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Continuing previous example

$Z = x_1 + 3x_4 - 2x_5 + 21$

$x_1, x_4, x_5 \geq 0$

inc. Z   inc Z   → optimum value using x_5

x_5 inc Z dec So, take x_5 is (Basic variable)

whenever x_1, x_4 changes value, it increases Z, ∴ no optimum using these

So, now x_5 is the basic variable
At one time, we can only replace one basic variable

∴ there is a possibility of improvement in objective fn.

Now, first x_2 & x_3 were basic variables
Now, we need to replace x_5 with one of them.

$x_1 = 0$, $x_4 = 0$

$$\left.\begin{aligned} x_3 + 2x_5 &= 6 \\ x_2 + 3x_5 &= 15 \end{aligned}\right\} \Rightarrow \begin{aligned} x_3 &= 6 - 2x_5 \geq 0 \\ x_2 &= 15 - 3x_5 \geq 0 \end{aligned}$$

$x_5 \leq \frac{6}{2} = 3$ → strict

$x_5 \leq \frac{15}{3} = 5$

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Set $x_5 = 3$
$x_3 = 0$ & $x_2 = 6 > 0$

$\therefore$ x_5 & x_2 will be our next basic
x_3 will be non-basic now.

Now soln is $(0, 6, 0, 0, 3) \rightarrow$ Feasible soln
$\hookrightarrow Z = 0 + 0 + 6 + 3(3)$
$Z = 15 \rightarrow$ Min.

$$Z = -4x_1 + x_2 + x_3 + 7x_4 + 3x_5$$
$$6 = -6x_1 + x_3 - 2x_4 + 2x_5$$
$$9 = 3x_1 + x_2 - x_3 + 8x_4 + x_5$$

Pivoting

Step - II

$$-x_1 + x_3 + x_4 = -15 + Z$$
$$-3x_1 + \tfrac{1}{2}x_3 - x_4 + x_5 = 3$$
$$6x_1 + x_2 - \tfrac{3}{2}x_3 + 9x_4 = 6$$

x_2, x_5 } Basic — Min. $Z = 15$
So soln is $(0, 6, 0, 0, 3)$ with $Z = 15$

Now check whether Z can be minimized more or not.

Step 3

$$Z = -x_1 + x_3 + x_4 + 15$$

$\hookrightarrow$ can Minimize Z

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$x_1 \rightarrow$ Basic Variable

Replace x_2 or x_5 by x_1

$x_3 = 0$, $x_4 = 0$

$-6x_1 + 2x_5 = 6 \Rightarrow -3x_1 + x_5 = 3$

$6x_1 + x_2 = 6$

$x_5 = 3 + 3x_1 \geq 0$

$x_2 = 6 - \underbrace{6x_1}_{} \geq 0$

Strict Condition

As $x_1 \not> 1 \therefore x_2 \leq 0$

which violate condition

$\Rightarrow x_1 \leq 1$

Now, substituting x_1 in

$x_2 = 0 \rightarrow$ Non-basic

$x_5 = 6$

x_1 & $x_5 \rightarrow$ New Basic Variables

Solⁿ is $(1, 0, 0, 0, 6)$

$\boxed{\text{Min } Z = 14}$

Thru Pivoting

$\frac{1}{6}x_2 + \frac{3}{4}x_3 + \frac{5}{2}x_4 = -14 + Z$

$\frac{1}{2}x_2 - \frac{1}{4}x_3 + \frac{7}{2}x_4 + x_5 = 6$

$x_1 + \frac{1}{6}x_2 - \frac{1}{4}x_3 + \frac{3}{2}x_4 = 1$

$(1, 0, 0, 0, 6) \longrightarrow$ Solⁿ (Basic feasible solⁿ)

$Z = 14$

$\longrightarrow$ Optimum Value.

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Step-4

$$Z = \frac{1}{6}x_2 + \frac{3}{4}x_3 + \frac{5}{2}x_4 + 14$$

Can't be further decreased
Algorithm stop.

Min Z = we see −ve sign
Max Z = " " +ve sign
will improve Z

Simplex Method
1) Standard form
2) Canonical form (feasible Basic solⁿ)
3) Move from one " " " to other
to get optimum solⁿ for Z

—— x —— x —— x ——

⇒ LP is in canonical form.
Let Assume m = no. of constraints,
n = " " variables
& mostly $n > m$

Let us assume $(x_1, x_2, \ldots x_m \geq 0) \rightarrow$ B.V.

$c_{m+1} x_{m+1} + \ldots + c_n x_n = Z_0 + Z$

$a_{1,m+1} x_{m+1} + \ldots + a_{1n} x_n = b_1$

$x_1 \ldots$

$0 \quad x_2$

$0 \quad 0 \quad x_3$

$0 \quad 0 \quad 0 \quad x_m + a_{m,m+1} x_{m+1} + \ldots + a_{mn} x_n = b_m$

$I \rightarrow$

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Rule :- For LP in (1), if $c_j \geq 0$, $j = m+1, --- n$ then
$\min Z = -Z_0$ &
basic solutions are
$(b_1, b_2 --- b_m, 0, 0, --- 0)$
Stop the algorithm if Rule is satisfied

Rule :- If one of ~~$c_j < 0$~~ $c_{ik} < 0$ (Strict condition), x_k shall enter to basic variable

$x_1 + a_{1k} x_k = b_1$
$x_2 + a_{2k} x_k = b_2$
⋮
$x_m + a_{mk} x_k = b_m$

we find a ~~strict condition~~

$\Rightarrow x_1 = b - a_k x_k \geq 0$