



January 18, 2021

Rule: If one of $a_{ik} < 0$ (strict condition),
 x_k shall enter as basic variable

$$\begin{array}{lcl} x_1 + a_{1k} x_k = b_1 & \Rightarrow & x_1 = b_1 - a_{1k} x_k \geq 0 \\ x_2 + a_{2k} x_k = b_2 & & x_2 = b_2 - a_{2k} x_k \geq 0 \\ \vdots & & \vdots \\ x_m + a_{mk} x_k = b_m & & \vdots \end{array}$$

I) $a_{ik} < 0$

~~If~~ $x_i = b_i - a_{ik} x_k \geq 0 \Rightarrow x_k \geq \frac{b_i}{a_{ik}}$

No limit \swarrow neg. value as $a_{ik} < 0$

x_k can be as ~~small~~ large as possible.

$Z = -Z_0 + \text{Non-basic} + c_k x_k + \dots$

As x_k tends to infinity ($x_k \rightarrow \infty$)

$Z \rightarrow -\infty = \text{bounded}$

II) If $a_{ik} = 0$
 $x_i = b_i \geq 0$ } Unbounded

If all $a_{ik} \leq 0$, then we stop
(Solⁿ Unbounded)



Result - Unbounded Case

For LP y \exists an index k , $m+1 \leq k \leq n$
such that $c_k < 0$ and $a_{ik} \leq 0$
 $\forall i = 1, \dots, m$

→ Bounded

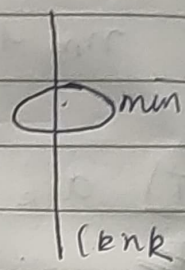
Case-3

If $c_k < 0$, $a_{ik} > 0$ for some i

$$i^{\text{th}} \text{ eq}^n \quad m_i + a_{ik} n_k = b_i$$

$$n_i = b_i - a_{ik} n_k \geq 0$$

$$\Rightarrow n_k \leq \frac{b_i}{a_{ik}}$$



Non-basic variable $\rightarrow \frac{b_t}{a_{tk}} = \min \left\{ \frac{b_i}{a_{ik}}, a_{ik} > 0 \right\}$

Basic Variable : $\left\{ \begin{array}{l} \{ n_i \}_{i=1, \dots, m} \\ n_k \end{array} \right\}$
 $i \neq t$

$$Z = -Z_0 + c_k n_k = -Z_0 + c_k \frac{b_t}{a_{tk}} \rightarrow Z \text{ dec.}$$

$$c_k < 0, \quad b_t > 0, \quad a_{tk} > 0$$

Result : $c_k < 0$ and at least one $a_{ik} > 0$

$$\frac{b_t}{a_{tk}} = \min \left\{ \frac{b_i}{a_{ik}}, a_{ik} > 0 \right\}$$

Replace n_t by n_k as basic variable

$$Z = -Z_0 + c_k n_k = -Z_0 + c_k \frac{b_t}{a_{tk}} \rightarrow \text{decreases}$$



$b_i > 0 \Rightarrow Z$ decreases strictly

$b_i = 0 \Rightarrow Z$ remains same

degenerate case $\{B.V. = 0\}$

No. of Basic variable \leq no. of choices B.V.

$${}^nC_m \rightarrow n \text{ eq B.V.}$$

$$m = \text{eqns}$$

$$\begin{array}{cccc} a_{11}n_1 & + a_{12}n_2 & + a_{13}n_3 & + a_{14}n_4 = b_1 \\ a_{21}n_1 & + a_{22}n_2 & + a_{23}n_3 & + a_{24}n_4 = b_2 \end{array}$$

6 choices

Simplex Alg. ($b_i > 0$) \rightarrow Converges

Ex: Min Z

$$n_1 + \dots + 2n_4 - n_5 = 10$$

$$n_2 + \dots - n_4 - 5n_5 = 20$$

$$+ n_3 + 6n_4 - 12n_5 = 18$$

$$\text{O.p.f.} \quad \dots - 2n_4 + 3n_5 = 60 + Z$$

n_1, n_2, n_3 are B.V.

