PRIVACY IN ONLINE SOCIAL NETWORKS

Part 2

Outline

- Trust transitivity analysis.
- Combining trust and reputation.
- Trust derivation based on trust comparisons.

TRUST TRANSITIVITY ANALYSIS

What is Trust Transitivity Analysis?

- Assume two agents A and B where A trusts B, and B believes that proposition x is true.
- Then by **transitivity**, **agent A will also believe that proposition x is true.**
- In our approach, **trust** and **belief** are formally expressed as **opinions**.
- **The transitive linking of these two opinions consists of discounting B’s opinion about x by A’s opinion about B, in order to derive A’s opinion about x.**

Representation of trust and disadvantages of TTA

- The **solid** arrows represent initial **direct trust**.
- The **dotted** arrow represents derived **indirect trust**.
- **Trust transitivity**, as trust itself, is a **human mental phenomenon**.

- **Disadvantages:**

1) The first is related to the **effect of A disbelieving that B** will give a good advice.
2) The second difficulty relates to the **effect of base rate trust in a transitive path**.

First Problem explained

1. Uncertainty Favoring Trust Transitivity

- A’s disbelief in the recommending agent B ⟶ A thinks that B ignores the truth value of x.
- As a result A also ignores the truth value of x.

Fig. 22.3 Principle of trust transitivity

First Problem explained

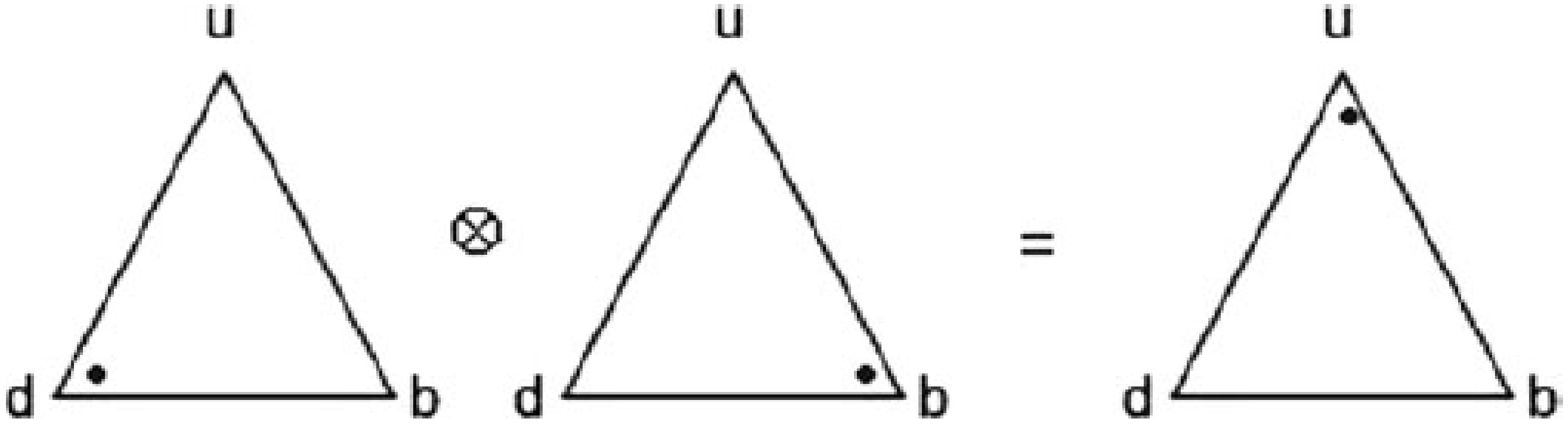


Fig. 22.4 Example of applying the discounting operator for independent opinions

Definition 22.1 (Uncertainty Favoring Discounting). Let A and B be two agents where A's opinion about B's recommendations is expressed as $\omega_B^A = \{b_B^A, d_B^A, u_B^A, a_B^A\}$, and let x be a proposition where B's opinion about x is recommended to A with the opinion $\omega_x^B = \{b_x^B, d_x^B, u_x^B, a_x^B\}$ Let $\omega_x^{A:B} = \{b_x^{A:B}, d_x^{A:B}, u_x^{A:B}, a_x^{A:B}\}$ be the opinion such that:

First Problem explained

$$\begin{cases} b_x^{A:B} = b_B^A b_x^B \\ d_x^{A:B} = d_B^A d_x^B \\ u_x^{A:B} = d_B^A + u_B^A + b_B^A u_x^B \\ a_x^{A:B} = a_x^B \end{cases}$$

then $\omega_x^{A:B}$ is called the uncertainty favoring discounted opinion of A. By using the symbol $\otimes$ to designate this operation, we get $\omega_x^{A:B} = \omega_B^A \otimes \omega_x^B$.

Problem - 1 :

$$\omega_B^A = \{0.1,\ 0.8,\ 0.1\}$$
$$\downarrow \quad \downarrow \quad \downarrow$$
$$b_B^A \quad d_B^A \quad u_B^A$$

$$\omega_x^B = \{0.8,\ 0.1,\ 0.1\}$$
$$\downarrow \quad \downarrow \quad \downarrow$$
$$b_x^B \quad d_x^B \quad u_x^B$$

$$\omega_x^{A:B} = \{ b_x^{A:B},\ d_x^{A:B},\ u_x^{A:B} \}$$

To find

1) $b_x^{A:B} = b_B^A\, b_x^B = 0.1 \times 0.8 = 0.08$

2) $u_x^{A:B} = d_B^A + u_B^A + b_B^A\, u_x^B$

$= 0.8 + 0.1 + 0.1 \times 0.1$

$= 0.9 + 0.01$

$= 0.91$

3) $d_x^{A:B} = 1 - [\, b_x^{A:B} + u_x^{A:B} \,]$

$= 1 - [0.08 + 0.91]$

$= 1 - 0.99$

$= 0.01$

First Problem explained

Your enemy's enemy is your friend

2. Opposite Belief Favoring

- A's disbelief in the recommending agent B means that A thinks that B consistently recommends the opposite of his real opinion about the truth value of x.

Definition 22.2 (Opposite Belief Favoring Discounting). Let A and B be two agents where A's opinion about B's recommendations is expressed as $\omega_B^A = \{b_B^A, d_B^A, u_B^A, a_B^A\}$, and let x be a proposition where B's opinion about x is recommended to A as the opinion $\omega_x^B = \{b_x^B, d_x^B, u_x^B, a_x^B\}$. Let $\omega_x^{A:B} = \{b_x^{A:B}, d_x^{A:B}, u_x^{A:B}, a_x^{A:B}\}$ be the opinion such that:

$$\begin{cases} b_x^{A:B} = b_B^A b_x^B + d_B^A d_x^B \\ d_x^{A:B} = b_B^A d_x^B + b_B^A d_x^B \\ u_x^{A:B} = u_B^A + (b_B^A + d_B^A) u_x^B \\ a_x^{A:B} = a_x^B \end{cases}$$

then $\omega_x^{A:B}$ is called the opposite belief favoring discounted recommendation from B to A. By using the symbol $\otimes$ to designate this operation, we get $\omega_x^{A:B} = \omega_B^A \otimes \omega_x^B$.

Second Problem explained

3. Base Rate Sensitive Transitivity

- A scenario!!!!!!!!!
- Imagine a stranger coming to a town which is know for its citizens being honest.
- The stranger is looking for a car mechanic, and asks the first person he meets to direct him to a good car mechanic.
- The stranger receives the reply that there are two car mechanics in town, David and Eric.
- David is cheap but does not always do quality work
- Eric might be a bit more expensive, but he always does a perfect job.
- Translated into the formalism of subjective logic, **the stranger has no other info about the person he asks than the base rate that the citizens in the town are honest.**

