Image Restoration

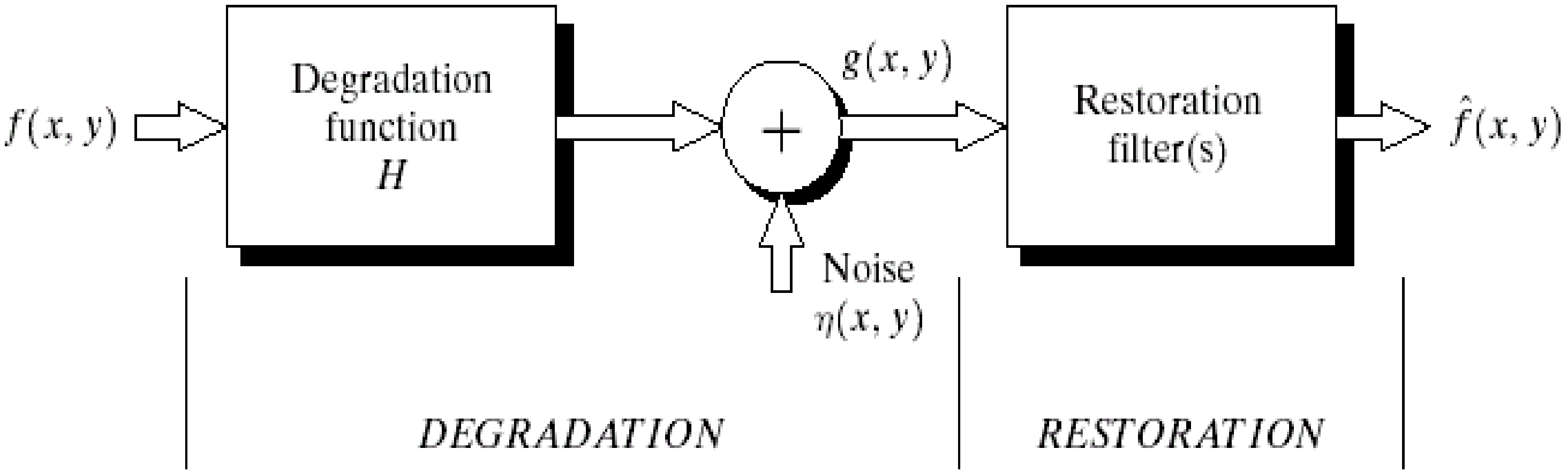
Preview

- Goal of image restoration
  - Improve an image in some predefined sense
  - Difference with image enhancement ?
- Features
  - Image restoration v.s image enhancement
  - Objective process v.s. subjective process
  - A prior knowledge v.s heuristic process

Image restoration

- Process which tries to restore an image which has been degraded by some knowledge of degradration.
- A prior knowledge of degradation phenomenon is considered.
- Find the phenomenon or what is the model that degrade the image
- Apply the inverse process to recover the original image
- Restoration can be applied in spatial and Frequency domain

A model of the image degradation/restoration process



$$g(x,y)=f(x,y)*h(x,y)+\eta(x,y)$$

$$G(u,v)=F(u,v)H(u,v)+N(u,v)$$

where *f is the original image, g is a degraded/noisy version* of the original image and *f˜ is a restored version.*

Image Restoration

- Image restoration removes a known degradation. If the degradation is linear and spatially-invariant

  $g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$

  $G(u,v) = H(u,v)\, F(u,v) + N(u,v)$

  where F - original image, H - degradation, N - additive noise and G - recorded image. Given H, an estimate of the original image is

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)} = F(u,v) + \frac{N(u,v)}{H(u,v)}.$$

  Notice that if $H \sim 0$, the noise will be amplified.

Degradation Models

Degradation Models:

- Image degradation can occur for many reasons, some typical degradation models are

$$h(i,j) = \begin{cases} 1 & ai + bj = 0 \\ 0 & otherwise \end{cases}$$

Motion Blur: due to camera panning or subject moving quickly.

$$h(i,j) = Ke^{-\left(\frac{i^2+j^2}{2\sigma}\right)}$$

Atmospheric Blur: long exposure

$$h(i,j) = \begin{cases} \frac{1}{L^2} & -\frac{L}{2} \le i, j \le \frac{L}{2} \\ 0 & otherwise \end{cases}$$

Uniform 2D Blur

$$h(i,j) = \begin{cases} \frac{1}{\pi R^2} & i^2 + j^2 \le R^2 \\ 0 & otherwise \end{cases}$$

Out-of-Focus Blur

Linear Position Invariant Degradations

Linear Position Invariant Degradations:

- The input output relationships before restoration is given by,

  $g(x,y) = H[f(x,y)] + \eta(x,y)$

- Let us assume, $\eta(x,y) = 0$ and H is linear, then

  $H[af_1(x,y) + bf_2(x,y)] = aH[f_1(x,y)] + bH[f_2(x,y)]$

  where a & b are scalars

- If a=1 and b=1 then

  $H[f_1(x,y) + f_2(x,y)] = H[f_1(x,y)] + H[f_2(x,y)]$

  which is called the property of additivity

Linear Position Invariant Degradations

Linear Position Invariant Degradations:

- With $f_2(x,y) = 0$,

  $H[af_1(x,y)] = aH[f_1(x,y)]$

  which is called the property of homogeneity.

Linear Position Invariant Degradations

An operator having the input – output relationship $g(x,y) = H[f(x,y)]$ is said to be position (or space) invariant if,

$$H[f(x-\alpha, y-\beta)] = g(x-\alpha, y-\beta)$$

- The response of an pt in the image should solely depend on the value of the pixel at that particular pt and response not depend on the position of the image

Noise models

- **Assuming degradation only due to additive noise (*H* = 1)**
- **Noise from sensors**
  - Electronic circuits
  - Light level
  - Sensor temperature
- **Noise from environment**
  - Lightening
  - Atmospheric disturbance
  - Other strong electric/magnetic signals

Noise models

- Source of noise
  - Image acquisition (digitization)
  - Image transmission
- Spatial properties of noise
  - Statistical behavior of the gray-level values of pixels
  - Noise parameters, correlation with the image
- Frequency properties of noise
  - Fourier spectrum
  - Ex. white noise (a constant Fourier spectrum)

Noise probability density functions

- Noises are taken as random variables
- Random variables
  - Probability density function (PDF)

Gaussian noise

- Math. tractability in spatial and frequency domain
- Electronic circuit noise and sensor noise

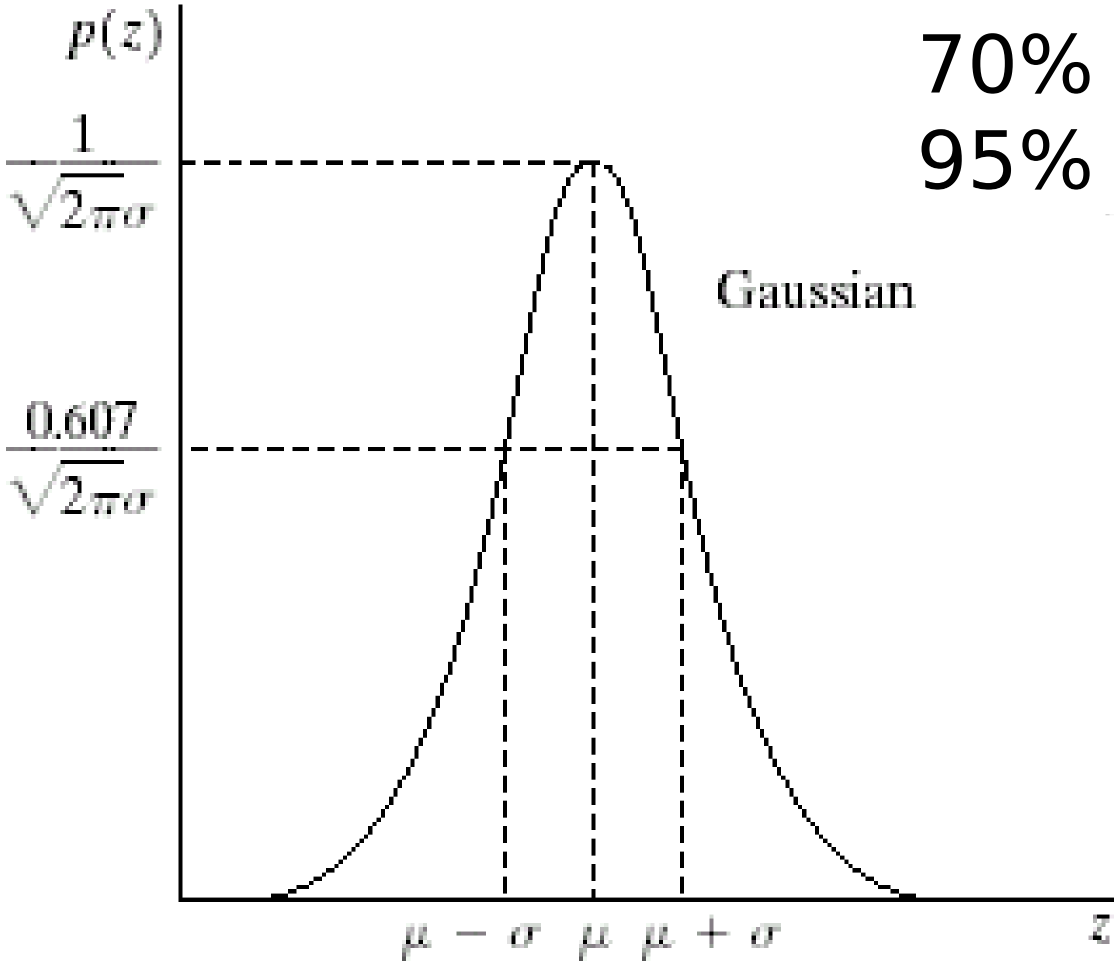
$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(z-\mu)^2/2\sigma^2}$$

mean

variance

Note: $\int_{-\infty}^{\infty} p(z)\,dz = 1$

Gaussian noise (PDF)



70% in [(μ–σ), (μ+σ)]
95% in [(μ–2σ), (μ+2σ)]

Uniform noise

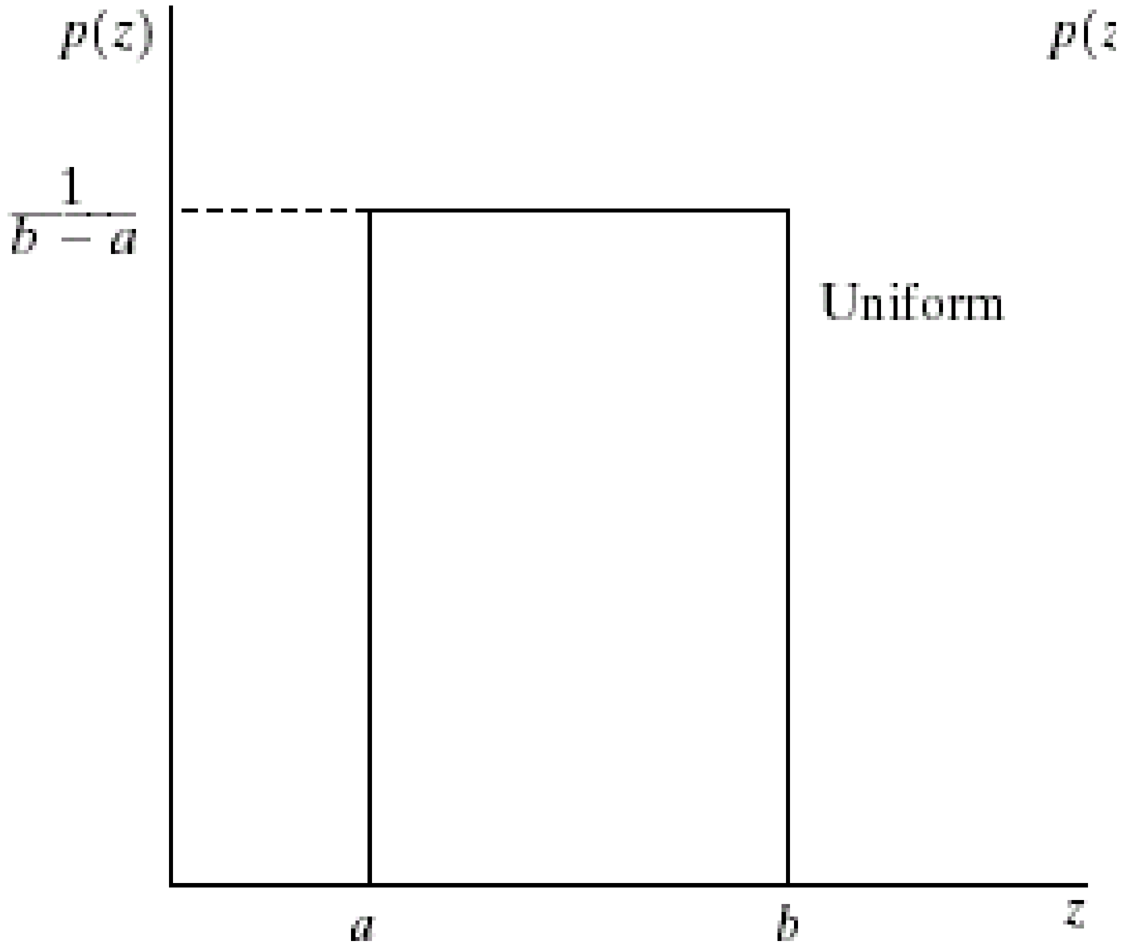
- Less practical, used for random number generator

$$p(z) = \begin{cases} \dfrac{1}{b-a} & \text{if } a \le z \le b \\ 0 & \text{otherwise} \end{cases}$$

