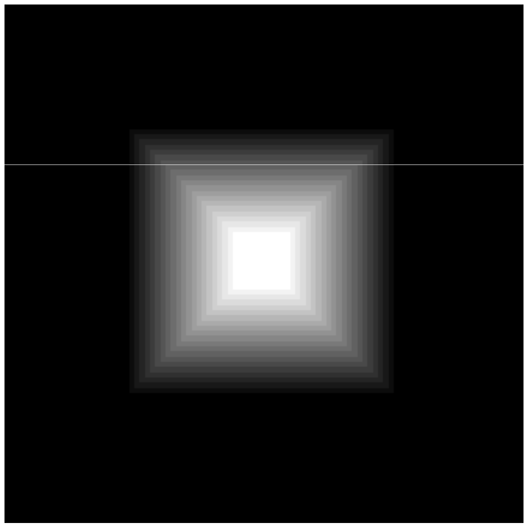
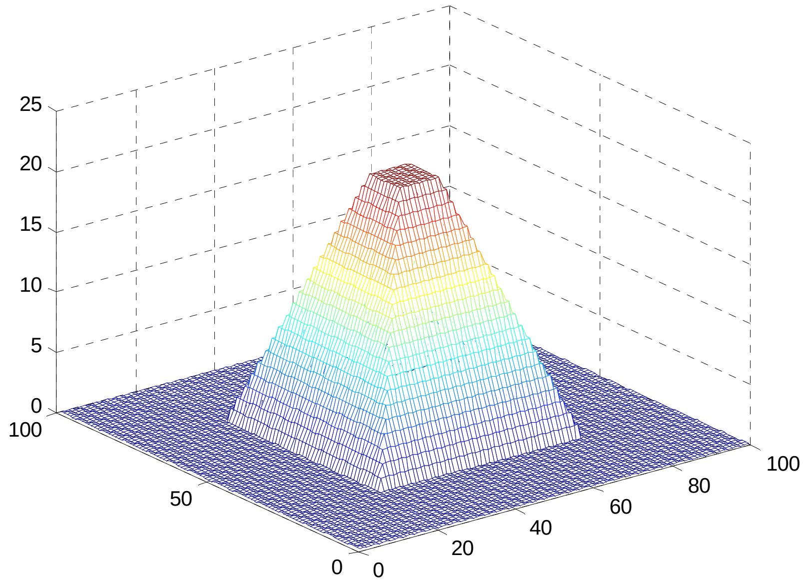
Segmentation by Morphological Watersheds

Introduction

- Based on visualizing an image in 3D

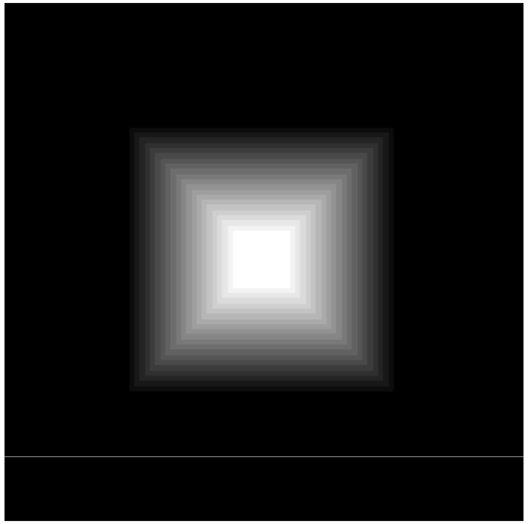




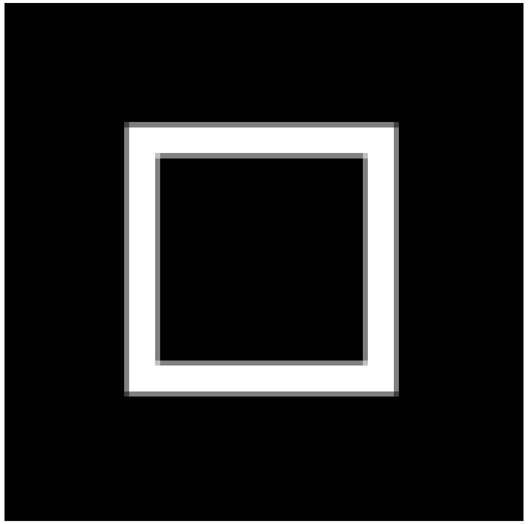
imshow(I,[ ])

