Sampling and Quantization

OVERVIEW

- **Image Formation model**
- **Sampling and Quantization**
- **Representing Digital Images**

Image Formation Model

- An image is defined by two dimensional function f(x,y).
- The value or amplitude of f at spatial coordinates (x,y) is a positive scalar quantity.
- When an image is generated from a physical process, its value are proportional to energy radiated by physical source.
- As a consequence, f(x,y) must be nonzero and finite
- The function f(x,y) may be characterized by two components:
  - Amount of source i(x,y) incident on the scene being viewed
  - Amount of illumination reflected r(x,y) by the objects in the scene.

Image Formation Model

- The two function combine as product to form f(x,y):
  - $f(x,y)=i(x,y)\ r(x,y)$
  - Where $0 < i(x,y) < \infty$ and $0 < r(x,y) < 1$

  r(x,y)=0 means total absorption r(x,y)=1 means total reflectance
- Intensity of a monochrome image f at (x_0,y_0) is **the gray level l** of the image at that point
  - $l=f\ (x_0,y_0)$
  - where $L_{min} \le l \le L_{max}$ && Lmin: Positive Lmax: Finite
- In practice: Lmin = Imin rmin and Lmax = Imax rmax
- The interval [Lmin, Lmax] is called as **gray scale** and can be shifted to numerical interval [0,L-1]

