DATE
PAGE

11 January 2021

Ex-

Ex - 2.3.4 (Pg 40)

We produce 2 Products
↳ P_1 and P_2

Materials (per lb)

Process	Labour (per hr)	A	B	P_1	P_2
1	20	160	30	35	55
2	30	100	35	45	42
3	10	200	60	70	0
4	25	75	80	0	90

Prod. req. Atleast 2100 units of P_1
1800 " " P_2

A : max 4 tons = 8000 lb
B : " 2 " = 4000 lb
labour = 1000 hr
Price : A : 3/lb
B : 7/lb

Variables : x_1, x_2, x_3, x_4 = no. of hr/week of process 1, 2, 3, 4.

Objective f^n

$$\text{Minimize } Z = (160x_1 + 100x_2 + 200x_3 + 75x_4) \times 3 + (30x_1 + 35x_2 + 60x_3 + 80x_4) \times 7$$

DATE
PAGE

$\Rightarrow$ Min $Z = 690x_1 + 545x_2 + 1020x_3 + 785x_4$

Constraints

$35x_1 + 45x_2 + 70x_3 \geq 2100$

$55x_1 + 42x_2 + 90x_4 \geq 1800$

$160x_1 + 100x_2 + 200x_3 + 75x_4 \leq 8000$

$30x_1 + 35x_2 + 60x_3 + 80x_4 \leq 4000$

$20x_1 + 30x_2 + 10x_3 + 25x_4 \leq 1000$

$x_1, x_2, x_3, x_4 \geq 0$

This is the LP Model for Problem I on part I (when no overtime is used)

Part-II

Overtime : Max 200 hr, with additional salary 30/hr

Introduce new variable x_5
$\hookrightarrow$ no. of hours of overtime

$x_5 \leq 200$
$\hookrightarrow$ Constraint added

Minimize fn
objective fn $Z =$ (same) $+ 30x_5$

Subject to P_1 :
P_2 :
A :
B :
} Constraints Same as Part-I (first 4)

Labor : $20x_1 + 30x_2 + 10x_3 + 25x_4 \leq 1000 + x_5$
$\hookrightarrow$ Extra work

$x_5 \leq 200$

$x_1, x_2, x_3, x_4, x_5 \geq 0$

DATE
PAGE

Standard form

$\leq, \geq, =$ } Any LPP model with these signs can be converted to Standard form

Min.^{z} the f^n

$\text{min } Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n - c_0$

subjected to

$a_{11}x_1 + \cdots + a_{1n}x_n = b_1$

$\vdots$

$a_{m1}x_1 + \cdots + a_{mn}x_n = b_m$

$x_i \geq 0, \; i = 1 \ldots n$

} $\geq, \leq$ is converted to $=$ sign

↳ Example for standard form

$\max Z = c_1x_1 + \cdots \; c_nx_n$

$\Rightarrow \min Z = -(c_1x_1 + \cdots + c_nx_n)$.

$\text{Min } Z = x_1 + 2x_2 + 3x_3$

subject to $x_1 + x_2 + x_3 \leq 10$

$x_1 - x_2 \geq 5$

$x_i \geq 0, \; i = 1, 2, 3$

① First check if Z (or obj. f^n) is minimum

✓ Here it is min

② Convert constraints into =

$x \leq q$ e.g. $4 \leq 10$

$x + x' = q$ $4 + \underbrace{6}_{\text{Introduce new variable}} = 10$

$x' \geq 0$

Surplus numbers or Slag variables are introduced in $\geq$, $\leq$ to make it $=$

Ily
$x \geq p$
$x - x' = p$

$x_1 + x_3 + x_3 \leq 10$ becomes
$x_1 + x_2 + x_3 + x_4 = 10$

$x_1 - x_2 \geq 5$ becomes
$x_1 - x_2 - x_5 = 5$

So new constraints are

$$\left.\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 10 \\ x_1 - x_2 \quad - x_5 &= 5 \\ x_i \geq 0 \quad i = 1, 2, \ldots 5 \end{aligned}\right\}$$

Slack variables
↳ For inequality with $\leq$, add $(+x')$ on LHS, $\leq \rightarrow =$, $x' \geq 0$
↳ For ineq. with $\geq$, add $(-x')$ on LHS, $\geq \rightarrow =$, $x' \geq 0$

In some cases one variable may be unrestricted, then we restrict it.
↳

Any no. can be converted ~~its~~ into a difference of any two negative numbers.

If one variable is unrestricted, we express it as

$$x = x' - x''$$

$$x', x'' \geq 0$$

Artificial Variables

E.g.

Max. $Z = 3x_1 - 2x_2 - x_3 + x_4 - 87$

with constraints

$$4x_1 - x_2 + x_4 \leq 6$$

$$-7x_1 + 8x_2 + x_3 \geq 7$$

$$x_1 + x_2 + 4x_4 = 12$$

$x_1, x_2, x_3 \geq 0$ $\quad$ x_4 is unrestricted.

