LINEAR PROGRAMMING

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INTRODUCTION

- Linear programming is a method to achieve the best outcome in a mathematical model.
- It is a special case of mathematical programming.
- Uses of programming here means choosing a course of action.
- Linear programming involves choosing a course of action when the mathematical model of the problem contains only linear function.

INTRODUCTION TO MANAGEMENT SCIENCE (MS) /LINEAR PROGRAMMING (LP)

- Management science is also referred as operations research, quantitative methods, quantitative analysis, decision sciences, and business analytics
- Management science is a scientific approach to solving management problems.
- It is used in a variety of organizations to solve many different types of problems.
- It encompasses a logical mathematical approach to problem solving.
- Management science involves a philosophy of problem solving in a logical manner.

ASSUMPTIONS OF LP

(i) There are a number of constraints or restrictions- expressible in quantitative terms.

(ii) The prices of input and output both are constant.

(iii) The relationship between objective function and constraints are linear.

(iv) The objective function is to be optimized i.e., profit maximization or cost minimization.

ADVANTAGES OF LP

LP has been considered an important tool due to following reasons:

1. LP makes logical thinking and provides better insight into business problems.

2. Manager can select the best solution with the help of LP by evaluating the cost and profit of various alternatives.

3. LP provides an information base for optimum allocation of scarce resources.

4. LP assists in making adjustments according to changing conditions.

5. LP helps in solving multi-dimensional problems.

LIMITATIONS OF LP

1. This technique could not solve the problems in which variables cannot be stated quantitatively.

2. In some cases, the results of LP give a confusing and misleading picture.
For example, the result of this technique is for the purchase of 1.6 machines. It is very difficult to decide whether to purchase one or two- machine because machine can be purchased in whole.

3. LP technique cannot solve the business problems of non-linear nature.

4. The factor of uncertainty is not considered in this technique.

5. This technique is highly mathematical and complicated.

6. If the numbers of variables or contraints involved in LP problems are quite large, then using costly electronic computers become essential, which can be operated, only by trained personnel.

7. Under this technique to explain clearly the objective function is difficult.

APPLICATIONS OF LP

LP technique is applied to a wide variety of problems listed below:

(a) Optimizing the product mix when the production line works under certain specification;

(b) Securing least cost combination of inputs;

(c) Selecting the location of Plant;

(d) Deciding the transportation route;

(e) Utilizing the storage and distribution centers;

(f) Proper production scheduling and inventory control;

(g) Solving the blending problems;

(h) Minimizing the raw materials waste;

(i) Assigning job to specialized personnel such as healthcare professionals.

LINEAR PROGRAMMING(LP) PROBLEM

- The maximization or minimization of some quantity is the objective in all linear programming problems.
- All LP have constraints that limit the degree to which the objective can be pursued.
- A feasible solution satisfies all the problem's constraint.
- An optional solution is feasible solution that result in the largest possible objective function value when maximizing 0r smallest when minimizing.

LINEAR PROGRAMMING(LP) PROBLEM

- Linear function are function in which each variable appears in a separate term raised to the first power and is multiplied by a constant(which could be 0)
- Linear constraints are linear function that restricted to be “less than or equal to”,”equal to” or “greater than or equal to” a constraint.
- If both the objective function and the constraint are linear, the problem is referred to a linear programing problem.

LP MODEL FORMULATION

- ***Linear programming*** *is a model that consists of linear relationships representing a firm's decision(s), given an objective and resource constraints.*
- *Steps:*
  1. Identify the **decision variables** – the unknown values that the model seeks to determine.
  2. Identify the **objective function** – the quantity we seek to minimize or maximize.
  3. Identify all appropriate **constraints** – limitations, requirements, or other restrictions that are imposed on any solution, either from practical or technological considerations or by management policy.
  4. Write the objective function and constraints as mathematical expressions.

SKLENKA SKI COMPANY: IDENTIFYING MODEL COMPONENTS

- SSC sells two snow ski models - Jordanelle & Deercrest
- Manufacturing requires fabrication and finishing.
- The fabrication department has 12 skilled workers, each of whom works 7 hours per day. The finishing department has 3 workers, who also work a 7-hour shift.
- Each pair of Jordanelle skis requires 3.5 labor-hours in the fabricating department and 1 labor-hour in finishing.
- The Deercrest model requires 4 labor-hours in fabricating and 1.5 labor-hours in finishing.
- The company operates 5 days per week.
- SSC makes a net profit of $50 on the Jordanelle model and $65 on the Deercrest model.