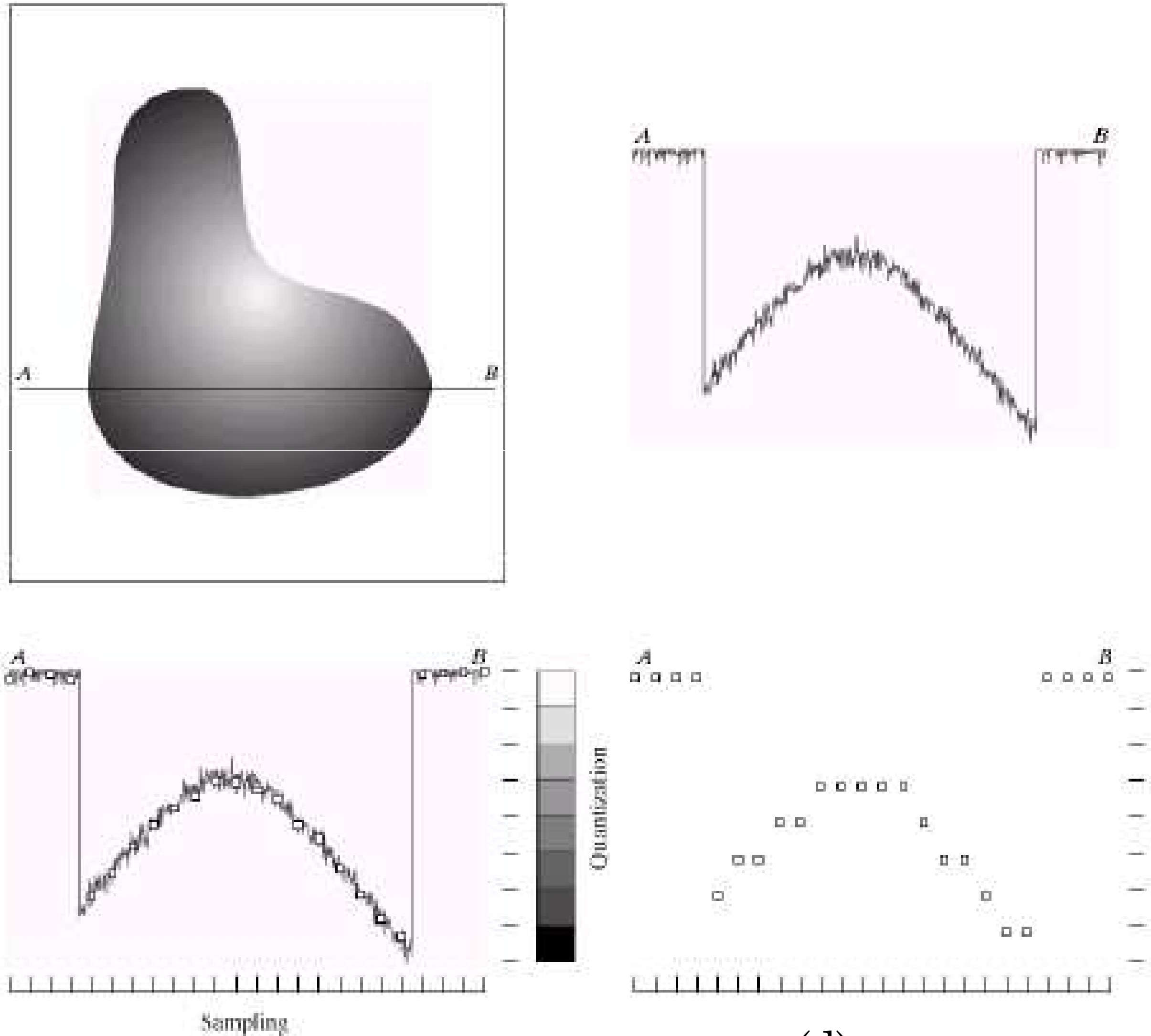
  Where l=0: black and l=L-1: white

Sampling and Quantization

- Digital image is created by converting the continuous sensed data into digital form, which involves two processes:
  - Sampling
  - quantization.
- An image may be continuous with respect to the x and y coordinates and also in amplitude.
- To convert to digital form digitize the functions in both coordinates and amplitude
- Digitizing the coordinate value is called **sampling**
- Digitizing the amplitude value is called **quantization**

Genearting the Digital Image

b)



c)

(d)

Genearting the Digital Image

- The one-dimensional function shown in Fig.(b) is a plot of amplitute (gray level) values of the continuous image along the line segment AB.
- To sample this function, we take equally spaced samples along line AB, as shown in Fig. (c).
  - Location of each sample is given by a vertical tick mark in the bottom part.
  - The samples are shown as small white squares superimposed on the function.
  - The set of these discrete locations gives the sampled function.
  - The values of the samples still span (vertically) as continuous range of gray-level values.

Genearting the Digital Image

- In order to form a digital function, the gray-level values also must be converted (quantized) into discrete quantities.
- The right side of Fig.(c) shows the gray-level scale divided into eight discrete levels, ranging from black to white.
- The vertical tick marks indicate the specific value assigned to each of eight gray levels.
- The continuous gray levels are quantized simply by assigning one of the eight discrete gray levels to each sample.
  - After sampling and quantization, we get

    $f : [1, . . . , N] \times [1, . . . , M] \longrightarrow [0, . . . , L].$
- Starting from the top of the image procedure is repeated to form two-dimensional digital image.
- Quality of digital images are based on the number of samples and discrete grey levels used.

