Modern Information Retrieval

Chapter 3

Modeling

Part I: Classic Models

- Introduction to IR Models
- Basic Concepts
- The Boolean Model
- Term Weighting
- The Vector Model
- Probabilistic Model

IR Models

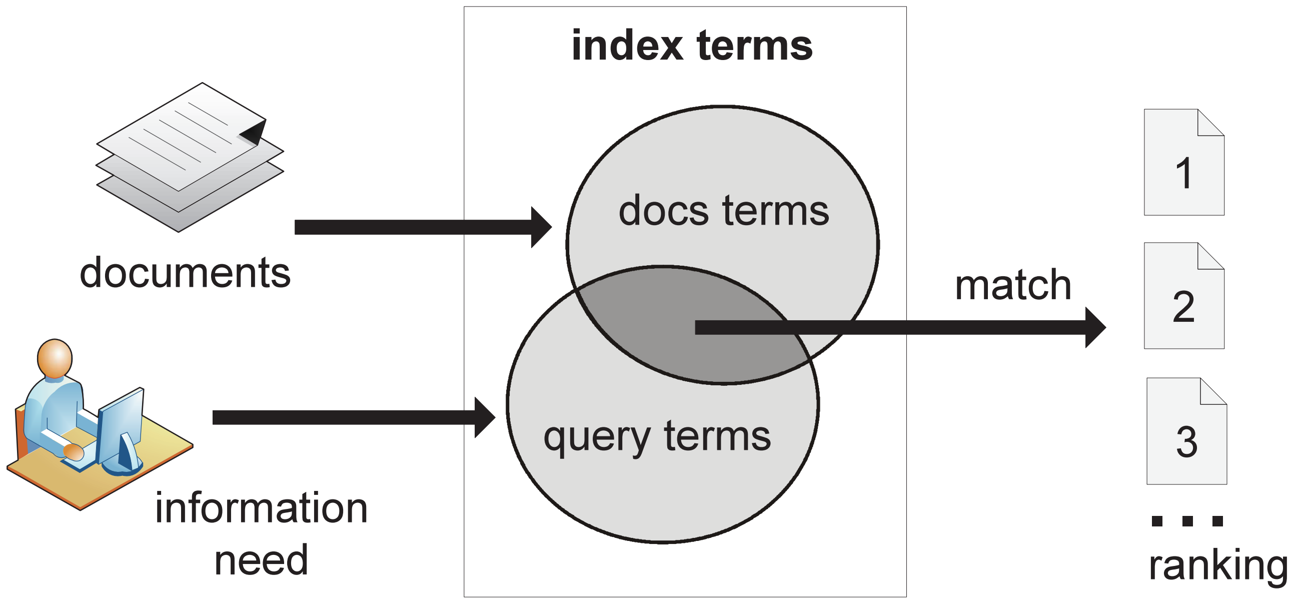
- **Modeling** in IR is a complex process aimed at producing a ranking function
  - **Ranking function**: a function that assigns scores to documents with regard to a given query
- This process consists of two main tasks:
  - The conception of a logical framework for representing documents and queries
  - The definition of a ranking function that allows quantifying the similarities among documents and queries

Modeling and Ranking

- IR systems usually adopt **index terms** to index and retrieve documents
- Index term:
  - In a restricted sense: it is a keyword that has some meaning on its own; usually plays the role of a noun
  - In a more general form: it is any word that appears in a document
- Retrieval based on index terms can be implemented efficiently
- Also, index terms are simple to refer to in a query
- Simplicity is important because it reduces the effort of query formulation

Introduction

- Information retrieval process

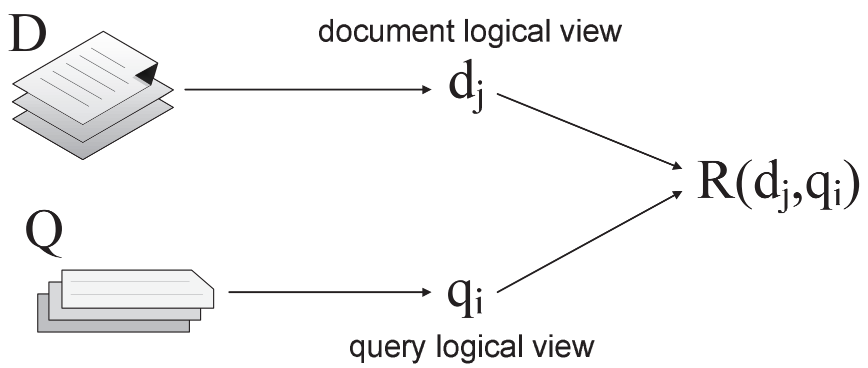


Introduction

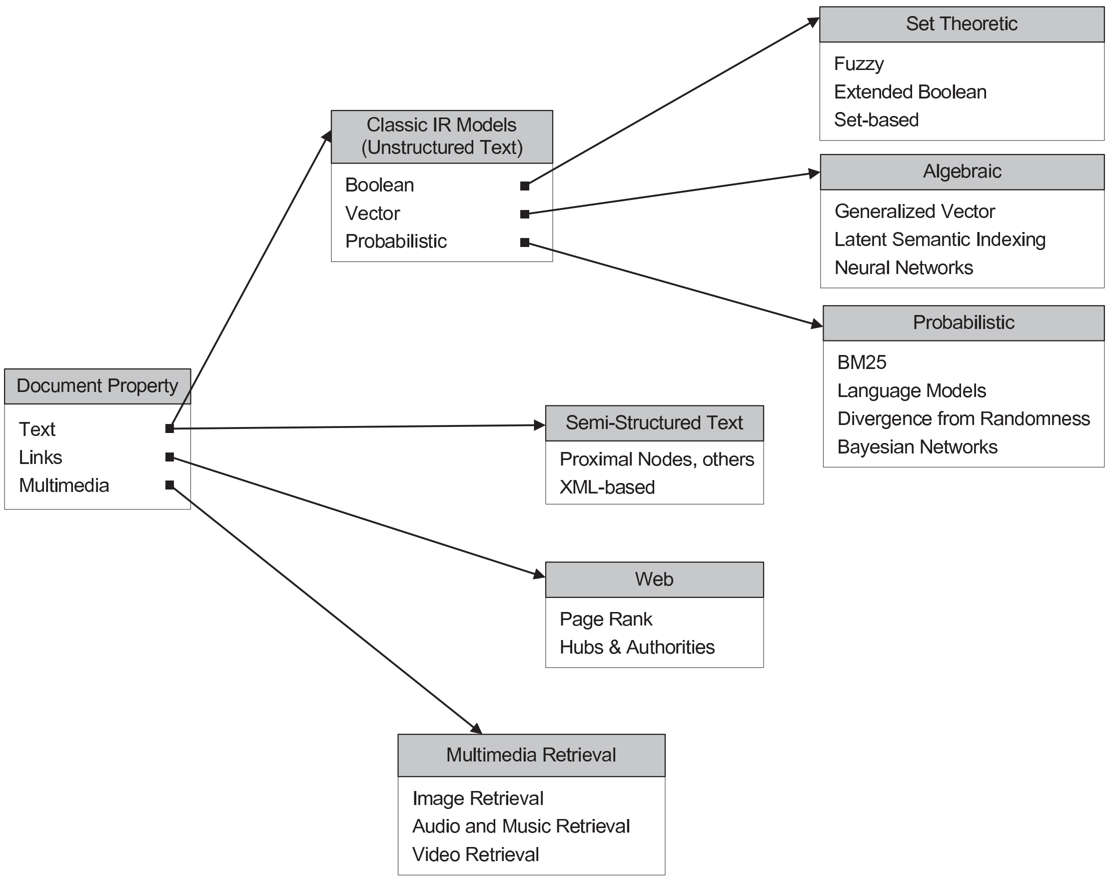
- A **ranking** is an ordering of the documents that (hopefully) reflects their **relevance** to a user query
- Thus, any IR system has to deal with the problem of predicting which documents the users will find relevant
- This problem naturally embodies a degree of uncertainty, or vagueness