CONTINUED

- Step 1: Identify the decision variables
- The company wants to determine how many of each model should be produced on a daily basis to maximize net profit.
- Define
  - Jordanelle = number of pairs of Jordanelle skis produced/day
  - Deercrest = number of pairs of Deercrest skis produced/day
- Clearly specify the dimensions of the variables!

CONTINUED

- Step 2: Identify the objective function
- SSC wishes to maximize net profit, and we are given the net profit figures for each type of ski.
  - SSC makes a net profit of $50 on the Jordanelle model and $65 on the Deercrest model.

CONTINUED

- Step 3: Identify the constraints
  - Look for clues in the problem statement that describe limited resources that are available, requirements that must be met, or other restrictions.
- Both the fabrication and finishing departments have limited numbers of workers, who work only 7 hours each day; this limits the amount of production time available in each department:
  - Fabrication: Total labor hours used in fabrication cannot exceed the amount of labor hours available.
  - Finishing: Total labor hours used in finishing cannot exceed the amount of labor hours available.
- The problem also notes that the company anticipates selling at least twice as many Deercrest models as Jordanelle models:
  - Number of pairs of Deercrest skis must be at leasttwice the number of parts of Jordanelle skis.
- Negative values of the decision variables cannot occur ("nonnegativity constraints")

TRANSLATING MODEL INFORMATION INTO MATHEMATICAL EXPRESSIONS

- Represent decision variables by descriptive names, abbreviations, or subscripted letters (X_1, X_2, etc.)
  - For mathematical formulations involving many variables, subscripted letters are often more convenient.
  - In spreadsheet models, we recommend using more descriptive names to make the models and solutions easier to understand.

SSC – MODELING THE OBJECTIVE FUNCTION

- Profit per pair of skis sold:
  $50 for Jordanelle skis, $65 for Deercrest skis

- Objective Function:
  Maximize total
  profit
  = 50 *Jordanelle* + 65 *Deercrest*

- Note how the dimensions verify that the expression is correct:
  - ($/pair of skis)(number of pairs of skis) = $.

TRANSLATING CONSTRAINTS MATHEMATICALLY

- Constraints are expressed as algebraic inequalities or equations, with all variables on the left side and constant terms on the right.
- Look for key words in word statements of constraints:
  - "Cannot exceed" translates mathematically as "≤"
  - "At least," would translate as "≥"
  - "Must contain exactly," would specify an "= " relationship.
- All constraints in optimization models must be one of these three forms.

CONSTRAINT FUNCTIONS

- A **constraint function** is the left-hand side of a constraint.
  - E.g.: Total labor-hours used in fabrication cannot exceed the amount of labor hours available.

SSC – MODELING THE CONSTRAINTS

- Fabrication constraint
  - Available fabrication labor hours: (12 workers)(7 hours/day) = 84 hours/day
  - Required fabrication labor hours per ski pair: 3.5 hours for Jordanelle, 4 hours for Deercrest
  - <u>Fabrication constraint</u>: 3.5 *Jordanelle* + 4 *Deercrest* ≤ 84
- Finishing constraint
  - Available finishing labor hours: (3 workers)(7 hours/day) = 21 hours/day
  - Required finishing labor hours per ski pair: 1 hour for Jordanelle; 1.5 hours for Deercrest
  - <u>Finishing constraint</u>: 1 *Jordanelle* + 1.5 *Deercrest* ≤ 21

CONTINUED

- Market mixture constraint
  - The number of pairs of Deercrest skis must be at least twice the number of Jordanelle skis.
  - *Deercrest* ≥ 2 *Jordanelle*,
  - or – 2 *Jordanelle* + 1 *Deercrest* ≥ 0
- Nonnegativity constraints:
  - *Jordanelle* ≥ 0
  - *Deercrest* ≥ 0

