Morphological operations

Morphological operations

- Mathematical morphology is a tool deals with structure
- Morphological image processing is used to extract image components for representation and description of region shape, such as boundaries, skeletons, and the convex hull
- Collection of non-linear operations related to the shape or morphological feature of the image
- Used for removing imperfections

Reflection & translation

- Reflection:

$$\hat{B} = \{\mathrm{w} | \mathrm{w} = -\mathrm{b}, \mathrm{b} \in B\}$$

B is a set of pixels representing objects in an image

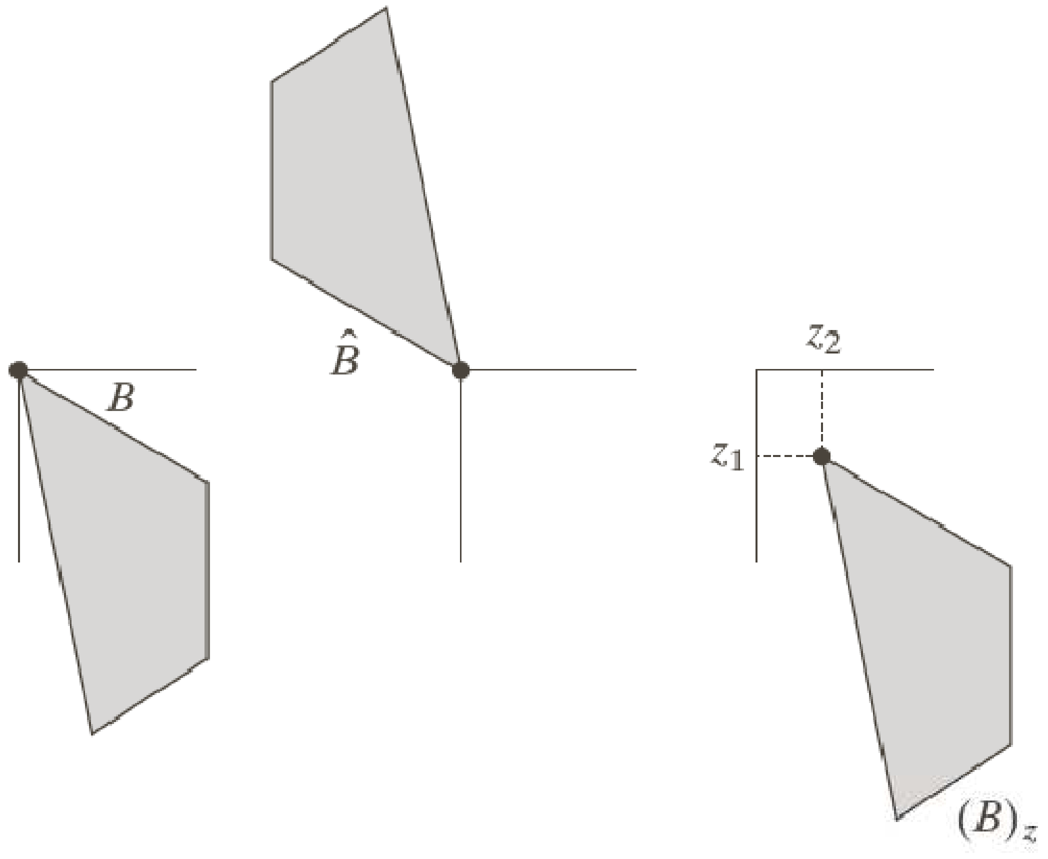
$\hat{B}$ is a set of points in B whose coordinates are replaced by (-x, -y)

- Translation:

