Assignment - 2

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Q.1

Any Worker can start their 4 consecutive work days on one of the seven days (Mon to Sun) & in one of the three eight hour shifts (Night, Day, Late)

Let,

Monday be Day 1	Night be Shift 1
Tuesday be Day 2	Day be Shift 2
⋮	Late be Shift 3
Sunday be Day 7	

Let A_{ij} denote the number of workers starting their 4 consecutive work days on day i ($i = 1, \ldots 7$) & shift j (1, 2, 3)

Total No. of Workers = 60 } Upper Limit

$$\sum_{i=1}^{7} \sum_{j=1}^{3} N_{ij} \leq 60$$

Now, Let B_{ij} represent the total no. of workers req. for a particular day & particular shift

B_{ij} is known (from the table)

e.g. $B_{11} = 5$

B_{ij} given from the table is the minimum no. of workers req. for a particular day & shift

So, constraints are :

Monday : $A_{1j} + A_{7j} + A_{6j} + A_{5j} \geq B_{1j}$

Tuesday : $A_{2j} + A_{1j} + A_{7j} + A_{6j} \geq B_{2j}$

Wednesday : $A_{3j} + A_{2j} + A_{1j} + A_{7j} \geq B_{3j}$

Thursday : $A_{4j} + A_{3j} + A_{2j} + A_{1j} \geq B_{4j}$

Friday : $A_{5j} + A_{4j} + A_{3j} + A_{2j} \geq B_{5j}$

Saturday : $A_{6j} + A_{5j} + A_{4j} + A_{3j} \geq B_{6j}$

Sunday : $A_{7j} + A_{6j} + A_{5j} + A_{4j} \geq B_{7j}$

$A_{ij} \geq 0$ $\qquad j = 1, 2, 3$

No. of workers starting ~~on~~ the shift j on day i either started on day i or on day $i-1$ or on day $i-2$ or on day $i-3$ (Consecutive 4 Days)

eg. Workers working on shift j on Day 5 either started on day 5 (Friday) or on day 4 (Thursday) or on day 3 (Wednesday) or on day 2 (Tuesday)

& The sum of these variables must be atleast the minimum no. of workers req. on that day & that shift (from the table)

The Manager is simply finding a feasible schedule & he might be interested in reducing the size of the workforce

$\therefore$ Min $Z = \sum_{i=1}^{7} \sum_{j=1}^{3} A_{ij}$

Some of the advantages / disadvantages of solving this problem as a linear program is :

1) The approach enables us to deal with the problem in a systematic fashion
2) This problem has the potential to reduce the size of workforce by more effectively matching the resources to the needs
3) This problem doesn't account to the fact that what happens when workers fail to report.
4) This problem doesn't account for change in shifts of the workers.
5) This problem also does not address the problem of change in shift patterns & day patterns.

To solve this problem we have 2 approach

1) Define the Dual of the problem & then solve it (Easy Method)
2) Define Surplus Variables & then Artificial Variables & then make $W = 0$ & then solve the LPP (Hard Method)

Q.2 Let A = Units of product A , B = Units of product B

The sales Volume for A is atleast 80% of the total sales of A & B

$\therefore \quad 0.8(A+B) \leq A$

$\Rightarrow \quad -0.2A + 0.8B \leq 0$

For Raw Material Availability

$2A + 4B \leq 240$

For sales Limit of A.

$A \leq 100$

~~Non~~ Feasibility Condition $A, B \geq 0$

$\therefore$ We need to maximize $Z = 20A + 50B$

subject to

$2A + 4B \leq 240$

$A \leq 100$

$-0.2A + 0.8B \leq 0$

$A, B \geq 0$

Introducing Slack Variables we get :

