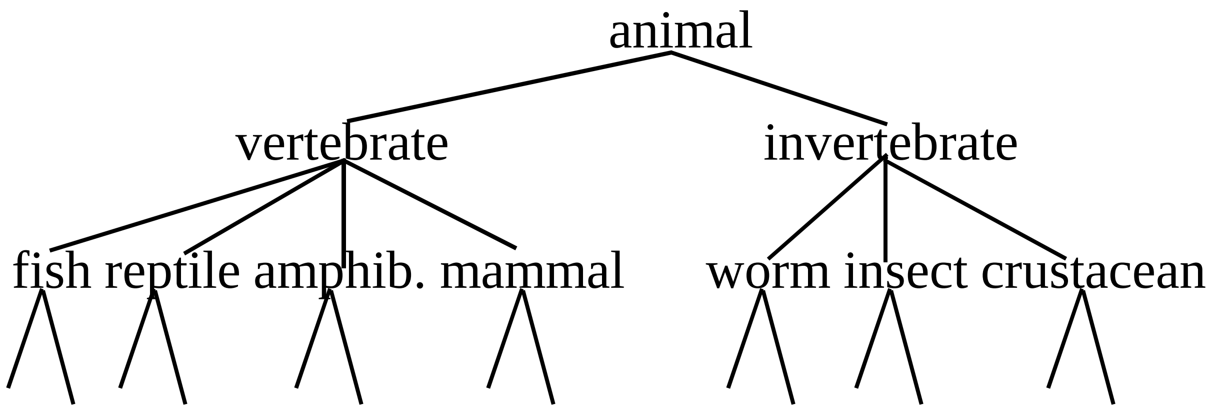
Hierarchical Methods

Hierarchical Clustering

- Works by grouping data objects at different levels as hierarchy or "trees" of clusters
- Used in data summarization and visualization
- Eg: organize employees into major groups such as executives, managers and staff. Further divided into subgroups as seniors officers, officers and trainees.
- All these groups form a hierarchy.

Hierarchical Clustering

- Build a tree-based hierarchical taxonomy (*dendrogram*) from a set of documents.



How could you do this with k-means?

Hierarchical Clustering Methods

- Hierarchical clustering methods can be further classified as
  - Agglomerative (Bottom-up)
  - Divisive (Top-Down)
- Needs a termination condition
- Does not require the number of clusters *k* in advance

Agglomerative hierarchical clustering

- This bottom-up strategy starts by placing each object in its own cluster
- Merges these atomic clusters into larger and larger clusters
- Until all of the objects are in a single cluster or until certain termination conditions are satisfied.
- Merges two cluster that are closest to each other (based on similarity measure)
- Clusters are merged per iteration each cluster contains one object requires at most n iterations.

Divisive Hierarchical Clustering

- This is top-down strategy does the reverse of agglomerative hierarchical clustering by starting with all objects in one cluster.
- It subdivides the clusters into smaller and smaller pieces
- Until each object form a cluster on its own or until it satisfies certain termination conditions
- Partitioning continues until each cluster is coherent enough
- Termination conditions :desired number of cluster or the diameter of each cluster is within a certain threshold.

