20 January, 2021

Ex - max $Z = 2x_1 + 3x_2 + 3x_3$

subject to

$3x_1 + 2x_2 \leq 60$

$-x_1 + x_2 + 4x_3 \leq 10$

$2x_1 - 2x_2 + 5x_3 \leq 50$

$x_i \geq 0$

Soln Convert into Standard Form

$\min Z = -(\max Z)$

$= -2x_1 - 3x_2 - 3x_3$

subject to (Introducing slack variables)

$3x_1 + 2x_2 + x_4 = 60$

$-x_1 + x_2 + 4x_3 + x_5 = 10$

$2x_1 - 2x_2 + 5x_3 + x_6 = 50$

$x_i \geq 0$

x_4, x_5, x_6 is giving a canonical form

Whenever $\leq$ sign, standard form has canonical form

Solving this $\longrightarrow$ NP Next Page

Max. Absolute Value

If same condition smaller will be taken x_2

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	x_1	x_2	x_3	x_4	x_5	x_6	b	
x_4	3	2	0	1	0	0	60	$60/2 = 30$
x_5	-1	1	4	0	1	0	10	$10/1 = 10$
x_6	2	-2	5	0	0	1	50	exit
Z	-2	-3	-3	0	0	0	0	
x_4	5	0	-8	1	-2	0	40	
x_2	-1	1	4	0	1	0	10	
x_6	0	0	13	0	2	1	70	
Z	-5	0	9	0	3	0	30	
x_1	1	0	-8/5	-1/5	-2/5	0	8	
x_2	0	1	12/5	1/5	3/5	0	18	
x_6	0	0	13	0	2	1	70	
Z	0	0	1	1	1	0	70	

$a_{ik} > 0$ not ≥ 0

Algorithm stops $\because$ no -ve sign

optimum soln

$$Z_{min} = -70 \text{ at } (8, 18, 0, 0, 70)$$

$$\therefore \boxed{Z_{max} = 70}$$

Degeneracy (One of the B.V. is 0)

Ex. max $Z = 4x_1 + 3x_2$

subj. to $2x_1 + 3x_2 \leq 8$

$3x_1 + 2x_2 \leq 12$

$x_i \geq 0$

Standard Form

$2x_1 + 3x_2 + x_3 = 8$

$3x_1 + 2x_2 + \quad + x_4 = 12$

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Sol^n We are solving this as max Z not by min Z

	x_1	x_2	x_3	x_4	b	
x_3	2	3	1	0	8	$8/2 = 4$
x_4	3	2	0	1	12	$12/3 = 4$ → exit
Z	4 ↑	3	0	0	0	→ For max Z
x_3	0	5/3	1	-2/3	$\boxed{0}$	we see the sign
x_1	1	2/3	0	1/3	4	exit
Z	0	1/3 ↑	0	-4/3	$\boxed{16}$	
x_2	0	1	3/5	-2/5	$\boxed{0}$	} Degenracy.
x_1	1	0	-2/5	3/5	4	
Z	0	0	-1/5	-6/5	16	

Zmin = 16

Degeneracy → Iterations increased, ∵ we get min. Z = 16 before only.
(b = 0 ∴ BV = 0)

We terminate simplex for following conditions:
1) Unbounded
2) Alternate Optimum
3) Infeasibility

Alternate Optimum sol^n

max $Z = 4x_1 + 3x_2$
subject to $8x_1 + 6x_2 \leq 25$
$3x_1 + 4x_2 \leq 15$
$x_i \geq 0$

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$$8x_1 + 6x_2 + x_3 = 25$$
$$3x_1 + 4x_2 + x_4 = 15$$
$$x_i \geq 0$$

	x_1	x_2	x_3	x_4	b	
x_3	8	6	1	0	25	25/8 → pivot
x_4	3	4	0	1	15	15/3
Z	4↑	3	0	0	0	
x_1	1	3/4	1/8	0	25/8	
x_4	0	7/4	-3/8	1	45/8	→ pivot
Z	$\boxed{0}$	0↑	-1/2	$\boxed{0}$	100/8	

$$Z = \frac{100}{8} \text{ at } \left(\frac{25}{8}, 0, 0, \frac{45}{8}\right)$$

$x_2 = 0$, The sol^n says we take x_1 & neglect x_2

But if we require a sol^n where we have a both x_1 & x_2. Enter 0↑ (Sol^n is alternate optimum)

Continuing

	x_1	x_2	x_3	x_4	b	
Z	0	0↑	-1/2	0	100/8	
x_1	1	0	2/7	-3/7	5/7	
x_2	0	1	-3/14	4/7	45/14	
Z	0	0	-1/2	0	25/4	→ or 100/8

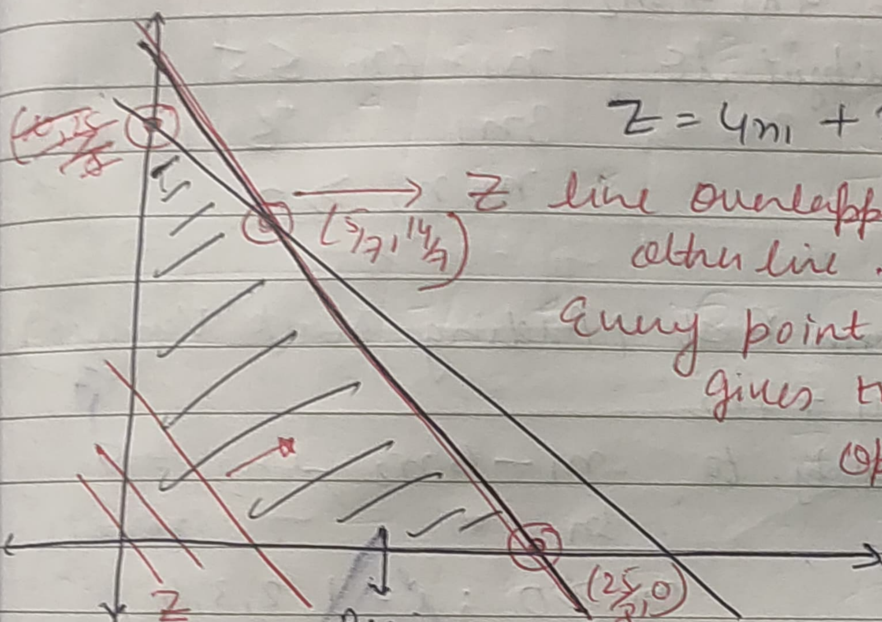
Optimum sol^n is same

(We don't have a problem whether slack variable > 0)

$Z = 25/4$ at $\left(\frac{5}{7}, \frac{45}{14}\right)$

Alternate Optimum

Graphically



$Z = 4x_1 + 3x_2 = K$

→ Z line overlapping with other line.
Every point on this line gives the same Optimum value

Region for our solⁿ

Problem ← for simplex

Simplex only gives optimum solⁿ for ~~optimum~~ corner points

In b/w points can also be used for optimum solⁿ