Histogram Processing

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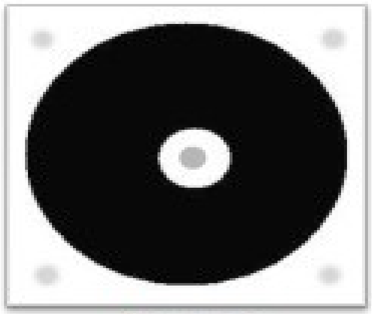
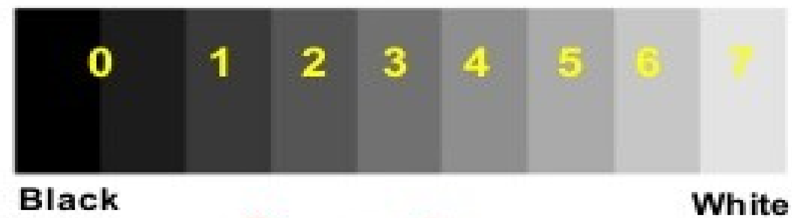
WHAT IS A HISTOGRAM?



Image

$$\begin{matrix} 0 & 7 & 3 & 2 & 3 \\ 0 & 0 & 0 & 6 & 7 \\ 7 & 7 & 2 & 2 & 0 \\ 1 & 1 & 0 & 4 & 1 \\ 0 & 0 & 7 & 4 & 1 \end{matrix}$$

Image matrix



Grey scale

Number of pixel with intensity value 0 [h(r0)] = 8
Similarly for **1** h(r1) = 4

WHAT IS A HISTOGRAM?

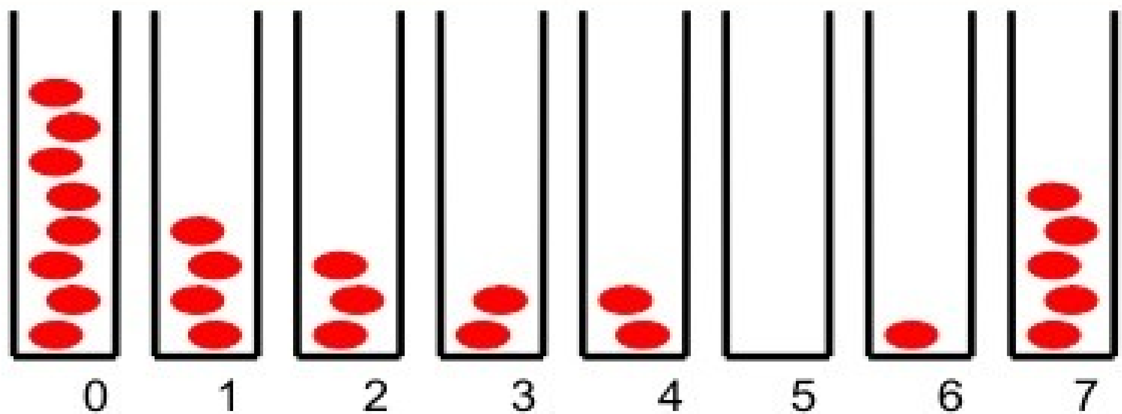
0	7	3	2	3
0	0	0	6	7
7	7	2	2	0
1	1	0	4	1
0	0	7	4	1

Image matrix

r	0	1	2	3	4	5	6	7
h(r)	8	4	3	2	2	0	1	5

h(r)

Number of pixels of intensity r



Intensity values **r**

HISTOGRAM

Histogram plots the number of pixels for each intensity value

WHAT IS A HISTOGRAM?

r	0	1	2	3	4	5	6	7
h(r)	8	4	3	2	2	0	1	5
p(r) h(r)/(5*5)	8/25	4/25	3/25	2/25	2/25	0/25	1/25	5/25

