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LPP Model already there from which we convert to dual → Primal

Matrix Form for LP Model & Dual

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \\ a_{m1} & \cdots & a_{mn} \end{bmatrix} \rightarrow \text{from constraints (coeff)}$$

(Coeff. Matrix)

$$B = \begin{bmatrix} b_1 \\ \vdots \\ b_m \end{bmatrix} \quad c = \begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \quad y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix}$$

Primal Model:

$$\max Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$$
$$= CX$$

subject to $Ax \leq b$, $X \geq 0$

Dual: $\min Z = b \cdot Y$

subject to

$$A^T \cdot y \geq c \qquad y \geq 0$$

llly for min in primal.

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Rules:

1. Convert any LP in max form.
If min given then convert it into max by multiplying -1 in objective f^n.

2. $\geq$ can be changed to $\leq$ by multiplying (-1) → In simplex we again convert the Eq^{ns} so that $b \geq 0$.

3. $=$

$X = Y$
$X \leq Y$
$X \geq Y$
$-X \leq -Y$

To convert $=$ into $<=$ we simply write $X \leq Y$ and $-X \leq -Y$

4. If we find the dual of a dual we get primal

Ex.

max $Z = 6x_1 + x_2 + 4x_3$

subject to $3x_1 + 7x_2 + x_3 \leq 15$

$x_1 - 2x_2 + 3x_3 = 20$ → $=$ sign convert in $<=$

$x_i \geq 0$

Sol^n

Primal:

max $Z = 6x_1 + x_2 + 4x_3$

subject to $3x_1 + 7x_2 + x_3 \leq 15$

$x_1 - 2x_2 + 3x_3 \leq 20$

$-x_1 + 2x_2 - 3x_3 \leq -20$

$x_i \geq 0$

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Dual:

$$\text{Min } Z = 15y_1 + 20y_2 - 20y_3$$

subject to

$$3y_1 + y_2 - y_3 \geq 6$$
$$7y_1 - 2y_2 + 2y_3 \geq 1$$
$$y_1 + 3y_2 - 3y_3 \geq 4$$
$$y_i \geq 0$$

In all the 3 eqns there is $y_2 - y_3$ so we can define a new variable y_4

$y_4 = y_2 - y_3$

but we don't know anything about y_4 (whether its +ve or -ve)

So, y_4 is unrestricted

$\therefore$ ~~Max Z =~~

$$\text{Min } Z = 15y_1 + 20y_4$$

subject to

$$3y_1 + y_4 \geq 6$$
$$7y_1 - 2y_4 \geq 1$$
$$y_1 + 3y_4 \geq 4$$
$$y_1, y_2, y_3 \geq 0$$

y_4 is unrestricted

Max $\xleftrightarrow{\text{Dual}}$ Min

i^{th} ($\leq$)	i^{th} variable Non-negative var.
i^{th} ($=$)	i^{th} variable is unrestricted
j^{th} non neg.	($\geq$) const.
j^{th} unrestricted	j^{th} constraint ($=$)

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Ex. Min $Z = x_1 - 2x_2 + 3x_3$

subject to

$4x_1 + 5x_2 - 6x_3 = 7$

$8x_1 - 9x_2 + 10x_3 \le 11$ } convert into $\geq$

$x_1, x_2 \geq 0$ x_3 is unrestricted

Soln Min $Z = x_1 - 2x_2 + 3x_3$

subject to

$4x_1 + 5x_2 - 6x_3 = 7 \rightarrow y_1$ unrestricted

$-8x_1 + 9x_2 - 10x_3 \geq -11 \rightarrow y_2$ non-neg.

$x_1, x_2 \geq 0$ x_3 is unrestricted

Dual: Max $Z = 7y_1 - 11y_2$

subject to

$4y_1 - 8y_2 \le 1$

$5y_1 + 9y_2 \le -2$

$-6y_1 - 10y_3 = 3$

y_1 is unrestricted, $y_2 \geq 0$

Max $Z = Cx$

Subject to $Ax \le b$

$x \geq 0$

Dual: Min $Z = b \cdot y$

subj. to $A^T y \geq c$

$y \geq 0$

Theorem 1: If x_0 is a feasible soln of (max)

Y_0 " " " " (min)

then $Cx_0 \le b \cdot Y_0$

↳ only feasible not optimum

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Proof:

We have $Ax_0 \leq b$, $x_0 \geq 0 \Rightarrow Ax_0 + u = b$,

$A^T y_0 \geq c$, $y_0 \geq 0 \Rightarrow A^T y_0 - v = c$

$u = b - Ax_0 \geq 0$ — ①

$v = A^T y_0 - c \geq 0$ — ②

$b = Ax_0 + u$ — ③

RHS

$$b \cdot y_0 = y_0^T b$$
$$= y_0^T (Ax_0 + u)$$
$$= y_0^T A x_0 + y_0^T u$$
$$=$$

This is a single value (y_0 is Column, A is Matrix, x_0 is Row)

$\therefore$ we can take its transpose

$$= (y_0^T A x_0)^T + y_0^T u$$
$$= y_0 \cdot u + x_0^T (A^T y_0)$$
$$= y_0 u + x_0^T (v + c)$$
$$= y_0 u + x_0^T v + x_0^T c$$
$$b y_0 = \cancel{y_0} \; y_0 \cdot u + x_0 v + x_0 \cdot c$$

$$\Rightarrow \boxed{b \cdot y_0 \geq x_0 c}$$

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$$b \cdot X_0 - X_0 \cdot C = \underbrace{Y_0 \cdot u + X_0 \cdot V}$$

Theorm 2: If z (max) is not bounded above, then (min) is not feasible

Hy If Z (min) is not bounded below them (max) is not feasible.