Concept learning

Machine Learning, Tom Mitchell

Mc Graw-Hill International Editions, 1997

(Chapters 1, 2).

Definition of concept learning

- *Task*: learning a category description (*concept*) from a set of positive and negative training examples.
  - Concept may be an event, an object …
- *Target function*: a boolean function $c: X \rightarrow \{0, 1\}$
- *Experience*: a set of training instances $D:\{\langle x, c(x)\rangle\}$
- A search problem for best fitting hypothesis in a hypotheses space
  - The space is determined by the choice of representation of the hypothesis (all boolean functions or a subset)

Sport example

- **Concept to be learned:**

  Days in which Aldo can enjoy water sport

  Attributes:

  Sky: ***Sunny, Cloudy, Rainy***

  AirTemp: ***Warm, Cold***

  Humidity: ***Normal, High***

  Wind: ***Strong, Weak***

  Water: ***Warm, Cool***

  Forecast: ***Same, Change***

- **Instances in the training set (out of the 96 possible):**

Sky	Temp	Humid	Wind	Water	Forecst	EnjoySpt
Sunny	Warm	Normal	Strong	Warm	Same	Yes
Sunny	Warm	High	Strong	Warm	Same	Yes
Rainy	Cold	High	Strong	Warm	Change	No
Sunny	Warm	High	Strong	Cool	Change	Yes

Hypotheses representation

- h is a set of constraints on attributes:
  - a specific value: e.g. $Water = Warm$
  - any value allowed: e.g. $Water = ?$
  - no value allowed: e.g. $Water = \emptyset$
- Example hypothesis:

$$
\begin{array}{cccccc}
Sky & AirTemp & Humidity & Wind & Water & Forecast \\
\langle Sunny, & ?, & ?, & Strong, & ?, & Same \rangle
\end{array}
$$

  Corresponding to boolean function:

  Sunny(Sky) ∧ Strong(Wind) ∧ Same(Forecast)

- H, hypotheses space, all possible h

Hypothesis satisfaction

- An instance x *satisfies* an hypothesis h iff all the constraints expressed by h are satisfied by the attribute values in x.

- Example 1:

  x_1: $\langle Sunny, Warm, Normal, Strong, Warm, Same\rangle$

  h_1: $\langle Sunny, ?, ?, Strong, ?, Same\rangle$ Satisfies? Yes

- Example 2:

  x_2: $\langle Sunny, Warm, Normal, Strong, Warm, Same\rangle$

  h_2: $\langle Sunny, ?, ?, \emptyset, ?, Same\rangle$ Satisfies? No

Formal task description

- **Given:**
  - X all possible days, as described by the attributes
  - A set of hypothesis H, a conjunction of constraints on the attributes, representing a function h: $X \rightarrow \{0, 1\}$

    [$h(x) = 1$ if x satisfies h; $h(x) = 0$ if x does not satisfy h]
  - A target concept: c: $X \rightarrow \{0, 1\}$ where

    $c(x) = 1$ iff $EnjoySport = Yes$;

    $c(x) = 0$ iff $EnjoySport = No$;
  - A training set of possible instances D: $\{\langle x, c(x)\rangle\}$
- **Goal:** find a hypothesis h in H such that

  $h(x) = c(x)$ for all x in D

  Hopefully h will be able to predict outside D…

The inductive learning assumption

- We can at best guarantee that the output hypothesis fits the target concept over the training data
- *Assumption*: an hypothesis that approximates well the training data will also approximate the target function over unobserved examples
- i.e. given a significant training set, the output hypothesis is able to make predictions

General to specific ordering

- **Consider:**

$$h_1 = \langle Sunny, ?, ?, Strong, ?, ? \rangle$$

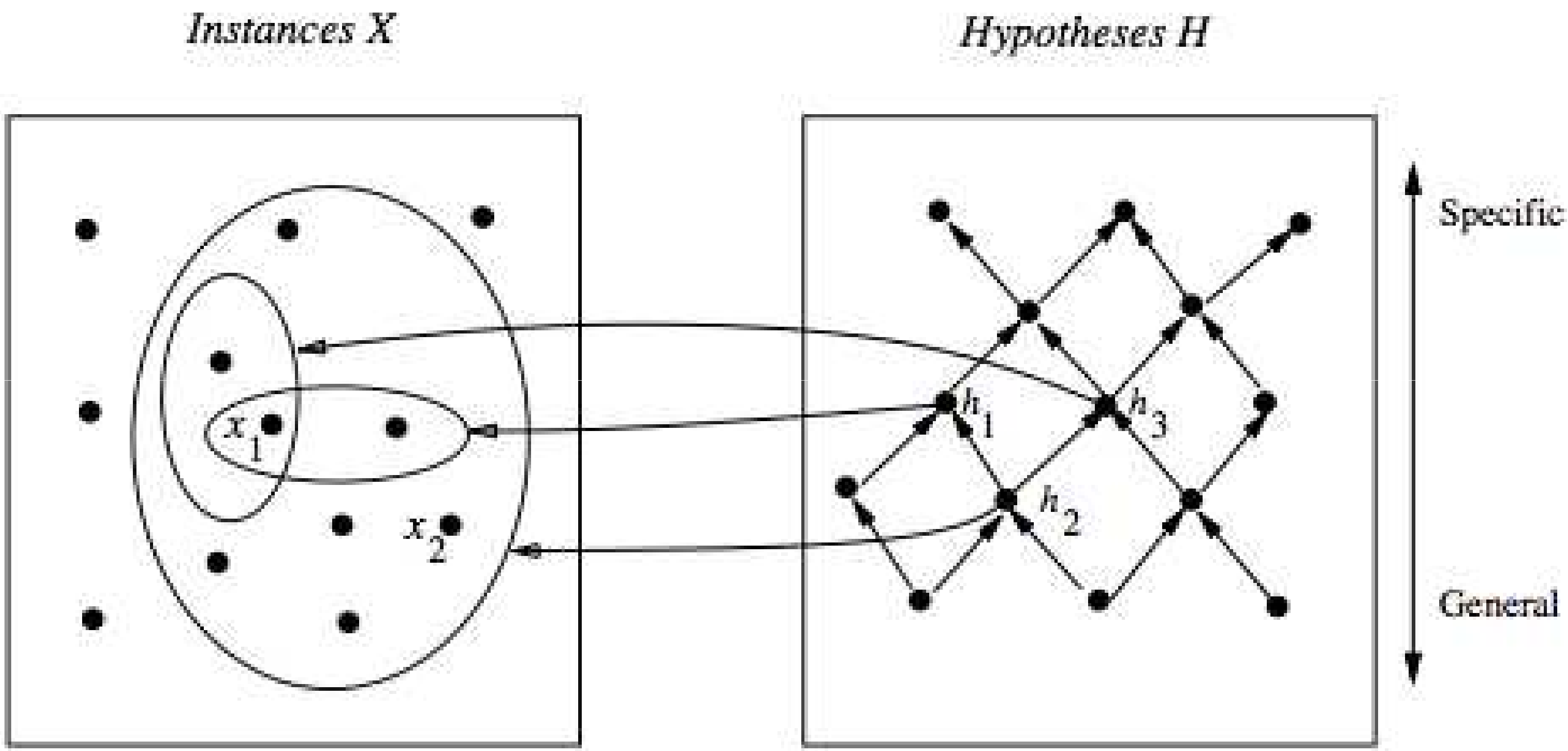
$$h_2 = \langle Sunny, ?, ?, ?, ?, ? \rangle$$

