Diffie-Hellman Key Exchange

- First published public-key algorithm
- A number of commercial products employ this key exchange technique
- Purpose is to enable two users to securely exchange a key that can then be used for subsequent symmetric encryption of messages
- The algorithm itself is limited to the exchange of secret values
- Its effectiveness depends on the difficulty of computing discrete logarithms

$$\log_x(1) = 0$$
$$\log_x(x) = 1$$

$$\log_x(yz) = log_x(y) + \log_x(z)$$
$$\log_x(y^r) = r \times \log_x(y)$$

$b \equiv r \pmod{p}$ for some r, where $0 \leq r \leq (p - 1)$

by the definition of modular arithmetic. It follows that for any integer b and a primitive root a of prime number p, we can find a unique exponent i such that

$$b \equiv a^i \pmod{p} \quad \text{where } 0 \leq i \leq (p - 1)$$

This exponent i is referred to as the **discrete logarithm** of the number b for the base $a \pmod{p}$. We denote this value as $\text{dlog}_{a,p}(b)$.[9]

Note the following:

$$\text{dlog}_{a,p}(1) = 0 \text{ because } a^0 \bmod p = 1 \bmod p = 1 \qquad \textbf{(8.13)}$$

$$\text{dlog}_{a,p}(a) = 1 \text{ because } a^1 \bmod p = a \qquad \textbf{(8.14)}$$

a	a^2	a^3	a^4	a^5	a^6	a^7	a^8	a^9	a^{10}	a^{11}	a^{12}	a^{13}	a^{14}	a^{15}	a^{16}	a^{17}	a^{18}
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	4	8	16	13	7	14	9	18	17	15	11	3	6	12	5	10	1
3	9	8	5	15	7	2	6	18	16	10	11	14	4	12	17	13	1
4	16	7	9	17	11	6	5	1	4	16	7	9	17	11	6	5	1
5	6	11	17	9	7	16	4	1	5	6	11	17	9	7	16	4	1
6	17	7	4	5	11	9	16	1	6	17	7	4	5	11	9	16	1
7	11	1	7	11	1	7	11	1	7	11	1	7	11	1	7	11	1
8	7	18	11	12	1	8	7	18	11	12	1	8	7	18	11	12	1
9	5	7	6	16	11	4	17	1	9	5	7	6	16	11	4	17	1
10	5	12	6	3	11	15	17	18	9	14	7	13	16	8	4	2	1
11	7	1	11	7	1	11	7	1	11	7	1	11	7	1	11	7	1
12	11	18	7	8	1	12	11	18	7	8	1	12	11	18	7	8	1
13	17	12	4	14	11	10	16	18	6	2	7	15	5	8	9	3	1
14	6	8	17	10	7	3	4	18	5	13	11	2	9	12	16	15	1
15	16	12	9	2	11	13	5	18	4	3	7	10	17	8	6	14	1
16	9	11	5	4	7	17	6	1	16	9	11	5	4	7	17	6	1
17	4	11	16	6	7	5	9	1	17	4	11	16	6	7	5	9	1
18	1	18	1	18	1	18	1	18	1	18	1	18	1	18	1	18	1

$$y = g^x \bmod p$$

ber can belong (mod n) is $\phi(n)$. If a number is of this order, it is referred to as a **primitive root** of n. The importance of this notion is that if a is a primitive root of n, then its powers

$$a, a^2, \ldots, a^{\phi(n)}$$

are distinct (mod n) and are all relatively prime to n. In particular, for a prime number p, if a is a primitive root of p, then

$$a, a^2, \ldots, a^{p-1}$$

are distinct (mod p). For the prime number 19, its primitive roots are 2, 3, 10, 13, 14, and 15.

