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7 January, 2020

In LPP
1. Constraints are linear
* 2. Minimum is attained at corner points (vertex)

Ex 2.2.1 (Pg 26)
Blending Model

	A	B	C	Cost
Feed 1	3	7	3	10
Feed 2	2	2	6	4
Min^n Req.	60	84	72	

$x \rightarrow$ # of feed 1
$y \rightarrow$ # of feed 2

$\left.\begin{array}{l}\text{Min } Z = 10x + 4y\end{array}\right\}$ Cost function
subject to
Constraints

$3x + 2y \geq 60$ — I
$7x + 2y \geq 84$ — II
$3x + 6y \geq 72$ — III
$x, y \geq 0$ — IV

2 variables, $\therefore$ we use graphical Method.

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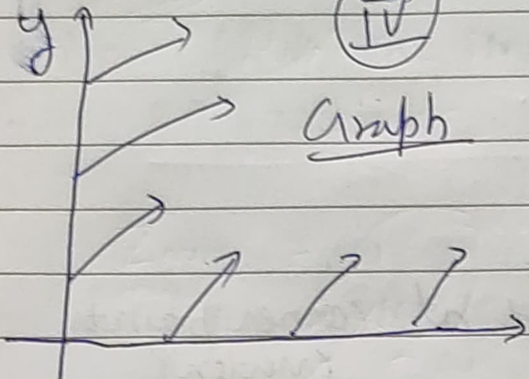
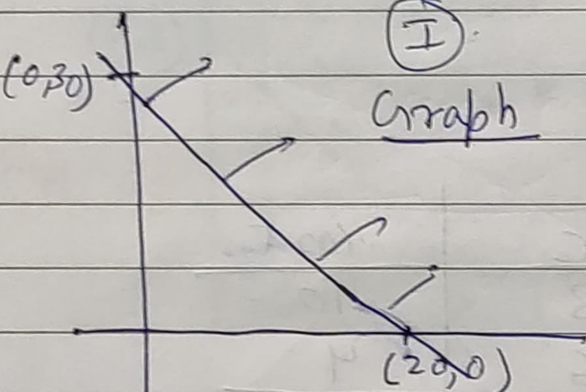
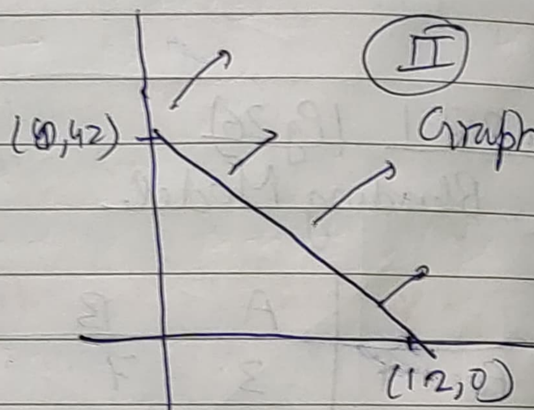
IV Graph



