Group 4 Presentation (2) MTH-404

By:Jeesun S.
Paul Ycay
Ali Syed



13.15 Corporate Profits In order to study the relationship of advertising and capital investment with corporate profits, the following data, recorded in units of \$100,000, were collected for 10 medium-sized firms in the same year. The variable y represents profit for the year, x_1 represents capital investment, and x_2 represents advertising expenditures.

| y | <i>X</i> ₁ | X 2 | <i>y</i> | <i>X</i> ₁ | <i>X</i> ₂ |
|----|-----------------------|------------|----------|-----------------------|-----------------------|
| 15 | 25 | 4 | 1 | 20 | 0 |
| 16 | 1 | 5 | 16 | 12 | 4 |
| 2 | 6 | 3 | 18 | 15 | 5 |
| 3 | 30 | 1 | 13 | 6 | 4 |
| 12 | 29 | 2 | 2 | 16 | 2 |

a. Using the model

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2$$

and an appropriate computer software package, find the least-squares prediction equation for these data.

- **b.** Use the overall F-test to determine whether the model contributes significant information for the prediction of y. Use $\alpha=.01$.
- c. Does advertising expenditure x_2 contribute significant information for the prediction of y, given that x_1 is already in the model? Use $\alpha = .01$.
- d. Calculate the coefficient of determination, R². What percentage of the overall variation is explained by the model?



Excel Data

| \mathbf{A} | Α | В | С | D | E | F | G | Н | I | J | К | L |
|--------------|----|---------|--------------------|---|---|---|---|---|---|---|---|---|
| 1 | y | x_{I} | \boldsymbol{x}_2 | | | | | | | | | |
| 2 | 15 | 25 | 4 | | | | | | | | | |
| 3 | 16 | 1 | 5 | | | | | | | | | |
| 4 | 2 | 6 | 3 | | | | | | | | | |
| 5 | 3 | 30 | 1 | | | | | | | | | |
| 6 | 12 | 29 | 2 | | | | | | | | | |
| 7 | 1 | 20 | 0 | | | | | | | | | |
| 8 | 16 | 12 | 4 | | | | | | | | | |
| 9 | 18 | 15 | 5 | | | | | | | | | |
| 10 | 13 | 6 | 4 | | | | | | | | | |
| 11 | 2 | 16 | 2 | | | | | | | | | |
| 12 | | | | | | | | | | | | |
| 13 | | | | | | | | | | | | |
| 14 | | | | | | | | | | | | |
| 15 | | | | | | | | | | | | |
| 16 | | | | | | | | | | | | |
| 17 | | | | | | | | | | | | |
| 18 | | | | | | | | | | | | |

Regression equation:

$$y = -8.18 + 0.29x_1 + 4.43x_2$$

| | Coefficie | Standard | | | Lower | Upper | Lower | Upper |
|-----------|-----------|----------|----------|----------|----------|----------|----------|----------|
| | nts | Error | t Stat | P-value | 95% | 95% | 99.0% | 99.0% |
| Intercept | -8.17702 | 4.205988 | -1.94414 | 0.092967 | -18.1226 | 1.768562 | -22.8958 | 6.541765 |
| x1 | 0.292132 | 0.135714 | 2.152556 | 0.068355 | -0.02878 | 0.613044 | -0.1828 | 0.767061 |
| x2 | 4.434303 | 0.800243 | 5.541193 | 0.000868 | 2.542028 | 6.326578 | 1.633864 | 7.234741 |
| | | | | | | | | |

Analysis of variance

| Source | DF | SS | MS | F | Р |
|-------------------|----|--------|--------|-------|-------|
| Regression | 2 | 355.22 | 177.61 | 16.28 | 0.002 |
| Residual Error | 7 | 76.38 | 10.91 | | |
| Total | 9 | 431.60 | | | |

| Source | DF | Seq SS |
|--------|----|--------|
| X1 | 1 | 20.16 |
| X2 | 1 | 335.05 |

A) From the analysis of variance, we can conclude that the least square prediction equation is $\hat{y}=-8.18+0.29x_1+4.43x_2$