$$(B)_z = \{c | c = b + z, for\ b \in B\}$$

$(B)_z$ is a set points in B whose coordinates are replaced by $(x+z_1, y+z_2)$

Reflection and translation

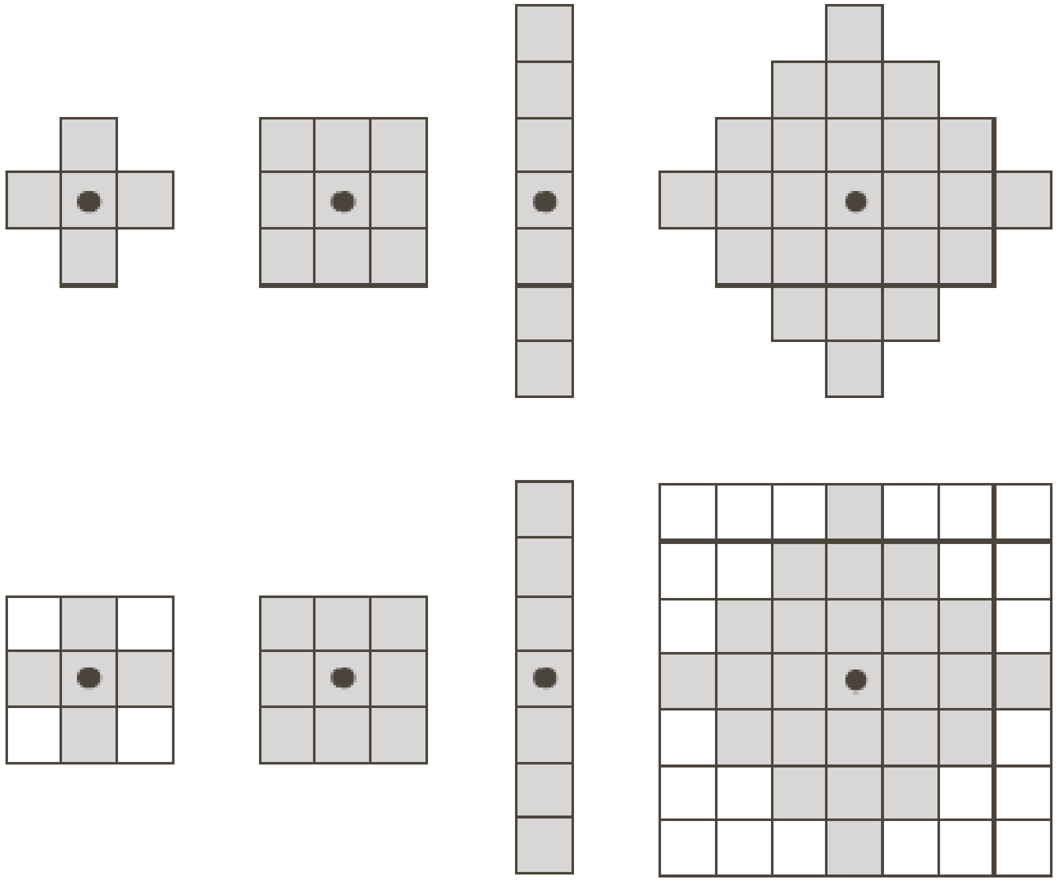


Courtesy: Gonzalez and woods, DIP

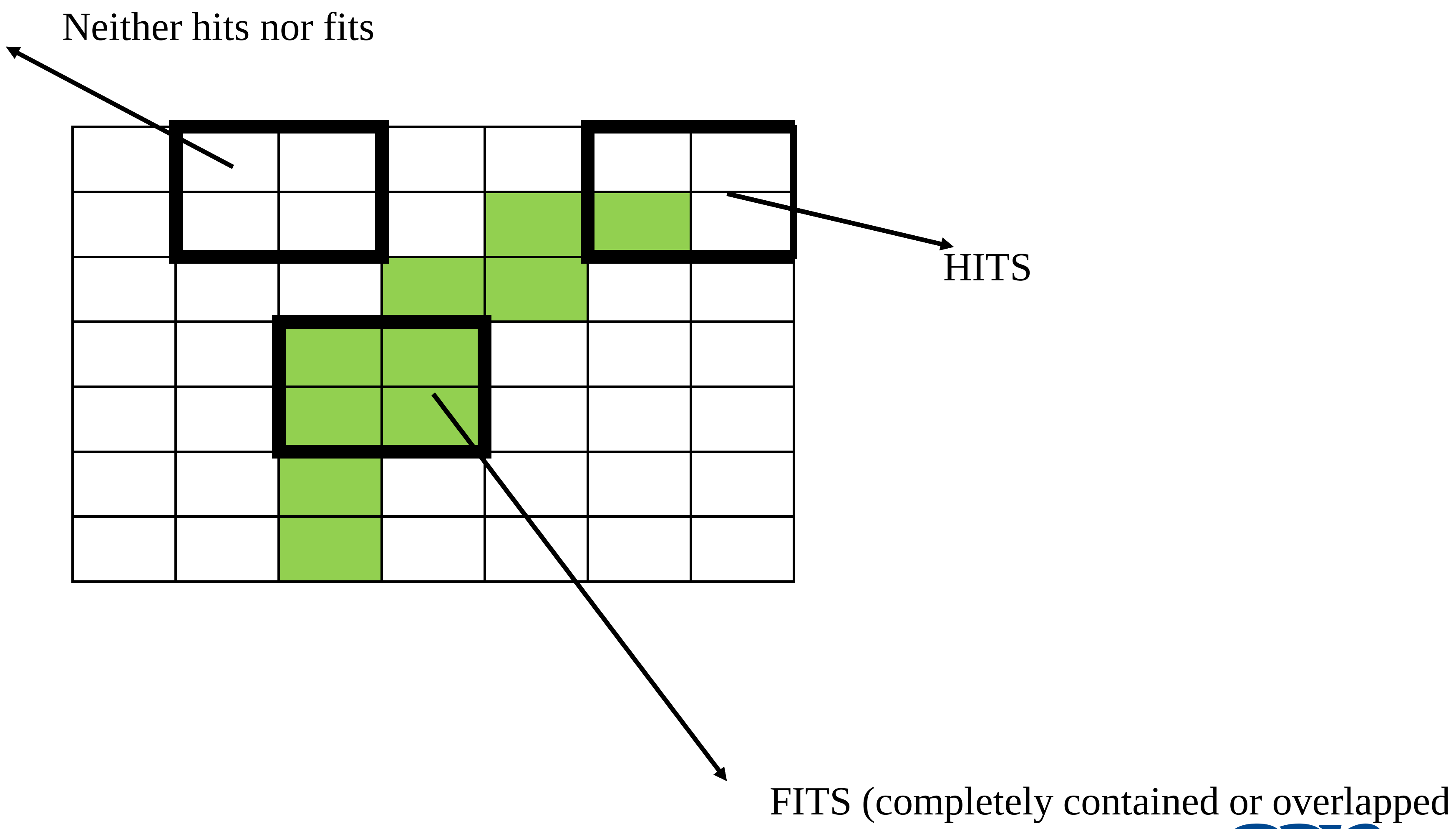
Structuring elements

- Set reflection and translatation are employed in morphology to formulate operations on "structuring element" (SE).
- They are small or subset images used to probe operation on the image under study
- Structuring element is placed in all possible pixels of the image and compared with the corresponding neighbourhood of pixels.
- Operations are based on whether the structuring element "fits" or "hits" (intersects) the neighbourhood.

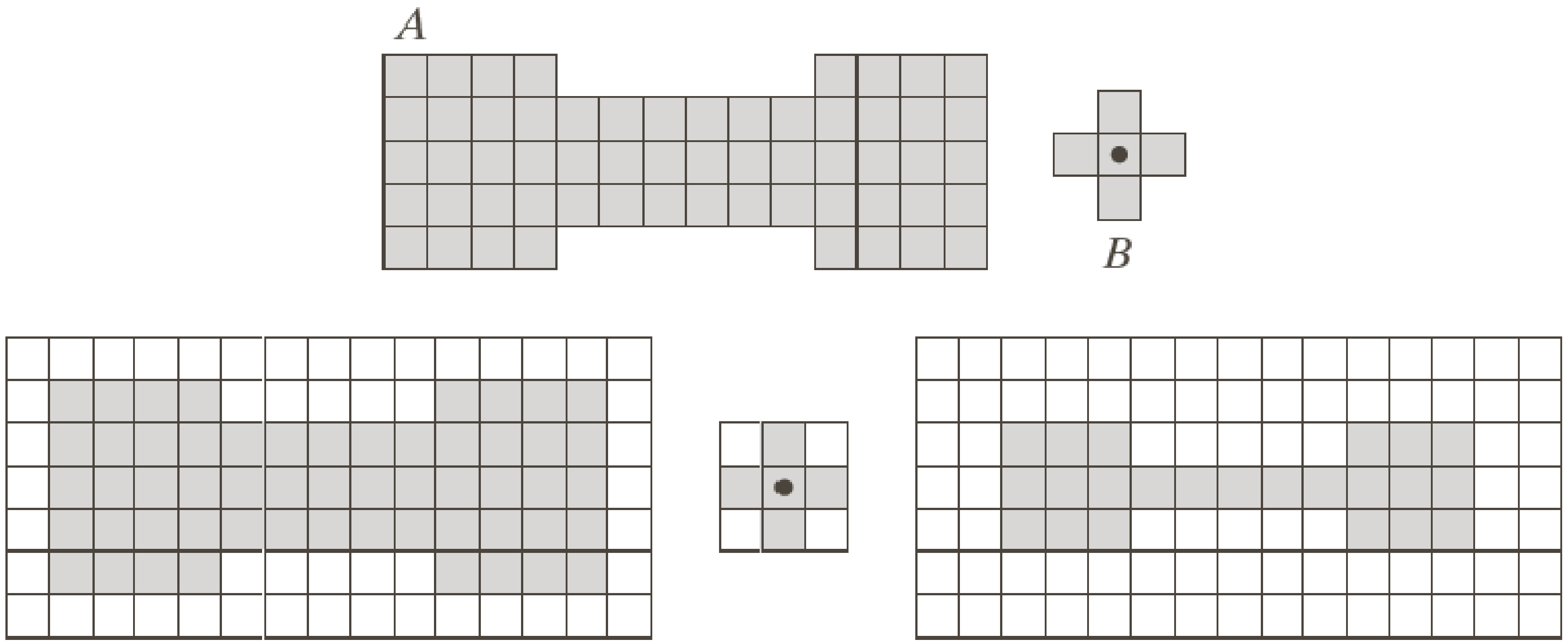
Structuring elements



Hits and fits



Erosion



Structuring element

- Is a small binary image
- Matrix dimension suggests the size of the structuring element
- Pattern of ones zeros specifies the shape of structuring element