- **Any instance classified positive by h_1 will also be classified positive by h_2**
- **h_2 is more general than h_1**
- **Definition:** $h_j \geq_g h_k \;\; iff \;\; (\forall x \in X)\, [(h_k = 1) \rightarrow (h_j = 1)]$

$\geq_g$ *more general or equal;* $\quad >_g$ *strictly more general*

- **Most general hypothesis:** $\langle ?, ?, ?, ?, ?, ? \rangle$
- **Most specific hypothesis:** $\langle \varnothing, \varnothing, \varnothing, \varnothing, \varnothing, \varnothing \rangle$

General to specific ordering: induced structure



x_1= <Sunny, Warm, High, Strong, Cool, Same>

x_2= <Sunny, Warm, High, Light, Warm, Same>

h_1= <Sunny, ?, ?, Strong, ?, ?>

h_2= <Sunny, ?, ?, ?, ?, ?>

h_3= <Sunny, ?, ?, ?, Cool, ?>

Find-S: finding the most specific hypothesis

- **Exploiting the structure we have alternatives to enumeration …**

1. **Initialize h to the most specific hypothesis in H**
2. **For each positive training instance:**

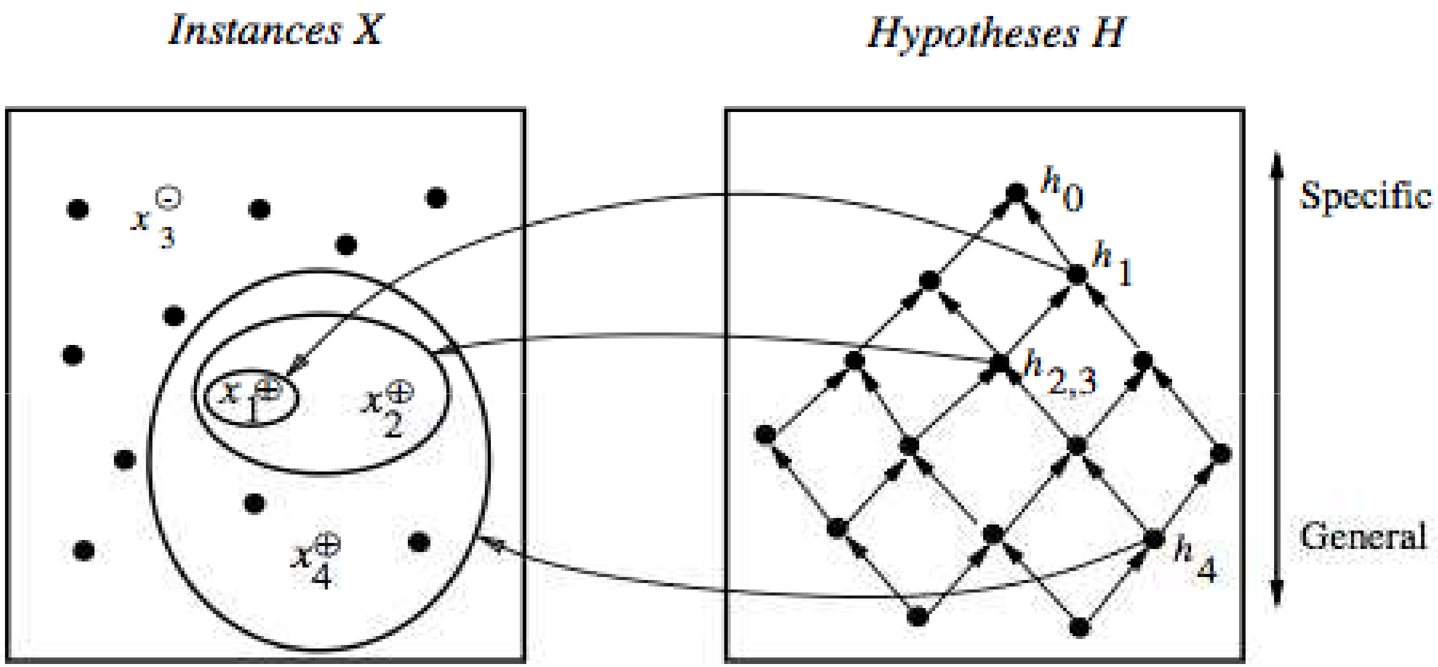
   for each **attribute constraint a in h:**

