MTH308

MP2

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Objective

For given a, b∈ℝ(a < b) and a positive integer n, choose a discrete computational grid X_{grid}= $\{x_j\}_{nj=1}$ with x_j∈[a, b], j= 1, . . . , n, and x_i != x_j for i != j. Let F_{grid} = $\{f(x_j)\}_{nj=1}$ be the function data obtained by evaluating a given infinitely differentiable functions f:[a, b]→ℝ that satisfies $f_{(k)}(a)$ =$f_{(k)}(b)$ = 0 for k = 0,1, . . . ,10 and $f_{(k)}(a)$!= $f_{(k)}(b)$.

We have to design and implement an approximation f_n [a, b]→ℝ to f based on the discrete data (X_{grid},F_{grid}).n.

Theory

As the first 10 derivatives are zero, we can assume that best approximation will be periodic function and so we will use trigonometric interpolation. We have used below formula

$$p(x) = a_0 + \sum_{k=1}^{K} a_k \cos(kx) + \sum_{k=1}^{K} b_k \sin(kx).$$

Discrete data points are chosen by affine translation of the Chebyshev nodes which are the roots of Chebyshev polynomial of the first kind of degree n.

$$T(x) = \sum_{k=0}^{n} a_k cos(k * cos^{-1}(x))$$

Error bound by using Chebyshev interpolation is

$$|f(x) - p(x)| \leq \left(\frac{(b-a)}{2}\right)^2 \frac{max_{x\in[a,b]} f^{(n+1)}(x)}{2^n(n+1)!}$$

Reference

1. Finding the Zeros of a Univariate Equation:Proxy Rootfinders, Chebyshev Interpolation,and the Companion Matrix by John P. Boyd

https://pdfs.semanticscholar.org/09c7/d626cd09de525b62eedf70cc91e43f26273e.pdf