0	0	0	0	0	0	0
0	0	0	1	0	0	0
0	0	1	1	1	0	0
0	1	1	**1**	1	1	0
0	0	1	1	1	0	0
0	0	0	1	0	0	0
0	0	0	0	0	0	0

0	0	0	0	0	0	0
0	0	0	1	0	0	0
0	0	0	1	0	0	0
0	1	1	**1**	1	1	0
0	0	0	1	0	0	0
0	0	0	1	0	0	0
0	0	0	0	0	0	0

Erosion

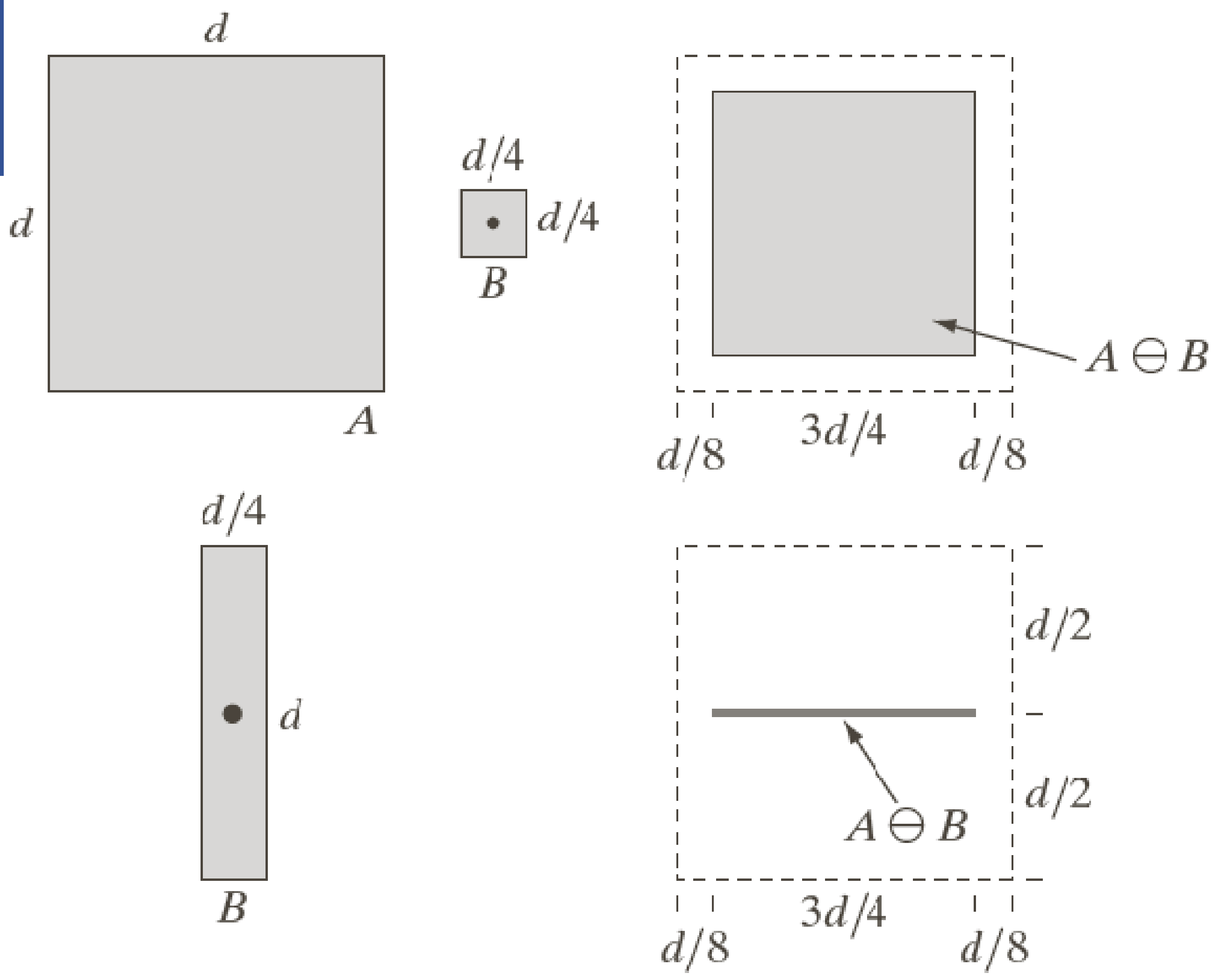
With A and B as sets in Z^2, the erosion of A by B, denoted $A \ominus B$, defined

$$A \ominus B = \{ z \mid (B)_Z \subseteq A \}$$

The set of all points z such that B, translated by z, is contained by A.

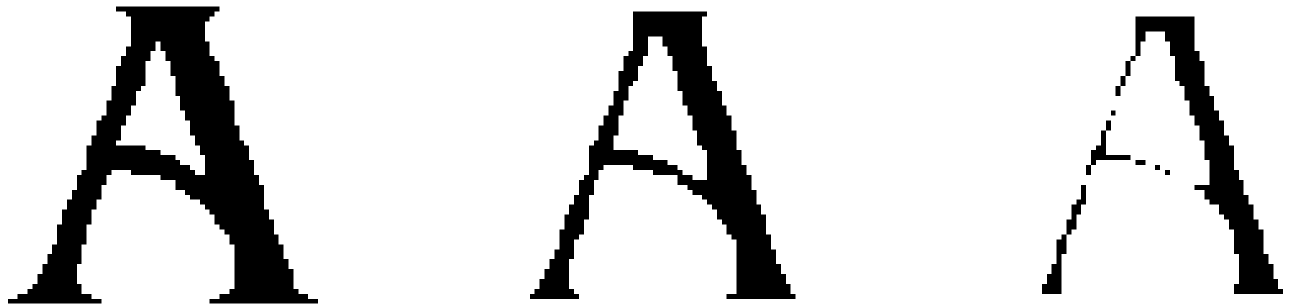
Erosion

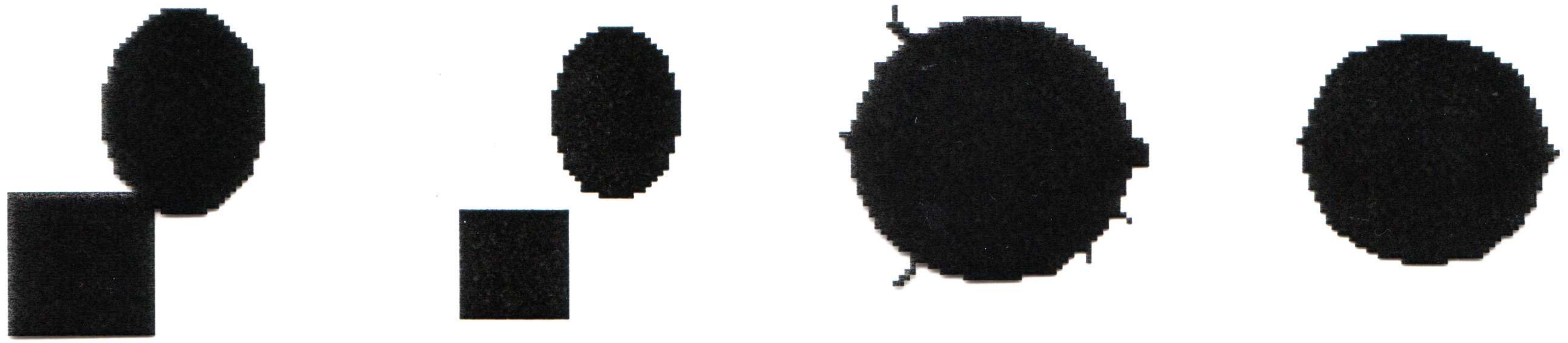
- B is the structuring element
- Structuring element contained in set A – the elements A and B completely overlap
- Erosion has a shrinking effect or is a thinning operation
- Origin of B visits every element in the set A
- For each location of the origin if B is completely contained in A the location is a member of a new set otherwise not.