B) Use the overall F-test to determine whether the model contributes significant information for the prediction of y. Use $\alpha = 0.01$ h(o): $\beta 1 = 0$

$$h(a)$$
: $\beta 1 \neq 0$

$$f = MSR/MSE$$

$$f = 177.61 / 10.91$$

$$f = 16.28$$

| | | | | | | ff ₁ | | | | |
|--------|-------|-------|------|------|------|-----------------|------|------|------|------|
| df_2 | ex | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 10 | 0.100 | 3.29 | 2.92 | 2.73 | 2.61 | 2.52 | 2,46 | 2.41 | 2.38 | 2.35 |
| | 0.050 | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 | 3,22 | 3.14 | 3.07 | 3.02 |
| | 0.025 | 6.94 | 5.46 | 4.83 | 4.47 | 4.24 | 4,07 | 3.95 | 3.85 | 3.78 |
| | 0.010 | 10.04 | 7.56 | 6.55 | 5.99 | 5.64 | 5,39 | 5.20 | 5.06 | 4.94 |
| | 0.005 | 12.83 | 9.43 | 8.08 | 7.34 | 6.87 | 6,54 | 6.30 | 6.12 | 5.93 |
| 11 | 0.100 | 3.23 | 2.86 | 2.66 | 2.54 | 2.45 | 2.39 | 2.34 | 2.30 | 2.2 |
| | 0.050 | 4.84 | 3.98 | 3.59 | 3.36 | 3.20 | 3.09 | 3.01 | 2.95 | 2.9 |
| | 0.025 | 6.72 | 5.26 | 4.63 | 4.28 | 4.04 | 3.88 | 3.76 | 3.66 | 3.5 |
| | 0.010 | 9.65 | 7.21 | 6.22 | 5.67 | 5.32 | 5.07 | 4.89 | 4.74 | 4.6 |
| | 0.005 | 12.23 | 8.91 | 7.60 | 6.88 | 6.42 | 6.10 | 5.86 | 5.68 | 5.5 |
| 12 | 0.100 | 3.18 | 2.81 | 2.61 | 2.48 | 2.39 | 2.33 | 2.28 | 2.24 | 2.2 |
| | 0.050 | 4.75 | 3.89 | 3.49 | 3.26 | 3.11 | 3.00 | 2.91 | 2.85 | 2.8 |
| | 0.025 | 6.55 | 5.10 | 4.47 | 4.12 | 3.89 | 3.73 | 3.61 | 3.51 | 3.4 |
| | 0.010 | 9.33 | 6.93 | 5.95 | 5.41 | 5.06 | 4.82 | 4.64 | 4.50 | 4.3 |
| | 0.005 | 11.75 | 8.51 | 7.23 | 6.52 | 6.07 | 5.76 | 5.52 | 5.35 | 5.2 |
| 3 | 0.100 | 3.14 | 2.76 | 2.56 | 2.43 | 2.35 | 2,28 | 2.23 | 2,20 | 2.1 |
| | 0.050 | 4.67 | 3.81 | 3.41 | 3.18 | 3.03 | 2,92 | 2.83 | 2,77 | 2.7 |
| | 0.025 | 6.41 | 4.97 | 4.35 | 4.00 | 3.77 | 3,60 | 3.48 | 3,39 | 3.3 |
| | 0.010 | 9.07 | 6.70 | 5.74 | 5.21 | 4.86 | 4,62 | 4.44 | 4,30 | 4.1 |
| | 0.005 | 11.37 | 8.19 | 6.93 | 6.23 | 5.79 | 5,48 | 5.25 | 5.08 | 4.9 |
| 4 | 0.100 | 3.10 | 2.73 | 2,52 | 2.39 | 2.31 | 2.24 | 2.19 | 2.15 | 2.1 |
| | 0.050 | 4.60 | 3.74 | 3,34 | 3.11 | 2.96 | 2.85 | 2.76 | 2.70 | 2.6 |
| | 0.025 | 6.30 | 4.86 | 4,24 | 3.89 | 3.66 | 3.50 | 3.38 | 3.29 | 3.2 |
| | 0.010 | 8.86 | 6.51 | 5,56 | 5.04 | 4.69 | 4.46 | 4.28 | 4.14 | 4.0 |