SSC LP MODEL

Maximize total profit = 50 *Jordanelle* + 65 *Deercrest*

$$3.5\ Jordanelle + 4\ Deercrest \leq 84$$

$$1\ Jordanelle + 1.5\ Deercrest \leq 21$$

$$-2\ Jordanelle + 1\ Deercrest \geq 0$$

$$Jordanelle \geq 0$$

$$Deercrest \geq 0$$

LP Exercises

3. Burger Office Equipment produces two types of desks, standard and deluxe. Deluxe desks have oak tops and more-expensive hardware and require additional time for finishing and polishing. Standard desks require 70 board feet of pine and 10 hours of labor, whereas deluxe desks require 60 board feet of pine, 18 square feet of oak, and 15 hours of labor. For the next week, the company has 5,000 board feet of pine, 750 square feet of oak, and 400 hours of labor available. Standard desks net a profit of $225, and deluxe desks net a profit of $320. All desks can be sold to national chains such as Staples or Office Depot.

a. Identify the decision variables, objective function, and constraints in simple verbal statements.
b. Mathematically formulate a linear programming model.

LP Exercises

1. Valencia Products makes automobile radar detectors and assembles two models: LaserStop and SpeedBuster. The firm can sell all it produces. Both models use the same electronic components. Two of these can be obtained only from a single supplier. For the next month, the supply of these is limited to 4,000 of component A and 3,500 of component B. The number of each component required for each product and the profit per unit are given in the table.

	Components Required/Unit		
	A	**B**	**Profit/unit**
LaserStop	18	6	\$24
SpeedBuster	12	10	\$40

a. Identify the decision variables, objective function, and constraints in simple verbal statements.
b. Mathematically formulate a linear programming model.

	A	B	C	D
1	Sklenka Skis			
2				
3	Data			
4		Product		
5	Department	Jordanelle	Deercrest	Limitation (hours)
6	Fabrication	3.5	4	84
7	Finishing	1	1.5	21
8				
9	Profit/unit	$ 50.00	$ 65.00	
10				
11				
12	Model			
13		Jordanelle	Deercrest	
14	Quantity Produced	0	0	Hours Used
15	Fabrication	0	0	0
16	Finishing	0	0	0
17				
18				Excess Deercrest
19	Market mixture			0
20				
21				Total Profit
22	Profit Contribution	$ -	$ -	$ -

	A	B	C	D
1	Sklenka Skis			
2				
3	Data			
4		Product		
5	Department	Jordanelle	Deercrest	Limitation (hours)
6	Fabrication	3.5	4	84
7	Finishing	1	1.5	21
8				
9	Profit/unit	50	65	
10				
11				
12	Model			
13		Jordanelle	Deercrest	
14	Quantity Produced	0	0	Hours Used
15	Fabrication	=B6*B14	=C6*C14	=B15+C15
16	Finishing	=B7*B14	=C7*C14	=B16+C16
17				
18				Excess Deercrest
19	Market mixture			=C14-2*B14

Decision variables Objective function Constraint functions

SYSTEM OF LINEAR EQUATIONS, BASIC, FEASIBLE, OPTIMAL AND DEGENERATE SOLUTIONS

- A feasible solution does not violate any of the constraints.
- An infeasible solution violates at least one of the constraints.
- The optimal solution is the best feasible solution
- Multiple optimal solutions can occur when the objective function is parallel to a constraint line.

IMPLEMENTING LINEAR OPTIMIZATION MODELS ON SPREADSHEETS

- Put the objective function coefficients, constraint coefficients, and right- hand values in a logical format in the spreadsheet.
  - For example, you might assign the decision variables to columns and the constraints to rows
- Define a set of cells (either rows or columns) for the values of the decision variables.
  - The names of the decision variables should be listed directly above the decision variable cells.
  - Use shading or other formatting to distinguish these cells.
- Define separate cells for the objective function and each constraint function (the left-hand side of a constraint).
  - Use descriptive labels directly above these cells.

EXAMPLE 13.5: A SPREADSHEET MODEL FOR SKLENKA SKIS

	A	B	C	D
1	Sklenka Skis			
2				
3	Data			
4		Product		
5	Department	Jordanelle	Deercrest	Limitation (hours)
6	Fabrication	3.5	4	84
7	Finishing	1	1.5	21
8				
9	Profit/unit	$ 50.00	$ 65.00	
10				
11				
12	Model			
13		Jordanelle	Deercrest	
14	Quantity Produced	0	0	Hours Used
15	Fabrication	0	0	0
16	Finishing	0	0	0
17				
18				Excess Deercrest
19	Market mixture			0
20				
21				Total Profit
22	Profit Contribution	$ -	$ -	$ -