   If **the constraint a is satisfied by x** *then do nothing*

   else **replace a in h by the next more general constraint satified by x (move towards a more general hp)**

§ **Output hypothesis h**

Find-S in action



x_1 = <Sunny Warm Normal Strong Warm Same>, +

x_2 = <Sunny Warm High Strong Warm Same>, +

x_3 = <Rainy Cold High Strong Warm Change>, -

x_4 = <Sunny Warm High Strong Cool Change>, +

h_0 = <∅, ∅, ∅, ∅, ∅, ∅>

h_1 = <Sunny Warm Normal Strong Warm San

h_2 = <Sunny Warm ? Strong Warm Same>

h_3 = <Sunny Warm ? Strong Warm Same>

h_4 = <Sunny Warm ? Strong ? ? >

Properties of *Find-S*

- *Find-S* is guaranteed to output the *most specific hypothesis* within *H* that is consistent with the *positive* training examples
- The final hypothesis will also be consistent with the negative examples
- Problems:
  - There can be more than one “most specific hypotheses”
  - We cannot say if the learner converged to the correct target
  - Why choose the most specific?
  - If the training examples are inconsistent, the algorithm can be mislead: no tolerance to rumor.
  - Negative example are not considered

Candidate elimination algorithm: the idea

- ***The idea*: output a description of the set of *all* *hypotheses* *consistent* with the training examples (correctly classify training examples).**
- ***Version space*: a representation of the set of hypotheses which are *consistent* with *D***
  - **an explicit list of hypotheses (List-Than-Eliminate)**
  - **a compact representation of hypotheses which exploits the *more_general_than* partial ordering (Candidate-Elimination)**

Version space

- The version space $VS_{H,D}$ is the subset of the hypothesis from H *consistent* with the training example in D

$$VS_{H,D} \equiv \{h \in H \mid Consistent(h, D)\}$$

- An hypothesis h is consistent with a set of training examples D iff $h(x) = c(x)$ for each example in D

$$Consistent(h, D) \equiv (\forall \langle x, c(x)\rangle \in D)\ h(x) = c(x))$$

Note: "x *satisfies* h" ($h(x)=1$) different from "h *consistent with* x"

In particular when an hypothesis h is consistent with a negative example $d = \langle x, c(x)=No\rangle$, then x must not satisfy h

The *List-Then-Eliminate* algorithm

Version space as list of hypotheses

1. *VersionSpace* $\leftarrow$ a list containing every hypothesis in *H*
2. For each training example, $\langle x, c(x) \rangle$

   Remove from *VersionSpace* any hypothesis h for which $h(x) \neq c(x)$

§ Output the list of hypotheses in *VersionSpace*

1. Problems
   1. The hypothesis space must be finite
   2. Enumeration of all the hypothesis, rather inefficient

A compact representation for *Version Space*

S: { <*Sunny, Warm,* ?, *Strong,* ?, ?> }

G: { <*Sunny,* ?, ?, ?, ?, ?>, <?, *Warm,* ?, ?, ?, ?> }

***Note*: The output of *Find-S* is just ⟨*Sunny, Warm, ?, Strong, ?, ?*⟩**

- **Version space represented by its most general members *G* and its most specific members *S (boundaries)***

General and specific boundaries

- The *Specific boundary*, *S*, of version space $VS_{H,D}$ is the set of its minimally general (most specific) members

  S $\equiv \{s \in H \mid Consistent(s, D) \wedge (\neg \exists s' \in H)[(s >_g s') \wedge Consistent(s', D)]\}$

  Note: any member of *S* is satisfied by all positive examples, but more specific hypotheses fail to capture some

- The *General boundary*, *G*, of version space $VS_{H,D}$ is the set of its maximally general members

  G $\equiv \{g \in H \mid Consistent(g, D) \wedge (\neg \exists g' \in H)[(g' >_g g) \wedge Consistent(g', D)]\}$

  Note: any member of *G* is satisfied by no negative example but more general hypothesis cover some negative example

Version Space representation theorem

- G and S completely define the Version Space
- *Theorem*: Every member of the version space (h consistent with D) is in S or G or lies between these boundaries

$$VS_{H,D}=\{h \in H \mid (\exists s \in S)\ (\exists g \in G)\ (g \geq_g h \geq_g s)\}$$

where $x \geq_g y$ means x is more general or equal to y

Sketch of proof:

$\Leftarrow$ If $g \geq_g h \geq_g s$, since s is in S and $h \geq_g s$, h is satisfied by all positive examples in D; g is in G and $g \geq_g h$, then h is satisfied by no negative examples in D; therefore h belongs to $VS_{H,D}$

$\Rightarrow$ It can be proved by assuming a consistent h that does not satisfy the right-hand side and by showing that this would lead to a contradiction

Candidate elimination algorithm-1

$S \leftarrow$ minimally general hypotheses in H,

$G \leftarrow$ maximally general hypotheses in H

Initially any hypothesis is still possible

$S_0 = \langle \emptyset, \emptyset, \emptyset, \emptyset, \emptyset, \emptyset \rangle \qquad G_0 = \langle ?, ?, ?, ?, ?, ? \rangle$

