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LPP

Semester - VI

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Q.3 Let x_1 denote our no. of units of product A
Let x_2 denote our no. of units of product B.

$\therefore$ Variables : x_1, x_2

Constraints : $2x_1 + 3x_2 \leq 1200$

$2x_1 + x_2 \leq 1000$

$0x_1 + 4x_2 \leq 800$

Objective f^n $\longrightarrow$ $Z = 3x_1 + 4x_2$ ( )

(a) Sol^n for each material is
(450, 100)

$\therefore$ No. of units of product A = 450
" " " " " B = 100

(b) Dual Prices or Shadow Prices for

Material A = $5/4$

Material B = $1/4$

Material C = 0

(c) Feasibility Ranges for shadow Price in (b) is

Sol^n is (450, 100)
Putting In Constraints

$900 + 300 \leq 1200 \Rightarrow =$

$900 + 100 \leq 1000 \Rightarrow =$

$4(100) \leq 800 \Rightarrow <$

$\therefore$ Matrial C is underused (using less than given value)

$\therefore$ We can increase our Material C upto 400 units

(d) Availability of Mat. A is increased to 1300 units

Means we are changing $\vec{b}$

Change in $\vec{b}$ does not affect LHS

Z_0^* & b^* changes.

So, Our initial b_1 was 1200

Now it is 1300

Let the change be denoted by λ

$\therefore \lambda = 100$

$b^* = B^{-1} b$

~~b^*~~ $B = [A^1 \quad A^5 \quad A^2]$

$$= \begin{bmatrix} 2 & 0 & 3 \\ 2 & 0 & 1 \\ 0 & 1 & 4 \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} -1/4 & 3/4 & 0 \\ -2 & 2 & 1 \\ 1/2 & -1/2 & 0 \end{bmatrix}$$

$b^* = B^{-1} b$

$$b^* = B^{-1}\begin{bmatrix}1200\\1000\\800\end{bmatrix} + B^{-1}\begin{bmatrix}100\\0\\0\end{bmatrix}$$

$$= \begin{bmatrix}450\\400\\100\end{bmatrix} + \begin{bmatrix}-1/4 & 3/4 & 0\\-2 & 2 & 1\\1/4 & -1/2 & 0\end{bmatrix}\begin{bmatrix}100\\0\\0\end{bmatrix}$$

$$= \begin{bmatrix}450\\400\\100\end{bmatrix} + \begin{bmatrix}-25\\-200\\25\end{bmatrix}$$

$$= \begin{bmatrix}425\\200\\125\end{bmatrix} \qquad b^* \geq 0$$

∴ The Basic feasible points will Remain same. Their value will only change

∴ New Optimum solⁿ is (425, 200)

(e) Material C is reduced to 350 units

∴ $\vec{b}$ is changing

Change in $\vec{b}$ doesn't affect LHS

$$b^* = B^{-1}b$$

$$= B^{-1}\begin{bmatrix}1200\\1000\\800\end{bmatrix} + B^{-1}\begin{bmatrix}0\\0\\-450\end{bmatrix}$$

$$= \begin{bmatrix} 450 \\ 400 \\ 100 \end{bmatrix} + \begin{bmatrix} -1/4 & 3/4 & 0 \\ -2 & 2 & 1 \\ 1/4 & -1/2 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -450 \end{bmatrix}$$

$$= \begin{bmatrix} 450 \\ 400 \\ 100 \end{bmatrix} + \begin{bmatrix} 0 \\ -450 \\ 0 \end{bmatrix} = \begin{bmatrix} 450 \\ -50 \\ 100 \end{bmatrix} \leq 0$$

We can't determine the new optimum solⁿ from the given info directly as $b^* \leq 0$ as for Simplex to work we need to have $b^* \geq 0$.

$\therefore$ Simplex will not work here

$\therefore$ We can determine the solⁿ using the dual simplex algorithm

(f) Let we introduce a variable M_1

$M_1 \rightarrow$ be the amount of material above 1200 units

$\therefore$ Max z becomes

$$z = 3x_1 + 4x_2 + 40M_1$$

and the constraints are

$$2x_1 + 3x_2 \leq 1200 + M_1$$

$$2x_1 + x_2 \leq 1000$$

$$4x_2 \leq 800$$

We have

$$B^{-1} = \begin{bmatrix} -1/4 & 3/4 & 0 \\ -2 & 2 & 1 \\ 1/4 & -1/2 & 0 \end{bmatrix}$$

$$C_B = \begin{bmatrix} 3 & 0 & 4 \end{bmatrix}$$

$$A^*(M) = B^{-1} A(M)$$

$$= \begin{bmatrix} 1/4 \\ 2 \\ -1/4 \end{bmatrix}$$

$$\cancel{C} \quad C_M = C_M - C_B A^*(M)$$

$$= 40 - \begin{bmatrix} 3 & 0 & 4 \end{bmatrix} \begin{bmatrix} 1/4 \\ 2 \\ -1/4 \end{bmatrix}$$

$$= 40 - \frac{3}{4} + \frac{4}{4}$$

$$= 41 - \frac{3}{4} \geq 0$$

$\therefore \begin{bmatrix} X^* \\ 0 \end{bmatrix}$ is an optimum sol^n

So, the Company should accept the offer.