Results of Sampling and Quantization

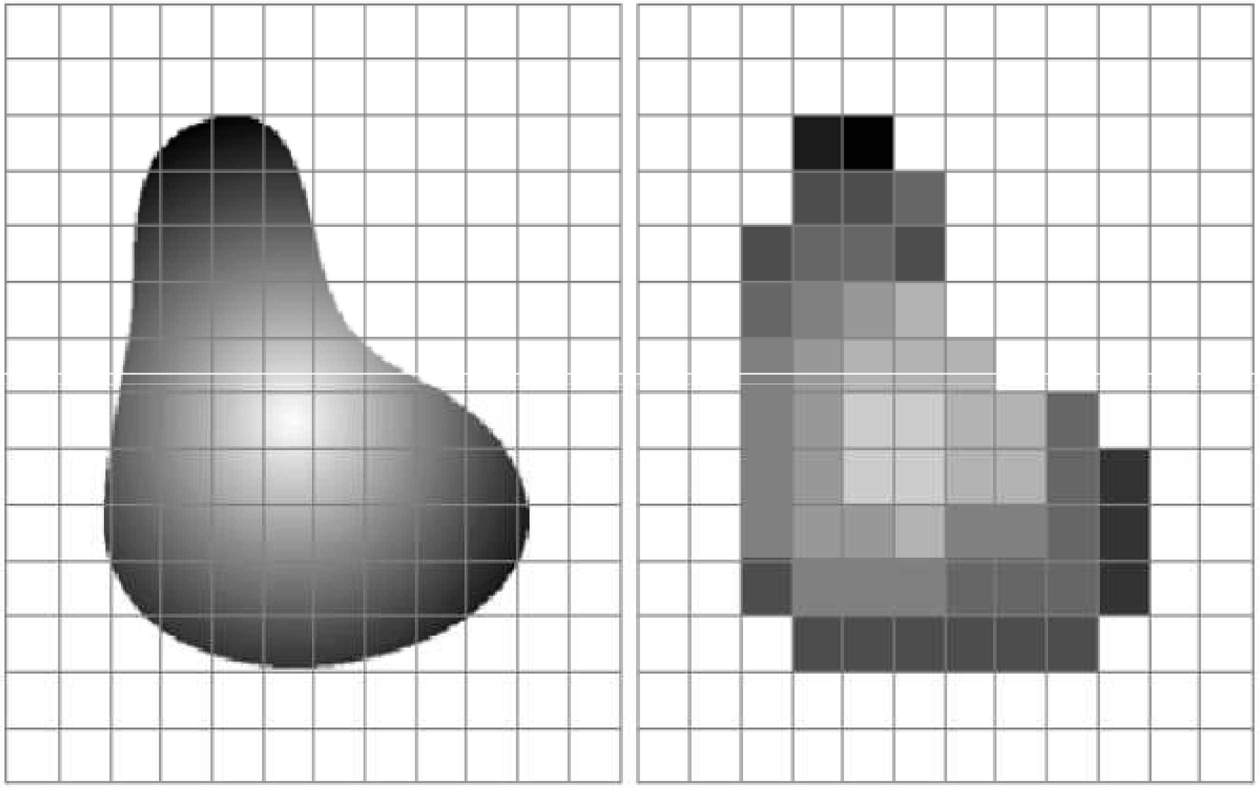
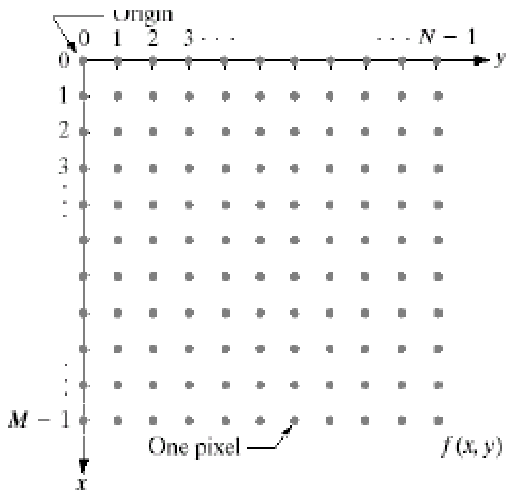


Image before sampling and quantization

Result of sampling and quantization

Representing the Digital Image



$$f(x,y)=\begin{bmatrix} f(0,0) & f(0,1) & . & . & f(0,N-1) \\ f(1,0) & f(1,1) & . & . & f(1,N-1) \\ . & . & . & . & . \\ . & . & . & . & . \\ f(M-1,0) & f(M-1,1) & . & . & f(M-1,N-1) \end{bmatrix}$$

Representing the Digital Image

- Image f(x,y) is sampled and the image has M rows and N columns
- The values of the coordinates (x,y) become discrete quantities.
- Each element of the matrix array is **called image element, picture element, pixel or pel.**
- The digitization process requires decisions on the values of **M, N and L (number of discrete intensity levels)**
- No restrictions on M and N other than: $M > 0$ and $N > 0$
- Due to processing, storage, and sampling hardware considerations, the number of gray levels typically is an integer power of 2, $L = 2^K$ where **k is number of bits require to represent a gray value**
- Assume that discrete levels are equally spaced and integers in [0,L-1]

