Infinitely Many Sol^ns of System of Eq^ns → Linearly Dependent

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Redundancy :

Phase-I → While solving phase-I, after pivot step, we have $w = 0$ but some artificial variables remains in the basic variable sol^n.

w = sum (Art. variables)

For an artificial variable, (A_k) which is in the basic sol^n.

1) $w = 0$

w is sum of A.V.

$\Rightarrow A_k = 0$

$\Rightarrow b_k = 0$ } Degenerate Case

We can change this variable with other & the objective f^n remains same.

For removing A_k, we can take any variable with non-zero value in that row.

eg

both can enter, it should be non-zero value

A_1	$-5/3$	0	$2/3$	0	0
z					0

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2) If the whole row is 0, we can't enter ~~a~~ ~~varia~~ other variable, ∴ there is redundancy & the eqns are linearly dependent

e.g.

A	0	0	0	0	0
Z					

Red.
or L.D.

~~Any 2 eqns are LD~~
~~& ∴ this row becomes~~
~~0.~~

In this case, we will remove the entire row from the simplex table & we can use the others which are in canonical form to optimize Z.

Here $w = 0$, No possibity for entering other variables
So we neglect / remove this row in the tableau.

Ex.
Case-I

$$\text{Max } Z = 2x_1 + x_2 + x_3 - x_4$$

subject to

$$x_1 - 2x_2 + 3x_3 + x_4 = 6$$

$$-x_1 + x_2 + 2x_3 + \frac{2}{3}x_4 = 4$$

$$x_i \geq 0$$

Equal signs ∴ Introduce A.V.

$$Min\ Z = 2x_1 + x_2 + x_3 - x_4$$

$$x_1 - 2x_2 + 3x_3 + x_4 + A_1 = 6$$

$$-x_1 + x_2 + 2x_3 + \tfrac{2}{3}x_4 + A_2 = 4$$

New objective fn

$$w = A_1 + A_2$$

subject to

$$x_i \geq 0$$

$$A_1, A_2 \geq 0$$

	x_1	x_2	x_3	x_4	A_1	A_2	b	
A_1	1	-2	3	1	1	0	6	Exit → 6/3=2
A_2	-1	1	2	2/3	0	1	4	4/2=2
w	0	1	-5↑	-5/3	0	0	-10	
x_3	1/3	-2/3	1	1/3	-1/3	0	2	
A_2	-5/3	7/3	0	0	5/3	0	0	
w	5/3	-7/3↑	0	0	5/3	0	0	
x_3	-1/7	0	1	1/3↑	Need		2	→ Exit
x_2	-5/7	1	0	0	not		0	
Z	10/7	0	0	-4/3	calculate		-2	
x_4	-3/7	0	3	1			6	
x_2	-5/7	1	0	0			0	
Z	6/7	0	4	0			6	

~~We can also~~ take this (Only non-zero)

$Z_{min} = -6$ for $(0, 0, 0, 6)$

So, enter other variable

w=0 But A_2 still in Basic Soln

If there is no objective f^n as in system of eq^ns We take our Phase - II Model like

i.e.

$$\left.\begin{array}{l} x_1 - 2x_2 + 3x_3 + x_4 + A_1 = 6 \\ -x_1 + x_2 + 2x_3 + \frac{2}{3}x_4 + A_2 = 4 \end{array}\right\}$$

$$w = A_1 + A_2$$

when $w = 0$, our objective f^n has attained min. value & after removing A.V., we get our solution.

Ex- Case-II

$$\left.\begin{array}{l} x_1 + 2x_2 + x_4 = 20 \\ 2x_1 + x_2 + x_3 = 10 \\ -x_1 + 4x_2 - 2x_3 + 3x_4 = 40 \end{array}\right\} \text{const.}$$

$$x_i \geq 0$$

Introduce A.V. A_1, A_2, A_3.