Alice **Bob**

Alice	Bob
Alice and Bob share a prime q and α, such that $\alpha < q$ and α is a primitive root of q	Alice and Bob share a prime q and α, such that $\alpha < q$ and α is a primitive root of q
Alice generates a private key X_A such that $X_A < q$	Bob generates a private key X_B such that $X_B < q$
Alice calculates a public key $Y_A = \alpha^{X_A} \bmod q$	Bob calculates a public key $Y_B = \alpha^{X_B} \bmod q$
Alice receives Bob's public key Y_B in plaintext	Bob receives Alice's public key Y_A in plaintext
Alice calculates shared secret key $K = (Y_B)^{X_A} \bmod q$	Bob calculates shared secret key $K = (Y_A)^{X_B} \bmod q$

Figure 10.1 Diffie-Hellman Key Exchange

$$
\begin{aligned}
K &= (Y_B)^{X_A} \bmod q \\
&= (\alpha^{X_B} \bmod q)^{X_A} \bmod q \\
&= (\alpha^{X_B})^{X_A} \bmod q \\
&= \alpha^{X_B X_A} \bmod q \\
&= (\alpha^{X_A})^{X_B} \bmod q \\
&= (\alpha^{X_A} \bmod q)^{X_B} \bmod q \\
&= (Y_A)^{X_B} \bmod q
\end{aligned}
$$

Key Exchange Protocols

- Users could create random private/public Diffie-Hellman keys each time they communicate
- Users could create a known private/public Diffie-Hellman key and publish in a directory, then consulted and used to securely communicate with them
- Vulnerable to Man-in-the-Middle-Attack
- Authentication of the keys is needed

Here is an example. Key exchange is based on the use of the prime number $q = 353$ and a primitive root of 353, in this case $\alpha = 3$. A and B select private keys $X_A = 97$ and $X_B = 233$, respectively. Each computes its public key:

A computes $Y_A = 3^{97} \bmod 353 = 40$.

B computes $Y_B = 3^{233} \bmod 353 = 248$.

After they exchange public keys, each can compute the common secret key:

A computes $K = (Y_B)^{X_A} \bmod 353 = 248^{97} \bmod 353 = 160$.

B computes $K = (Y_A)^{X_B} \bmod 353 = 40^{233} \bmod 353 = 160$.

We assume an attacker would have available the following information:

$$q = 353; \alpha = 3; Y_A = 40; Y_B = 248$$

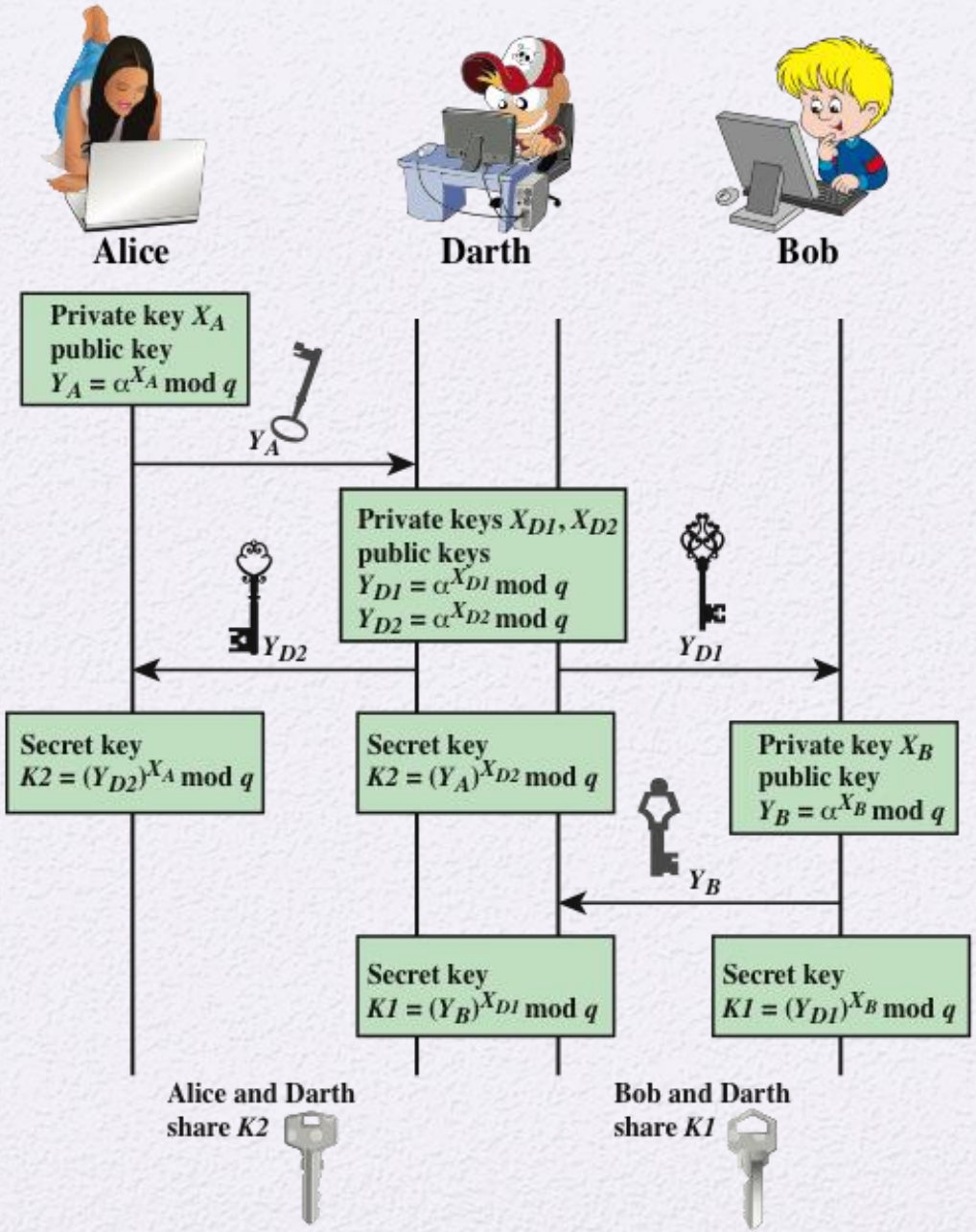


Figure 10.2 Man-in-the-Middle Attack

1. Alice sends an encrypted message M: E($K2$, M).
2. Darth intercepts the encrypted message and decrypts it to recover M.
3. Darth sends Bob $E(K1, M)$ or E($K1$, M'), where M' is any message. In the first case, Darth simply wants to eavesdrop on the communication without altering it. In the second case, Darth wants to modify the message going to Bob.

1. Venona project
2. A50, A51, E0

Key Distribution Technique

- Term that refers to the means of delivering a key to two parties who wish to exchange data without allowing others to see the key
- For symmetric encryption to work, the two parties to an exchange must share the same key, and that key must be protected from access by others
- Frequent key changes are desirable to limit the amount of data compromised if an attacker learns the key

Symmetric Key Distribution

Given parties A and B, key distribution can be achieved in a number of ways:

- **A can select a key and physically deliver it to B**
- **A third party can select the key and physically deliver it to A and B**
- **If A and B have previously and recently used a key, one party can transmit the new key to the other, encrypted using the old key**
- **If A and B each has an encrypted connection to a third party C, C can deliver a key on the encrypted links to A and B**

