8 February, 2021

Q: max $Z = 3x_1 + 5x_2 + 9x_3$

subject to $4x_1 + 12x_2 + 15x_3 = 90 \rightarrow y_1$

$-x_1 + 2x_2 + 3x_3 = 12 \rightarrow y_2$

$x_i \geq 0$

Unrestricted

Dual: min $Z = 90y_1 + 12y_2$

subject to $4y_1 - y_2 \geq 3$

$12y_1 + 2y_2 \geq 5$

$15y_1 + 3y_2 \geq 9$

y_1, y_2 unrestricted

Substitute this:

$$\left.\begin{array}{l} y_1 = y_1' - y_1'' \\ y_2 = y_2' - y_2'' \end{array}\right\} \begin{array}{l} y_1', y_1'', \\ y_2', y_2'' \end{array} \geq 0$$

To solve add surplus variable (y_3, y_4, y_5) & to convert in canonical form we need A.V. A_1, A_2, A_3

So, to solve this, we need 10 variables

Can be solved by A.V A_1 & A_2

Solve any one to get the solution of other.

So, solving primal

	x_1	x_2	x_3	x_4	x_5	b
x_4	4	12	15	1	0	90
x_5	-1	2	3	0	1	12
Z	-3	-5	-9	0	0	0
w	-3	-14	-18	0	0	-102
x_4	9	2	0	1	-5	30
x_3	-1/3	2/3	1	0	1/3	4
Z	-6	1	0	0	3	36
w	-9	-2	0	0	6	-30
x_1	1	2/9	0	1/9	-5/9	10/3
x_3	0	29/27	1	1/27	4/27	46/9
Z	0	7/3	0	2/3	-1/3	56
w	0	0	0	1	1	0

need to calculate A.V. pivoting so as to calculate the sol^n for Dual

$\therefore$ Sol^n of Dual is $\left(\frac{2}{3}, -\frac{1}{3}\right)$

& Min Z = 56

w = 0
Proceed to II-Phase
But no -ve value for entry

$\therefore$ Max Z = +56 at $\left(\frac{10}{3}, 0, \frac{46}{9}\right)$

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Complimentary Slackness Theorm

max $Z = C \cdot X$ subject to $An \leq b$
$n \geq 0$

Dual:
min $Z = b \cdot Y$ subject to $A^T y \geq c$,
$Y \geq 0$

→ Let X_0, Y_0 be feasible solⁿ for (max), (min) problem respectively then

$$b \cdot Y_0 - c \cdot x_0 = (b - An_0) \cdot Y_0 + (A^T Y_0 - c) \cdot X_0$$

→ If ~~$b \cdot X_0$~~ $b \cdot Y_0 = c X_0$, ⇒ X_0, Y_0 are our optimal solutions. ∴ max = min (objective fⁿ)

Let X^* & Y^* be our optimal solⁿˢ instead of X_0 & Y_0.
As these are our optimal solⁿˢ, we have $c n_0 = b y_0$

∴

$$\underbrace{(\underset{\geq 0}{b - Ax^*})\underset{\geq 0}{y^*}}_{\geq 0} + \underbrace{(\underset{\geq 0}{A^T y^* - c})\underset{\geq 0}{x^*}}_{\geq 0} = 0$$

⇒ Sum of two +ve no. is 0 only when both are zero