For each training example d, do:

If d is a *positive* example:

- § Remove from G any h inconsistent with d
- § $Generalize(S, d)$

If d is a *negative* example:

- § Remove from S any h inconsistent with d
- § $Specialize(G, d)$

Note: when $d = \langle x, No \rangle$ is a negative example, an hypothesis h is inconsistent with *d* iff h *satisfies* x

Candidate elimination algorithm-2

Generalize(S, d): d is *positive*

For each hypothesis s in S not consistent with d:

- Remove s from S
- Add to S all minimal generalizations of s consistent with d and having a generalization in G
- Remove from S any hypothesis with a more specific h in S

Specialize(G, d): d is *negative*

For each hypothesis g in G not consistent with d: i.e. g *satisfies d,*

- Remove g from G *but d is negative*
- Add to G all minimal specializations of g consistent with d and having a specialization in S
- Remove from G any hypothesis having a more general hypothesis in G

Example: initially

S_0: $\langle \varnothing, \varnothing, \varnothing, \varnothing, \varnothing. \varnothing \rangle$

G_0 $\langle ?, ?, ?, ?, ?, ? \rangle$

Example:

after seing ⟨*Sunny,Warm, Normal, Strong, Warm, Same* ⟩ +

S_0: ⟨∅, ∅, ∅, ∅, ∅. ∅⟩

S_1: ⟨***Sunny,Warm, Normal, Strong, Warm, Same***⟩

G_0, G_1 **⟨?, ?, ?, ?, ?, ?⟩**

Example:

after seing ⟨*Sunny,Warm, High, Strong, Warm, Same*⟩ +

S_1: ⟨***Sunny,Warm, Normal, Strong, Warm, Same***⟩

S_2: ⟨***Sunny,Warm, ?, Strong, Warm, Same***⟩

G_1, G_2 ⟨**?, ?, ?, ?, ?, ?**⟩

Example:

after seing ⟨*Rainy, Cold, High, Strong, Warm, Change*⟩–

S_2, S_3: ***⟨Sunny, Warm, ?, Strong, Warm, Same⟩***

G_3: ***⟨Sunny, ?, ?, ?, ?, ?⟩ ⟨?, Warm, ?, ?, ?, ?⟩ ⟨?, ?, ?, ?, ?, Same⟩***

G_2: ***⟨?, ?, ?, ?, ?, ?⟩***

Example:

after seing ⟨*Sunny, Warm, High, Strong, Cool Change* ⟩ +

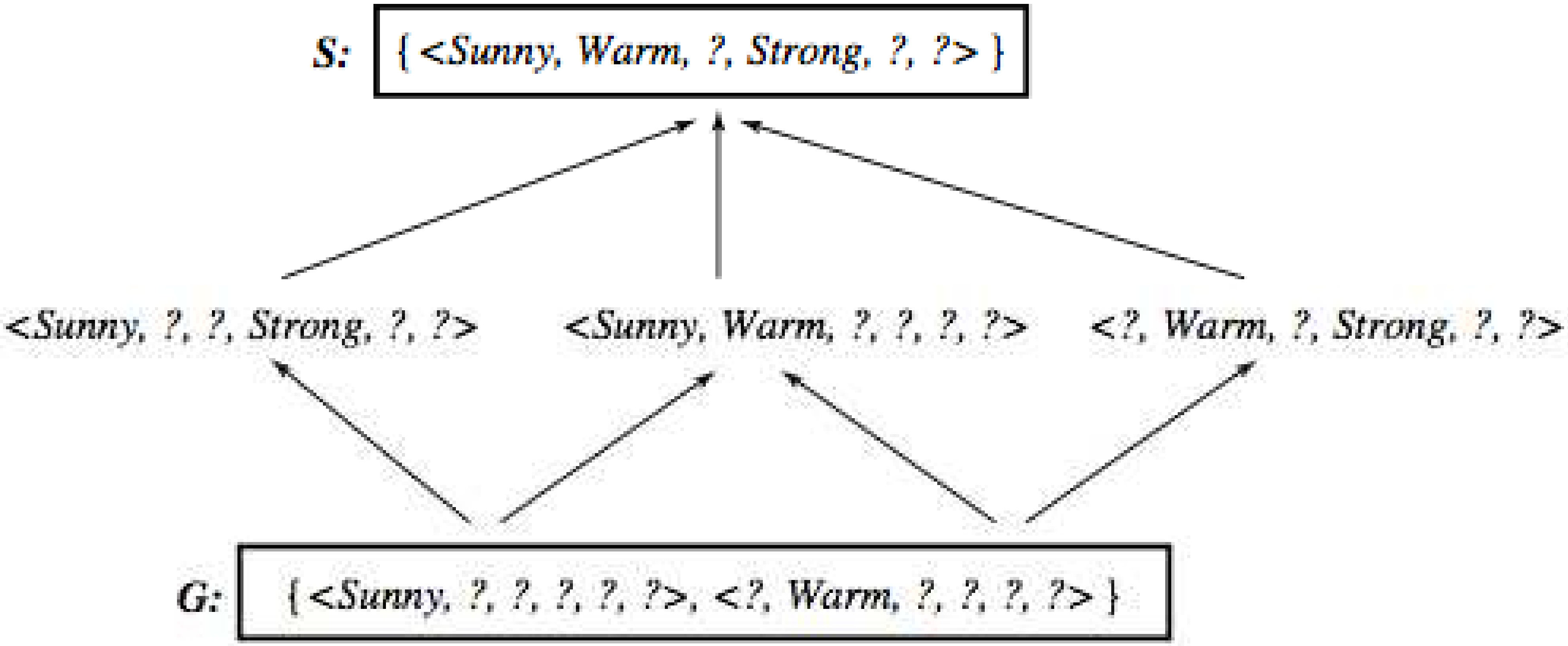
S_3 ⟨*Sunny, Warm, ?, Strong, Warm, Same*⟩