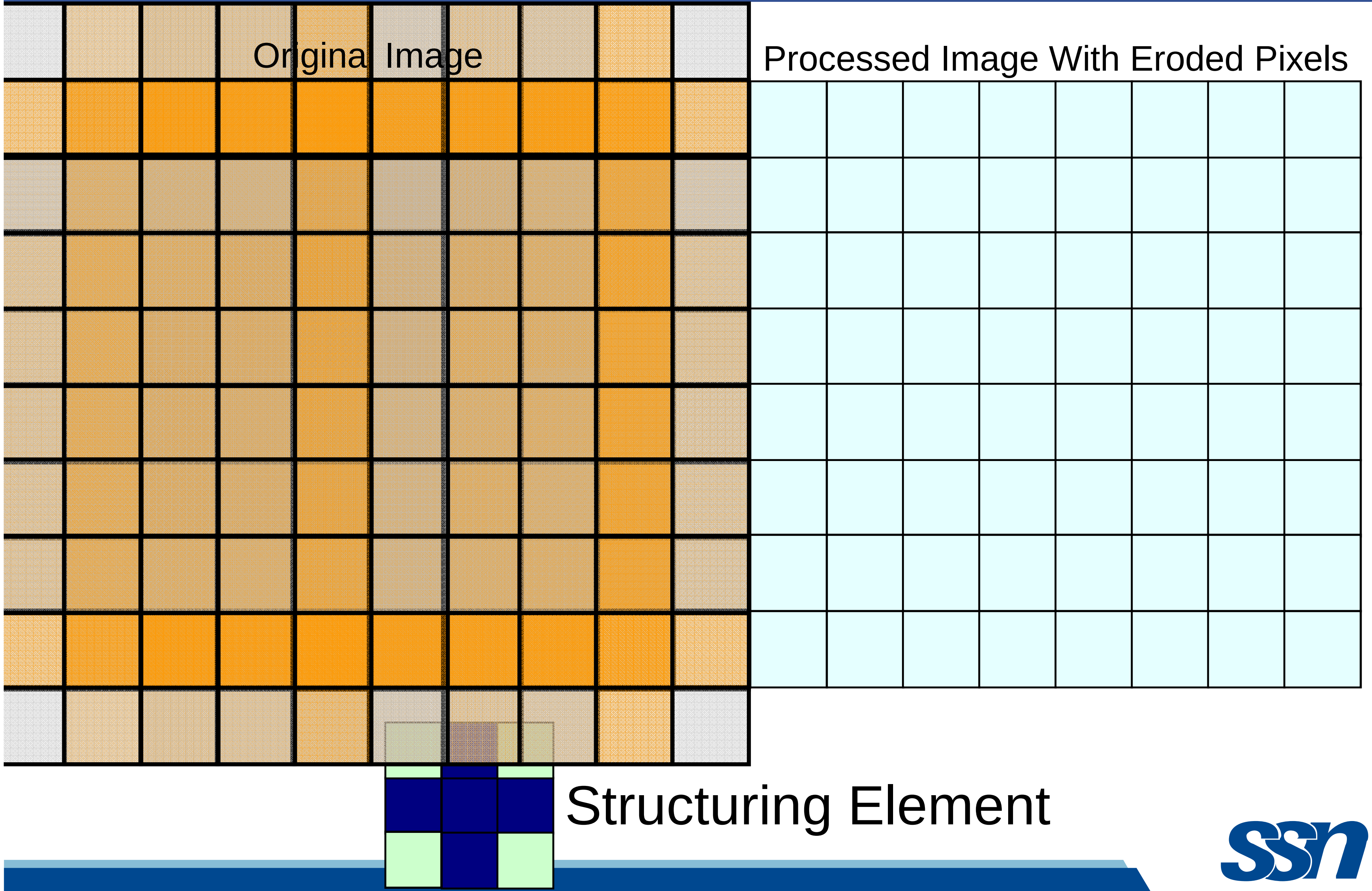
Courtesy: Gonzalez and woods, DIP

Erosion examples



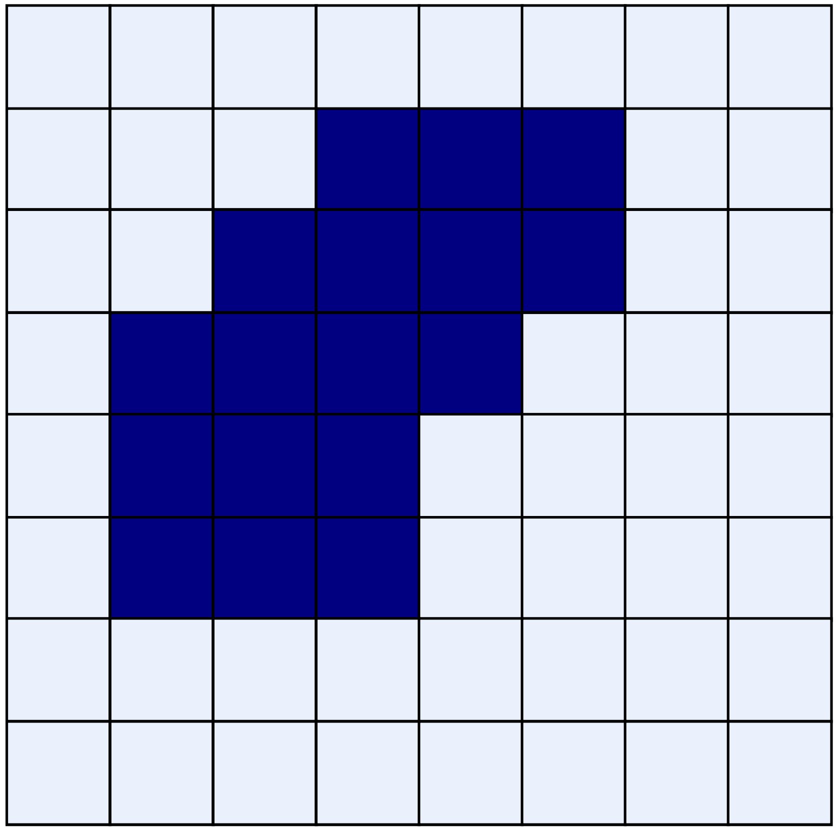


Erosion Example

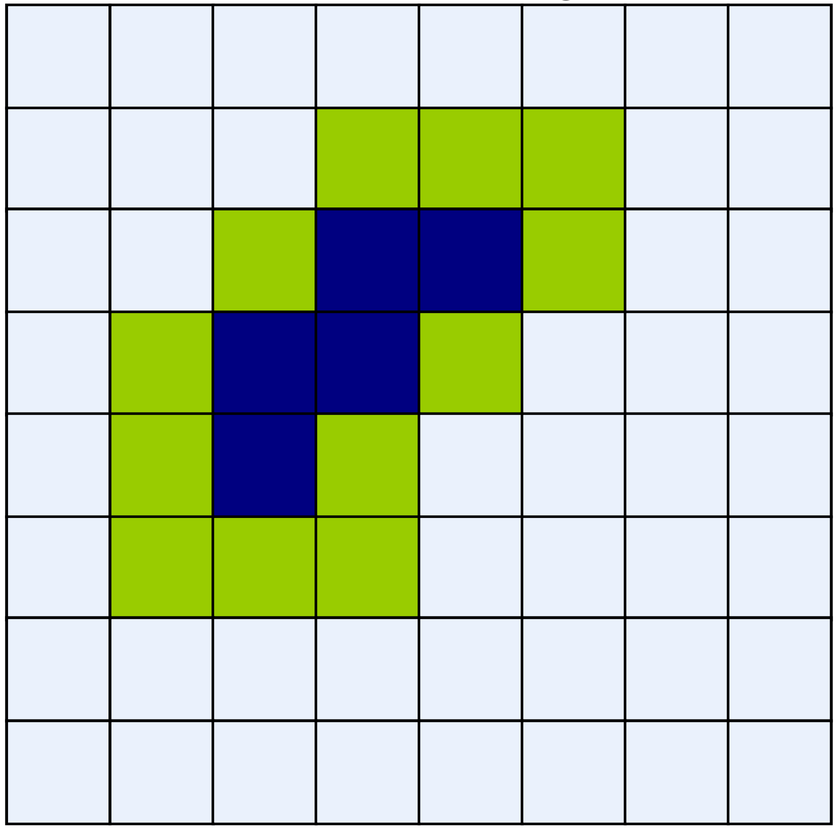


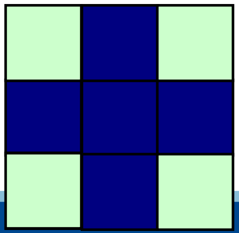
Erosion Example

Original Image

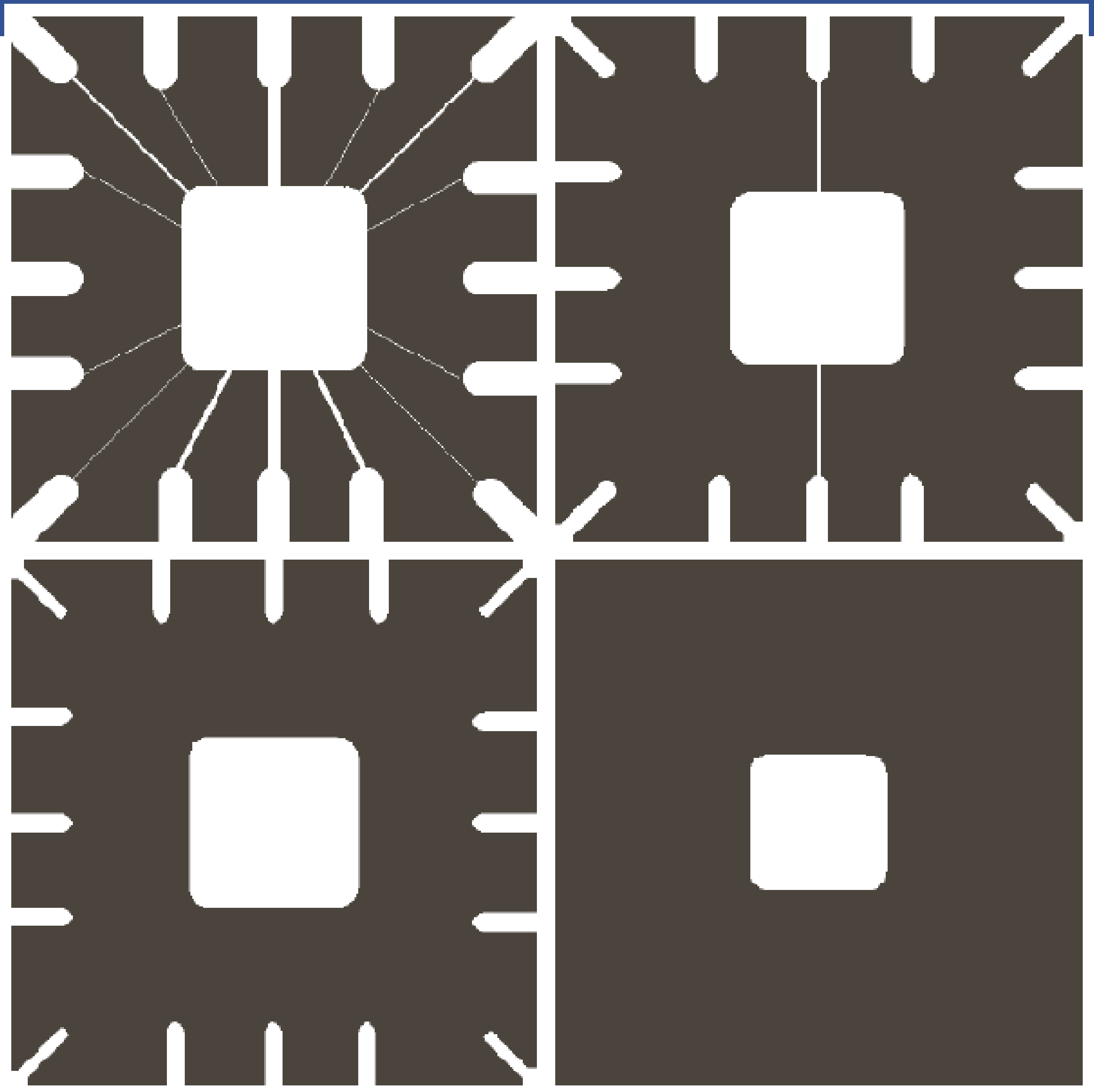


Processed Image





Structuring Element



a b
c d

FIGURE 9.5 Using erosion to remove image components. (a) A 486 × 486 binary image of a wire-bond mask. (b)–(d) Image eroded using square structuring elements of sizes 11 × 11, 15 × 15, and 45 × 45, respectively. The elements of the SEs were all 1s.

