PART - A.

1. DFA $M = (Q, \Sigma, \delta, F, \delta)$

Q - Set of states.

Σ - set of input symbols.

δ - $q_0 \in Q$ initial state

$F \subseteq Q$ - set of final states

$\delta : Q \times \Sigma \rightarrow Q$ is a transition function.

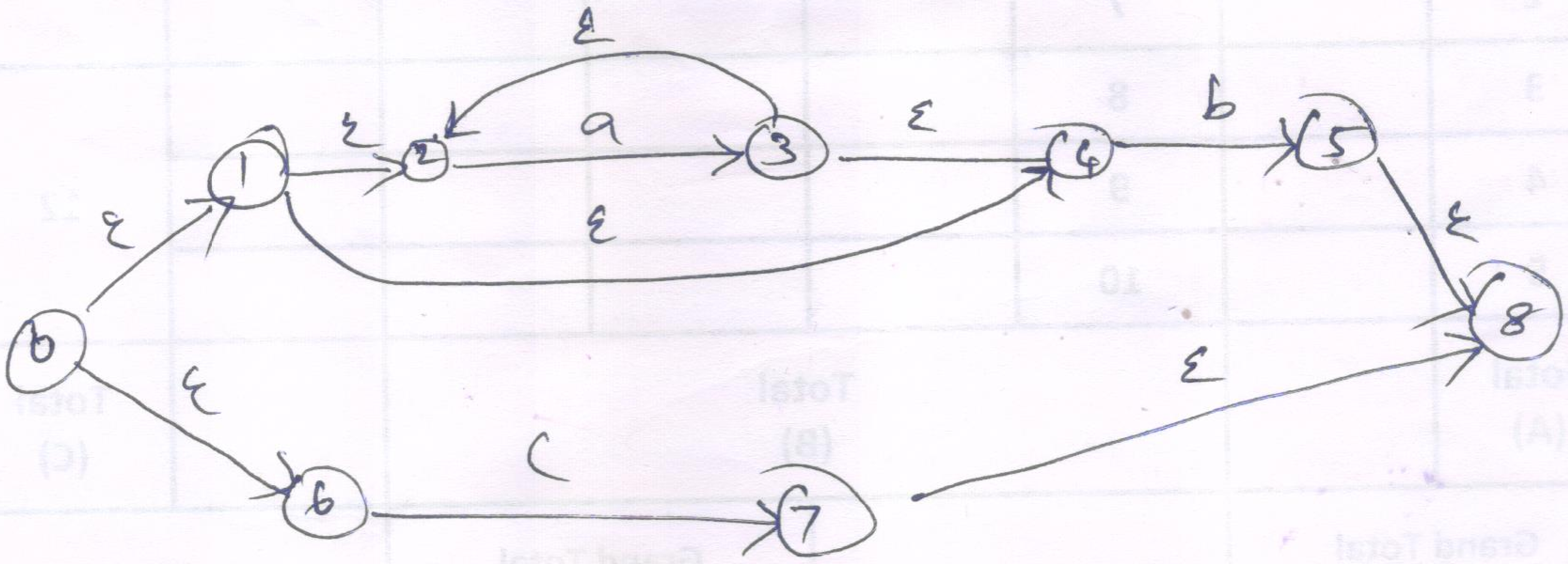
2. (i) Regular sets are closed under union, concatenation & closure.

(ii) The class of regular sets are closed under complementation

(iii) intersection (iv) set difference (v) Reverse.

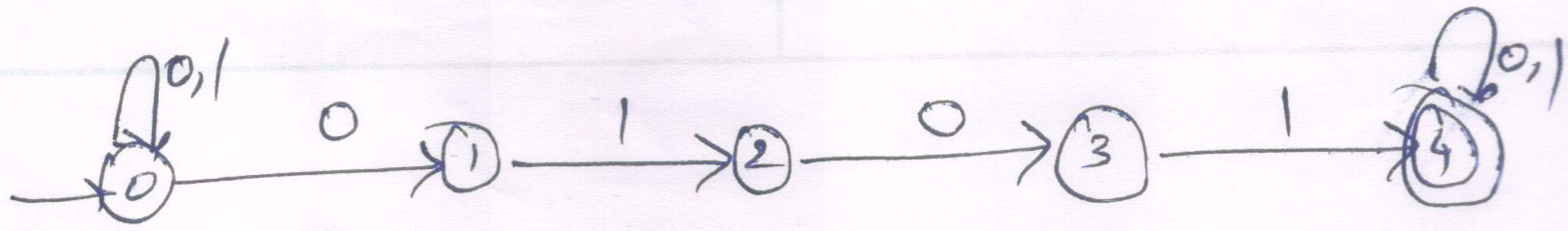
3. ϵ-NFA $a^* b / c$

$(a^* b) / c$



4. $00(000)^*$

5.