$2A + 4B + C = 240$

$A + D = 100$

$-0.2A + 0.8B + E = 0$

$A, B \geq 0$

$C, D, E \geq 0$

Max $Z = 20A + 50B$

So, we have to solve the following LPP Model:

$$\text{Min } Z = -20A - 50B \quad \text{subject to}$$

$$2A + 4B + C = 240$$

$$A + D = 100$$

$$-0.2A + 0.8B + E = 0$$

$$A, B, C, D, E \geq 0$$

	A	B	C	D	E	RHS
C	2	4	1	0	0	240
D	1	0	0	1	0	100
E	-1/5	4/5	0	0	1	0
	-20	-50	0	0	0	0
C	3	0	1	0	-5	240
D	1	0	0	1	0	100
B	-1/4	1	0	0	5/4	0
	-65/2	0	0	0	125/2	0
A	1	0	1/3	0	-5/3	80
D	0	0	-1/3	1	5/3	20
B	0	1	1/12	0	5/6	20
	0	0	65/6	0	25/3	2600

$\text{Min } Z = -2600$

$\therefore$ $\text{Max } Z = 2600$ at $(80, 20)$

$\therefore$ In order to maximize the profit the company should sell 80 units of product A & 20 units of product B

Profit (Max) = \$2600

Q.3 Let
P = Time allocated to play
W = Time allocated to work

ATQ, we have to minimize Jack's pleasure from both work & play.

$$\text{Min } Z = 2P + W$$

{ Play is twice as much fun as work }

<u>Constraints</u> :

1) Jack has to study as much as he plays
$W \geq P$

2) He can't play more than 4 hrs
$P \leq 4$

3) Total Available Time = 10 hrs
$W + P \leq 10$

$W, P \geq 0$

<u>The LP Model is</u> :

$$\text{Min } Z = 2P + W$$

subject to

$W + P \leq 10$

$P - W \leq 0$

$P \leq 4$

$W, P \geq 0$

Introducing slack variables :

$\text{Min } Z = W + 2P \quad \text{subj. to}$

$$W + P + S_1 = 10$$
$$-W + P + S_2 = 0$$
$$P + S_3 = 4$$

$$W, P, S_1, S_2, S_3 \geq 0$$

$\boxed{\text{No feasible sol}^n \text{ as we can't enter a variable in place of slack variables}}$

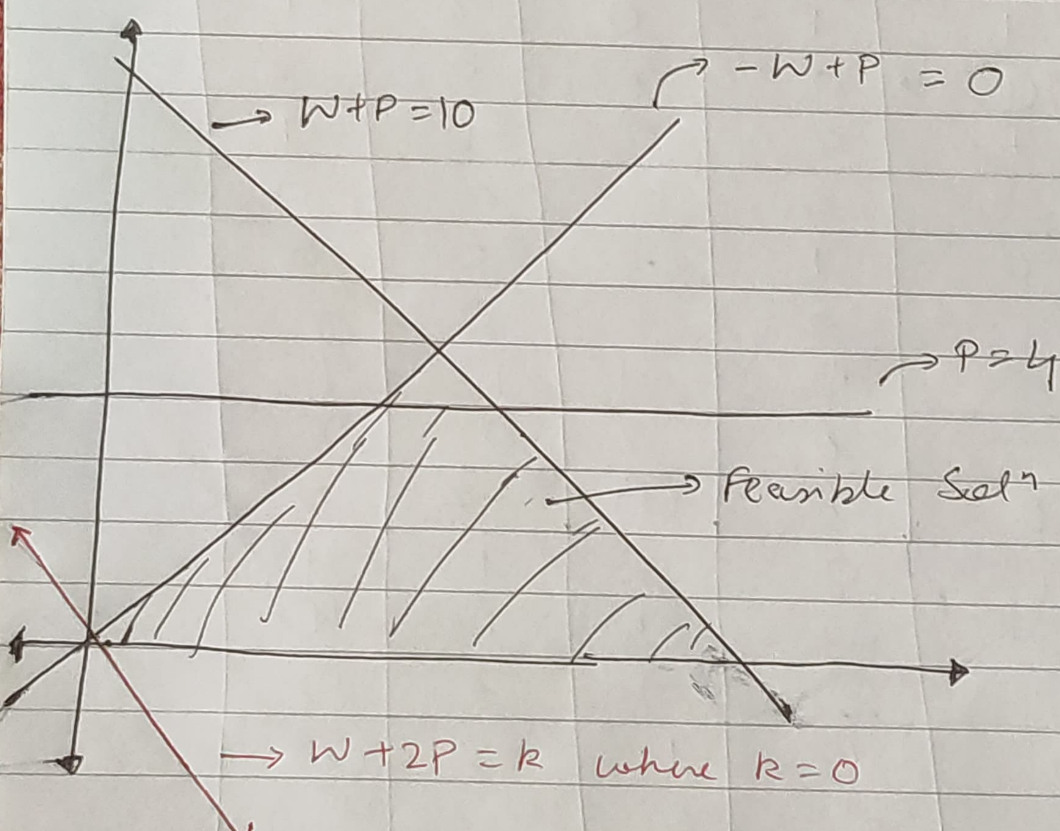
But if we were to maximize his pleasure then the sol^n is follows :