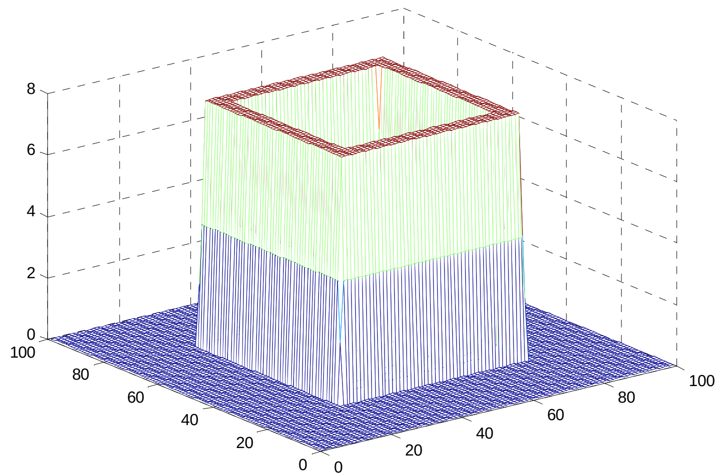
mesh(I)

Introduction



- Instead of working on an image itself, this technique is often applied on its gradient image.
  – In this case, each object is distinguished from the background by its up-lifted edges



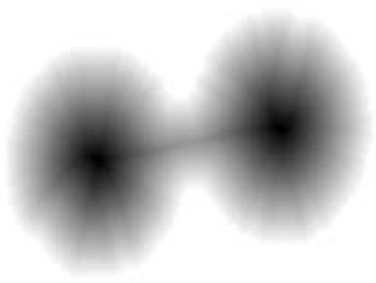


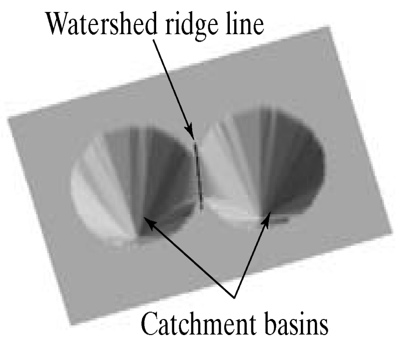
Basic Definitions

- I: 2D gray level image
- D_I: Domain of I
- Path P of length l between *p* and *q* in I
  - A (l +1)-tuple of pixels (p_0=p,p_1,…,p_l=q) such that p_i,p_{i+1} are adjacent (4 adjacent, 8 adjacent, or m adjacent, see Section 2.5)
- l(P): The length of a given path P
- Minimum
  - A minimum *M* of I is a connected plateau of pixels from which it is impossible to reach a point of lower altitude without having to climb