S_4 ⟨***Sunny, Warm, ?, Strong, ?, ?***⟩

G_4: ⟨**Sunny, ?, ?, ?, ?, ?**⟩ ⟨**?, Warm, ?, ?, ?, ?**⟩

G_3: ⟨*Sunny, ?, ?, ?, ?, ?*⟩ ⟨*?, Warm, ?, ?, ?, ?*⟩ ⟨*?, ?, ?, ?, ?, Same*⟩

Learned Version Space



Observations

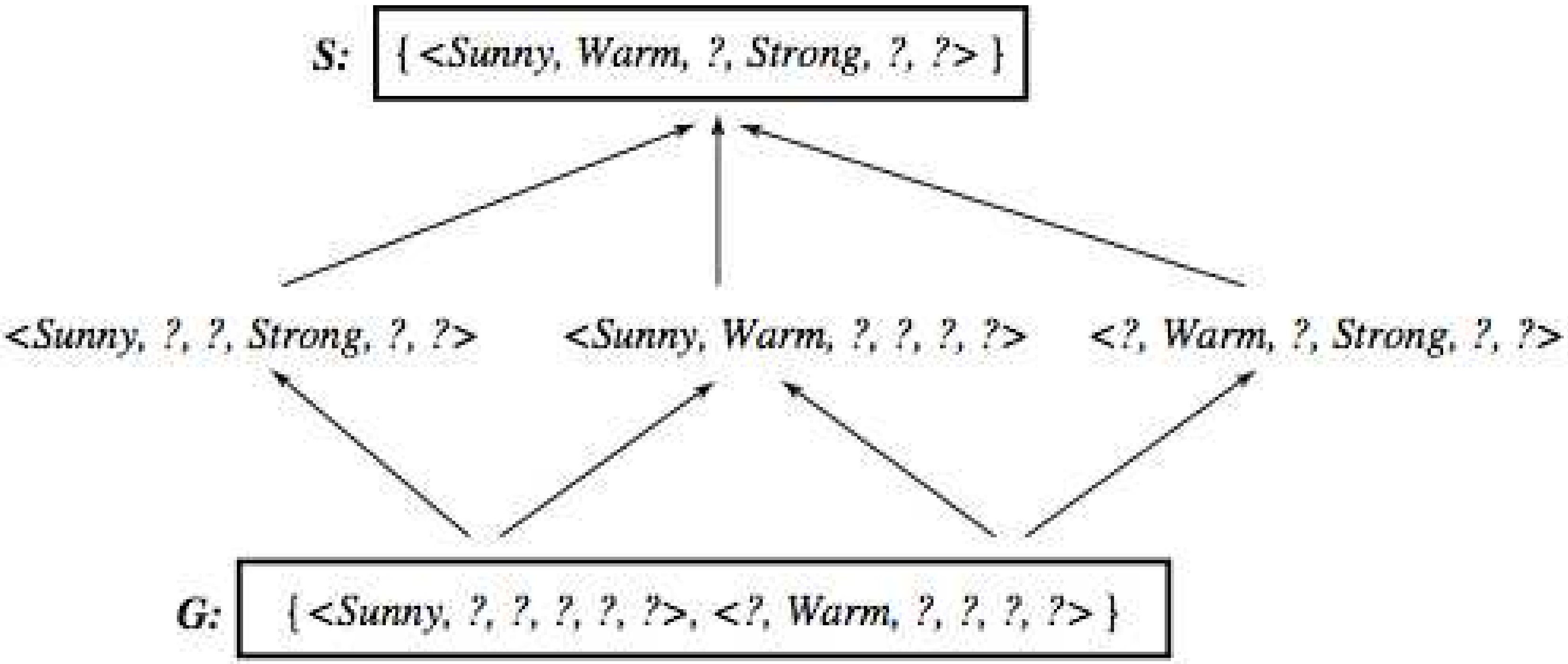
- The learned Version Space correctly describes the target concept, provided:
  1. There are no errors in the training examples
  2. There is some hypothesis that correctly describes the target concept
- If S and G converge to a single hypothesis the concept is exactly learned
- In case of errors in the training, useful hypothesis are discarded, no recovery possible
- An empty version space means no hypothesis in H is consistent with training examples

Ordering on training examples

- The learned version space does not change with different orderings of training examples
- Efficiency does
- Optimal strategy (if you are allowed to choose)
  - Generate instances that satisfy half the hypotheses in the current version space. For example:

    $\langle Sunny, Warm, Normal, Light, Warm, Same\rangle$ satisfies 3/6 hyp.
  - Ideally the VS can be reduced by half at each experiment
  - Correct target found in $\lceil log_2|VS|\rceil$ experiments

