Canonical LR Parser

Introduction

- In SLR method, the state i makes a reduction by $A \rightarrow \alpha$ when the current token is a:
  - if the $A \rightarrow \alpha.$ in the I_i and a is FOLLOW(A)

- In some situations, βA cannot be followed by the terminal a in a right-sentential form when $\beta\alpha$ and the state i are on the top stack. This means that making reduction in this case is not correct.

$S \rightarrow AaAb$ $\qquad S \Rightarrow AaAb \Rightarrow Aab \Rightarrow ab$ $\qquad S \Rightarrow BbBa \Rightarrow Bba \Rightarrow ba$

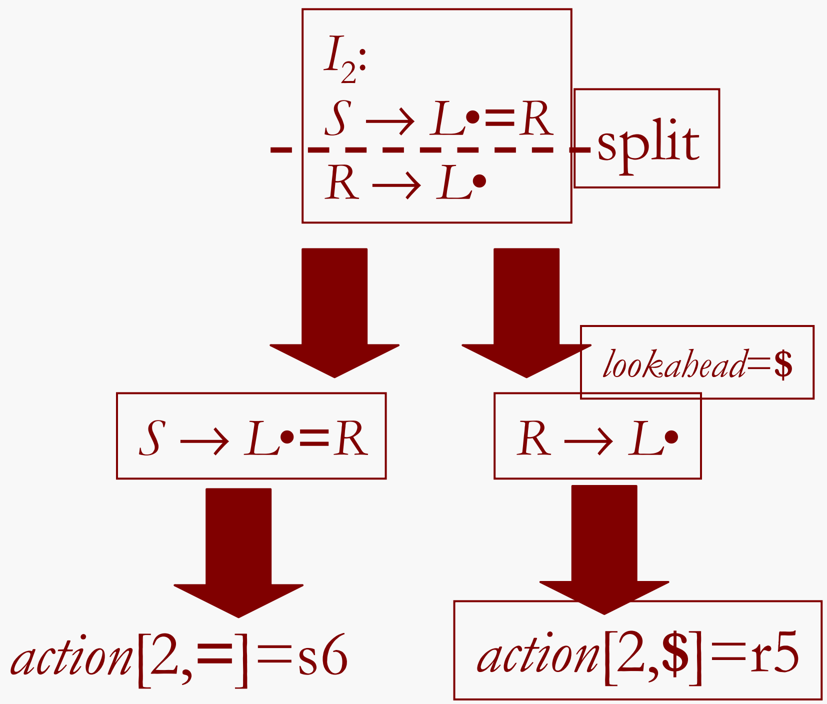
$S \rightarrow BbBa$

$A \rightarrow \varepsilon$ $\qquad Aab \Rightarrow \varepsilon\, ab$ $\qquad Bba \Rightarrow \varepsilon\, ba$

$B \rightarrow \varepsilon$ $\qquad AaAb \Rightarrow Aa\, \varepsilon\, b$ $\qquad BbBa \Rightarrow Bb\, \varepsilon\, a$

Introduction

- Split the SLR states by adding LR(1) lookahead
- Unambiguous grammar
  1. $S \to L = R$
  2. $S \to R$
  3. $L \to * R$
  4. $L \to \mathbf{id}$
  5. $R \to L$



Should not reduce on **=**, because no right-sentential form begins with R=

LR(1) Item

- To avoid some of invalid reductions, the states need to carry more information.
- Extra information is put into a state by including a terminal symbol as a second component in an item.

- A LR(1) item is:

  $A \rightarrow \alpha.\beta,a$ where **a** is the look-head of the LR(1) item

  (**a** is a terminal or end-marker.)

LR(1) Item (cont.)