	A	B	C	D
1	Sklenka Skis			
2				
3	Data			
4		Product		
5	Department	Jordanelle	Deercrest	Limitation (hours)
6	Fabrication	3.5	4	84
7	Finishing	1	1.5	21
8				
9	Profit/unit	50	65	
10				
11				
12	Model			
13		Jordanelle	Deercrest	
14	Quantity Produced	0	0	Hours Used
15	Fabrication	=B6*B14	=C6*C14	=B15+C15
16	Finishing	=B7*B14	=C7*C14	=B16+C16
17				
18				Excess Deercrest
19	Market mixture			=C14-2*B14
20				
21				Total Profit
22	Profit Contribution	=B9*B14	=C9*C14	=B22+C22

Decision variables   Objective function   Constraint functions

CORRESPONDENCE BETWEEN THE MODEL AND THE SPREADSHEET

Maximize *Jordanelle* + 65 *Deercrest*

3.5 *Jordanelle* + 4 *Deercrest* ≤ 84

1 *Jordanelle* + 1.5 *Deercrest* ≤ 21

−2 *Jordanelle* + 1 *Deercrest* ≥ 0

Jordanelle ≥ 0

Deercrest ≥ 0

Maximize D22 = B9* *B14* + C9* *C14*

D15 = B6* *B14* + C6* *C14* ≤ D6

D16 = B7* *B14* + C7* *C14* ≤ D7

D19 = *C14* - 2* *B14* ≥ 0

B14 ≥ 0

C14 ≥ 0

	A	B	C	D
1	Sklenka Skis			
2				
3	Data			
4		Product		
5	Department	Jordanelle	Deercrest	Limitation (hours)
6	Fabrication	3.5	4	84
7	Finishing	1	1.5	21
8				
9	Profit/unit	$ 50.00	$ 65.00	
10				
11				
12	Model			
13		Jordanelle	Deercrest	
14	Quantity Produced	0	0	Hours Used
15	Fabrication	0	0	0
16	Finishing	0	0	0
17				
18				Excess Deercrest
19	Market mixture			0
20				
21				Total Profit
22	Profit Contribution	$ -	$ -	$ -

	A	B	C	D
1	Sklenka Skis			
2				
3	Data			
4		Product		
5	Department	Jordanelle	Deercrest	Limitation (hours)
6	Fabrication	3.5	4	84
7	Finishing	1	1.5	21
8				
9	Profit/unit	50	65	
10				
11				
12	Model			
13		Jordanelle	Deercrest	
14	Quantity Produced	0	0	Hours Used
15	Fabrication	=B6*B14	=C6*C14	=B15+C15
16	Finishing	=B7*B14	=C7*C14	=B16+C16
17				
18				Excess Deercrest
19	Market mixture			=C14-2*B14
20				
21				Total Profit
22	Profit Contribution	=B9*B14	=C9*C14	=B22+C22

USING THE SUMPRODUCT FUNCTION

- In Excel, the pairwise sum of products of terms can easily be computed using the SUMPRODUCT function.
  - B9* B14 + C9*C14 = SUMPRODUCT(B9:C9,B14:C14)
- This often simplifies the model-building process, particularly when many variables are involved.

EXCEL FUNCTIONS TO AVOID IN LINEAR OPTIMIZATION

- Several common functions in Excel can cause difficulties when attempting to solve linear programs using Solver because they are discontinuous (or "nonsmooth") and do not satisfy the conditions of a linear model.
- These include:
  - IF
  - MAX
  - INT
  - ROUND
  - COUNT

SOLVING LINEAR OPTIMIZATION MODELS

- A **feasible solution** to an optimization problem is any solution that satisfies all of the constraints.
- An **optimal solution** is the best of all the feasible solutions.
- Software for determining optimal solutions
  - *Solver* ("standard *Solver*") is a free add-in packaged with Excel for solving optimization problems.
  - *Premium Solver*, which is a part of *Analytic Solver Platform* has better functionality, accuracy, reporting, and interface.