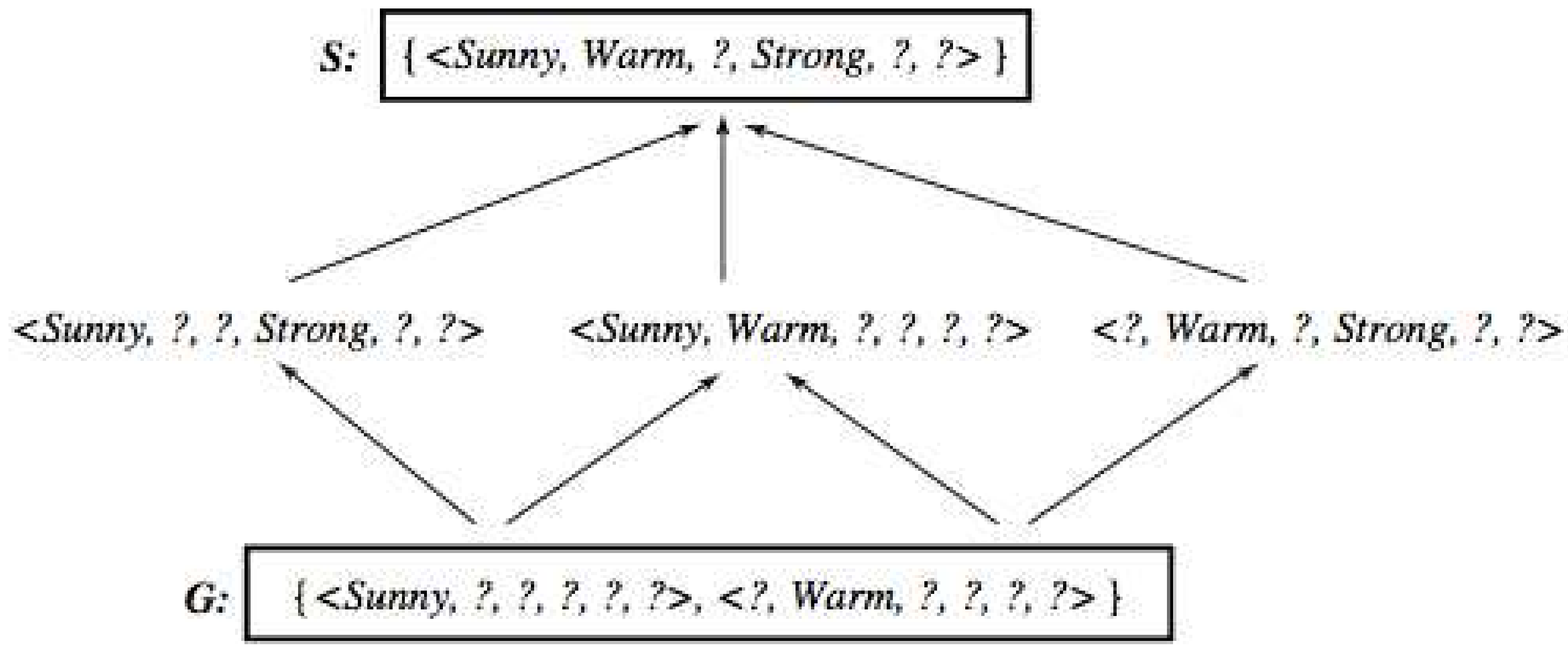
Use of partially learned concepts



$$\langle Sunny\ Warm\ Normal\ Strong\ Cool\ Change \rangle$$

Classified as *positive* by all hypothesis, since satisfies any hypothesis in S

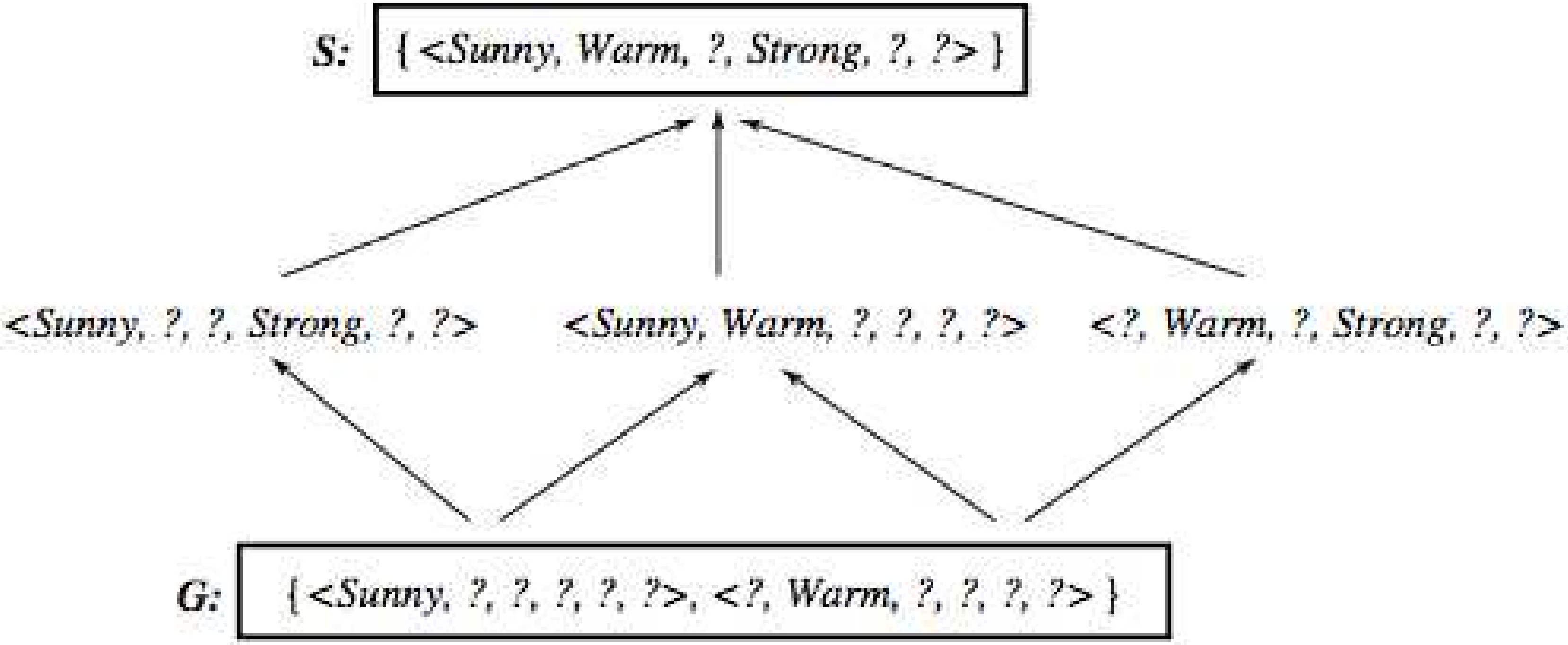
Classifying new examples



$$\langle Rainy\ Cool\ Normal\ Light\ Warm\ Same \rangle$$