IR Models

- An **IR model** is a quadruple $[\mathbf{D}, \mathbf{Q}, \mathcal{F}, R(q_i, d_j)]$ where
  1. $\mathbf{D}$ is a set of logical views for the documents in the collection
  2. $\mathbf{Q}$ is a set of logical views for the user queries
  3. $\mathcal{F}$ is a framework for modeling documents and queries
  4. $R(q_i, d_j)$ is a ranking function

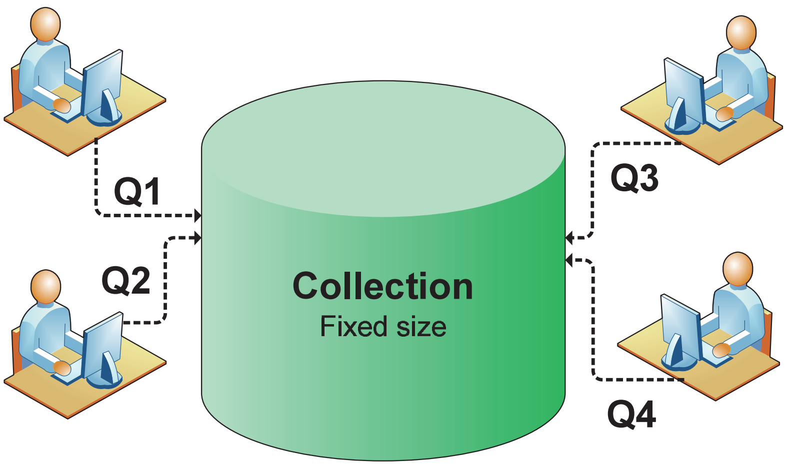


A Taxonomy of IR Models



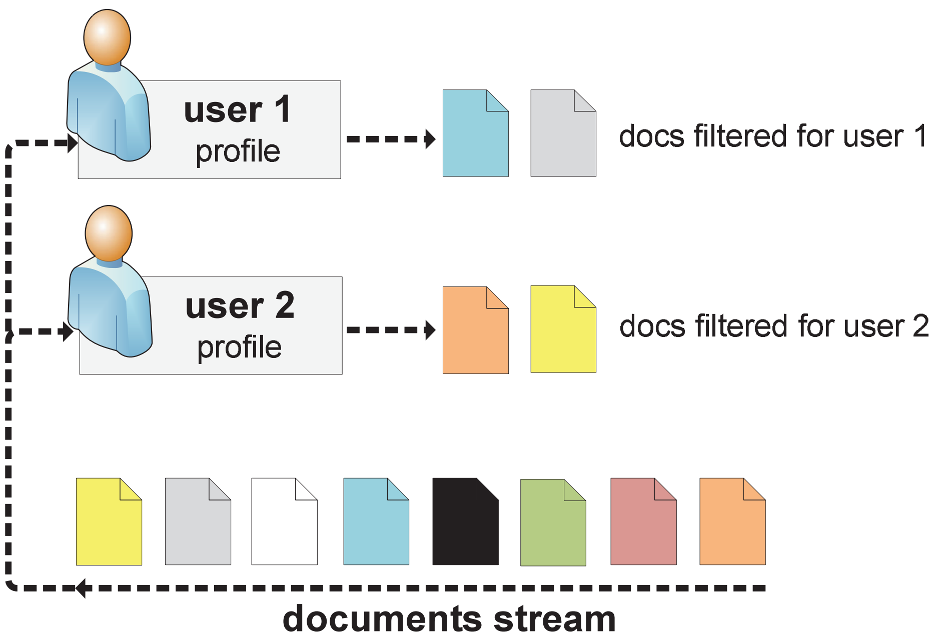
Retrieval: Ad Hoc x Filtering

- Ad Hoc Retrieval:



Retrieval: Ad Hoc x Filtering

- Filtering



Basic Concepts

- Each document is represented by a set of representative keywords or index terms
- An index term is a word or group of consecutive words in a document
- A pre-selected set of index terms can be used to summarize the document contents
- However, it might be interesting to assume that all words are index terms (full text representation)

Basic Concepts

- Let,
  - t be the number of index terms in the document collection
  - k_i be a generic index term
- Then,
  - The **vocabulary** $V = \{k_1, \ldots, k_t\}$ is the set of all distinct index terms in the collection

$$V = \boxed{k_1 \quad k_2 \quad k_3 \quad \ldots \quad k_t}$$

vocabulary of *t*
index terms

Basic Concepts

- Documents and queries can be represented by **patterns of term co-occurrences**

$$V = \boxed{k_1 \quad k_2 \quad k_3 \; \ldots \; k_t}$$

$$\boxed{1 \quad 0 \quad 0 \; \ldots \; 0}$$

pattern that represents documents (and queries) with the term k_1 and no other

$$\vdots$$

$$\boxed{1 \quad 1 \quad 1 \; \ldots \; 1}$$

pattern that represents documents (and queries) with all index terms

- Each of these patterns of term co-occurence is called a **term conjunctive component**
- For each document d_j (or query q) we associate a unique term conjunctive component $c(d_j)$ (or $c(q)$)

The Term-Document Matrix

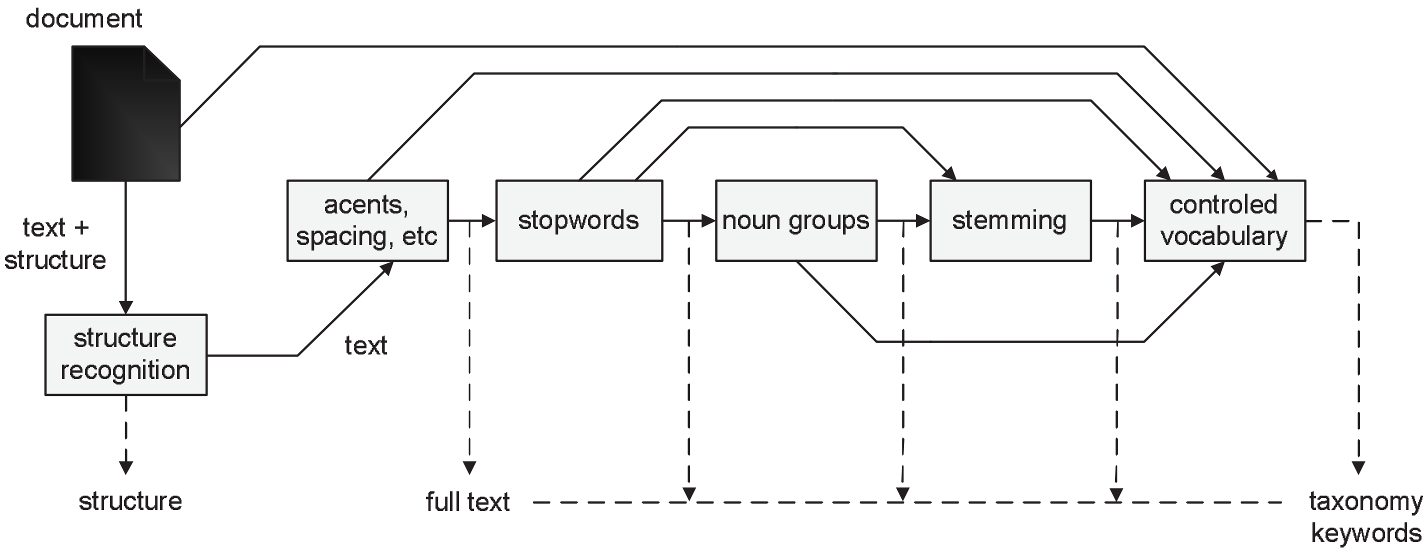
- The occurrence of a term k_i in a document d_j establishes a relation between k_i and d_j
- A **term-document relation** between k_i and d_j can be quantified by the frequency of the term in the document
- In matrix form, this can written as

