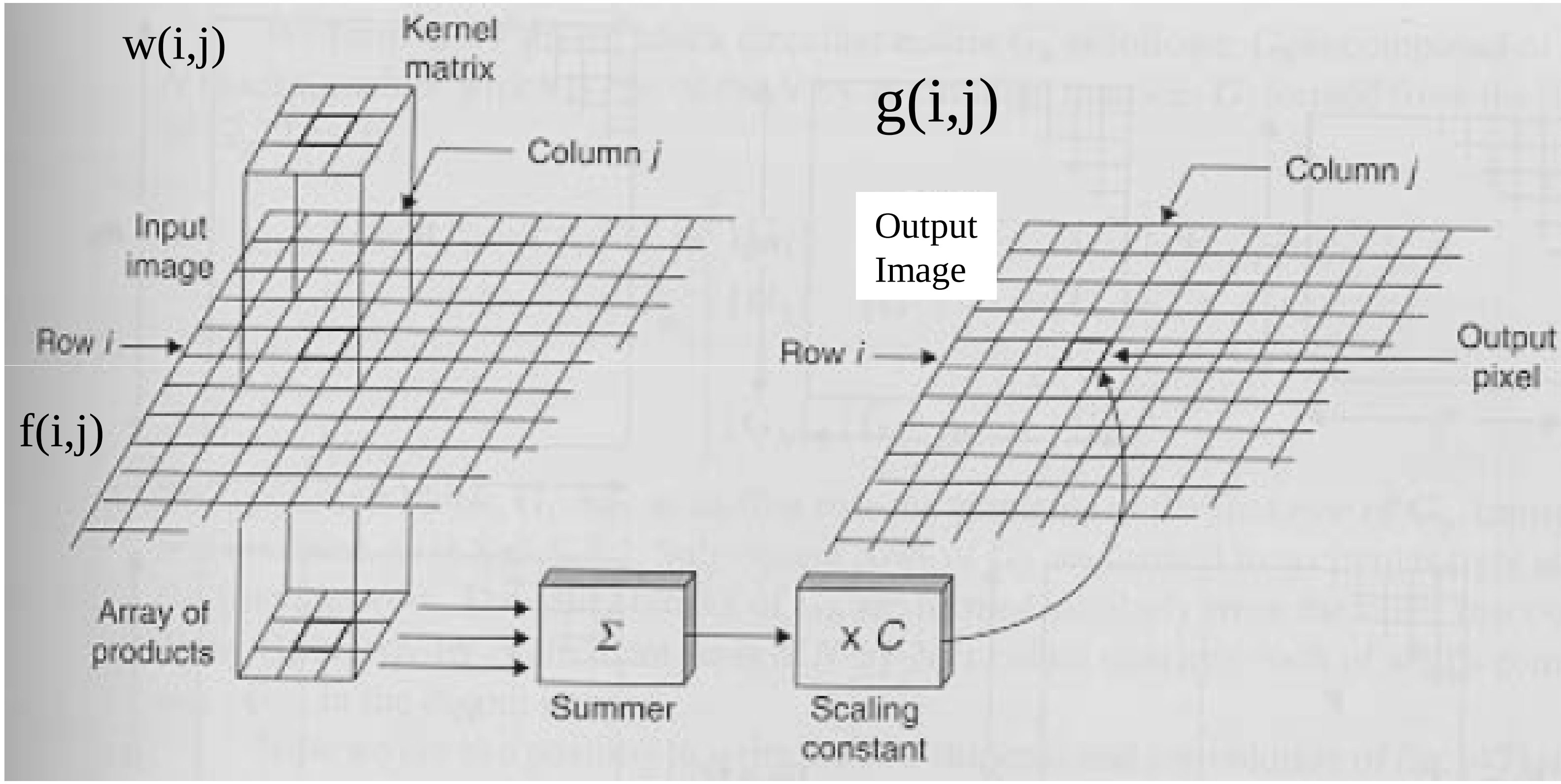
pad with zeroes



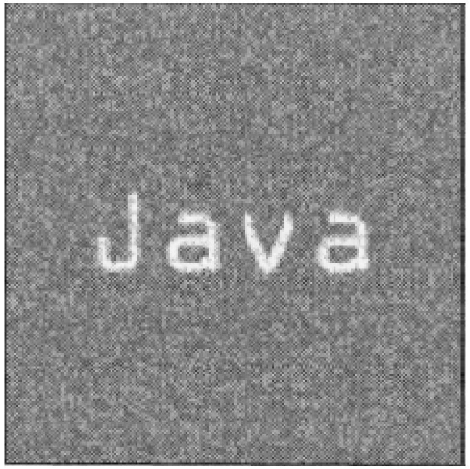
or

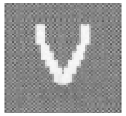


Correlation



Correlation (cont'd)





Often used in applications where we need to measure the similarity between images or parts of images
(e.g., template matching).

Convolution

Similar to correlation except that the mask is first ***flipped*** both horizontally and vertically.

$$g(i,j) = w(i,j) * f(i,j) = \sum_{s=-K/2}^{K/2} \sum_{t=-K/2}^{K/2} w(s,t) f(i-s, j-t)$$

Note: if w(i, j) is symmetric, that is w(i, j)=w(-i,-j), then convolution is equivalent to correlation!

Example

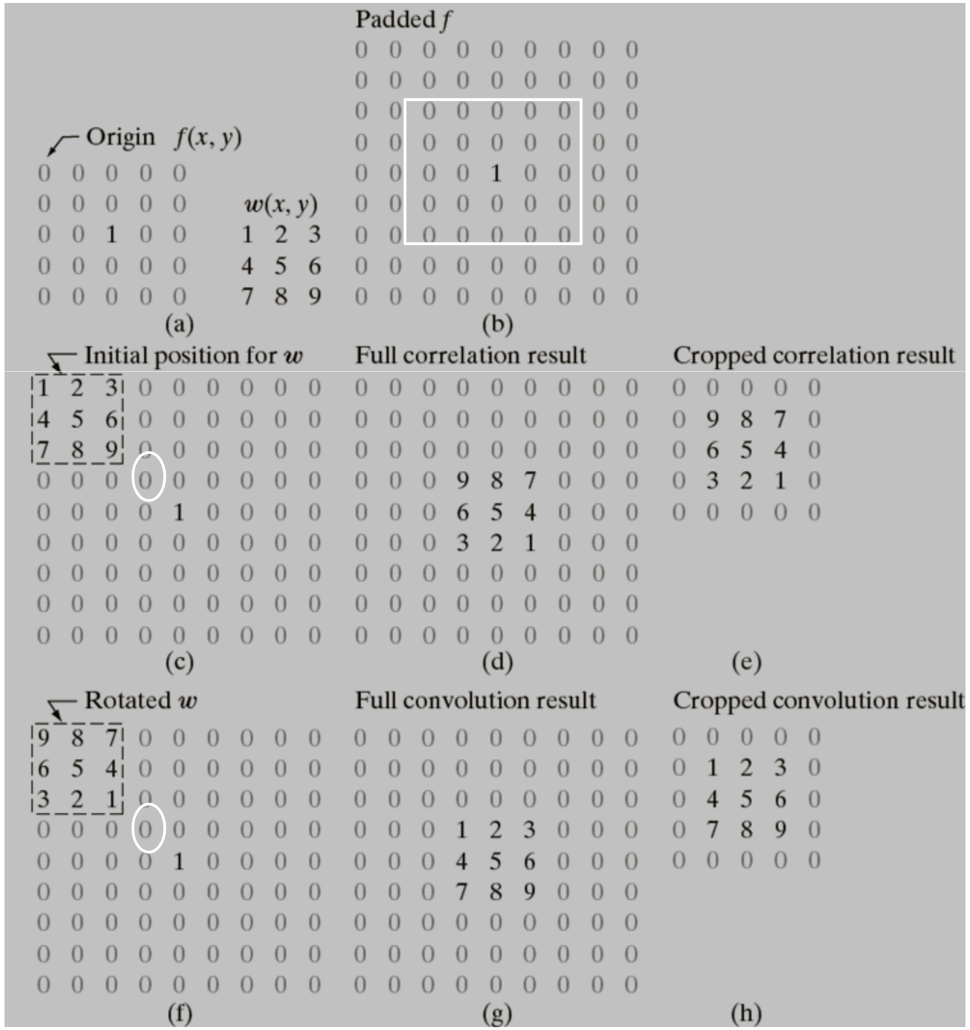
	Correlation	Convolution	
	Origin f / w	Origin f / w rotated 180°	
(a)	0 0 0 1 0 0 0 0 / 1 2 3 2 8	0 0 0 1 0 0 0 0 / 8 2 3 2 1	(i)
(b)	0 0 0 1 0 0 0 0 1 2 3 2 8 Starting position alignment	0 0 0 1 0 0 0 0 8 2 3 2 1	(j)
(c)	Zero padding 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 1 2 3 2 8	0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 8 2 3 2 1	(k)
(d)	0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 1 2 3 2 8 Position after one shift	0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 8 2 3 2 1	(l)
(e)	0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 1 2 3 2 8 Position after four shifts	0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 8 2 3 2 1	(m)
(f)	0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 1 2 3 2 8 Final position	0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 8 2 3 2 1	(n)
(g)	Full correlation result 0 0 0 8 2 3 2 1 0 0 0 0	Full convolution result 0 0 0 1 2 3 2 8 0 0 0 0	(o)
(h)	Cropped correlation result 0 8 2 3 2 1 0 0	Cropped convolution result 0 1 2 3 2 8 0 0	(p)