Basic Definitions

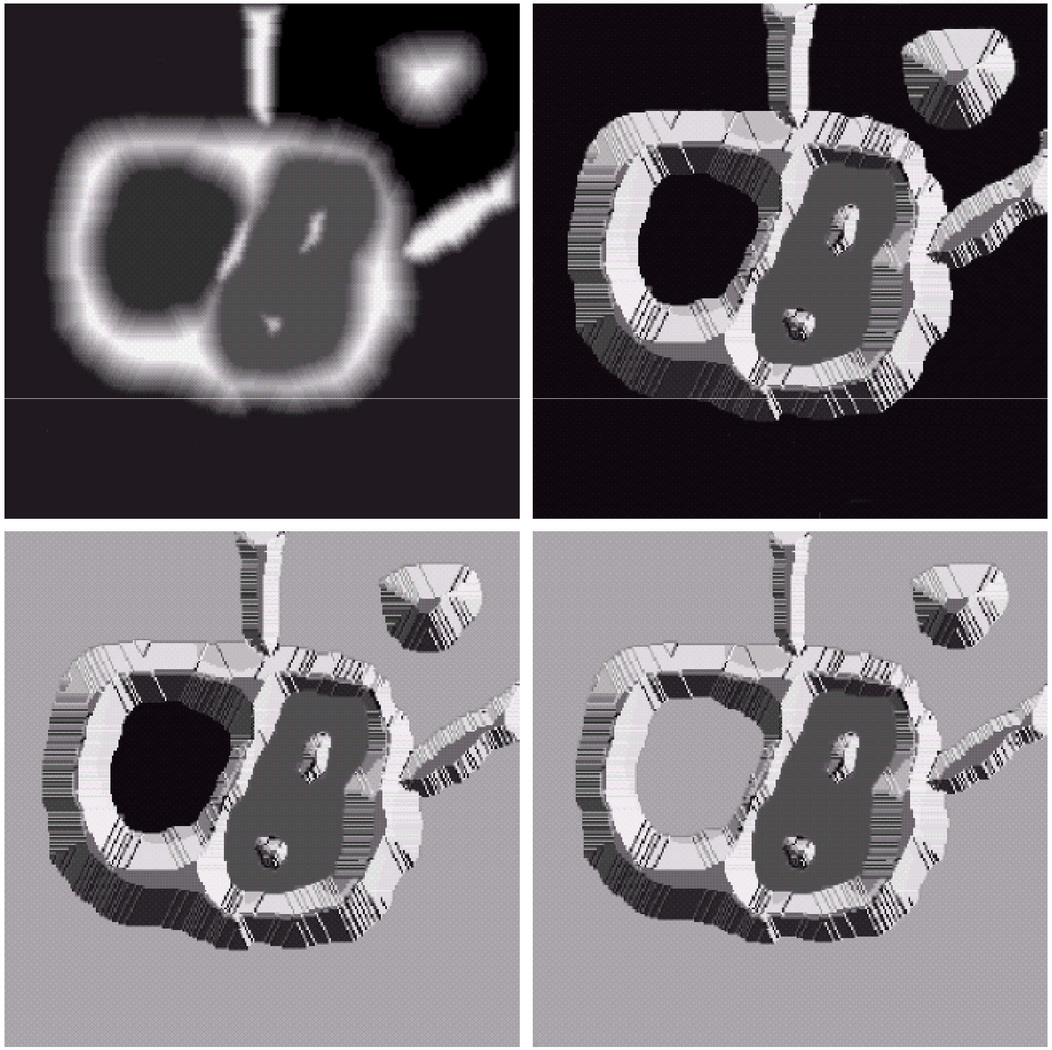
- Instead of working on an image itself, this technique is often applied on its gradient image.
- Three types of points
  - **Points belonging to a regional minimum**
  - **Catchment basin / watershed of a regional minimum**
    - **Points at which a drop of water will certainly fall to a single minimum**
  - **Divide lines / Watershed lines**
    - **Points at which a drop of water will be equally likely to fall to more than one minimum**
    - **Crest lines on the topographic surface**
- This technique is to identify all the third type of points for segmentation