$$\begin{array}{c c} & \begin{array}{cc} d_1 & d_2 \end{array} \\ \begin{array}{c} k_1 \\ k_2 \\ k_3 \end{array} & \left[\begin{array}{cc} f_{1,1} & f_{1,2} \\ f_{2,1} & f_{2,2} \\ f_{3,1} & f_{3,2} \end{array} \right] \end{array}$$

where each $f_{i,j}$ element stands for the frequency of term k_i in document d_j

Basic Concepts

- Logical view of a document: from full text to a set of index terms



The Boolean Model

The Boolean Model

- Simple model based on **set theory** and **boolean algebra**
- Queries specified as boolean expressions
  - quite intuitive and precise semantics
  - neat formalism
  - example of query

$$q = k_a \wedge (k_b \vee \neg k_c)$$

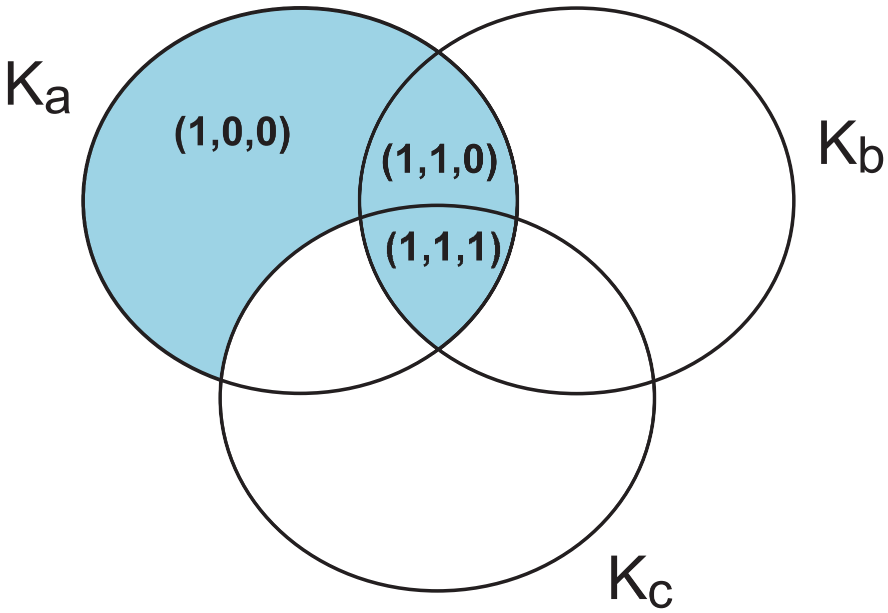
- Term-document frequencies in the term-document matrix are all binary
  - $w_{ij} \in \{0, 1\}$: weight associated with pair (k_i, d_j)
  - $w_{iq} \in \{0, 1\}$: weight associated with pair (k_i, q)

The Boolean Model

- A term conjunctive component that satisfies a query q is called a **query conjunctive component** $c(q)$
- A query q rewritten as a disjunction of those components is called the **disjunct normal form** q_{DNF}
- To illustrate, consider
  - query $q = k_a \wedge (k_b \vee \neg k_c)$
  - vocabulary $V = \{k_a, k_b, k_c\}$
- Then
  - $q_{DNF} = (1,1,1) \vee (1,1,0) \vee (1,0,0)$
  - $c(q)$: a conjunctive component for q

The Boolean Model

- The three conjunctive components for the query $q = k_a \wedge (k_b \vee \neg k_c)$



The Boolean Model

- This approach works even if the vocabulary of the collection includes terms not in the query
- Consider that the vocabulary is given by $V = \{k_a, k_b, k_c, k_d\}$
- Then, a document d_j that contains only terms k_a, k_b, and k_c is represented by $c(d_j) = (1, 1, 1, 0)$
- The query $[q = k_a \wedge (k_b \vee \neg k_c)]$ is represented in disjunctive normal form as

$$
\begin{aligned}
q_{DNF} \quad = \quad & (1,1,1,0) \vee (1,1,1,1) \vee \\
& (1,1,0,0) \vee (1,1,0,1) \vee \\
& (1,0,0,0) \vee (1,0,0,1)
\end{aligned}
$$

The Boolean Model

- The similarity of the document d_j to the query q is defined as

$$sim(d_j, q) = \begin{cases} 1 & \text{if } \exists c(q) \mid c(q) = c(d_j) \\ 0 & \text{otherwise} \end{cases}$$

- The Boolean model predicts that each document is either relevant or non-relevant

Drawbacks of the Boolean Model

- Retrieval based on binary decision criteria with no notion of partial matching
- No ranking of the documents is provided (absence of a grading scale)
- Information need has to be translated into a Boolean expression, which most users find awkward
- The Boolean queries formulated by the users are most often too simplistic
- The model frequently returns either too few or too many documents in response to a user query

Term Weighting

Term Weighting

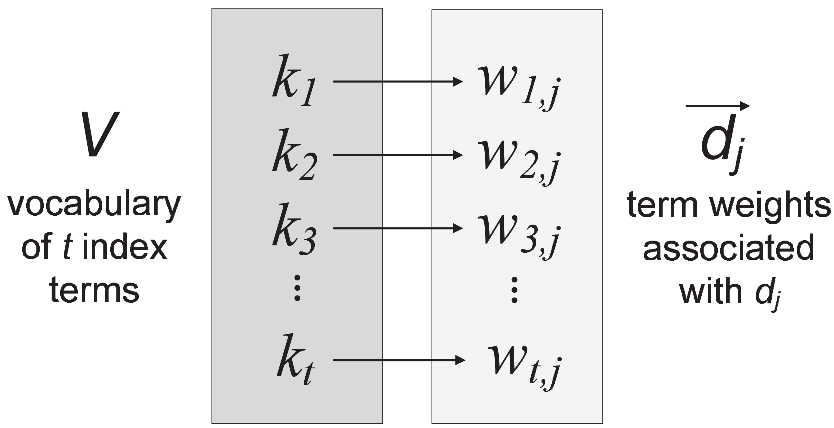
- The terms of a document are not equally useful for describing the document contents
- In fact, there are index terms which are simply vaguer than others
- There are properties of an index term which are useful for evaluating the importance of the term in a document
  - For instance, a word which appears in all documents of a collection is completely useless for retrieval tasks

Term Weighting

- To characterize term importance, we associate a weight $w_{i,j} > 0$ with each term k_i that occurs in the document d_j
  - If k_i that does not appear in the document d_j, then $w_{i,j} = 0$.
- The weight $w_{i,j}$ quantifies the importance of the index term k_i for describing the contents of document d_j
- These weights are useful to compute a rank for each document in the collection with regard to a given query