6. $(0/1)^* \; 1 \; (0/1)^*$

7. $\bar{\delta}(q, wa) = \bigcup_{P \in \bar{\delta}(q,w)} \delta(P, a)$

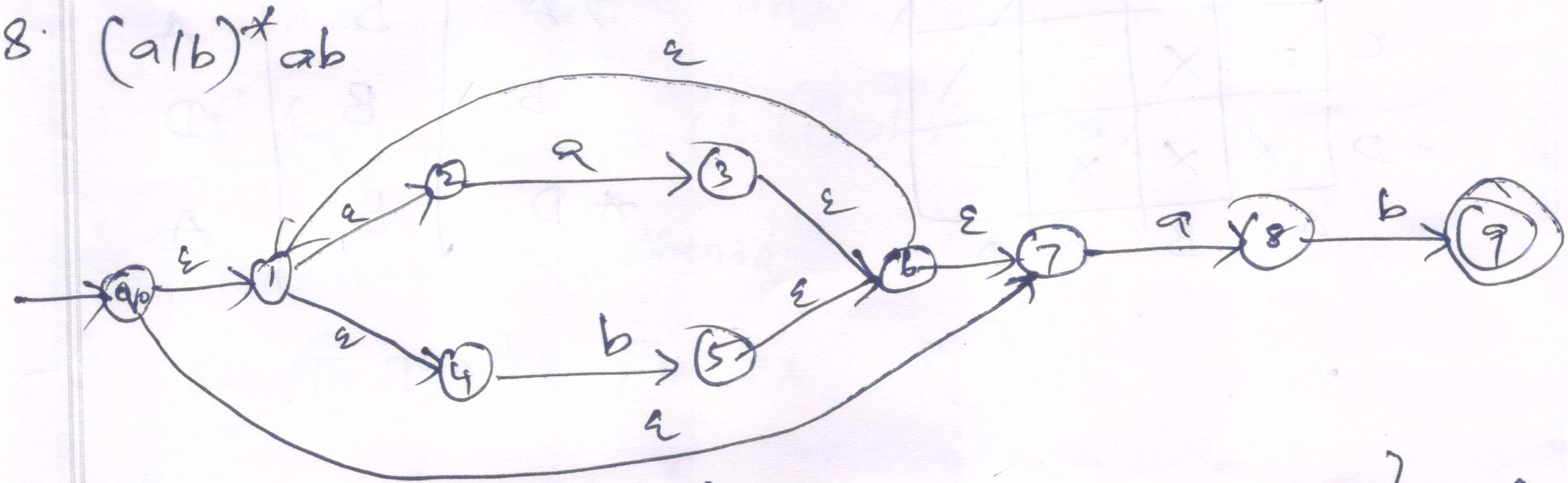
for each $w \in \Sigma^*$

$a \in \Sigma$ and

$P \in (q, w)$

Part-B

8. $(a/b)^* ab$



$\varepsilon\text{-closure}(q_0) = \{q_0, q_1, q_2, q_4, q_7\} = A$

$Mov(A, a) = \{q_3, q_8\}$

$Mov(A, b) = \{q_5\}$

$\varepsilon\text{-closure}(Mov(A, a)) = \{q_3, q_8, q_6, q_1, q_2, q_4, q_7\} = B$

$\varepsilon\text{-closure}(Mov(A, b)) = \{q_5, q_6, q_7, q_1, q_2, q_4\} = C$

$Mov(B, a) = \{q_3, q_8\}$ $\quad \varepsilon\text{-closure}(Mov(B, a)) = B$

$Mov(B, b) = \{q_5, q_9\}$

$\varepsilon\text{-closure}(Mov(B, b)) = \{q_5, q_9, q_6, q_7, q_1, q_2, q_4\} = D$

$Mov(C, a) = \{q_3, q_8\} = B$

$Mov(C, b) = \{q_5\} = C$

$Mov(D, a) = \{q_3, q_8\}$ $\quad$ $\varepsilon\text{-closure}(Mov(D, a)) = B$.

$Mov(D, b) = \{q_5\}$ $\quad$ $\varepsilon\text{-closure}(Mov(D, b)) = C$.

δ	a	b
$\rightarrow$ A	B	C
B	B	D
C	B	C
*D	B	C

	A	B	C	D
$\rightarrow$ A	=			
B	X	=		
C	=	X	=	
*D	X	X	X	=

δ	a	b
$\rightarrow$ A	B	A
B	B	D
*D	B	A

9\.
a. A language L is accepted by some DFA iff L is accepted by some NFA.