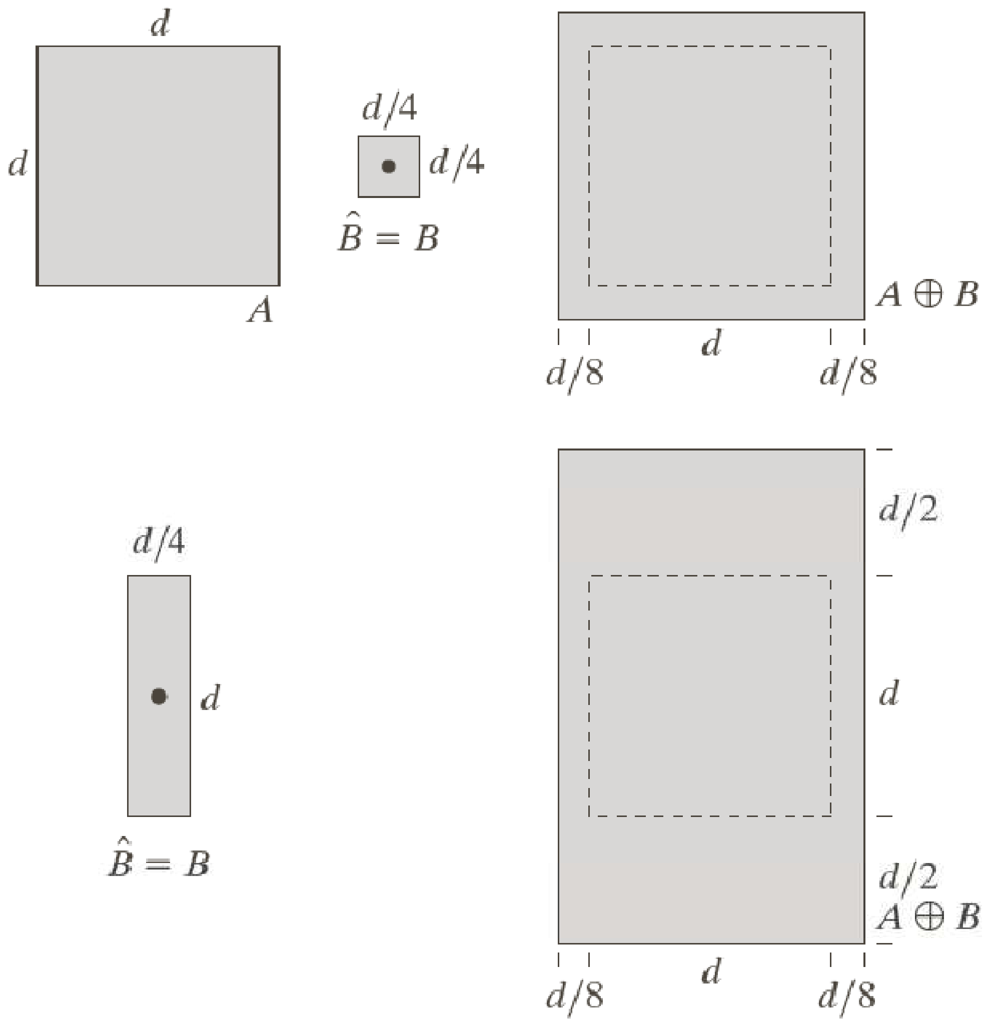
Dilation

- Equation based on reflecting B about its origin and shifting this reflection by z

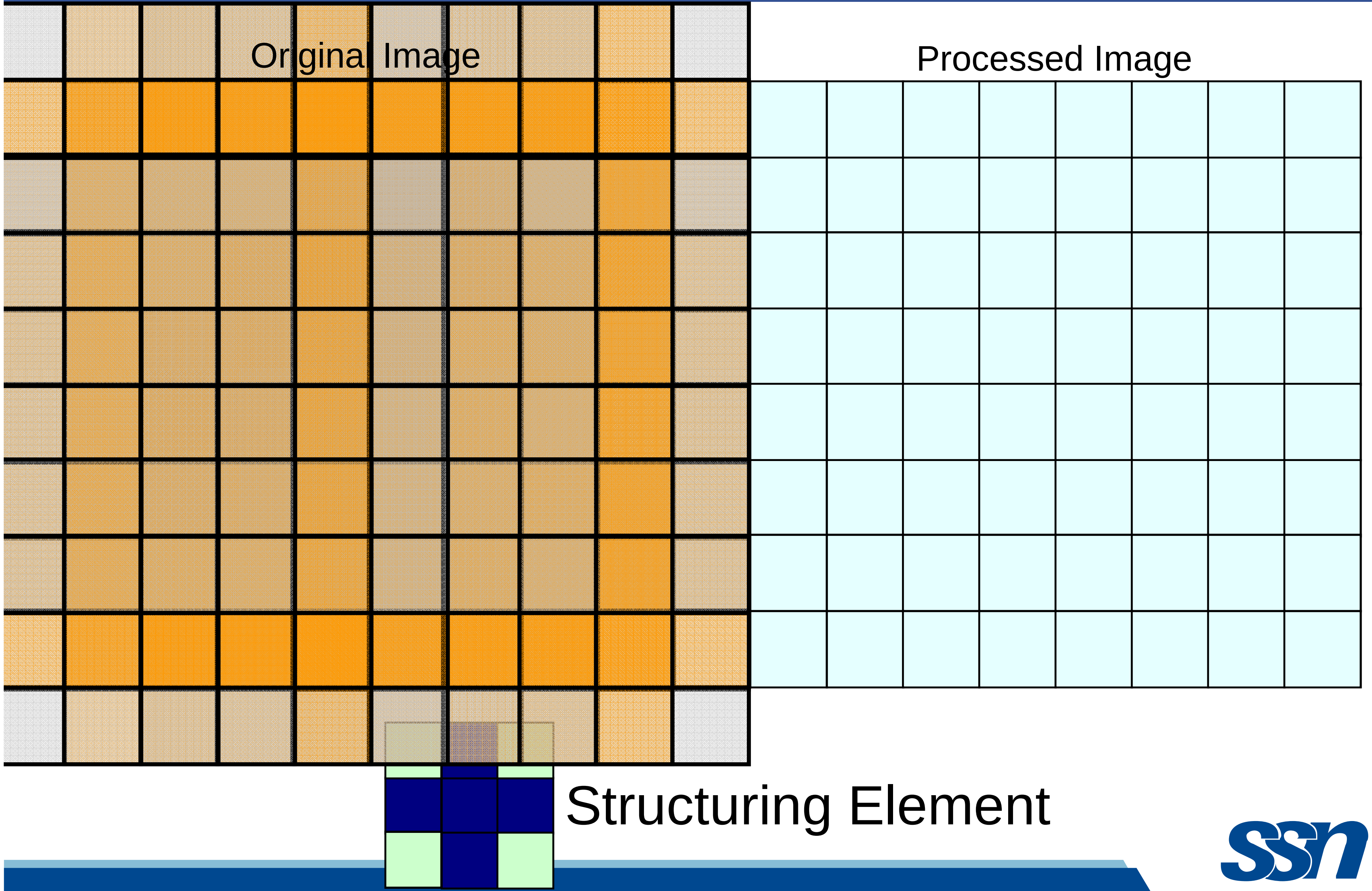
With A and B as sets in Z^2, the dilation of A by B, denoted $A \oplus B$, is defined as

$$\mathrm{A} \oplus \mathrm{B} = \left\{ z \mid \left(\hat{B} \right)_z \cap A \neq \varnothing \right\}$$

Dilation example (Gonzalez)