Term Weighting

- Let,
  - k_i be an index term and d_j be a document
  - $V = \{k_1, k_2, ..., k_t\}$ be the set of all index terms
  - $w_{i,j} \geqslant 0$ be the weight associated with (k_i, d_j)
- Then we define $\vec{d_j} = (w_{1,j}, w_{2,j}, ..., w_{t,j})$ as a weighted vector that contains the weight $w_{i,j}$ of each term $k_i \in V$ in the document d_j



Term Weighting

- The weights $w_{i,j}$ can be computed using the **frequencies of occurrence** of the terms within documents
- Let $f_{i,j}$ be the frequency of occurrence of index term k_i in the document d_j
- The **total frequency of occurrence** F_i of term k_i in the collection is defined as

$$F_i = \sum_{j=1}^{N} f_{i,j}$$

  where N is the number of documents in the collection

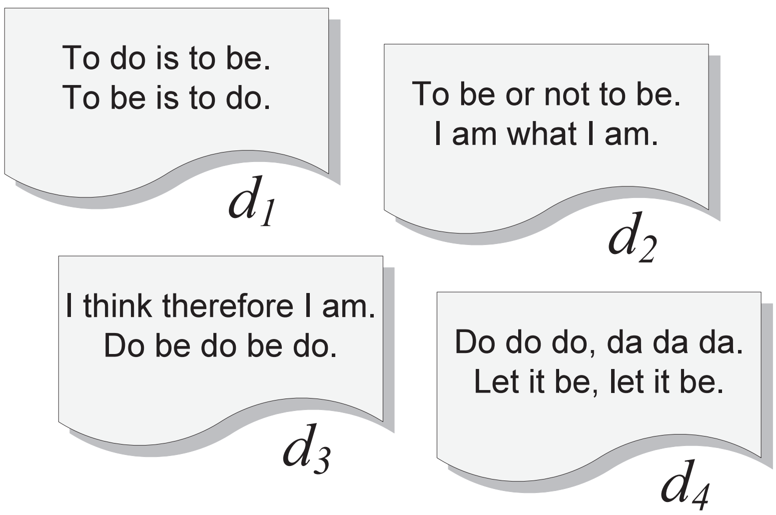
Term Weighting

- The **document frequency** n_i of a term k_i is the number of documents in which it occurs
  - Notice that $n_i \leq F_i$.
- For instance, in the document collection below, the values $f_{i,j}$, F_i and n_i associated with the term *do* are

$$f(do, d_1) = 2$$
$$f(do, d_2) = 0$$
$$f(do, d_3) = 3$$
$$f(do, d_4) = 3$$

$$F(do) = 8$$

$$n(do) = 3$$



Term-term correlation matrix

- For classic information retrieval models, the index term weights are assumed to be **mutually independent**
  - This means that $w_{i,j}$ tells us nothing about $w_{i+1,j}$
- This is clearly a simplification because occurrences of index terms in a document are not uncorrelated
- For instance, the terms **computer** and **network** tend to appear together in a document about **computer networks**
  - In this document, the appearance of one of these terms attracts the appearance of the other
  - Thus, they are correlated and their weights should reflect this correlation.

Term-term correlation matrix

- To take into account term-term correlations, we can compute a correlation matrix
- Let $\vec{M} = (m_{ij})$ be a term-document matrix $t \times N$ where $m_{ij} = w_{i,j}$
- The matrix $\vec{C} = \vec{M}\vec{M}^t$ is a term-term correlation matrix
- Each element $c_{u,v} \in \mathbf{C}$ expresses a correlation between terms k_u and k_v, given by

$$c_{u,v} = \sum_{d_j} w_{u,j} \times w_{v,j}$$

- Higher the number of documents in which the terms k_u and k_v co-occur, stronger is this correlation

Term-term correlation matrix

- Term-term correlation matrix for a sample collection

$$
\underbrace{
\begin{array}{cc}
 & \begin{array}{cc} d_1 & d_2 \end{array} \\
\begin{array}{c} k_1 \\ k_2 \\ k_3 \end{array} &
\left[\begin{array}{cc}
w_{1,1} & w_{1,2} \\
w_{2,1} & w_{2,2} \\
w_{3,1} & w_{3,2}
\end{array}\right]
\end{array}
\qquad
\begin{array}{cc}
 & \begin{array}{ccc} k_1 & k_2 & k_3 \end{array} \\
\begin{array}{c} d_1 \\ d_2 \end{array} &
\left[\begin{array}{ccc}
w_{1,1} & w_{2,1} & w_{3,1} \\
w_{1,2} & w_{2,2} & w_{3,2}
\end{array}\right]
\end{array}
\\
\mathbf{M} \qquad\qquad \times \qquad\qquad \mathbf{M}^T
}
$$

$$\Downarrow$$

$$
\begin{array}{cc}
 & \begin{array}{ccc} k_1 & k_2 & k_3 \end{array} \\
\begin{array}{c} k_1 \\ k_2 \\ k_3 \end{array} &
\left[\begin{array}{ccc}
w_{1,1}w_{1,1} + w_{1,2}w_{1,2} & w_{1,1}w_{2,1} + w_{1,2}w_{2,2} & w_{1,1}w_{3,1} + w_{1,2}w_{3,2} \\
w_{2,1}w_{1,1} + w_{2,2}w_{1,2} & w_{2,1}w_{2,1} + w_{2,2}w_{2,2} & w_{2,1}w_{3,1} + w_{2,2}w_{3,2} \\
w_{3,1}w_{1,1} + w_{3,2}w_{1,2} & w_{3,1}w_{2,1} + w_{3,2}w_{2,2} & w_{3,1}w_{3,1} + w_{3,2}w_{3,2}
\end{array}\right]
\end{array}
$$

TF-IDF Weights

TF-IDF Weights

- TF-IDF term weighting scheme:
  - Term frequency (TF)
  - Inverse document frequency (IDF)
  - Foundations of the most popular term weighting scheme in IR

Term-term correlation matrix

- **Luhn Assumption**. The value of $w_{i,j}$ is proportional to the term frequency $f_{i,j}$
  - That is, the more often a term occurs in the text of the document, the higher its weight
- This is based on the observation that high frequency terms are important for describing documents
- Which leads directly to the following tf weight formulation:

$$tf_{i,j} = f_{i,j}$$

Term Frequency (TF) Weights

- A variant of tf weight used in the literature is

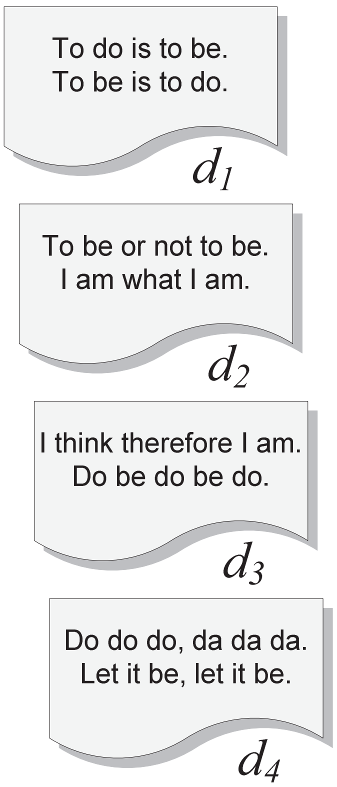
$$tf_{i,j} = \begin{cases} 1 + \log f_{i,j} & \texttt{if} \quad f_{i,j} > 0 \\ 0 & \texttt{otherwise} \end{cases}$$

  where the log is taken in base 2

- The log expression is a the preferred form because it makes them directly comparable to idf weights, as we later discuss