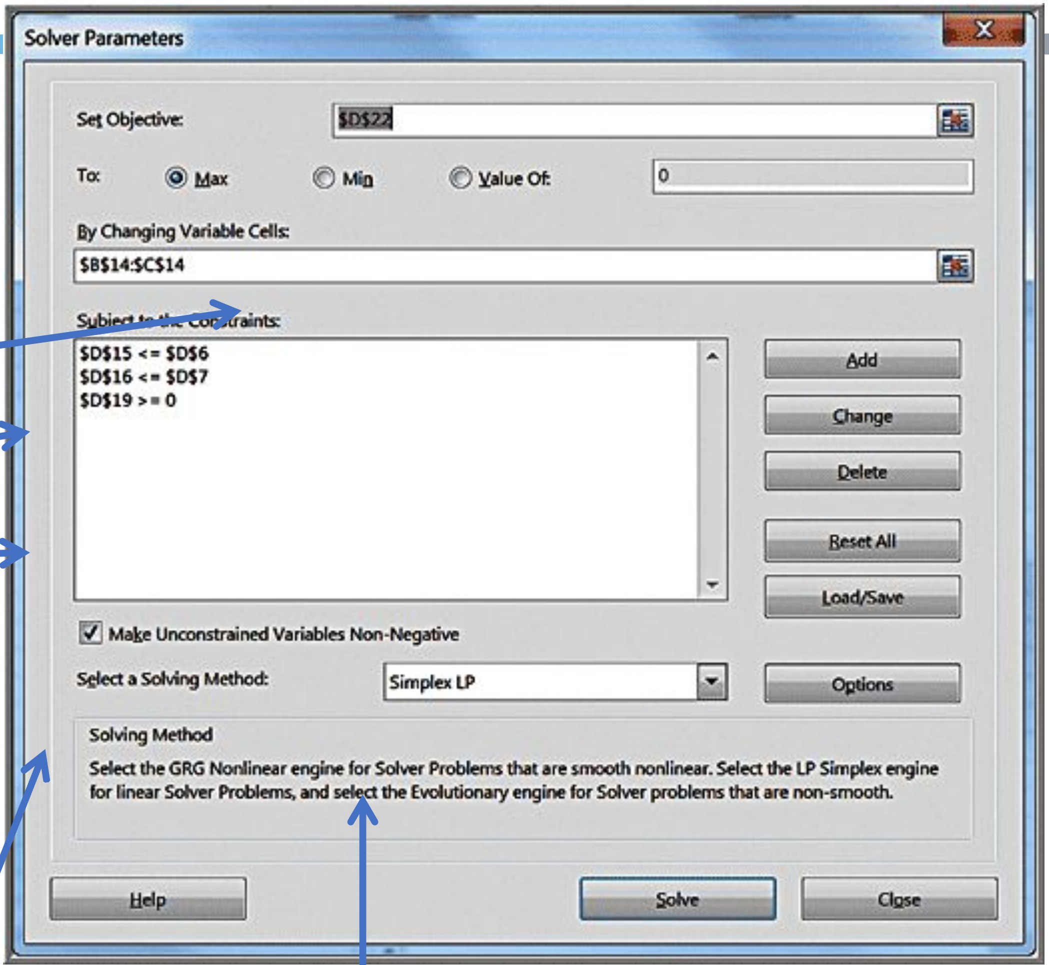
USING THE STANDARD SOLVER

- *Data* > *Analysis* > *Solver* in the Excel ribbon
- Use the *Solver Parameters* dialog to define the objective, decision variables, and constraints from your spreadsheet model.

THE SSC PROBLEM

- *Solver Parameters* dialog Objective function cell Decision variables cells Constraints

to enter click *Add* and fill in the *Add Constraint* dialog:

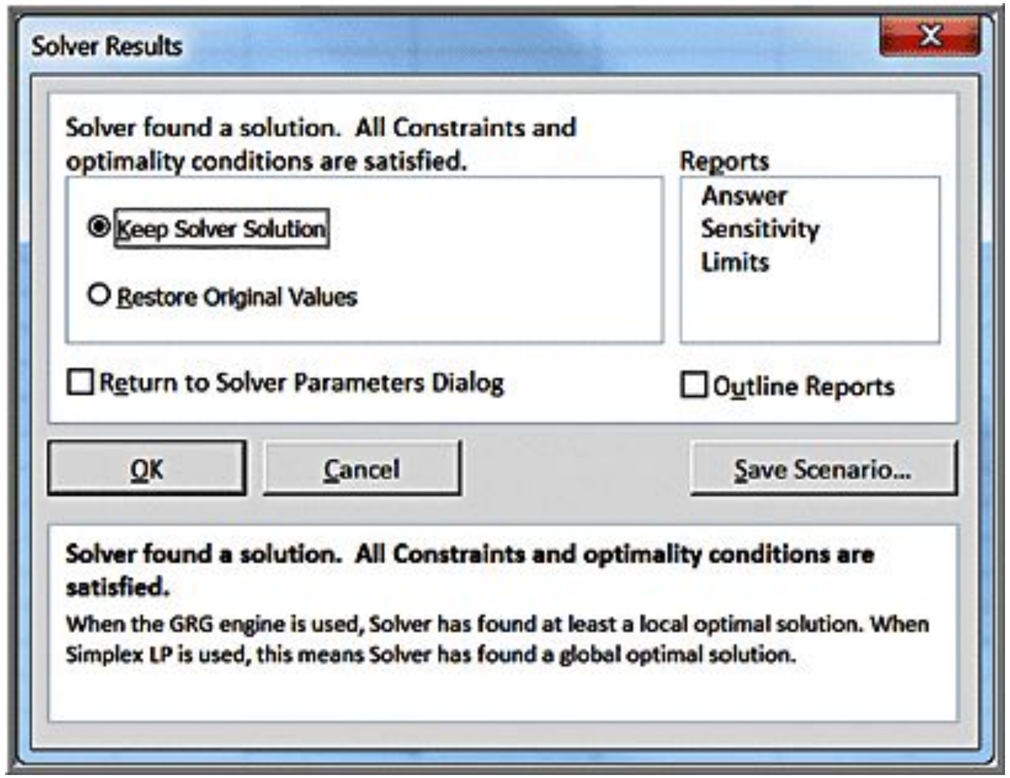


Check box for Nonnegativity

Always select “Simplex LP”

SOLVER RESULTS DIALOG

- Three reports:
  Answer, Sensitivity,
  and Limits
  - To add them to your Excel workbook, click on the ones you want and then click *OK*.
- <u>Do not check</u> the box *Outline Reports*; this is an Excel feature that produces the reports in "outlined format."



OPTIMAL SOLUTION TO SSC PROBLEM

	A	B	C	D
1	Sklenka Skis			
2				
3	Data			
4		Product		
5	Department	Jordanelle	Deercrest	Limitation (hours)
6	Fabrication	3.5	4	84
7	Finishing	1	1.5	21
8				
9	Profit/unit	$ 50.00	$ 65.00	
10				
11				
12	Model			
13		Jordanelle	Deercrest	
14	Quantity Produced	5.25	10.5	Hours Used
15	Fabrication	18.375	42	60.375
16	Finishing	5.25	15.75	21
17				
18				Excess Deercrest
19	Market mixture			0
20				
21				Total Profit
22	Profit Contribution	$ 262.50	$ 682.50	$ 945.00

INTRODUCTION TO THE SIMPLEX METHOD

- Simplex method is an approach to solving linear programming models by hand using slack variables,
  tableaus, and pivot variables as a means to finding the optimal solution of an optimization problem.

- Simplex method steps:
  - Standard form
  - Introducing slack variables
  - Creating the tableau
  - Pivot variables
  - Creating a new tableau
  - Checking for optimality
  - Identify optimal values

QUES

Maximize $z = 40x1+50X2$ Subject to $x1+2x2 <= 40$ $4X1 + 3x2 <= 120$ $X1, x2 >= 0$
Step1: Standard form $x1+2x2 +s1 = 40$ $4X1+3x2 +s2 = 120$ $X1, x2, s1, s2 >= 0$ $z -40x1-50X2 =0$ Step2: Slack variables are s1, s2

1 Starting simplex tableau

	Basic	z	x1	x2	s1	s2	Solution if x1=0,x2=0 (first corner point)	
Objective equation	z							ratio of solution and x2
	s1							
	s2							

2 Select most positive co-efficient of objective function or most negative of objective equation as entering variable (x1) and test the condition of optimality

	Basic	entering x2	Solution	ratio or intercept	Remarks
leave	s1				
	s2				

EXAMPLE

- A firm can produce three types of cloth A, B and C. Three kinds of wool is required for it, say red, green and blue wools. One unit length of type A cloth needs 2 yards of red wool, 5 yards of blue wools, one unit length of type B cloth needs 3 yards of red wool, 4 yards of green wool, and 2 yards of blue wool, and one unit length of type C cloth needs 6 yards of green and 5 yards of blue wools. The firm has only a stock of 10 yards of red wool, 12 yards of green wool, and 17 yards of blue wool. It is assumed that the income obtained from one unit length of type A, B and C are Rs 4.00, 5.00 and 6.00 respectively. Determine how the firm should use the available material, so as to maximize the income from the finished cloths.
- Determine Decision Variables, objective function, and const.
- Solve using solver