	W	P	S_1	S_2	S_3	RHS
S_1	1	1	1	0	0	10
S_2	-1	1	0	1	0	0
S_3	0	1	0	0	1	4
	-1	-2	0	0	0	0
S_1	2	0	1	-1	0	10
P	-1	1	0	1	0	0
S_3	1	0	0	-1	1	4
	-3	0	0	2	0	0
S_1	0	0	1	1	-2	2
P	0	1	0	0	1	4
W	1	0	0	-1	1	4
	0	0	0	-1	3	12
S_2	0	0	1	1	-2	2
P	0	1	0	0	1	4
W	1	0	1	0	-1	6
	0	0	1	0	1	14

So, In order to maximize his pleasure
~~ate~~ Jack has to
Work: 6 hours
Play : 4 hours
So that he can have 14 units of pleasure.

Note: In this ques, we had to find minimum of Z but that is not possible as there is no entering variable
∴ We have found max Z

If we take Min Z then lets analyze it with Graph



Min $Z=0$ at $(0,0)$

~~Q4~~ Let

x_1: no. of Hifi-1 units
x_2: no. of Hifi-2 units

Time taken per unit for
Workstation 1 : $6x_1 + 4x_2$
Workstation 2 : $5x_1 + 5x_2$
Workstation 3 : $4x_1 + 6x_2$

Total Maintenance Time for
W_1 = 10% of 480 = 48
W_2 = 14% of 480 = 67.2
W_3 = 12% of 480 = 57.6

Idle Time for
W_1 = Total Time − Time taken per unit − Maintenance time
$= 480 - (6x_1 + 4x_2) - 48$
$W_2 = 480 - (5x_1 + 5x_2) - 67.2$
$W_3 = 480 - (4x_1 + 6x_2) - 57.6$

Total Idle Time = Idle Time of $W_1 + W_2 + W_3$
$= 1267.2 - 15(x_1 + x_2)$

∴ We have to minimize Z where
$Z = 1267.2 - 15(x_1 + x_2)$
↳ Min.

Constraints

Idle Time can't be less than 0

$\therefore$ W_1 : $480 - (6x_1 + 4x_2) - 48 \geq 0$

$6x_1 + 4x_2 \leq 432$

||ly W_2 : $5x_1 + 5x_2 \leq 412.8$

||ly W_3 : $4x_1 + 6x_2 \leq 422.4$

$\therefore$ The LP Model is :

