UT2 - Answer key.

Part-A.

① $G = (V, T, P, S)$

② $L = \{ (ab)^n \mid n \geq 1 \}$

③ 1. Acceptance by empty stack

$L(M) = \{ w \mid (q_0, w, z_0) \vdash^* (p, \lambda, \lambda), \ p \in Q \}$

2. Acceptance by final state

$L(M) = \{ w \mid (q_0, w, z_0) \vdash^* (p, \lambda, \alpha), \ p \in F, \ \alpha \in \Gamma^* \}$

④ $S \rightarrow 0S0 \mid 1S1 \mid \lambda \mid 0 \mid 1$

⑤ yes, stack for push & pop operations, Auxiliary memory.

⑥ Null production:

$S \rightarrow \lambda$

Unit production:

$A \rightarrow B$

⑦ $S \Rightarrow aB$

$\Rightarrow aaBB \ (B \rightarrow aBB)$

$\Rightarrow aaa\underline{BB}B \ (B \rightarrow aBB)$

$\Rightarrow aaabbB \ (B \rightarrow b)(B \rightarrow b)$

$\Rightarrow aaabbabB \ (B \rightarrow b)$

$\Rightarrow aaabbabbS \ (B \rightarrow bS)$

$\Rightarrow aaabbabbbA \ (S \rightarrow bA)$

$\Rightarrow aaabbabbba \ (A \rightarrow a)$

Part-B.

8(a) $L = \{ww^R \mid w \in (0+1)^*\}$

To push 'w':

$\delta(q_0, 0, z_0) = (q_1, 0z_0)$

$\delta(q_0, 1, z_0) = (q_1, 1z_0)$

$\delta(q_1, 0, 0) = (q_1, 00)$

$\delta(q_1, 1, 1) = (q_1, 11)$

$\delta(q_1, 0, 1) = (q_1, 01)$

$\delta(q_1, 1, 0) = (q_1, 10)$

To check for 'w^R':

$\delta(q_1, \lambda, 0) = (q_2, 0)$

$\delta(q_1, \lambda, 1) = (q_2, 1)$

To match 'w' with 'w^R':

$\delta(q_2, 0, 0) = (q_2, \lambda)$

$\delta(q_2, 1, 1) = (q_2, \lambda)$

Termination:

$\delta(q_2, \lambda, z_0) = (q_3, z_0)$

$q_3 \in F$.

8. (b). ID – (q, w, x)

$q \in Q$, w is input string, x – stack content.

$\delta(q_0, 1001, z_0) \vdash (q_1, 001, 1z_0)$

$\vdash (q_1, 01, 01z_0)$

$\vdash (q_2, 01, 01z_0)$

$\vdash ((q_2, 1, 1z_0)$

$\vdash (q_2, \lambda, z_0)$

$\vdash (q_3, \lambda)$.

9. (a). Ambiguous grammar:

Any $w \in L(G)$, $L(G)$ has two (or) more left or right most derivations.

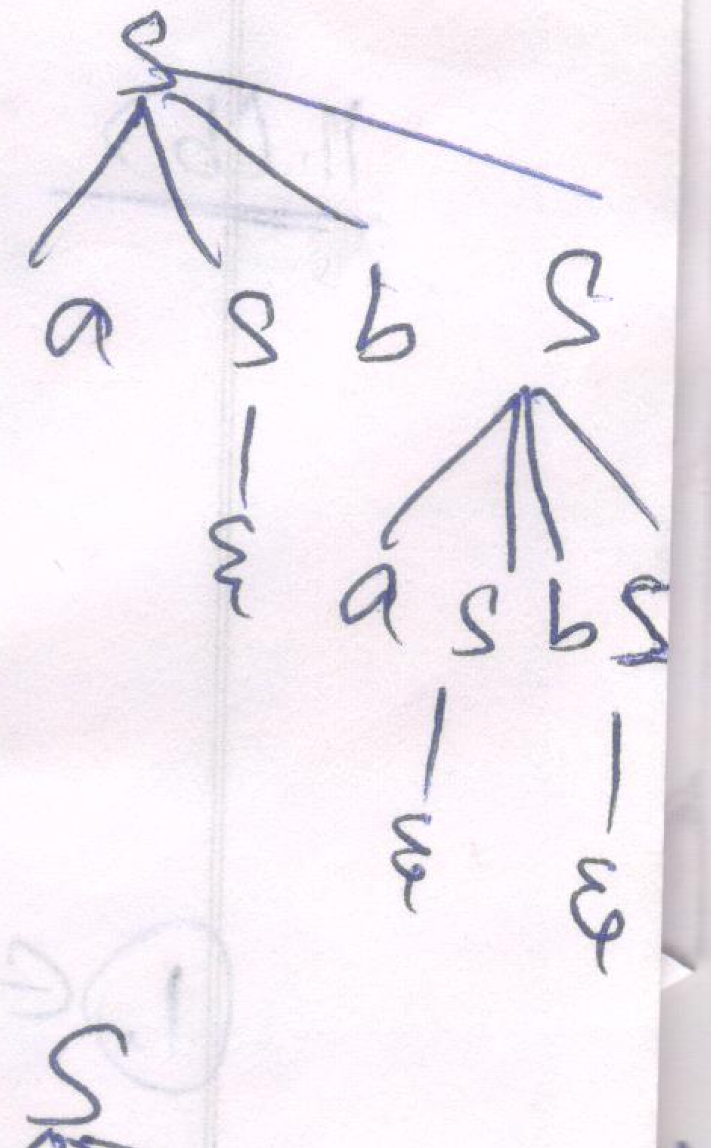
9. (b). $S \to aSbS \mid bSaS \mid \epsilon$ Let $w = abab$