Definition 22.3 (Base Rate Sensitive Discounting). The base rate sensitive discounting of a belief $\omega_x^B = \{b_x^B, d_x^B, u_x^B, a_x^B\}$ by a belief $\omega_B^A = \{b_B^A, d_B^A, u_B^A, a_B^A\}$ produces the transitive belief $\omega_x^{A:B} = \{b_x^{A:B}, d_x^{A:B}, u_x^{A:B}, a_x^{A:B}\}$ where

$$\begin{cases} b_x^{A:B} = E\left(\omega_B^A\right) b_x^B \\ d_x^{A:B} = E\left(\omega_B^A\right) d_x^B \\ u_x^{A:B} = 1 - E\left(\omega_B^A\right)\left(b_x^B + d_x^B\right) \\ a_x^{A:B} = a_x^B \end{cases}$$

where the probability expectation value $E\left(\omega_B^A\right) = b_B^A + a_B^A u_B^A$.

Second Problem explained

4. Mass Hysteria

- **Mass hysteria** can be caused by **people not** being **aware of dependence between opinions**.
- Let’s take for example; person A recommend an opinion about a particular statement x to a group of other persons.
- Without being aware of the fact that the opinion came from the same origin, these persons can recommend their opinions to each other as illustrated in Figure below.

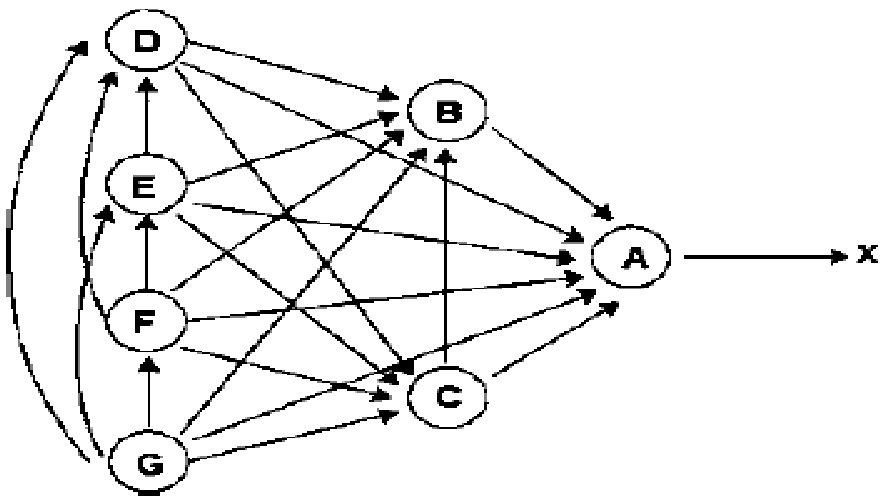


Fig. 22.5 The effects of unknown dependence

Second Problem explained

- ***<u>Problem with Mass Hysteria:</u>***
- The arrows represent trust so that for example B → A can be interpreted as saying that B trusts A to recommend an opinion about statement x.
- It can be seen that A recommends an opinion about x - 6 other agents, and that G receives six recommendations in all.
- If G assumes the recommended opinions to be independent and takes the consensus between them, his opinion can become abnormally strong and in fact even stronger than A’s opinion.

Second Problem explained

Solution for Mass Hysteria:

- Taking all the possible recommendations into account, we should first create a relatively complex trust graph.
- Analyzing the whole graph of dependent paths, as if they were independent.
- Retain only the non-redundant edges.
- If there are edges like $G \rightarrow B \rightarrow A$, $G \rightarrow C \rightarrow A$ and $G \rightarrow A$, retain only the edge $G \rightarrow A$.

