UNIT TEST II - Answer key

Part - A.

1. CFG $G = (N, T, P, S)$

   $N \rightarrow$ finite set of NT's

   $T \rightarrow$ finite set of terminals.

   $P \rightarrow$ finite set of productions of the form

   $\alpha \rightarrow \beta$

   $\alpha \in N$, $\beta \in (N \cup T)^*$ — Ex. Any CFG.

   $S \rightarrow$ start symbol, $S \in N$

2. $L = \{ab, abab \cdots\}$

   $L = \{(ab)^n \mid n \geq 1\}$.

3. (i) Acceptance by empty stack

   $M \rightarrow PDA$

   $L_E(M) = \{w \mid (q_0, w, Z_0) \vdash^* (p, \varepsilon, \varepsilon)$ for some p in $Q\}$

   (ii) Acceptance by Final state

   $L_F(M) = \{w \mid (q_0, w, Z_0) \vdash^* (p, \varepsilon, \gamma)$ for some p in F & $\gamma \in \Gamma^*\}$

4. $S \rightarrow 0S0 \mid 1S1 \mid 1 \mid 0 \mid \varepsilon$

5. PDA is an NFA with a memory in the form of stack. The stack stores unbounded limit of information. But there is a restriction on the usage of memory ~~(......)~~. (LIFO)

6. Null production. $(A \rightarrow \varepsilon)$

   This is of the form $A \rightarrow \varepsilon$ where $A \in N$ and ε is empty string.

7. $S \xrightarrow{lm} a\underline{B}$

$\xrightarrow{lm} a\,\underline{a}BB$

$\xrightarrow{lm} aa\,\underline{a}BBB.$

$\xrightarrow{lm} aaa\,\underline{b}BB$

$\xrightarrow{lm} aaab\,b\underline{S}B$

$\xrightarrow{lm} aaabb\,\underline{a}BB$

$\xrightarrow{lm} aaabb\,a\underline{b}B$

$\xrightarrow{lm} aaabb\,abb\underline{S}$

$\xrightarrow{lm} aaabbabb\underline{bA}$

$\xrightarrow{lm} aaabbabbb\underline{a}$

Part - B

8. a. 6marks.

$\delta(q_0, 0, Z) = \{(q_0, 0Z)\}$

$\delta(q_0, 1, Z) = \{(q_0, 1Z)\}$

$\delta(q_0, 0, 0) = \{(q_0, 00)\}$

$\delta(q_0, 1, 0) = \{(q_0, 10)\}$

$\delta(q_0, 0, 1) = \{(q_0, 01)\}$

$\delta(q_0, 1, 1) = \{(q_0, 11)\}$

$\delta(q_0, \varepsilon, 0) = \{(q_1, 0)\}$

$\delta(q_0, \varepsilon, 1) = \{(q_1, 1)\}$

$\delta(q_0, \varepsilon, Z) = \{(q_1, Z)\}$

(4marks)

$\delta(q_1, 0, 0) = \{(q_1, \varepsilon)\}$

$\delta(q_1, 1, 1) = \{(q_1, \varepsilon)\}$

$\delta(q_1, \varepsilon, Z) = \{(q_2, Z)\}.$

+ transition diagram (1mark) + 7 tuples. (1mark)

$(q_0, 1001, Z) \vdash (q_0, 001, 1Z)$

$\vdash (q_0, 01, 01Z)$

ID Definition (2marks.)

ID is a triple (q, w, γ) where

(2marks)

$q \rightarrow$ current state

$w \rightarrow$ remaining i/p.

$\gamma \rightarrow$ stack content.

$(q_1, aw, bx) \vdash (q_2, w, yx)$ is possible if

$$\delta(q_1, a, b) = \{(q_2, y)\}.$$

9.

a. (2marks) A grammar G is ambiguous if there is a word $w \in L(G)$ having atleast 2 diff. parse trees.

i.e. $\exists$ a string w with more than 1 parse tree (or) with left most der

(or)

" " " with RM derivation

b. ~~Ambing~~ Ambiguous

(6marks)