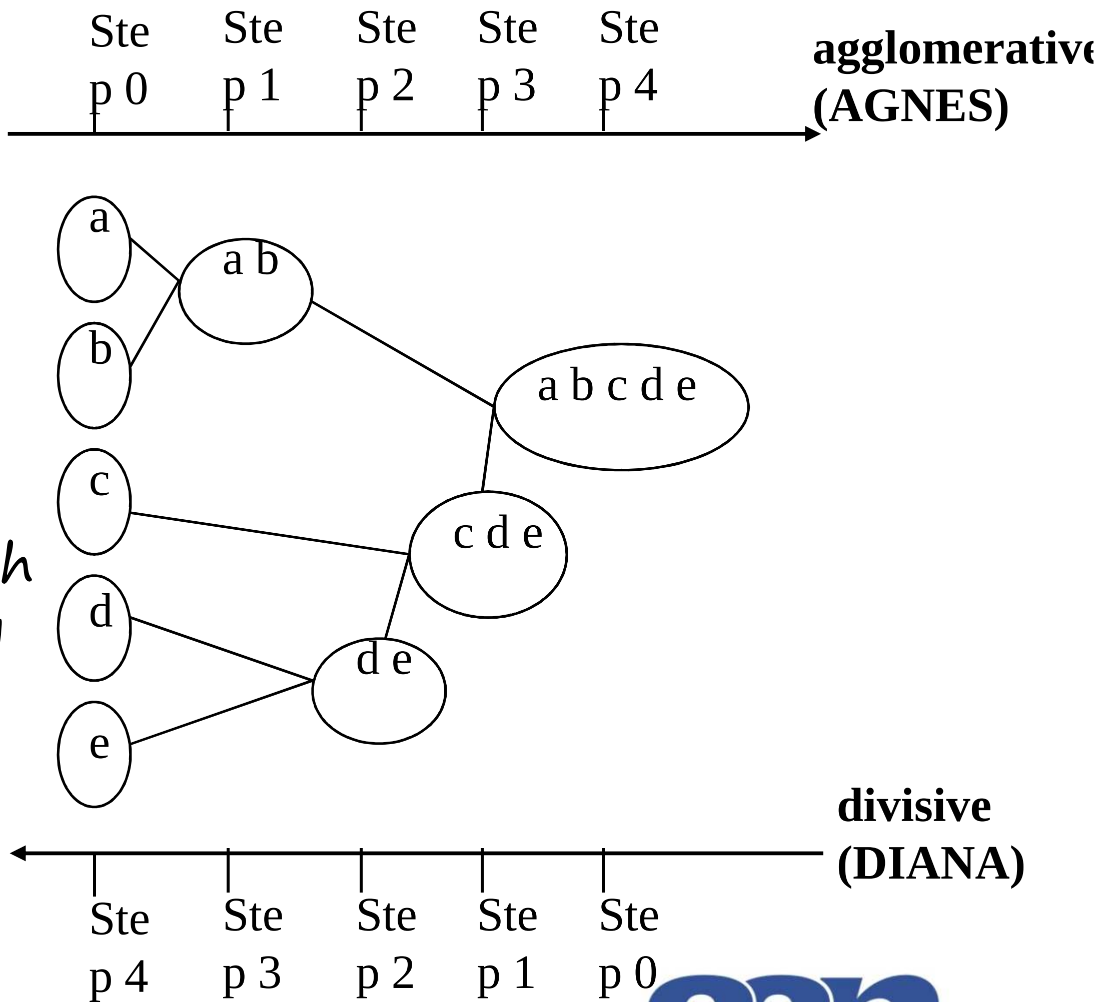
Agglomerative Vs Divisive Hierarchical clustering

AGNES : Agglomerative NESting

- Places each object into cluster of its own

- Clusters are merged step-by-step according to some criterion

- Uses single-linkage approach
  - Cluster represented by all objects
  - Similarity between objects are measured by the similarity of closet pair of data points different clusters