6 marks

~~Proof~~

Let $M = (Q, \Sigma, q_0, F, \delta)$ be NFA accepting L,

We construct $M' = (Q', \Sigma, q_0', F', \delta')$ where

$Q' = 2^{Q}$

$q_0' = [q_0]$

F' set of final states.

we define

$\delta'([q_1, q_2 \cdots q_i], a) = \delta(q_1, a) \cup \delta(q_2, a) \cdots \cup \delta(q_i, a)$

equivalently.

$$\delta'([q_1, q_2 \, q_i], a) = [p_1 p_2 \ldots p_j]$$

iff

$$\delta(\{q_1, q_2, \ldots q_i\}, a) = \{p_1, p_2 \cdots p_j\}$$

Basic

$$q_0' = [q_0]$$

x must be ε.

Induction.

xa length $m+1$.

$$\delta'([q_0], xa) = \delta'(\delta'([q_0], x), a)$$

by induction hypothesis

$$\delta'([q_0], x) = [p_1, p_2 \cdots p_j]$$

iff

$$\delta(q_0, x) = \{p_1, p_2, \cdots p_j\}$$

By definition,

$$\delta([p_1 \, p_2 \cdots p_j], a) = [r_1 r_2 \cdots r_k]$$

iff

$$\delta(\{p_1, p_2, \cdots p_j\}, a) = \{r_1, r_2 \cdots r_k\}$$

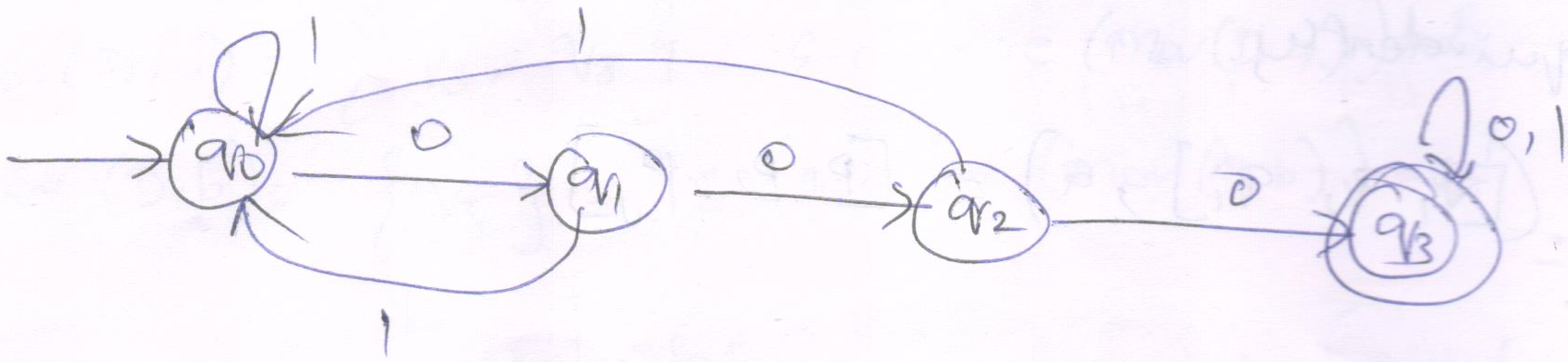
Then

$$\delta'([q_0], xa) = [r_1 r_2 \cdots r_k]$$

iff $\delta(q_0, xa) = \{r_1 r_2 \cdots r_k\}$

$\therefore \; L(m) = L(m')$

4 marks b



10. a.

8 marks.

ε-NFA $\longrightarrow$ NFA.

δ	ε	0	1
$\rightarrow p$	$\{r\}$	$\{q\}$	$\{p, r\}$
q	ϕ	$\{p\}$	ϕ
$* r$	$\{p, q\}$	$\{r\}$	$\{p\}$

$$\delta'(p, 0) = \hat{\delta}(p, 0)$$

$$= \varepsilon\text{-closure}(\delta(\hat{\delta}(p, \varepsilon), 0))$$

$$= \varepsilon\text{-closure}(\delta(\{p, r\}, 0))$$

$$= \varepsilon\text{-closure}(r\})$$

$$= \{p, q, r\}$$

$$\delta'(p, 1) = \hat{\delta}(p, 1)$$

$$= \varepsilon\text{-closure}(\delta(\hat{\delta}(p, \varepsilon), 1))$$

$$= \varepsilon\text{-closure}(\delta(\{p, r\}), 1))$$

$$= \varepsilon\text{-closure}(p)$$

$$= \cancel{\{q, r\}} = \{p, q, r\}$$

$$\delta'(q, 0) = \hat{\delta}(q, 0)$$

$$= \varepsilon\text{-closure}(\delta(\hat{\delta}(q, \varepsilon), 0))$$

$= \varepsilon\text{-closure}\ (\cancel{\phi}q)$

$= \cancel{\phi}\ \{p, q, r\}$

$\delta'(q, 1) = \hat{\delta}(q, 1)$

$= \varepsilon\text{-closure}\ (\delta(\hat{\delta}(q, \varepsilon), 1))$

$= \varepsilon\text{-closure}\ (\delta(q, 1))$ $= \varepsilon\text{-closure}\ (\phi$