LMD1: $S \Rightarrow aSbS$

$\Rightarrow abS$ $(S \to \epsilon)$

$\Rightarrow abaSbS$ $(S \to aSbS)$

$\Rightarrow abab$ $(S \to \epsilon)$



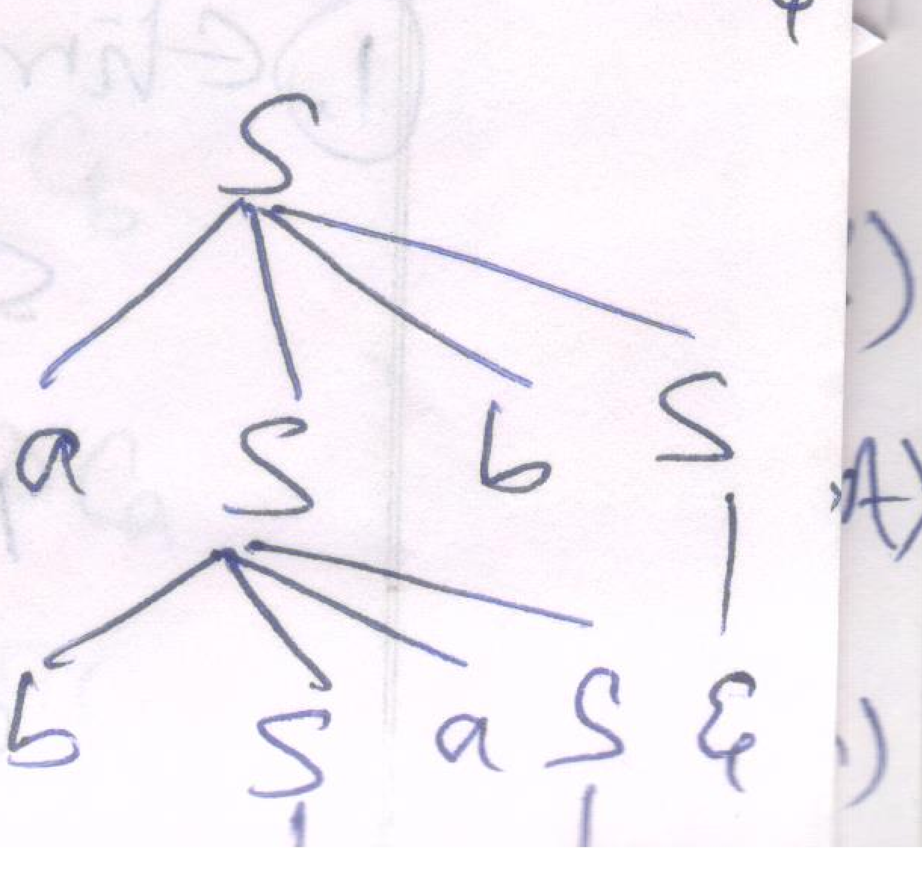
LMD2: $S \Rightarrow aSbS$

$\Rightarrow abSaSbS$ $(S \to bSaS)$

$\Rightarrow abaSbS$ $(S \to \epsilon)$

$\Rightarrow ababS$ $(S \to \epsilon)$

$\Rightarrow abab$ $(S \to \epsilon)$



(10) $G = (V, T, P, S)$ $S \overset{*}{\Rightarrow} \alpha$, iff there is a derivation tree which yield α.

