LR Parsers

LR Parsers

- The most powerful shift-reduce parsing (yet efficient) is:

LR(k) parsing.

left to right scanning

right-most derivation

k lookhead (k is omitted ➔ it is 1)

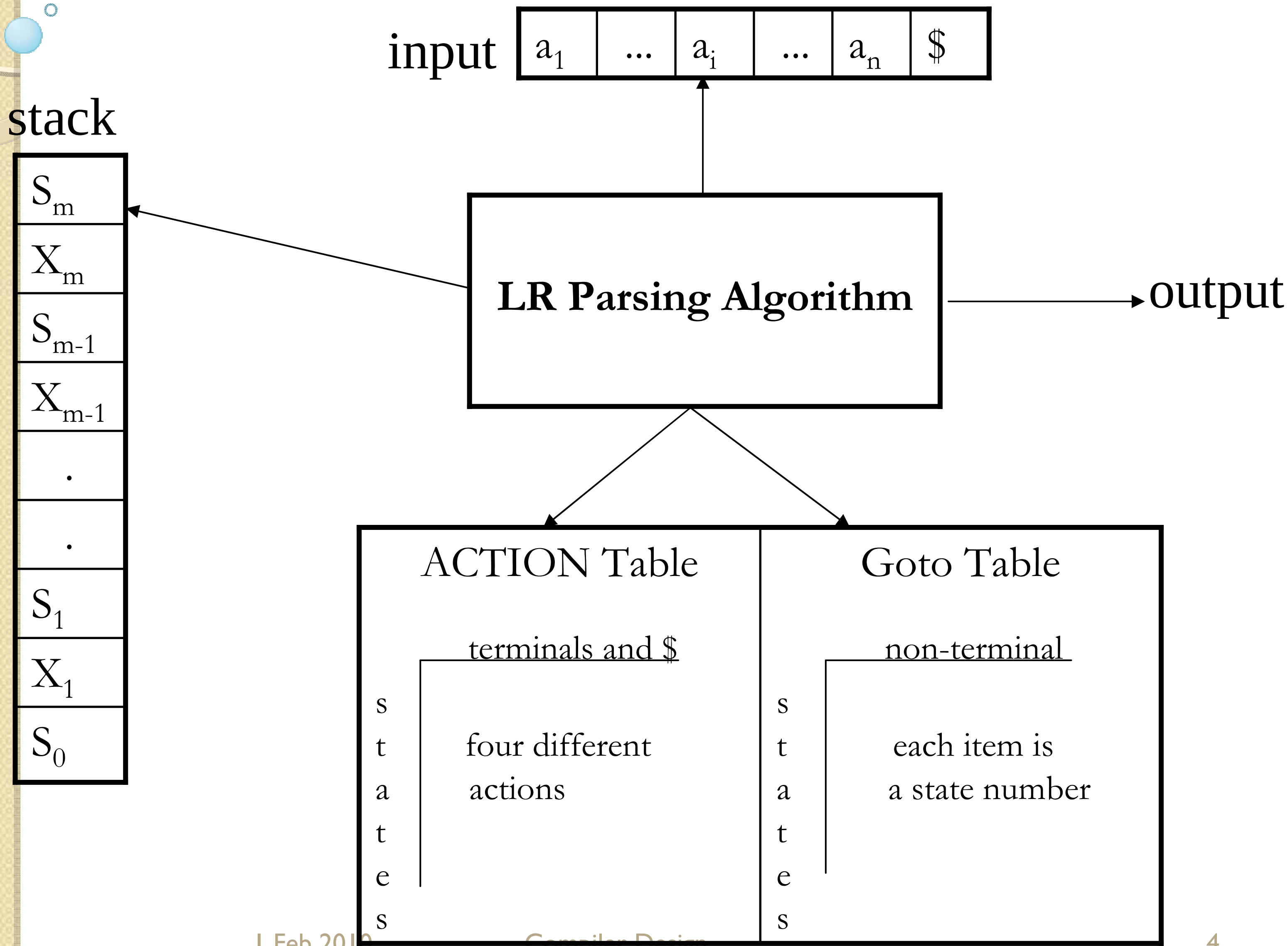
- LR parsing is attractive because:
  - LR parsing is most general non-backtracking shift-reduce parsing, yet it is still efficient.
  - The class of grammars that can be parsed using LR methods is a proper superset of the class of grammars that can be parsed with predictive parsers.

