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13 January 2021

We can consider any basic variable

But feasible $x_i \geq 0$

↳ Feasible Basic

eg. In previous eg.

$x_2, x_4 \rightarrow$ Basic

but not feasible

$\boxed{x_4 = -3} \rightarrow \times$ (Violates Non-negativity condition $x_i \geq 0$)

$x_2 = 9$

Visualization

Geometrically,

$$x_1 + x_2 + 2x_3 + x_4 = 6$$

$$3x_2 + x_3 + 8x_4 = 3$$

↳ Writing this in Vector form

$$\underset{a_1}{x_1\begin{bmatrix} 1 \\ 0 \end{bmatrix}} + \underset{a_2}{x_2\begin{bmatrix} 1 \\ 3 \end{bmatrix}} + \underset{a_3}{x_3\begin{bmatrix} 2 \\ 1 \end{bmatrix}} + \underset{a_4}{x_4\begin{bmatrix} 1 \\ 8 \end{bmatrix}} = \underset{b}{\begin{bmatrix} 6 \\ 3 \end{bmatrix}}$$

↳ what we are doing is expressing b as a linear combination of these vectors.

So, we have to write b as a positive linear combination of these vectors.

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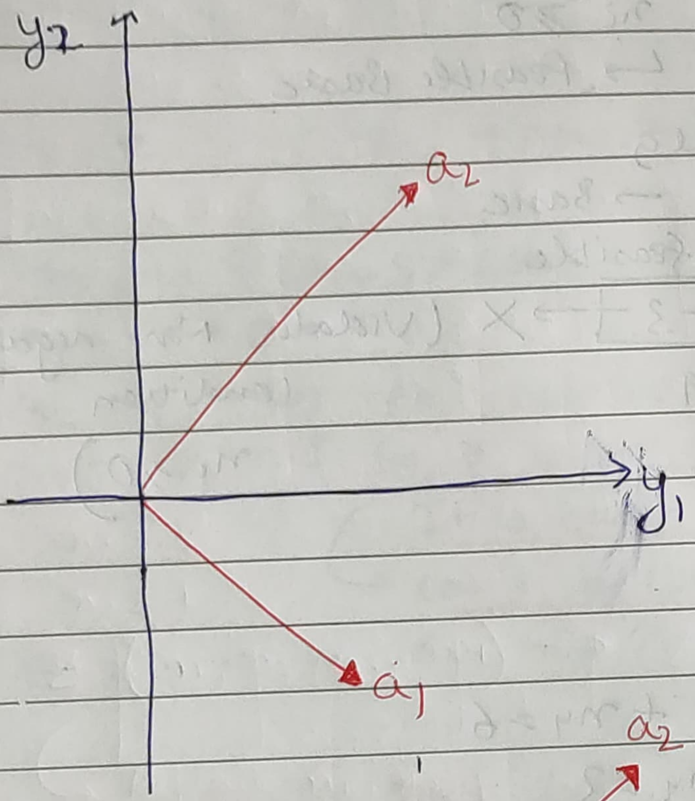
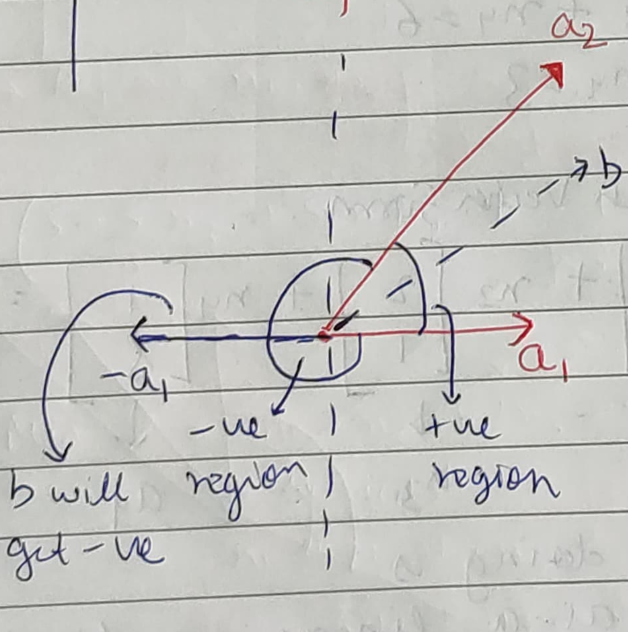
Write b as a non-negative combination of a_1, a_2, a_3, a_4.





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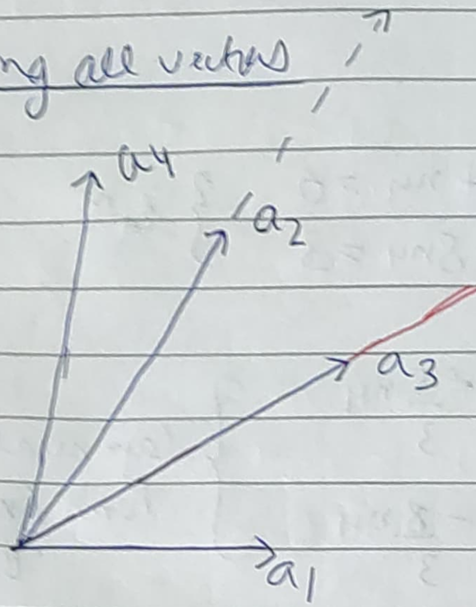
Drawing all vectors



b lies in b/w a_1 & a_2

$\underbrace{a_1 \,\&\, a_2}_{\text{can be used}}$

a_2 & a_4 can't be used $\because$ b doesn't lie in their region.

a_4, a_1 can be used

a_3, a_2 " " "

a_1, a_3 " " "

Here we can see that b can be expressed in the form of a_3 only.

