Tutorial 2

Coloring, Covering and Partitioning

August 2016

Answer any FIVE questions.

1. Prove that the chromatic number of a graph will not exceed by more than one the maximum degree of the vertices in a graph.
2. Show that the chromatic number of a graph can not exceed its diameter.
3. For a simple graph G, define $\alpha(G)$ (domination number) and $\beta(G)$ (independence number) and show that $\alpha(G) \leq \beta(G)$.
4. In the class we saw an example of uniquely colorable graph. Can you give another (different) example of uniquely colorable graph.
5. Prove the following: An n-vertex graph is a tree if and only if its chromatic polynomial is $P_n(\lambda) = \lambda(\lambda - 1)^{n-1}$.
6. Find the chromatic polynomial for the graph given in Figure 1.
7. Sketch two different (i.e., non-isomorphic) graphs that have the same chromatic polynomial.
8. Sketch a bipartite graph that does not satisfy the condition in Theorem 8-8 (in the text book) and yet has a complete matching.

   Theorem 8-8 asserts the following: In a bipartite graph a complete matching of V_1 into V_2 exists if (but not only if) there is a positive integer m for which the following condition is satisfied:

   $$\text{degree of every vertex in } V_1 \geq m \geq \text{degree of every vertex in } V_2.$$

9. Suppose that the graph in Figure 2 represents the street map of a part of a city. Each of the vertices is a potential trouble spot and must be kept under surveillance of a patrol car. Every patrol car covers exactly one street. What is the minimum number of patrol cars needed to cover every vertex ?

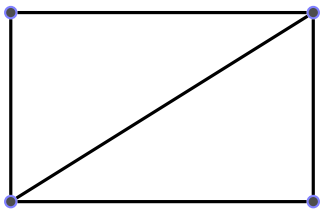

Figure 1: Graph for Question 6

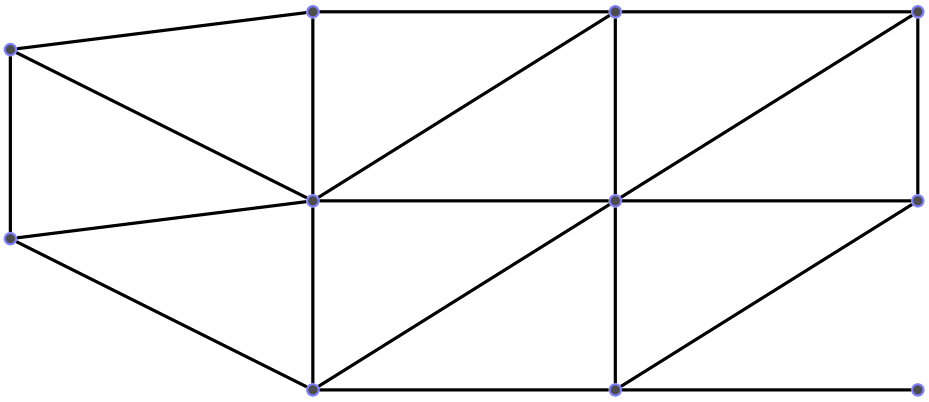

Figure 2: Graph for Question 9

10. Six reporters Asif (A), Becky (B), Chris (C), David (D), Emma (E), and Fred (F) are to be assigned to six news stories Business (1), Crime (2), Financial (3), Foreign (4), Local (5) and Sport (6). The table shows possible allocations of reporters to news stories. Find a maximal matching for this problem. Is there a complete matching ? If not, find the deficiency.

	1	2	3	4	5	6
A					X	
B	X			X		
C	X	X		X		
D					X	
E			X		X	X
F				X		