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Change In $\vec{b}$ vector (RHS)

$A^* = B^{-1} A$

$b^* = B^{-1} b$

$Z_0^* = Z_0 - C_B b^*$

$C^* = C - C_B A^*$

So, a change in $\vec{b}$ vector will cause a change in b^* and Z_0^*.
∴ Left side remain same
Right side will change

- Change in b does not effect the L.H.S
- Z_0^* & b^* changes

When changing $\vec{b}$, b^* should be ≥ 0
If $b^* < 0$ we need some other algorithm

Continuing our previous example

Ex: Max $Z = 11x_1 + 4x_2 + x_3 + 15x_4$

Subj. to

$3x_1 + x_2 + 2x_3 + 4x_4 \leq 28$

$8x_1 + 2x_2 - x_3 + 7x_4 \leq 50$

$x_i \geq 0$

Initial & final Tableau are given:

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	x_1	x_2	x_3	x_4	x_5	x_6	b
x_5	3	1	2	4	1	0	28
x_6	8	2	-1	7	0	1	50
	-11	-4	-1	-15	0	0	0

} Initial

Changing this into 50+α

Final Tableau ↓

	x_1	x_2	x_3	x_4	x_5	x_6	b
x_4	-2	0	5	1	2	-1	6
x_2	11	1	-18	0	-7	4	4
	3	0	2	0	2	1	106

} Final Tableau

- Change b_2 into $50 + \alpha$
then b^* is changing

$$b^* = B^{-1} b$$

$$b^* = \begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix} \begin{bmatrix} 28 \\ 50+\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix} \begin{bmatrix} 28 \\ 50 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix} \begin{bmatrix} 0 \\ \alpha \end{bmatrix}$$

$$= \begin{bmatrix} 6 \\ 4 \end{bmatrix} + \begin{bmatrix} -\alpha \\ 4\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 6-\alpha \\ 4+4\alpha \end{bmatrix}$$

$b^* \geq 0 \longrightarrow$ for Simplex (feasibility)

$\Rightarrow \quad 6 - \alpha \geq 0$

$4 + 4\alpha \geq 0$

$\Rightarrow \alpha \leq 6$

$\Rightarrow \alpha \geq -1$

$\therefore \quad -1 \leq \alpha \leq 6$

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If $-1 \le \alpha \le 6$ then ~~sum~~ optimal points will not ~~be~~ change, only b^* & z_0^* will change
But the Basic feasible points remain same. (Their value only changes)

~~$b^* =$~~

$$z_0^* = z_0 - [-15\ -4]\begin{bmatrix} 6-\alpha \\ 4+4\alpha \end{bmatrix}$$

Value will change
not the optimum points.

$= 106 + \alpha$ → optimum solⁿ is also changes accordly to change in b.
e.g. One unit increase in b_2 increases z_0^* by one unit

~~$X^* = [0, 6-\alpha, 0,$~~

$X^* = (0, 4+4\alpha, 0, 6-\alpha)$
↳ Points

e.g. Let $b_2 = 50 \to 53$ } $\alpha = 3$

$$b^* = B^{-1}b = \begin{bmatrix} 6-\alpha \\ 4+4\alpha \end{bmatrix} = \begin{bmatrix} 3 \\ 16 \end{bmatrix} \ge 0$$
satisfying

$X^* = (0, 16, 0, 3)$ → Feasible Basic optimum Point

$z_0^* = 0 - C_B b^*$
$= -[-15\ -4]\begin{bmatrix} 3 \\ 16 \end{bmatrix} = 109$
$= 106 + 3$

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Considering Dual :
The optimal value for dual can be found out using the optimal value for Primal.
→ slack variables is the soln for Dual.

Here $(y_1^*, y_2^*) = \cancel{(2, 1)} = (2, 1)$

For Dual
$\text{Min } z = b_1 y_1^* + b_2 y_2^*$
↳ change in $\vec{b_1}$ changes objective f^n in Dual.

c is same ∴ Points are same
$y^* = (2, 1)$ → Same
$z = y^* b^* = y^* \begin{bmatrix} 28 \\ 50 + \alpha \end{bmatrix}$

$= \begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} 28 \\ 50 + \alpha \end{bmatrix} = 2 \cdot 28 + 1 \cdot (50 + \alpha)$

$= 106 + \alpha$

Shadow Price

$= 106 + 1\alpha$

$b^* < 0$
↳ when limit is violated
Let $b_2 = 57 \quad \alpha = 7$
$b^* < 0$

$b_2 = 50 \longrightarrow 57$

$b^* = B^{-1} b$

$= \begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix} \begin{bmatrix} 28 \\ 57 \end{bmatrix}$

