Additional Concepts

1. Amdahl’s Law

Amdahl's Law

- Validity of the single processor approach to achieving large scale computing capabilities

INTRODUCTION

If F is the fraction of a calculation that is sequential, and (1-F) is the fraction that can be parallelized, then the maximum speed-up that can be achieved by using P processors is $1/(F+(1-F)/P)$.

Examples

- if 90% of a calculation can be parallelized (i.e. 10% is sequential) then the maximum speed-up which can be achieved on 5 processors is

  $1/(F+(1-F)/P) = 1/(0.1+(1-0.1)/5)$

  $= 3.6$

(i.e. the program can theoretically run 3.6 times faster on five processors than on one)

Examples

- If 90% of a calculation can be parallelized then the maximum speed-up on 10 processors is 1/(0.1+(1-0.1)/10) or 5.3 (i.e. investing twice as much hardware speeds the calculation up by about 50%).
- If 90% of a calculation can be parallelized then the maximum speed-up on 20 processors is 1/(0.1+(1-0.1)/20) or 6.9 (i.e. doubling the hardware again speeds up the calculation by only 30%).

Amdahl's Law

Speedup due to enhancement E:

$$\text{Speedup(E)} = \frac{\text{ExTime w/o E}}{\text{ExTime w/ E}} = \frac{\text{Performance w/E}}{\text{Performance w/o E}}$$

Suppose that enhancement E accelerates a fraction F of the task by a factor S, and the remainder of the task is unaffected, then:

ExTime(E) =

Speedup(E) =

Amdahl's Law

$$\text{ExTime}_E = \text{ExTime} \times \left[(1 - \text{Fraction}_E) + \frac{\text{Fraction}_E}{\text{Speedup}_E} \right]$$

$$\text{Speedup} = \frac{\text{ExTime}}{\text{ExTime}_E} = \frac{1}{(1 - \text{Fraction}_E) + \dfrac{\text{Fraction}_E}{\text{Speedup}_E}}$$

$$= \frac{1}{(1 - F) + F/S}$$

Amdahl's Law

Floating point instructions are improved to run 2 times(100% improvement); but only 10% of actual instructions are FP

$$\text{Speedup} = \frac{1}{(1-F) + F/S}$$

$$= \frac{1}{(1-0.1) + 0.1/2} = \frac{1}{0.95} = 1.053$$

5.3% improvement

Example

- Suppose we want to achieve a speedup of 80 with 100 processors. What is the fraction of original computation that can be sequential?

$$\text{Speedup} = \frac{1}{(1-F) + F/S}$$

$$80 = \frac{1}{(1-x) + x/100}$$

$$(1-x) + x/100 = \frac{1}{80}$$

$$x = 99.74$$

$$\text{Sequential code} = 1 - 99.74$$

$$= 0.25\ \%$$

Example Problem

- Suppose we enhance a machine by making all floating-point instructions run five times faster. If the execution time of some benchmark before the floating-point enhancement is 10 seconds, what will the speedup be if half of the 10 seconds is spent executing floating-point instructions?

Example Problem

- Recall that according to Amdahl's law
  Exec. time after improvement = Exec. Time unaffected +
  (Exec.time effected) / (Speed improvement)
- Since total execution time is 10 sec, then floating point time is 5 sec and the remaining time is 5 sec.
  So affected time is 5 sec and unaffected time is 5 sec.
  Given : speed improvement is 5 times faster
- Exec. time after improvement = 5 + 5/5 = 6 sec.
- Speedup = (Exec. Time Before) / (Exec. Time After) = 10/6 = 1.67

MAXIMUM THEORETICAL SPEED-UP

- Amdahl's Law is a statement of the maximum theoretical speed-up you can ever hope to achieve. The actual speed-ups are always less than the speed-up predicted by Amdahl's Law