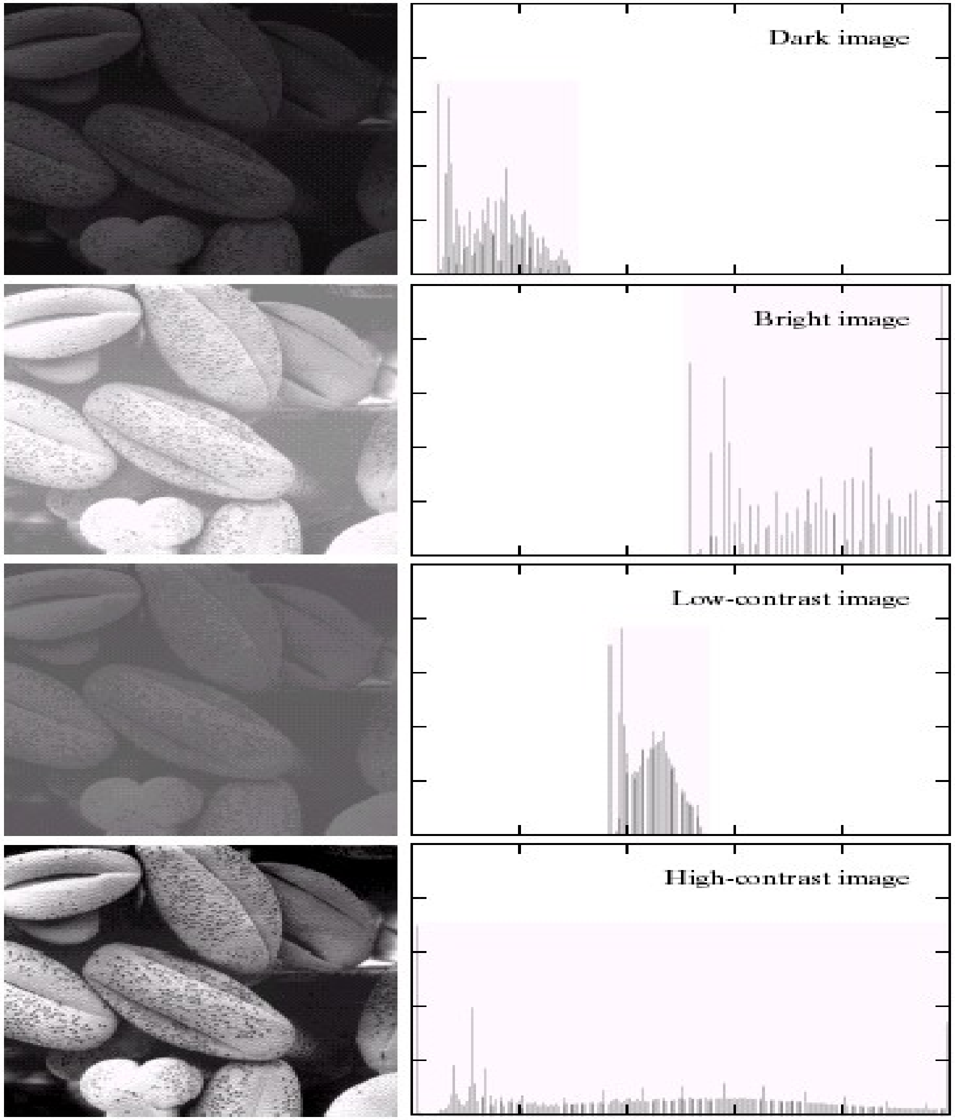
HISTOGRAM - ***h(r)*** - Y axis - ***number*** of intensities

NORMALIZED *HISTOGRAM* - ***p(r)*** - Y axis - ***probability*** of intensities

HISTOGRAM

- Histogram: of a digital image with gray levels 0 to L-1 is a discrete function of $\mathbf{h(r_k) = n_k}$
    - $\mathbf{r_k}$ **– k th gray level**
    - $\mathbf{n_k}$ **– number of pixels having gray levels** $\mathbf{r_k}$
- Normalized histogram $\mathbf{p(r_k) = n_k/n}$
  - n – total number of pixels in the image
- Normalized histogram gives an estimate of probability of occurrence of each gray level.
- Sum of all the components in an normalized histogram is 1.
- Useful in compression, segmentation

HISTOGRAM



- Horizontal axis- gray level values r_k
- Vertical axis- $\mathbf{h(r_k) = n_k}$ **or**

$\mathbf{p(r_k) = n_k/n}$

- Dark image-> components of the histogram on the lowside of gray level
- Bright image-> high side
- Low contrast-> centered towards middle
- High contrast-> board range of gray scale
- Image whose pixels tends to be distrubuted uniformly will have apperance of high contrast.
- Possible to develop a transformation function to achieve uniform distribution of pixels