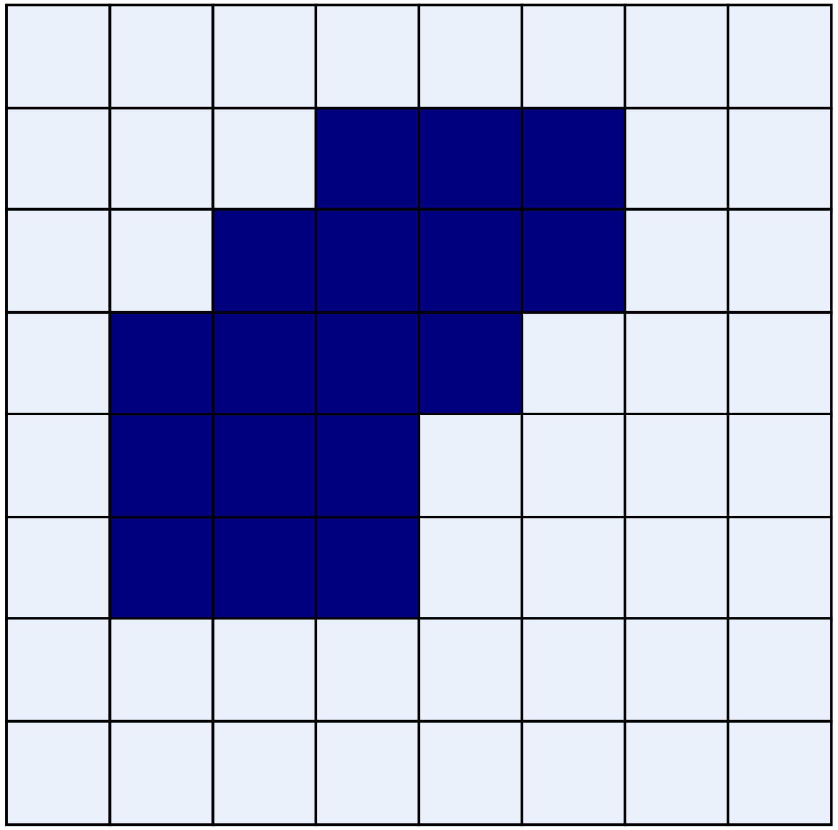


Dilation Example

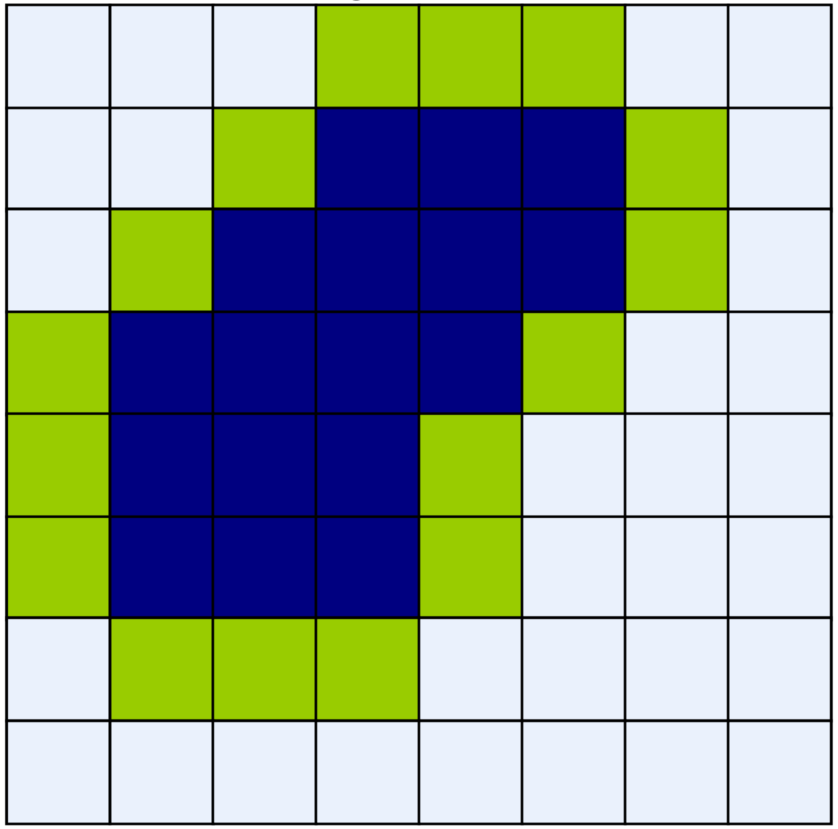


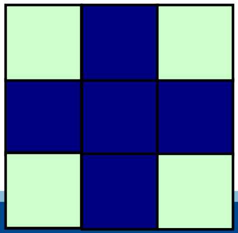
Dilation Example

Original Image



Processed Image With Dilated Pixels





Structuring Element

Dilation

- Growing or thickening effect on the objects of a binary image
- The extent of thickening is a function of shape of the structuring element used
- superimpose the structuring element on top of the input image so that the origin of the structuring element coincides with the input pixel position.
- If at least one pixel in the structuring element coincides with a foreground pixel in the image underneath, then the input pixel is set to the foreground value.
- If all the corresponding pixels in the image are background, however, the input pixel is left at the background value.

Opening and closing (compound operations)

- Opening: generally smoothens the contour of an object and breaks narrow isthmuses and eliminates thin protrusions

- Closing: also smoothens the sections of contours but as opposed to opening it generally
  - fuses narrow breaks and long thin gulfs and
  - eliminates small holes and fills the gaps in the contour