| | 0.005 | 11.06 | 7.92 | 6,68 | 6.00 | 5.56 | 5.26 | 5.03 | 4.86 | 4.7 |
| 5 | 0.100 | 3.07 | 2.70 | 2.49 | 2.36 | 2.27 | 2.21 | 2.16 | 2.12 | 2.0 |
| | 0.050 | 4.54 | 3.68 | 3.29 | 3.06 | 2.90 | 2.79 | 2.71 | 2.64 | 2.5 |
| | 0.025 | 6.20 | 4.77 | 4.15 | 3.80 | 3.58 | 3.41 | 3.29 | 3.20 | 3.1 |
| | 0.010 | 8.68 | 6.36 | 5.42 | 4.89 | 4.56 | 4.32 | 4.14 | 4.00 | 3.8 |
| | 0.005 | 10.80 | 7.70 | 6.48 | 5.80 | 5.37 | 5.07 | 4.85 | 4.67 | 4.5 |
| 6 | 0.100 | 3.05 | 2.67 | 2.46 | 2.33 | 2.24 | 2.18 | 2.13 | 2.09 | 2.0 |
| | 0.050 | 4.49 | 3.63 | 3.24 | 3.01 | 2.85 | 2.74 | 2.66 | 2.59 | 2.5 |
| | 0.025 | 6.12 | 4.69 | 4.08 | 3.73 | 3.50 | 3.34 | 3.22 | 3.12 | 3.0 |
| | 0.010 | 8.53 | 6.23 | 5.29 | 4.77 | 4.44 | 4.20 | 4.03 | 3.89 | 3.7 |
| | 0.005 | 10.58 | 7.51 | 6.30 | 5.64 | 5.21 | 4.91 | 4.69 | 4.52 | 4.3 |
| 7 | 0.100 | 3.03 | 2.64 | 2.44 | 2.31 | 2.22 | 2.15 | 2.10 | 2.06 | 2.0 |
| | 0.050 | 4.45 | 3.59 | 3.20 | 2.96 | 2.81 | 2.70 | 2.61 | 2.55 | 2.4 |
| | 0.025 | 6.04 | 4.62 | 4.01 | 3.66 | 3.44 | 3.28 | 3.16 | 3.06 | 2.9 |
| | 0.010 | 8.40 | 6.11 | 5.18 | 4.67 | 4.34 | 4.10 | 3.93 | 3.79 | 3.6 |
| | 0.005 | 10.38 | 7.35 | 6.16 | 5.50 | 5.07 | 4.78 | 4.56 | 4.39 | 4.2 |
| 8 | 0.100 | 3.01 | 2.62 | 2.42 | 2.29 | 2.20 | 2.13 | 2.08 | 2.04 | 2.0 |
| | 0.050 | 4.41 | 3.55 | 3.16 | 2.93 | 2.77 | 2.66 | 2.58 | 2.51 | 2.4 |
| | 0.025 | 5.98 | 4.56 | 3.95 | 3.61 | 3.38 | 3.22 | 3.10 | 3.01 | 2.9 |
| | 0.010 | 8.29 | 6.01 | 5.09 | 4.58 | 4.25 | 4.01 | 3.84 | 3.71 | 3.6 |
| | 0.005 | 10.22 | 7.21 | 6.03 | 5.37 | 4.96 | 4.66 | 4.44 | 4.28 | 4.1 |
| 9 | 0.100 | 2.99 | 2.61 | 2.40 | 2.27 | 2.18 | 2.11 | 2.06 | 2.02 | 1.9 |
| | 0.050 | 4.38 | 3.52 | 3.13 | 2.90 | 2.74 | 2.63 | 2.54 | 2.48 | 2.4 |
| | 0.025 | 5.92 | 4.51 | 3.90 | 3.56 | 3.33 | 3.17 | 3.05 | 2.96 | 2.8 |
| | 0.010 | 8.18 | 5.93 | 5.01 | 4.50 | 4.17 | 3.94 | 3.77 | 3.63 | 3.5 |
| | 0.005 | 10.07 | 7.09 | 5.92 | 5.27 | 4.85 | 4.56 | 4.34 | 4.18 | 4.0 |
| 0 | 0.100 | 2.97 | 2.59 | 2.38 | 2.25 | 2.16 | 2.09 | 2.04 | 2.00 | 1.9 |
| | 0.050 | 4.35 | 3.49 | 3.10 | 2.87 | 2.71 | 2.60 | 2.51 | 2.45 | 2.3 |
| | 0.025 | 5.87 | 4.46 | 3.86 | 3.51 | 3.29 | 3.13 | 3.01 | 2.91 | 2.8 |
| | 0.010 | 8.10 | 5.85 | 4.94 | 4.43 | 4.10 | 3.87 | 3.70 | 3.56 | 3.4 |
| | 0.005 | 9.94 | 6.99 | 5.82 | 5.17 | 4.76 | 4.47 | 4.26 | 4.09 | 3.9 |