I. $A \overset{*}{\Rightarrow} \alpha$, iff A derives an A-tree with α.

$$A \rightarrow x_1 x_2 \cdots\cdots x_m$$
$$\Rightarrow \alpha_1\ x_2 \cdots\cdots x_m$$
$$\overset{*}{\Rightarrow} a_1, a_2 \cdots\cdots a_m = \alpha.$$

II. A tree with yield α derives $A \Rightarrow \alpha$.

$$A \rightarrow x_1 x_2 \cdots x_m \overset{k-1}{\Rightarrow} \alpha$$

<u>case (i)</u> $x_i = a_i$

<u>case (ii)</u> $x_i \overset{*}{\Rightarrow} \alpha_i$

<u>(11)(a)</u> <u>CNF:</u>

$$S \rightarrow a \mid AB \mid \lambda$$

$S \rightarrow \lambda$, if S does not appears on the RHS of any production.

<u>11.(b)</u> <u>Given:</u> $S \rightarrow AACD$
$A \rightarrow aAb \mid \lambda$
$C \rightarrow aC \mid a$
$D \rightarrow aDa \mid bDb \mid \lambda$.

(1.) Eliminating null production.

$S \rightarrow AACD \mid ACD \mid AAC \mid AC \mid C \mid CD$

$A \rightarrow aAb \mid ab$

② Eliminating unit Production

$S \rightarrow AACD \mid ACD \mid AAC \mid AC \mid CD \mid aC \mid a$

$A \rightarrow aAb \mid ab$

$C \rightarrow aC \mid a$

$D \rightarrow aDa \mid bDb \mid aa \mid bb$

③ Replacing the terminals of RHS with non terminal.

$S \rightarrow AC \mid CD \mid C_aC \mid a \mid AACD \mid ACD \mid AAC$

$A \rightarrow C_aAC_b \mid C_aC_b$

$C \rightarrow C_aC \mid a$

$D \rightarrow C_aDC_a \mid C_bDC_b \mid C_aC_a \mid C_bC_b$

④ restricting the no. of variables on the RHS.

$S \rightarrow AC \mid CD \mid C_aC \mid a$ $\quad S \rightarrow AD_1$

$S \rightarrow AD_2 \,,\, D_2 \rightarrow CD$ $\quad D_1 \rightarrow AD_2$

$S \rightarrow AD_3 \,,\, D_3 \rightarrow AC$ $\quad D_2 \rightarrow CD$

$A \rightarrow C_aC_b \mid C_aD_4$ $\quad D_4 \rightarrow AC_b$

$C \rightarrow C_aC \mid a$

$D \rightarrow C_aC_a \mid C_bC_b \mid$ $\quad D_5 \rightarrow DC_a$

$C_aD_5 \mid C$ D $\quad D_6 \rightarrow DC_b$

12.(a). GNF:

$S \rightarrow a$

$S \rightarrow a\alpha \quad \alpha \in V^*$

$S \rightarrow \lambda$ iff S does not appears on the RHS and $\lambda \in L(G)$.

12.(b) Given:

$S \rightarrow AB$

$A \rightarrow BS \mid b$

$B \rightarrow SA \mid a$

① Replacing $S = A_1$, $A = A_2$, $B = A_3$.

$A_1 \rightarrow A_2 A_3$

$A_2 \rightarrow A_3 A_1 \mid b$

$A_3 \rightarrow A_1 A_2 \mid a$

② substitute A_1 in A_3

$A_1 \rightarrow A_2 A_3$

$A_2 \rightarrow A_3 A_1 \mid b$

$A_3 \rightarrow A_2 A_3 A_2 \mid a$

③ substitute A_2 in A_3.

$A_1 \rightarrow A_2 A_3$

$A_2 \rightarrow A_3 A_1 \mid b$

$A_3 \rightarrow A_3 A_1 A_3 A_2 \mid b A_3 A_2 \mid a$

Lemma 2 of GNF.

$A \rightarrow A\alpha \mid \beta_1 \mid \beta_2$

(4) 'A' Production rule: $A \rightarrow \beta_1 | \beta_2 | \dots | \beta_s$

$A \rightarrow \beta_1 B | \beta_2 B | \dots | \beta_s B$

$$\boxed{\begin{array}{l} A_3 \rightarrow bA_3A_2 \mid a \\ A_3 \rightarrow bA_3A_2B_3 \mid aB_3 \end{array}}$$

'B' production rule: $B \rightarrow \alpha_1 | \alpha_2 | \dots | \alpha_r$

$B \rightarrow \alpha_1 B | \alpha_2 B | \dots | \alpha_r B$

$B_3 \rightarrow A_1A_3A_2$

$B_3 \rightarrow A_1A_3A_2B_3$

(5) Substitute A_3 in A_2, since A_3 Productions are in GNF after lemma 2.