HISTOGRAM Processing

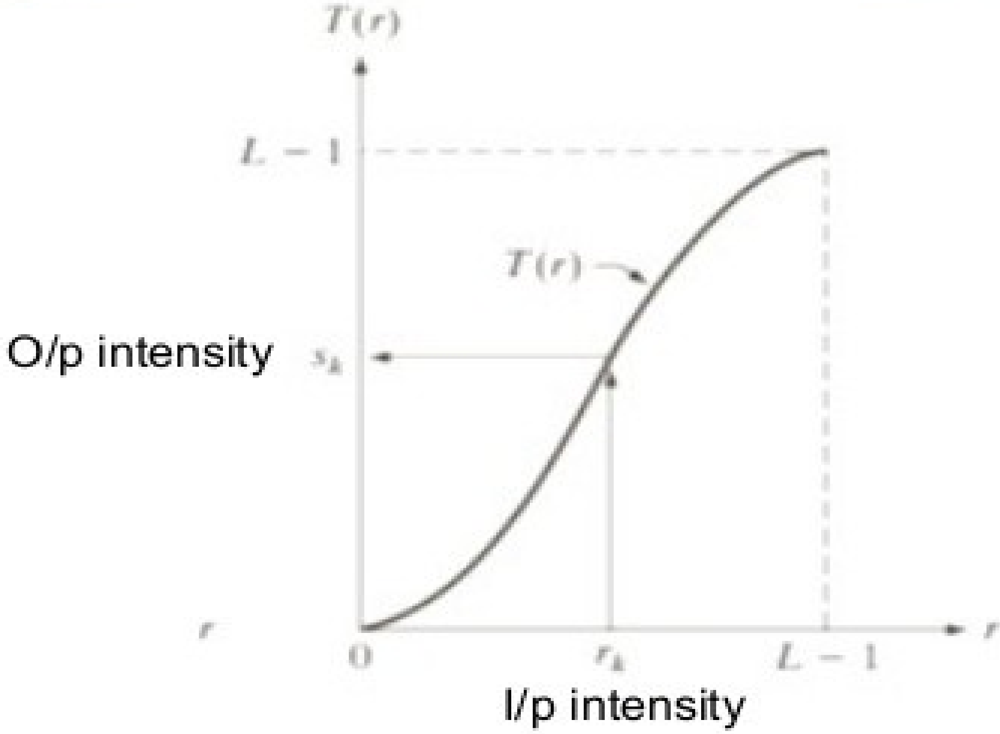
To achieve the contrast enhancement two different histogram processing can be done:

- Histogram Equalization
- Histogram Matching

Histogram Equalization

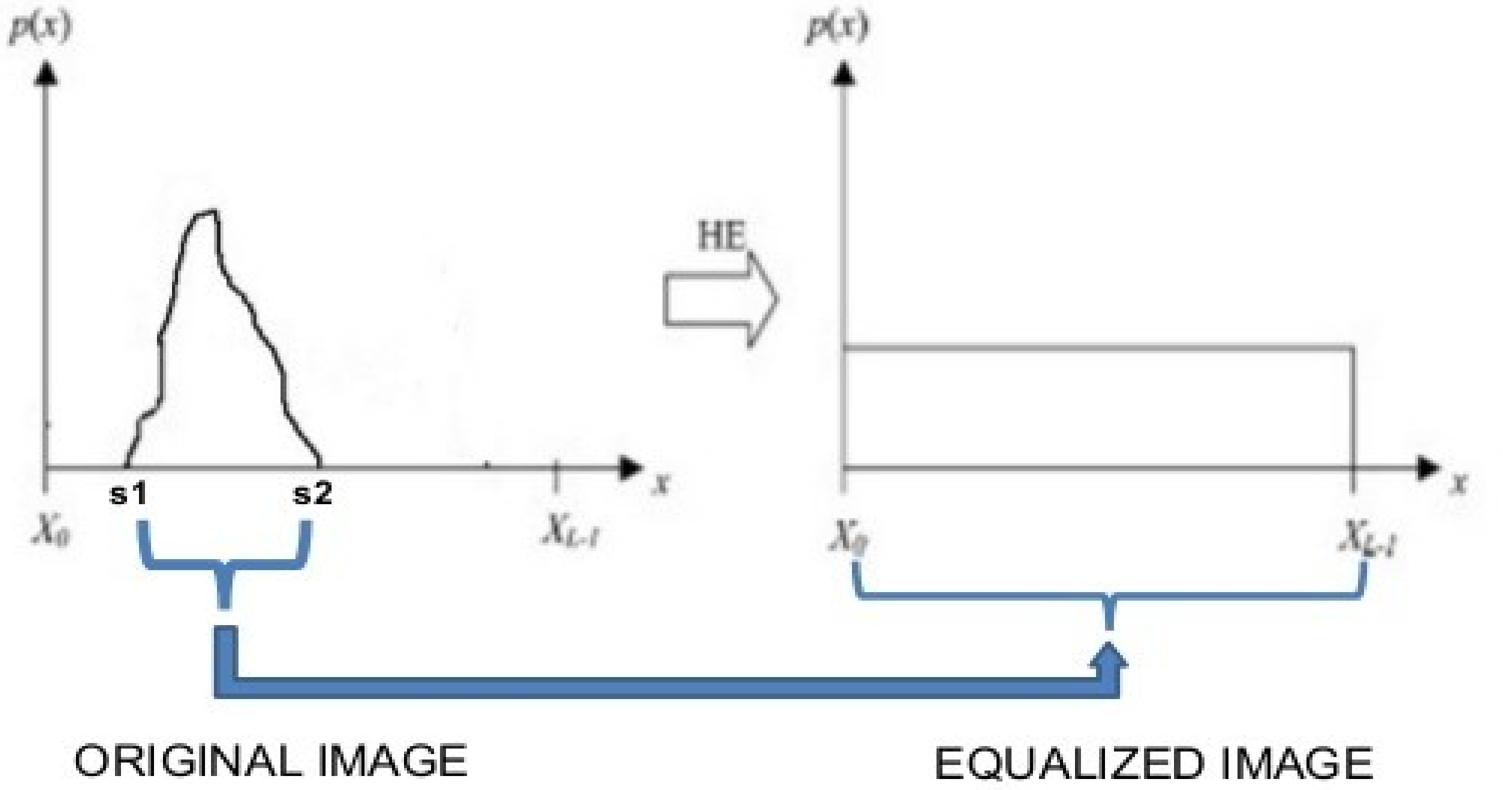
- The goal of histogram equalization to obtain uniform histogram for the output image
- To transform the gray levels of the image so that the histogram of the resultant image has histogram equalized or uniformly distributed.
- Used for modifying the contrast

