Exercise Set 2

Cut-Sets, Connectivity, Separability and Planarity

Attempt all questions.

1. Show that if the vertex set of a connected graph $G = (V, E)$ is partitioned into two non-empty sets X and Y, the disconnecting set $F = (X, Y)$ consisting of all edges of G joining two vertices in X with vertices in Y is a cut-set if the subgraph $G' = (V, E - F)$ has exactly two components.
2. Prove that with respect to a given spanning tree T, a branch b_i that determines a fundamental cut-set S is contained in every fundamental circuit associated with the chords in S and no other.
3. Prove that with respect to a given spanning tree T, a chord c_i that determines a fundamental circuit Γ is contained in every fundamental cut-set associated with the branches in Γ and no other.
4. Pick an arbitrary spanning tree in the graph given in Figure 4.6 of the text book and list all fundamental cut-sets of the graph with respect to this tree.
5. With the fundamental cut-sets computed above and using ring-sum operation on graphs generate all cut-sets of the given graph.
6. Prove that in a connected graph G the complement of a cut-set in G does not contain a spanning tree and the complement of a spanning tree does not contain a cut-set.
7. Prove that in a connected graph G a vertex v is a cut-vertex if and only if there exists two (or more) edges x and y incident on v such that no circuit in G includes both x and y.
8. (a) Define circuit correspondence in graphs. Illustrate with an example.
   (b) Show that 1-isomorphic graphs have circuit correspondence.
   (c) Show that 2-isomorphic graphs have circuit correspondence.
9. Prove that every connected graph with three or more vertices has at least two vertices which are not cut-vertices.
10. Show that a graph G is nonseparable if every vertex pair in G can be placed in some circuit in G.
11. Prove that an Euler graph can not have a cut-set with an odd number of edges.
12. Prove Theorem 5-8 of the text book.
13. Describe the algorithm to detect planarity (or non-planarity) of a simple graph. Use it to show that Petersen graph is non-planar.

14. Construct a graph G with following properties: Edge connectivity of $G = 4$, vertex connectivity of $G = 3$, and degree of every vertex of $G \geq 5$.

15. In a connected graph G, let Q be a set of edges with the following properties:

    (a) Q has an even number of edges in common with every cut-set of G.

    (b) There is no proper subset of Q that satisfies the above property.

    Show that Q is a circuit.

16. (a) Describe 2-isomorphism of graphs with an example.

    (b) Show that if two graphs are 2-isomorphic then they have the same the same rank and nullity.

17. Show (by drawing them) that two graphs with the same rank and nullity need not be 2-isomorphic.

18. Let us define a new term called edge isomorphism as follows: Two graphs G_1 and G_2 are edge isomorphic if there is a one-to-one correspondence between the edges of G_1 and G_2 such that two edges are incident (at a common vertex) in G_1 if and only if the corresponding edges are also incident in G_2. Discuss the properties of edge isomorphism. Construct an example to show that edge-isomorphic graphs are not isomorphic.