FIGURE 3.29 Illustration of 1-D correlation and convolution of a filter with a discrete unit impulse. Note that correlation and convolution are functions of *displacement*.

Example

Correlation:

Convolution:



Nonlinear spatial filtering

- Nonlinear spatial filters also operate on neighborhoods, and the mechanics of sliding a mask past an image are the same as linear filters
- The filtering operation is based **conditionally on the values of the pixels** in the neighborhood under consideration
- They do not explicitly use co-efficients in the sum-of-products
- Eg: Median filter for noise reduction (computation of variance)

Type of smoothing filtering

There are 2 types of smoothing spatial filters

- Smoothing Linear Filters
- Order-Statistics Filters

Smoothing Spatial Filters

- They are used for blurring and for noise reduction.
- Blurring is used to remove the small details from an image prior to object extraction and bridging of small gaps in lines or curves.

Smoothing Linear Filters

- Linear spatial filter is simply the **average of the pixels** contained in the neighborhood of the filter mask.
- Sometimes called "averaging filters" and also referred to a low pass filters
- The idea is replacing the value of every pixel in an image by the average of the gray levels in the neighborhood defined by the filter mask.

Two 3x3 Smoothing Linear Filters

$$\frac{1}{9}\times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Standard average

$$\frac{1}{16}\times \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

Weighted average

3* 3 Smoothening FIlters

- It is the average of the gray levels of the pixels in the 3*3 neighborhood under mask

$$R = 1/9 \sum_{1}^{9} z\, i$$

- If the coefficients are equal in the spatial averaging filter and is called as box filter

3* 3 smoothening filters

- Weighted average filter used to indicate that pixels are multiplied by different coefficients
- Pixels at center are multiplied by a higher value other pixels are inversely weighted as function of their distance from the center of the mask
- The diagonal terms are further away from center and are weighted less
- Giving more weights to center and reducing the value of the coefficients as a function of increasing distance helps in reducing blurring in the smoothening process.