Mean: $\mu = \dfrac{a+b}{2}$

Variance: $\sigma^2 = \dfrac{(b-a)^2}{12}$

Uniform PDF



Impulse (salt-and-pepper) nosie

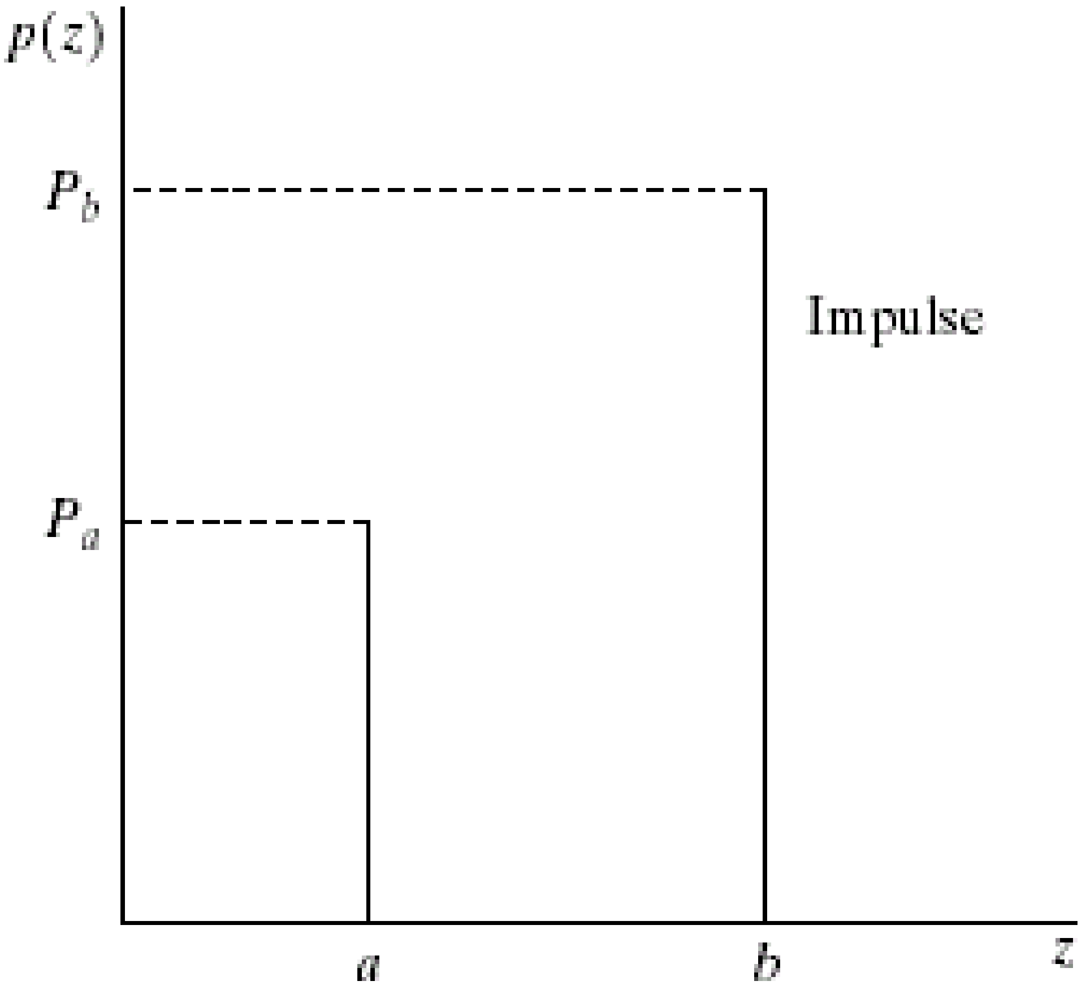
- Quick transients, such as faulty switching during imaging

$$p(z)=\begin{cases} P_a & \text{for } z=a \\ P_b & \text{for } z=b \\ 0 & \text{otherwise} \end{cases}$$

If either P_a or P_b is zero, it is called *unipolar.*
Otherwise, it is called bipoloar.

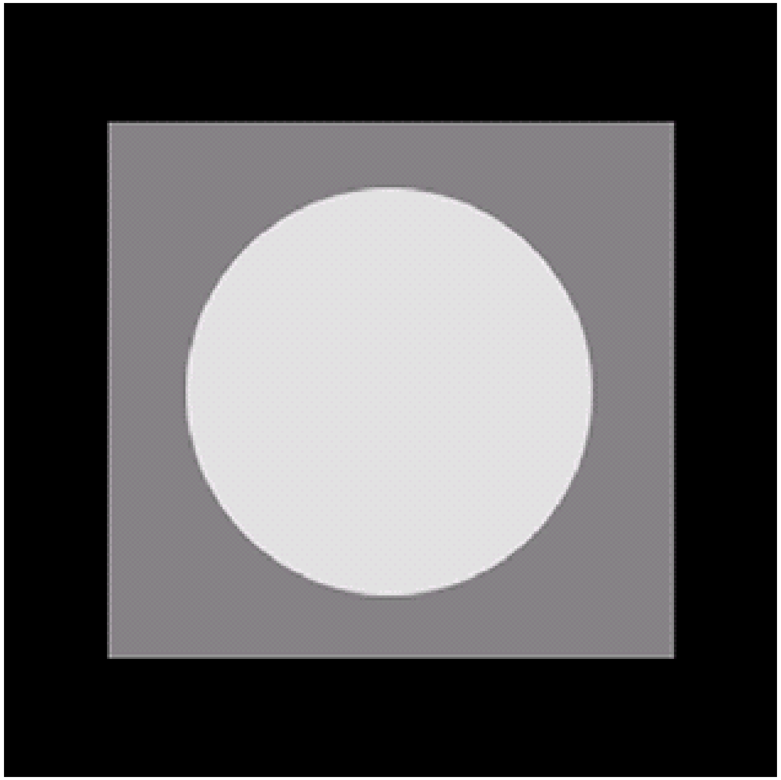
- In practical, impulses are usually stronger than ima
signals. Ex., a=0(black) and b=255(white) in 8-bit i

Impulse (salt-and-pepper) nosie PDF

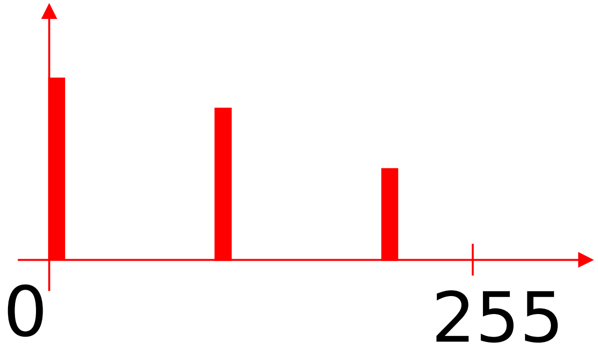


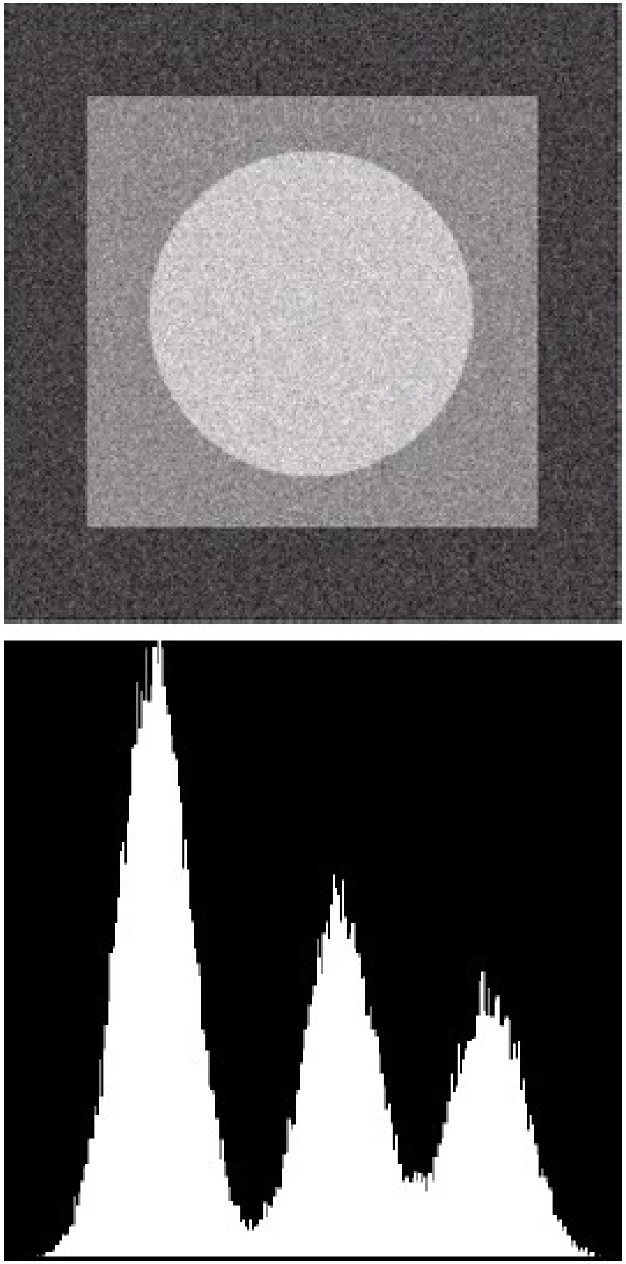
Test for noise behavior

- Test pattern

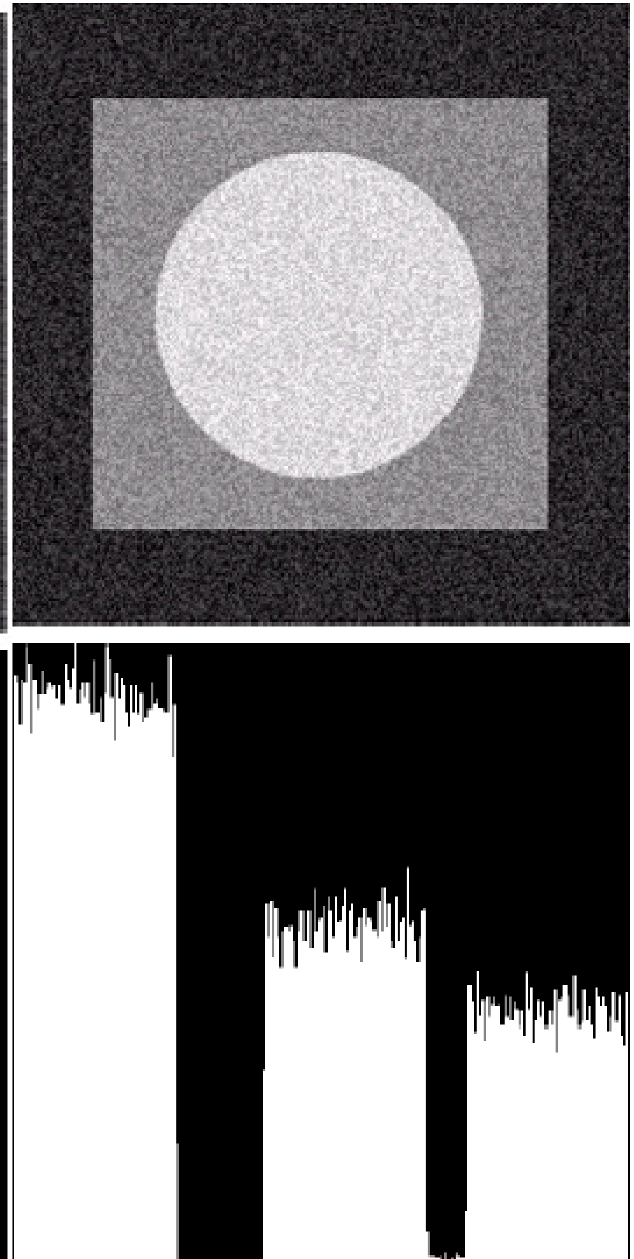


Its histogram:

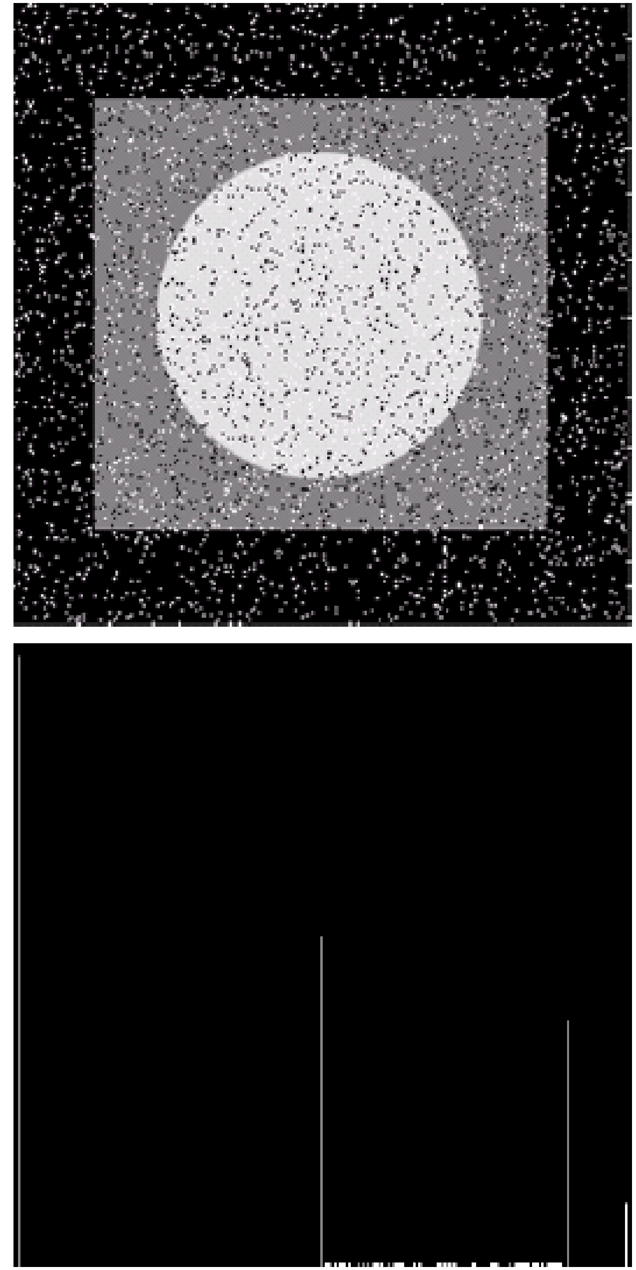




Gaussian



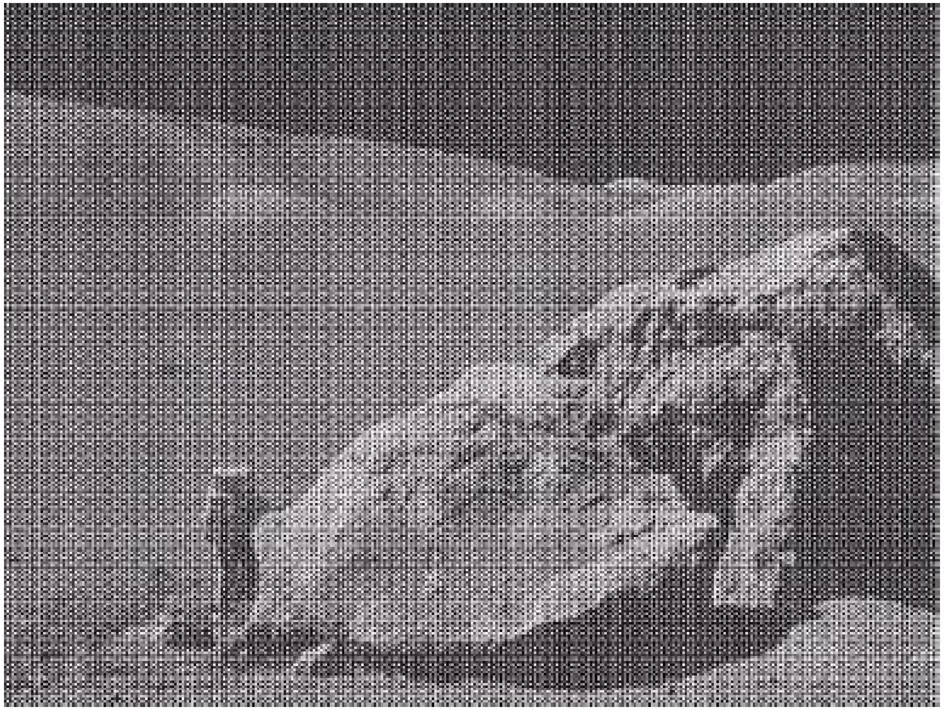
Uniform

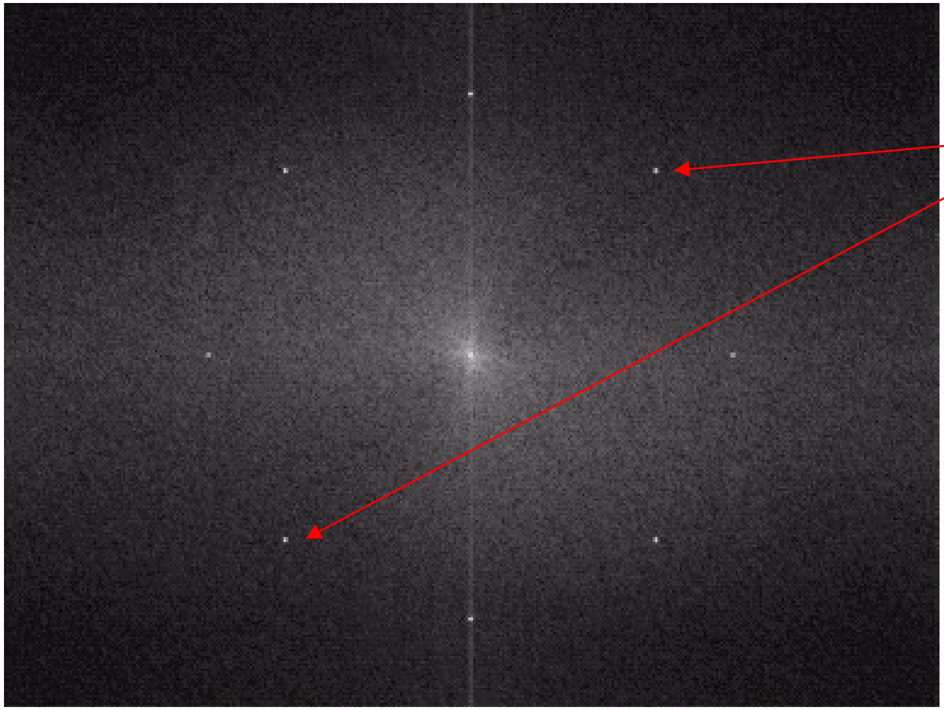


Salt & Pepper

Periodic noise

- Arise from electrical or electromechanical interference during image acquisition
- Spatial dependence
- Observed in the frequency domain



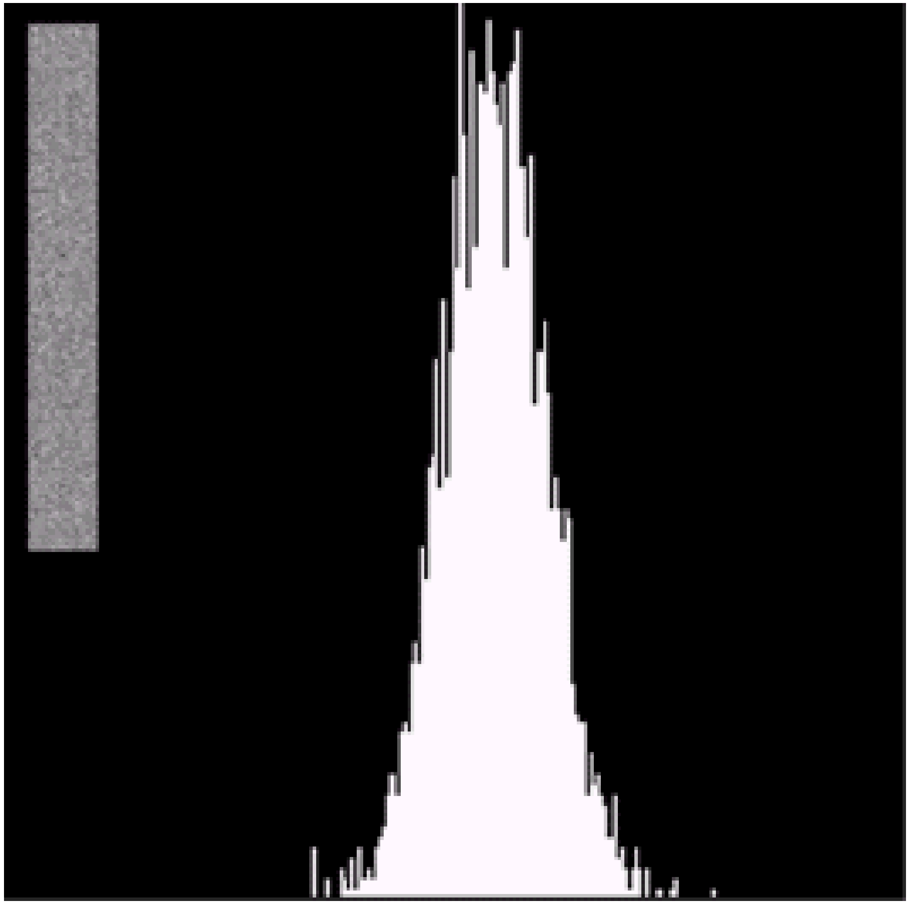


Sinusoidal noise:
Complex conjugat
pair in frequency
domain

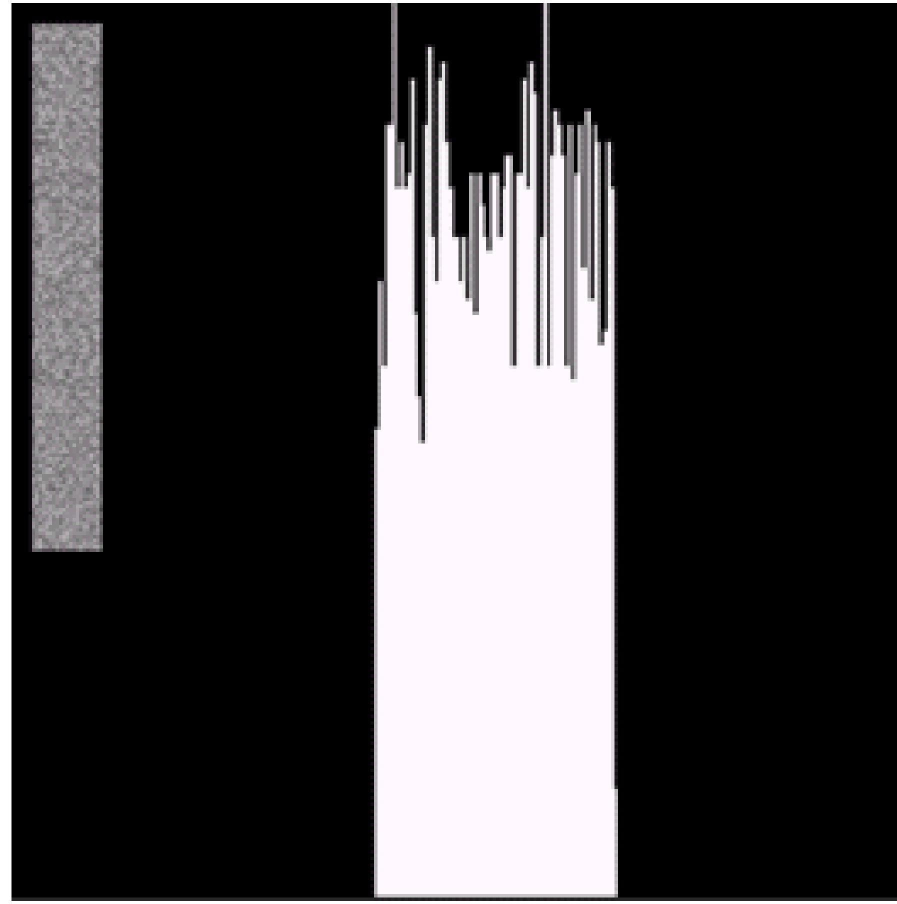
Estimation of noise parameters

- Periodic noise
  - Observe the frequency spectrum
- Random noise with unknown PDFs
  - Case 1: imaging system is available
    - Capture images of “flat” environment
  - Case 2: noisy images available
    - Take a strip from constant area
    - Draw the histogram and observe it
    - Measure the mean and variance