Representing the Digital Image

- Number b of bits required to store an image:
- $b = M \times N \times k$ if $M = N$ then $b = N^2 k$
- Image with 2^k gray levels => "k-bit image" (ex: 256 → 8-bit image)
- The range of values spanned by the gray scale is called **dynamic range of the image**
- Contrast = difference in intensity between the highest and the lowest intensity levels in an image
- High dynamic range => high contrast expected
- Low dynamic range => dull, washed-out gray look

Representing the Digital Image

The number storage bits for various values of N and k

TABLE 2.1
Number of storage bits for various values of *N* and *k*.

N/k	1 ($L = 2$)	2 ($L = 4$)	3 ($L = 8$)	4 ($L = 16$)	5 ($L = 32$)	6 ($L = 64$)	7 ($L = 128$)	8 ($L = 256$)
32	1,024	2,048	3,072	4,096	5,120	6,144	7,168	8,192
64	4,096	8,192	12,288	16,384	20,480	24,576	28,672	32,768
128	16,384	32,768	49,152	65,536	81,920	98,304	114,688	131,072
256	65,536	131,072	196,608	262,144	327,680	393,216	458,752	524,288
512	262,144	524,288	786,432	1,048,576	1,310,720	1,572,864	1,835,008	2,097,152
1024	1,048,576	2,097,152	3,145,728	4,194,304	5,242,880	6,291,456	7,340,032	8,388,608
2048	4,194,304	8,388,608	12,582,912	16,777,216	20,971,520	25,165,824	29,369,128	33,554,432
4096	16,777,216	33,554,432	50,331,648	67,108,864	83,886,080	100,663,296	117,440,512	134,217,728
8192	67,108,864	134,217,728	201,326,592	268,435,456	335,544,320	402,653,184	469,762,048	536,870,912

Spatial and Gray – Level Resolution:

- Sampling is the principal factor determining the spatial resolution of an image or no of samples to generate digital image
- It is the smallest detectable detail in an image.
- Resolution is the dots(pixels) per unit distance.
- Gray level resolution is the smallest discernible change in gray level, which is a subjective process
- The number of gray levels is usually an integer of power 2 due to hardware consideration.
- Common bits are 8,16 bits in gray level
- Eg: An image whose intensity is quantized into 256 levels has 8 bits of intensity resolution.