THEORY BEHIND HISTOGRAM EQUALIZATION



- TRANSFORMATION FUNCTION THAT MAPS THE INPUT INTENSITY TO ALL AVAILABLE INTENSITIES.

THEORY BEHIND HISTOGRAM EQUALIZATION



Histogram Equalization

- Assuming the pixel values are continuous functions
  - **p(s) ds = p(r) dr**
- That is probability density of the transformed variable ‘s’ is determined by the gray level distribution of the input image and by the chosen transformation function
  - **s = T(r)**
- T should satisfies the following conditions
  - T(r) is single valued and monotonically increasing where r is in the range [0,1]
  - T(r) also varies in the range [0,1]
-

HISTOGRAM EQUALIZATION Continuous case

- The first requirement is to ensure that inverse transformation exist and monotonicity ensures the order of increasing intensities (black to white)
- The second requirement is to ensure that resulting gray levels are in the same range as input levels (onto)
- The inverse transformation from s back to r is denoted by $r = T^{-1}(s)$

HISTOGRAM EQUALIZATION Continuous case

- The gray levels in an image can be viewed as random variables in the interval [0, 1] and their pdf calculated
- If **p_r** and **p_s** are two different probability distributions on r and s (of input and transformed image) respectively,

Then the probability theory says if **p_r (r) and** T(r) is known then PDF **$p_s(s)$** of the transformed image at gray level s obtained using

- **$p_s(s) = p_r(r)|dr/ds|$**
- So the pdf of s depends on pdf of r and the transformation function.

HISTOGRAM EQUALIZATION Continuous case

Consider the CDF to be the transformation function. i.e.

$$s = T(r) = \int_0^r pr(w)\,dw$$

- W is dummy variable.This T(r) is single valued and monotonically increasing also the integration of a pdf is a pdf in the same range. So both constraints are statisfied.