Observe the histogram



Gaussian



uniform

Measure the mean and variance

- Histogram is an estimate of PDF

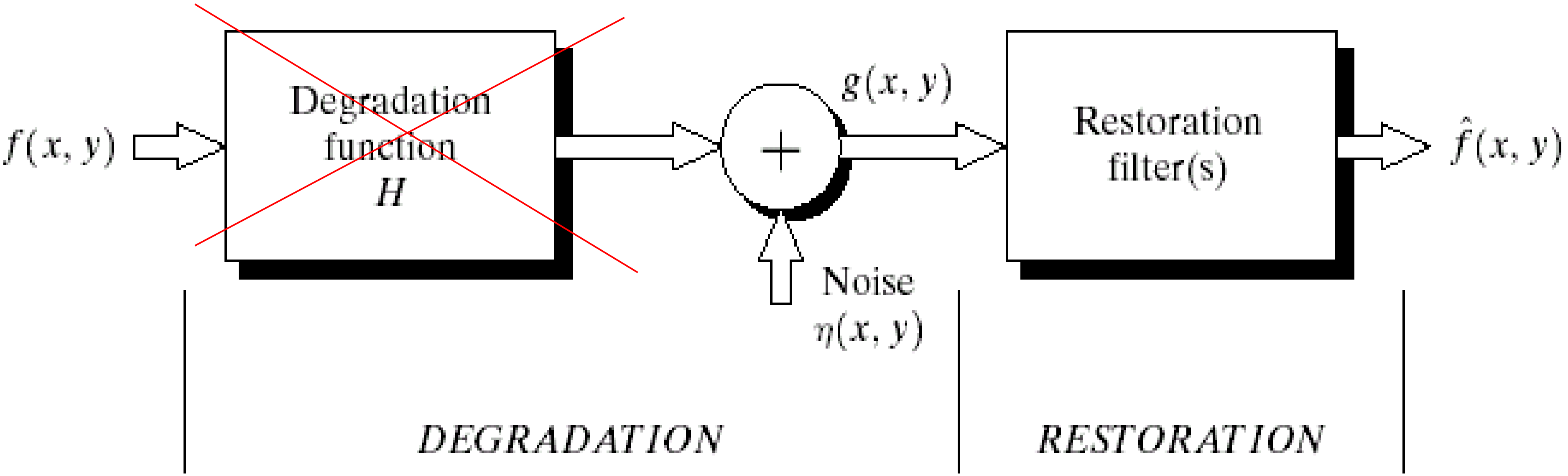
$$\begin{cases} \mu = \sum_{z_i \in S} z_i\, p(z_i) \\ \sigma^2 = \sum_{z_i \in S} (z_i - \mu)^2\, p(z_i) \end{cases}$$

$\Leftrightarrow$ $\begin{cases} \text{Gaussian: } \mu, \sigma \\ \text{Uniform: } a, b \end{cases}$

Outline

- A model of the image degradation / restoration process
- Noise models
- Restoration in the presence of noise only – spatial filtering
- Periodic noise reduction by frequency domain filtering
- Linear, position-invariant degradations
- Estimating the degradation function
- Inverse filtering

Additive noise only



$$
\begin{cases}
g(x,y)=f(x,y)+\boldsymbol{\eta(x,y)} \\
G(u,v)=F(u,v)+N(u,v)
\end{cases}
$$

Spatial filters for de-noising additive noise

- Skills similar to image enhancement
- Mean filters
- Order-statistics filters
- Adaptive filters

Mean filters

- Arithmetic mean

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$$

Window centered at (x,y)

- Geometric mean

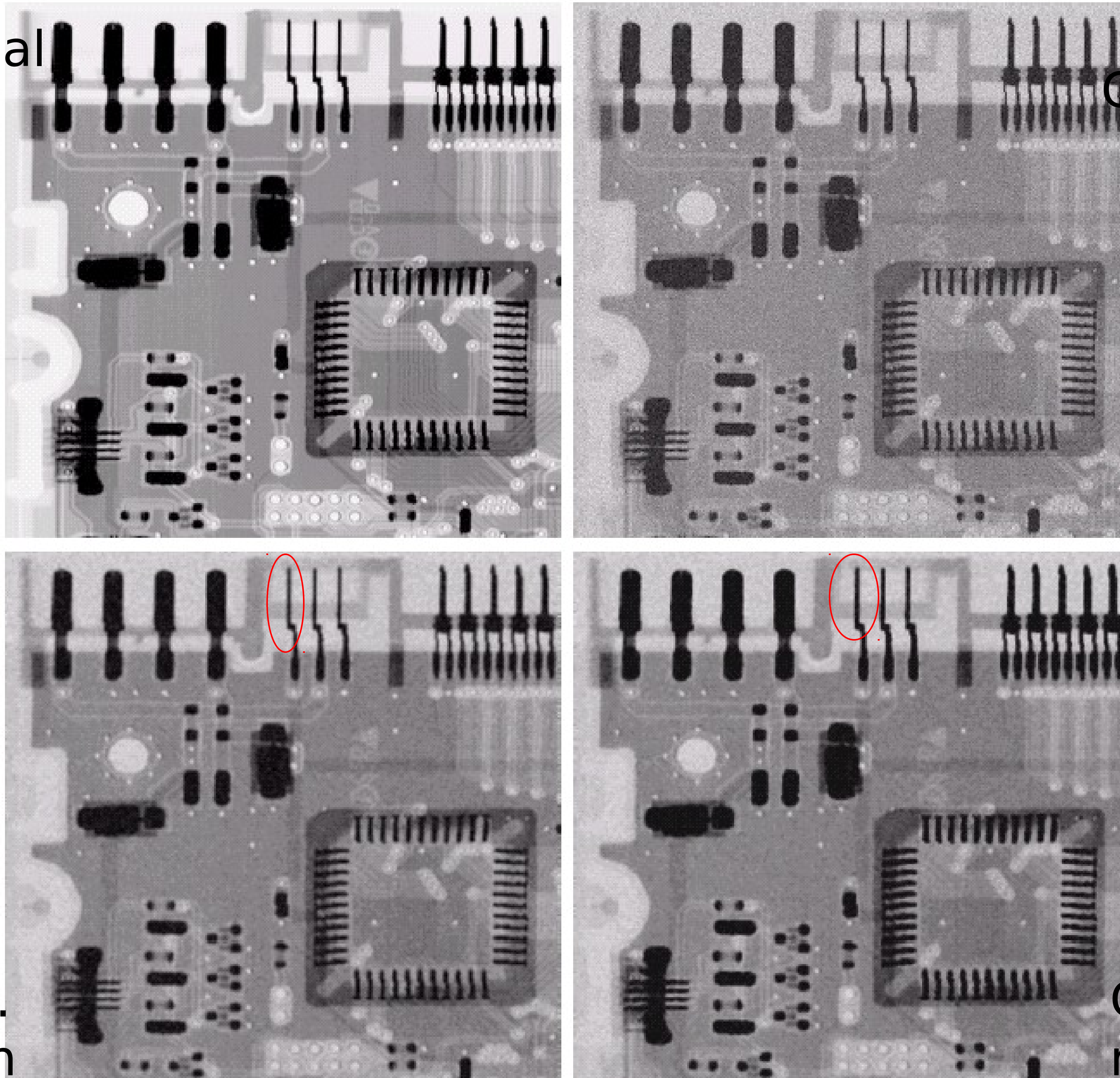
$$\prod_{(s,t)\in S_{xy}}¿\; g(s,t)$$
$$¿$$
$$¿$$
$$¿\,1/mn$$
$$\hat{f}(x,y) = ¿$$
$$¿$$

original

Noisy Gaussian

$\mu=0$

$\sigma=20$



Arith. mean

Geometri mean

Mean filters (cont.)

- Harmonic mean filter

$$\hat{f}(x,y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

- Contra-harmonic mean filter

$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

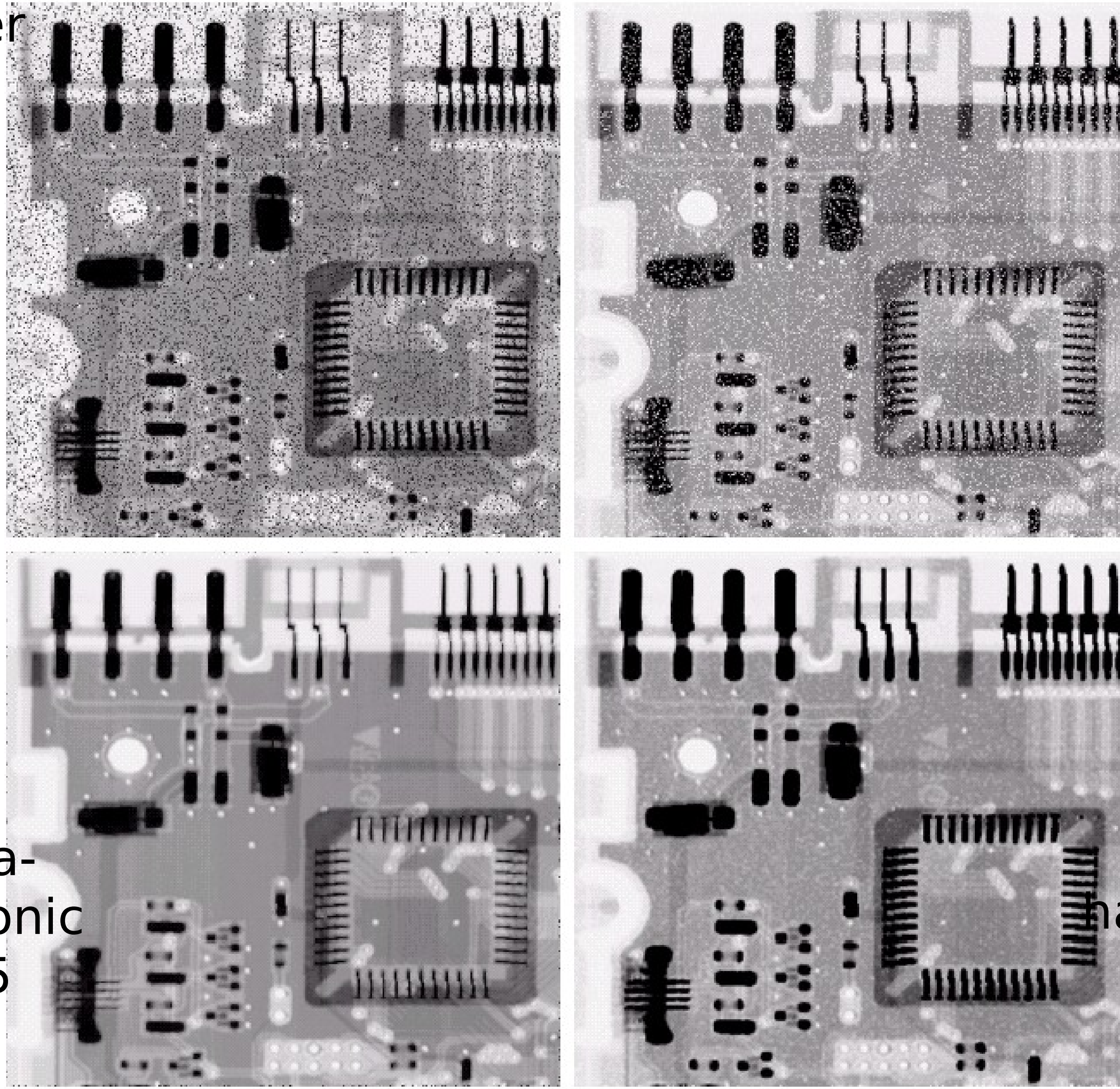
Q=-1, harmonic

Q=0, airth. mean

Q=+, ?