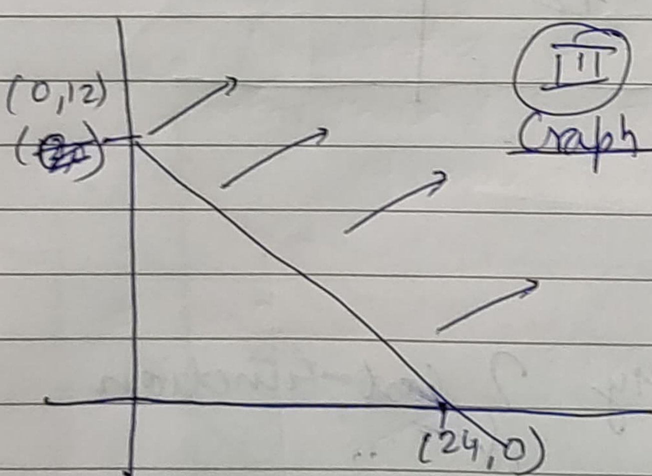
I Graph



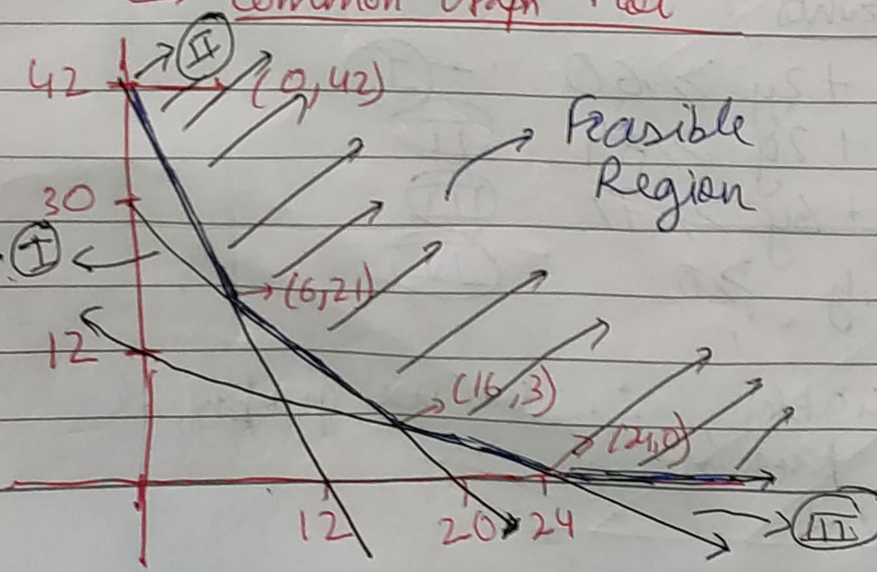
II Graph



III Graph

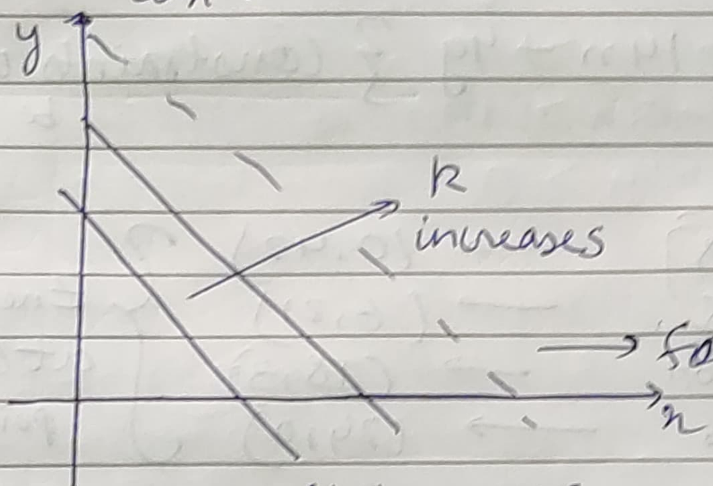


→ Common Graph Plot



Now minimize
$Z = 10x + 14y$

Cost f^n



$Z = 10x + 4y$

$10x + 4y = k$

→ for diffrent k

slope $= -\frac{10}{4}$

Corner Points

$(0,42)$, $(6,21)$ $(16,3)$ $(24,0)$

Checking Z at Corner Points

$Z_{min} = 10x + 4y$
$= 10(0) + 4(42)$ for $(0,42)$
$= 168$

$Z_{min} = 10(6) + 4(21)$ for $(6,21)$
$= 60 + 84 = 144$

$Z_{min} = 10(16) + 4(3)$ for $(16,3)$
$= 160 + 12 = 172$

$Z_{min} = 10(24) + 0$ for $(24,0)$
$= 240$

* So, 6 units of feed 1 & 21 units of feed 2
Min Cost $= 144$

* let we change price for
feed 1 : 14 cents / lb
feed 2 : 4 cents / lb

$\Big\}$ Cost f^n will change
$Z_{new} = 14x + 4y$

$\text{Min}^n \ \underset{new}{Z} = 14x + 4y$ } Constraints will be same

$Z_{min\ new} = 168 \longrightarrow (0,42)$
$= 168 \longrightarrow (6,21)$
$= 264 \longrightarrow (18,3)$
$= 336 \longrightarrow (24,0)$