ds/dr=dT(r)/dr=d/dr $\{\int pr(w)\,dw\}=p_r(r)$

-

HISTOGRAM EQUALIZATION Continuous case

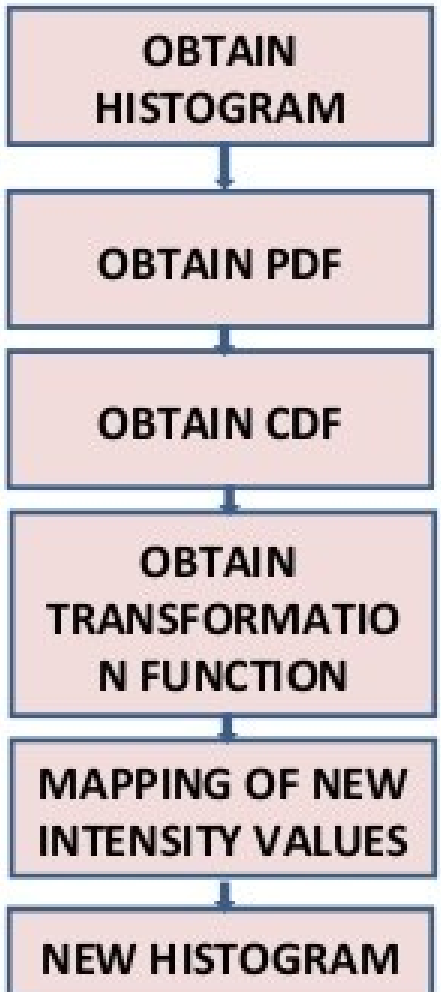
Substiuting the value of ds/dr we get

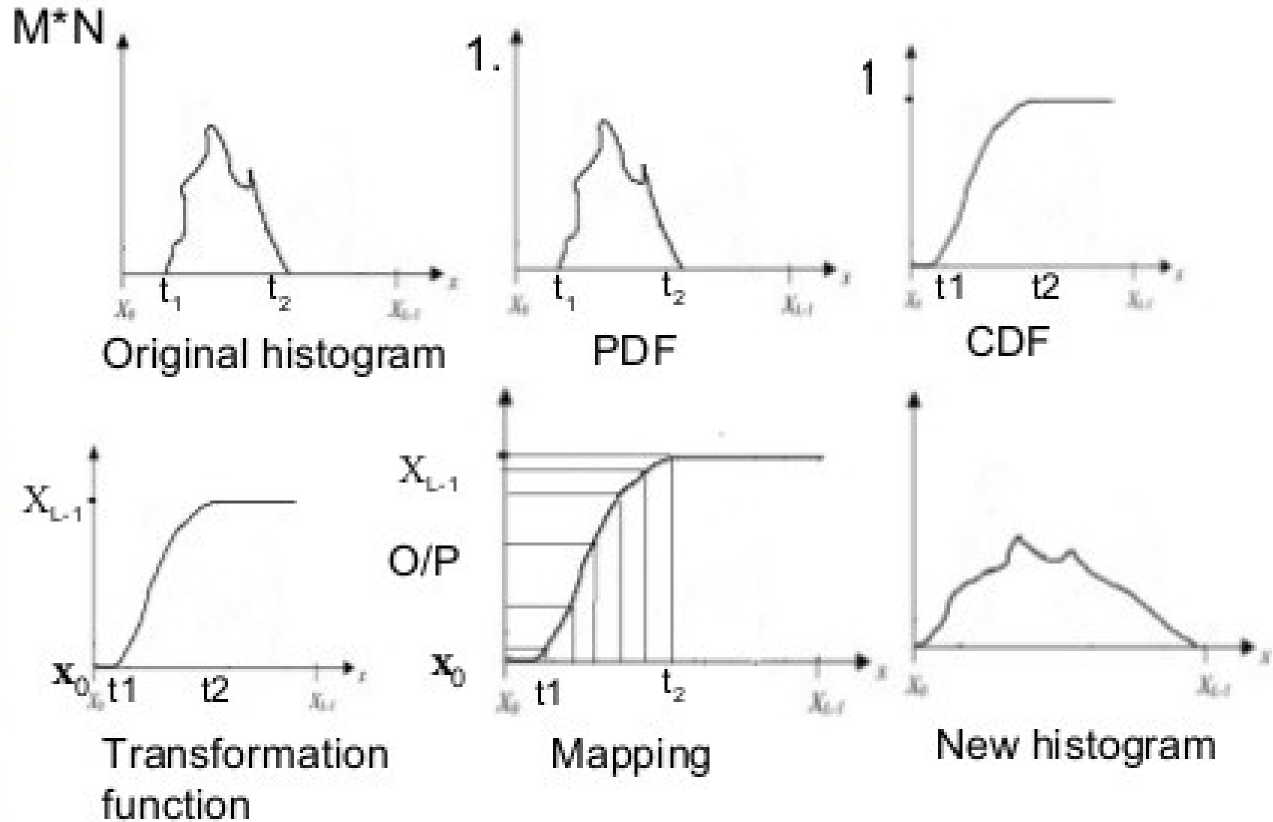
$p_s(s)=p_r(r)\ |dr/ds|\ =p_r(r)|1/p_r(r)|=1$

$p_s(s)$ is a pdf that is 0 outside the interval [0,1] and 1 in the interval [0,1] :- a uniform density.