Effect of Spatial Resolution

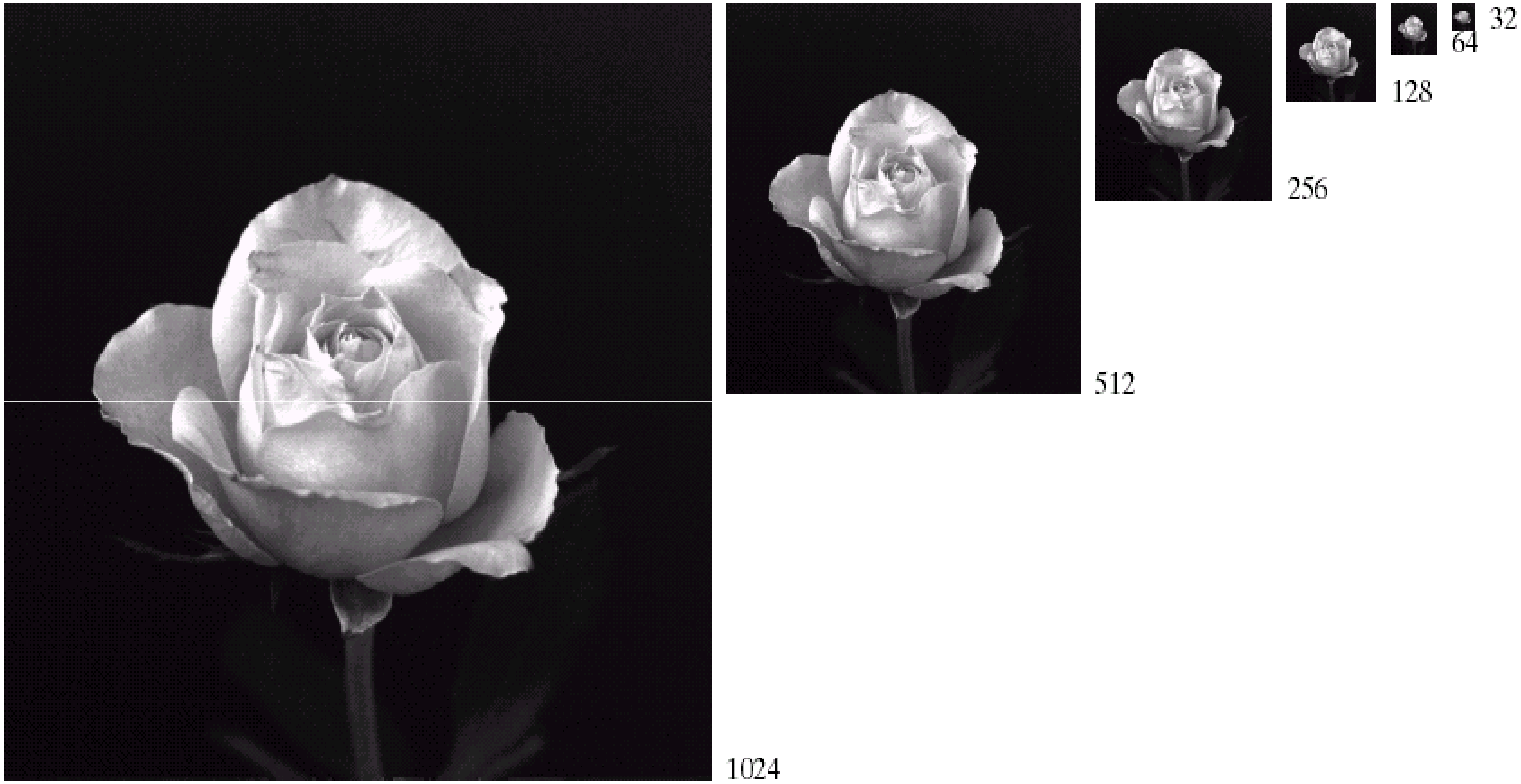


FIGURE 2.19 A 1024 × 1024, 8-bit image subsampled down to size 32 × 32 pixels. The number of allowable gray levels was kept at 256.