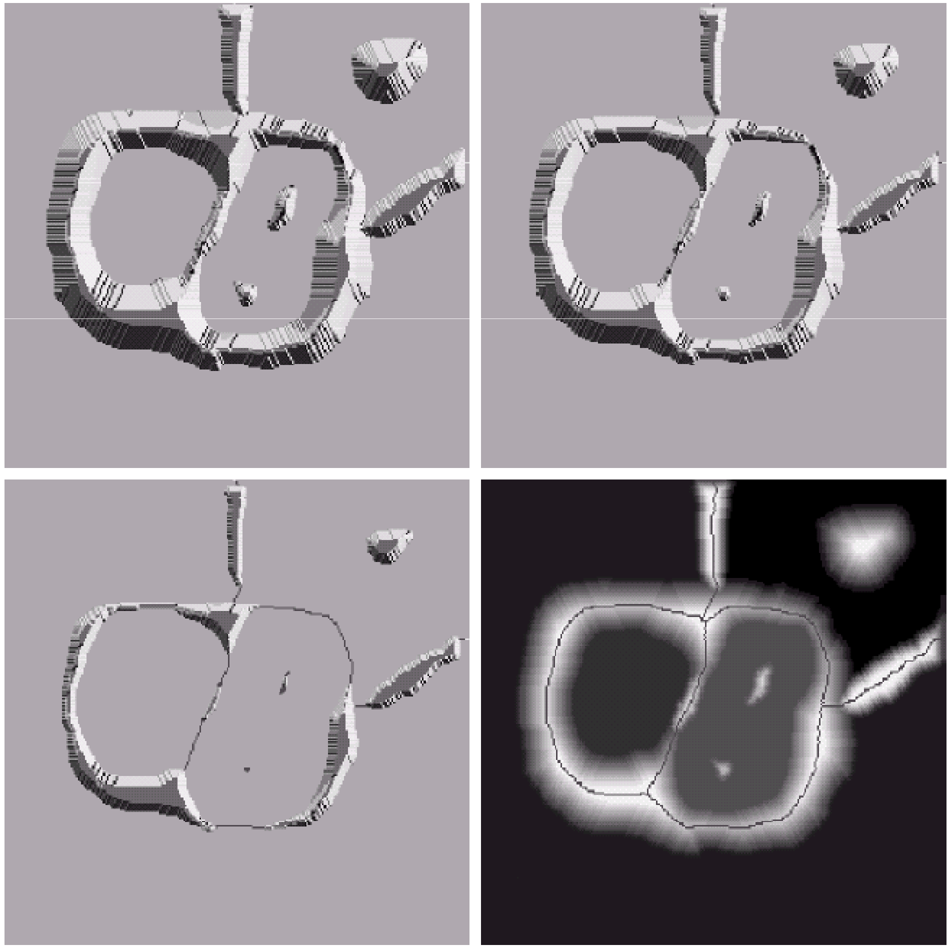


Basic Steps

1. Piercing holes in each regional minimum of I
2. The 3D topography is flooded from below gradually
3. When the rising water in distinct catchment basins is about to merge, a dam is built to prevent the merging



Basic Steps



e f
g h

FIGURE 10.44 *(Continued)* (e) Result of further flooding. (f) Beginning of merging of water from two catchment basins (a short dam was built between them). (g) Longer dams. (h) Final watershed (segmentation) lines. (Courtesy of Dr. S. Beucher, CMM/Ecole des Mines de Paris.)

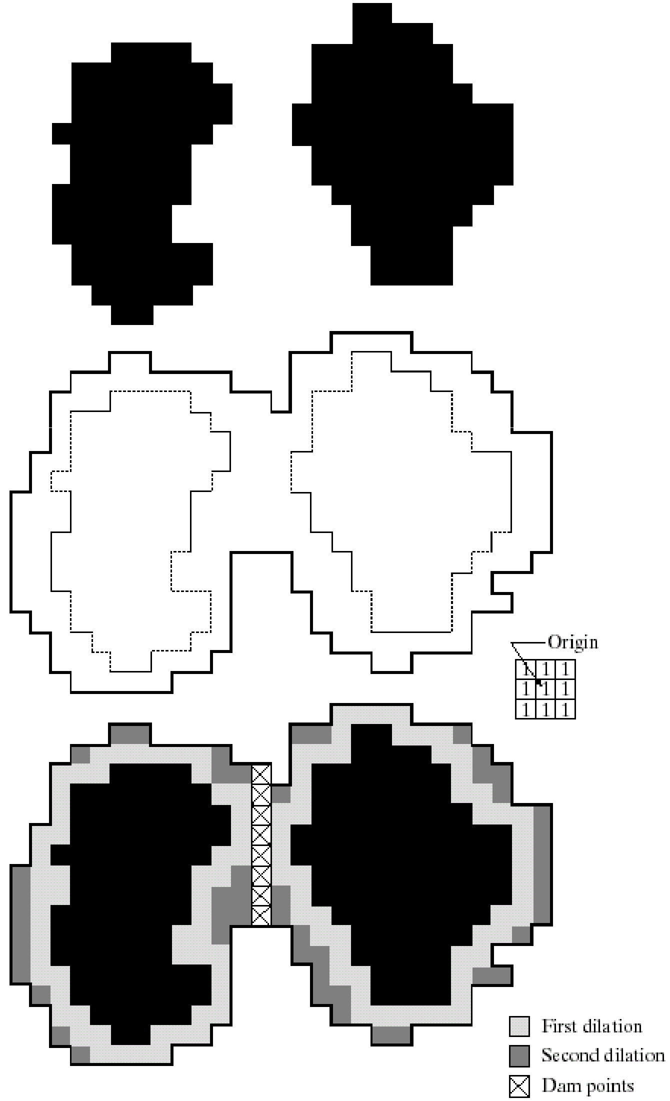
3. The dam boundaries correspond to the watershed lines to be extracted by a watershed segmentation algorithm
4. Eventually only constructed dams can be seen from above