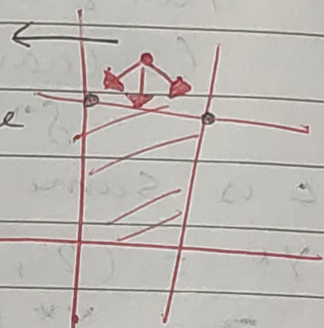
$= \begin{pmatrix} -1 \\ 32 \end{pmatrix} \rightarrow$ negative ≤ 0

$\hookrightarrow$ Basic Sol^n is not feasible

Now:

We will force this point to come to feasible region

(Motive: To go towards feasible Region)



The Dual Simplex Algorithm

Rules to start this algorithm:

1) LP should be in canonical form but some $\underbrace{b_i\text{'s can be zero}}_{b_i < 0}$. {If $b_i \geq 0$, Simplex}

2) $c_i \geq 0$

Qn: min $z = 10x_1 + 5x_2 + 4x_3$

subj. to $3x_1 + 2x_2 - 3x_3 \geq 3$

$4x_1 \qquad + 2x_3 \geq 10$

$x_i \geq 0 \qquad i = 1, 2, 3$

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Sol^n:
$$\text{Min } Z = 10x_1 + 5x_2 + 4x_3$$
$$3x_1 + 2x_2 - 3x_3 - x_4 = 3$$
$$4x_1 + 2x_3 - x_5 = 10$$
$$x_i \geq 0$$

Introduce A.V. → Simplex

~~for~~ Dual Simplex (Multiply by -1 both sides)

$$-3x_1 - 2x_2 + 3x_3 + 1x_4 = -3$$
$$-4x_1 - 2x_3 + 1x_5 = -10$$
$$x_i \geq 0$$

↳ Can't Apply Simplex $b_i \leq 0$

Basic Sol^n = (0, 0, -3, -10)
↳ But this sol^n is not feasible

	x_1	x_2	x_3	x_4	x_5	b
x_4	-3	-2	3	1	0	-3
x_5	-4	0	-2	0	1	-10 →
	10	5	4 ↑	0	0	0
x_4	-9	-2	0	1	3/4	-18 →
x_3	2	0	1	0	-1/2	5 ✓
	2 ↑	5	0	0	2	-20
x_1	1	2/9	0	-1/9	-1/4	2
x_3	0	-4/9	1	2/9	-1/6	1
	0	41/9	0	2/9	7/3	-24

All rules satisfying for Dual Algorithm

+ve

start Simplex Method

No -ve value to enter

∴ Min Z = 24 at (2, 0, 1)

Dual Sol^n (2/9, 7/3)

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We will Jump from One Basic soln to other using pivot.
So, Where to Pivot ?

Algorithm Steps
1) find the Row with -ve b_i
(In Simplex we find column)
Can take any -ve b_i
Let $b_2 = -10 \longrightarrow$ Row
2) After selecting a row, the variable corresponding to that row will exit.
How to enter & which variable to enter ?
3) find the Column { We take -ve value here }
4) find Ratio of c_j / a_{ij} { select largest }
Choose only -ve a_{ij}
Values will be -ve. So, largest in +ve
or smallest in ~~+ve~~ -ve
eg -1 -4 ✗
Absolute Value Min.

Let we define Dual for same Example
Max $Z = 3y_1 + 10y_2$
subject to
$3y_1 + 4y_2 \le 10$
$2y_1 \le 5$
$-3y_1 + 2y_2 \le 4$

Standard Form
Min $Z = -3y_1 - 10y_2$
subj. to
$3y_1 + 4y_2 + y_3 = 10$
$2y_1 + y_4 = 5$
$-3y_1 + 2y_2 + y_5 = 4$

	y_1	y_2	y_3	y_4	y_5	b	
y_3	3	4	1	0	0	10	$= 10/4 = 2.5$
y_4	2	0	0	1	0	5	
y_5	-3	2	0	0	1	4	$=2$ →
	-3	-10	0	0	0	0	
y_3	9	0	1	0	-2	2	→
y_4	2	0	0	1	0	5	
y_2	-3/4	1	0	0	1/2	2	
	-18	0	0	0	5	20	
y_1	~~2/9~~ 1	0	-2/9	0	4/9	41/9	
y_4	~~-1/9~~ 0	0	1/9	1	-2/9	2/9	
y_2	~~-1/4~~ 0	1	1/14	0	1/16	7/3	
	0	0	2	0	1	24	

↳ Dual & Primal

Primal × (−1) $=$ $\underbrace{\text{Dual}}_{\text{Relation}}$

Connection b/w Dual & Primal

↳ (−1) × Primal sol^n = Dual