COMBINING TRUST AND REPUTATION

Combining trust and reputation

- A bijective mapping can be defined between multinomial reputation scores and opinions.
- It is possible to interpret these two mathematical representations as equivalent.
- The mapping can symbolically be expressed as:

$$\omega \leftrightarrow \vec{R}$$

THEOREM - Equivalence Between Opinions and Reputations

Theorem 22.1. Equivalence Between Opinions and Reputations. *Let* $\omega = (\vec{b}, u, \vec{a})$ *be an opinion, and* $\vec{R}$ *be a reputation, both over the same state space* X *so that the base rate* $\vec{a}$ *also applies to the reputation. Then the following equivalence holds (22.3):*

Case when u ≠ 0 and u = 0

For $u \neq 0$:

$$\begin{cases} \vec{b}(x_i) = \frac{\vec{R}(x_i)}{C+\sum_{i=1}^{k}\vec{R}(x_i)} \\ u = \frac{C}{C+\sum_{i=1}^{k}\vec{R}(x_i)} \end{cases} \Longleftrightarrow \begin{cases} \vec{R}(x_i) = \frac{C\vec{b}(x_i)}{u} \\ u + \sum_{i=1}^{k}\vec{b}(x_i) = 1 \end{cases} \quad (22.13)$$

For $u = 0$:

$$\begin{cases} \vec{b}(x_i) = \eta(x_i) \\ u = 0 \end{cases} \Longleftrightarrow \begin{cases} \vec{R}(x_i) = \eta(x_i)\sum_{i=1}^{k}\vec{R}(x_i) = \eta(x_i)\infty \\ \sum_{i=1}^{k} m(x_i) = 1 \end{cases} \quad (22.14)$$

Case 1 : u = 0

- The case $u = 0$ reflects an infinite amount of aggregate ratings.
- Here the parameter Ŋ determines the relative proportion of infinite ratings among the rating levels.
- **Case 1-a** : In case $u = 0$ and $\eta(x_i) = 1$ for a particular rating level x_i,
  - $\vec{R}(x_i) = \infty$
  - all the other rating parameters **are finite**.
- **Case 1-b :** $\eta(x_i) - 1/k$ for all $i = 1 \ldots k$,
  - all the rating parameters **are equally infinite**.

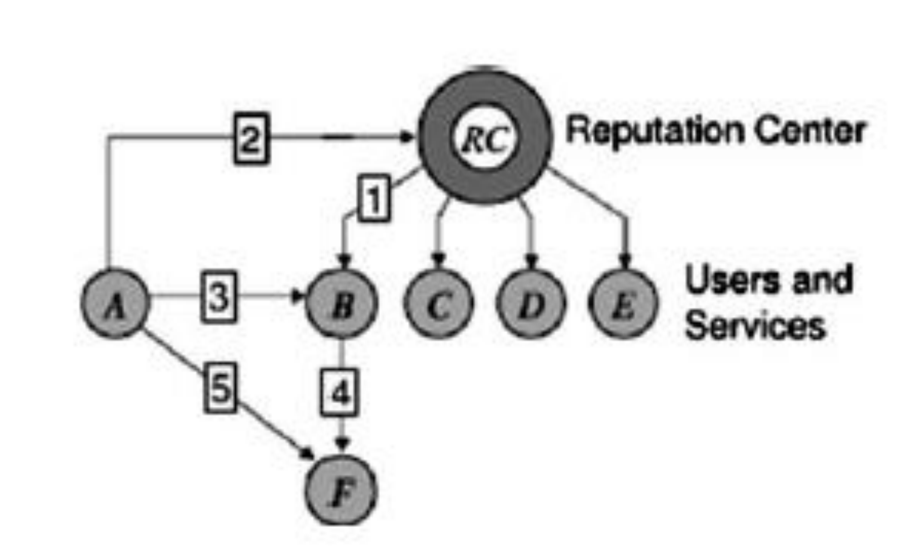
Multinomial ratings → Binomial Trust

- Multinomial aggregate ratings can be used to derive binomial trust in the form of an opinion.
- This is done by first converting the multinomial ratings to binomial ratings according to equation 22.15, and then to apply Theorem 22.1.
- <u>**Basics:**</u>
  - *Let the multinomial reputation model have k rating levels* $x_i\,;\, \mathrm{i} = 1, \ldots k$
  - $\vec{R}(x_i)$ represents the ratings on each level x_i
  - *let σ represent the point estimate reputation score*
  - *Let the binomial reputation model have **positive** and **negative** ratings **r** and **s** respectively.*

$$\begin{cases} r = \sigma \sum_{i=1}^{k} \vec{R}_y(x_i) \\ s = \sum_{i=1}^{k} \vec{R}_y(x_i) - r \end{cases} \qquad (22.15)$$