- Thus the transformation T(r) yields a random variable s characterized by a uniform pdf.
- T(r) depends on $p_r(r)$ but always $p_s(s)$ is always a uniform pdf.
- In discrete case r takes discrete values r_k , k=0,1...L-1 and probability of occurrence of a gray level r_k in an image is approximated by:
  - $\mathbf{p_r(r_k) = n_k/n}$ **where k = 0,1...L - 1**

GLOBAL HISTOGRAM EQUALIZATION





Histogram equalization

Steps involved:

- Find the number of pixels in each gray level and total number of pixels
- Find the probability mass function of all the pixels in the image
- Find the cumulative distribution function
- Multiply the cumulative distribution function by L-1, L being

the maximum gray level, to obtain new gray levels

- Map the new gray level values to the pixels with old gray levels.

- Mapping each pixel with level r_k in the input image into the corresponding pixel with level s_k in the output image

Gray level values	Number of pixels	Probability mass function	Cumulative distribution function
0	790	0.19	0.19
1	1023	0.25	0.44
2	850	0.21	0.65
3	656	0.16	0.81
4	329	0.08	0.89
5	245	0.06	0.95
6	122	0.03	0.98
7	81	0.02	1.00

$$s_k = T(r_k) = \sum_{j=0}^{k} p(r_j)$$

$$p(r_j) = \frac{n_j}{n} \quad k = 0,1,2.....L\text{-}1$$

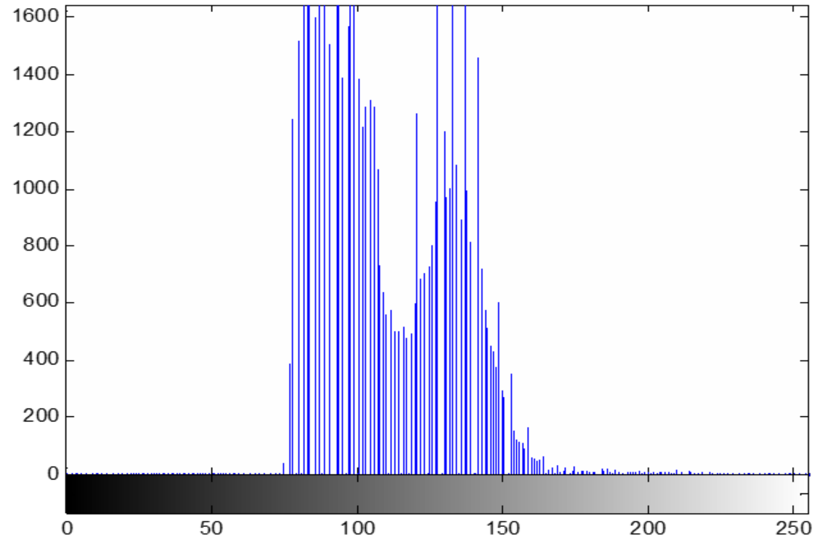
Gray level values	Number of pixels	Probability mass function	Cumulative distribution function	CDF * L-1
0	790	0.19	0.19	1.33 = 1
1	1023	0.25	0.44	3.08 = 3
2	850	0.21	0.65	4.55 = 5
3	656	0.16	0.81	5.67 = 6
4	329	0.08	0.89	6.23 = 6
5	245	0.06	0.95	6.65 = 7
6	122	0.03	0.98	6.86 = 7
7	81	0.02	1.00	7 = 7