Sol Convert to Standard form

1] Convert to Minimum

$$\text{Min } Z = -(\text{Max } Z)$$

$$= -(3x_1 - 2x_2 - x_3 + x_4 - 87)$$

$$= -3x_1 + 2x_2 + x_3 - x_4 + 87$$

Constraints:

$$4x_1 - x_2 + x_4 + x_5 = 6$$

$$-7x_1 + 8x_2 + x_3 - x_6 = 7$$

$$x_1 + x_2 + 4x_4 = 12$$

Converted to Standard form

As x_4 is unrestricted

$$x_4 = x_4' - x_4'' \qquad x_4', x_4'' \geq 0$$

$\therefore$

$$4x_1 - x_2 + (x_4' - x_4'') + x_5 = 6$$

$$-7x_1 + 8x_2 + x_3 - x_6 = 7$$

$$x_1 + x_2 + 4(x_4' - x_4'') = 12$$

$$\text{Min } Z = -3x_1 + 2x_2 + x_3 - (x_4' - x_4'') + 87$$

$x_1, x_2, x_3, x_4', x_4'', x_5, x_6 \geq 0$

Eg. Solving for LPP model (finding feasible region)

$x_1 + 2x_2 + x_3 = 4$
$2x_1 + 2x_2 + 3x_3 = 7$
$x_1 + x_2 + 4x_3 = 6$

} Pivoting

Diagonal element is 1 & values above it & below it should be converted to 0

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 2 & 2 & 3 & 7 \\ 1 & 1 & 4 & 6 \end{array}\right]$$

$R_2 \to R_2 - 2R_1 \qquad R_3 \to R_3 - R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & -2 & 1 & -1 \\ 0 & -1 & 3 & 2 \end{array}\right]$$

$R_2 \to -\frac{R_2}{2}$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & 1 & -1/2 & 1/2 \\ 0 & -1 & 3 & 2 \end{array}\right]$$

$R_3 \to R_3 + R_2$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & 1 & -1/2 & 1/2 \\ 0 & 0 & 5/2 & 5/2 \end{array}\right]$$

DATE
PAGE

$R_1 \to R_1 - 2R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ 0 & 1 & -1/2 & -1/2 \\ 0 & 0 & 5/2 & 5/2 \end{array}\right]$$

$R_3 \to 2/5 R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ 0 & 1 & -1/2 & 1/2 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$R_1 \to R_1 - 2R_3$
$R_2 \to R_2 + \frac{1}{2} R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

↳ Any pivot can be considered

Eg: $x_1 + x_2 + 2x_3 + x_4 = 6$
$3x_2 + x_3 + 8x_4 = 3$

Pivoting

$x_1 + \quad + 5/3\, x_3 - 5/3\, x_4 = 5$
$x_2 + 1/3\, x_3 + 8/3\, x_4 = 1$

Basic Variables

x_1 & x_2 } Pivots

Cannonical Form

Solⁿ is not Unique } Infinitely Many

form after Pivoting → Canonical form

Standard forms → Cannonical form

First Basic ↓

Feasible Basic

So continuing ~~pre~~ previous example

Basic Variables $= x_1, x_2$ } As they can be converted in canonical forms

Non " " $= x_3, x_4$

~~Can take~~ ~~Infinite~~ Values (x_1, x_2)

$$\left\{\begin{aligned} x_1 &= 5 - \frac{5}{3}x_3 + \frac{5}{3}x_4 \\ x_2 &= 1 - \frac{1}{3}x_3 - \frac{8}{3}x_4 \end{aligned}\right.$$

There are infinitely many solⁿ, we take one arbitrary solⁿ

Let $x_3, x_4 = 0$
$x_1 = 5$
$x_2 = 1$
} Particular solⁿ

One Particular Value → Check Z } Repeat until max. or min. is achieved

$(5, 1, 0, 0)$ ⏌ solⁿ of system of variables

$x_1, x_2, x_3, x_4 \geq 0$

→ ~~Feasible Basic solⁿ~~

DATE
PAGE

Steps
1) Standard form
2) Canonical form (Pivoting)
3) Non Basic = 0 } Basic Sol^n
Basic = RHS value
4) If it satisfies non-negativity condition, then it is feasible basic sol^n

(I) We can have many basic variables
(II) Basic variables may be feasible or non-feasible

Let choose x_1 & x_4 as pivot elements in previous examples.

$$x_1 + \frac{5}{8}x_2 + \frac{15}{8}x_3 = \frac{45}{8}$$

$$\frac{3}{8}x_2 + \frac{1}{8}x_3 + x_4 = \frac{3}{8}$$

Basic :- x_1, x_4
Let $x_2, x_3 = 0$
$x_1 = 45/8$
$x_4 = 3/8$
$x_1 \geq 0, \quad x_4 \geq 0$

} Basic feasible Sol^n (Non-negativity condition)

$$\left(\frac{45}{8}, 0, 0, 3/8\right)$$

DATE
PAGE

Now,

x_4, x_2 as pivot element

$$\frac{8}{5}x_1 + x_2 + 3x_3 = 9$$

$$-\frac{3}{5}x_1 \quad - x_3 + x_4 = -3$$

x_2, x_4 } Basic Variables

$x_1, x_3 = 0$

$x_2 = 9$
$x_4 = -3$ } Non-negativity not satisfied
So, it is not feasible basic sol^n
It is basic sol^n
→ can't be used for optimal sol^n

So, a sol^n can be basic but it need not be feasible basic sol^n [Non-negativity sol^n]