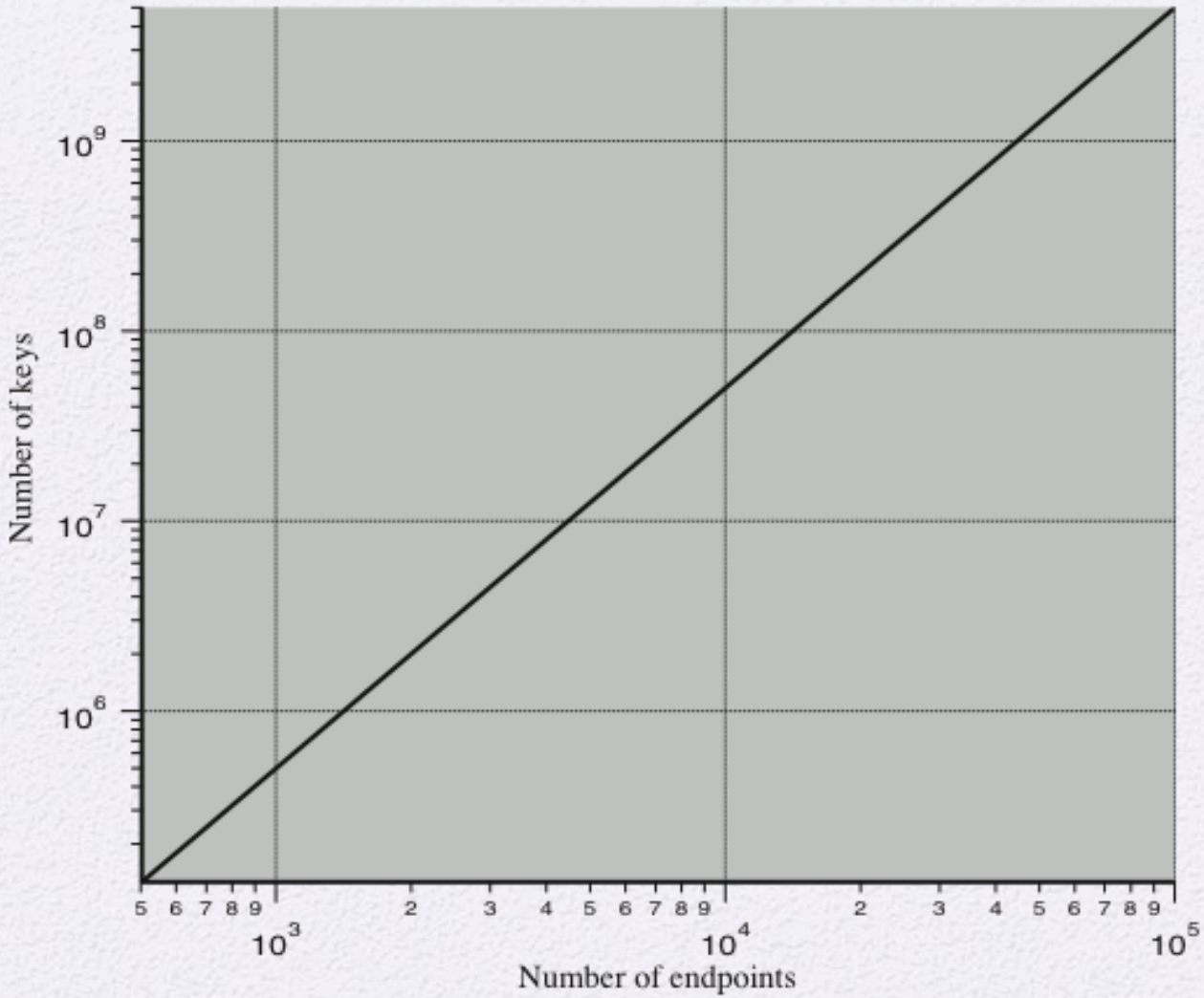