Term Frequency (TF) Weights

- Log tf weights $tf_{i,j}$ for the example collection



Vocabulary		$tf_{i,1}$	$tf_{i,2}$	$tf_{i,3}$	$tf_{i,4}$
1	to	3	2	-	-
2	do	2	-	2.585	2.585
3	is	2	-	-	-
4	be	2	2	2	2
5	or	-	1	-	-
6	not	-	1	-	-
7	I	-	2	2	-
8	am	-	2	1	-
9	what	-	1	-	-
10	think	-	-	1	-
11	therefore	-	-	1	-
12	da	-	-	-	2.585
13	let	-	-	-	2
14	it	-	-	-	2

Inverse Document Frequency

- We call **document exhaustivity** the number of index terms assigned to a document
- The more index terms are assigned to a document, the higher is the probability of retrieval for that document
  - If too many terms are assigned to a document, it will be retrieved by queries for which it is not relevant
- **Optimal exhaustivity**. We can circumvent this problem by optimizing the number of terms per document
- Another approach is by weighting the terms differently, by exploring the notion of **term specificity**

Inverse Document Frequency

- **Specificity** is a property of the term semantics
  - A term is more or less specific depending on its meaning
  - To exemplify, the term *beverage* is less specific than the terms *tea* and *beer*
  - We could expect that the term *beverage* occurs in more documents than the terms *tea* and *beer*
- Term specificity should be interpreted as a statistical rather than semantic property of the term
- **Statistical term specificity**. The inverse of the number of documents in which the term occurs

Inverse Document Frequency

- Terms are distributed in a text according to Zipf's Law
- Thus, if we sort the vocabulary terms in decreasing order of document frequencies we have

$$n(r) \sim r^{-\alpha}$$

  where $n(r)$ refer to the rth largest document frequency and α is an empirical constant
- That is, the document frequency of term k_i is an exponential function of its rank.

$$n(r) = Cr^{-\alpha}$$

  where C is a second empirical constant

Inverse Document Frequency

- Setting $\alpha = 1$ (simple approximation for english collections) and taking logs we have

$$\log n(r) = \log C - \log r$$

- For $r = 1$, we have $C = n(1)$, i.e., the value of C is the largest document frequency
  - This value works as a normalization constant
- An alternative is to do the normalization assuming $C = N$, where N is the number of docs in the collection

$$\log r \sim \log N - \log n(r)$$

Inverse Document Frequency

- Let k_i be the term with the rth largest document frequency, i.e., $n(r) = n_i$. Then,

  $$idf_i = \log \frac{N}{n_i}$$

  where idf_i is called the **inverse document frequency** of term k_i

- Idf provides a foundation for modern term weighting schemes and is used for ranking in almost all IR systems

Inverse Document Frequency

- Idf values for example collection

To do is to be.
To be is to do.

d_1

To be or not to be.
I am what I am.

d_2

I think therefore I am.
Do be do be do.

d_3

Do do do, da da da.
Let it be, let it be.

d_4

	term	n_i	$idf_i = \log(N/n_i)$
1	to	2	1
2	do	3	0.415
3	is	1	2
4	be	4	0
5	or	1	2
6	not	1	2
7	I	2	1
8	am	2	1
9	what	1	2
10	think	1	2
11	therefore	1	2
12	da	1	2
13	let	1	2
14	it	1	2

TF-IDF weighting scheme

- The best known term weighting schemes use weights that combine idf factors with term frequencies
- Let $w_{i,j}$ be the term weight associated with the term k_i and the document d_j
- Then, we define

$$w_{i,j} = \begin{cases} (1 + \log f_{i,j}) \times \log \frac{N}{n_i} & \texttt{if} \;\; f_{i,j} > 0 \\ 0 & \texttt{otherwise} \end{cases}$$

which is referred to as a **tf-idf weighting scheme**

TF-IDF weighting scheme

- Tf-idf weights of all terms present in our example document collection

To do is to be.
To be is to do.

d_1

To be or not to be.
I am what I am.

d_2

I think therefore I am.
Do be do be do.

d_3

Do do do, da da da.
Let it be, let it be.

d_4

		d_1	d_2	d_3	d_4
1	to	3	2	-	-
2	do	0.830	-	1.073	1.073
3	is	4	-	-	-
4	be	-	-	-	-
5	or	-	2	-	-
6	not	-	2	-	-
7	I	-	2	2	-
8	am	-	2	1	-
9	what	-	2	-	-
10	think	-	-	2	-
11	therefore	-	-	2	-
12	da	-	-	-	5.170
13	let	-	-	-	4
14	it	-	-	-	4

Variants of TF-IDF

- Several variations of the above expression for tf-idf weights are described in the literature
- For tf weights, five distinct variants are illustrated below

	tf weight
binary	{0,1}
raw frequency	$f_{i,j}$
log normalization	$1 + \log f_{i,j}$
double normalization 0.5	$0.5 + 0.5 \frac{f_{i,j}}{max_i f_{i,j}}$
double normalization K	$K + (1 - K) \frac{f_{i,j}}{max_i f_{i,j}}$

Variants of TF-IDF

- Five distinct variants of idf weight

	idf weight
unary	1
inverse frequency	$\log \frac{N}{n_i}$
inv frequency smooth	$\log(1 + \frac{N}{n_i})$
inv frequeny max	$\log(1 + \frac{max_i n_i}{n_i})$
probabilistic inv frequency	$\log \frac{N - n_i}{n_i}$

Variants of TF-IDF

- Recommended tf-idf weighting schemes

weighting scheme	document term weight	query term weight
1	$f_{i,j} * \log \frac{N}{n_i}$	$(0.5 + 0.5 \frac{f_{i,q}}{max_i \ f_{i,q}}) * \log \frac{N}{n_i}$
2	$1 + \log f_{i,j}$	$\log(1 + \frac{N}{n_i})$
3	$(1 + \log f_{i,j}) * \log \frac{N}{n_i}$	$(1 + \log f_{i,q}) * \log \frac{N}{n_i}$

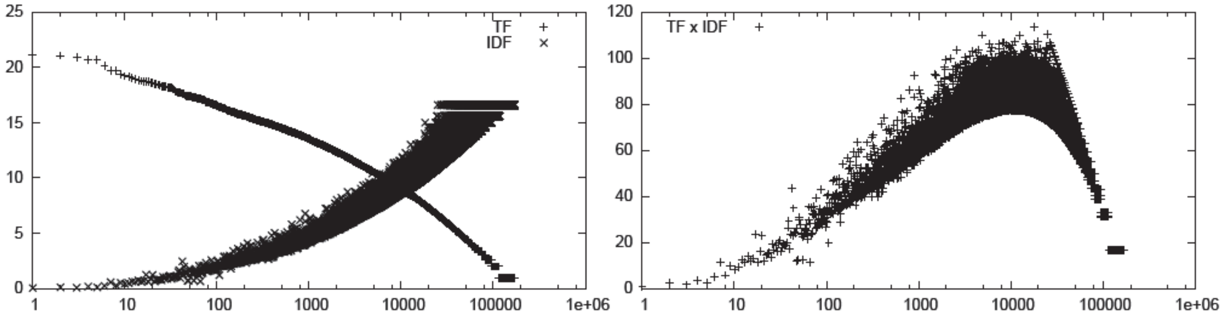
TF-IDF Properties

- Consider the tf, idf, and tf-idf weights for the *Wall Street Journal* reference collection
- To study their behavior, we would like to plot them together
- While idf is computed over all the collection, tf is computed on a per document basis. Thus, we need a representation of tf based on all the collection, which is provided by the term collection frequency F_i
- This reasoning leads to the following tf and idf term weights:

$$tf_i = 1 + \log \sum_{j=1}^{N} f_{i,j} \qquad\qquad idf_i = \log \frac{N}{n_i}$$

