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(2) Unbounded Soln

Max $Z = 4x_1 + 3x_2$

subject to $x_1 - 6x_2 \leq 5$

$3x_1 \leq 11$

$x_1, x_2 \geq 0$

- Standard form first (Adding slack variables)

$Z = 4x_1 + 3x_2$

subj. to $x_1 - 6x_2 + 1x_3 = 5$

$3x_1 + 1x_4 = 11$

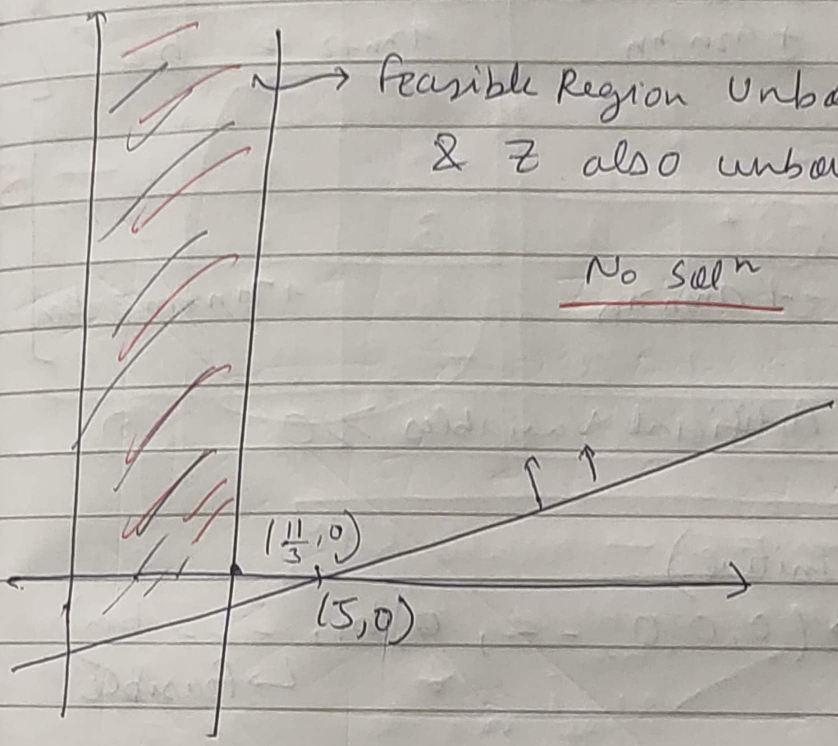
$x_i \geq 0 \; ; \; i = 1, 2, 3, 4$

	x_1	x_2	x_3	x_4	b	
x_3	1	-6	1	0	5	5/1
x_4	3	0	0	1	11	11/3 → exit
Z	4 ↑	3	0	0	0	
x_3	0	-6	1	-1/3	4/3	
x_1	1	0	0	1/3	11/3	
Z	0	3 ↑	0	-4/3	14/3	→ Entering Variable coming, But Exit Variable Not coming

→ Unbounded

(no +ve value so that one of the basic variable is exited)

Graphically,



③ Infeasible Soln → No optimum value for Z

Artificial Variables

Initial Canonical Form → to achieve → add artificial variables

$a_{11}x_1 + - - - + a_{1n}x_n = b_1$
⋮
$a_{m1}x_1 + - - - + a_{mn}x_n = b_m$ } Already In Standard form

$b_i \geq 0$ → If not true multiply by −ve

↓ Introduce Artificial Variables

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Introduce Artificial Variables, $x_{n+1}, \ldots x_{n+m}$ (≥ 0)

$$\left.\begin{aligned}
a_{11}x_1 + \cdots + a_{1n}x_n + x_{n+1} &= b_1 \\
a_{21}x_1 + \cdots + a_{2n}x_n \quad + x_{n+2} &= b_2 \\
&\vdots \\
a_{m1}x_1 + \cdots + a_{mn}x_n \quad + x_{n+m} &= b_m
\end{aligned}\right\} \circledast$$

Artificial Variables ≥ 0

Basic Soln (Initial)

$\hookrightarrow (0, 0, 0, \ldots, 0, b_1, \ldots b_m)$

$\hookrightarrow$ feasible

① Two-phase Method

Let we define an objective f^n

$\text{Min}^n \; w = x_{n+1} + x_{n+2} + \cdots + x_{n+m}$

Now, $-d_1x_1 - d_2x_2 - \cdots - d_nx_n + \underbrace{x_{n+1} + \cdots + x_{n+m}}_{w} = -w_0$

$\hookrightarrow$ expressing in terms of non-basic

Basically adding all constraints

where

After adding all consts.