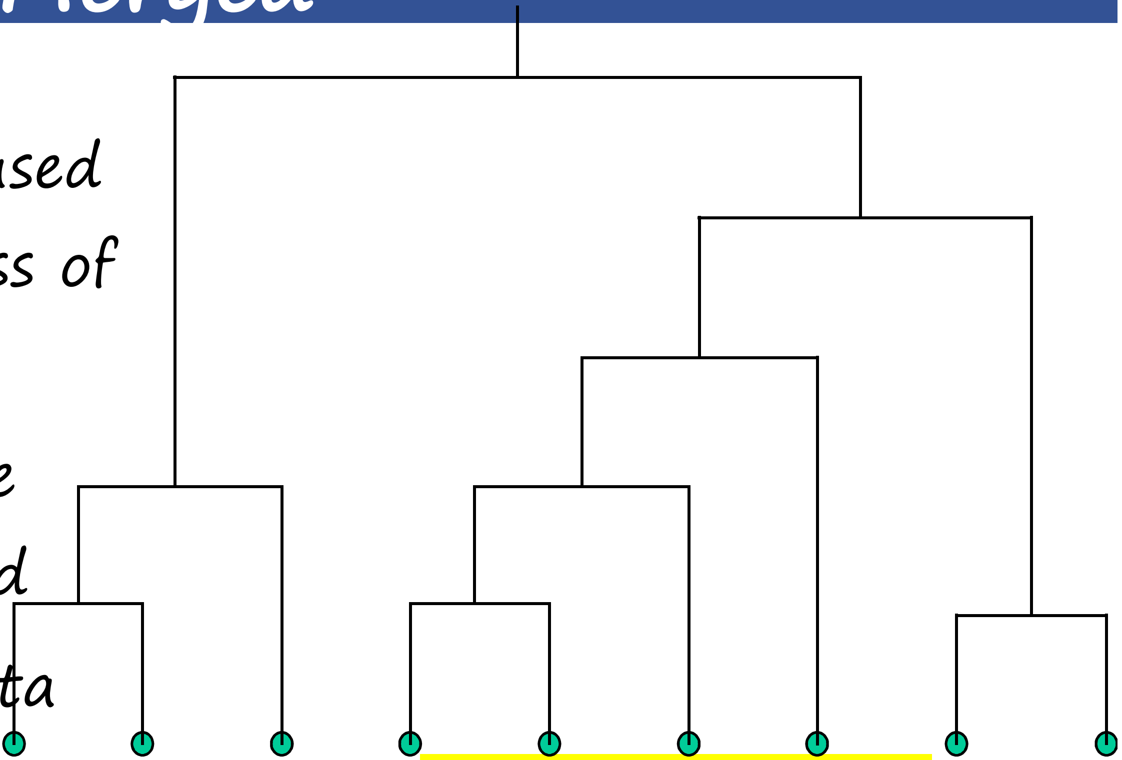


Agglomerative Vs Divisive Hierarchical clustering

- **DIANA(Divisive ANAlysis)**, the divisive method, proceeds in the contrasting way.
- All the objects are used to form one initial cluster.
- The cluster is split according to some principle such as the maximum Euclidean distance between the closest neighboring objects in the cluster.
- The cluster-splitting process repeats until, eventually, each new cluster contains only a single object.

Dendrogram: Shows How Clusters are Merged

- Dendrogram: Tree structure commonly used to reprsent the process of hierarchical clustering
- Shows how objects are grouped or partitioned
- A clustering of the data objects is obtained by cutting the dendrogram at the desired level, then each connected component forms a cluster



Hierarchical clustering generates a dendrogram (a hierarchy of clusters)

Distance Measures in Algorithmic methods

- Four widely used measures: let (p,p') be the points, mi is the mean for cluster Ci and ni is the number of objects in Ci.
- Measures are called as linkage measures

$$\textbf{Minimum distance:} \quad dist_{min}(C_i, C_j) = \min_{p \in C_i, p' \in C_j} \{|p - p'|\}$$

$$\textbf{Maximum distance:} \quad dist_{max}(C_i, C_j) = \max_{p \in C_i, p' \in C_j} \{|p - p'|\}$$

$$\textbf{Mean distance:} \quad dist_{mean}(C_i, C_j) = |m_i - m_j|$$

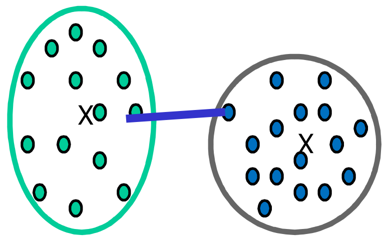
$$\textbf{Average distance:} \quad dist_{avg}(C_i, C_j) = \frac{1}{n_i n_j} \sum_{p \in C_i, p' \in C_j} |p - p'|$$

