DATE
PAGE

11 February, 2021

Continuing previous example

$$\min Z = \begin{cases} 31c_2 & \frac{c_1}{c_2} \geq \frac{5}{2} \quad (S_z \leq S_1) \\ 10c_1 + 6c_2 & \text{if } \frac{3}{5} \leq \frac{c_1}{c_2} \leq \frac{5}{2} \quad (S_1 \leq S_z \leq S_2) \\ 20c_1 & \text{if } \frac{c_1}{c_2} \leq \frac{3}{5} \quad (S_z \geq S_2) \end{cases}$$

Now, Let the total requirment also changes (i.e. A Req = 124 & B. Req = 60)

$\cancel{3x_1 + 5x_2 \geq b_2}$.

$10x_1 + 4x_2 \geq b_1$

$3x_1 + 5x_2 \geq b_2$

↳ Dual for the problem:

Max $Z = b_1y_1 + b_2y_2$

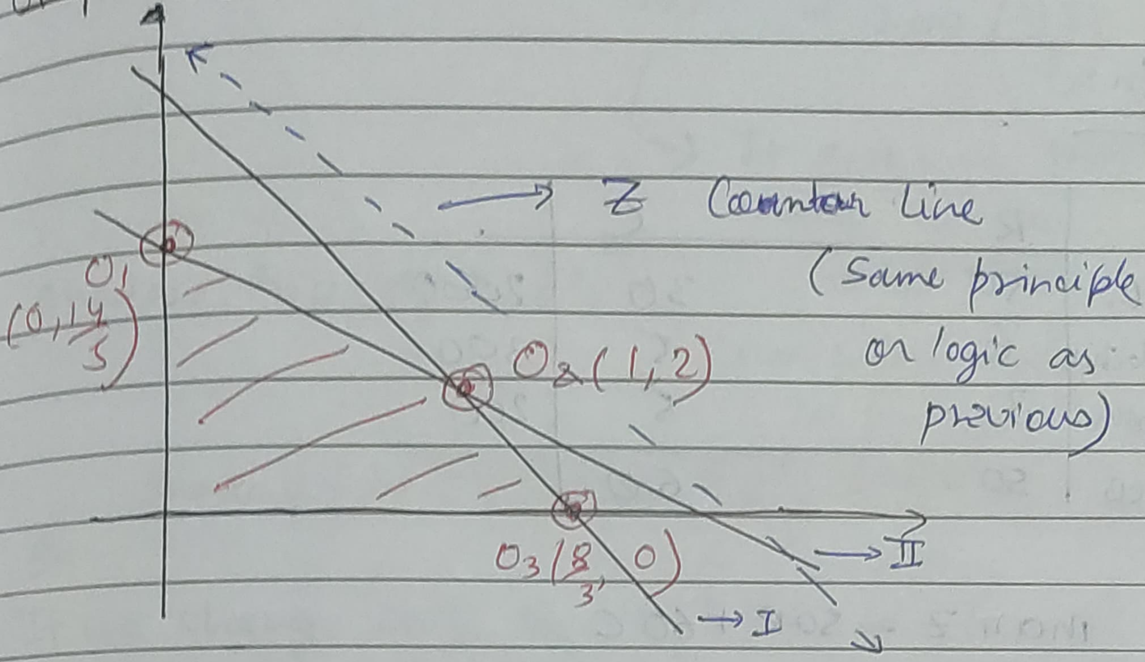
Subject to $10y_1 + 3y_2 \leq 16$

$4y_1 + 5y_2 \leq 14$

$y_1, y_2 \geq 0$

DATE
PAGE

Graphically



$S_1 = \dfrac{-10}{3}$, $S_2 = -\dfrac{4}{5}$, $S_Z = \dfrac{-b_1}{b_2}$

If $S_2 \le S_Z \Rightarrow O_1$

$S_1 \le S_Z \le S_2 \Rightarrow O_2$

$S_2 \le S_1 \Rightarrow O_3$

min value = (Dual)
$$\begin{cases} \frac{14}{5} b_2 & \text{if } \frac{b_1}{b_2} \le \frac{4}{5} \\ b_1 + 2b_2 & \text{if } \frac{4}{5} \le \frac{b_1}{b_2} \le \frac{10}{3} \\ \frac{8}{3} b_1 & \text{if } \frac{b_1}{b_2} \geq \frac{10}{3} \end{cases}$$