    LL(1)-Grammars $\subset$ LR(1)-Grammars
  - An LR-parser can detect a syntactic error as soon as it is possible to do so a left-to-right scan of the input.

LR Parsers

- covers wide range of grammars.
- SLR – simple LR parser
- CLR – most general LR parser
- LALR – intermediate LR parser (look-head LR parser)
- SLR, LR and LALR work same (they used the same algorithm), only their parsing tables are different.

LR Parsing Algorithm



Steps

- Construct an augmented grammar G' for any grammar G
- Construct LR(0) collection of items
- Construct DFA
- Find FOLLOW for the non terminals
- Construct the parsing table

Keywords

- Viable Prefixes
- Augmented Grammar
- Item
- CLOSURE(I)
- GOTO(I,X)

Viable Prefixes

- Prefix of a right sentential form that appears on the stack of a Shift-Reduce parser
- It is always possible to add terminal symbols to the end of a viable prefix to obtain a right sentential form

Augmented Grammar

- G' is G with a new production rule S'→S where S' is the new starting symbol.

Grammar:

$E \rightarrow E + T$

$E \rightarrow T$

$T \rightarrow T * F$

$T \rightarrow F$

$F \rightarrow (E)$

$F \rightarrow \textbf{id}$

Augmented Grammar:

$E' \rightarrow E$

$E \rightarrow E + T$

$E \rightarrow T$

$T \rightarrow T * F$

$T \rightarrow F$

$F \rightarrow (E)$

$F \rightarrow \textbf{id}$

ITEM or LR(0) Items of a Grammar

- An *LR*(0) *item* of a grammar G is a production of G with a • at some position of the right-hand side
- Thus, a production

  $A \to X\ Y\ Z$

  has four items:

  $[A \to \bullet\ X\ Y\ Z]$
  $[A \to X \bullet\ Y\ Z]$
  $[A \to X\ Y \bullet\ Z]$
  $[A \to X\ Y\ Z \bullet]$
- Note that production $A \to \varepsilon$ has one item $[A \to \bullet]$

Significance of •

- Tells us how much of the production is seen in the grammar while in the process of parsing.
- $[A \to X \bullet Y\, Z]$
  - Seen a string derivable from X in the input
  - Expect to see the string derivable from $Y\, Z$ next in the input.

The Closure Operation

- If ***I*** is a set of LR(0) items for a grammar G, then ***closure(I)*** is the set of LR(0) items constructed from I by the two rules:

1. Initially, every LR(0) item in I is added to closure(I).
2. If $A \rightarrow \alpha.B\beta$ is in closure(I) and $B \rightarrow \gamma$ is a production rule of G; then $B \rightarrow .\gamma$ will be in the closure(I). We will apply this rule until no more new LR(0) items can be added to closure(I).

The Closure Operation (Example)

$closure(\{[E' \to \bullet E]\}) =$

$\{ [E' \to \bullet E] \}$ ⇒ $\{ [E' \to \bullet E]$
$[E \to \bullet E + T]$
$[E \to \bullet T] \}$ ⇒ $\{ [E' \to \bullet E]$
$[E \to \bullet E + T]$
$[E \to \bullet T]$
$[T \to \bullet T * F]$
$[T \to \bullet F] \}$ ⇒ $\{ [E' \to \bullet E]$
$[E \to \bullet E + T]$
$[E \to \bullet T]$
$[T \to \bullet T * F]$
$[T \to \bullet F]$
$[F \to \bullet \mathbf{(}\ E\ \mathbf{)}]$
$[F \to \bullet\ \mathbf{id}] \}$

Add $[E \to \bullet \gamma]$

Add $[T \to \bullet \gamma]$

Add $[F \to \bullet \gamma]$

Grammar:

$E \to E + T \mid T$

$T \to T * F \mid F$

$F \to \mathbf{(}\ E\ \mathbf{)}$

$F \to \mathbf{id}$

GOTO Operation

- If I is a set of LR(0) items and X is a grammar symbol (terminal or non-terminal), then GOTO(I, X) is defined as follows:
  - If $A \rightarrow \alpha.X\beta$ in I
    then every item in **CLOSURE({$A \rightarrow \alpha X.\beta$})** will be in GOTO(I,X).

