

# Tutorial 2

## Counting

Answer any 3 Questions

1. (a) Find all partitions of 7.  
(b) Show that the number of partitions of  $n \in \mathbb{Z}^+$  where no summand is divisible by 4 equals the number of partitions of  $n$  where no even summand is repeated (although odd summands may or may not be repeated).
2. (a) Of 1000 applicants for a mountain-climbing trip in the Himalayas, 450 get altitude sickness, 622 are not in good enough shape, and 30 have allergies. An applicant qualifies if and only if this applicant does not get altitude sickness, is in good shape, and does not have allergies. If there are 111 applicants who get altitude sickness and are not in good enough shape, 14 who get altitude sickness and have allergies, 18 who are not in good enough shape and have allergies, and 9 who get altitude sickness, are not in good enough shape, and have allergies, how many applicants qualify ?  
(b) Use the principle of inclusion-exclusion to compute the number of solutions of the equation  $x_1 + x_2 + x_3 + x_4 = 17$ , where  $x_1 \leq 3$ ,  $x_2 \leq 4$ ,  $x_3 \leq 5$  and  $x_4 \leq 8$  ?
3. (a) Use generating functions to find the number of ways to choose a dozen bagels from three varieties—egg, salty, and plain—if at least two bagels of each kind but no more than three salty bagels are chosen.  
(b) Use generating functions and summation operator to find a formula to express  $0^2 + 1^2 + 2^2 + \cdots + n^2$  as a function of  $n$ .
4. (a) Find a recurrence relation for the number of strictly increasing sequences of positive integers that have 1 as their first term and  $n$  as their last term, where  $n$  is a positive integer. That is, sequences  $a_1, a_2, \dots, a_k$ , where  $a_1 = 1$ ,  $a_k = n$ , and  $a_j < a_{j+1}$  for  $j = 1, 2, \dots, k - 1$ . What are the initial conditions? How many sequences are there when  $n$  is an integer with  $n \geq 2$  ?  
(b) A string that contains only 0s, 1s, and 2s is called a **ternary string**. Find a recurrence relation for the number of ternary strings of length  $n$  that do not contain two consecutive 0s. What are the initial conditions? How many ternary strings of length six do not contain two consecutive 0s ?