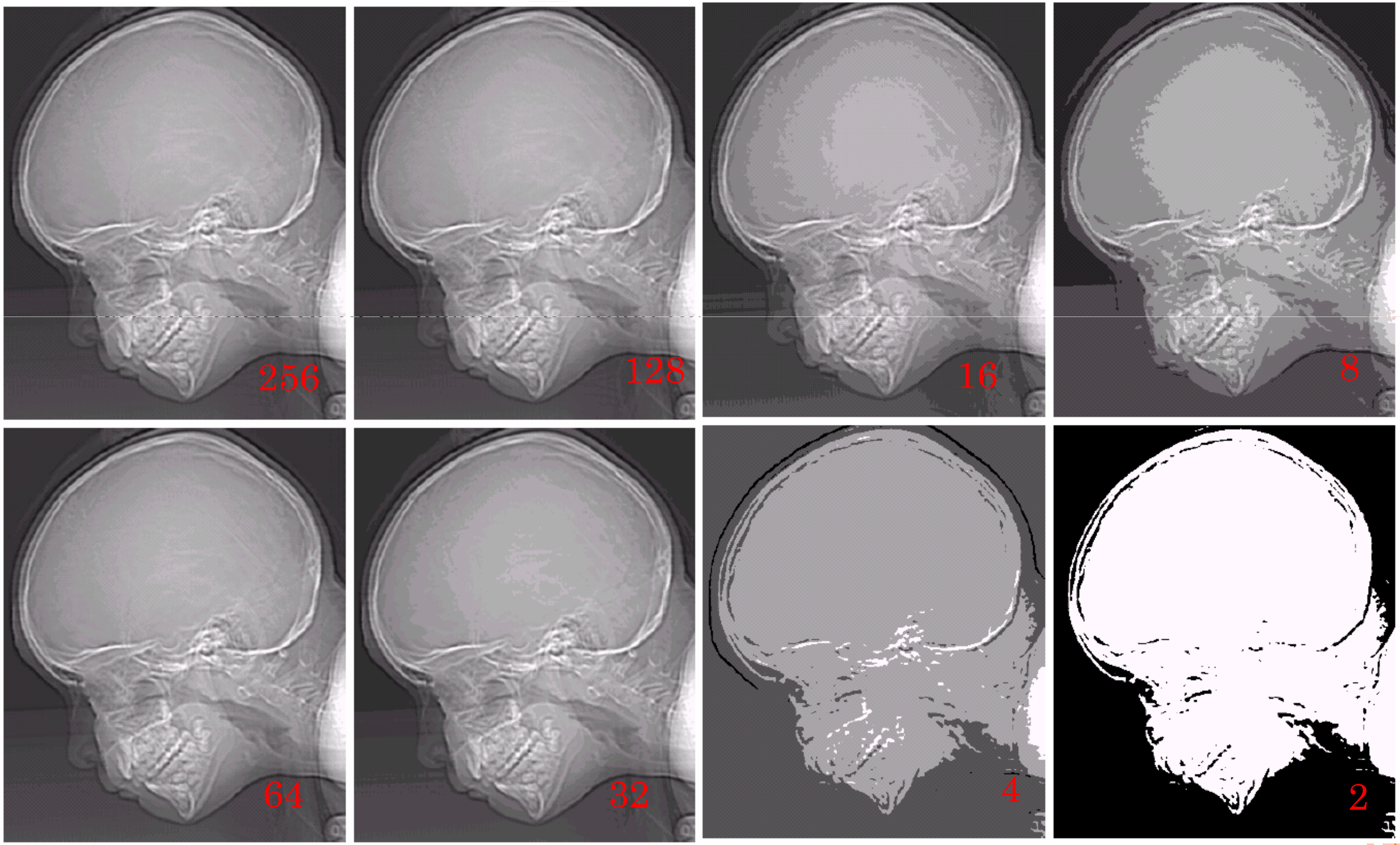
Effect of Spatial Resolution

a b c
d e f

FIGURE 2.20 (a) 1024 × 1024, 8-bit image. (b) 512 × 512 image resampled into 1024 × 1024 pixels by row and column duplication. (c) through (f) 256 × 256, 128 × 128, 64 × 64, and 32 × 32 images resampled into 1024 × 1024 pixels.

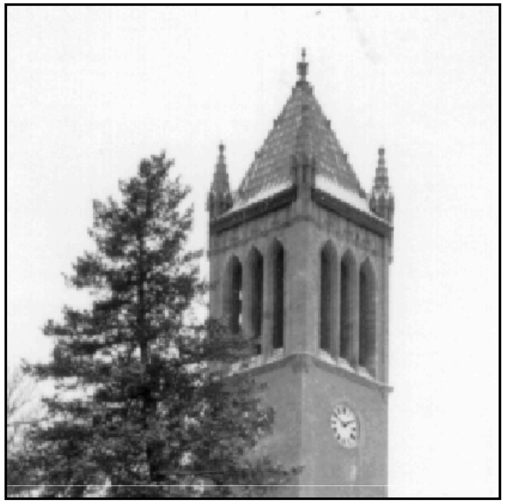
- Detail loss is very small in case of (a), (b) and (c).
- Increment of graininess in other images