Smoothing Linear Filters

The general implementation for filtering an MxN image with a weighted averaging filter of size mxn is given by the expression

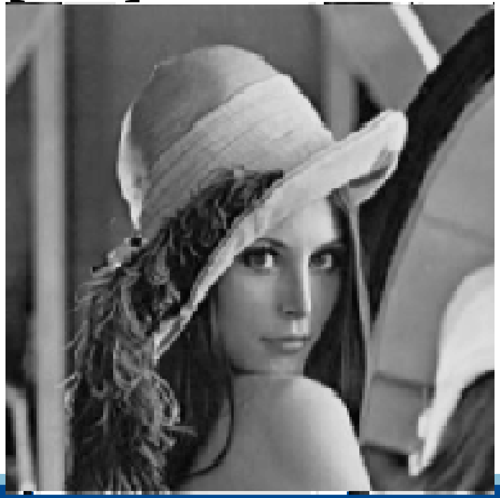
$$g(x,y)=\frac{\sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t)f(x+s,y+t)}{\sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t)}$$

where $a=(m-1)/2$ and $b=(n-1)/2$

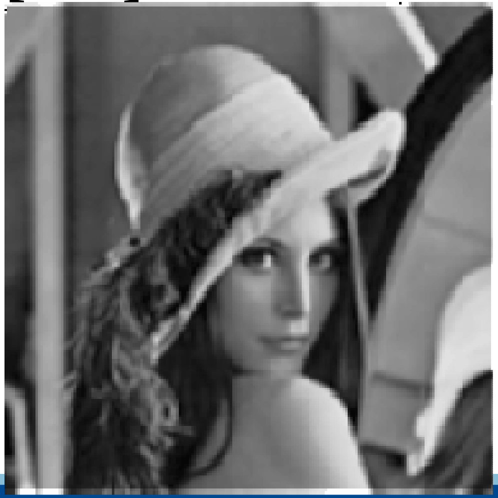
Result of Smoothing Linear Filters

Original Image

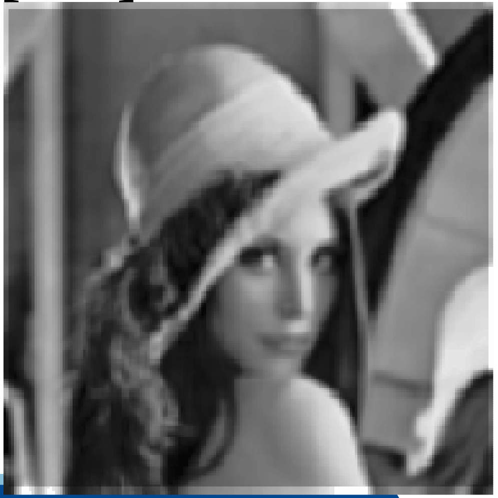
[3x3]



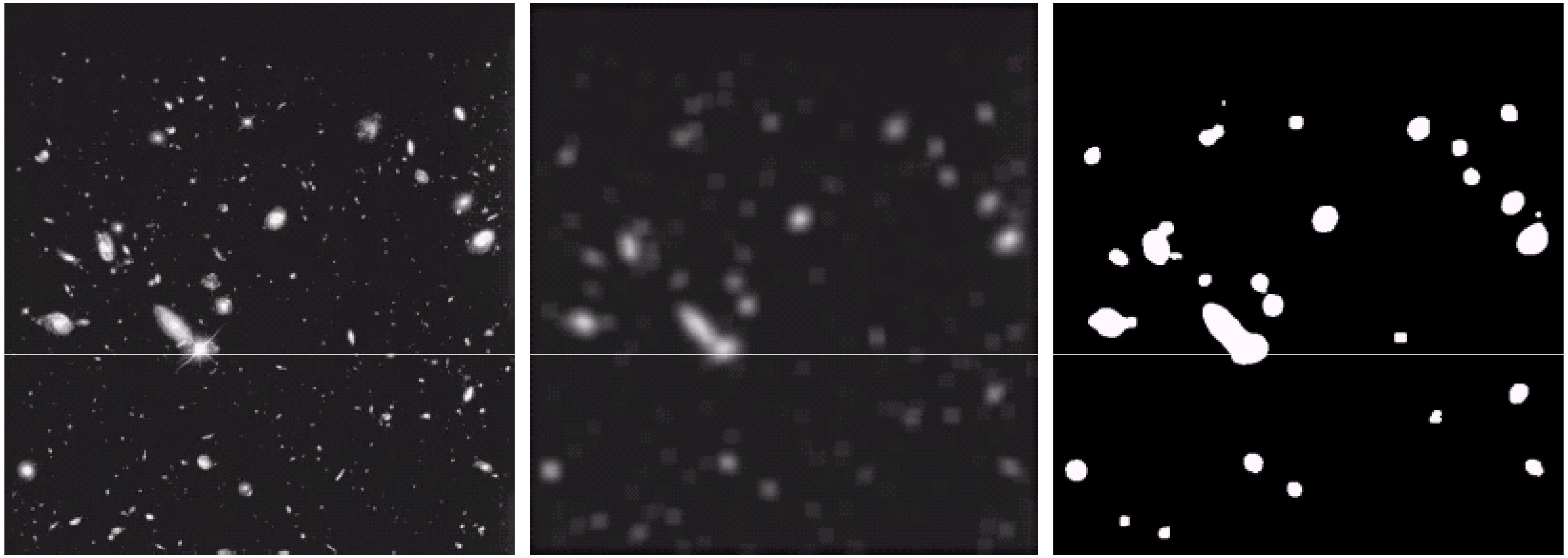
[5x5]