Pepper Noise 黑點

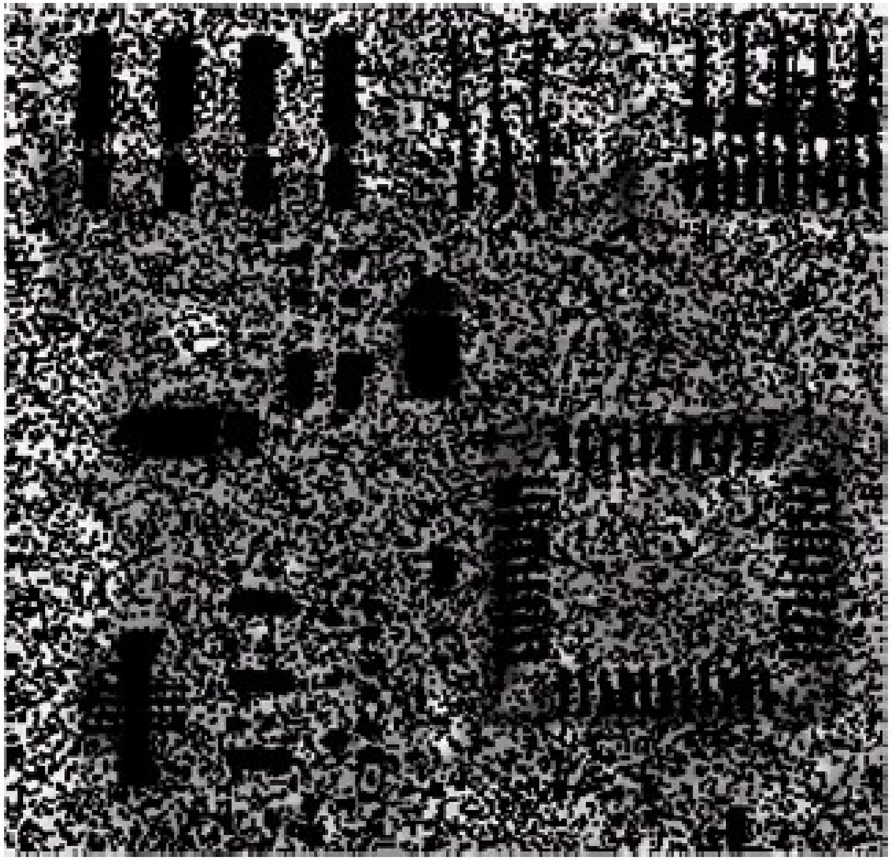
Salt Noise 白點



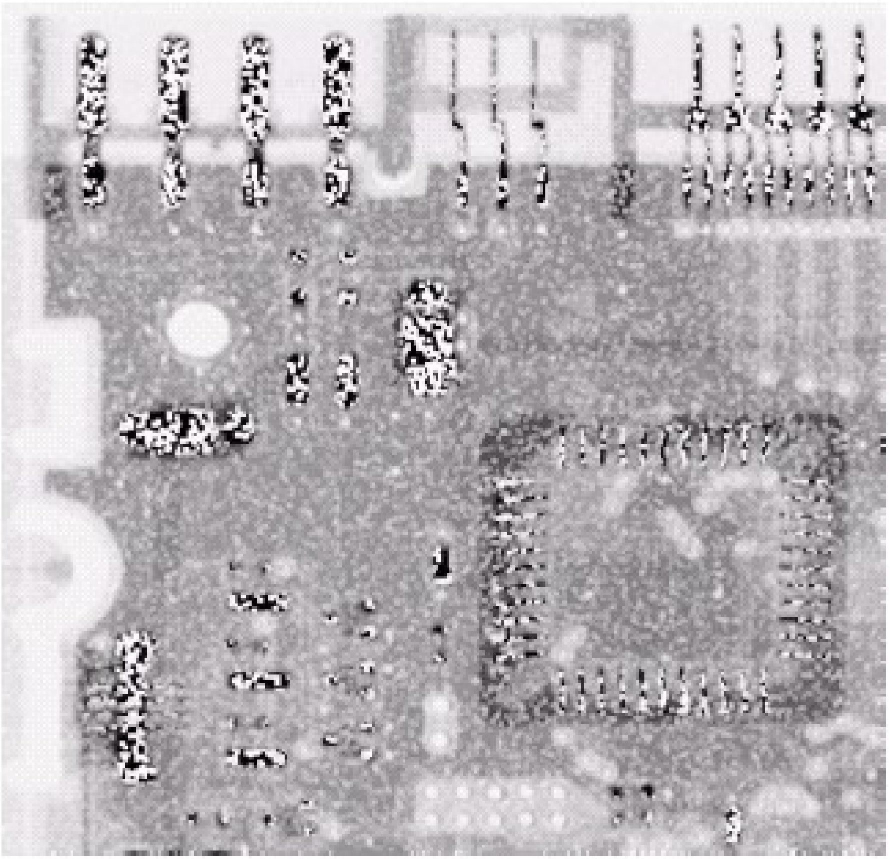
Contra-harmonic Q=1.5

Contra-harmonic Q=-1.5

Wrong sign in contra-harmonic filter



Q=-1.5



Q=1.5

Order-statistics filters

- Based on the ordering(ranking) of pixels
  - Suitable for unipolar or bipolar noise (salt and pepper noise)
- Median filters
- Max/min filters
- Midpoint filters
- Alpha-trimmed mean filters

Order-statistics filters

- Median filter

$$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{median}\{g(s,t)\}$$

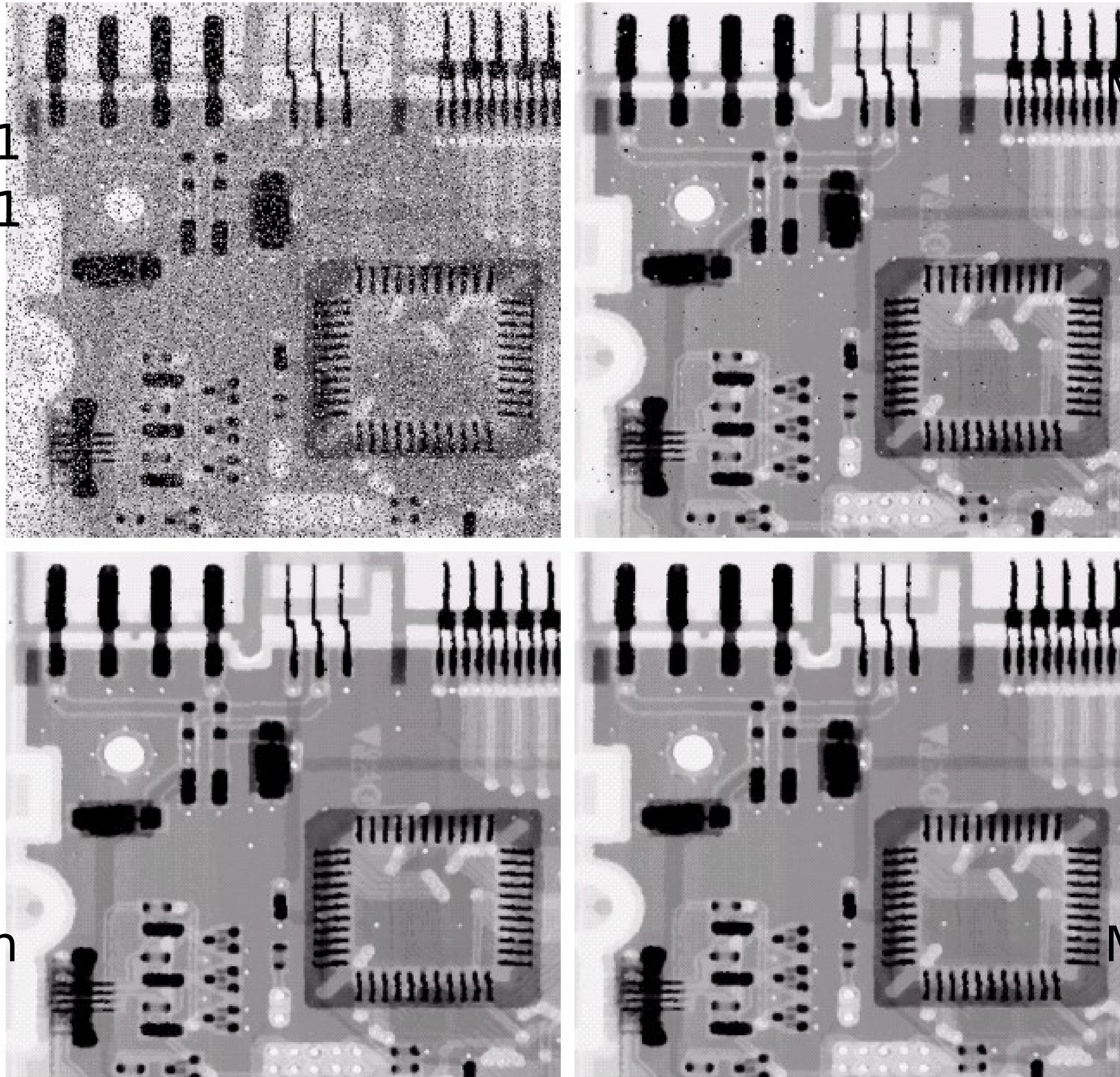
- Max/min filters

$$\hat{f}(x,y) = \max_{(s,t)\in S_{xy}}\{g(s,t)\}$$

$$\hat{f}(x,y) = \min_{(s,t)\in S_{xy}}\{g(s,t)\}$$

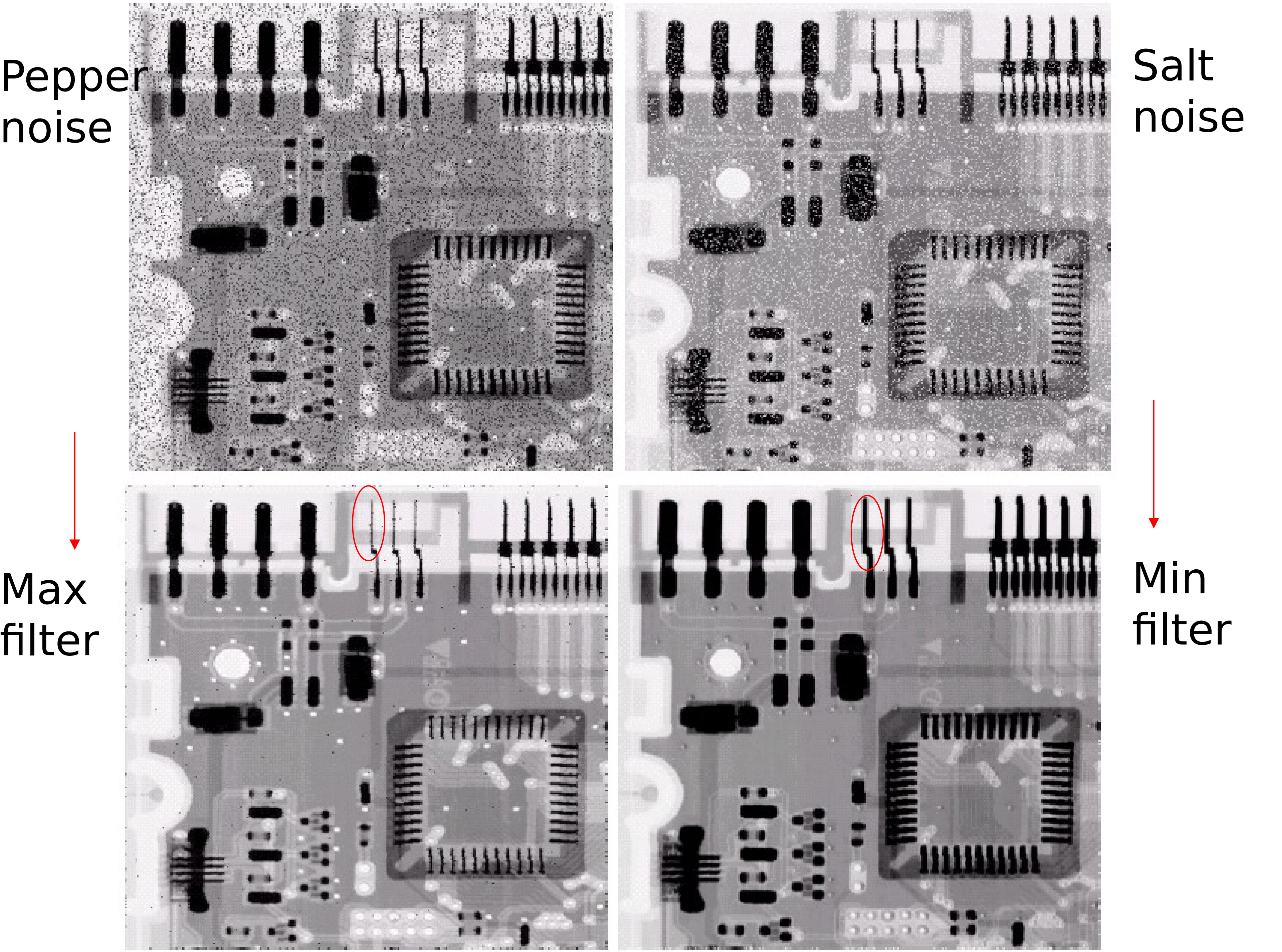
bipolar
Noise
$P_a = 0.1$
$P_b = 0.1$

3x3
Median
Filter
Pass 1



3x3
Median
Filter
Pass 2

3x3
Median
Filter
Pass 3


Pepper noise
Salt noise
Max filter
Min filter

Order-statistics filters (cont.)

- Midpoint filter

$$\hat{f}(x,y)=\frac{1}{2}\left[\max_{(s,t)\in S_{xy}}\{g(s,t)\}+\min_{(s,t)\in S_{xy}}\{g(s,t)\}\right]$$

- Alpha-trimmed mean filter
  - Delete the d/2 lowest and d/2 highest gray-level pixels

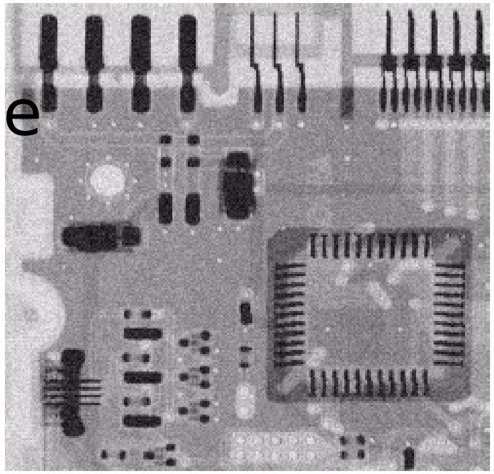
$$\hat{f}(x,y)=\frac{1}{mn-d}\sum_{(s,t)\in S_{xy}} g_r(s,t)$$