Image Quantization

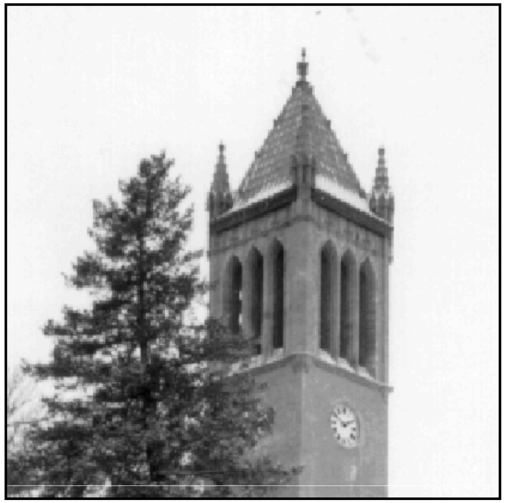


Effect of Quantization Levels

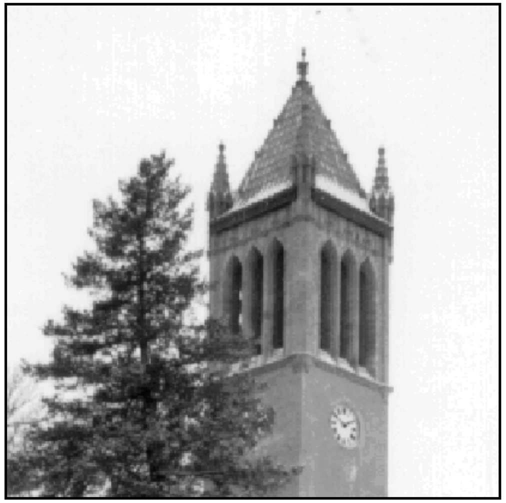
256 gray levels (8 bits per pixel)



256 gray levels (7 bits per pixel)



Where k=7 to 1

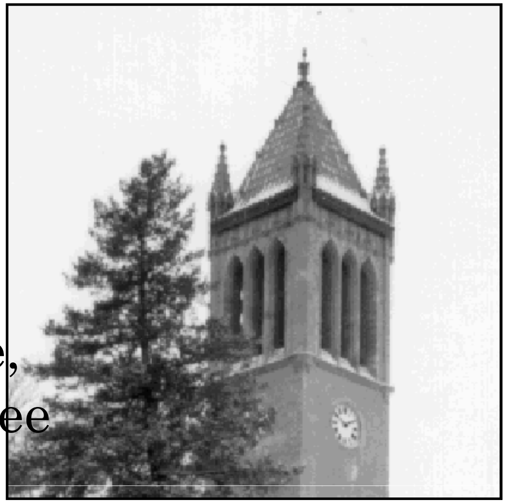


256 gray levels (6 bits per pixel)

256 gray levels (5 bits per pixel)

Effect of Quantization Levels (cont.)

In this image,
it is easy to see
false contour



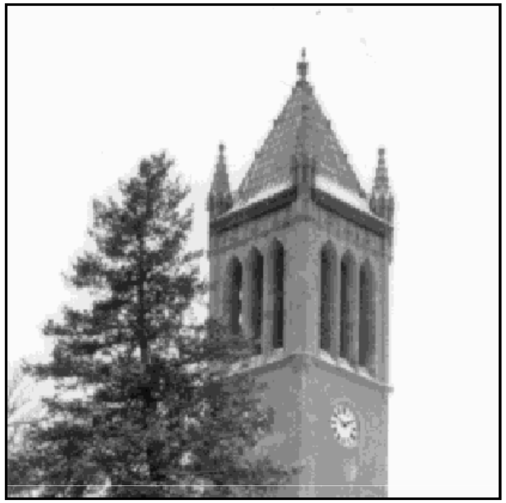


Image Interpolation

- Interpolation is a basic tool used in tasks as zooming, shrinking, rotation and geometric rotations.
- Interpolation is the process of using known data to estimate values at unknown locations.
- Zooming corresponds to oversampling and shrinking corresponds to undersampling.
- Zooming requires two steps:
  - Creation of new pixel locations
  - Assignment of gray levels to those new locations.
- Achieved using nearest neighbor interpolation, bilinear interpolation and bicubic interpolation