TF-IDF Properties

- Plotting tf and idf in logarithmic scale yields



(a) (b)

- We observe that tf and idf weights present power-law behaviors that balance each other
- The terms of intermediate idf values display maximum tf-idf weights and are most interesting for ranking

Document Length Normalization

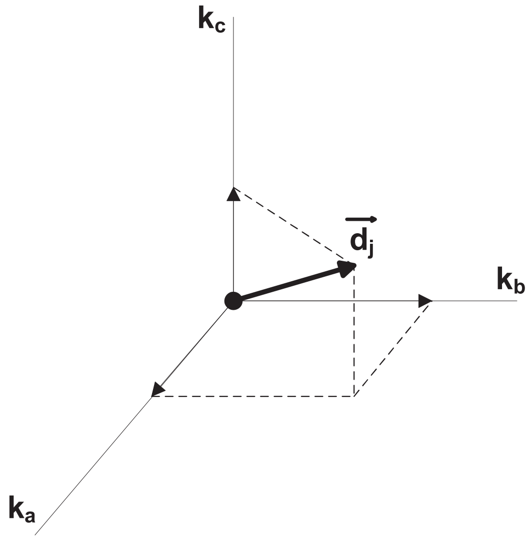
- Document sizes might vary widely
- This is a problem because longer documents are more likely to be retrieved by a given query
- To compensate for this undesired effect, we can divide the rank of each document by its length
- This procedure consistently leads to better ranking, and it is called **document length normalization**

Document Length Normalization

- Methods of document length normalization depend on the representation adopted for the documents:
  - **Size in bytes**: consider that each document is represented simply as a stream of bytes
  - **Number of words**: each document is represented as a single string, and the document length is the number of words in it
  - **Vector norms**: documents are represented as vectors of weighted terms

Document Length Normalization

- Documents represented as vectors of weighted terms
  - Each term of a collection is associated with an orthonormal unit vector $\vec{k}_i$ in a t-dimensional space
  - For each term k_i of a document d_j is associated the term vector component $w_{i,j} \times \vec{k}_i$



Document Length Normalization

- The document representation $\vec{d_j}$ is a vector composed of all its term vector components

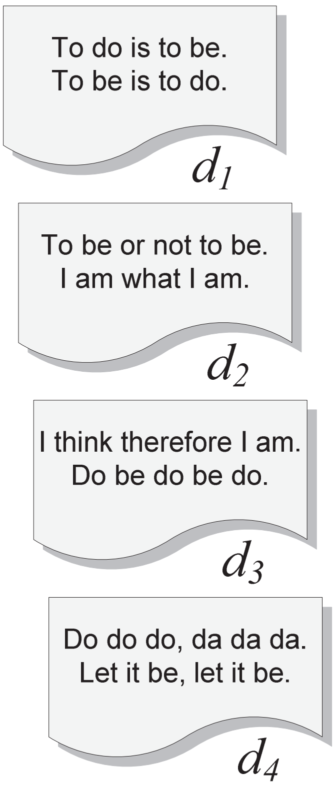
$$\vec{d_j} = (w_{1,j}, w_{2,j}, ..., w_{t,j})$$

- The document length is given by the norm of this vector, which is computed as follows

$$|\vec{d_j}| = \sqrt{\sum_i^t w_{i,j}^2}$$

Document Length Normalization

- Three variants of document lengths for the example collection



	d_1	d_2	d_3	d_4
size in bytes	34	37	41	43
number of words	10	11	10	12
vector norm	5.068	4.899	3.762	7.738

The Vector Model

The Vector Model

- Boolean matching and binary weights is too limiting
- The vector model proposes a framework in which partial matching is possible
- This is accomplished by assigning non-binary weights to index terms in queries and in documents
- Term weights are used to compute a **degree of similarity** between a query and each document
- The documents are **ranked** in decreasing order of their degree of similarity

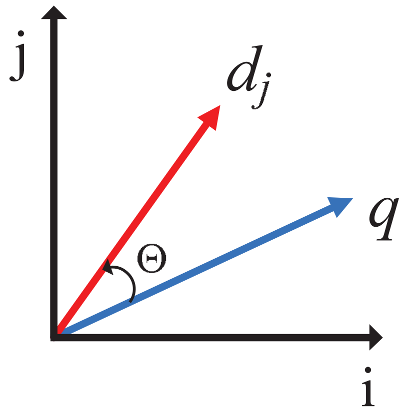
The Vector Model

- For the vector model:
  - The weight $w_{i,j}$ associated with a pair (k_i, d_j) is positive and non-binary
  - The index terms are assumed to be all mutually independent
  - They are represented as unit vectors of a t-dimensionsal space (t is the total number of index terms)
  - The representations of document d_j and query q are t-dimensional vectors given by

$$\vec{d_j} = (w_{1j}, w_{2j}, \ldots, w_{tj})$$
$$\vec{q} = (w_{1q}, w_{2q}, \ldots, w_{tq})$$

The Vector Model

- Similarity between a document d_j and a query q



$$cos(\theta) = \frac{\vec{d_j} \bullet \vec{q}}{|\vec{d_j}| \times |\vec{q}|}$$

$$\boxed{sim(d_j, q) = \frac{\sum_{i=1}^{t} w_{i,j} \times w_{i,q}}{\sqrt{\sum_{i=1}^{t} w_{i,j}^2} \times \sqrt{\sum_{j=1}^{t} w_{i,q}^2}}}$$

Since $w_{ij} > 0$ and $w_{iq} > 0$, we have $0 \leqslant sim(d_j, q) \leqslant 1$

The Vector Model

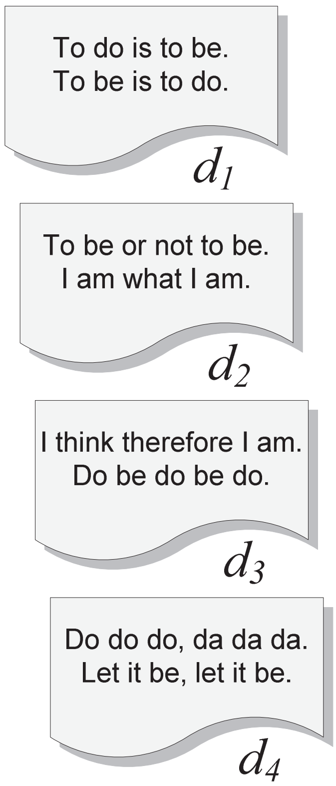
- Weights in the Vector model are basically tf-idf weights

$$
\begin{aligned}
w_{i,q} &= (1 + \log f_{i,q}) \times \log \frac{N}{n_i} \\
w_{i,j} &= (1 + \log f_{i,j}) \times \log \frac{N}{n_i}
\end{aligned}
$$

- These equations should only be applied for values of term frequency greater than zero
- If the term frequency is zero, the respective weight is also zero

The Vector Model

- Document ranks computed by the Vector model for the query "to do" (see tf-idf weight values in Slide 43)



doc	rank computation	rank
d_1	$\frac{1*3+0.415*0.830}{5.068}$	0.660
d_2	$\frac{1*2+0.415*0}{4.899}$	0.408
d_3	$\frac{1*0+0.415*1.073}{3.762}$	0.118
d_4	$\frac{1*0+0.415*1.073}{7.738}$	0.058

The Vector Model

- Advantages:
  - term-weighting improves quality of the answer set
  - partial matching allows retrieval of docs that approximate the query conditions
  - cosine ranking formula sorts documents according to a degree of similarity to the query
  - document length normalization is naturally built-in into the ranking
- Disadvantages:
  - It assumes independence of index terms

Probabilistic Model

Probabilistic Model

- The probabilistic model captures the IR problem using a probabilistic framework
- Given a user query, there is an **ideal answer set** for this query
- Given a description of this ideal answer set, we could retrieve the relevant documents
- Querying is seen as a specification of the **properties** of this ideal answer set
  - But, what are these properties?