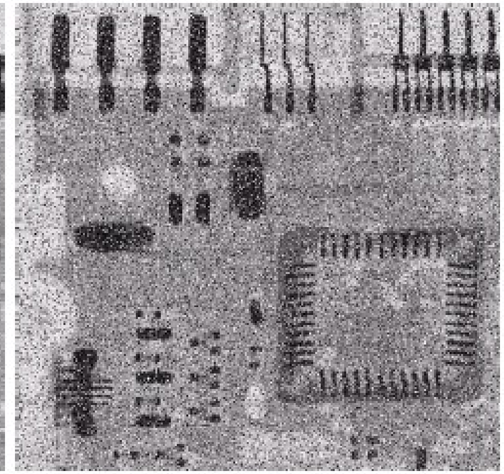
Middle (mn-d) pixels

Uniform noise

$\mu=0$

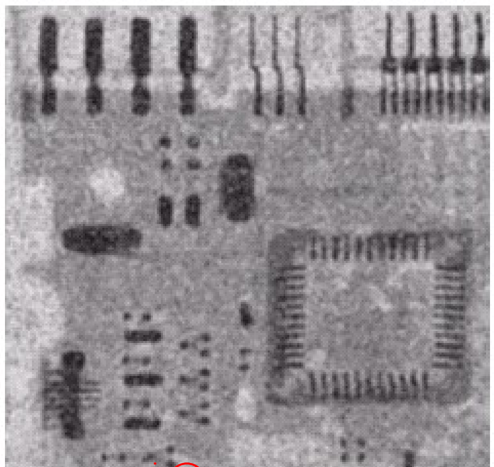
$\sigma^2=800$

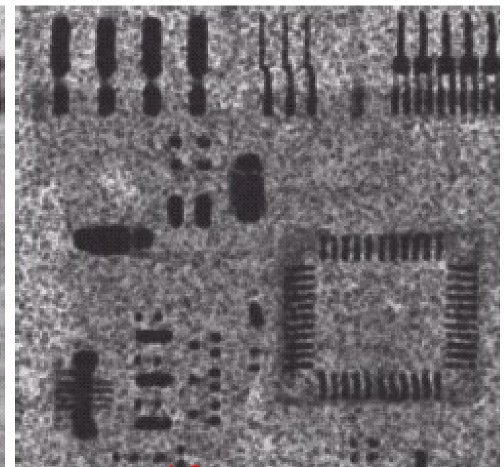




Left +
Bipolar Noise
$P_a = 0.1$
$P_b = 0.1$

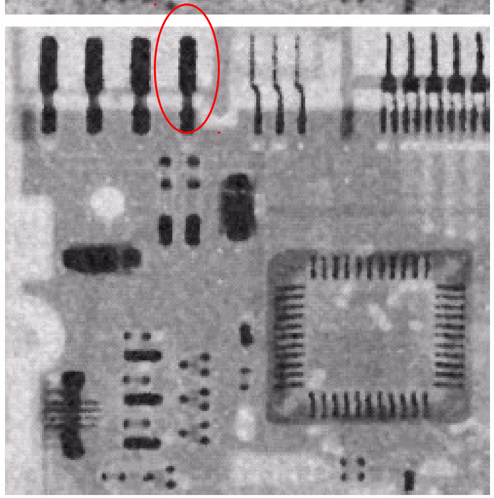
5x5
Arith. Mean
filter

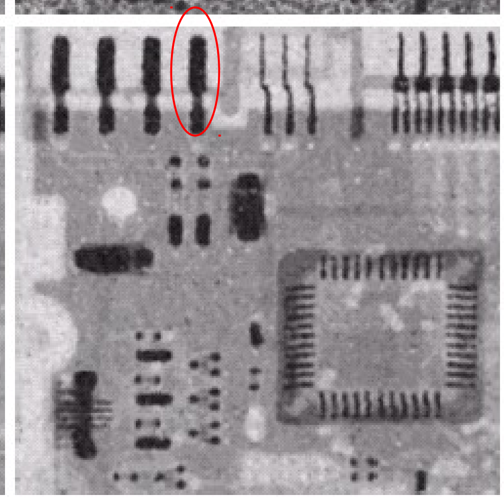




5x5
Geometric
mean

5x5
Median
filter





5x5
Alpha-trim.
Filter
d=5

Adaptive filters

- Adapted to the behavior based on the statistical characteristics of the image inside the filter region S_{xy}
- Improved performance v.s increased complexity
- Example: Adaptive local noise reduction filter

Adaptive local noise reduction filter

- Simplest statistical measurement
  - Mean and variance
- Known parameters on local region S_{xy}
  - g(x,y): noisy image pixel value
  - σ^2_η: noise variance (assume known a prior)
  - m_L : local mean
  - σ^2_L: local variance

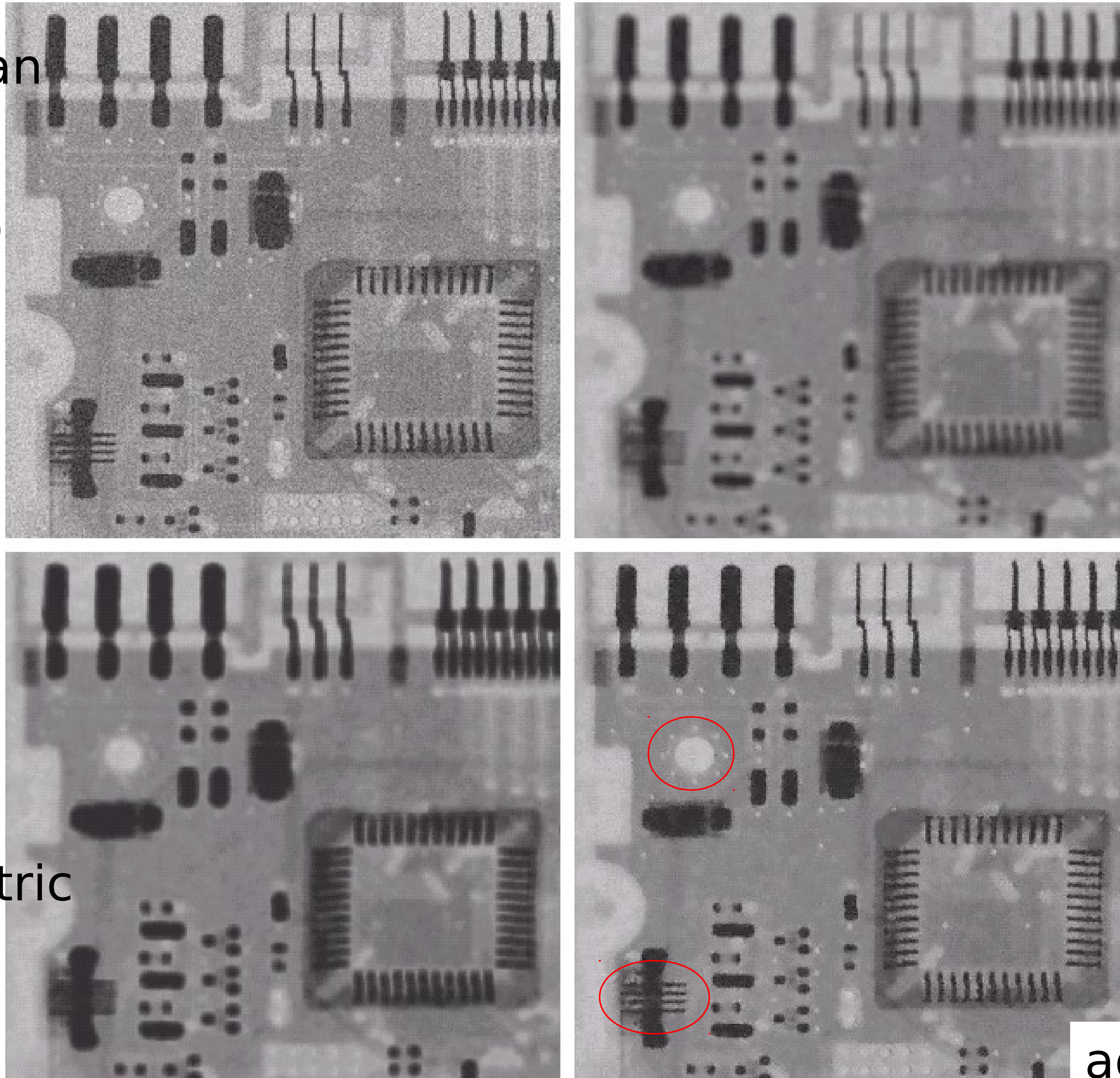
Adaptive local noise reduction filter (cont.)

- Analysis: we want to do
  - If σ^2_η is zero, return g(x,y)
  - If $\sigma^2_L > \sigma^2_\eta$, return value close to g(x,y)
  - If $\sigma^2_L = \sigma^2_\eta$, return the arithmetic mean m_L
- Formula

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma^2_\eta}{\sigma^2_L}\left[g(x,y) - m_L\right]$$

Gaussian noise
$\mu=0$
$\sigma^2=1000$

Arith. mean 7x7



Geometric mean 7x7

adaptive

Outline

- A model of the image degradation / restoration process
- Noise models
- Restoration in the presence of noise only – spatial filtering
- Periodic noise reduction by frequency domain filtering
- Linear, position-invariant degradations
- Estimating the degradation function
- Inverse filtering

Periodic noise reduction

- Pure sine wave
  - Appear as a pair of impulse (conjugate) in the frequency domain

$$\begin{cases} f(x,y) = A\sin(u_0 x + v_0 y) \\ F(u,v) = -j\dfrac{A}{2}\left[\delta(u-\dfrac{u_0}{2\pi}, v-\dfrac{v_0}{2\pi}) - \delta(u+\dfrac{u_0}{2\pi}, v+\dfrac{v_0}{2\pi})\right] \end{cases}$$

Periodic noise reduction (cont.)

- Bandreject filters
- Bandpass filters
- Notch filters
- Optimum notch filtering

