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Rule: If one of $c_k < 0$ (strict condition), x_k shall enter as basic variable

$$\left.\begin{array}{l} x_1 + a_{1k} x_k = b_1 \\ x_2 + a_{2k} x_k = b_2 \\ \vdots \\ x_m + a_{mk} x_k = b_m \end{array}\right\} \Rightarrow \begin{array}{l} x_1 = b_1 - a_{1k} x_k \geq 0 \\ x_2 = b_2 - a_{2k} x_k \geq 0 \\ \vdots \end{array}$$

I) $a_{ik} < 0$

$$x_i = b_i - a_{ik} x_k \geq 0 \quad \Rightarrow x_k \geq \left\{\frac{b_i}{a_{ik}}\right\}$$

No limit

x_k can be as ~~small~~ large as possible.

neg. value as $a_{ik} < 0$

$Z = -Z_0 + \text{Non-basic} + c_k x_k + \cdots$

As x_k tends to infinity ($x_k \to \infty$)

$Z \to -\infty$ = bounded

II) If $a_{ik} = 0$

$x_i = b_i \geq 0$ } Unbounded

If all $a_{ik} \leq 0$, then we stop (Sol^n Unbounded)

Result – Unbounded Case

For LP if $\exists$ an index k, $m+1 \le k \le n$
such that $c_k < 0$ and $a_{ik} \le 0$
$\forall\, i = 1, \ldots n$

→ **Bounded**

Case-3 If $c_k < 0$, $a_{ik} > 0$ for some i

i^{th} Eq^n $\quad x_i + a_{ik} x_k = b_i$

$$x_i = b_i - a_{ik} x_k \geq 0$$

$$\Rightarrow x_k \le \frac{b_i}{a_{ik}}$$

min

$c_k x_k$

Non-basic variable → $\dfrac{b_t}{a_{tk}} = \min \left\{ \dfrac{b_i}{a_{ik}},\ a_{ik} > 0 \right\}$

Basic Variable: $\left\{ \{x_i\}_{\substack{i=1,\ldots m \\ i \neq t}} \quad x_k \right\}$

$$z = -z_0 + c_k x_k = -z_0 + c_k \frac{b_t}{a_{tk}} \rightarrow z \text{ dec.}$$

$c_k < 0$, $b_t > 0$, $a_{tk} > 0$

Result: $c_k < 0$ and atleast one $a_{ik} > 0$

$$\frac{b_t}{a_{tk}} = \min \left\{ \frac{b_i}{a_{ik}},\ a_{ik} > 0 \right\}$$

Replace x_t by x_k as basic variable

$$z = -z_0 + c_k x_k = -z_0 + c_k \frac{b_t}{a_{tk}}$$

↳ decreases

$b_i > 0 \Rightarrow Z$ decreases strictly
$b_i = 0 \Rightarrow Z$ remains same

degenerate case $\{B.V. = 0\}$

No. of Basic variable $\leq$ no. of choices B.V.

$^nC_m \rightarrow n$ ~~eq~~ B.V.
m - eq^{ns}

$$a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 = b_1$$
$$a_5x_1 + a_6x_2 + a_7x_3 + a_8x_4 = b_2$$

6 choices

Simplex Alg. $(b_i > 0) \rightarrow$ converges

Ex: ~~Min Z~~

$$x_1 + 2x_4 - x_5 = 10$$
$$x_2 - x_4 - 5x_5 = 20$$
$$x_3 + 6x_4 - 12x_5 = 18$$
O.f^n $\leftarrow$ $-2x_4 + 3x_5 = 60 + Z$

x_1, x_2, x_3 are B.V.

New B.V. as it will dec Z more

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Now for $a_{ik} > 0$ (Case-3)

I Eq^n & III eq^n

$\frac{b_i}{a_{ik}}$ for I → $10/2 = 5$

III → $18/6 = 3$

$x_1 \quad + 2x_4 - x_5 = 10$

$x_2 \quad - x_4 - 5x_5 = 20$

$x_3 + 6x_4 - 12x_5 = 18$ → x_3 exit

$-2x_4 + 3x_5 = 60 + Z$

$a_{ik} > 0$

Bounded

→ then find $\frac{b_i}{a_{ik}}$

Select eq^n for Min. one { x_3 will exit & x_4 will enter }

x_3 : Non-basic variable

x_4 : Basic Variable

$x_1 \quad -1/3\, x_3 + \quad + 3x_5 = 4$ → x_1 exit

$x_2 + 1/6\, x_3 + \quad - 7x_5 = 23$

$1/6\, x_3 + x_4 - 2x_5 = 3$

$1/3\, x_3 \quad - x_5 = 66 + Z$

New Basic

New Sol^n (4, 23, 0, 3, 0)

$Z_{min} = 66$

x_1 → Non-Basic Variable

x_5 → New Basic Variable

$\frac{1}{3}x_1 \quad -\frac{1}{9}x_3 \quad + x_5 = \frac{4}{3}$

$\frac{7}{3}x_1 + x_2 \quad -\frac{11}{18}x_3 \quad = \frac{97}{3}$

$\frac{2}{3}x_1 \quad -\frac{1}{18}x_3 \quad + x_4 = \frac{17}{3}$

$\frac{1}{3}x_1 \quad \frac{1}{9}x_3 \quad = \frac{202}{3} + Z$

Soln $\left(0, \frac{97}{3}, 0, \frac{17}{3}, \frac{4}{3}\right) \longrightarrow \frac{-202}{3}$ (Z min)

stop
as can't be
decreased

Simplex Tableau

	x_1	x_2	x_3	x_4	x_5	b	
x_1	1	0	0	2	−1	10 $\frac{10}{2}=5$	
x_2	0	1	0	−1	−5	20	
x_3	0	0	1	6	−12	18 $\frac{18}{6}=3$	→ Exit x_3
Z	0	0	0	−2	3	60	
x_1	1	0	−1/3	0	3	4	→ Exit x_1
x_2	0	1	1/6	0	−7	23	
x_4	0	0	1/3	1	−2	3	
Z	0	0	1/3	0	−1	66	
x_5	1/3	0	−1/9	0	1	4/3	
x_2	7/5	1	−11/18	0	0	97/3	
x_4	2/3	0	−1/18	1	0	17/3	
Z	1/3	0	1/9	0	0	202/3	

No −ve sign in Z
stop Algorithm

$Z_{min} = \frac{-202}{3}$ at $\left(0, \frac{97}{3}, 0, \frac{17}{3}, \frac{4}{3}\right)$

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$$4x_1 + 3x_2 + x_3 + x_4 =$$

$$6x_1 + 2x_2 + 4x_3 + x_4 =$$

$$3x_1 + x_2 + x_3 + 4x_4 = 0$$

How do we find initial canonical form?

How " " " " basic variables