$$w = A_1 + A_2 + A_3$$

Min Z (given) $= x_1 + 4x_2 + 3x_3 + 2x_4$

So, $w = A_1 + A_2 + A_3$

$$20 = x_1 + 2x_2 + x_4 + A_1$$

$$10 = 2x_1 + x_2 + x_3 + A_2$$

$$40 = -x_1 + 4x_2 - 2x_3 + 3x_4 + A_3$$

$$x_i \geq 0, \quad A_i \geq 0$$

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	x_1	x_2	x_3	x_4	A_1	A_2	A_3	b	
A_1	1	2	0	1	1	0	1	20	$= 20/2 = 10$
A_2	2	1	1	0	0	1	0	10	$= 10/1 = 10$
A_3	-1	4	-2	3	0	0	0	40	$= 40/4 = 10$
Z	1	4	3	0	0	0	0	0	
W	-2	-7	1	-4	0	0	0	-70	
x_2	1/2	1	0	1/2	1/2	0	0	10	
A_2	3/2	0	1	-1/2	-1/2	1	0	0	
A_3	-3	0	-2	1	-2	0	1	0	Same value
Z	-1	0	3	-2	-2	0	0	-40	
W	3/2	0	1	-1/2	7/2	0	0	0	
x_2	2	1	1	0	0	1	0	10	
x_4	-3	0	-2	1	1	-2	0	0	
A_3	0	0	0	0	-3	2	1	0	
Z	-7	0	-1	0	0	-4	0	-40	
W	0	0	0	0	4	-1	0	0	

The row has become 0
& ∴ we can't enter
any other variable

So, Neglect this row or
remove this row
for Phase-II

feasible
soln, but A.V.
still in Basic
So, exit them
& enter other
variables

Phase-II

$$\text{Min } Z = -7x_1 - x_3 \quad \text{subject to}$$

$$10 = 2x_1 + 1x_2 + 1x_3$$

$$0 = -3x_1 - 2x_3 + 1x_4$$

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Phase-II

	x_1	x_2	x_3	x_4	b
x_2	2	1	1	0	10 → Exit
x_4	-3	0	-2	1	0
Z	-7	0	-1	0	-40
x_1	1	1/2	1/2	0	5
x_4	0	3/2	-1/2	1	15
Z	0	7/2	5/2	0	-5

$Z = -5$ at $(5, 0, 0, 15)$

Duality

from Pg-9 of copy or Ex. 2.2.1 (Pg 26)

Blending Model (Model-I)

Nutrition

	A	B	C	Cost
feed 1	3	7	3	10
feed 2	2	2	6	4
Min^m Req.	60	84	72	

Let x_1 = # of feed 1

x_2 = " " " 2

$\text{Min } Z = 10x_1 + 4x_2$

subject to

$3x_1 + 2x_2 \geq 60$

$7x_1 + 2x_2 \geq 84$

$3x_1 + 6x_2 \geq 72$

$x_i \geq 0$

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After solving thru Simplex, we have
Min $Z = 144$ at $(6, 24)$

# of feed 1 , # of feed 2

Model - 2

So, lets say the farmer goes to the market & sees a shopkeeper with a new tablet or powder of feed 1 & feed 2.
Now, the shopkeeper wants to maximize his profit but at the same time the quantity & the cost of farmer should be same.
So, the shopkeeper wants to fin the price of the products to maximize its profit.

Variables :- $y_1, y_2, y_3 \rightarrow$ Price for A, B, C Nutrients

Objective Max. Profit

$$\text{Max. } Z = \underbrace{60y_1}_{\text{Min Req. of A}} + \underbrace{84y_2}_{B} + \underbrace{72y_3}_{C}$$

subject to
Constraints :

$$3y_1 + 7y_2 + 3y_3 \leq 10$$
$$2y_1 + 2y_2 + 6y_3 \leq 4$$
$$y_i \geq 0$$

We have change this model according to our needs

This model is dependent on above model

$\therefore$ Model - II is dual of I
Dependent on each other.

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So, when will farmer come to this shop?
↳ Only when the cost of farmer (Min $Z = 144$) is equal to shopkeeper's profit.

∴ In Model II

$$60y_1 + 84y_2 + 72y_3 = 144$$

↳ ∵ One is Minimize f^n (Cost of farmer) & other " Maximize " (Profit of Shopkeeper)
∴ When they are equal, only then both cond. will be satisfied.

So ~~after~~ solving Model - II

Eq^{ns} will become (After adding Slack Variables) :

$$60y_1 + 84y_2 + 72y_3 = Z$$
$$3y_1 + 7y_2 + 3y_3 + y_4 = 10$$
$$2y_1 + 2y_2 + 6y_3 + y_5 = 4$$
$$y_i \geq 0$$

$$i = 1, \ldots 5$$

Now, we need to find the sol of above model and when $Z = 144$ we will get the max. profit for the shopkeeper.