$= \cancel{\{q\}}\ \phi$

$\delta'(q, 0) = \hat{\delta}(r, 0)$

$= \varepsilon\text{-closure}\ (\delta(\hat{\delta}(r, \varepsilon), 0))$

$= \varepsilon\text{-closure}\ (\delta(\{r, p, q\}\ 0)$

$= \varepsilon\text{-closure}\ (\{p, q, r\})$

$= \{p, q, r\}$

$\delta'(r, 1) = \hat{\delta}(r, 1)$

$= \varepsilon\text{-closure}\ (\delta(\hat{\delta}(r, \varepsilon), 1))$

$= \varepsilon\text{-closure}\ (\delta(\{p, q, r\}, 1))$

$= \varepsilon\text{-closure}\ (\{p, \cancel{q}\ r\})$

$= \{p, q, r\}$

δ	0	1
p	$\{p, q, r\}$	$\{p, q, r\}$
q	$\{p, q, r\}$	ϕ
r	$\{p, q, r\}$	$\{p, q, r\}$

+ transition diag.

b

2marks.

~~$\delta(q$~~

$\delta(p, 001) = \delta(\delta(p,0), 01)$

$= \delta(\{p, q, r\}, 01)$

$= \delta((\delta(p,0) \cup \delta(q,0) \cup \delta(r,0)), 1)$

$= \delta(\delta(p, q, r\}, 1)$

$= \{p, q, r\}$

$\therefore$ Accepted.

11. a) NFA to DFA.

6marks

δ	0	1
$\rightarrow p$	$\{q, s\}$	$\{q\}$
$* q$	$\{r\}$	$\{q, r\}$
r	$\{s\}$	$\{p\}$
$* s$	-	$\{p\}$