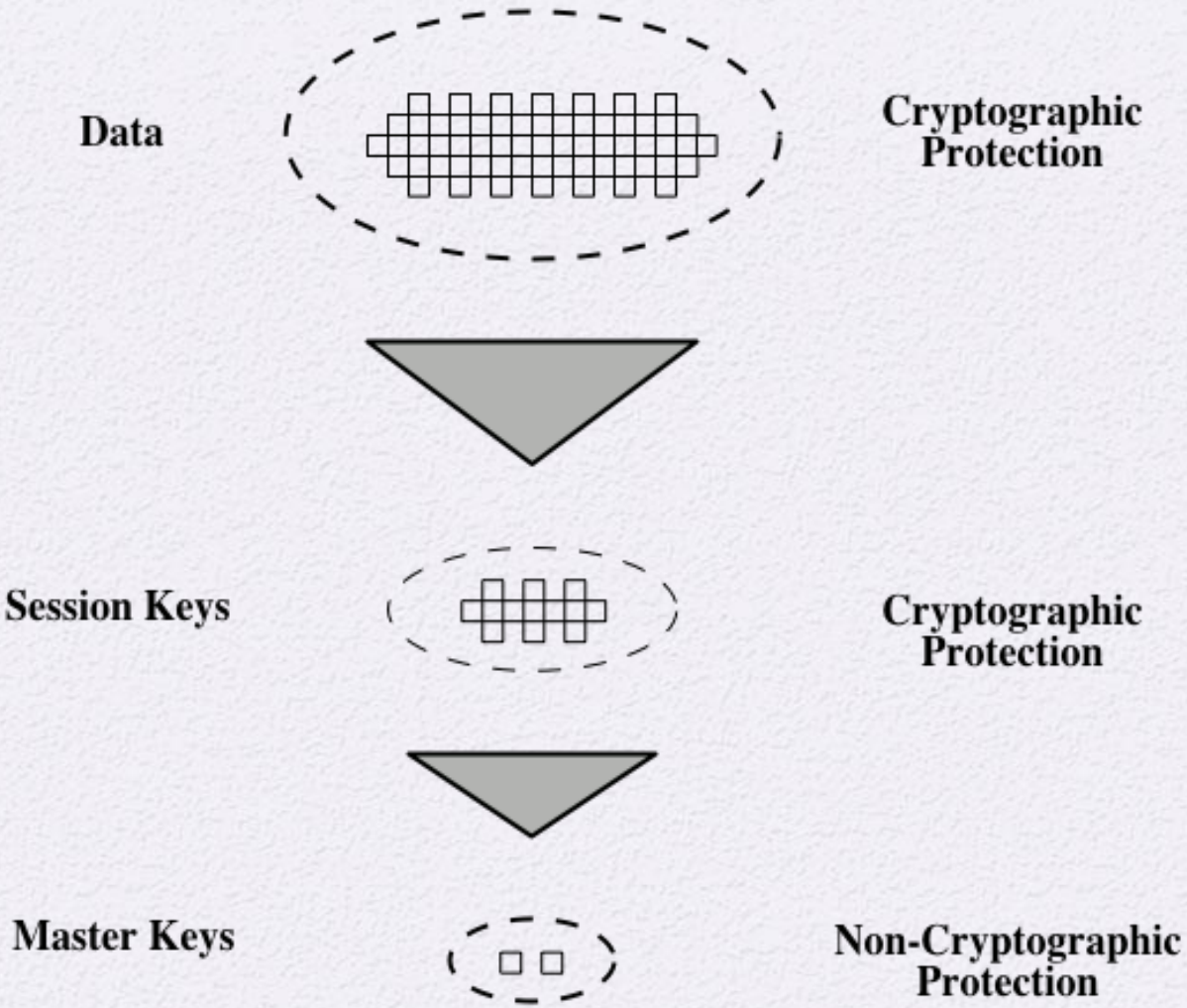


