1. Consider the linear programming problem of

$$\text{Maximizing } 2x_1 + 2x_2$$
$$\text{subject to}$$
$$x_1 + x_3 + x_4 \leq 1$$
$$x_2 + x_3 - x_4 \leq 1$$
$$x_1 + x_2 + 2x_3 \leq 3$$
$$x_i \geq 0, i = 1,2,3,4$$

a) Determine the dual problem. (2)
b) Show that $X^* = (1,1,0,0)$ and $Y^* = (1,1,1)$ are feasible solutions to the primal and dual problems, respectively. (3)
c) Is Y^* an optimal solution to the dual? (2)

2. On most university campuses students are contracted by academic departments to do errands, such as answering the phone and typing. The need for such service fluctuates during work hours (8:00 A.M. to 5:00 P.M.). In the IE department, the minimum number of students needed is 2 between 8:00 A.M. and 10:00 A.M., 3 between 10:01 A.M. and 11:00 A.M., 4 between 11:01 A.M. and 1:00 P.M. and 3 between 1:01P.M. and 5:00 P.M. Each student is allotted 3 consecutive hours (except for those starting at 3:01, who work for 2 hours and those who starts at 4:01, who work for one hour). Because of their flexible schedule, students can usually report to work at any hour during the work day, except that no student wants to start working at lunch time (12:00 noon). Determine the minimum number of students the IE department should employ and specify the time of the day at which they should report to work. (6)

3. A company produces two products that use three material A, B and C resp. The following table provide the data of the situation

	Product 1	Product 2	availability
Mat A	2	3	1200
Mat B	2	1	1000
Mat C	0	4	800
Unit Price	3	4	

Let the final optimal table associates to the modal is

Basic/ Variables	x_1	x_2	x_3	x_4	x_5	b
x_1	1	0	-1/4	3/4	0	450
x_5	0	0	-2	2	1	400
x_2	0	1	1/2	-1/2	0	100
z	0	0	5/4	1/4	0	1750

a) Determine the solution of each material. (1)

b) Determine the dual prices for the Mat A, Mat B, and Mat C. (1)

c) Determine the feasibility ranges for the dual prices obtained in (b). (1)

d) If the availability of Mat A is increased to 1300 units, find the new optimum solution.(3)

e) If the Mat C is reduced to 350 units, will it be possible to determine the new optimum solution directly from the given information? Explain.   (3)

f) A new contractor is offering to sell Company additional Mat A at 40 Rs. each, but only if company would purchase at least 500 units. Should Company accept the offer? (3)