$\delta'(p, 0) = [qs]$

$\delta'(p, 1) = [q]$

$\delta'([qs], 0) = [r]$

$\delta'([qs], 1) = [p, qr]$

$\delta'([q], 0) = [r]$

$\delta'([q], 1) = [qr]$

$\delta'([r], 0) = [s]$

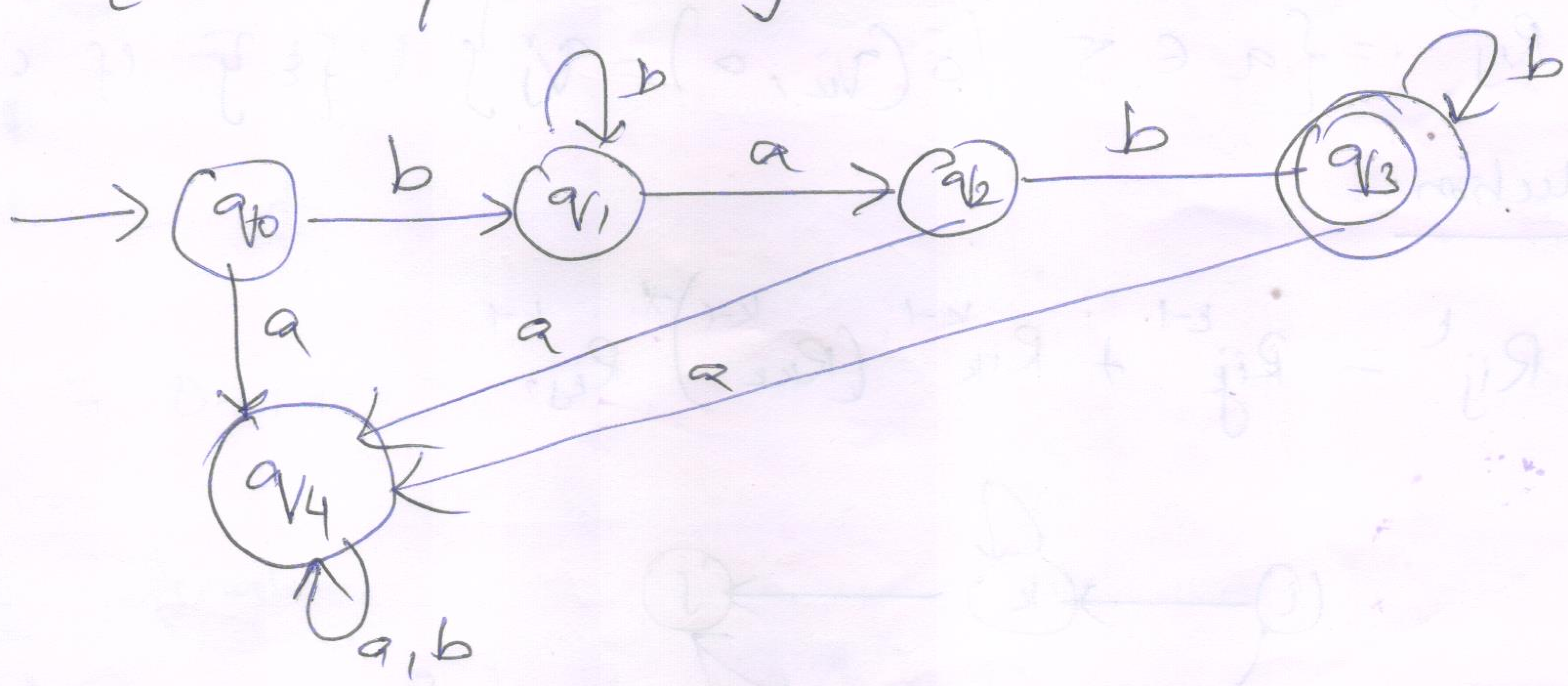
$\delta'([r], 1) = [p]$

⋮

δ'	0	1
→ [p]	[qs]	[q]
* [q]	[r]	[qr]
[r]	[s]	[p]
* [s]	φ	[p]
* [qs]	[r]	[pqr]
* [qr]	[rs]	[pqr]
* [rs]	[s]	[p]
* [pqr]	[qrs]	[pqr]
* [qrs]	[rs]	[pqr]

11.
b.
4marks.

$L = \{ b^m a b^n / m, n > 0 \}$



12. a. 8 marks.

DFA $\rightarrow$ RE R_{ij}^{k}

Theorem.

If $L = L(m)$ for some DFA $m = (Q, \Sigma, \delta, S, F)$ then there is a RE r such that $L = L(r)$

Proof.

Let L be the set accepted by the DFA

Given a DFA $m = (Q, \Sigma, \delta, S, F)$ where

$Q = \{q_1, q_2 \ldots q_n\}$ is $|Q| = n$

$R_{ij}^{k} \rightarrow \ldots\ldots$

Basis

$k = 0 \rightarrow$ no intermediate states.

$R_{ij}^{0} \rightarrow \ldots\ldots$

case (i)

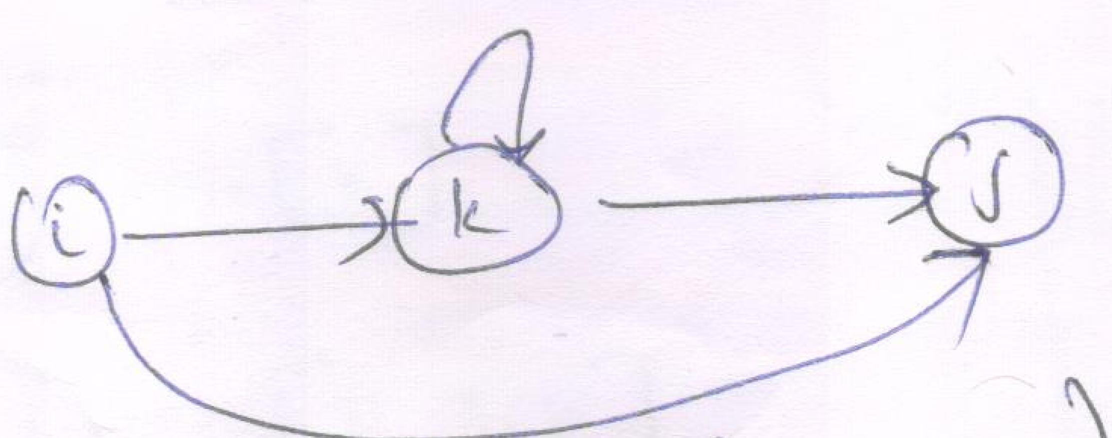
$$R_{ij}^{0} = \{a \in \Sigma \mid \delta(q_i, a) = q_j\} \quad \text{if } i \neq j$$

Case (ii)

$$R_{ij}^{0} = \{a \in \Sigma \mid \delta(q_i, a) = q_j\} \cup \{\varepsilon\} \quad \text{if } i = j$$

