



MAEKAWA'S DISTRIBUTED MUTEX ALGORITHM

Reference: 1. Mukesh Singhal & N.G. Shivaratri, Advanced
Concepts in Operating Systems,
2. George Coulouris, Jean Dollimore and Tim Kindberg,
“Distributed Systems Concepts and Design”, Fifth Edition,
Pearson Education, 2012

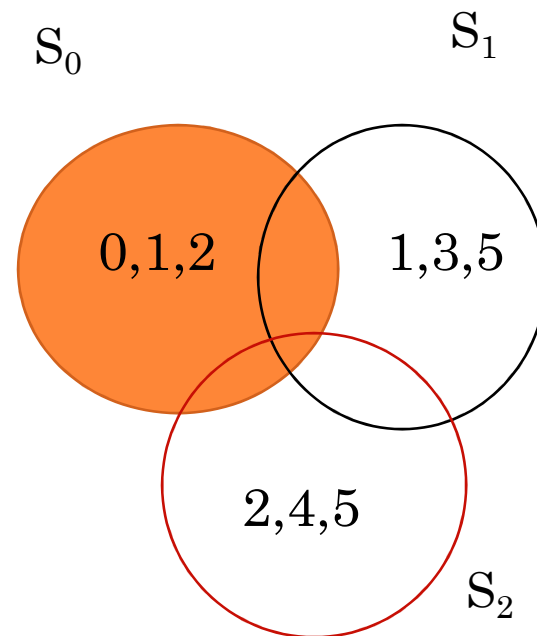
MAEKAWA'S ALGORITHM

- With each process i , associate a subset S_i . Divide the set of processes into subsets that satisfy the following two conditions:

$$i \in S_i$$

$$\forall i, j : 0 \leq i, j \leq n-1 \mid S_i \cap S_j \neq \emptyset$$

- Main idea.** Each process i is required to receive permission from S_i **only**. Correctness requires that multiple processes will never receive permission from all members of their respective subsets.



MAEKAWA'S ALGORITHM

Example. Let there be **seven** processes 0, 1, 2, 3, 4, 5, 6

$$S_0 = \{0, 1, 2\}$$

$$S_1 = \{1, 3, 5\}$$

$$S_2 = \{2, 4, 5\}$$

$$S_3 = \{0, 3, 4\}$$

$$S_4 = \{1, 4, 6\}$$

$$S_5 = \{0, 5, 6\}$$

$$S_6 = \{2, 3, 6\}$$

MAEKAWA'S ALGORITHM

Version 1 {Life of process I}

1. Send timestamped **request** to each process in S_i .
2. Request received \rightarrow send **ack** to process with the **lowest timestamp**. Thereafter, "lock" (i.e. **commit**) yourself to that process, and keep others waiting.
3. Enter CS if you receive an **ack** from **each member** in S_i .
4. To exit CS, send **release** to every process in S_i .
5. Release received \rightarrow **unlock** yourself. Then send ack to the next process with the lowest timestamp.

$S_0 = \{0, 1, 2\}$

$S_1 = \{1, 3, 5\}$

$S_2 = \{2, 4, 5\}$

$S_3 = \{0, 3, 4\}$

$S_4 = \{1, 4, 6\}$

$S_5 = \{0, 5, 6\}$

$S_6 = \{2, 3, 6\}$

MAEKAWA'S ALGORITHM-VERSION 1

ME1. *At most one process can enter its critical section at any time.*

Let i and j attempt to enter their Critical Sections

$S_i \cap S_j \neq \emptyset$ implies there is a process $k \in S_i \cap S_j$

Process k will **never** send ack to both.

So it will act as the arbitrator and establishes ME1

$$S_0 = \{0, 1, 2\}$$

$$S_1 = \{1, 3, 5\}$$

$$S_2 = \{2, 4, 5\}$$

$$S_3 = \{0, 3, 4\}$$

$$S_4 = \{1, 4, 6\}$$

$$S_5 = \{0, 5, 6\}$$

$$S_6 = \{2, 3, 6\}$$

MAEKAWA'S ALGORITHM-VERSION 1

ME2. No deadlock. Unfortunately deadlock is possible! Assume 0, 1, 2 want to enter their critical sections.

From $S_0 = \{0, 1, 2\}$, 0, 2 send *ack* to 0, but 1 sends *ack* to 1;

From $S_1 = \{1, 3, 5\}$, 1, 3 send *ack* to 1, but 5 sends *ack* to 2;

From $S_2 = \{2, 4, 5\}$, 4, 5 send *ack* to 2, but 2 sends *ack* to 0;

Now, 0 waits for 1 (to send a release), 1 waits for 2 (to send a release), , and 2 waits for 0 (to send a release), . So deadlock is possible!