Figure 14.1 Number of Keys Required to Support Arbitrary Connections Between Endpoints



However, only *N master keys are required,*
one for each entity. Thus, master keys can be distributed in some non-cryptographic way, such as physical delivery.

Figure 14.2 The Use of a Key Hierarchy

- Public announcement of public keys
- Publicly available directory
- Public-key authority
- Public-key certificates

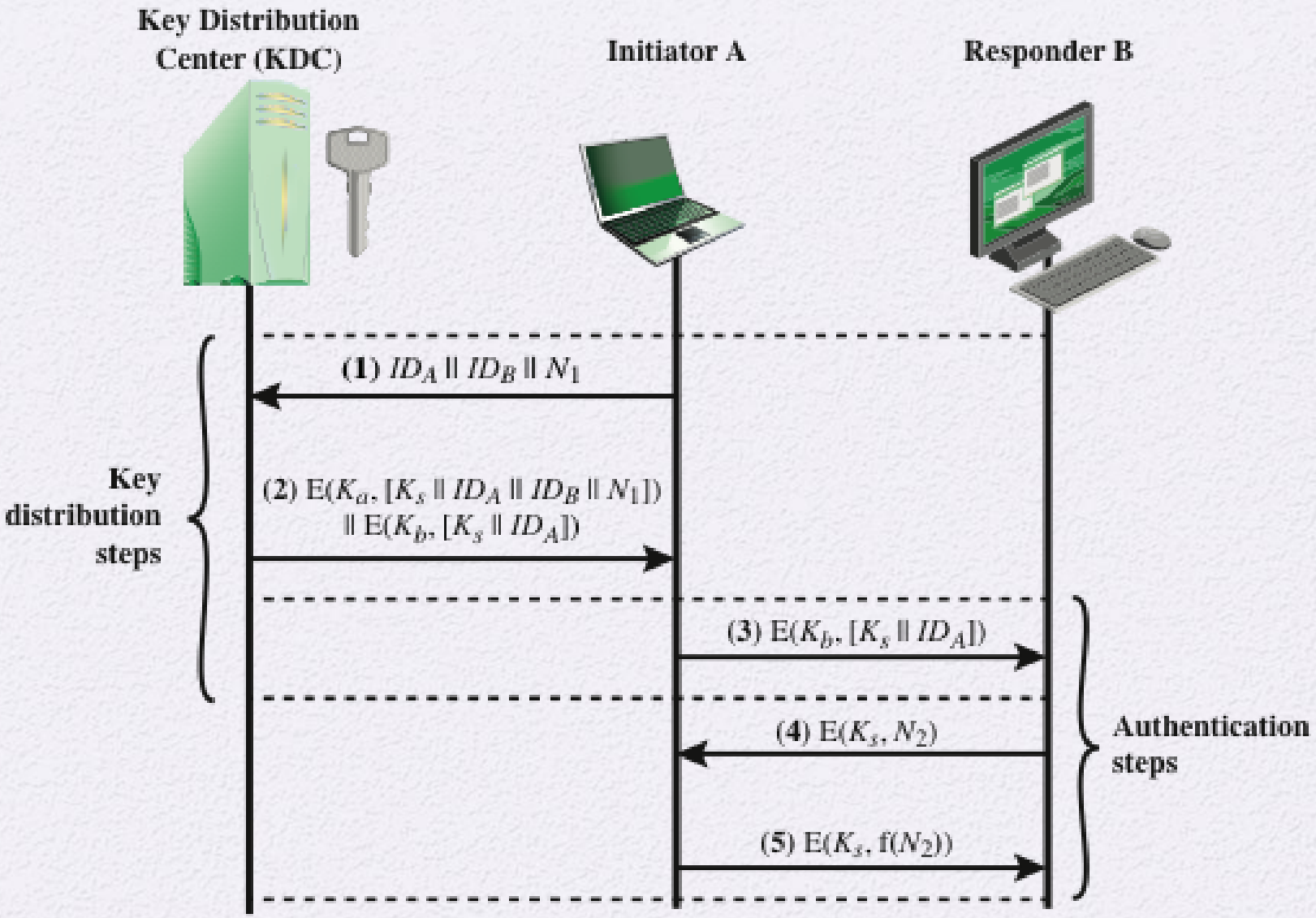


Figure 14.3 Key Distribution Scenario

Hierarchical Key Control

- For communication among entities within the same local domain, the local KDC is responsible for key distribution
  - If two entities in different domains desire a shared key, then the corresponding local KDC's can communicate through a global KDC
- The hierarchical concept can be extended to three or more layers
- Scheme minimizes the effort involved in master key distribution because most master keys are those shared by a local KDC with its local entities
  - Limits the range of a faulty or subverted KDC to its local area only

Session Key Lifetime

For connection-oriented protocols one choice is to use the same session key for the length of time that the connection is open, using a new session key for each new session

A security manager must balance competing considerations:

- The more frequently session keys are exchanged, the more secure they are
- The distribution of session keys delays the start of any exchange and places a burden on network capacity

For a connectionless protocol there is no explicit connection initiation or termination, thus it is not obvious how often one needs to change the session key