Induction

$$R_{ij}^{k} = R_{ij}^{k-1} + R_{ik}^{k-1} (R_{kk}^{k-1})^{*} R_{kj}^{k-1}$$



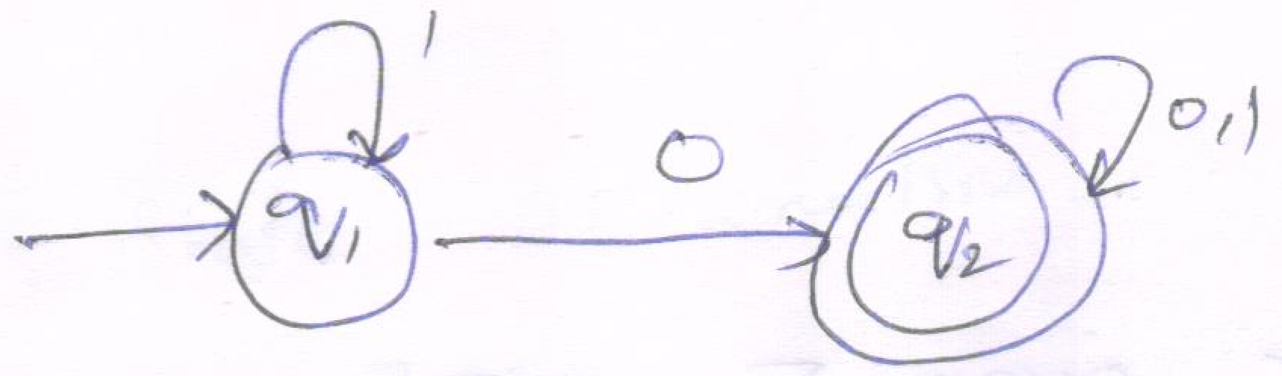
$$L(m) = \{w \in \Sigma^{*} \mid \delta(q_1, w) = q_i) \in F\}$$