Probabilistic Model

- An initial set of documents is retrieved somehow
- The user inspects these docs looking for the relevant ones (in truth, only top 10-20 need to be inspected)
- The IR system uses this information to refine the description of the ideal answer set
- By repeating this process, it is expected that the description of the ideal answer set will improve

Probabilistic Ranking Principle

- The probabilistic model
  - Tries to estimate the probability that a document will be relevant to a user query
  - Assumes that this probability depends on the query and document representations only
  - The ideal answer set, referred to as R, should maximize the probability of relevance
- But,
  - How to compute these probabilities?
  - What is the sample space?

The Ranking

- Let,
  - R be the set of relevant documents to query q
  - $\overline{R}$ be the set of non-relevant documents to query q
  - $P(R|\vec{d}_j)$ be the probability that d_j is relevant to the query q
  - $P(\overline{R}|\vec{d}_j)$ be the probability that d_j is non-relevant to q
- The similarity $sim(d_j, q)$ can be defined as

$$sim(d_j, q) = \frac{P(R|\vec{d}_j)}{P(\overline{R}|\vec{d}_j)}$$

The Ranking

- Using Bayes' rule,

$$sim(d_j, q) = \frac{P(\vec{d_j}|R,q) \times P(R,q)}{P(\vec{d_j}|\overline{R},q) \times P(\overline{R},q)} \sim \frac{P(\vec{d_j}|R,q)}{P(\vec{d_j}|\overline{R},q)}$$

  where

  - $P(\vec{d_j}|R,q)$: probability of randomly selecting the document d_j from the set R
  - $P(R,q)$: probability that a document randomly selected from the entire collection is relevant to query q
  - $P(\vec{d_j}|\overline{R},q)$ and $P(\overline{R},q)$: analogous and complementary

The Ranking

- Assuming that the weights $w_{i,j}$ are all binary and assuming independence among the index terms:

$$sim(d_j, q) \sim \frac{(\prod_{k_i|w_{i,j}=1} P(k_i|R,q)) \times (\prod_{k_i|w_{i,j}=0} P(\overline{k}_i|R,q))}{(\prod_{k_i|w_{i,j}=1} P(k_i|\overline{R},q)) \times (\prod_{k_i|w_{i,j}=0} P(\overline{k}_i|\overline{R},q))}$$

  where

  - $P(k_i|R,q)$: probability that the term k_i is present in a document randomly selected from the set R
  - $P(\overline{k}_i|R,q)$: probability that k_i is not present in a document randomly selected from the set R
  - probabilities with $\overline{R}$: analogous to the ones just described

The Ranking

- To simplify our notation, let us adopt the following conventions
  - $p_{iR} = P(k_i|R, q)$
  - $q_{iR} = P(k_i|\overline{R}, q)$
- Since
  - $P(k_i|R, q) + P(\overline{k}_i|R, q) = 1$
  - $P(k_i|\overline{R}, q) + P(\overline{k}_i|\overline{R}, q) = 1$

  we can write:

$$sim(d_j, q) \sim \frac{(\prod_{k_i|w_{i,j}=1} p_{iR}) \times (\prod_{k_i|w_{i,j}=0}(1 - p_{iR}))}{(\prod_{k_i|w_{i,j}=1} q_{iR}) \times (\prod_{k_i|w_{i,j}=0}(1 - q_{iR}))}$$

The Ranking

- Taking logarithms, we write

$$
\begin{aligned}
sim(d_j, q) \quad \sim \quad & \log \prod_{k_i | w_{i,j}=1} p_{iR} + \log \prod_{k_i | w_{i,j}=0} (1 - p_{iR}) \\
& - \log \prod_{k_i | w_{i,j}=1} q_{iR} - \log \prod_{k_i | w_{i,j}=0} (1 - q_{iR})
\end{aligned}
$$

The Ranking

- Summing up terms that cancel each other, we obtain

$$
\begin{aligned}
sim(d_j, q) \quad \sim \quad & \log \prod_{k_i | w_{i,j}=1} p_{iR} + \log \prod_{k_i | w_{i,j}=0} (1 - p_{ir}) \\
& - \log \prod_{k_i | w_{i,j}=1} (1 - p_{ir}) + \log \prod_{k_i | w_{i,j}=1} (1 - p_{ir}) \\
& - \log \prod_{k_i | w_{i,j}=1} q_{iR} - \log \prod_{k_i | w_{i,j}=0} (1 - q_{iR}) \\
& + \log \prod_{k_i | w_{i,j}=1} (1 - q_{iR}) - \log \prod_{k_i | w_{i,j}=1} (1 - q_{iR})
\end{aligned}
$$

The Ranking

- Using logarithm operations, we obtain

$$\begin{aligned} sim(d_j, q) \sim\ & \log \prod_{k_i | w_{i,j}=1} \frac{p_{iR}}{(1-p_{iR})} + \log \prod_{k_i} (1-p_{iR}) \\ & + \log \prod_{k_i | w_{i,j}=1} \frac{(1-q_{iR})}{q_{iR}} - \log \prod_{k_i} (1-q_{iR}) \end{aligned}$$

- Notice that two of the factors in the formula above are a function of all index terms and do not depend on document d_j. They are constants for a given query and can be disregarded for the purpose of ranking

The Ranking

- Further, assuming that
  - $\forall\ k_i \notin q,\ \ p_{iR} = q_{iR}$

  and converting the log products into sums of logs, we finally obtain

$$\boxed{sim(d_j, q) \ \sim\ \sum_{k_i \in q \wedge k_i \in d_j} \log \left(\frac{p_{iR}}{1 - p_{iR}}\right) + \log \left(\frac{1 - q_{iR}}{q_{iR}}\right)}$$

  which is a key expression for ranking computation in the probabilistic model

Term Incidence Contingency Table

- Let,
  - N be the number of documents in the collection
  - n_i be the number of documents that contain term k_i
  - R be the total number of relevant documents to query q
  - r_i be the number of relevant documents that contain term k_i
- Based on these variables, we can build the following contingency table

	relevant	non-relevant	all docs
docs that contain k_i	r_i	$n_i - r_i$	n_i
docs that do not contain k_i	$R - r_i$	$N - n_i - (R - r_i)$	$N - n_i$
all docs	R	$N - R$	N

Ranking Formula

- If information on the contingency table were available for a given query, we could write
  - $p_{iR} = \frac{r_i}{R}$
  - $q_{iR} = \frac{n_i - r_i}{N - R}$
- Then, the equation for ranking computation in the probabilistic model could be rewritten as

$$sim(d_j, q) \quad \sim \quad \sum_{k_i[q,d_j]} \log \left(\frac{r_i}{R - r_i} \times \frac{N - n_i - R + r_i}{n_i - r_i} \right)$$

where $k_i[q, d_j]$ is a short notation for $k_i \in q \wedge k_i \in d_j$

Ranking Formula

- In the previous formula, we are still dependent on an estimation of the relevant dos for the query
- For handling small values of r_i, we add 0.5 to each of the terms in the formula above, which changes $sim(d_j, q)$ into

$$\sum_{k_i[q,d_j]} \log \left(\frac{r_i + 0.5}{R - r_i + 0.5} \times \frac{N - n_i - R + r_i + 0.5}{n_i - r_i + 0.5} \right)$$