Dam Construction

- Based on binary morphological dilation
- At each step of the algorithm, the binary image in obtained in the following manner
  1. Initially, the set of pixels with minimum gray level are 1, others 0.
  2. In each subsequent step, we flood the 3D topography from below and the pixels covered by the rising water are 1s and others 0s. (See previous slides)

Notations

- M1, M2:
  - Sets of coordinates of points in the two regional minima
- $C_{n-1}(M_1)$, $C_{n-1}(M_2)$
  - Sets of coordinates of points in the catchment basins associated with M1 M2 at stage n-1 of flooding (catchment basins up to the flooding level)
- C[n-1]
  - Union of $C_{n-1}(M_1)$, $C_{n-1}(M_2)$

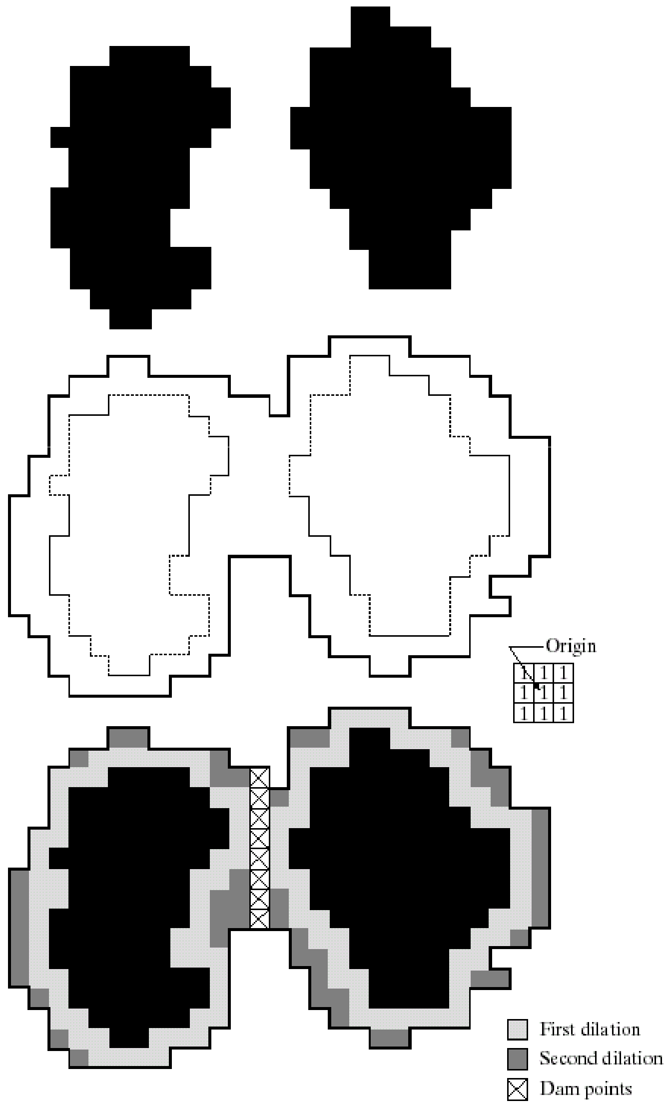


Dam Construction

Repeatedly dilate $C_{n-1}(M_1)$, $C_{n-1}(M_2)$
By the 3×3 structuring element shown,
subject to the following condition

