

Assignment 3
September 2016
Answer any FIVE questions

1. Suppose that a_1, a_2, \dots, a_N is a sequence of real numbers. A subsequence of this sequence is a sequence of the form $a_{i_1}, a_{i_2}, a_{i_m}$, where $1 \leq i_1 < i_2 < \dots < i_m \leq N$. A sequence is called **strictly increasing** if each term is larger than the one that precedes it, and it is called **strictly decreasing** if each term is smaller than the one that precedes it. Using pigeonhole principle, prove the following:

Every sequence of $n^2 + 1$ distinct real numbers contains a subsequence of length $n + 1$ that is either strictly increasing or strictly decreasing.

2. Give a combinatorial as well as an inductive proof for the following assertion:

Let n and r be non-negative integers with $r \leq n$. Then

$$\binom{n+1}{r+1} = \sum_{j=r}^n \binom{j}{r}.$$

3. How many different paths in the xyz -plane are there from $(0, 0, 0)$ to $(5, 5, 5)$, if a path proceeds one step at a time by going either one space along x (X) or one space along y (Y) or one space along z (Z)? Generalize the above result.
4. Using generating functions, find a formula to express $0^3 + 1^3 + 2^3 + \dots + n^3$ as a function of n .
5. Define exponential generating functions. Find the exponential generating function for the number of ways to arrange $n \geq 0$ letters selected from the word **MISSOURI**. What is the exponential generating function if the arrangement must contain at least two **I**'s ?
6. What is a Ferrers graph ? Using Ferrers graph, show that the number of partitions of an integer n into summands not exceeding m is equal to the number of partitions of n into at most m summands.
7. Show that the number of partitions of a positive integer n where no summand appears more than twice equals the number of partitions of n where no summand is divisible by 3.
8. If a die is rolled 12 times, what is the probability that the sum of rolls is 30 ?
9. In how many ways can Traci select n marbles from a large supply of blue, red, and yellow marbles (all of the same size) if the selection must include even number of blue ones ?
10. Determine the number of integer solutions to $x_1 + x_2 + x_3 + x_4 = 25$ where $0 \leq x_i \leq 10$ for all $1 \leq i \leq 4$.