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	x_1	x_2	x_3	x_4	x_5	b	
x_4	3	7	3	1	0	10	$10/7$ →
x_5	2	2	6	0	1	4	$4/2 = 2$
Z	60	84	72	0	0	0	
x_2	$3/7$	1	$3/7$	$1/7$	0	$10/7$	$= 10/3$
x_5	$8/7$	0	$36/7$	$-2/7$	1	$8/7$	$= 8/36$ →
Z	24	0	36	-12	0	-120	
x_2	$1/3$	1	0	$1/6$	$-1/12$	$4/3$	$= 4$
x_3	$2/9$	0	1	$-1/12$	$7/36$	$2/9$	$= 1$ out
Z	16	0	0	0	-10	-128	
x_2	0	1	$-3/2$	$+1/24$	$-1/18$	1	
x_1	1	0	$-9/2$	$-9/24$	$7/18$	1	
	0	0	-72	-6	-21	-144	

Soln of Model I

No Entering Variable

Max. Z at 144 at (1, 1, 0)

So, $y_1 = 1$; $y_2 = 1$; $y_3 = 0$

Initially giving C at free but after sometime he can increase this to increase his profit

Optimum Soln Remains same but the model is changed completely.

Observations (when writing dual)

1) Objective fn changes from minimum to maximum
2) Greater than ($\geq$) changes to less than ($\leq$)
3) No. of variables & no. of constraints are interchanged.

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4) When we write the initial const. of both Models in matrix form, then they are transpose of each other.
∴ Coefficient Matrix Transpose

5) Coefficient of Objective f^n & RHS values of constraints are also switched

6) Min of I Model = Max. of II Model

7) Whenever we are solving dual model then the slack variable value will be the solution for the first Model
eg. In this case we can see (6, 21) at (x_4, x_5) which is sol^n of Model-I
So, Optimum Basic sol^n (for Model-I) is the coefficient of slack variable in Z f^n when Max. is reached.
∴ No need to solve 2 models
Solve only dual

Definition
Max form:

$$\text{Max } Z = c_1x_1 + \cdots + c_nx_n$$

subject to

$$a_{11}x_1 + \cdots a_{1n}x_n \le b_1$$

$$\vdots \qquad \vdots$$

$$a_{m1}x_1 + \cdots + a_{mn}x_n \le b_m$$

$$x_i \ge 0$$

When f^n is max constraints should have <= sign, Otherwise we need to make it <=

Multiply by negative.

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Min form:

$$\text{Min } Z = c_1x_1 + \cdots + c_nx_n$$

subject to $a_{11}x_1 + \cdots + a_{m1}x_n \geq b_1$

$\vdots$

$a_{m1}x_1 + \cdots + a_{mn}x_n \geq b_m$

$x_i \geq 0$

If the constraints are not according to the needed signs, we need to convert it so that we can write dual.

When the f^n is to be min. then the constraints sign should be $\geq$

Changing to Dual

1) Max form

$$\text{Min } Z = b_1y_1 + b_2y_2 + \cdots + b_my_m$$

↳ for m constraints, we will have m variables in dual

subject to:

$a_{11}y_1 + \cdots \quad a_{m1}y_m \geq c_1$

$\vdots$

$a_{1n}y_1 + \cdots + a_{mn}y_n \geq c_n$

$y_i \geq 0$

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2) Min form

Max $z = b_1 y_1 + b_2 y_2 + \cdots + b_m y_m$

subject to :

$a_{1n} y_1 + \cdots + a_{m1} y_1 \le c_1$

$\vdots$

$a_{1n} y_1 + \cdots + a_{mn} y_n \le c_n$

* If = sign comes, then we convert that in <= or >=

Ex. Max $Z = x_1 + 3x_2 + 5x_3$

subject to $x_1 + 2x_2 + 3x_3 \le 10$

$4x_1 + 5x_2 + x_3 \le 15$

$x_i \ge 0$

Dual : Min $Z = 10y_1 + 15y_2$

subject to

$y_1 + 4y_1 \ge 1$

$2y_1 + 5y_2 \ge 3$

$3y_1 + y_2 \ge 5$

$y_i \ge 0$