Classified as *negative* by all hypothesis, since does not satisfy any hypothesis in G

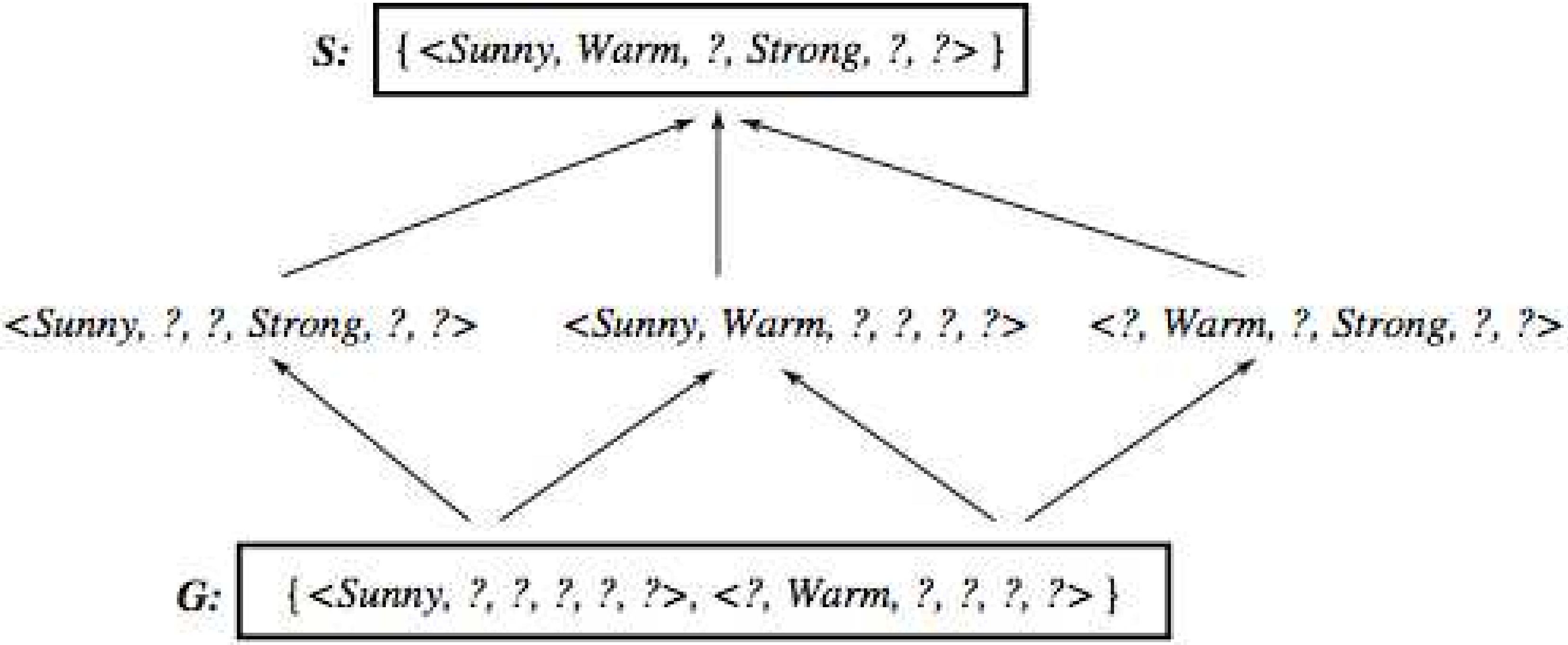
Classifying new examples



$$\langle Sunny\ Warm\ Normal\ Light\ Warm\ Same \rangle$$

Uncertain classification: half hypothesis are consistent, half are not consistent