[7x7]



Smoothing Spatial Filters
Another Example



a b c

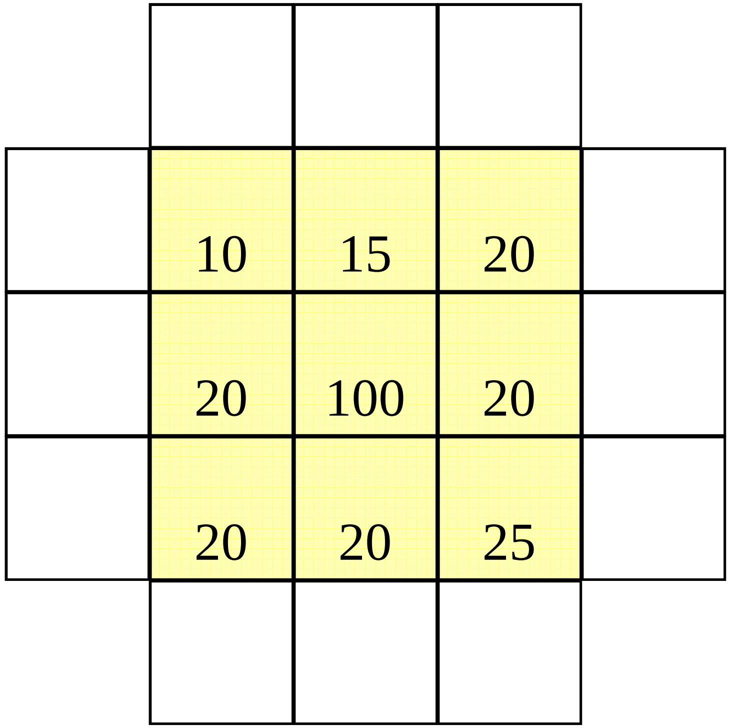
FIGURE 3.36 (a) Image from the Hubble Space Telescope. (b) Image processed by a 15 × 15 averaging mask. (c) Result of thresholding (b). (Original image courtesy of NASA.)

The application of spatial averaging is to blur an image for the purpose of getting Object of interest such that the intensity of smaller objects are blended with back ground and larger objects will become blob like

Order-Statistics Filters

- Order-statistics filters are nonlinear spatial filters whose response is based on ordering (ranking) the pixels contained in the image area encompassed by the filter, and then replacing the value of the center pixel with the value determined by the ranking result.
- Best-known "median filter"
- Median filters provide excellent noise reduction causing less blurring than linear smoothing filters of similar size.

Process of Median filter



10, 15, 20, 20, 20, 20, 20, 25, 100

5th

- Crop region of neighborhood
- Sort the values of the pixel in our region
- In the MxN mask the median is MxN div 2 +1

Order-Statistics Filters

- Median filters are particularly effective in the presence of impulse noise (salt and pepper noise)

- Similarly Max filter is used to find the brightest points in an image and Min filter is used to find the opposite.