

Code : 303102

BCA 1st Semester Theory Examination, 2017

Basic Mathematics

Time : 3 Hrs

Full Marks : 60

Instructions :

- (i) The questions are of equal value.
- (ii) There are **Seven Questions** in this paper.
- (iii) Attempt Five questions in all.
- (iv) **Question Nos. 1 & 2 is compulsory.**

1. Answer any six of the following:

- (a) A function $f(x)$ is continuous the interval $[0, 2]$. It is known that $f(0) = f(2) = -1$ and $f(1) = 1$. which one of the following statements must be true?
- (i) There exists a y in interval $(0, 1)$ such that $f(y) = f(y+1)$.
 - (ii) For every y in the interval $(0, 1)$, $f(y) = f(2-y)$.
 - (iii) The maximum value of the function in the interval $(0, 2)$ is 1.
 - (iv) There exists a y in the interval $(0, 1)$ such that $f(y) = f(2-y)$.
- (b) Find the number of positive integers lying between 1 and 100 (both inclusive) but NOT divisible by 2, 3 and 5.
- (c) The power set of countably infinite set is:
- (i) Countable

- (ii) Uncountable
 - (iii) None of these
 - (d) For a set A , the power set is denoted by 2^A . If $A = \{5, \{6\}, \{7\}\}$, find all the sub-sets of 2^A .
 - (e) Let $A = \{1, 2, 3\}$ and $B = \{2, 3, 1\}$. Find $A-B$.
 - (f) Let $P(A)$ denote the power set of A . If $P(A) \subseteq B$ then
 - (i) $2^{|A|} \leq |B|$
 - (ii) $2^{|A|} \geq |B|$
 - (iii) $2^{|A|} < |B|$
 - (iv) None of these
 - (g) Find the slope of $x^2y = 8$ at the point $(2, 2)$.
 - (h) Find the second derivative of $x^3 - 5x^2 + x = 0$.
 - (i) If $f(x) = x^3 + 3x^2 + 4x + 5$ and $g(x) = 5$, then find $g(f(x))$.
 - (j) Find the value of $\int_0^x x^2 dx$
2. (a) Prove that, for all sets A , B and C

$$(A - B) \cap (C - B) = (A \cap C) - B$$
.
- (b) Let : $X \rightarrow Y$ and $g : Y \rightarrow Z$. Let $h = g \circ f : X \rightarrow Z$. Suppose g is one-to-one and onto. Now, if f is one-to-one, then prove that h is one-to-one.
- (c) If $y = (t^2 + 2)^2$ and $t = x^{1/2}$, determine dy/dx .

(d) At $t = 0$, a particle starts at rest and moves along a line in such a way that at time t its acceleration is $24t^2$ feet per second per second. Through how many feet does the particle move during the first 2 seconds?

(e) If $F(x) = \int_0^x e^{-t^2} dt$, then find $F'(x)$.

3. The number of bacteria in a culture is growing at a rate of $3000e^{\frac{2}{5}t}$ per unit of time t . At $t=0$, the number of bacteria was present 7,500. Find the number present at $t=5$.
4. If $\frac{dy}{dx} = 4y$ and if $y=4$ when $x=0$, then $y=?$
5. If $y = e^{nx}$, then find $\frac{d^n y}{dx^n}$.
6. What is the area of the region completely bounded by the curve $y = -x^2 + x + 6$ and the line $y = 4$?
7. The line segment connecting $(x, 6)$ and $(9, y)$ is bisected by the point $(7, 3)$. Find the values of x and y .