$$= \bigcup_{q_j \in F} R_{ij}^{n}$$

$R_{ij}^{n} \rightarrow q_{\cdot} \ldots \quad F = \ldots\ldots$

$$L(m) = R_{ij_1}^{n} + R_{ij_2}^{n} + \ldots R_{1j_p}^{n}$$

b



5marks

$\underline{k=0}$

$R_{11} = 1+\varepsilon$

$R_{12} = 0$

$R_{22} = 0+1+\varepsilon$

$R_{21} = \phi$

$\underline{k=1}$

$R_{11}^{1} = R_{11}^{0} + R_{11}^{0} (R_{11}^{0})^* R_{11}^{0}$

$= (1+\varepsilon) + (1+\varepsilon)(1+\varepsilon)^* (1+\varepsilon)$

$= (1+\varepsilon)(\varepsilon + (1+\varepsilon)^* (1+\varepsilon))$

$= (1+\varepsilon) 1^*$

$R_{12}^{1} = R_{12}^{0} + R_{11}^{0} (R_{11}^{0})^* R_{12}^{0}$

$= 0 + (1+\varepsilon)(1+\varepsilon)^* 0$

$= 1^* \cdot 0$

$R_{21}^{\phi} = \phi$

$R_{22}^{1} = 0+1+\varepsilon$

$\underline{R_{12}^{2}}$ final

$R_{12}^{2} = R_{12}^{1} + R_{12}^{1} (R_{22}^{1})^* R_{22}$

$= 1^*0 + (1^*0)(0+1+\varepsilon)^* (0+1*\varepsilon)$

$= 1^*0 (\varepsilon + (0+1+\varepsilon)^* (0+1+\varepsilon))$

$= 1^*0 (0+1+\varepsilon)^*$

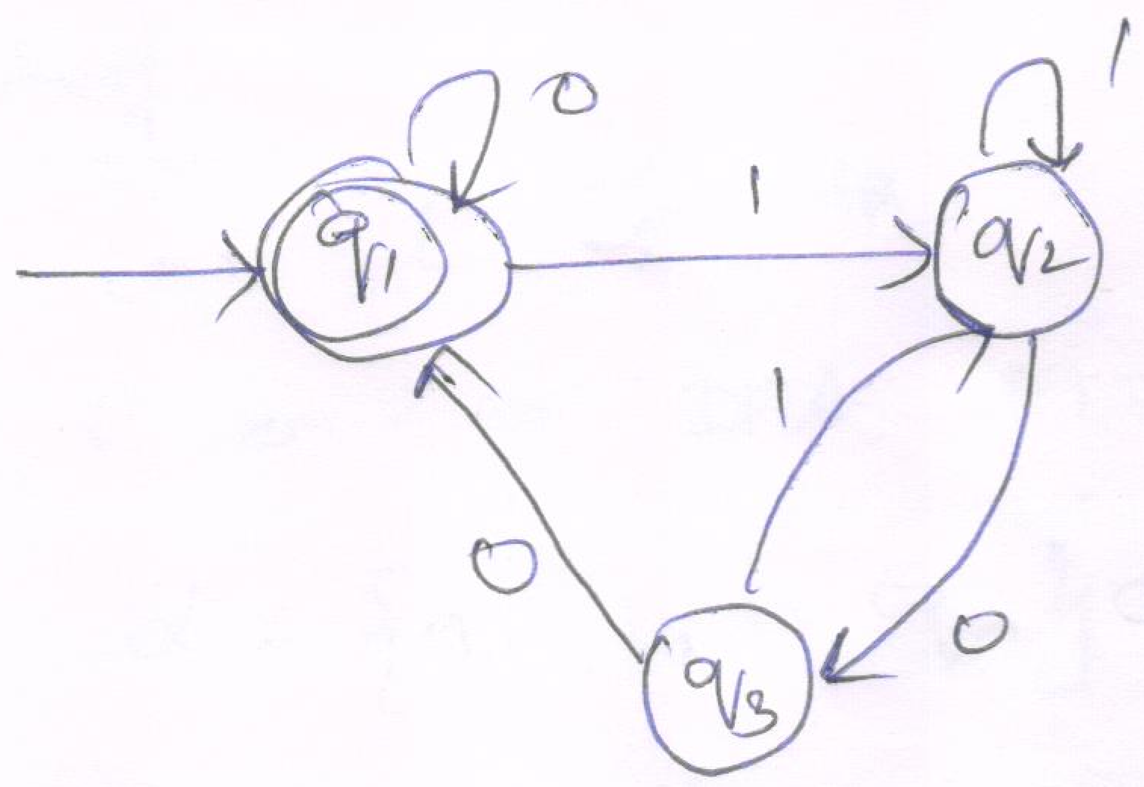
$\boxed{1^*0(0+1)^*}$

13.

a. Theorem

6 marks.

Let P & Q be 2 RE over Σ. If P does not contain ε, then the equation is $R = Q + RP$ has a solution ie $R = QP^*$



$q_1 = q_1 0 + q_2 \phi + q_3 0 + \varepsilon$

$q_1 = q_1 0 + q_3 0 + \varepsilon$ ——— (1)

$q_2 = q_1 1 + q_2 1 + q_3 1$ ——— (2)

$q_3 = q_1 \phi + q_2 0 + \phi$

$q_3 = q_2 \cdot 0$ ——— (3)

(3) in (2)

$q_2 = q_1 1 + q_2 1 + q_2 01$

$q_2 = q_1 1 + q_2 (1+01)$ $\quad R = Q + RP$

$\therefore q_2 = q_1 1 (1+01)^*$

$q_1 = q_1 0 + q_2 0 \cdot 0 + \varepsilon$

$= q_1 0 + q_1 1 (1+01)^* 00 + \varepsilon$

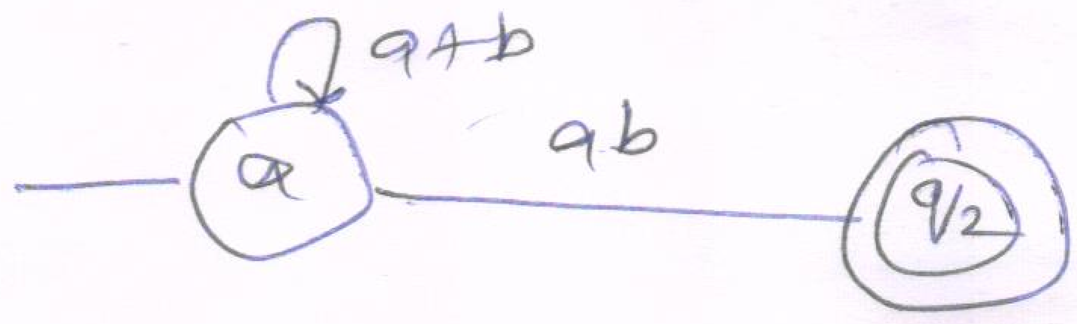
$q_1 = \varepsilon (0 + 1(1+01)^* 00)^*$