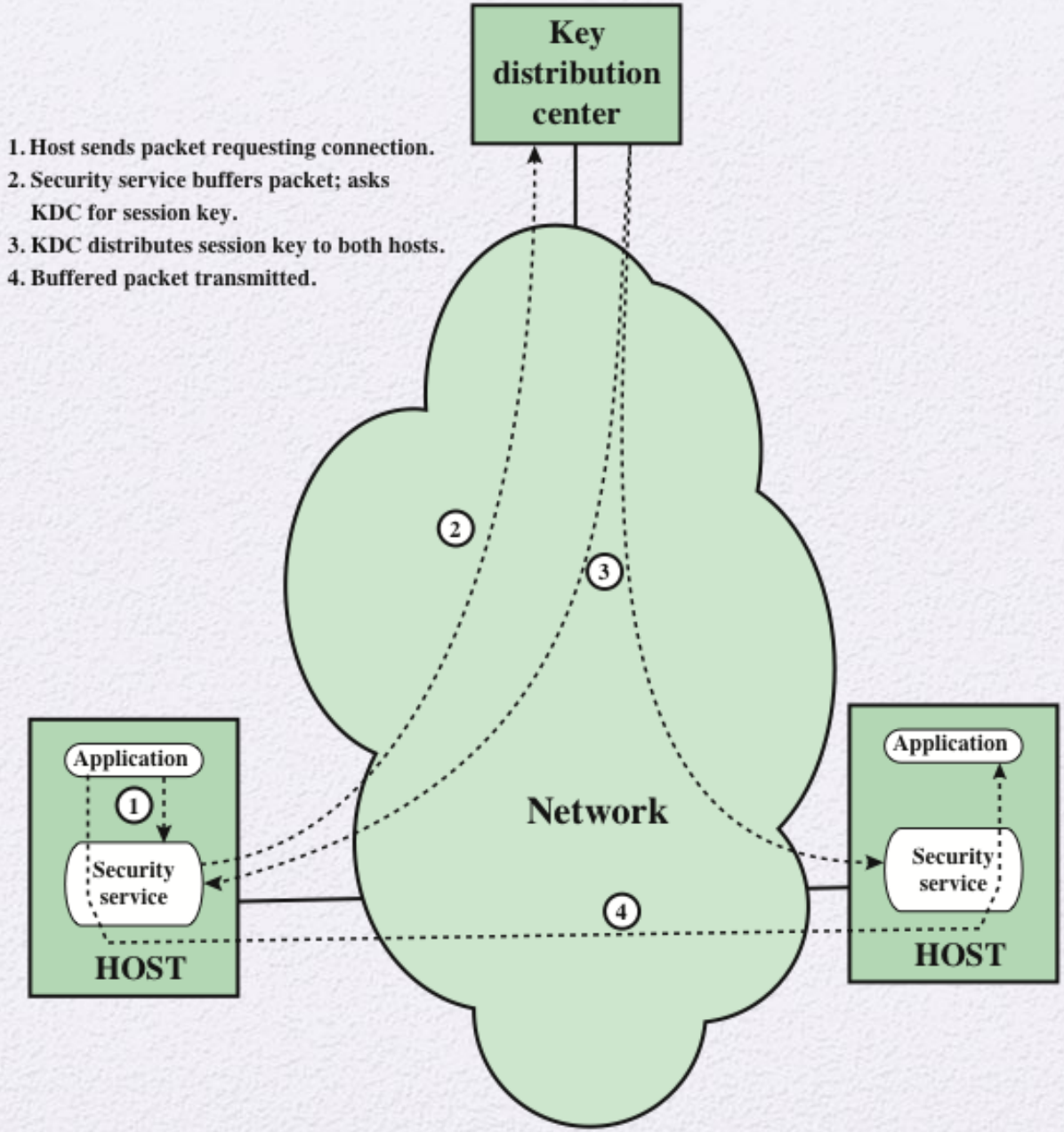


Figure 14.4 Automatic Key Distribution for Connection-Oriented Protocol

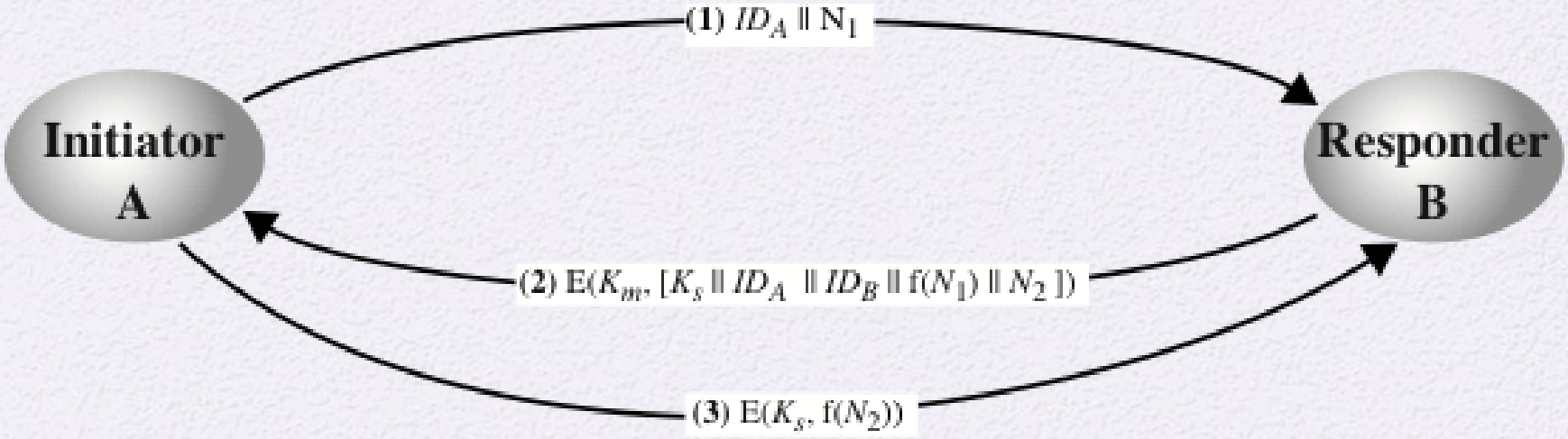


Figure 14.5 Decentralized Key Distribution

1. A issues a request to B for a session key and includes a nonce, N_1.
2. B responds with a message that is encrypted using the shared master key. The response includes the session key selected by B, an identifier of B, the value $f(N_1)$, and another nonce, N_2.
3. Using the new session key, A returns $f(N_2)$ to B.

Thus, although each node must maintain at most $(n - 1)$ master keys, as many session keys as required may be generated and used. Because the messages transferred using the master key are short, cryptanalysis is difficult. As before, session keys are used for only a limited time to protect them.

