"perfect numbers"

19
28
37
46
55
64
73
82
91

9

9

109
118
127
136
145
154
163
172
181
190

10

1009
1018
1027
1036
⋮
1090
1108
1117
⋮

19

209
217
226 ; 235
271
280 $10+30\cdot 9$

9

$307 = 10+297 = 10+\underbrace{(30+3)}_{33}\cdot 9$

361

370

$\boxed{10+n9}$:

Not:

100, 199, 289.. 299 ~~379~~ 379 829 919

$10+10\cdot 9$ | $10+21\cdot 9$ | $10+31\cdot 9$ | $10+369$ | $10+41\cdot 9$ | $10+101\cdot 9$

8 [30 … 7]

⋮

3 [802, 811, 820]

2 [901, 910]

$10+91\cdot 9 = \cancel{10+801}$
$10+819 =$
$= 10+91\cdot 9$

$91\cdot 9 = 91(10-1)$

829

$10+101\cdot 9 = \underline{919}$

$101\cdot 9 = 1010-101 =$
$= 909$

$10+\boxed{111\cdot 9} = \cancel{1008}$
1009

$111\cdot 9 = 999$

$10+121\cdot 9 = 1099$

~~while i < n:~~

$a = [0, 1, 9]$ 1007|11

100 7|0 2 ; 1007|20 ; ~~1007|30~~ x 9

1008|01   1008|10

1009|00

1010|08

~~a(end-1)+1~~

more increment

~~a[: s = sum(a(1:end-2)) + a(end-1)~~ +1

~~bn =~~ ~~b2=~~

~~if s < 10 , bn = 10 - s~~

~~if s >= 10~~

let's generate these numbers recursively

$a_1 = [0, | 1, 9]$; $a_j = [0, | 9, 1]$

$s = sum(a[1:end-3])$

$s + a[end-1] < 9$: ~~$x = a[end-1]+1$~~ $x = a[end-1]+1$

$z = 10 - s - x$

... |++?

~~[0, 10, 1]~~

~~return~~

~~return~~ $[a[1:end-2], x, z]$

$s + a[end-1] = 9$: $x = 0, z = 1$

or

$x = 1, z = 0$.

1007 02
1007 20
1008 00
1009
1006

1007 02
~~1007~~ 11
1007 20
~~1007~~ 29
2

1005 04
1005 13
22
31
40
4

s=7

~~1000~~ 1204 | 03
11
21
30
1

$\overline{xy} + 9$

$\overline{a|xy}$ $a = 1 \ldots$

index

$s \Leftarrow$ sum of digits of a

if $n \le 9$, return $\boxed{19 + (n-1) \cdot 9}$

$\boxed{p = 10 - s + 1}$ # num of ~~magic~~ perf. nums indexed by a

assume $n > 9$

$a = 1$; $i = 9$; $s = 1$

n = 23 → 280
29 → 307

n = 4

while $i < n$:

s = sum_digits(a)

$i = i + 10 - s + 1$

a++

n = 12

1

~~i = 9 + 10 = 19~~ + 1 = 19

a = 2

~~found~~ a

~~a = 1~~

#return 10

a--

$i -= 10 - s + 1$

~~a = 1~~

~~i = 19 - 10 = 9~~ i = 1

$k = n - i$ ~~-1~~

k = 3

~~10 - s + 9 \cdot (n - i)~~

return:

$a \, \overline{\underbrace{(10 - s) + 9 (k-1)}_{1st}}$

$\overline{9 + 27}$

↳ $a \cdot 100 + \otimes$

$\underline{a=1}$

$s=1$

$i=9+10-1+1=19$

$\underline{a=2}$

$s=2$

$i=19+10-2+1=\underline{28}$

$\underline{a=3}$

$s=3$

$i=28+7+1=36$

$\underline{a=4}$

PE $312 // 10 = 31$

• $312-31\cdot10 = 2$

$p = n // 10$

if $p > 0$, $s += n - 10p$

else return s

$\underset{n=23}{\longrightarrow}$ $a=2$

$i = 28-(10-2+1) = 19.$

$k = 23-19-1 = 3.$

$\boxed{2}\ \underbrace{10-2+27}_{35} \Rightarrow 230$

$\underset{n=29}{\longrightarrow}$ $a=3$

$i = \cancel{28}\ 36-10+3-1 = \underline{28}.$

$k = 0 = 29-28-1$

$\boxed{3}\ \underbrace{10-3}_{7}\ 4$

$7 \downarrow 07 \Rightarrow 307.$

$\underset{n=5}{\longrightarrow}$ $a=0 ; s=0$

$i = 9-10+0-1 = -2$

$k = 5+2-1 = 6.$

$\boxed{0}\ 10 \cancel{-1} + 9\cdot5 = 9+45 = 54$

$i = 9-11+s = -2+s$

$k = n+2-s-1 = n+1-s$