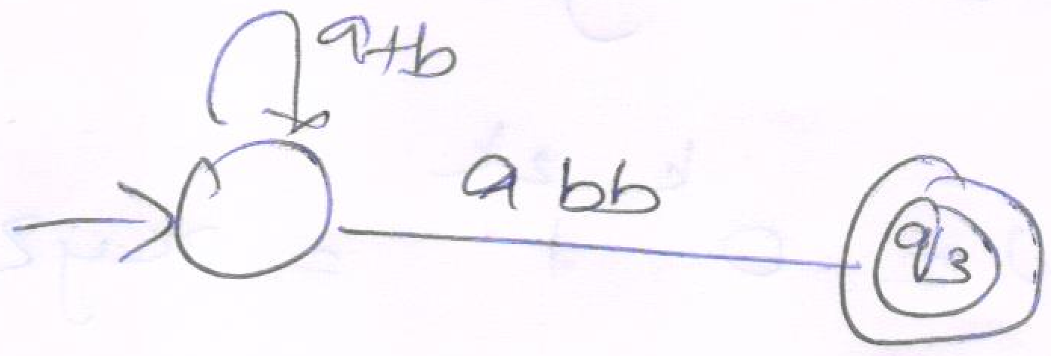
$\boxed{(0 + 1(1+01)^* 00)^*}$

b

4 marks

State elimination.





$(a+b)^* ab$ $\qquad$ $(a+b)^* abb$

$(a+b)^* abb + (a+b)^* ab$

14.

a.

4 marks. Let L be RL. Then there is a constant n such that for every string w in L such ~~that~~ $|w| \geq n$ break w into 3 strings

$w = xyz$ such that

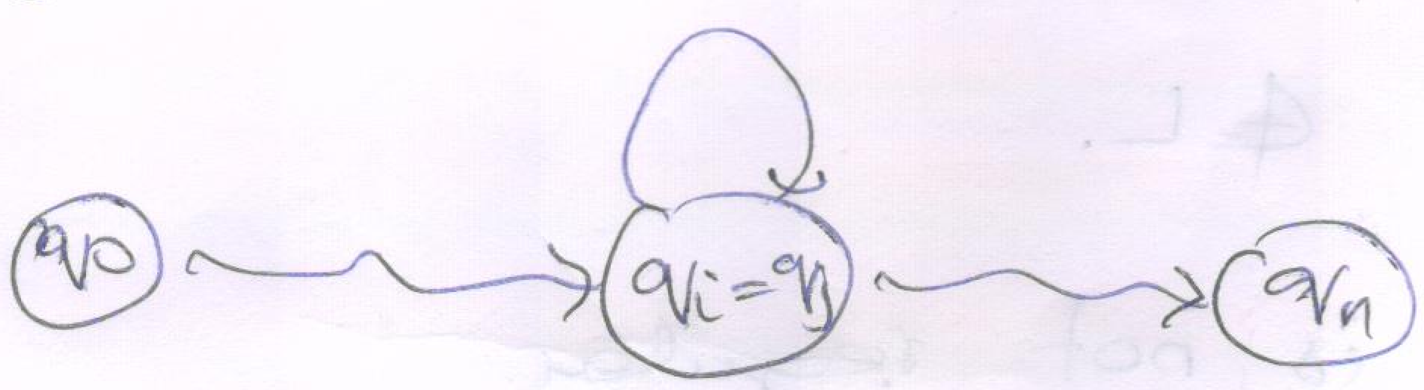
$y \neq \varepsilon$ i.e. $|y| > 0$,

$|xy| \leq n$ $\forall i \geq 0$ $xy^iz \in L$

Proof.

explanation

$\delta(q_0, a_1a_2 \cdots a_i) = q_i$



$\delta(q_0, a_1a_2 \cdots a_i) = q_i = q_j$

$\delta(q_i, a_{i+1} \cdots a_j) = q_i$

$\delta(q_j, a_{j+1} \cdots a_m) = q_n$

$\therefore$ $\delta(q_i, y^i) = q_i$ $\forall$ $i \geq 0$

b. 6 marks

$L = \{0^n 1^{2n}\}$

$w = 0^k 1^{2k} = xyz$

$xy = 0^p \quad p < n$

$y = 0^q \quad q < n$

$z = 0^{k-p} 1^{2k}$

$i=0. \quad xyz = xy^{i} y^{i-1} z$

$=$

$0^p (0^q)^{i-1} 0^{k-p} 1^{2k}$

$\rightarrow \cancel{0^p} 0^{-q} 0^{\cancel{k-p}} 1^{2k}$

$0^{k-q} 1^{2k} \notin L \qquad i=1 \qquad \in L.$

$i=2$

$\cancel{0^p} 0^{q(2-1)} 0^{k-\cancel{p}} 1^{2k}$

$0^{k+q} 1^{2k} \notin L.$

$\therefore$ L is not regular.