Combining Trust and Reputation

- It is possible to analyze trust networks based on both trust relationships and reputation scores.
- In the diagram below, Agent A needs to derive a measure of trust in agent F .



1. Agent B has reputation score $\vec{R}_B^{RC}$ (arrow 1)

2. Agent A has trust ω_{RC}^{A} in the Reputation Centre (arrow 2),
3. So that A can derive a measure of trust in B (arrow 3).
4. Agent B's trust in F (arrow 4) can be recommended to A so that A can derive a measure of trust in F (arrow 5).
5. Mathematically this can be expressed as:

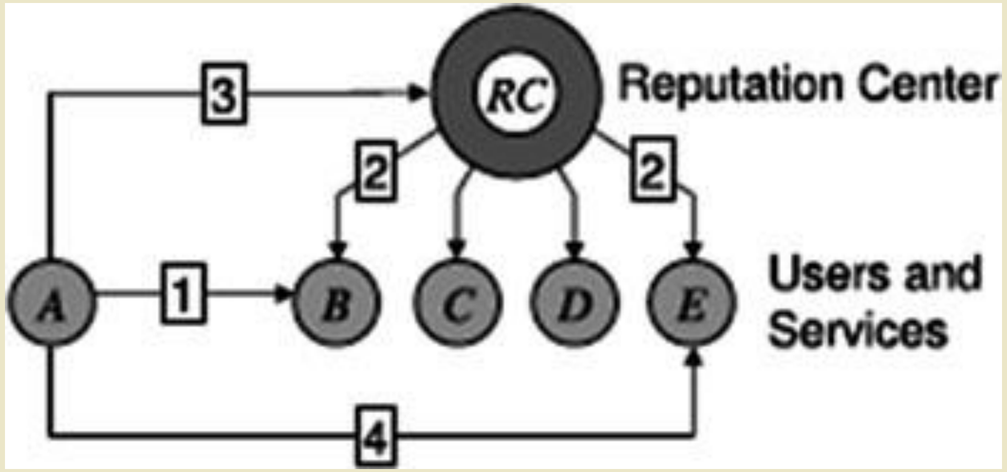
$$\omega_F^A = \omega_{RC}^A \otimes \vec{R}_B^{RC} \otimes \omega_F^B \qquad (22.16)$$

TRUST DERIVATION BASED ON TRUST COMPARISONS

Trust Comparisons and Mutual Trust

- Different agents have different trust in the same entity.
- This could affect the mutual trust between the two agents.
- **Arrow 1:** A's trust $\omega_B^A \psi$
- **Arrow 2:** B's reputation score $\bar{R}_B^{RC}$
- As a result, **A** will derive a **reduced trust value**

in the **Reputation Centre** (**Arrow 3**).



Now the task is = **A needs to derive a trust value in E.**

Trust Derivation – A needs to derive a trust value in E !!!!!

- **Reduced trust value** must be taken into **account** when using ***RC*’s** **reputation score** for **computing trust in E** .
- The **operator** for **deriving trust** based **on trust conflict** produces a **binomial opinion** over the binary state space $\{x, \bar{x}\}$
- **x** is a **proposition** that can be interpreted as **x:** ***“RC provides reliable reputation scores”*** and **X (bar)** is its complement.
- **Binomial opinions** have the special notation $\mathbf{w_x} = (\mathbf{b},\mathbf{d},\boldsymbol{u},\mathbf{a})$ where d represents disbelief in proposition x.
- **difference in trust values** α **semantics of the state space.**

Comparison of polarized and average reputation scores.