So $b = 3a_3 + 0 \cdot a_1$ (or a_2 or a_4)

$x_3 = 3$

$x_1 = 0$ → Degenerate Sol^n as Basic Variable comes out to be 0

Optimum Sol^n

$x_1 + x_2 + 2x_3 + x_4 = 6$
$3x_2 + x_3 + 8x_4 = 3$ } Eq^n

$x_1 = 5 - \frac{5}{3}x_3 + \frac{5}{3}x_4$
$x_2 = 1 - \frac{1}{3}x_3 - \frac{8}{3}x_4$
} Canonical form (x_1, x_2 Basic Variable) & also feasible

$x_1 = 5$
$x_2 = 1$

$Z(x_1, x_2, x_3, x_4) \rightarrow Z$ is a f^n of x_1, x_2, x_3, x_4

↳ Replace x_1, x_2

$Z\left(5 - \frac{5}{3}x_3 + \frac{5}{3}x_4,\ 1 - \frac{1}{3}x_3 - \frac{8}{3}x_4,\ x_3,\ x_4\right)$

Ex. Min $Z = x_1 - x_2 + 2x_3 - 5x_4$
↳ subject to
$x_1 + x_2 + 2x_3 + x_4 = 6$
$3x_2 + x_3 + 8x_4 = 3$
$x_i \geq 0$

Min $Z = 5 - \frac{5}{3}x_3 + \frac{5}{3}x_4 - 1 + \frac{1}{3}x_3 + \frac{8}{3}x_4$
$+ 2x_3 - 5x_4$

Min $Z = \frac{2}{3}x_3 - \frac{2}{3}x_4 + 4$

$x_3, x_4 \rightarrow$ Non basic

Min $Z = 4$ } for $x_1, x_2 = (5, 1)$

$\hookrightarrow$ Feasible soln $\rightarrow$ ~~Optim~~ value is 4

$\hookrightarrow$ This is an extra step

Instead, we can do is put the Z fn & pivot along with constraints.

So now we will pivot these 3 eqns

$x_1 - x_2 + 2x_3 - 5x_4$

$x_1 + x_2 + 2x_3 + x_4 = 6$

$3x_2 + x_3 + 8x_4 = 3$

Summary

1) The standard LP is in canonical form with set of basic variables if

- The system of constraints is in canonical form with basic variables
- The associated basic variables is feasible ($x_i \geq 0$)
- The objective fn is expressed using non-basic

Eg → Min. $Z = -4x_1 + x_2 + x_3 + 7x_4 + 3x_5$

subject to const.

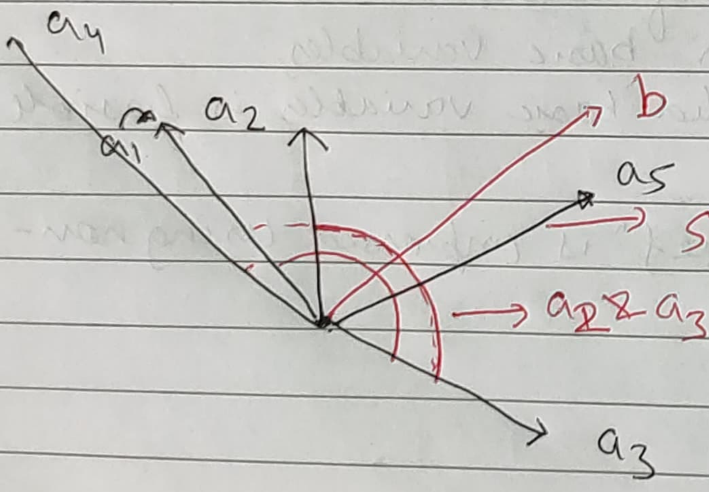
$$-6x_1 + x_3 - 2x_4 + 2x_5 = 6$$
$$3x_1 + x_2 - x_3 + 8x_4 + x_5 = 9$$
$$x_i \geq 0 \quad i = 1, \dots 5$$

1] Convert to Standard form
(Already in stand. form)

2] Convert to Canonical form

Convert to Vector (∵ 2 constraints)

$$x_1\begin{bmatrix} -6 \\ 3 \end{bmatrix} + x_2\begin{bmatrix} 0 \\ 1 \end{bmatrix} + x_3\begin{bmatrix} 1 \\ -1 \end{bmatrix} + x_4\begin{bmatrix} -2 \\ 8 \end{bmatrix} + x_5\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 9 \end{bmatrix}$$



So ∴ we can consider x_2 & x_3 as basic variable

Canonical form:

$$6 = -6x_1 + x_3 - 2x_4 + 2x_5$$
$$15 = -3x_1 + x_2 + 6x_4 + 3x_5$$

$x_3 = 6$, $x_2 = 15$ $\qquad (0, 15, 6, 0, 0)$

Now Z also pivoting

$$Z - 21 = 5x_1 + 0x_2 + 0x_3 + 3x_4 - 2x_5$$
$$6 = -6x_1 \quad + x_3 - 2x_4 + 2x_5$$
$$15 = -3x_1 + x_2 + \quad + 6x_4 + 3x_5$$

↪ $(0, 15, 6, 0, 0)$

↪ feasible Basic

Optimum $\boxed{\text{Min } Z = 21}$ →