Opening and closing

- Opening a set A by a structuring element B is denoted by

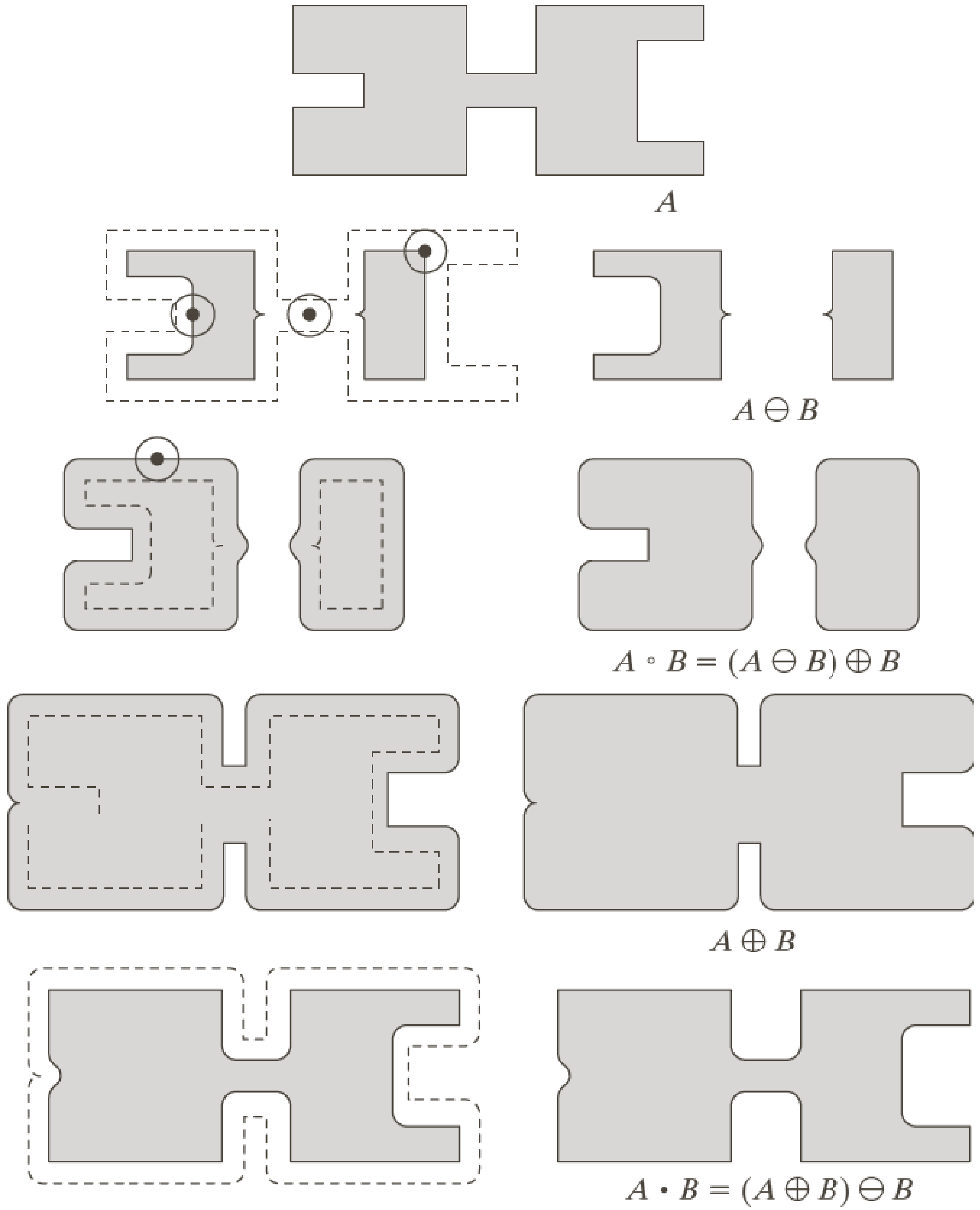
$A \circ B = (A \ominus B) \oplus B$

Erosion followed by dilation

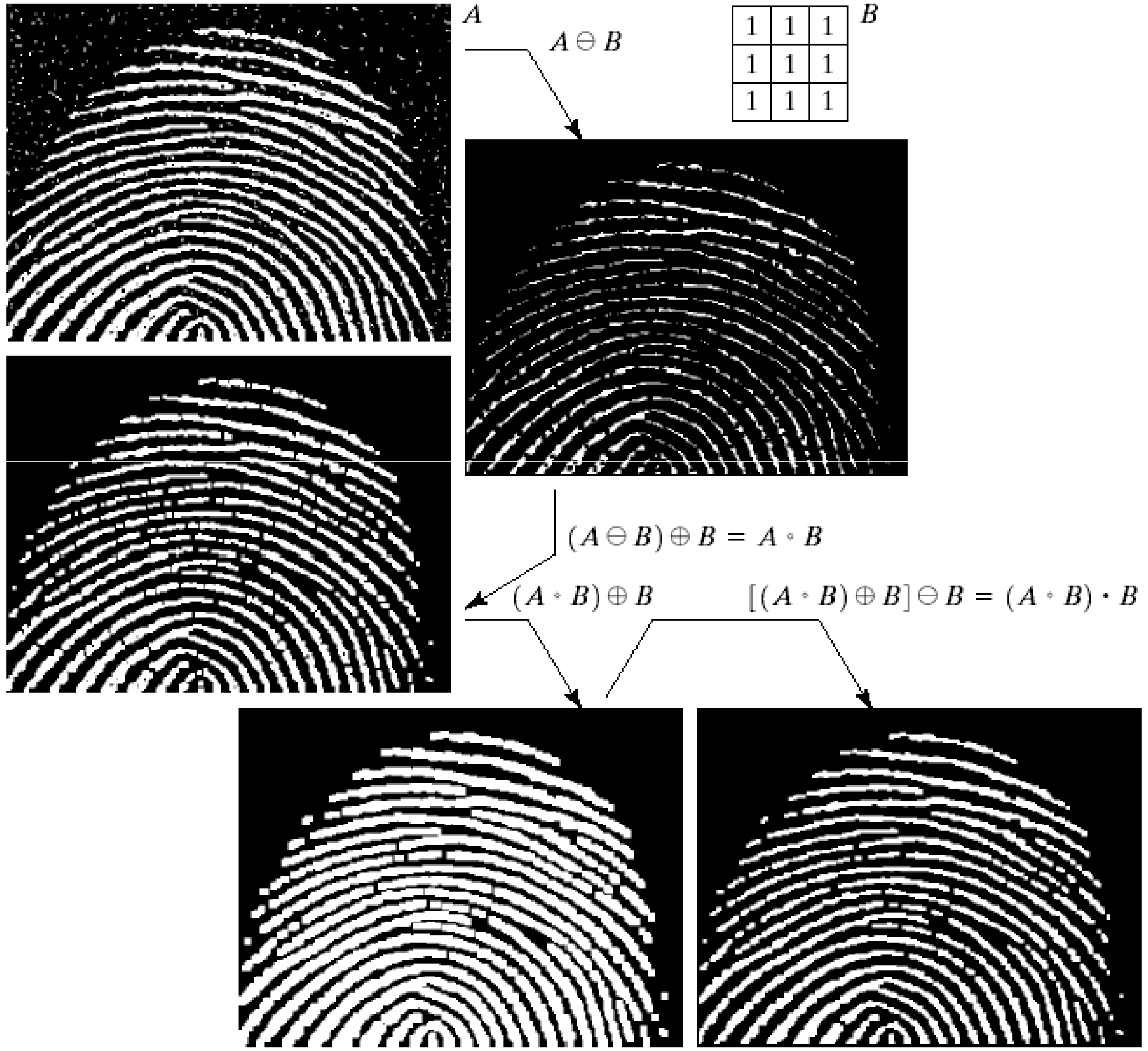
- Closing a set A by a structuring element is denoted by

$A \cdot B = (A \oplus B) \ominus B$

Dilation followed by erosion


A
A ⊖ B
A ∘ B = (A ⊖ B) ⊕ B
A ⊕ B
A • B = (A ⊕ B) ⊖ B

Morphological processing – example



A
$A \ominus B$
B
1 1 1
1 1 1
1 1 1
$(A \ominus B) \oplus B = A \circ B$
$(A \circ B) \oplus B$
$[(A \circ B) \oplus B] \ominus B = (A \circ B) \bullet B$

Hit or miss transformation

- The hit-and-miss transform is a general binary morphological operation that can be used to look for particular patterns of foreground and background pixels in an image.

- As with other binary morphological operators it takes as input a binary image and a structuring element, and produces another binary image as output.

Hit or miss transform

- Both the structuring element and the image will have both foreground and background pixels
- In erosion and dilation the ‘O’ pixel considered to be don’t cares or simply fillers
- Foreground pixels are 1’s and background pixels are O’s

Hit or miss

- Translate the origin of the structuring element to all points in the image
- Compare the elements in the structuring element and the image
- If the foreground and background elements of structuring element exactly coincide with the foreground and background pixels of the image then the pixel underneath the origin should be replaced by foreground pixel value
- If it doesn't match replace it by background pixel value.

Example (hit or miss)

	1	
0	1	1
0	0	

	1	
1	1	0
	0	0

	0	0
1	1	0
	1	

0	0	
0	1	1
	1