$S \rightarrow aSbS \mid bSaS \mid \varepsilon$

abab

$S \xRightarrow{lm} aSbS$

$\xRightarrow{lm} a\varepsilon bS$

$\xRightarrow{lm} abaSbS$

$\xRightarrow{lm} aba\varepsilon bS$

$\xRightarrow{lm} abab.$

$S \xRightarrow{lm} aSbS$

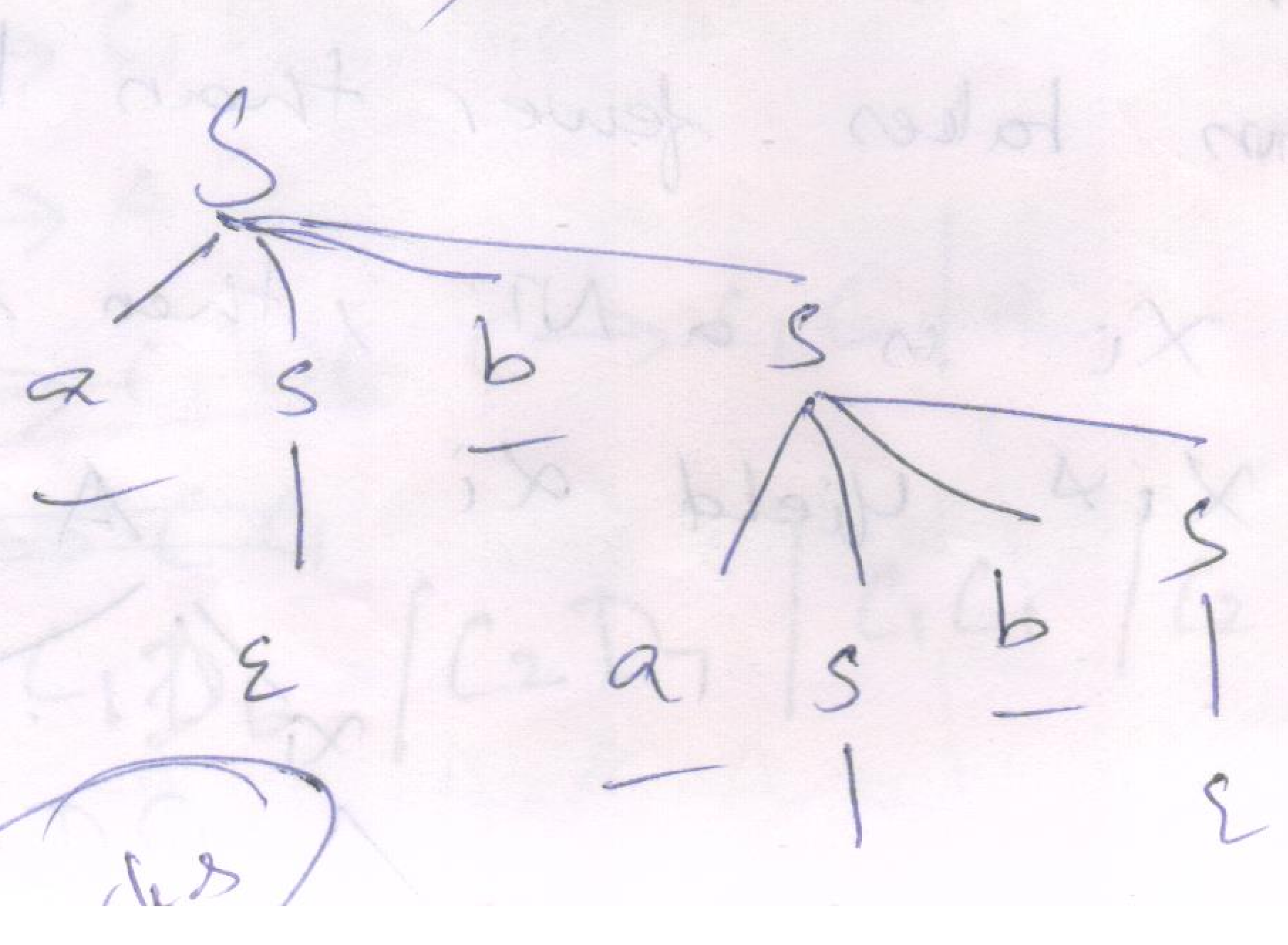
$\xRightarrow{lm} abSaSbS$

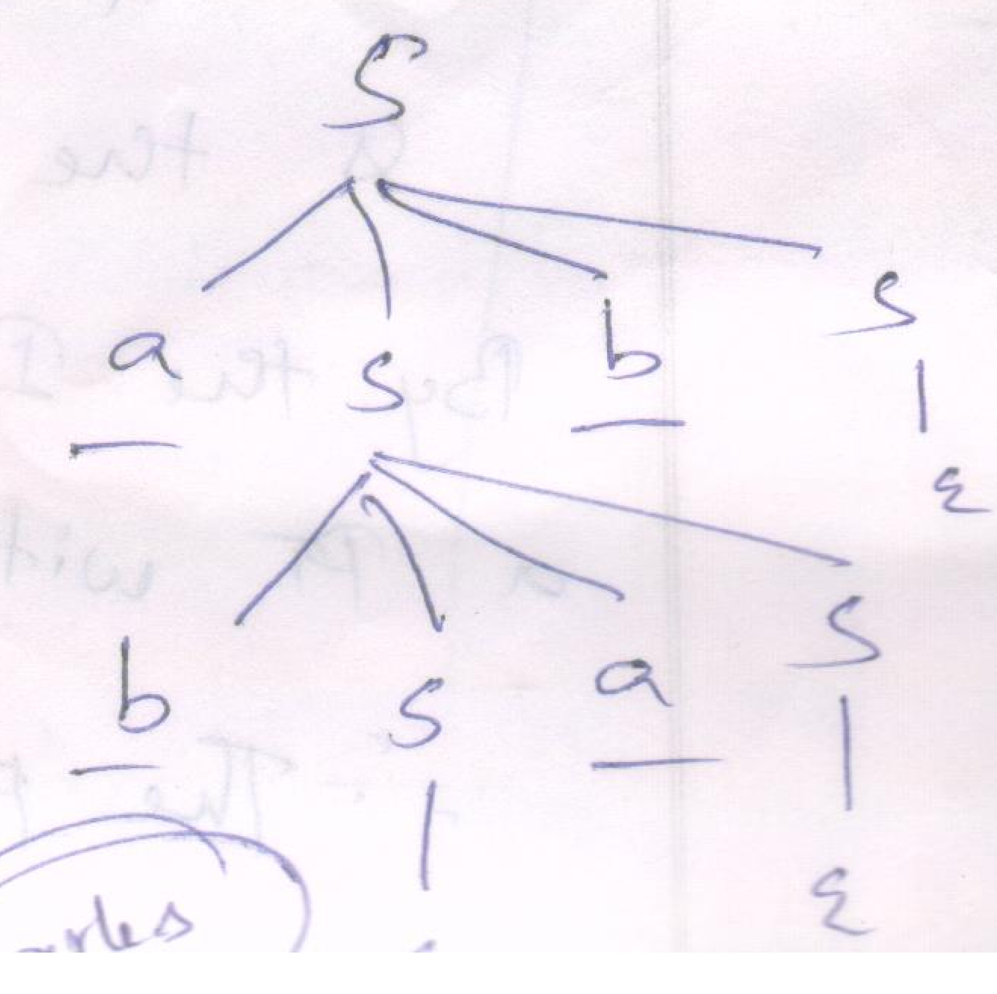
$\xRightarrow{lm} ab\varepsilon aSbS$

$\xRightarrow{lm} aba\varepsilon bS$

$\xRightarrow{lm} abab\varepsilon$

$\xRightarrow{lm} abab$





Language generated by the grammar

(2 marks) L = {equal number of a's & b's over $\Sigma = \{a, b\}$}