Kinds of wool	Types of cloth			Stock of wool (yards)
	A	*B*	*C*	
Red	2	3	0	10
Green	0	4	6	12
Blue	5	2	5	17
Income from one unit of clothes (Rs)	4.00	5.00	6.00	

- Decision Variables Let the firm produce $x1$, $x2$, $x3$ yards of three types of cloth A, B and C respectively. Therefore, $x1$, $x2$ and $x3$ can be treated as decision variables.
- Objective Function Since the profit per unit length of type A, B and C are given and we have to maximize the profit, therefore, we have Max $Z = 4x1 + 5x2 + 6x3$ …(1)
- Constraints As per the statement of given problem, we have
- $2x1+ 3x2+ 0x3 <= 10$
- $0x1+4 x2+ 6x3 <= 12$
- $5x1+ 2x2+5x3 <= 17$
- Non-negative Restrictions $x1, x2, x3 \geq 0$

EXAMPLE

- A manufacturer produces three models I, II and III of a certain product. He uses two types of raw materials (A and B) of which 5000 and 8000 units respectively are available. Raw material of type A requires 3, 4 and 6 units of each model. Whereas type B requires 6, 4 and 8 of model I, II and III respectively. The labour time of each unit of model I is twice that of model II and three times of model III. The entire labour force of the factory can produce equivalent of 3000 units of model I. A market survey indicates that the minimum demand of three models is 600, 400 and 350 units respectively. However, the ratios of number of units produced must be equal to 3 : 2 : 5. Assume that the profit per unit of models I, II and III are Rs 80, 50, and 120 respectively. Formulate this problem as linear programming model to determine the number of units of each product which will maximize the profit.

GRAPHICAL METHOD OF SOLVING LINEAR PROGRAMMING PROBLEMS

Graphical method is applicable to find the simple linear programming problem with two decision variables. Various steps for solving the problems are given below:

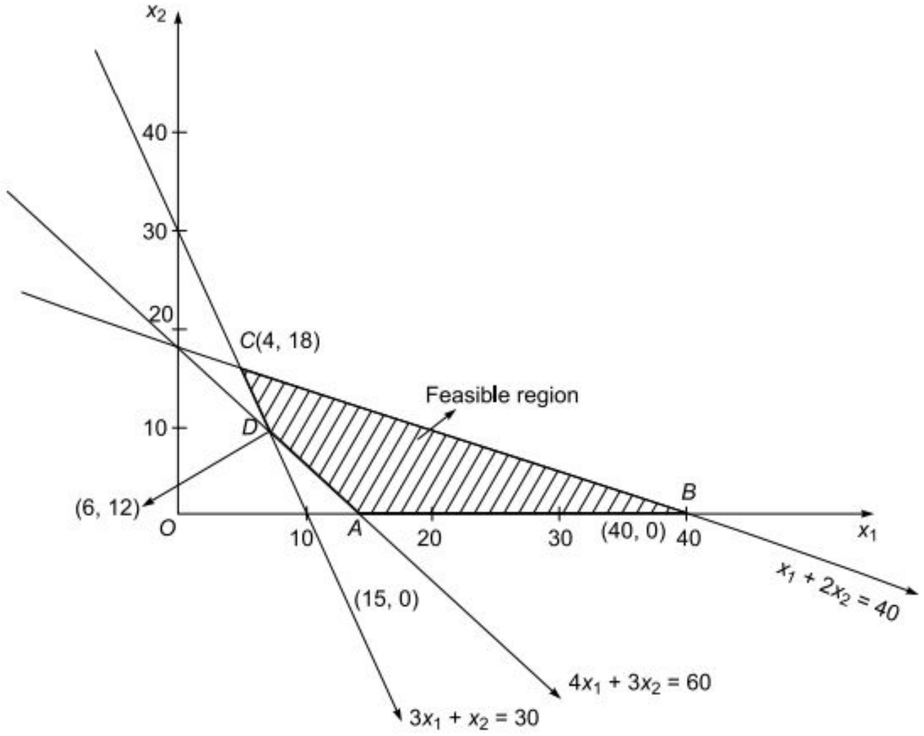
- 1. Consider each inequality constraint as equation.
- 2. Plot each equation on the graph such that each will geometrically respect a straight line.
- 3. Identify the feasible region. If the inequality constraint corresponding to that line is $<=$, then the region below the line in the first quadrant is to be shaded. For the inequality constraint $\geq$, then the region above the line in the first quadrant is shaded. The points lying in common region will satisfy all the constraints simultaneously. This common region is called feasible region.
- 4. Locate the corner points of the feasible region.
- 5. Draw the straight line to represent the objective function.
- 6. Test the objective function at each corner point of the feasible region and choose the point, where objective function obtains optimal value.