Image Interpolation

- **Nearest neighbor interpolation:**
  - Each new location is assigned with the intensity of its nearest neighbor in the orginal image
  - Produces severe distortions of straight edges
- **Bilinear interpolation :**
  - Use the four nearest neighbors to estimate the intensity at given location.
  - The assigned intensity value obtained using
    - $v(x',y') = ax' + by' + cx'y' + d$
- **Bicubic interpolation :**
  - Involves sixteen nearest neighbors of a point
- Image shrinking is done by row – column deletion.

Aliasing and Moiré Pattern

- All signals (functions) can be shown to be made up of a linear combination sinusoidal signals (sines and cosines) of different frequencies.
- For physical reasons, there is a highest frequency component in all real world signals.
- Theoretically,
  - if a signal is sampled at more than twice its highest frequency component, then it can be reconstructed exactly from its samples.
  - But, if it is sampled at less than that frequency (called undersampling), then aliasing will result.
  - This causes frequencies to appear in the sampled signal that were not in the original signal.
  - The Moiré pattern shown in Figure 2.24 is an example. The vertical low frequency pattern is a new frequency not in the original patterns.

Aliasing and Moiré Pattern
The effect of aliased frequencies

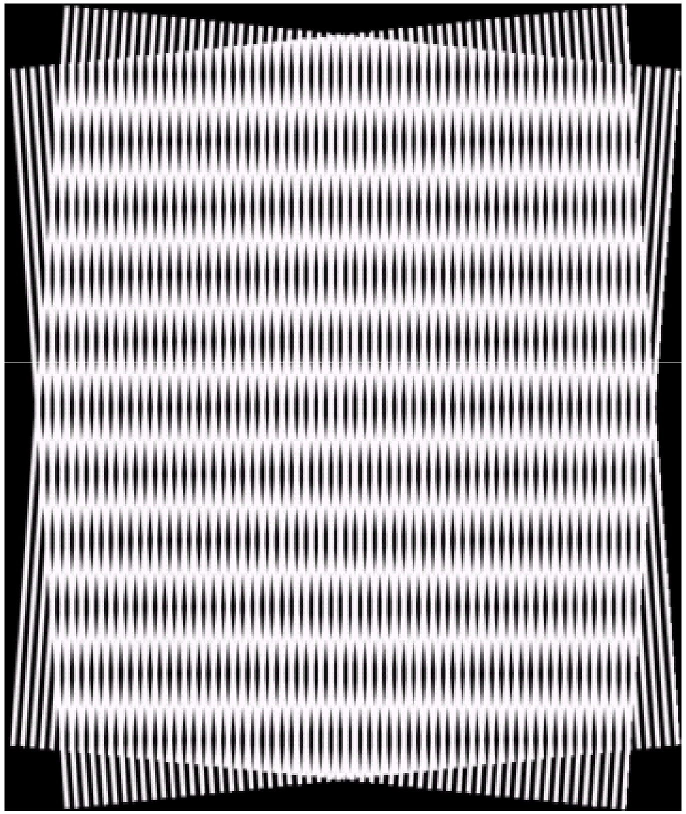


FIGURE 2.24 Illustration of the Moiré pattern effect.

Aliasing and Moiré Pattern

- Note that subsampling of a digital image will cause undersampling if the subsampling rate is less than twice the maximum frequency in the digital image.

- Sampling at a higher sampling rate (usually twice or more) than necessary to prevent aliasing is called oversampling.

- The principal approach for reducing the aliasing effects on an image is to reduce its high – frequency components by blurring the image prior to sampling.