10. (10 marks) If $A \stackrel{*}{\Rightarrow} \alpha$, then there is a parse tree with root A and yield α.

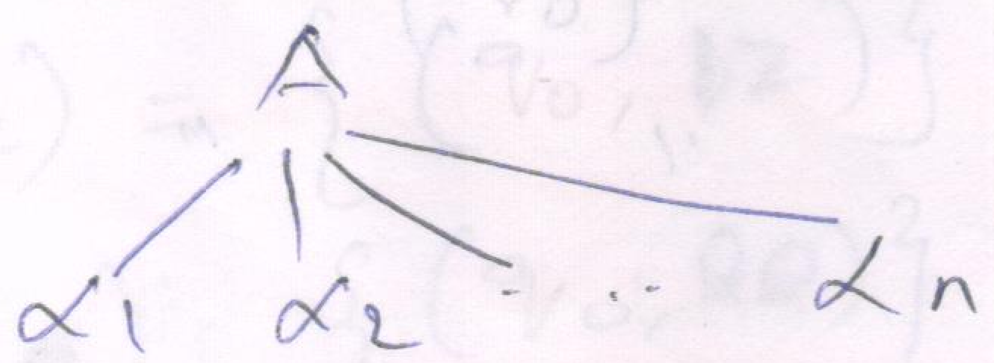
i Given a derivation of a terminal string α, prove the existence of a parse tree.

* The proof is an induction on the length of the derivation.

Basis

If $A \stackrel{*}{\Rightarrow} \alpha_1 \alpha_2 \ldots \alpha_n$ by one-step derivation then there must be a parse tree

$(A \rightarrow \alpha_1 \alpha_2 \ldots \alpha_n)$



Induction

Assume for derivation of fewer than $k > 1$ steps, & let $A \stackrel{*}{\Rightarrow} \alpha$ be a k-step derivation.

1st step $A \Rightarrow X_1 X_2 \ldots X_n$.