→ min of primal problem

DATE
PAGE

Shadow Price : Sol^n of the dual

Ex:

	R	C	
Al.	50	30	2000
Machine	6	5	300
Labour	3	5	200
Price	50	60	

$$\max Z = 50R + 60C$$

subject to

$$50R + 30C \le 2000$$
$$6R + 5C \le 300$$
$$3R + 5C \le 200$$
$$R, C \ge 0$$

$\hookrightarrow$ $Z_{max} = 2750$ at $(25, 25)$

Dual:

$$\min Z = 2000y_1 + 300y_2 + 200y_3$$

subject to

$$50y_1 + 6y_2 + 3y_3 \ge 50$$
$$30y_1 + 5y_2 + 5y_3 \ge 60$$
$$y_i \ge 0$$

$\hookrightarrow$ Z_{min} ~~at~~ $= 2750$

at $\left(\frac{7}{16}, 0, \frac{75}{8}\right)$

We can use Shadow price only to a point where our optimality is not gone.

DATE________

PAGE________

$Z_{max} = Z_{min} = 2000\left(\frac{7}{16}\right) + 300(0) + 200\left(\frac{75}{8}\right)$

It gives us the change in price difference.

Shadow Price of Dual

(No Information in primal)

Shadow Price $= \left(\frac{7}{16}, 0, \frac{75}{8}\right) = y^*$

If we change 2000 to 2001 unit in primal problem, then for using one extra unit, the price difference will be given by $\frac{7}{16}$

Dual Soln $\rightarrow$ (y_1, y_2, y_3) is price & in the primal problem it is the shadow price.
We can't see these price in primal but when we solved that in dual, we get the unit through which the change in Z fn can be calculated.

$\rightarrow$ $\therefore$ Shadow Price

$\frac{7}{16}$ tells us the difference in price in the original problem.

$Z_{min} = Z_{max}$ $\rightarrow$ $7/16$ is the price of resources in primal.

Resource limit or resources changed in primal $\rightarrow$ Price diff. $7/16$.

$\|$ly Labour $\rightarrow$ $\frac{75}{8}$, Machine Time $\rightarrow$ [illegible]

DATE
PAGE

Shadow Price = $\left(\frac{7}{16}, 0, \frac{75}{8}\right) = y^*$

Doesn't Affect Any Change ~~is~~

Why 0? Bcuz in solⁿ, the Slack variable is Basic (x_4)

$50R + 30C \leq 2000 \rightarrow =$

$6R + 5C \leq 300 \rightarrow <$

$3R + 5C \leq 200 \rightarrow =$

Bcuz the slack is not in basic

Here the slack is in Basic Solⁿ (~~Ato~~

∴ Strictly less than

It means that when manufacturing boats or products, our ~~labour~~ ~~is~~ Machine is underused. (using less than given value)

∴ Extra units is there

∴ If we change 300, no affect as we are using only around 275.

So, we can increase 300 and it doesn't affect the model.

Shadow Price when zero → Doesn't affect Model, can change without affecting model.

DATE ____

PAGE ____

Matrix form of Simplex Algorithm

$\min z + z_0 = c\,x$ → row vector

column vector

∴ a number

subject to

$Ax = b$, $x \geq 0$

A	b
c	

A	b
c	z_0

(Initial)

→

A^*	b^*
c^*	z_0^*

(final)

We find relations b/w initial & final tables, e.g. If we change something in c, then how that change will be reflected in c^*.

We need not do the full pivoting, we will just relations b/w initial & final.

Let for m constraints, there be x_{j1}, x_{j2} --- --- x_{jm} Basic Variables.

Let $A^{(j)}$ = j^{th} col of A

A^{J_1} -- -- -- A^{J_m} → Basic Variable columns $= B$

DATE______
PAGE______

$\therefore B = [A^{J_1} | \cancel{AJ}\ A^{J_2} | \text{-----} | A^{J_m}]$

↳ Columns of Basic

$C_B = [C_{j_1} \text{ --- } C_{j_m}]$ → C values corresponding to Basic variables

Results

1) $A^* = B^{-1} A$

2) $\cancel{B}^* = B^{-1} b$

3) $C^* = C - C_B B^{-1} A = C - C_B A^*$

4) $Z_0^* = Z_0 - C_B B^{-1} b = Z_0 - C_B \cancel{B}^*$