Example

$I = \{ \quad E' \rightarrow .E, \quad E \rightarrow .E+T, \quad E \rightarrow .T,$
$\quad T \rightarrow .T*F, \quad T \rightarrow .F,$
$\quad F \rightarrow .(E), \quad F \rightarrow .id \}$

$GOTO(I, E) = \{ E' \rightarrow E., E \rightarrow E.+T \}$
$GOTO(I, T) = \{ E \rightarrow T., T \rightarrow T.*F \}$
$GOTO(I, F) = \{ T \rightarrow F. \}$
$GOTO(I, () = \{ F \rightarrow (.E), E \rightarrow .E+T, E \rightarrow .T, T \rightarrow .T*F,$
$\quad T \rightarrow .F, F \rightarrow .(E), F \rightarrow .id \}$
$GOTO(I, id) = \{ F \rightarrow id. \}$

Construction of The Canonical LR(0) Collection

- To create the SLR parsing tables for a grammar G, we will create the canonical LR(0) collection of the grammar G'.

- ***Algorithm***:

  C is { closure({S'→.S}) }

  repeat the followings until no more set of LR(0) items can be added to ***C***.

  for each I in ***C*** and each grammar symbol X

  if goto(I,X) is not empty and not in ***C***

  add goto(I,X) to ***C***

- goto function is a DFA on the sets in C.

Canonical LR(0) Collection - Example

I_0: $E' \rightarrow .E$
$E \rightarrow .E+T$
$E \rightarrow .T$
$T \rightarrow .T*F$
$T \rightarrow .F$
$F \rightarrow .(E)$
$F \rightarrow .id$

I_1: $E' \rightarrow E.$
$E \rightarrow E.+T$

I_2: $E \rightarrow T.$
$T \rightarrow T.*F$

I_3: $T \rightarrow F.$

I_4: $F \rightarrow (.E)$
$E \rightarrow .E+T$
$E \rightarrow .T$
$T \rightarrow .T*F$
$T \rightarrow .F$
$F \rightarrow .(E)$
$F \rightarrow .id$

I_5: $F \rightarrow id.$

I_6: $E \rightarrow E+.T$
$T \rightarrow .T*F$
$T \rightarrow .F$
$F \rightarrow .(E)$
$F \rightarrow .id$

I_7: $T \rightarrow T*.F$
$F \rightarrow .(E)$
$F \rightarrow .id$

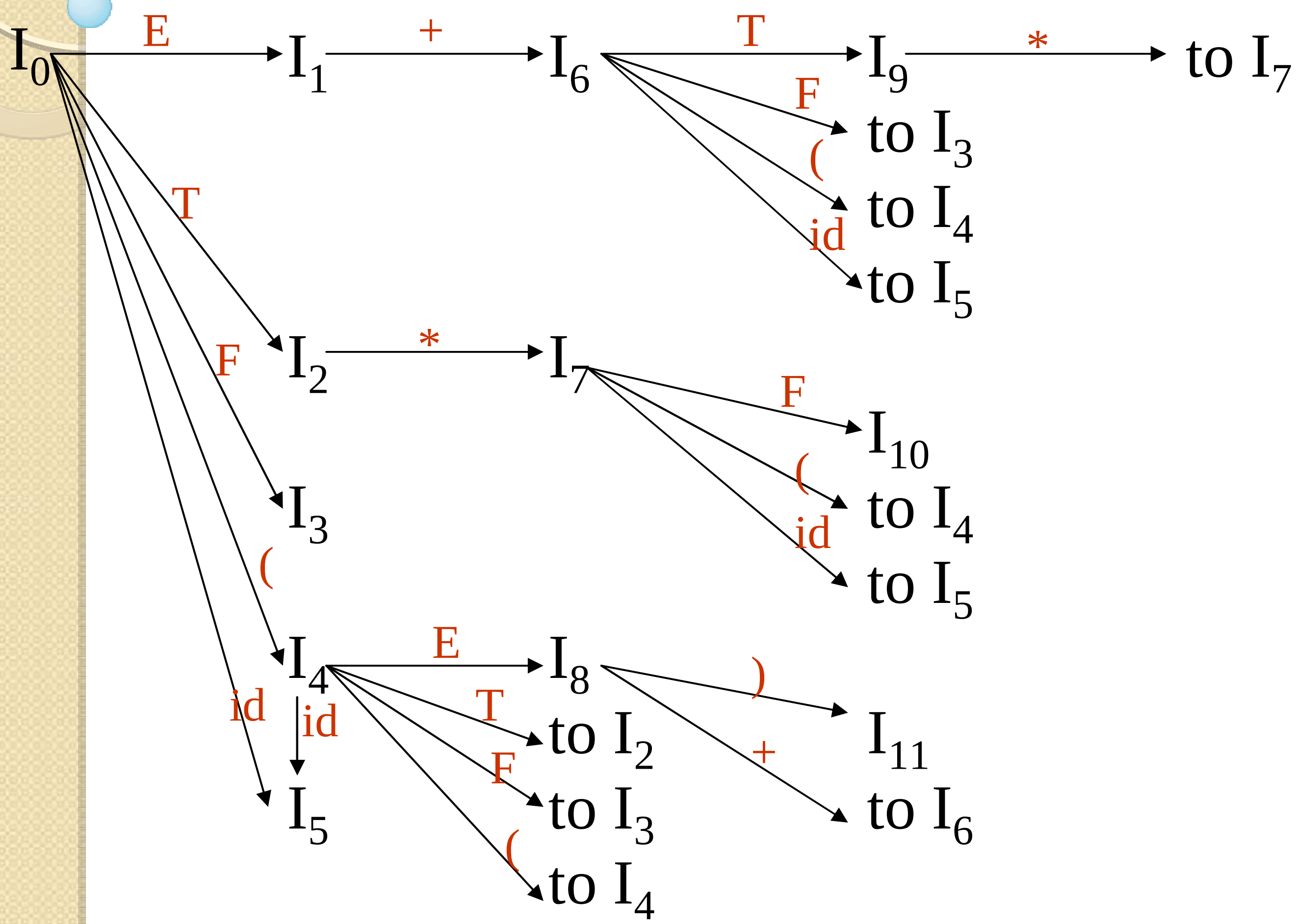
I_8: $F \rightarrow (E.)$
$E \rightarrow E.+T$