$\text{Min } Z = 1267.2 - 15(x_1 + x_2)$ Subject to

$6x_1 + 4x_2 \leq 432$

$5x_1 + 5x_2 \leq 412.8$

$4x_1 + 6x_2 \leq 422.4$

$x_1, x_2 \geq 0$

Introducing Slack Variables

So, our constraints become

$6x_1 + 4x_2 + x_3 = 432$

$5x_1 + 5x_2 + x_4 = 412.8$

$4x_1 + 6x_2 + x_5 = 422.4$

$-15x_1 - 15x_2 = -1267.2 + Z$

	x_1	x_2	x_3	x_4	x_5	RHS
x_3	6	4	1	0	0	432
x_4	5	5	0	4	0	412.8
x_5	4	6 →	0	0	1	422.4
	-15	-15	0	0	0	-1267.2
x_3	10/3	0	1	0	-2/3	752/5
x_4	5/3 →	0	0	1	-5/6	304/5
x_2	2/3	1	0	0	1/6	352/5
	-5	0	0	0	5/2	-1056/5
x_3	0	0	1	-2	1	144/5
x_1	1	0	0	3/5	-1/2	912/25
x_2	0	1	0	-2/5	1/2	1132/25
	0	0	0	3	0	-144/5

So, the optimal min of x_1 & x_2 is

$x_1 = 912/25 =$

$x_2 = 1152/25$

So that the idle time is minimized

Min $Z = 144/5$

$= 28.8$ minutes

Q.5 Let

x_i = Amount of product i produced on Machine X per week
($i = 1, 2, 3, 4$)

y_i = Amount of product i produced on machine Y per week
($i = 2, 3, 4$)

$x_i, y_i \geq 0$

We have not defined y_1 as product 1 must be processed on both machines X and Y

Floor Space

$0.1x_1 + 0.15(x_2 + y_2) + 0.5(x_3 + y_3) + 0.05(x_4 + y_4) \leq 50$

Customer Req.

$x_2 + y_2 = 2(x_3 + y_3)$

Available Time

$10x_1 + 12x_2 + 13x_3 + 8x_4 \leq 0.95(35)(60)$
↳ 5% time Machine X not working & working week is 35 hrs long.

$\|^{ly}$,

$27x_1 + 19y_2 + 33y_3 + 23y_4 \leq 0.93(35)(60)$
↳ Machine Y

Max Profit $Z = 10x_1 + 12(x_2 + y_2) + 17(x_3 + y_3) + 8(x_4 + y_4)$

$\therefore$ The LPP Model is

$$\text{Max } Z = 10x_1 + 12x_2 + 17x_3 + 8x_4 + 12y_2 + 17y_3 + 8y_4$$

subject to

$$0.1x_1 + 0.15x_2 + 0.5x_3 + 0.05x_4 + 0.15y_2 + 0.5y_3 + 0.05y_4 \leq 50 ;$$

$$\cancel{x_2+y_2} \quad x_2 - 2x_3 + y_2 - 2y_3 = 0 ;$$

$$10x_1 + 12x_2 + 13x_3 + 8x_4 \leq 1995 ;$$

$$27x_1 + 19y_2 + 33y_3 + 23y_4 \leq 1953 ;$$

Q.6 Let

x : amount of metal M_1

y : amount of metal M_2

We have to minimize the cost

$$\text{Min } Z = 100x + 80y$$

Constraints

1) Aluminium must be atleast 3% & atmost 6%

$\therefore 30 \le 0.06x + 0.03y \le 60$

$$\therefore \left\{ \begin{array}{l} 0.03 \le \dfrac{0.06x + 0.03y}{x+y} \le 0.06 \\ x+y = 1000 \text{ (given)} \end{array} \right\}$$

2) Silicon must be in range 3% - 5%

$\therefore 30 \le 0.03x + 0.06y \le 50$

$$\left\{ \begin{array}{l} \because 0.03 \le \dfrac{0.03x + 0.06y}{x+y} \le 0.05 \\ x+y = 1000 \end{array} \right\}$$