$S_0 = \{0, 1, 2\}$

$S_1 = \{1, 3, 5\}$

$S_2 = \{2, 4, 5\}$

$S_3 = \{0, 3, 4\}$

$S_4 = \{1, 4, 6\}$

$S_5 = \{0, 5, 6\}$

$S_6 = \{2, 3, 6\}$

MAEKAWA'S ALGORITHM-VERSION 2

Avoiding deadlock

If processes always receive messages **in increasing order of timestamp**, then deadlock “could be” avoided. But this is too strong an assumption.

Version 2 uses three ***additional*** messages:

- *failed*
- *inquire*
- *relinquish*

$S_0 = \{0, 1, 2\}$

$S_1 = \{1, 3, 5\}$

$S_2 = \{2, 4, 5\}$

$S_3 = \{0, 3, 4\}$

$S_4 = \{1, 4, 6\}$

$S_5 = \{0, 5, 6\}$

$S_6 = \{2, 3, 6\}$

MAEKAWA'S ALGORITHM-VERSION 2

New features in version 2

- Send **ack** and set **lock** as usual.
- If **lock is set** and a request with a larger timestamp arrives, send **failed** (*you have no chance*). If the incoming request has a lower timestamp, then send **inquire** (*are you in CS?*) to the locked process.
- Receive **inquire** and at least one **failed** message → send **relinquish**. The recipient resets the lock.

$S_0 = \{0, 1, 2\}$

$S_1 = \{1, 3, 5\}$

$S_2 = \{2, 4, 5\}$

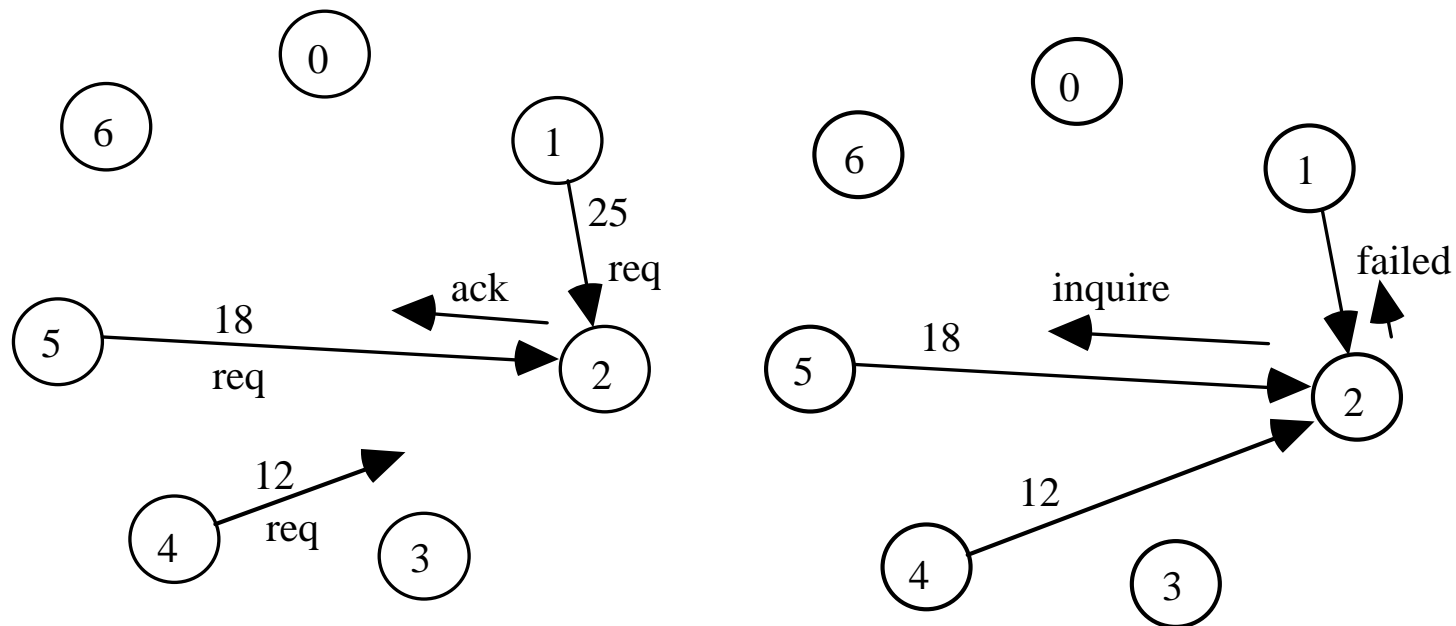
$S_3 = \{0, 3, 4\}$

$S_4 = \{1, 4, 6\}$

$S_5 = \{0, 5, 6\}$

$S_6 = \{2, 3, 6\}$

MAEKAWA'S ALGORITHM-VERSION 2



COMMENTS

- Let $K = |S_i|$. Let each process be a member of D subsets. When $N = 7$, $K = D = 3$. When $K=D$, $N = K(K-1)+1$. So $K = O(\sqrt{N})$
- The message complexity of Version 1 is $3\sqrt{N}$. Maekawa's analysis of Version 2 reveals a complexity of $7\sqrt{N}$
- *Sanders identified a bug in version 2 ...*