$A_2 \rightarrow A_3A_1 \mid b$

$$\boxed{\begin{array}{l} A_2 \rightarrow bA_3A_2A_1 \mid aA_1 \mid bA_3A_2B_3A_1 \mid \\ \qquad aB_3A_1 \mid b \end{array}}$$

(6) Substitute A_2 in A_1

$A_1 \rightarrow A_2A_3$

$$\boxed{\begin{array}{l} A_1 \rightarrow bA_3A_2A_1A_3 \mid aA_1A_3 \mid bA_3A_2B_3A_1A_3 \mid \\ \qquad aB_3A_1A_3 \mid bA_3 \end{array}}$$

⑦ Substitute A_1 in B_3

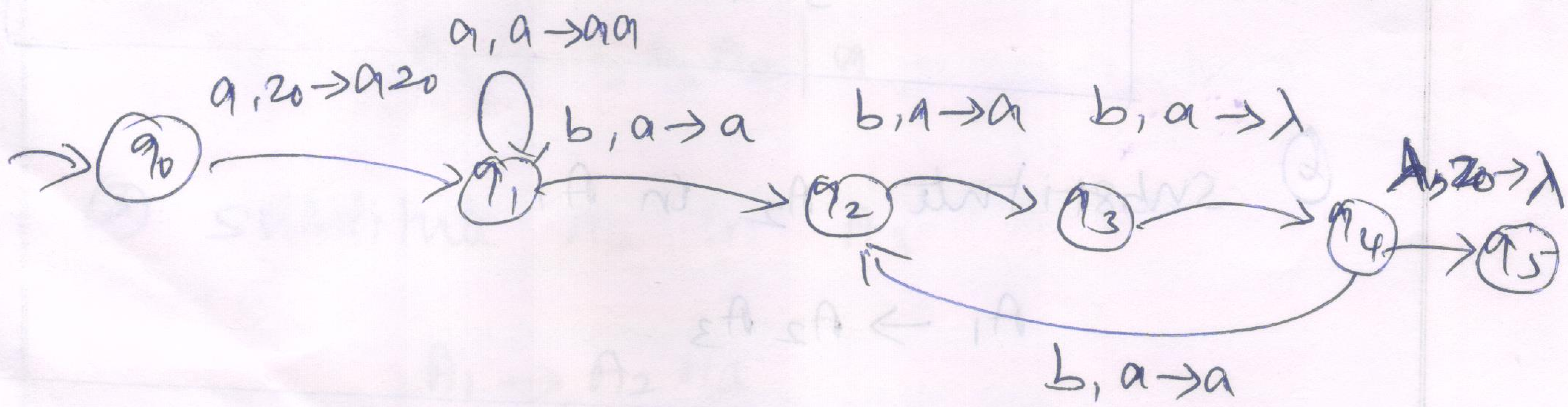
$$B_3 \to A_1 A_3 A_2$$

$$\boxed{\begin{aligned} B_3 \to\ & bA_3A_2A_1A_3A_3A_2 \mid aA_1A_3A_3A_2 \mid \\ & bA_3A_2B_3A_1A_3A_3A_2 \mid aB_3A_1A_3A_3A_2 \mid \\ & bA_3A_3A_2 \end{aligned}}$$

$$B_3 \to A_1 A_3 A_2 B_3$$

$$\boxed{\begin{aligned} B_3 \to\ & bA_3A_2A_1A_3A_3A_2B_3 \mid aA_1A_3A_3A_2B_3 \mid \\ & bA_3A_2B_3A_1A_3A_3A_2B_3 \mid aB_3A_1A_3A_3A_2B_3 \mid \\ & bA_3A_3A_2B_3 \end{aligned}}$$

13.(a). $L = \{a^n b^{3n} \mid n \geq 1\}$



$w = abb$

$\delta(q_0, abb, z_0) \vdash (q_1, bb, az_0)$

$\vdash (q_2, b, az_0)$

$\vdash (q_3, \lambda, az_0)$ → Rule not defined & stack is not empty

Invalid string.

$w = aabbbbbb$

$\delta(q_0, aabbbbbb, z_0) \vdash (q_1, abbbbbb, az_0)$

$\vdash (q_1, bbbbbb, aaz_0)$

$\vdash (q_2, bbbbb, aaz_0)$

$\vdash (q_3, bbbb, aaz_0)$

$\vdash (q_4, bbb, az_0)$

$\vdash (q_2, bb, az_0)$

$\vdash (q_3, b, az_0)$

$\vdash (q_4, \lambda, z_0)$

$\vdash (q_5, \lambda, \lambda)$

Stack becomes empty.

valid string.