MTH308

MP1

Porush Yadav 160487

AIM:

Given a simple closed curve of period 1 and a random point, find the point on the curve which is nearest to the given point.

Algorithm used:

Newton raphson to find the solution of f(t) = 0.

Idea

Distance between any general point in curve and (x0, y0) = sqrt((x(t) - x(0))^2 + (y(t) - y(0))^2)

To minimize the distance, differentiation of distance must be zero.

So we have to solve f(t) = 0 where

f(t) = (x(t) - x0) * dXdt + (yt) - y0) * dYdt

So I will apply newton raphson method to solve this function.

Method:

I first found the convexity of the curve i.e. whether the curve is concave or convex.

To check whether curve is convex or not, I calculated curvature of the curve by partition the interval [0, 1] into 200 points and then calculating curvature at each point.

If curvature at t=0 is positive and it always remains positive then curve is convex and if at t=0, curvature is negative and it always remains negative then curve is convex and in all remaining cases it will be concave.

If my curve is convex then I will partition the interval [0, 1] into 100 partition and then just compare the distance of those 100 points with (x0, y0) and whichever point is closest to (x0, y0) I will apply newton raphson at the point.

If my curve is concave then I will partition the interval [0, 1] into 1000 partition and then at each point I will apply newton raphson to find local maximas or local minimas and then the value of t which will be closest to (x0, y0) will be our required t.

Initial problems with approach:

What if Newton raphson converges to a point which is local minima but not a global minima or what if newton raphson converges to a point which is local maxima.

Proof of correctness:

1. When the curve is convex then we know that where will only one minima and one maxima. So when I found the point from the interval closest to (x0, y0) and then apply newton raphson on the point, it will converge to only global minima.
2. When the curve is concave, I have made 1000 partition to ensure that I will get as close and possible to the actual solution of f(x) = 0 because the probability of reaching the global minima increases with increasing the number of partitions.