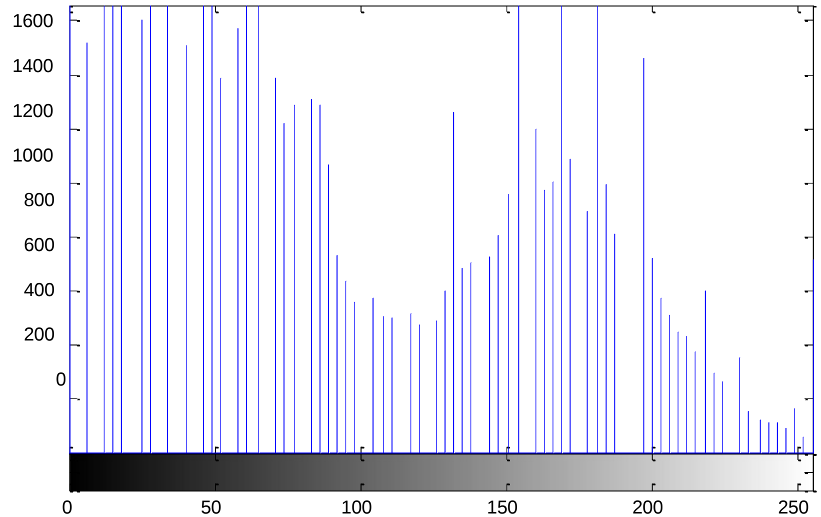
Mapping

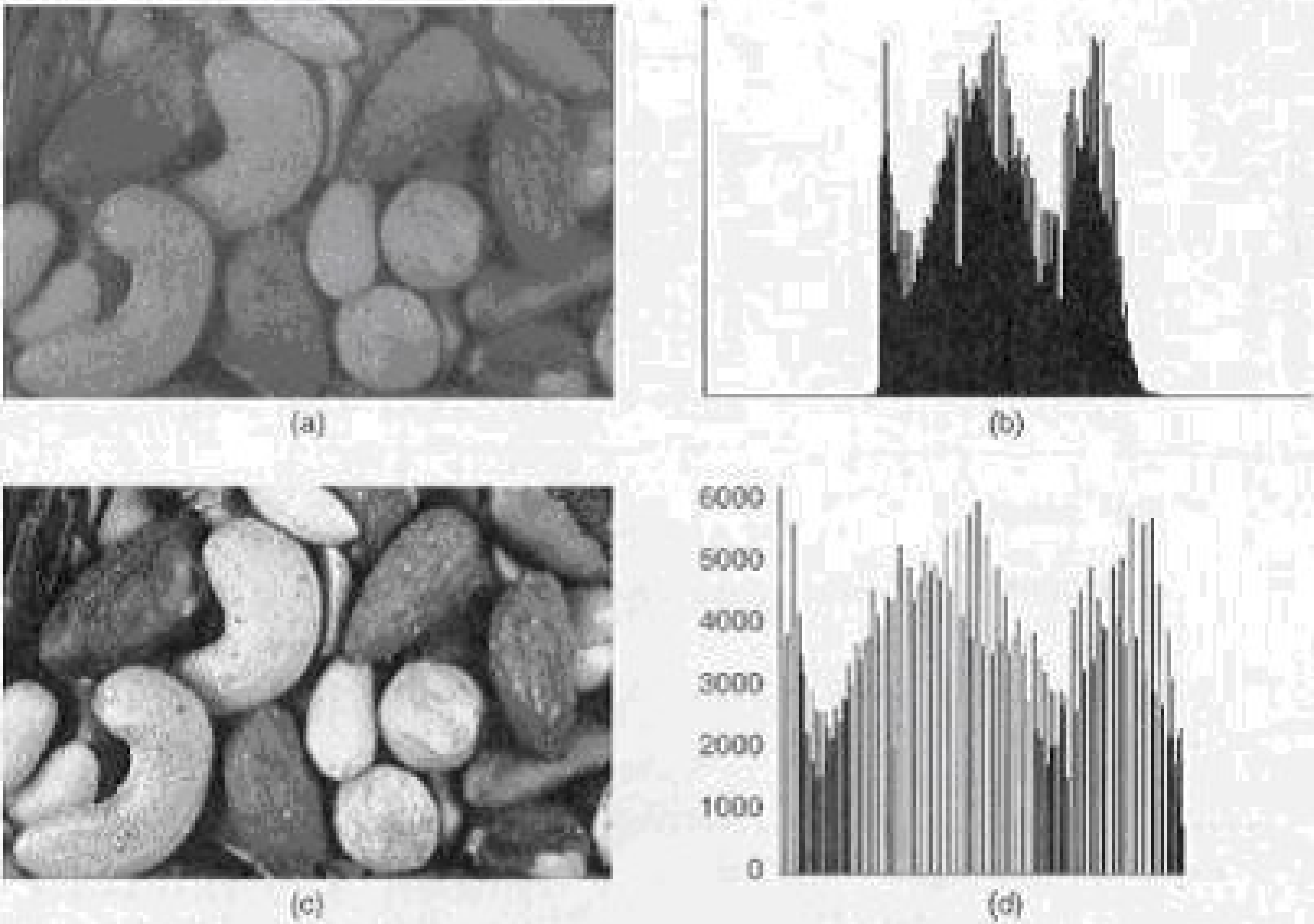
- Mapping:

Old gray level values	New gray level values	Frequency of occurrence
0	1	790
1	3	1023
2	5	850
3	6	656
4	6	329
5	7	245
6	7	122
7	7	81

Histogram equalization







(a)
(b)
(c)
6000
5000
4000
3000
2000
1000
0
(d)

Histogram matching or specification

- Specify the shape of the histogram we wish the processed image should have
- The method used to generate a processed image that has a specified histogram is called histogram matching or histogram specification
- Steps involved:
  - Equalize the histogram of the input image
  - Equalize the specified histogram
  - Relate the two equalized histograms

Histogram matching

$$s_k = \mathrm{T}(r_k) = \sum_{j=0}^{k} p(r_j)$$

$$\mathrm{Min}(s_k - v_k)$$

$$Z = G^{-1}(s_k)$$

$$v_k = \mathrm{G}(z_k) = \sum_{j=0}^{k} p(z_j) = s_k$$

Matching

- Steps involved:
  - Find histogram and CDF of input image

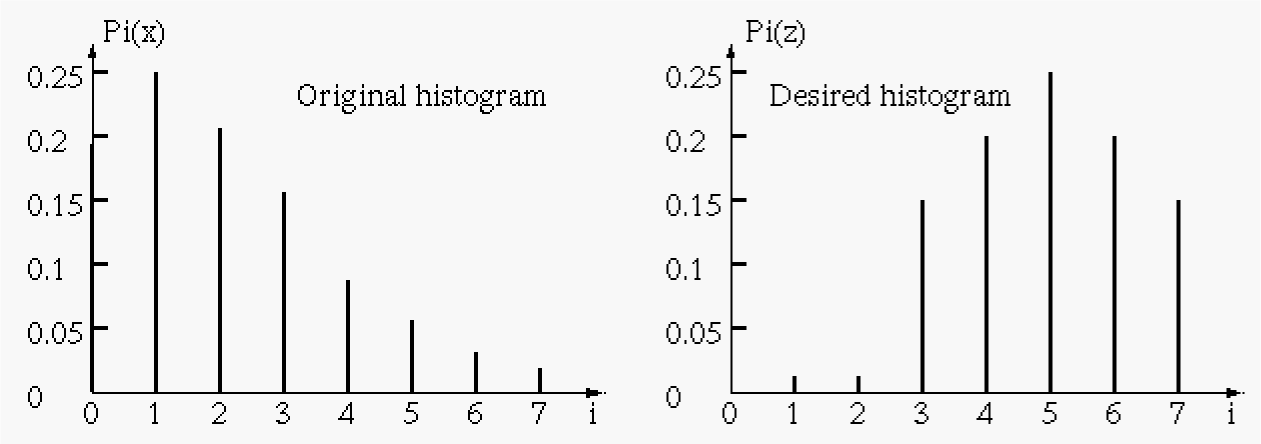
$$s_k = T(r_k) = \sum_{j=0}^{k} p(r_j)$$

$$p(r_j) = \frac{n_j}{n} \quad k = 0,1,2.....L-1$$

$$v_k = T(z_k) = \sum_{j=0}^{k} p(z_j)$$

  - Specify the desired histogram, find its cumulative and CDF
  - Relate the two mapping function and to build a look up table and for overall mapping
  - For each input level find the out put level

Example



Pi(x)
Original histogram
0.25
0.2
0.15
0.1
0.05
0
0 1 2 3 4 5 6 7 i
Pi(z)
Desired histogram
0.25
0.2
0.15
0.1
0.05
0
0 1 2 3 4 5 6 7 i

Example

Gray level values	Number of pixels	Probability mass function	Cumulative distribution function
0	790	0.19	0.19
1	1023	0.25	0.44
2	850	0.21	0.65
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6	122	0.03	0.98
7	81	0.02	1.00

Gray level values	Probability mass function	Cumulative distribution function
0	0	0
1	0	0
2	0	0
3	0.15	0.15
4	0.2	0.35
5	0.3	0.65
6	0.2	0.85
7	0.15	1.0

Example

Input gray level	Cumulative distribution function	Cumulative distribution function	Output gray level
0	0.19	0.0	3
1	0.44	0.0	4
2	0.65	0.0	5
3	0.81	0.15	6
4	0.89	0.35	6
5	0.95	0.65	7
6	0.98	0.85	7
7	1.0	1.0	7

i	0	1	2	3	4	5	6	7
j	3	4	5	6	6	7	7	7