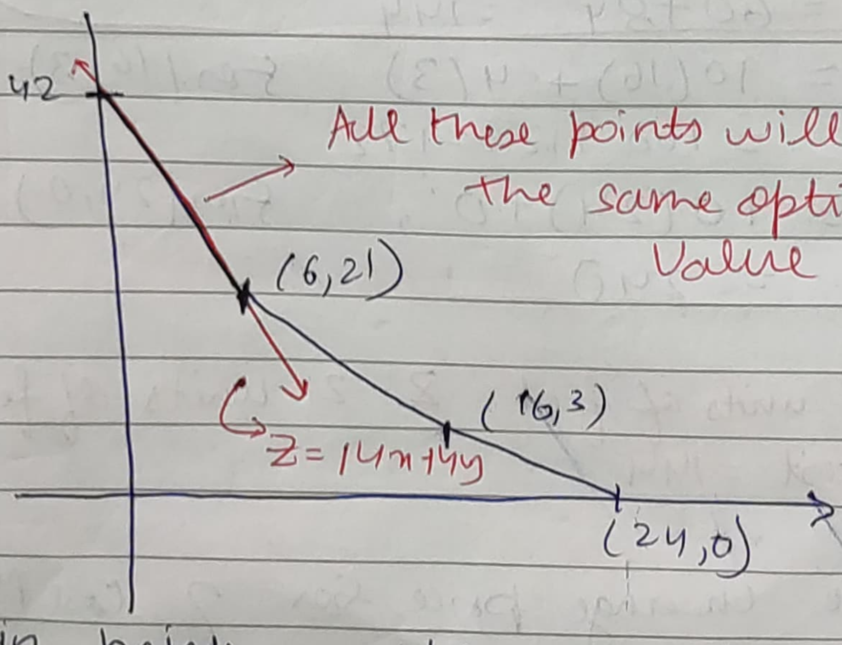
} Z_{new} at corner points

→ 2 min. points

Why? ∵ $14x + 4y = k$ has slope $= -\frac{7}{2}$

& the constraint $7x + 2y \geq 84$, has also slope $= -7/2$

So the constraint line & Z are overlapping.
So, more than 1 points, ~~so~~
Optimal value → Unique



* ① Min, points could be more than one
* ② The optimal value is unique
③ The points will be adjacent (Corner points)