- The state space consists of five rating levels.

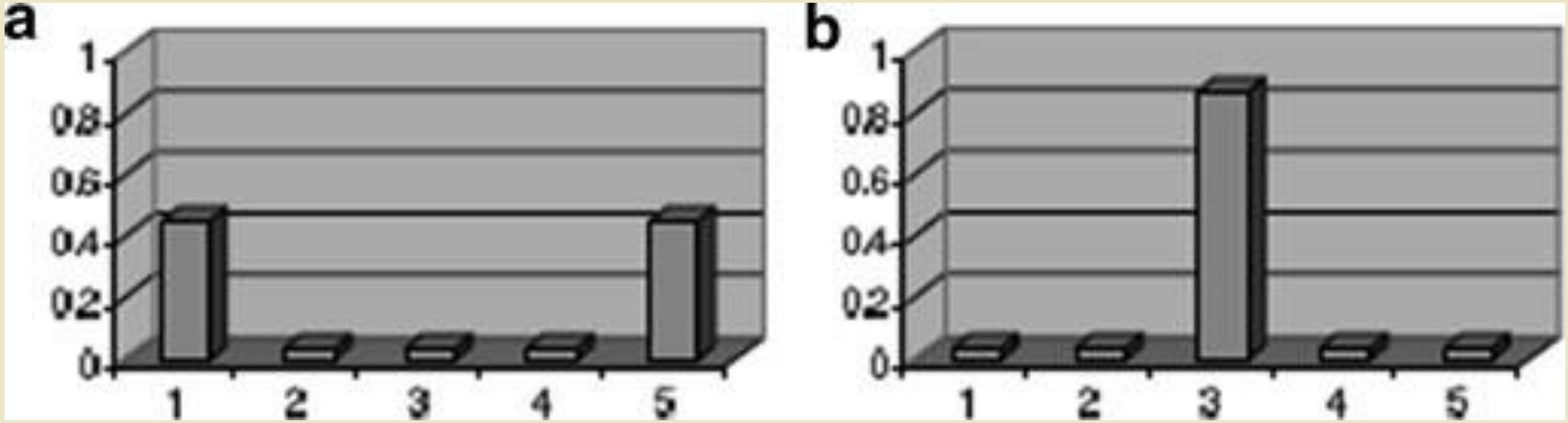


Fig. 22.10 Comparison of polarized and average reputation scores. **(a)** Reputation score from polarized ratings **(b)** Reputation score from avg ratings

Defining Operators – Mutual trust / Mutual Distrust

- We will define an operator which derives trust based on point estimates as defined by (22.11).
- **Two agents** having **similar point estimates** about the **same agent** or proposition should induce **mutual trust**.
- **Dissimilar point estimates** should induce **mutual distrust**.

Definition of Trust Derivation Based on Trust Comparison

Definition 22.4 (Trust Derivation Based on Trust Comparison). Let ω_B^A and ω_B^{RC} be two opinions on the same state space Bψ with a set rating levels. A's trust in RCψ based on the similarity between their opinions is defined as:

$$\omega_{RC}^A = \omega_B^A \downarrow \omega_B^{RC} \text{ where } \begin{cases} d_{RC}^A = \left|\sigma\left(\vec{R}_B^A\right) - \sigma\left(\vec{R}_B^{RC}\right)\right| \\ u_{RC}^A = Max\left[u_B^A, u_B^{RC}\right] \\ b_{RC}^A = 1 - b_{RC}^A - u_{RC}^A \end{cases}$$

Inferences:

- Disbelief in *RC* is proportional to the greatest difference in point estimates between the two opinions. Also, the uncertainty is equal to the greatest uncertainty of the two opinions.
- A is able to derive trust in E expressed as:

$$\omega_E^A = \omega_{RC}^A \otimes \omega_E^{RC} \qquad (22.17)$$

Thank You