Simple Secret Key Distribution

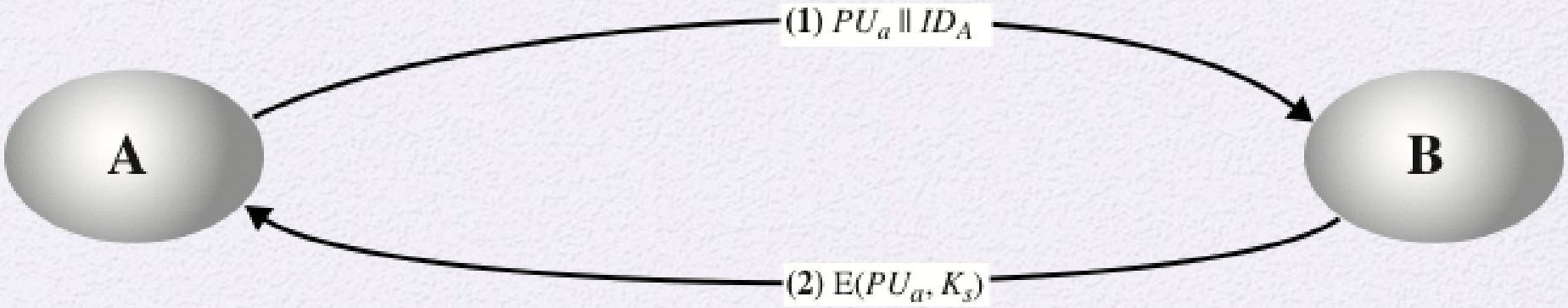


Figure 14.7 Simple Use of Public-Key Encryption to Establish a Session Key

Man in the middle attack

1. A generates a public/private key pair $\{PU_a, PR_a\}$ and transmits a message intended for B consisting of PU_a and an identifier of A, ID_A.
2. D intercepts the message, creates its own public/private key pair $\{PU_d, PR_d\}$ and transmits $PU_s||ID_A$ to B.
3. B generates a secret key, K_s, and transmits $E(PU_s, K_s)$.
4. D intercepts the message and learns K_s by computing $D(PR_d, E(PU_d, K_s))$.
5. D transmits $E(PU_a, K_s)$ to A.