I_9: $E \rightarrow E+T.$
$T \rightarrow T.*F$

I_{10}: $T \rightarrow T*F.$

I_{11}: $F \rightarrow (E).$

DFA of GOTO Function



Constructing SLR Parsing Table

1. Construct the canonical collection of sets of LR(0) items for G'.
   - $C \leftarrow \{I_0, \ldots, I_n\}$
2. Create the parsing ACTION table as follows
   - If a is a terminal, $A \rightarrow \alpha.a\beta$ in I_i and $GOTO(I_i,a)=I_j$ then ACTION[i,a] is ***shift j.***
   - If $A \rightarrow \alpha.$ is in I_i, then ACTION[i,a] is ***reduce*** $\boldsymbol{A \rightarrow \alpha}$ for all a in FOLLOW(A) where $A \neq S'$.
   - If $S' \rightarrow S.$ is in I_i, then ACTION[i,$] is ***accept***.
   - If any conflicting actions generated by these rules, the grammar is not SLR(1).
3. Create the parsing GOTO table
   - for all non-terminals A, if $GOTO(I_i,A)=I_j$ then GOTO[i,A]=j
4. All entries not defined by (2) and (3) are errors.
5. Initial state of the parser contains $S' \rightarrow .S$

SLR Parsing Table

state	id	+	*	(	)	$	*E*	*T*	*F*
	ACTION						*GOTO*		
0	s5			s4			1	2	3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			
4	s5			s4			8	2	3
5		r6	r6		r6	r6			
6	s5			s4				9	3
7	s5			s4					10
8		s6			s11				
9		r1	s7		r1	r1			
10		r3	r3		r3	r3			
11		r5	r5		r5	r5			

LR Parsing Algorithm

Set ip to point to the first symbol of w$;
Repeat forever begin

  let S be the state on the top of the stack and a be the symbol pointed to by ip;
  if ACTION [S, a]=shift S' then
    push a then S' on the top of the stack
    advance ip to the next input symbol
  else if ACTION [S, a]=reduce $A \rightarrow \beta$ then
    pop $2^*|\beta|$ symbols on the top of the stack
    let s' be the state now on the top of the stack
    Push A then GOTO[S',A] on the top of the stack
    Output the production $A \rightarrow \beta$
  else if ACTION [S, a]= accept then
    return
  else
    error()

end

Example LR Parsing Table

Grammar:

1. $E \rightarrow E$ **+** T
2. $E \rightarrow T$
3. $T \rightarrow T$ ***** F
4. $T \rightarrow F$
5. $F \rightarrow$ **(** E **)**
6. $F \rightarrow$ **id**