$d_1 = -(a_{11} + a_{21} + \cdots a_{m1})$

$d_i = -(a_{1i} + a_{2i} + \cdots + a_{mi})$

$i = 1, 2, 3, \ldots n$

$w_0 = -(b_1 + b_2 + \cdots + b_m)$

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$\Rightarrow w + w_0 = d_1 x_1 + d_2 x_2 + \cdots + d_n x_n$ (1)

(1) & (2) is in canonical form

$w_{min} \geq 0$

$\because$ Artificial variables are ≥ 0

We need to evaluate $\boxed{w_{min} = 0}$ to get a feasible solⁿ for removing artificial variables.

If $w_{min} > 0 \Rightarrow$ LP model is not feasible

e.g.

$$x_1 - x_2 = 1$$
$$2x_1 + x_2 - x_3 = 3$$
$$x_i \geq 0$$

Introduce Artificial Variables (x_4 & x_5)

$x_4, x_5 \geq 0$

& Define Objective f^n for these A.V.

Let $\boxed{\min.\ w = x_4 + x_5}$

Canonical form

$$-3x_1 + 0x_2 + x_3 = -4 + w$$

Subj to

$$x_1 - x_2 + x_4 = 1$$
$$2x_1 + x_2 - x_3 + x_5 = 3$$
$$x_i \geq 0$$

Using (1)

I Model

Divide the table after the first phase

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	x_1	x_2	x_3	x_4	x_5	b	
x_4	1	-1	0	1	0	1	1/2 out
x_5	2	1	-1	0	1	3	3/2
w	-3 ↑	0	1	0	0	-4	
x_1	1	-1	0	1	0	1	
x_5	0	3	-1	-2	1	1	→ out
w	0	-3 ↑	1	3	1	-1	
x_1	1	0	1/3	1/3	1/3	4/3	
x_2	0	1	-1/3	-2/3	1/3	1/3	
w	0	0	0	1	1	0	

$\boxed{w_{min} = 0 \rightarrow \text{feasible sol}^n}$

We will never enter AV in basic

Need not do pivoting

Very Important in Phase I

If $\neq 0$, then no feasible

I Model

Now, min Z

subject to $x_1 + 1/3\, x_3 = 4/3$
$x_2 - 1/3\, x_3 = 1/3$
$x_i \geq 0$

} Second Phase

	x_1	x_2	x_3	b
x_1	1	0	1/3	4/3
x_2	0	1	-1/3	1/3

} Solve this

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Eg. Min $Z = 2x_1 - 3x_2 + x_3 + x_4$

subj. to $x_1 - 2x_2 - 3x_3 - 2x_4 = 3$

$x_1 - x_2 + 2x_3 + x_4 = 11$

$x_i \geq 0$

Phase-I

A.V. x_5, x_6, $w = x_5 + x_6$

$-2x_1 + 3x_2 + x_3 + x_4 = -14 + w$

$x_1 - 2x_2 - 3x_3 - 2x_4 + x_5 = 3$

$x_1 - x_2 + 2x_3 + x_4 + x_6 = 11$

Initial Canonical form

	x_1	x_2	x_3	x_4	x_5	x_6	b
x_5	1	-2	-3	-2	1	0	3
x_6	1	-1	2	1	0	1	11
w	-2	+3	1	1	0	0	-14
x_1	1	-2	-3	-2	Need not do here for A.V.		3
x_6	0	1	5	3			8
w	0	-1	-5	-3			-8
x_1	1	0	7	4			19
x_2	0	1	5	3			8
w	0	0	0	0			$\boxed{0}$

Starting Point for Phase-II

we can enter ~~exit~~ any variable here & x_6 will be exited

∵ they all are -ve

If we go by rule, x_5 will be entered, but we are entering -1 here

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for the problem,

$Z_{min} = 14$

at $(19, 8, 0, 0)$