The feasible is empty – not feasible

The sol^n is unbounded

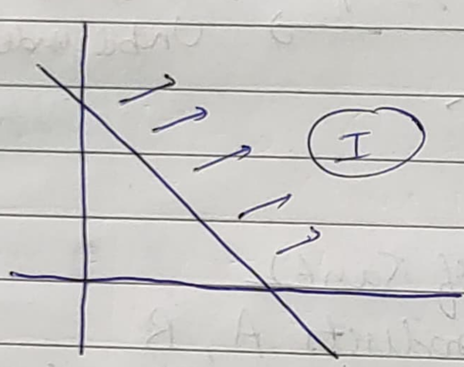
Example min $Z = x + y$

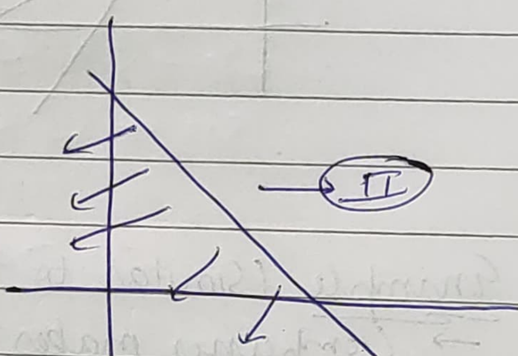
subject to

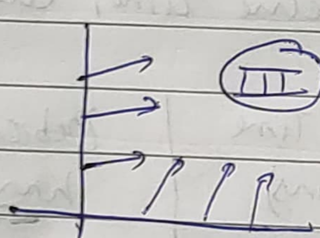
$x + y \geq 10$ — I

$x + y \leq 9$ — II

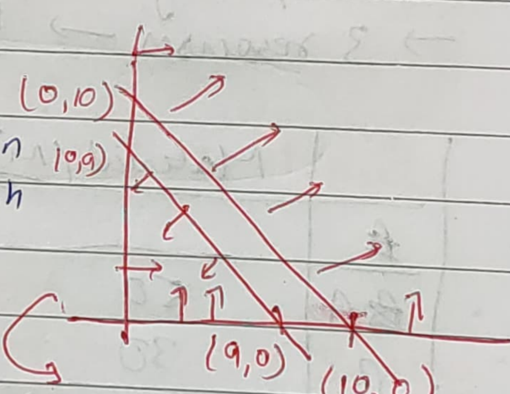
$x, y \geq 0$ — III







↳ Common Graph



So, no common or feasible region

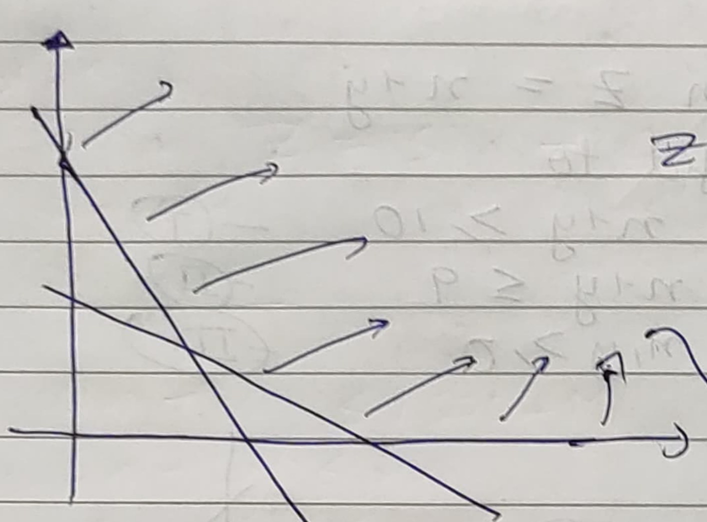
↳ No sol^n

~~Min Z~~ Maximize $Z = x + y$

$2x + y \geq 10$

$x + 3y \geq 10$

$x, y \geq 0$



$Z = x + y = k$

As we increase k Z increases

(No boundary)

~~Unbound~~

Unbounded solⁿ

Example (Similar to that of Jan6)

→ Company makes 2 products A, B

→ 3 resources → Material, Machine Time, Labour

	Material (M)	Machine Time T (mins)	Labour (L) hrs
~~A~~			
~~B~~A	50	6	3
B	30	5	5
Max.	2000	300	200

Profit

A → 50 per unit

B → 60 " "

How much produce A & B to maximize profit?

Sol^n

Variables : A → # units of A (x)
B → # " " B (y)

Maximize $Z = 50x + 60y$
subject to

Constraints

$$50x + 30y \leq 2000$$
$$6x + 5y \leq 300$$
$$3x + 5y \leq 200$$
$$x, y \geq 0$$

↳ Lets say, we change Materials limit
let that be 1500
Upto 1500, cost f^n is same
Above 1500, we incur extra costs & my profit decreases.

when we got
↳ 1501 → 20 cents

Let we introduce a new variable M_1
M_1 → be the amount of material above 1500 materials units (lb)

$M_1 \geq 0$

$\hookrightarrow M_1 = 0$ if we use less than 1500 lb

$\hookrightarrow M_1 > 0 \rightarrow$ More than 1500 lb

Cost f^n

max $Z = 50x + 60y - \underline{20} M_1$

$\hookrightarrow$ See units accordingly (Z in cents whatever is given, change accordingly)

subject to

<u>Constraints</u>

$50x + 30y \leq 1500 + M_1$

$6x + 5y \leq 300$

$3x + 5y \leq 200$

$x, y \geq 0$

$M_1 \geq 0$