- This formula is considered as the classic ranking equation for the probabilistic model and is known as the Robertson-Sparck Jones Equation

Ranking Formula

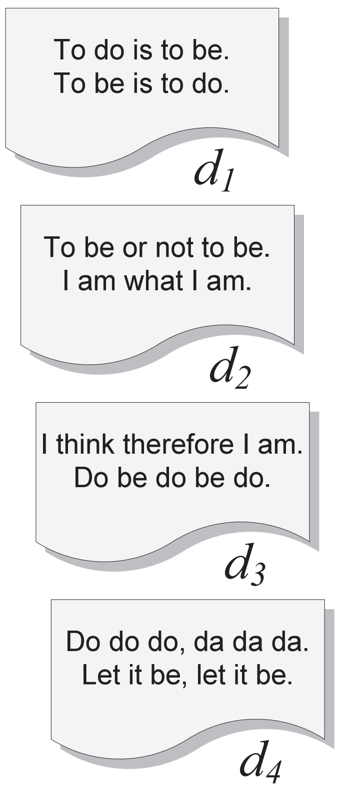
- The previous equation cannot be computed without estimates of r_i and R
- One possibility is to assume $R = r_i = 0$, as a way to boostrap the ranking equation, which leads to

$$\boxed{sim(d_j, q) \sim \sum_{k_i[q,d_j]} \log\left(\frac{N-n_i+0.5}{n_i+0.5}\right)}$$

- This equation provides an idf-like ranking computation
- In the absence of relevance information, this is the equation for ranking in the probabilistic model

Ranking Example

- Document ranks computed by the previous probabilistic ranking equation for the query "to do"



doc	rank computation	rank
d_1	$\log\frac{4-2+0.5}{2+0.5} + \log\frac{4-3+0.5}{3+0.5}$	- 1.222
d_2	$\log\frac{4-2+0.5}{2+0.5}$	0
d_3	$\log\frac{4-3+0.5}{3+0.5}$	- 1.222
d_4	$\log\frac{4-3+0.5}{3+0.5}$	- 1.222

Ranking Example

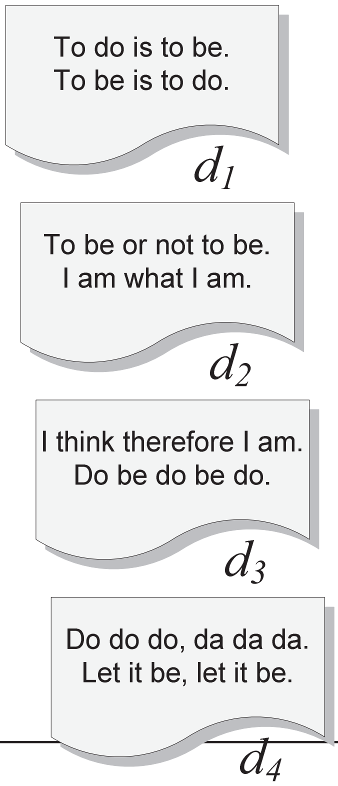
- The ranking computation led to negative weights because of the term “do”
- Actually, the probabilistic ranking equation produces negative terms whenever $n_i > N/2$
- One possible artifact to contain the effect of negative weights is to change the previous equation to:

$$sim(d_j, q) \sim \sum_{k_i[q,d_j]} \log \left(\frac{N + 0.5}{n_i + 0.5} \right)$$

- By doing so, a term that occurs in all documents ($n_i = N$) produces a weight equal to zero

Ranking Example

- Using this latest formulation, we redo the ranking computation for our example collection for the query “to do” and obtain



doc	rank computation	rank
d_1	$\log \frac{4+0.5}{2+0.5} + \log \frac{4+0.5}{3+0.5}$	1.210
d_2	$\log \frac{4+0.5}{2+0.5}$	0.847
d_3	$\log \frac{4+0.5}{3+0.5}$	0.362
d_4	$\log \frac{4+0.5}{3+0.5}$	0.362

Estimaging r_i and R

- Our examples above considered that $r_i = R = 0$
- An alternative is to estimate r_i and R performing an initial search:
  - select the top 10-20 ranked documents
  - inspect them to gather new estimates for r_i and R
  - remove the 10-20 documents used from the collection
  - rerun the query with the estimates obtained for r_i and R
- Unfortunately, procedures such as these require human intervention to initially select the relevant documents

Improving the Initial Ranking

- Consider the equation

$$sim(d_j, q) \sim \sum_{k_i \in q \wedge k_i \in d_j} \log \left(\frac{p_{iR}}{1 - p_{iR}} \right) + \log \left(\frac{1 - q_{iR}}{q_{iR}} \right)$$

- How obtain the probabilities p_{iR} and q_{iR} ?
- Estimates based on assumptions:
  - $p_{iR} = 0.5$
  - $q_{iR} = \frac{n_i}{N}$ where n_i is the number of docs that contain k_i
  - Use this initial guess to retrieve an initial ranking
  - Improve upon this initial ranking

Improving the Initial Ranking

- Substituting p_{iR} and q_{iR} into the previous Equation, we obtain:

$$sim(d_j, q) \quad \sim \quad \sum_{k_i \in q \wedge k_i \in d_j} \log \left(\frac{N - n_i}{n_i} \right)$$

- That is the equation used when no relevance information is provided, without the 0.5 correction factor
- Given this initial guess, we can provide an initial probabilistic ranking
- After that, we can attempt to improve this initial ranking as follows

Improving the Initial Ranking

- We can attempt to improve this initial ranking as follows
- Let
  - D : set of docs initially retrieved
  - D_i : subset of docs retrieved that contain k_i
- Reevaluate estimates:
  - $p_{iR} = \frac{D_i}{D}$
  - $q_{iR} = \frac{n_i - D_i}{N - D}$
- This process can then be repeated recursively

Improving the Initial Ranking

$$sim(d_j, q) \sim \sum_{k_i \in q \wedge k_i \in d_j} \log \left(\frac{N - n_i}{n_i} \right)$$

- To avoid problems with $D = 1$ and $D_i = 0$:

$$p_{iR} = \frac{D_i + 0.5}{D + 1}; \qquad q_{iR} = \frac{n_i - D_i + 0.5}{N - D + 1}$$

- Also,

$$p_{iR} = \frac{D_i + \frac{n_i}{N}}{D + 1}; \qquad q_{iR} = \frac{n_i - D_i + \frac{n_i}{N}}{N - D + 1}$$

Pluses and Minuses

- Advantages:
  - Docs ranked in decreasing order of probability of relevance
- Disadvantages:
  - need to guess initial estimates for p_{iR}
  - method does not take into account tf factors
  - the lack of document length normalization

Comparison of Classic Models

- Boolean model does not provide for partial matches and is considered to be the weakest classic model
- There is some controversy as to whether the probabilistic model outperforms the vector model
- Croft suggested that the probabilistic model provides a better retrieval performance
- However, Salton *et al* showed that the vector model outperforms it with general collections
- This also seems to be the dominant thought among researchers and practitioners of IR.