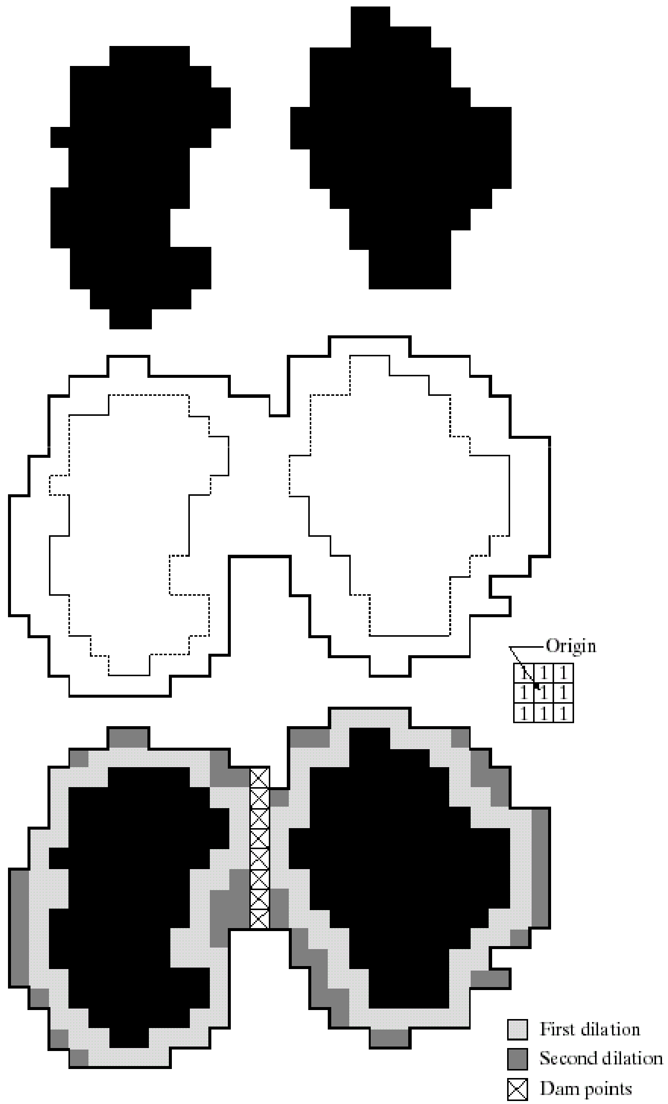
- Constrained to q (center of the structuring element can not go beyond q during dilation)
- The dilation cannot be performed on points that would cause the sets being dilated to merge

- Note in the figure, the condition (1) was satisfied by every point and cond (2) was not satisfied result in expanding the boundary .
- In second dilation cond(1) was not satisfied and cond(2) was satisfied resulting in broken perimeter



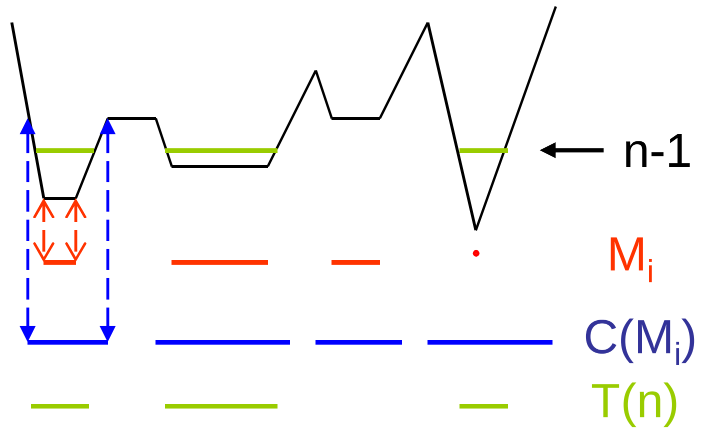
Dam Construction

- The points in q that satisfy both conditions describe the 1-pixel thick connected path shown in hatched lines.
- This path constitutes the desired separating dam at stage n of flooding.
- Construction of dam at this level of flooding sets all points in the path to a value greater than the maximum intensity value of the image. Usually max+1
- The dams built by this procedure are desired segmentation boundaries of connected components.