- When β (in the LR(1) item $A \rightarrow \alpha.\beta,a$) is not empty, the look-head does not have any affect.
- When β is empty ($A \rightarrow \alpha.,a$), we do the reduction by $A \rightarrow \alpha$ only if the next input symbol is **a** (not for any terminal in FOLLOW(A)).
- A state will contain $A \rightarrow \alpha.,a_1$ where $\{a_1,...,a_n\} \subseteq$ FOLLOW(A)

$$\begin{array}{c} A \rightarrow \alpha.,a_1 \\ \ldots \\ A \rightarrow \alpha.,a_n \end{array}$$

Canonical Collection of Sets of LR(1) Items

- The construction of the canonical collection of the sets of LR(1) items are similar to the construction of the canonical collection of the sets of LR(0) items, except that *closure* and *goto* operations work a little bit different.

closure(I) is: (where I is a set of LR(1) items)

- every LR(1) item in I is in closure(I)
- if $A \rightarrow \alpha.B\beta,a$ in closure(I) and $B \rightarrow \gamma$ is a production rule of G; then $B \rightarrow .\gamma,b$ will be in the closure(I) for each terminal b in FIRST(βa) .

goto operation

- If I is a set of LR(1) items and X is a grammar symbol (terminal or non-terminal), then goto(I,X) is defined as follows:
  - If $A \rightarrow \alpha.X\beta,a$ in I
    then every item in **closure({$A \rightarrow \alpha X.\beta,a$})** will be in goto(I,X).

Construction of Canonical LR(1) Collection

- ***Algorithm***:

  C is { closure({S'→.S,\$}) }

  repeat the followings until no more set of LR(1) items can be added to ***C***.

  for each I in ***C*** and each grammar symbol X

  if goto(I,X) is not empty and not in ***C***

  add goto(I,X) to ***C***

- goto function is a DFA on the sets in C.

Short Notation for Sets of LR(1) Items

- A set of LR(1) items containing the following items

$$A \rightarrow \alpha.\beta,a_1$$

$$\ldots$$

$$A \rightarrow \alpha.\beta,a_n$$

can be written as

$$A \rightarrow \alpha.\beta,a_1/a_2/\ldots/a_n$$

Example LR(1) Items

- Unambiguous LR(1) grammar:

$$S \rightarrow L = R$$
$$S \rightarrow R$$
$$L \rightarrow * \, R$$
$$L \rightarrow \textbf{id}$$
$$R \rightarrow L$$

- Augment with $S' \rightarrow S$
- LR(1) items (next slide)

LR(1) Items

I_0: $[S' \rightarrow \bullet S, \$]$ goto$(I_0,S)=I_1$
$[S \rightarrow \bullet L{=}R, \$]$ goto$(I_0,L)=I_2$
$[S \rightarrow \bullet R, \$]$ goto$(I_0,R)=I_3$
$[L \rightarrow \bullet {*}R, {=}/\$]$ goto$(I_0,*)=I_4$
$[L \rightarrow \bullet \mathbf{id}, {=}/\$]$ goto$(I_0,\mathbf{id})=I_5$
$[R \rightarrow \bullet L, \$]$ goto$(I_0,L)=I_2$

I_1: $[S' \rightarrow S\bullet, \$]$

I_2: $[S \rightarrow L\bullet{=}R, \$]$ goto$(I_0,=)=I_6$
$[R \rightarrow L\bullet, \$]$

I_3: $[S \rightarrow R\bullet, \$]$

I_4: $[L \rightarrow {*}\bullet R, {=}/\$]$ goto$(I4,R)=I7$
$[R \rightarrow \bullet L, {=}/\$]$ goto$(I4,L)=I8$
$[L \rightarrow \bullet {*}R, {=}/\$]$ goto$(I4,*)=I4$
$[L \rightarrow \bullet \mathbf{id}, {=}/\$]$ goto$(I4,\mathbf{id})=I5$

I_5: $[L \rightarrow \mathbf{id}\bullet, {=}/\$]$

Canonical LR(1) Collection

$S' \rightarrow S$
1) $S \rightarrow L=R$
2) $S \rightarrow R$
3) $L \rightarrow {}^*R$
4) $L \rightarrow id$
5) $R \rightarrow L$