3) Carbon must be b/w 3 & 7%

$$\therefore 0.03 \le \frac{0.04x + 0.03y}{x+y} \le 0.07$$

$$\Rightarrow 30 \le 0.04x + 0.03y \le 70$$

$\because x+y = 1000$

$x, y \ge 0$

So, the LPP Model is :

$$\text{Min } Z = 100x + 80y$$

subject to

$$x + y = 1000$$
$$0.06x + 0.03y \geq 30$$
$$0.06x + 0.03y \leq 60$$
$$0.09x + 0.06y \geq 30$$
$$0.03x + 0.06y \leq 50$$
$$0.04x + 0.03y \geq 30$$
$$0.04x + 0.03y \leq 70$$

$$x, y \geq 0$$

Upon solving the above problem with LP Assistant we get :

$$x = \frac{1000}{3}$$

$$y = \frac{2000}{3}$$

and ~~Max Z =~~

$$\text{Min } Z = \frac{260000}{3}$$

The LP Assistant program is attached with this assignment

Q7

Let

x_1 = no. of units of single family homes
x_2 = no. of units of double family homes
x_3 = no. of units of triple family homes
x_4 = no. of recreation areas

We have to maximize the net return

So, $\text{Max } Z = 10000x_1 + 12000x_2 + 15000x_3$

Now,

Constraints :

1) Land Use :

15% of the acreage will be allocated to the streets & utility easements.

Total Land Available = 85% of 800 acre
= 680 acre

Now, Minimum lot sizes for single, double & triple family homes are 2, 3 & 4 acres & for recreation areas we have 1 acre

$\therefore \quad 2x_1 + 3x_2 + 4x_3 + 1x_4 \leq 680$

2) Single family homes

Only single, double & triple family homes can be constructed with single family homes accounting for atleast 50% of the total

$\therefore \quad \dfrac{x_1}{x_1+x_2+x_3} \geq 0.5 \quad$ or $\quad x_1 - x_2 - x_3 \geq 0$

3) Recreation Areas

Recreation areas of 1 acre each must be established at rate of one area per 200 families

$\Rightarrow \cancel{x_4 \geq x_1 + x_2 + x_3}$

$$\Rightarrow x_4 \geq \frac{x_1 + 2x_2 + 3x_3}{200}$$

or

$$200x_4 - x_1 - 2x_2 - 3x_3 \geq 0$$

4) Capital

The country charges a min. of \$100,000 for the project.

$$1000x_1 + 1200x_2 + 1400x_3 + 800x_4 \geq 100000$$

5) Water Consumption

$$400x_1 + 600x_2 + 840x_3 + 450x_4 \leq 200000$$

$\therefore$ The LPP Model is :

$$\text{Max } Z = 10000x_1 + 12000x_2 + 15000x_3$$

subject to

$$2x_1 + 3x_2 + 4x_3 + x_4 \leq 680$$

$$x_1 - x_2 - x_3 \geq 0$$

$$-x_1 - 2x_2 - 3x_3 + 200x_4 \geq 0$$

$$10x_1 + 12x_2 + 14x_3 + 8x_4 \geq 1000$$

$$40x_1 + 60x_2 + 84x_3 + 45x_4 \leq 20000$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Solving the problem:
Introducing necessary slack or surplus variables.

$$\text{Max } Z = 10000x_1 + 12000x_2 + 15000x_3$$

subject to

$$2x_1 + 3x_2 + 4x_3 + x_4 + x_5 = 680$$
$$x_1 - x_2 - x_3 - x_6 + A_1 = 0$$
$$-x_1 - 2x_2 - 3x_3 + 200x_4 - x_7 + A_2 = 0$$
$$10x_1 + 12x_2 + 14x_3 + 8x_4 - x_8 + A_3 = 1000$$
$$40x_1 + 60x_2 + 84x_3 + 45x_4 + x_9 = 20000$$

$$x_i \geq 0 \quad i = 1, \ldots 9$$

As the program was lengthy, so it is attached along with this Assignment.

When solving the above problem with LP Assistant we get

$$\left.\begin{aligned} x_1 &= \frac{136000}{401} \\ x_2 &= 0 \\ x_3 &= 0 \\ x_4 &= \frac{680}{401} \\ \& \text{ Max } Z &= \frac{136000000}{401} \end{aligned}\right\} \text{Sol}^n$$