Bandreject filters

* Reject an isotropic frequency

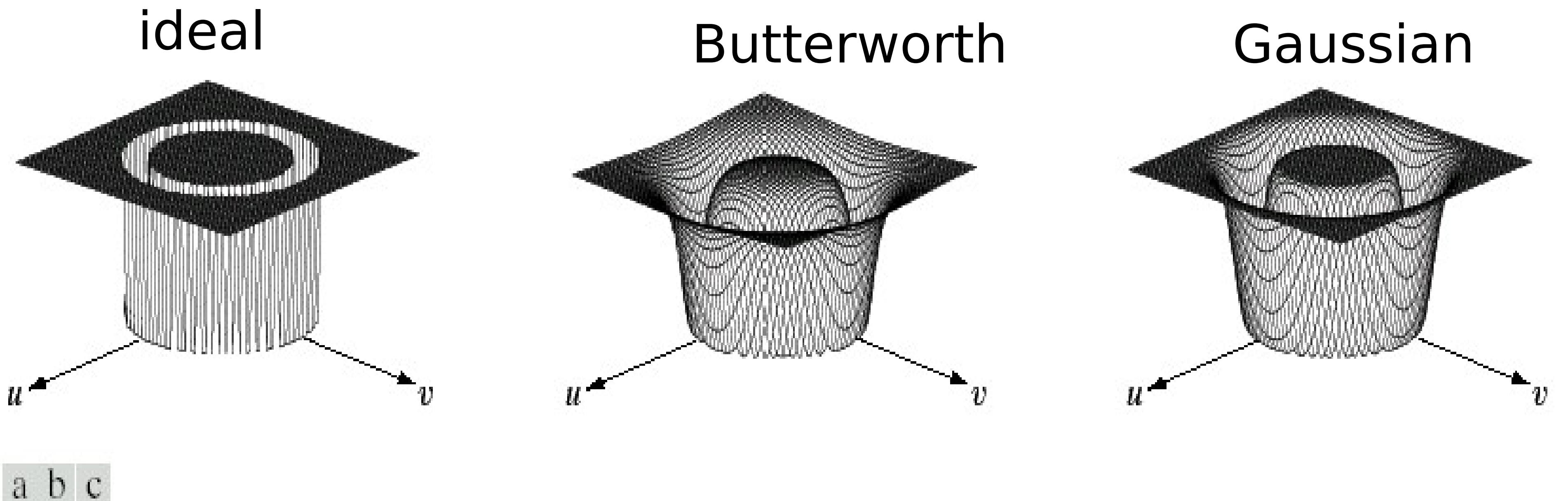
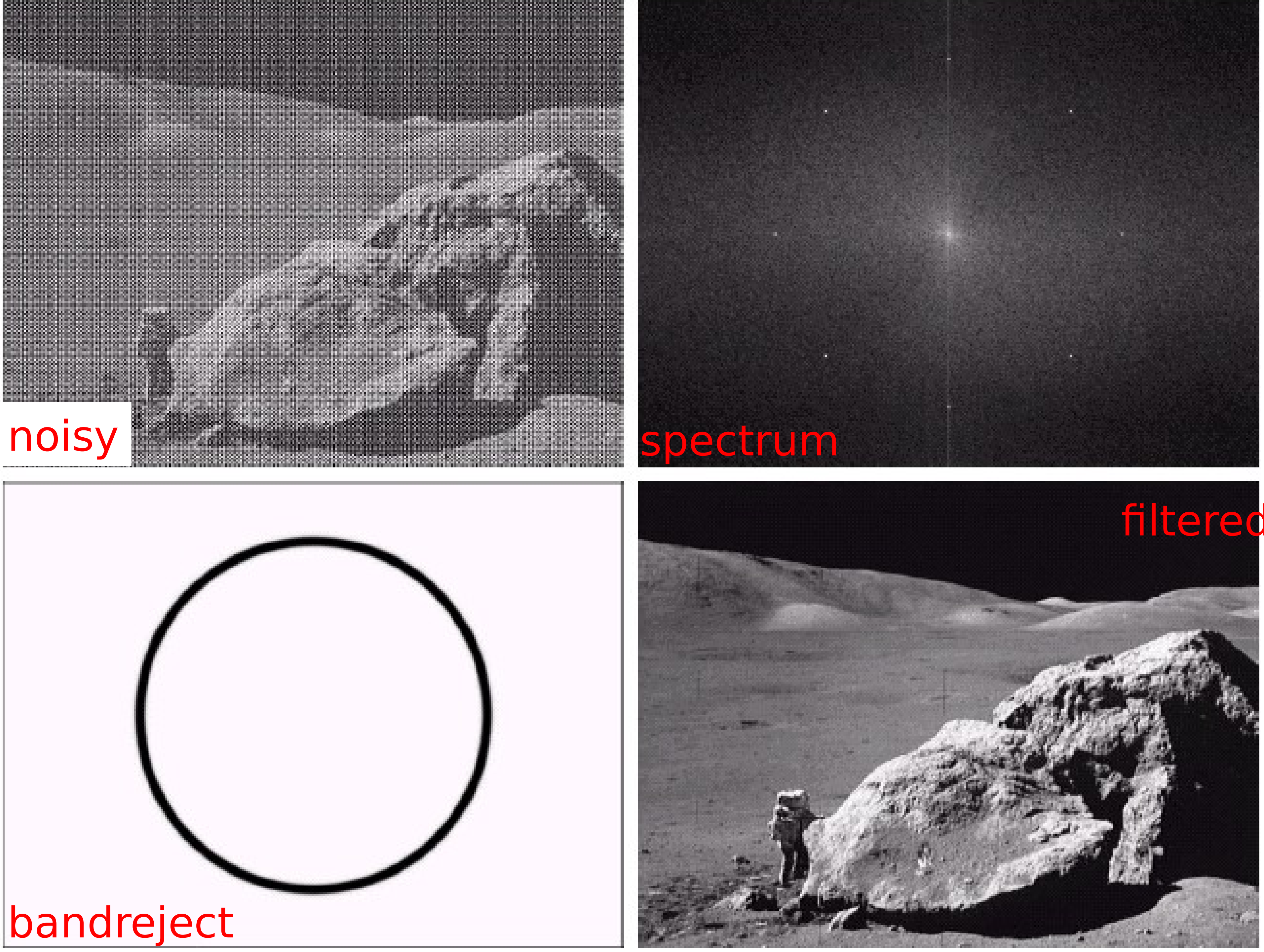
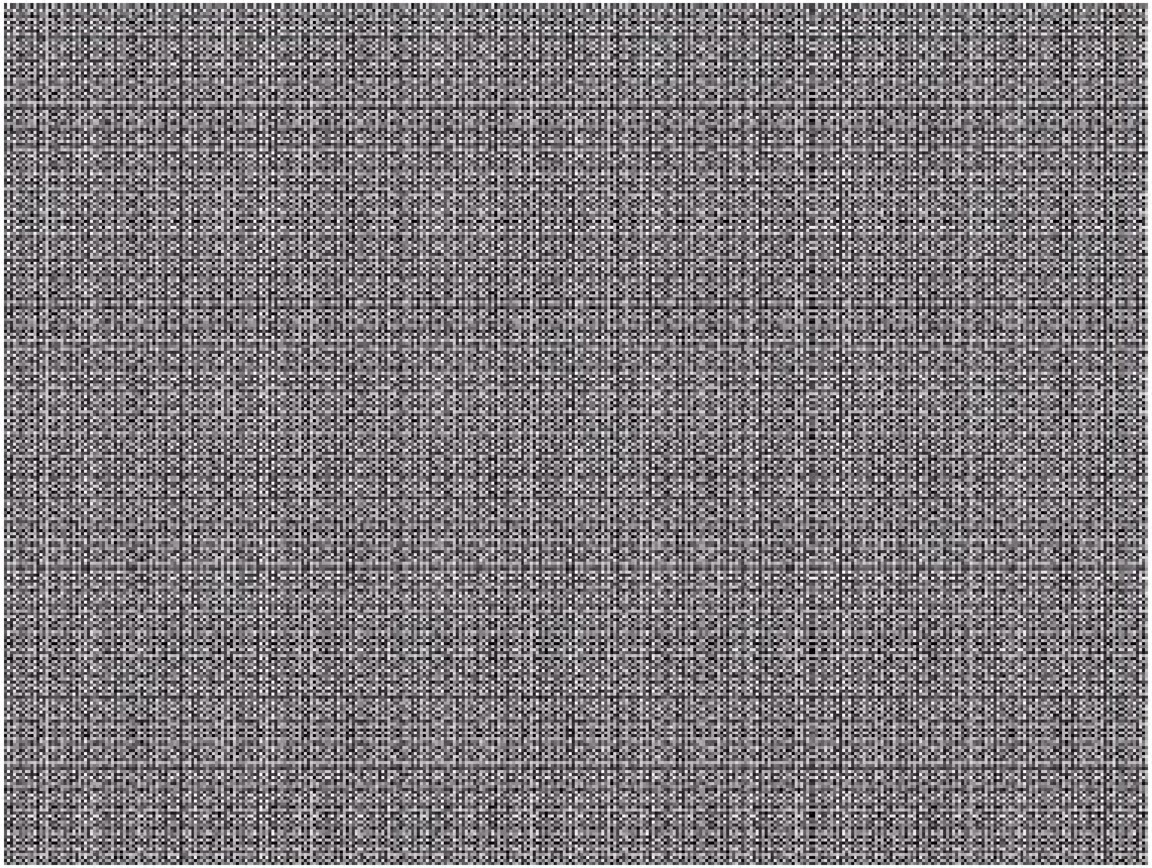


FIGURE 5.15 From left to right, perspective plots of ideal, Butterworth (of order 1), and Gaussian bandreject filters.


noisy
spectrum
bandreject
filtered

Bandpass filters

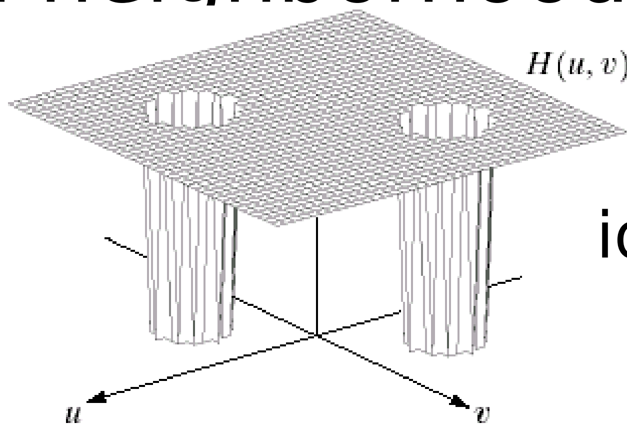
- $H_{bp}(u,v)=1- H_{br}(u,v)$



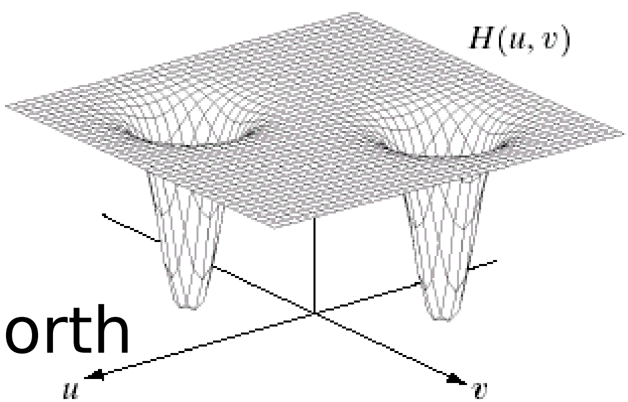
$$\mathfrak{I}^{-1}\left\{G(u,v)H_{bp}(u,v)\right\}$$

Notch filters

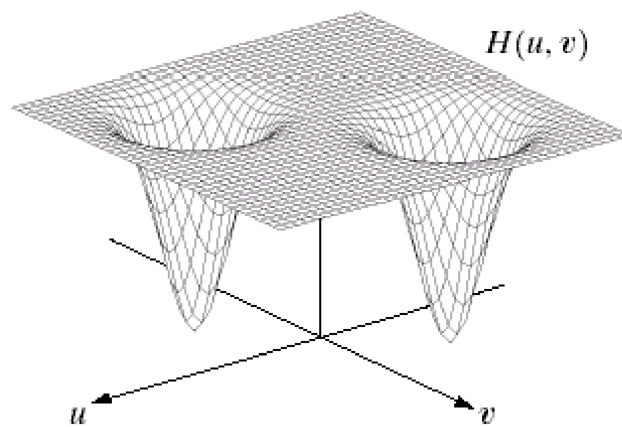
- Reject(or pass) frequencies in predefined neighborhoods about a ce



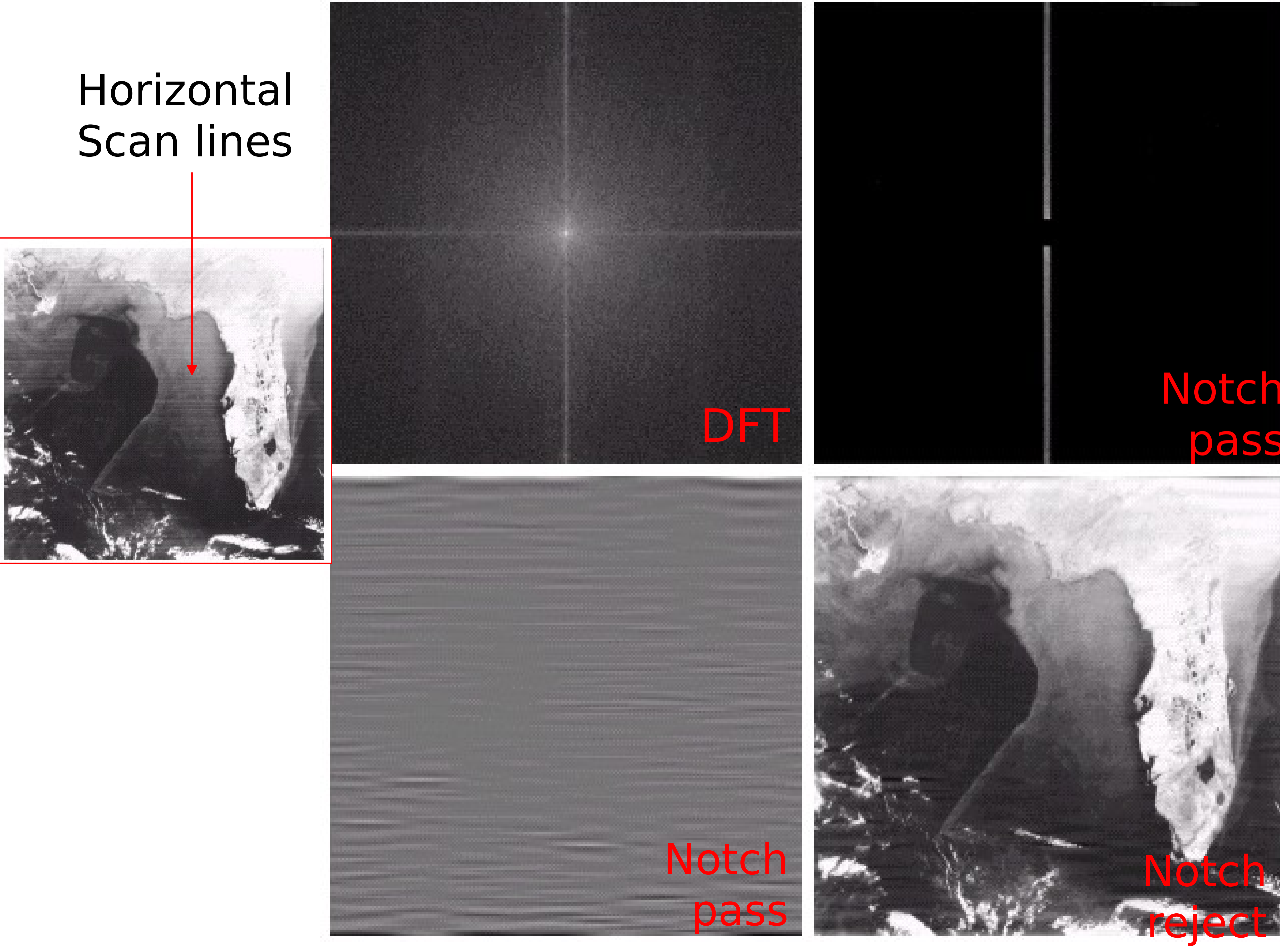
ideal



Butterworth



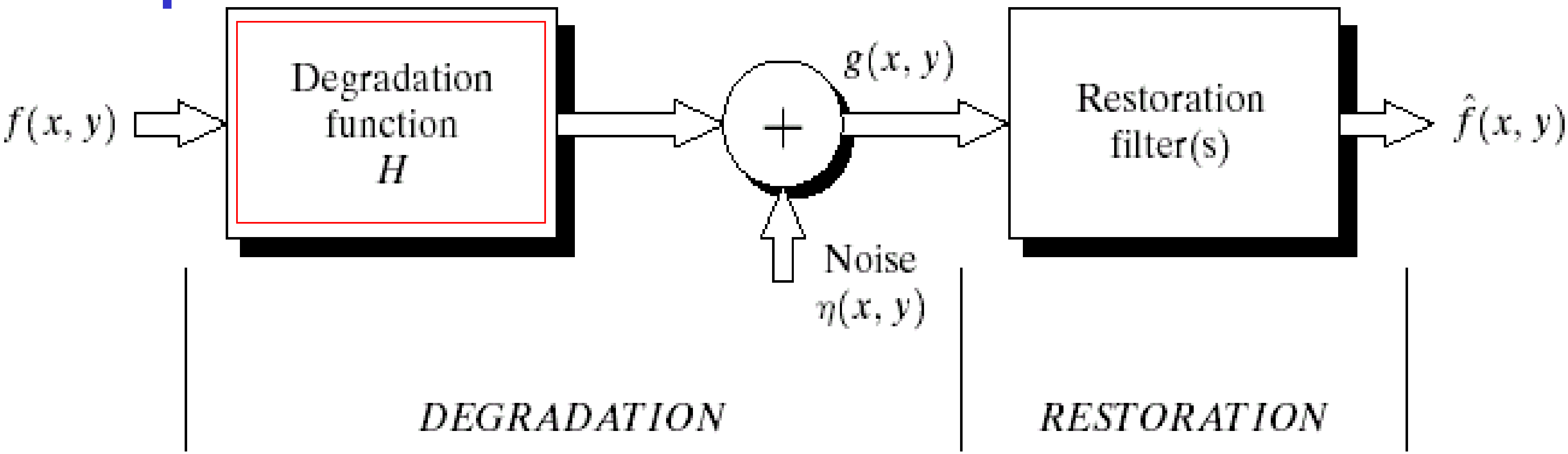
Gaussian


Horizontal Scan lines
DFT
Notch pass
Notch pass
Notch reject

Outline

- A model of the image degradation / restoration process
- Noise models
- Restoration in the presence of noise only – spatial filtering
- Periodic noise reduction by frequency domain filtering
- Linear, position-invariant degradations
- Estimating the degradation function
- Inverse filtering