I_0: $S' \rightarrow .S,\$$
$S \rightarrow .L=R,\$$
$S \rightarrow .R,\$$
$L \rightarrow .{}^*R,\$/=$
$L \rightarrow .id,\$/=$
$R \rightarrow .L,\$$

$I_0 \xrightarrow{S} I_1$, $I_0 \xrightarrow{*} I_4$, $I_0 \xrightarrow{L} I_2$, $I_0 \xrightarrow{R} I_3$, $I_0 \xrightarrow{id} I_5$

I_1: $S' \rightarrow S.,\$$

I_2: $S \rightarrow L.=R,\$$ $\rightarrow$ to I_6
$R \rightarrow L.,\$$

I_3: $S \rightarrow R.,\$$

I_4: $L \rightarrow {}^*.R,\$/=$
$R \rightarrow .L,\$/=$
$L \rightarrow .{}^*R,\$/=$
$L \rightarrow .id,\$/=$

$\xrightarrow{R}$ to I_7
$\xrightarrow{L}$ to I_8
$\xrightarrow{*}$ to I_4
$\xrightarrow{id}$ to I_5

I_5: $L \rightarrow id.,\$/=$

I_6: $S \rightarrow L=.R,\$$
$R \rightarrow .L,\$$
$L \rightarrow .{}^*R,\$$
$L \rightarrow .id,\$$

$\xrightarrow{R}$ to I_9
$\xrightarrow{L}$ to I_{10}
$\xrightarrow{*}$ to I_{11}
$\xrightarrow{id}$ to I_{12}

I_7: $L \rightarrow {}^*R.,\$/=$

I_8: $R \rightarrow L.,\$/=$

I_9: $S \rightarrow L=R.,\$$

I_{10}: $R \rightarrow L.,\$$

I_{11}: $L \rightarrow {}^*.R,\$$
$R \rightarrow .L,\$$
$L \rightarrow .{}^*R,\$$
$L \rightarrow .id,\$$

$\xrightarrow{R}$ to I_{13}
$\xrightarrow{L}$ to I_{10}
$\xrightarrow{*}$ to I_{11}
$\xrightarrow{id}$ to I_{12}

I_{12}: $L \rightarrow id.,\$$

I_{13}: $L \rightarrow {}^*R.,\$$

I_4 and I_{11}

I_5 and I_{12}

I_7 and I_{13}

I_8 and I_{10}

Canonical LR Parsing Tables

1. Augment the grammar with $S' \to S$
2. Construct the set $C=\{I_0,I_1,\ldots,I_n\}$ of LR(1) items
3. If $[A \to \alpha \bullet a\beta,\ b] \in I_i$ and $goto(I_i,a)=I_j$ then set $action[i,a]=$shift j
4. If $[A \to \alpha \bullet,\ a] \in I_i$ then set $action[i,a]=$reduce $A \to \alpha$ (apply only if $A \neq S'$)
5. If $[S' \to S \bullet,\ \$]$ is in I_i then set $action[i,\$]=$accept
6. If $goto(I_i,A)=I_j$ then set $goto[i,A]=j$
7. Repeat 3-6 until no more entries added
8. The initial state i is the I_i holding item $[S' \to \bullet S,\$]$

Parsing Table

Grammar:

1. $S' \rightarrow S$
2. $S \rightarrow L = R$
3. $S \rightarrow R$
4. $L \rightarrow * R$
5. $L \rightarrow$ **id**
6. $R \rightarrow L$

	id	*****	**=**	**$**	*S*	*L*	*R*
0	s5	s4			1	2	3
1				acc			
2			s6	r6			
3				r3			
4	s5	s4				8	7
5			r5	r5			
6	s12	s11				10	9
7			r4	r4			
8			r6	r6			
9				r2			
10				r6			
11	s12	s11				10	13
12				r5			
13				r4			