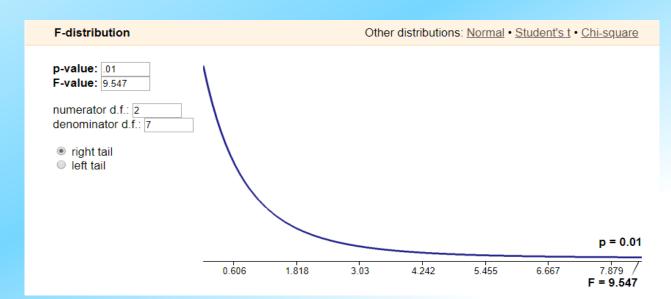
 f_a = 0.01 with df_1 =2 and df_2 =7 is non existent in table 6, so we go to the p-value. Since the p-value, from ANOVA table was 0.002 < α = 0.01, we reject the Null **Hypothesis** and conclude that there is a significant evidence for the prediction of y.

F-Test

$$F_{.01,2,7} = 9.55$$

$$\because$$
 our $f > f_a$

We Can conclude that the regression to be highly significant. At least, one of the predictor variables is contributing significant information for the prediction of the response variable y. That is, profits for the year.



c. Does advertising expenditure x_2 contribute significant information for the prediction of y, given that x_1 is already in the model? Use $\alpha = 0.01$.

| | Coefficie | Standard | | | Lower | Upper | Lower | Upper |
|-----------|-----------|----------|----------|----------|----------|----------|----------|----------|
| | nts | Error | t Stat | P-value | 95% | 95% | 99.0% | 99.0% |
| Intercept | -8.17702 | 4.205988 | -1.94414 | 0.092967 | -18.1226 | 1.768562 | -22.8958 | 6.541765 |
| x1 | 0.292132 | 0.135714 | 2.152556 | 0.068355 | -0.02878 | 0.613044 | -0.1828 | 0.767061 |
| x2 | 4.434303 | 0.800243 | 5.541193 | 0.000868 | 2.542028 | 6.326578 | 1.633864 | 7.234741 |
| | | | | | | | | |

h(o): = x_2 is not significant h(a): = x_2 is significant

From the table above, the t-stat associated with x_2 is 5.54. Looking in the table, the p-value associated with x_2 is 0.0008, which is less than $\alpha = 0.01$. Since it is less than, we can reject the null hypothesis and claim the significance of x_2 . Calculating the p-value by hand was difficult since df of 7 with t stat 5.54 exceeded the t-table, thus excel had to interpret the p-value.