Classifying new examples



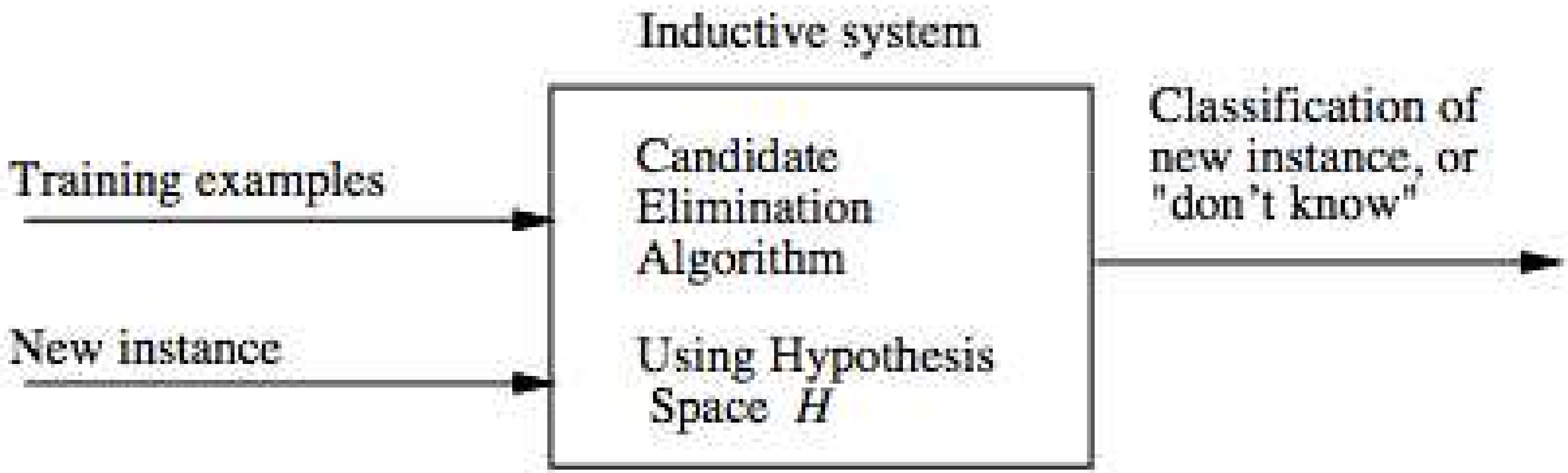
$\langle$*Sunny, Cold, Normal, Strong, Warm, Same*$\rangle$

4 hypothesis not satisfied; 2 satisfied
Probably a negative instance. Majority vote?

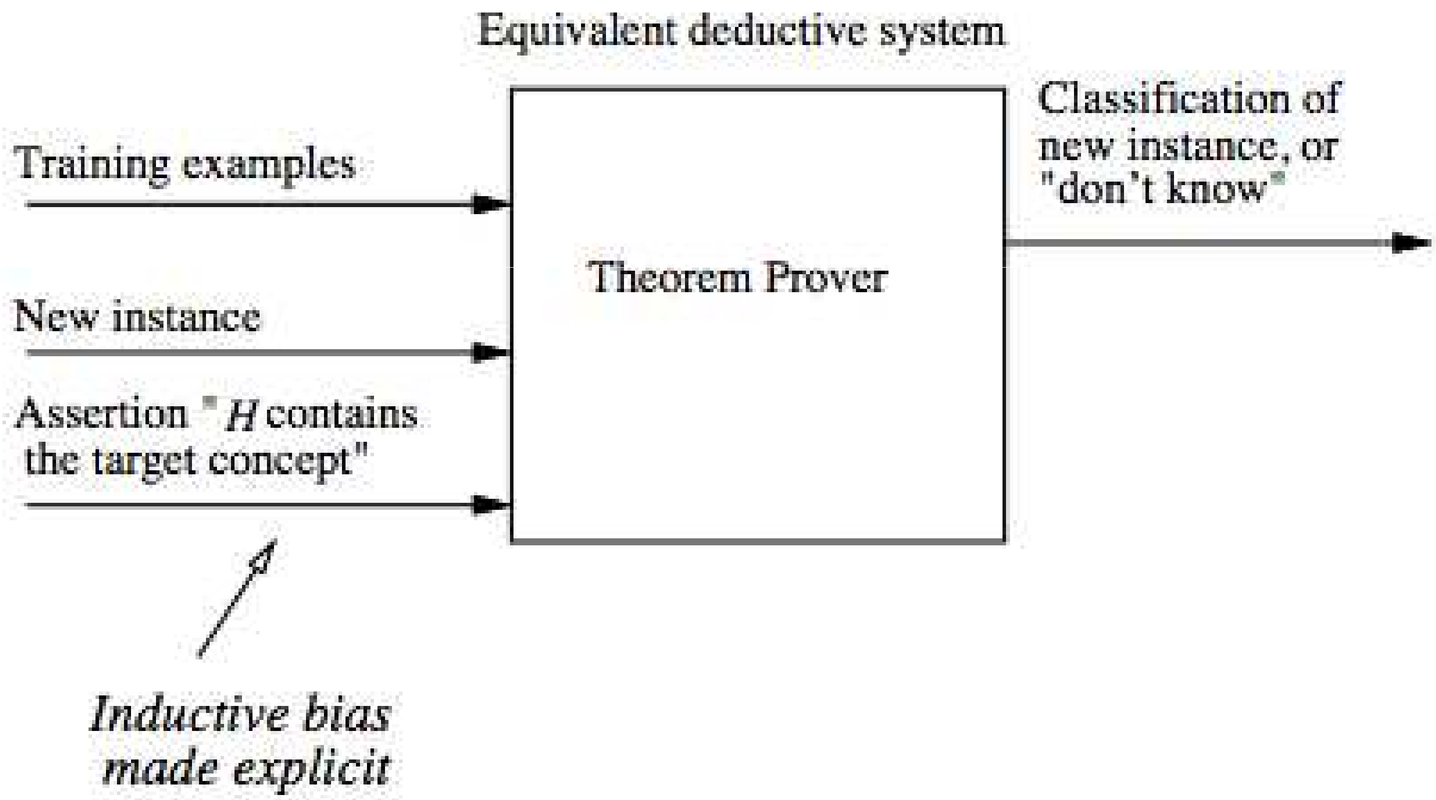
Hypothesis space and bias

- What if H does not contain the target concept?
- Can we improve the situation by extending the hypothesis space?
- Will this influence the ability to generalize?
- These are general questions for inductive inference, addressed in the context of Candidate-Elimination
- Suppose we include in H every possible hypothesis … including the ability to represent disjunctive concepts

Inductive system



Equivalent deductive system



Bibliography

- Machine Learning, Tom Mitchell, Mc Graw-Hill International Editions, 1997 (Cap 2).