New B.V.

as it will dec

Z more

Now for $a_{ik} > 0$ (Case-3)

I e_{q^n} & III e_{q^n}

$\frac{b_i}{a_{ik}}$

for

$$I \rightarrow 10/2 = 5$$

$$III \rightarrow 18/6 = 3$$

$$\begin{array}{rcl} n_1 & +2n_4 & -n_5 = 10 \\ n_2 & -n_4 & -5n_5 = 20 \\ n_3 & +6n_4 & -12n_5 = 18 \rightarrow n_3 \text{ end} \\ \text{air} > 0 & \uparrow -2n_4 & +3n_5 = 60 + Z \end{array}$$

Bounded

then find $\frac{b_i}{a_{ik}}$

Select e_{q^n} for Min. one $\{ n_3 \text{ will exit} \}$
 $\& n_4 \text{ will enter}$

n_3 : Non-basic Variable

n_4 : Basic Variable

$$\begin{array}{rcl} n_1 & -1/3 n_3 + & +3n_5 = 4 \rightarrow n_1 \text{ exit} \\ n_2 & +1/6 n_3 + & -7n_5 = 23 \\ & 1/6 n_3 + n_4 & -2n_5 = 3 \\ & 1/3 n_3 & -n_5 = 66 + Z \end{array}$$

New Basic

New Solⁿ (4, 23, 0, 3, 0)

$$Z_{\min} = 66$$

$n_1 \rightarrow$ Non-Basic Variable

$n_5 \rightarrow$ New Basic Variable



$$\begin{array}{rclcl}
 \frac{1}{3}n_1 & -\frac{1}{9}n_3 & +n_5 & = \frac{4}{3} \\
 \frac{7}{3}n_1 + n_2 & -\frac{11}{18}n_3 & & = \frac{97}{3} \\
 \frac{2}{3}n_1 & -\frac{1}{18}n_3 & +n_4 & = \frac{17}{3} \\
 \frac{1}{3}n_1 & \frac{2}{9}n_3 & & = \frac{202}{3} + z
 \end{array}$$

\hookrightarrow Solⁿ $\left(0, \frac{97}{3}, 0, \frac{17}{3}, \frac{4}{3}\right) \rightarrow \frac{-202}{3} (z \text{ min})$

stop
as can't be
decreased

Simpler Tableau

	n_1	n_2	n_3	n_4	n_5	b
n_1	1	0	0	✓ 2	-1	10 $\frac{10}{2}=5$
n_2	0	1	0	✗ -1	-5	20
n_3	0	0	1	✓ 6	-12	18 $\frac{18}{6}=3 \rightarrow$ Exit n_3
Z	0	0	0	⊖ 2	3	60
n_1	1	0	-1/3	0	✓ 3	4 \rightarrow Exit n_1
n_2	0	1	1/6	0	✗ -7	23
n_4	0	0	1/3	1	✗ -2	3
Z	0	0	1/3	0	⊖ 1	(66)
n_5	1/3	0	-1/9	0	1	4/3
n_2	7/5	1	-11/18	0	0	97/3
n_4	2/3	0	-1/18	1	0	17/3
Z	1/3	0	1/9	0	0	(202/3)

No -ve sign in Z
Step Algorithm

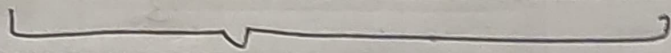
$z \text{ min} = \frac{-202}{3}$ at $\left(0, \frac{97}{3}, 0, \frac{17}{3}, \frac{4}{3}\right)$



$$4n_1 + 3n_2 + n_3 + n_4 =$$

$$6n_1 + 2n_2 + 4n_3 + n_4 =$$

$$3n_1 + n_2 + n_3 + 4n_4 = 0$$



How do we find initial canonical form?

How " " " " basic variables