	ACTION						*GOTO*		
state	**id**	**+**	*****	**(**	**)**	**$**	*E*	*T*	*F*
0	s5			s4			1	2	3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			
4	s5			s4			8	2	3
5		r6	r6		r6	r6			
6	s5			s4				9	3
7	s5			s4					10
8		s6			s11				
9		r1	s7		r1	r1			
10		r3	r3		r3	r3			
11		r5	r5		r5	r5			

Shift & GOTO 5

Reduce by production #1

Example LR Parsing

Grammar:

1. $E \rightarrow E + T$
2. $E \rightarrow T$
3. $T \rightarrow T * F$
4. $T \rightarrow F$
5. $F \rightarrow (E)$
6. $F \rightarrow$ **id**

Stack	Input	Action
$ 0	**id*id+id$**	shift 5
$ 0 **id** 5	***id+id$**	reduce 6 goto 3
$ 0 *F* 3	***id+id$**	reduce 4 goto 2
$ 0 *T* 2	***id+id$**	shift 7
$ 0 *T* 2 * 7	**id+id$**	shift 5
$ 0 *T* 2 * 7 **id** 5	**+id$**	reduce 6 goto 10
$ 0 *T* 2 * 7 *F* 10	**+id$**	reduce 3 goto 2
$ 0 *T* 2	**+id$**	reduce 2 goto 1
$ 0 *E* 1	**+id$**	shift 6
$ 0 *E* 1 + 6	**id$**	shift 5
$ 0 *E* 1 + 6 **id** 5	**$**	reduce 6 goto 3
$ 0 *E* 1 + 6 *F* 3	**$**	reduce 4 goto 9
$ 0 *E* 1 + 6 *T* 9	**$**	reduce 1 goto 1
$ 0 *E* 1	**$**	accept

SLR(1) Grammar

- An LR parser using SLR(1) parsing tables for a grammar G is called as the SLR(1) parser for G.
- If a grammar G has an SLR(1) parsing table, it is called SLR(1) grammar (or SLR grammar in short).
- Every SLR grammar is unambiguous, but every unambiguous grammar is not a SLR grammar.

shift/reduce and reduce/reduce conflicts

- If a state does not know whether it will make a shift operation or reduction for a terminal, we say that there is a **shift/reduce conflict**.

- If a state does not know whether it will make a reduction operation using the production rule `i` or `j` for a terminal, we say that there is a **reduce/reduce conflict**.

- If the SLR parsing table of a grammar G has a conflict, we say that that grammar is not SLR grammar.

Conflict Example

$S \rightarrow L=R$

$S \rightarrow R$

$L \rightarrow *R$

$L \rightarrow id$

$R \rightarrow L$

I_0: $S' \rightarrow .S$
$S \rightarrow .L=R$
$S \rightarrow .R$
$L \rightarrow .*R$
$L \rightarrow .id$
$R \rightarrow .L$

I_1: $S' \rightarrow S.$

I_2: $S \rightarrow L.=R$
$R \rightarrow L.$

I_3: $S \rightarrow R.$

I_4: $L \rightarrow *.R$
$R \rightarrow .L$
$L \rightarrow .*R$
$L \rightarrow .id$

I_5: $L \rightarrow id.$

I_6: $S \rightarrow L=.R$
$R \rightarrow .L$
$L \rightarrow .*R$
$L \rightarrow .id$

I_7: $L \rightarrow *R.$

I_8: $R \rightarrow L.$

I_9: $S \rightarrow L=R.$

Problem

FOLLOW(R)={=,$}

= → shift 6

→ reduce by $R \rightarrow L$

shift/reduce conflict

Conflict Example

$S \rightarrow AaAb$

$S \rightarrow BbBa$

$A \rightarrow \varepsilon$

$B \rightarrow \varepsilon$

I_0: $S' \rightarrow .S$

$S \rightarrow .AaAb$

$S \rightarrow .BbBa$

$A \rightarrow .$

$B \rightarrow .$

Problem

FOLLOW(A)={a,b}

FOLLOW(B)={a,b}

a → reduce by $A \rightarrow \varepsilon$

→ reduce by $B \rightarrow \varepsilon$

reduce/reduce conflict

b → reduce by $A \rightarrow \varepsilon$

→ reduce by $B \rightarrow \varepsilon$

reduce/reduce conflict