Watershed Transform

- Denote $M_1, M_2, \ldots, M_R$ as the sets of the coordinates of the points in the regional minima of an (gradient) image g(x,y)
- Denote $C(M_i)$ as the coordinates of the points in the catchment basin associated with regional minimum M_i.
- Denote the minimum and maximum gray levels of g(x,y) as *min* and *max*
- Denote T[n] as the set of coordinates (s,t) for which g(s,t) < n
- T(n) represents set of coordinates of points in g(x,y) lying below the plane g(x,y)=n
- Flood the topography in integer flood increments from n=*min*+1 to n=*max*+1
- At each flooding, the topography is viewed as a binary image

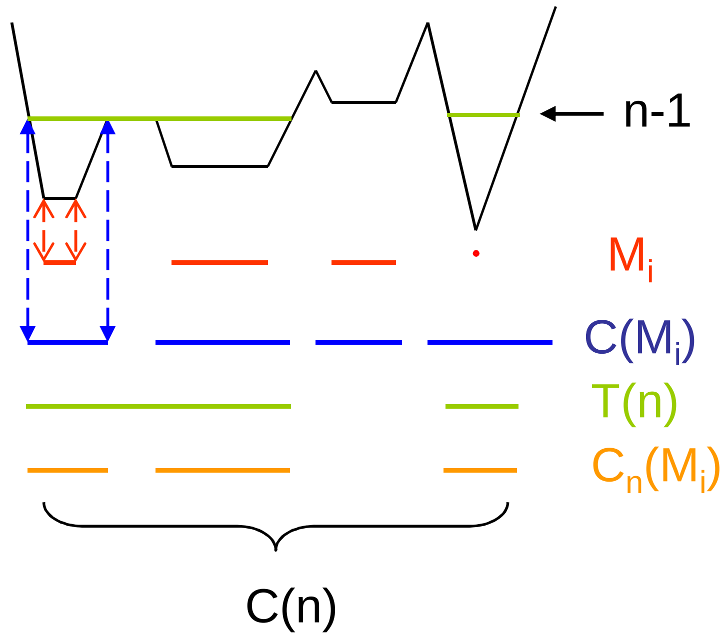


Watershed Transform

- Denote $C_n(M_i)$ as the set of coordinates of points in the catchment basin associated with minimum M_i at flooding stage n.
  - $C_n(M_i) = C(M_i) \cap T[n]$
  - $C_n(M_i) = \subseteq T[n]$
- Denote C[n] as the union of the flooded catchment basin portions at stage n:

$$C[n] = \bigcup_{i=1}^{R} C_n(M_i) \text{ and } C[\max+1] = \bigcup_{i=1}^{R} C(M_i)$$

- At each step n, assume C[n-1] has been constructed. The goal is to obtain C[n] from C[n-1]



Watershed Transform

- Denote Q[n] as the set of connected components in T[n].
- For each $q \in Q[n]$, there are three possibilities
  1. $q \cap C[n-1]$ is empty (q_1)
     - A new minimum is encountered
     - q is incorporated into C[n-1] to form C[n]
  2. $q \cap C[n-1]$ contains one connected component of C[n-1] (q_2)
     - q is incorporated into C[n-1] to form C[n]
  3. $q \cap C[n-1]$ contains more than one connected components of C[n-1] (q_3)
     - A ridge separating two or more catchment basins has been encountered
     - A dam has to be built within q to prevent overflow between the catchment basins
  4. Repeat the procedure until n=max+1