SOLVE

- Solve the following L.P.P. by graphical method
- Min $Z = 20x1 + 10x2$
- Subject to
- $x1 + 2x2 <= 40$ $x= 40, y =20$
- $3x1 + x2 \geq 30$ $x1=10, x2 =30$
- $4x1 + 3x2 \geq 60$ $x1= 15, x2= 20$
- $x1, x2 \geq 0$

- Convert all the inequalities of the constraints into equations, we have
- $x_1 + 2x_2 = 40$
- $3x_1 + x_2 = 30$
- $4x_1 + 3x_2 = 60$
- $x_1 + 2x_2 = 40$ passes through (0, 20) (40, 0)
- $3x_1 + x_2 = 30$ passes through (0, 30) (10, 0)
- $4x_1 + 3x_2 = 60$ passes through (0, 20) (15, 0)
- Plot above equations on graph, we have

HERE FEASIBLE REGION IS ABCD. THE COORDINATES OF ABCD ARE A(15, 0) B(40, 0), C(4, 18), D(6, 12)



Now

Corner Points	Coordinate	Value of Z
A	(15, 0)	300
B	(40, 0)	800
C	(4, 18)	260
D	(6, 12)	240

Therefore, minimum value of Z occurs at $D(6, 12)$. Hence, optimal solution is $x_1 = 6$, $x_2 = 12$.

- Solve the following L.P.P. using graphical methods
- Max $Z = 6x1 + 8x2$
- Subject to
- $5x1 + 10x2 <= 60$
- $4x1 + 4x2 <= 40$
- $x1, x2 \geq 0$

- Decision Variables Let the firm produce x1, x2, x3 yards of three types of cloth A, B and C respectively. Therefore, x1, x2 and x3 can be treated as decision variables.
- Objective Function Since the profit per unit length of type A, B and C are given and we have to maximize the profit, therefore, we have Max Z = 4x1 + 5x2 + 6x3 …(1)
- Constraints As per the statement of given problem, we have
- 0 4 6 12 5 2 5 17
- 2x1+ 3x2+ 0x3 <= 10
- 0x1+4 x2+ 6x3 <= 12
- 5x1+ 2x2+5x3 <= 17
- Non-negative Restrictions x1, x2, x3 ≥ 0

DUALITY

- Every LPP is associated with another unique LPP. The original problem is called "primal" while the other one is called "dual".
- An interesting feature of duality is that if the optimal solution of one is known then the optimal solution of the other one is readily available.
- If the primal problem contains a large number of constraints and a few decision variables, then solving dual problem is the ideal case to obtain solution of the primal, as it can reduce computational burden.

- Steps to convert into dual form of LPP:
  - If any of the constraints in the primal problem is a perfect equality, then the corresponding dual variable is unrestricted in sign.
  - If any variable of the primal problem is unrestricted in sign, then the corresponding constraint of the dual will be an equality.

DUALITY

Example: **Primal**

Given the LPP

Max $Z = 2x1 + 3x2 + 4x3$

subject to

$1x1 - 5x2 + 3x3 \leq 7$

$-2x1 + 5x2 \leq -3$

$3x2 - x3 \leq 5$

$x1; x2; x3 \geq 0$

Formulate the **dual** of the LPP/Primal Given the LPP

Min $W = 7y1 + 3y2 + 5y3$

subject to

$1y1 + 2y2 \geq 2$

$-5y1 - 5y2 + 3y3 \geq 3$

$3y1 - y3 \geq 4$

$y1; y2; y3 \geq 0$

THANK YOU