Q.1 Max $Z = 2x_1 + 2x_2$

subject to

$x_1 + x_3 + x_4 \leq 1$

$x_2 + x_3 - x_4 \leq 1$

$x_1 + x_2 + 2x_3 \leq 3$

$x_i \geq 0$

$i = 1, 2, 3, 4$

Sol^n Let y_1, y_2, y_3 be the variables for dual

(a) Dual Problem is

$y_1 + 0y_2 + y_3 \geq 2$

$0y_1 + y_2 + y_3 \geq 2$

$y_1 + y_2 + 2y_3 \geq 0$

$y_1 - y_2 + 0y_3 \geq 0$

Min $Z = y_1 + y_2 + 3y_3$

(b) Checking whether $X^* = (1, 1, 0, 0)$ is a feasible sol^n to primal

Putting the X^* in ~~LF~~ Primal we get

$1 + 0 \leq 1$ $1 \leq 1 \rightarrow$ True

$1 + 0 - 0 \leq 1$ $1 \leq 1 \rightarrow$ True

$1 + 1 + 0 \leq 3$ $2 \leq 3 \rightarrow$ True

All constraints are satisfying

$\therefore X^*$ is a feasible sol^n of primal hecking whether Y^* is a feasible sol^n of dual

$Y^* = (1,1,1)$

$1+1 \geq 2$	$2 \geq 2 \rightarrow$ True
$0+1+1+1 \geq 2$	$2 \geq 2 \rightarrow$ True
$1+1+1+2 \geq 0$	$4 \geq 0 \rightarrow$ True
$1-1+0 \geq 0$	$0 \geq 0 \rightarrow$ True

All constraints are satisfying

$\therefore$ Y^* is a feasible sol^n of Dual

(c) Y^* is a feasible sol^n. We have to check whether its optimal or not

Slack for Dual is $\begin{bmatrix} 0 \\ 0 \\ -4 \\ 0 \end{bmatrix}$

Let $X^* = (x_1^*, x_2^*, x_3^*, x_4^*)$

By Complimentary slack Theorm

$x_3^* = 0$

Now Y^* (slack of Primal) $= 0$

$\therefore$ $V_1 = V_2 = V_3 = 0$

$\therefore$ $Max\ z = 2x_1 + 2x_2$

subj. to $x_1 + x_3 + x_4 \leq 1$

$x_2 + x_3 - x_4 \leq 1$

$x_1 + x_2 + 2x_3 \leq 3$ $\quad x_i \geq 0$

$x_3^* = 0$

$x_1 + x_4 = 1$

$x_2 - x_4 = 1$

$x_1 + x_2 = 3$

$\therefore$ ~~As~~ These Eq^{ns} are linearly dependent

$\therefore$ No sol^n

$\therefore$ No X^* corresponding to Y^*

$\therefore$ Y^* is not optimal

Q.2 Let

x_1 = no. of students who start at 8:00
x_2 = " " " " " " 9:00
x_3 = " " " " " " 10:00
x_4 = " " " " " " 11:00
x_5 = " " " " " " 1:00
x_6 = " " " " " " 2:00
x_7 = " " " " " " 3:00
x_8 = " " " " " " 4:00

ATQ

$x_1 \geq 2$

$x_1 + x_2 \geq 2$

$x_1 + x_2 + x_3 \geq 3$

$x_2 + x_3 + x_4 \geq 4$

$x_3 + x_4 \geq 4$

$x_4 + x_5 \geq 3$

$x_4 + x_5 + x_6 \geq 3$

$x_6 + x_7 + x_8 \geq 3$

$x_5 + x_6 + x_7 \geq 3$

$\text{Min } Z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8$

Now, we need to introduce surplus variables and then artificial variables to solve this problem or, we need to do this problem using The Dual Simplex Algorithm

We will solve this problem using Dual. (1)

Defining Dual for above problem :

~~$y_1 \leq 1$~~

$y_1 + y_2 + y_3 \leq 1$

$y_2 + y_3 + y_4 \leq 1$

$y_3 + y_4 + y_5 \leq 1$

$y_4 + y_5 + y_6 + y_7 \leq 1$

$y_6 + y_7 + y_8 \leq 1$

$y_7 + y_8 + y_9 \leq 1$

$y_8 + y_9 \leq 1$

$y_9 \leq 1$

$$\text{Max } Z = 2y_1 + 2y_2 + 3y_3 + 4y_4 + 4y_5 + 3y_6 + 3y_7 + 3y_8 + 3y_9$$

Adding slack variables and converting into standard form

$y_1 + y_2 + y_3 + y_{10} = 1$

$y_2 + y_3 + y_4 + y_{11} = 1$

$y_3 + y_4 + y_5 + y_{12} = 1$

$y_4 + y_5 + y_6 + y_7 + y_{13} = 1$

$y_6 + y_7 + y_8 + y_{14} = 1$

$y_7 + y_8 + y_9 + y_{15} = 1$

$y_8 + y_9 + y_{16} = 1$

$y_9 + y_{17} = 1$

On solving this we get ~~soln~~ $Z = 9$

∴ The Minimum no. of students the IE Department should employ is 9

and the time of Day at which they should report is :

1) 2 Students Report at 8:00 am

2) 1 Student Report at 10:00 am

3) 3 Students Report at 11:00 am

4) 3 students Report at 2:00 pm