A model of the image degradation /restoration process



$$\begin{cases} g(x,y)=f(x,y)*h(x,y)+\eta(x,y) \\ G(u,v)=F(u,v)H(u,v)+N(u,v) \end{cases}$$

If linear, position-invariant syst

Linear, position-invariant degradation

Properties of the degradation function H

- Linear system
  - $H[af_1(x,y)+bf_2(x,y)]=aH[f_1(x,y)]+bH[f_2(x,y)]$
- Position(space)-invariant system
  - $H[f(x,y)]=g(x,y)$
  - $\Leftrightarrow H[f(x-\alpha, y-\beta)]=g(x-\alpha, y-\beta)$
- c.f. 1-D signal
  - LTI (linear time-invariant system)

Linear, position-invariant degradation model

- Linear system theory is ready
- Non-linear, position-dependent system
  - May be general and more accurate
  - Difficult to solve compuatationally
- Image restoration: find H(u,v) and apply inverse process
  - Image deconvolution

Estimating the degradation function

- Estimation by Image observation
- Estimation by experimentation
- Estimation by modeling

Estimation by image observation

- Take a window in the image
  - Simple structure
  - Strong signal content
- Estimate the original image in the window

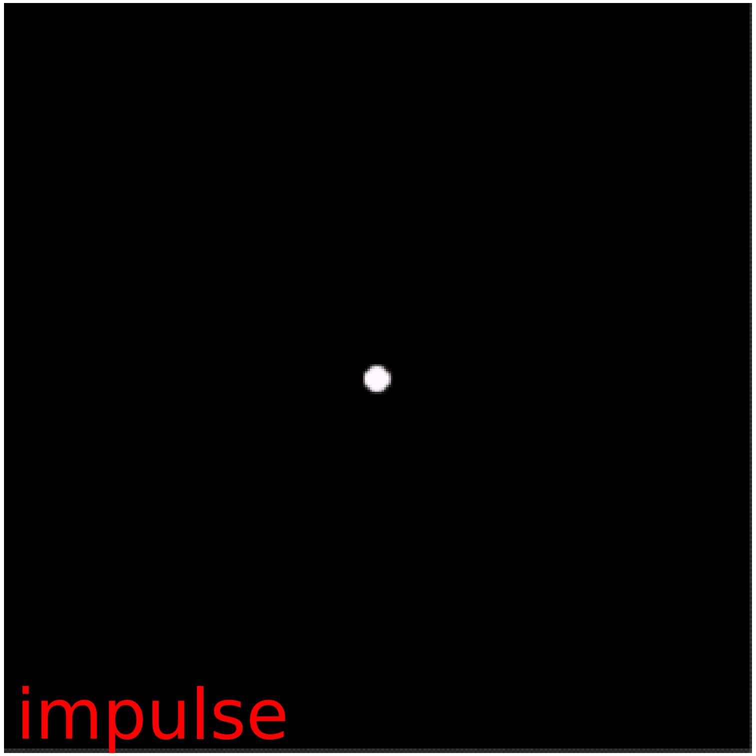
$$H_s(u,v) = \frac{G_s(u,v)}{\hat{F}_s(u,v)}$$

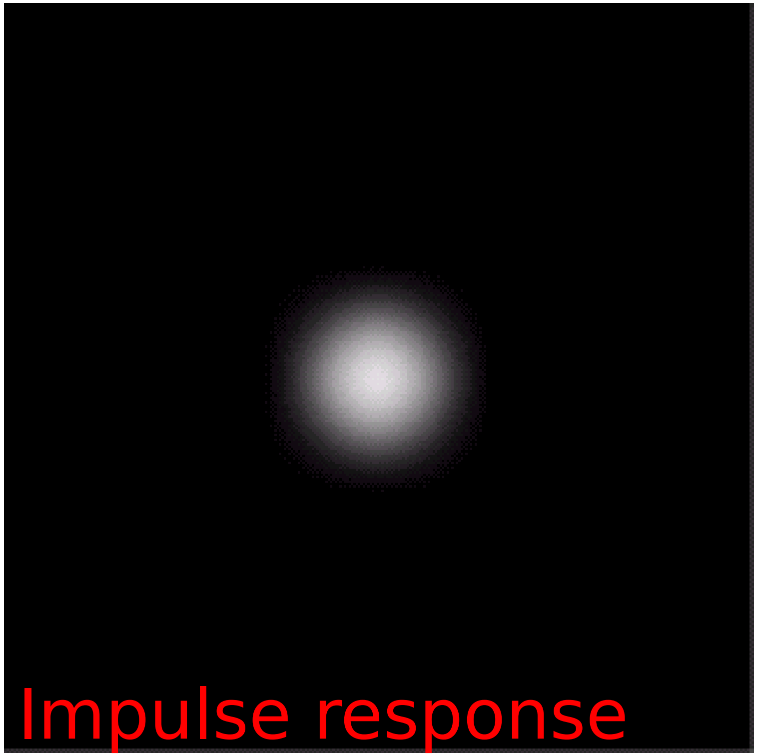
known

estimate

Estimation by experimentation

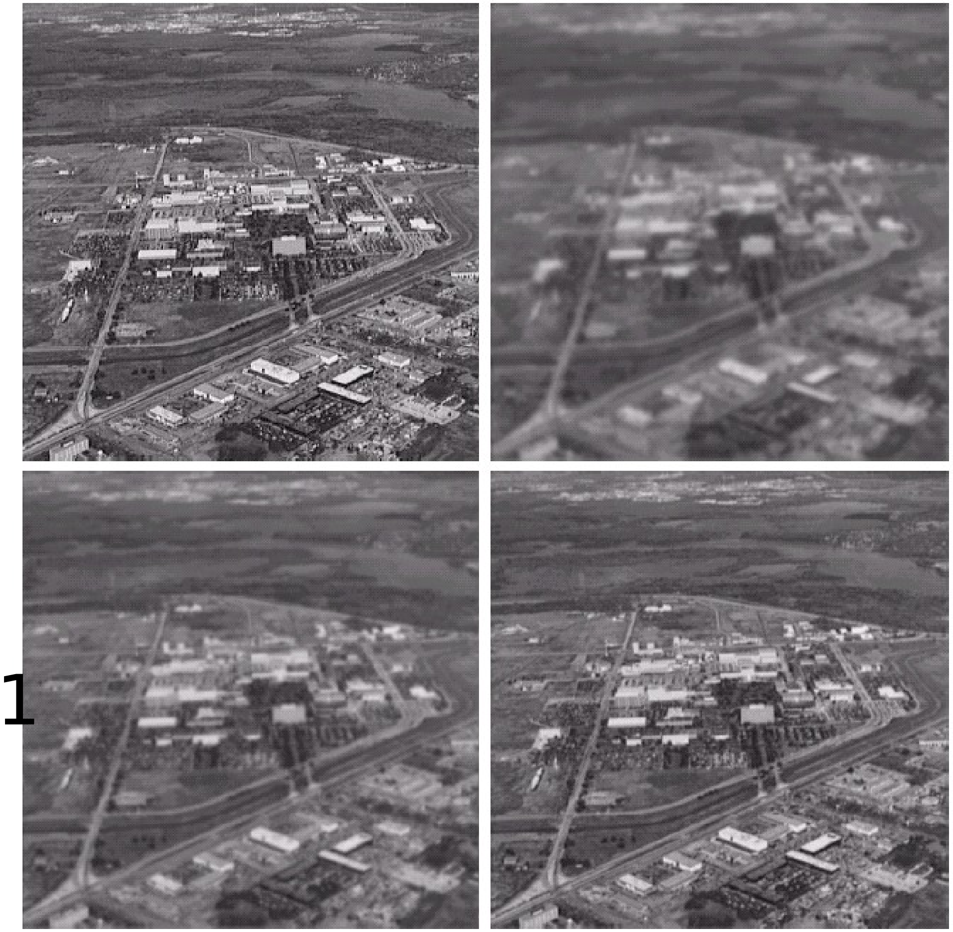
- If the image acquisition system is ready
- Obtain the impulse response





Estimation by modeling (1)

- Ex. Atmospheric model $H(u,v)=e^{-k(u^2+v^2)^{5/6}}$



original

k=0.0025

k=0.001

k=0.00025

Estimation by modeling (2)

- Derive a mathematical model
- Ex. Motion of image

$$g(x,y)=\int_0^T f\left(x-x_0(t),\, y-y_0(t)\right)dt$$

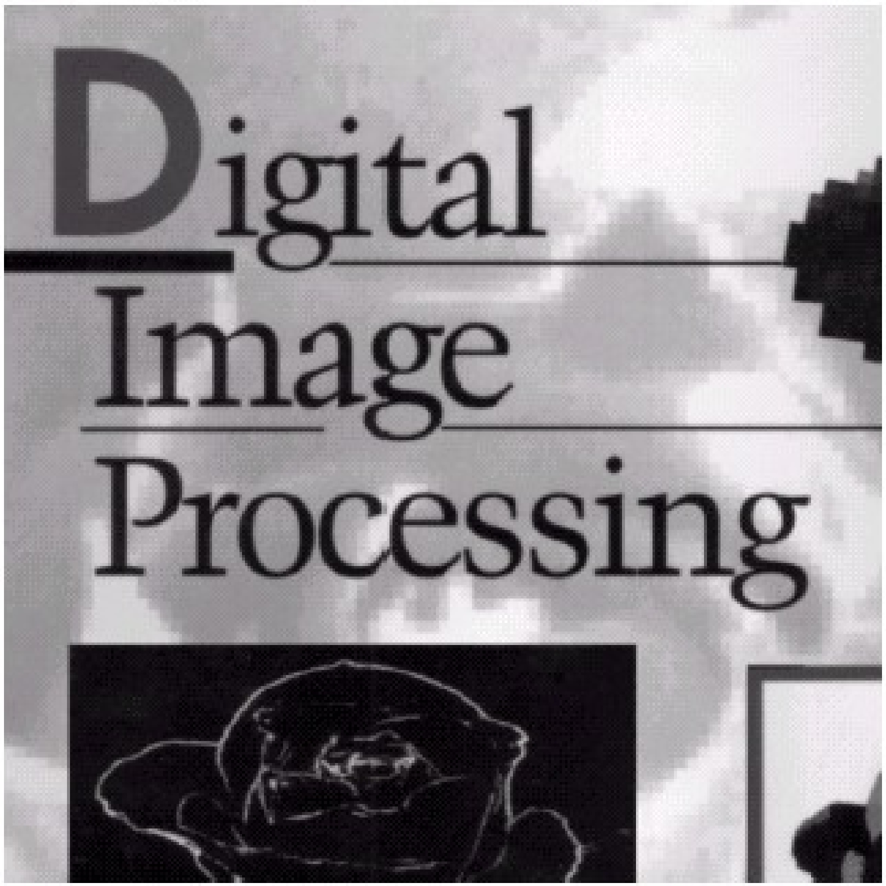
Planar motion

Fourier transform

$$G(u,v)=F(u,v)\int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]}dt$$

Estimation by modeling: example

original



Apply motion model

Inverse filtering

- With the estimated degradation function H(u,v)

G(u,v)=F(u,v)H(u,v)+N(u,v)

Unknown noise

=> $$\hat{F}(u,v)=\frac{G(u,v)}{H(u,v)}=F(u,v)+\frac{N(u,v)}{H(u,v)}$$

Estimate of original image

Problem: 0 or small values

Sol: limit the frequency
around the origin

Full inverse filter for k=0.0025

Cut Outside 40%

Cut Outside 70%

Cut Outside 85%