Basics of Spatial filtering

Spatial filtering

- Operating on the neighbourhood with a sub-image of size same as that of the neighbourhood.
- The sub-image is called kernel, window, template, filter or mask
- Can be used to smooth, blur, sharpen and to find edges of an image
- The values in the sub-image are called "coefficients" and not pixels
- Pickup a center pixel in an array. Apply filter in the neighbourhood. Move the center.

Spatial filtering

$$g(x,y) = \sum_{s=-K/2}^{K/2} \sum_{t=-K/2}^{K/2} w(s,t)\, f(x+s, y+t)$$

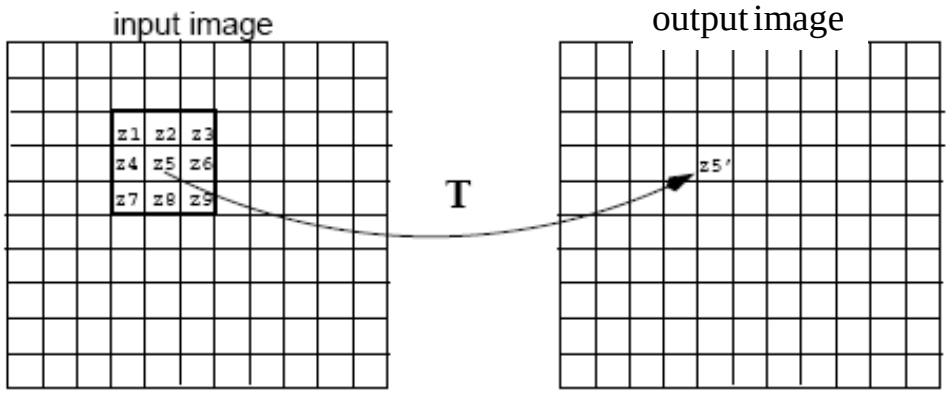
$$g(x,y) = \sum_{s=-1}^{1} \sum_{t=-1}^{1} w(s,t)\, f(x+s, y+t)$$

$$g(x,y) = w(x,y) * f(x,y) = \sum_{s=-K/2}^{K/2} \sum_{t=-K/2}^{K/2} w(s,t)\, f(x-s, y-t)$$

Linear and non-linear filtering

Convolution
Correlation

Area or Mask Processing Methods



$$g(x,y) = T[f(x,y)]$$

T operates on a neighborhood of pixels

z5′ = R = w1z1 + w2z2 + ... + z9w9

w1	w2	w3
w4	w5	w6
w7	w8	w9

Spatial filter

- When the center of the pixel reaches the border of the image
  - Zero padding
  - Replicate the pixel values
  - Use only the portion which coincides with the pixels
  - Smaller mask
- Smoothing spatial filters
  - Used for blurring and noise reduction
  - Removing the small details and bridging small gaps in lines and curves

Averaging filter

- Replacing the center pixel value by the average of its neighbourhood
- Low pass filtering or averaging
- Reduces sharp transitions in gray levels
- Noise reduction (has abrupt or sharp transitions)
- Edges are also represented by sharp transitions. Smoothing results in blurring the edges which is undesirable
- Box filter (all coefficients are 1)
- Weighted average

Spatial filtering

$(1/9) *$

1	1	1
1	1	1
1	1	1

Box filter, averaging

$(1/16) *$

1	2	1
2	4	2
1	2	1

Weighted average

Median filtering

- Order statistics filter
- Non-linear filter
- Response is based on the ordering of the pixels
- Replacing the value of the center pixel with the value determined by the ordering
- Median filter replaces the center pixel value by the median of the gray levels in the neighbourhood
- Median – arrange the pixel values in ascending order and the center value being the median
- Always odd order
- Forcing the points to have values close to the nedighbours

Sharpening

- Differentiation, differencing
- First differencing
  - Zero for flat segments
  - Non-zero for the onset of ramps
- Second differencing
  - Zero for flat segments
  - Must be zero for constant slope

Second derivatives in enhancement – The Laplacian

- *Isotropic filters*
  - *Response is independent of the direction of the discontinuities in the image*
  - *Isotropic filters are rotation invariant, that is , rotating the image and then applying the filter gives the same result for vice versa process*
- *Simplest isotropic derivative operator is Laplacian*
  - $\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y}$
  - *Linear operator*

Laplacian operator

- $\frac{\partial^2 f}{\partial x^2} = f(x+1,y) + f(x-1,y) - 2f(x,y)$
- $\frac{\partial^2 f}{\partial y^2} = f(x,y+1) + f(x,y-1) - 2f(x,y)$
- $\nabla^2(f) = f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)$

0	1	0
1	-4	1
0	1	0

1	1	1
1	-5	1
1	1	1

0	-1	0
-1	4	-1
0	-1	0

-1	-1	-1
-1	4	-1
-1	-1	-1

Laplacian for image enhancement

- Laplacian is a derivative operator and therefore highlights gray-level discontinuities and deemphasizes the slowly varying gray levels.
- Background features can be recovered by adding the original image.

$$g(x,y) = f(x,y) - \nabla^2 f(x, \quad)$$

$$g(x,y) \; y \quad (\quad) \qquad (\quad)$$

$$= f \;\; x,y \;\; + \nabla^2 f \;\; x,$$

$$y$$

Unsharp masking and high-boost filtering

- Unsharp masking: Subtracting a blurred version of an image from the image itself to obtain a sharpened image is called unsharp masking.

$$f_s(x,y) = f(x,y) - f'(x,y)$$

- High-boost filtering:

$$f_{hb}(x,y) = Af(x,y) - f'(x,y)$$

$$f_{hb}(x,y) = (A-1)f(x,y) + f(x,y) - f'(x,y)$$

$$f_{hb}(x,y) = (A-1)f(x,y) + f_s(x,y)$$