| | cum. prob | t.50 | t .75 | t.80 | t .85 | t.90 | t.95 | t .975 | t.99 | t .995 | t .999 | t .9995 |
|---|-----------|-------|-------|-------|-------|--------|----------|--------|-------|--------|--------|---------|
| | one-tail | 0.50 | 0.25 | 0.20 | 0.15 | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 | 0.001 | 0.0005 |
| | two-tails | 1.00 | 0.50 | 0.40 | 0.30 | 0.20 | 0.10 | 0.05 | 0.02 | 0.01 | 0.002 | 0.001 |
| • | df | 1.00 | 0.00 | 0.40 | 0.00 | 0.20 | 0.10 | 0.00 | 0.02 | 0.01 | 0.002 | 0.001 |
| | 1 | 0.000 | 1.000 | 1.376 | 1.963 | 3.078 | 6.314 | 12.71 | 31.82 | 63.66 | 318.31 | 636.62 |
| | 2 | 0.000 | 0.816 | 1.061 | 1.386 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | 22.327 | 31.599 |
| | 3 | 0.000 | 0.765 | 0.978 | 1.250 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 10.215 | 12.924 |
| | 4 | 0.000 | 0.741 | 0.941 | 1.190 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | 7.173 | 8.610 |
| | 5 | 0.000 | 0.727 | 0.920 | 1.156 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 5.893 | 6.869 |
| | 6 | 0.000 | 0.718 | 0.906 | 1.134 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 | 5.208 | 5.959 |
| | 7 | 0.000 | 0.711 | 0.896 | 1.119 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | 4.785 | 5.408 |
| | 8 | 0.000 | 0.706 | 0.889 | 1.108 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 | 4.501 | 5.041 |
| | 9 | 0.000 | 0.703 | 0.883 | 1.100 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 | 4.297 | 4.781 |
| | 10 | 0.000 | 0.700 | 0.879 | 1.093 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 4.144 | 4.587 |
| | 11 | 0.000 | 0.697 | 0.876 | 1.088 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 4.025 | 4.437 |
| | 12 | 0.000 | 0.695 | 0.873 | 1.083 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 3.930 | 4.318 |
| | 13 | 0.000 | 0.694 | 0.870 | 1.079 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 | 3.852 | 4.221 |
| | 14 | 0.000 | 0.692 | 0.868 | 1.076 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 3.787 | 4.140 |
| | 15 | 0.000 | 0.691 | 0.866 | 1.074 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 3.733 | 4.073 |
| | 16 | 0.000 | 0.690 | 0.865 | 1.071 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 | 3.686 | 4.015 |
| | 17 | 0.000 | 0.689 | 0.863 | 1.069 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 | 3.646 | 3.965 |
| | 18 | 0.000 | 0.688 | 0.862 | 1.067 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 | 3.610 | 3.922 |
| | 19 | 0.000 | 0.688 | 0.861 | 1.066 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 | 3.579 | 3.883 |
| | 20 | 0.000 | 0.687 | 0.860 | 1.064 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 | 3.552 | 3.850 |
| | 21 | 0.000 | 0.686 | 0.859 | 1.063 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 | 3.527 | 3.819 |
| | 22 | 0.000 | 0.686 | 0.858 | 1.061 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 | 3.505 | 3.792 |
| | 23 | 0.000 | 0.685 | 0.858 | 1.060 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 | 3.485 | 3.768 |
| | 24 | 0.000 | 0.685 | 0.857 | 1.059 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 | 3.467 | 3.745 |
| | 25 | 0.000 | 0.684 | 0.856 | 1.058 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 | 3.450 | 3.725 |
| | 26 | 0.000 | 0.684 | 0.856 | 1.058 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 | 3.435 | 3.707 |
| | 27 | 0.000 | 0.684 | 0.855 | 1.057 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 | 3.421 | 3.690 |
| | 28 | 0.000 | 0.683 | 0.855 | 1.056 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 | 3.408 | 3.674 |
| | 29 | 0.000 | 0.683 | 0.854 | 1.055 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 | 3.396 | 3.659 |
| | 30 | 0.000 | 0.683 | 0.854 | 1.055 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 | 3.385 | 3.646 |
| | 40 | 0.000 | 0.681 | 0.851 | 1.050 | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 | 3.307 | 3.551 |
| | 60 | 0.000 | 0.679 | 0.848 | 1.045 | 1.296 | 1.671 | 2.000 | 2.390 | 2.660 | 3.232 | 3.460 |
| | 80 | 0.000 | 0.678 | 0.846 | 1.043 | 1.292 | 1.664 | 1.990 | 2.374 | 2.639 | 3.195 | 3.416 |
| | 100 | 0.000 | 0.677 | 0.845 | 1.042 | 1.290 | 1.660 | 1.984 | 2.364 | 2.626 | 3.174 | 3.390 |
| | 1000 | 0.000 | 0.675 | 0.842 | 1.037 | 1.282 | 1.646 | 1.962 | 2.330 | 2.581 | 3.098 | 3.300 |
| | Z | 0.000 | 0.674 | 0.842 | 1.036 | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | 3.090 | 3.291 |
| | | 0% | 50% | 60% | 70% | 80% | 90% | 95% | 98% | 99% | 99.8% | 99.9% |
| | | | | | | Confid | dence Le | evel | | | | |

d. Calculate the coefficient of determination, R². What R percentage of the overall variation is explained by the model?

r²=SSR/Total SS

=355.251/431.6

=0.823102409

Standard Error = 3.30335 R-Sq = 82.3% R-Sq(adj) = 77.2%

•• A 82% overall variation is explained by the model, meaning this model works well with the given data

| Regression Statistics | | | | | | | | | |
|-----------------------|----------|--|--|--|--|--|--|--|--|
| Multiple | | | | | | | | | |
| R | 0.907204 | | | | | | | | |
| R Square | 0.823019 | | | | | | | | |
| Adjusted | | | | | | | | | |
| R Square | 0.772453 | | | | | | | | |
| Standard | | | | | | | | | |
| Error | 3.30335 | | | | | | | | |
| Observat | | | | | | | | | |
| ons | 10 | | | | | | | | |

