TWO n-BODY SOLVERS

- In an n-body problem, we need to find the positions and velocities of a collection of interacting particles over a period of time.
- Ex: An astrophysicist might want to know the positions and velocities of a collection of stars, while a chemist might want to know the positions and velocities of a collection of molecules or atoms.
- An n-body solver is a program that finds the solution to an n-body problem by simulating the behavior of the particles.
- The input to the problem is the mass, position, and velocity of each particle at the start of the simulation, and the output is typically the position and velocity of each particle at a sequence of user-specified times, or simply the position and velocity of each particle at the end of a user-specified time period.

1. The problem.

- Let's write an n-body solver that simulates the motions of planets or stars. We'll use Newton's second law of motion and his law of universal gravitation to determine the positions and velocities. Thus, if particle ***q*** has position $\mathbf{s}_q(t)$ at time ***t,*** and particle ***k*** has position $\mathbf{s}_k(t)$, then the force on particle ***q*** exerted by particle ***k*** is given by:

$$\mathbf{f}_{qk}(t) = -\frac{Gm_q m_k}{\left|\mathbf{s}_q(t) - \mathbf{s}_k(t)\right|^3}\left[\mathbf{s}_q(t) - \mathbf{s}_k(t)\right]. \quad \rightarrow 1$$

- Here, G is the gravitational constant ($6.673 \times 10^{-11}\mathrm{m}^3/(\mathrm{kg}\cdot\mathrm{s}^2)$), and m_q and m_k are the masses of particles ***q*** and ***k***, respectively. Also, the notation $|\mathbf{s}_q(t) - \mathbf{s}_k(t)|$ the distance from particle ***k*** to particle ***q***. Note that in general the positions, the velocities, the accelerations, and the forces are vectors, so we're using boldface to represent these variables. We'll use an italic font to represent the other, scalar, variables, such as the time t and the gravitational constant G.

- We can use formula 1 to find the total force on any particle by adding the forces due to all the particles. If our n particles are numbered 0, 1, 2, … ,n-1, then the total force on particle q is given by

$$\mathbf{F}_q(t) = \sum_{\substack{k=0\\k\neq q}}^{n-1}\mathbf{f}_{qk} = -Gm_q\sum_{\substack{k=0\\k\neq q}}^{n-1}\frac{m_k}{\left|\mathbf{s}_q(t) - \mathbf{s}_k(t)\right|^3}\left[\mathbf{s}_q(t) - \mathbf{s}_k(t)\right]. \quad \rightarrow 2$$

- The acceleration of an object is given by the second derivative of its position and that Newton's second law of motion states that the force on an object is given by its mass multiplied by its acceleration, so if the acceleration of particle q is $\mathbf{a}_q(t)$, $\mathbf{F}_q(t) = m_q\mathbf{a}_q(t) = m_q\mathbf{s}_q''(t)$, where $\mathbf{S}_q$''(t) is the second derivative of the position $\mathbf{S}q(t)$. Thus the acceleration of particle q:

$$\mathbf{s}_q''(t) = -G\sum_{\substack{j=0\\ j\neq q}}^{n-1} \frac{m_j}{\left|\mathbf{s}_q(t)-\mathbf{s}_j(t)\right|^3}\left[\mathbf{s}_q(t)-\mathbf{s}_j(t)\right].$$

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- Newton's laws give us a system of differential equations—equations involving derivatives—and our job is to find at each time t of interest the position $s_{\mathbf{q}}(t)$.

and

$V_{\mathbf{q}}(t)= s_{\mathbf{q}}'(t)$. We want to find the positions and velocities at the times

t = 0, Δt,2 Δt,3 Δt …..T Δt

- Here, Δt and T are specified by the user, so the input to the program will be n, the number of particles, Δt, T, and, for each particle, its mass, its initial position, and its initial velocity.

- In a fully general solver, the positions and velocities would be three-dimensional vectors, but in order to keep things simple, we'll assume that the particles will move in a plane, and we'll use two-dimensional vectors.

- The output of the program will be the positions and velocities of the n particles
at the timesteps 0, Δt,2 Δt,3 Δt …. or just the positions and velocities at.T Δt.