Hierarchical clustering Agglomerative

- Input: a pairwise matrix involved in all instances in S
- Algorithm
  1. Place each instance of S in its own cluster (singleton), creating the list of clusters L (initially, the leaves of T):
     $L = S_1, S_2, S_3, \ldots, S_{n-1}, S_n$.
  2. Compute a **merging cost function** between every pair of elements in L to find the two closest clusters $\{S_i, S_j\}$ which will be the cheapest couple to merge.
  1. Remove S_i and S_j from L.
  1. Merge S_i and S_j to create a new internal node S_{ij} in T which will be the parent of S_i and S_j in the resulting tree.
  1. Go to **Step 2** until there is only one set remaining.

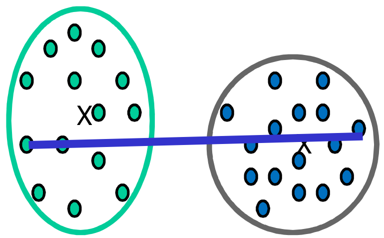
Hierarchical clustering Agglomerative

- **single-linkage clustering (also called the connectedness or minimum method):**
  - considers the **shortest** distance between one cluster and another cluster from any member of one cluster to any member of the other cluster.
  - Local similarity-based: Emphasizing more on close regions, ignoring the overall structure of the cluster
  - Capable of clustering non-elliptical shaped group of objects
  - Sensitive to noise and outliers



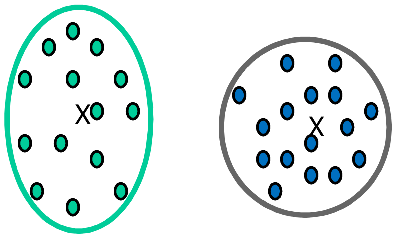
Hierarchical clustering
Agglomerative

- **Complete-linkage clustering** (also called the diameter or maximum method):
  - considers the **greatest** distance between from any member of one cluster to any member of the other cluster. (dissimilar members)
  - Global in behavior, obtaining compact shaped clusters
  - Sensitive to outliers



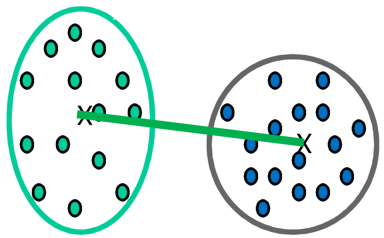
Single Link vs. Complete Link in Hierarchical Clustering

- **Average-linkage clustering:**
  - Considers the distance between one cluster and another cluster to be equal to the **average** distance from any member of one cluster to any member of the other cluster.
  - Expensive to compute



Agglomerative Clustering: Average vs. Centroid Links

- Agglomerative clustering with **centroid link**
  - **Centroid link**: The distance between the centroids of two clusters



$$c_{a\cup b} = \frac{N_a c_a + N_b c_b}{N_a + N_b}$$

Hierarchical clustering: example

	BA	FI	MI	NA	RM	TO
BA	0	662	877	255	412	996
FI	662	0	295	468	268	400
MI	877	295	0	754	564	138
NA	255	468	754	0	219	869
RM	412	268	564	219	0	669
TO	996	400	138	869	669	0

Hierarchical Agglomerative clustering: example

	BA	FI	MI/TO	NA	RM
BA	0	662	877	255	412
FI	662	0	295	468	268
MI/TO	877	295	0	754	564
NA	255	468	754	0	219
RM	412	268	564	219	0

	BA	FI	MI/TO	NA/RM
BA	0	662	877	255
FI	662	0	295	268
MI/TO	877	295	0	564
NA/RM	255	268	564	0

Hierarchical Agglomerative clustering: example

	BA/NA/RM	FI	MI/TO
BA/NA/RM	0	268	564
FI	268	0	295
MI/TO	564	295	0

	BA/FI/NA/RM	MI/TO
BA/FI/NA/RM	0	295
MI/TO	295	0

Hierarchical clustering: example using single linkage