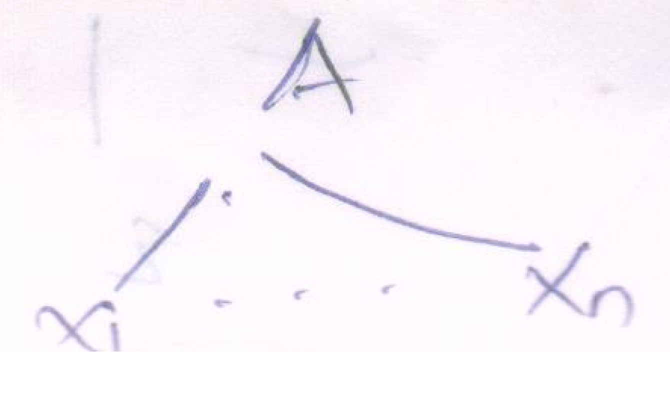
α can be divided so the 1st portion is derived from X_1, the next from $X_2 \ldots$

If X_i is a terminal, $\alpha_i = X_i$

ii $X_i \stackrel{*}{\Rightarrow} \alpha_i$ $\forall i$ such that X_i is a variable.

& the derivation takes fewer than k-steps

By the IH, if X_i is a NT, then, there is a PT with root X_i & yield α_i



a. ~~D~~ CNF →

2 marks → A CFG is in CNF if every production is of the form $A \to a$ or $A \to BC$ & $S \to \varepsilon$ is in G if $\varepsilon \in L(G)$. When ε is in $L(G)$, assume that S does not appear on RHS of any production.

b. 8 marks.

$S \to AACD$

$A \to aAb \mid \varepsilon$

$C \to aC \mid a$

$D \to aDa \mid bDb \mid \varepsilon$

After eliminating ε & unit productions.

$S \to AACD \mid ACD \mid AAC \mid CD \mid AC \mid aC \mid a$

$A \to aAb \mid ab$

$C \to aC \mid a$

$D \to aDa \mid bDb \mid aa \mid bb$

$C_1 \to a$

$C_2 \to b.$

Final.

$S \to AD_1 \mid AD_3 \mid AD_4 \mid CD \mid AC \mid C_1C \mid a$

$D_1 \to AD_2$

$D_2 \to CD$

$D_3 \to CD$

$D_4 \to AC$

$A \to C_1D_5 \mid C_1C_2$

$D_5 \to AC_2$

~~$C \to C_1D_6$~~ $C \to C_1C \mid a.$

~~$D_6 \to CC_1$~~

$D \to C_1D_6 \mid C_2D_7 \mid C_1C_1 \mid C_2C_2$

$D_6 \to DC_1$

$D_7 \to DC_2$

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12. a. GNF

2 marks. A CFG G is in GNF if every production is of the form $A \to a\alpha$ where $\alpha \in N^*$ and $a \in T$ (α may be ε) & $S \to \varepsilon$ is in G if $\varepsilon \in L(G)$ where S does not appear on RHS of any production.

b. 8 marks. $S \to AB$

$A \to BS / b$

$B \to SA / a$

$A_1 \to A_2 A_3$

$A_2 \to A_3 A_1 / b$

$A_3 \to A_1 A_2 / a$

$A_3 \to A_2 A_3 A_2 / a$ $\underline{j > i}$

$A_3 \to A_3 A_1 A_3 A_2 / b A_3 A_2 / a$

eliminate LR

$A_3 \to b A_3 A_2 / a / b A_3 A_2 B_3 / a B_3$

$B_3 \to A_1 A_3 A_2 / A_1 A_3 A_2 B_3$.

~~After substituting~~.

Final

$A_1 \to b A_3 A_2 A_1 A_3 / a A_1 A_3 / b A_3 A_2 B_3 A_1 A_3 / a B_3 A_1 A_3 / b A_3$

$A_2 \to b A_3 A_2 A_1 / a A_1 / b A_3 A_2 B_3 A_1 / a B_3 A_1 / b$

$A_3 \to b A_3 A_2 / a / b A_3 A_2 B_3 / a B_3$

$B_3 \to b A_3 A_2 A_1 A_3 A_3 A_2 / a A_1 A_3 A_3 A_2 / b A_3 A_2 B_3 A_1 A_3 A_3 A_2 /$

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Additional Sheet

13.

a. 6marks

$\delta(q_0, a, z) = \{(q_1, aaaz)\}$

$\delta(q_1, a, a) = \{(q_1, aaaa)\}$

$\delta(q_1, b, a) = \{(q_2, \varepsilon)\}$

$\delta(q_2, b, a) = \{(q_2, \varepsilon)\}$

$\delta(q_2, \varepsilon, z) = \{(q_2, \varepsilon)\}$. 4marks

+

transition diagram. 1mark. + 6 tuples. 1marks

b. 2marks

abb

$(q_0, abb, z) \vdash (q_1, bb, aaaz)$

$\vdash (q_2, b, aaz)$

$\vdash (q_2, \varepsilon, az)$

cannot empty.

∴ not accepted.

marks

aabbbbbb.

can empty.

∴ Accepted.