28 January, 2021

(2) Big M Method

E.g. $\quad Max\ Z = -3x_1 - 4x_2$

$2x_1 + 3x_2 \geq 8$

$5x_1 + 2x_2 \geq 12$

$x_1, x_2 \geq 0$

⇒ We can't multibly by −1 ∵ infeasibility cond. becomes false

First convert into standard form

$2x_1 + 3x_2 - x_3 = 8$

$5x_1 + 2x_2 - x_4 = 12$

$x_i \geq 0$

} Adding Surplus Variable (Slack Variables)

↳ Adding A.V.

$max\ Z = -3x_1 - 4x_2 + 0x_3 + 0x_4 - MA_1 - MA_2$

$2x_1 + 3x_2 - x_3 + A_1 = 8$

$5x_1 + 2x_2 - x_4 + A_2 = 12$

$x_i \geq 0$

M is a large value or → ∞

	x_1	x_2	x_3	x_4	A_1	A_2	b
A_1	2	3	−1	0	1	0	8
A_2	5	2	0	−1	0	1	12 → exit
Z	7M−3 ↑	5M−4	−M	−M	0	0	20M
A_1	0	17/5	−1	2/5	1	−2/5	16/5
x_1	1	2/5	0	−1/5	0	1/5	12/5
Z	0	$\frac{11M}{5} - \frac{14}{5}$	−M	$\frac{2M}{5} - \frac{3}{5}$	0	$-\frac{7M}{5} + \frac{13}{5}$	
x_2	0	1	−5/11	2/11	5/11	−2/11	16/11
x_1	1	0	2/11	−3/11	−2/11	3/11	20/11
Z	0	0	−14/11	−1/11	−M−14/11	−M−1/11	−124/11

	x_1	x_2	x_3	x_4	A_1	A_2	b
x_2	0	1	$-5/11$	$2/11$	$5/11$	$-2/11$	$16/11$
x_1	1	0	$2/11$	$3/11$	$-2/11$	$3/11$	$20/11$
Z	0	0	$-14/11$	$-1/11$	$-M+M/11$	$-M+1/11$	$-124/11$

$$\boxed{\begin{array}{l} Z = \frac{124}{11} \\ \text{at } \left(\frac{20}{11}, \frac{16}{11}, 0, 0\right) \end{array}}$$

Eg. Max $Z = 4x_1 + 3x_2$

$x_1 + 4x_2 \leq 3 \quad \Rightarrow \quad x_1 + 4x_2 + x_3 = 3$

$3x_1 + x_2 \geq 12 \quad\quad 3x_1 + x_2 - x_4 + A_1 = 12$

$x_1, x_2 \geq 0$

$x_i \geq 0 \quad A_1 \geq 0$

Max $Z = 4x_1 + 3x_2 - MA_1$

	x_1	x_2	x_3	x_4	A_1	b
x_3	1	4	1	0	0	3
A_1	3	1	0	-1	1	12
Z	$3M+4 \uparrow$	$M+3$	0	$-M$	0	
x_1	1	4	1	0	0	3
A_1	0	-11	-3	-1	1	3
Z	0	$-11M-13$	$-3M-4$	$-M$	0	

No entering variable

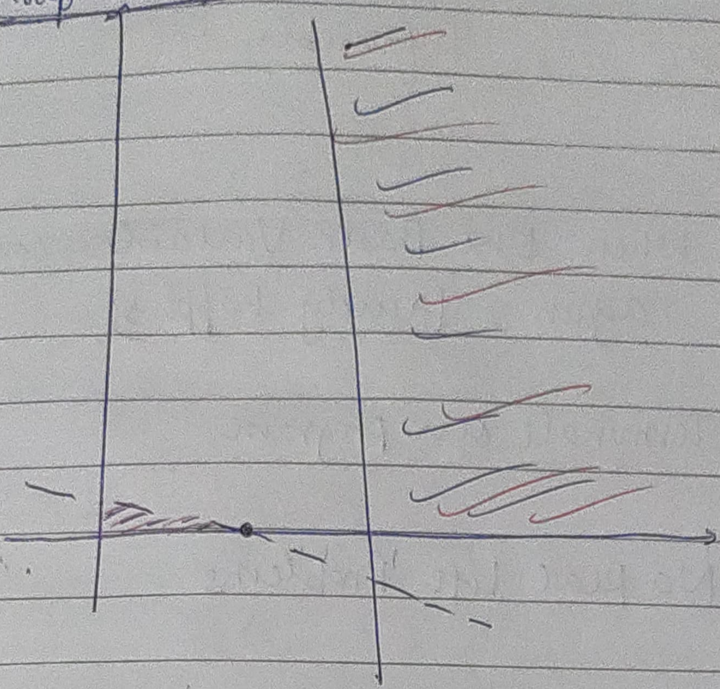
M still in basic variable

Infeasible

$-M \to -\infty$

optimum is Negative Infinity

Graphically,



Infeasible

Eg. Two phase Method

$$\text{Min } Z = x_1 + x_2 + x_3$$

subject to

$$x_1 + 2x_2 + x_3 \leq 11$$
$$-x_1 + 2x_3 \geq 4$$
$$x_1 - x_2 + 2x_3 = 4$$

Standard form

$$x_1 + 2x_2 + x_3 + x_4 = 1$$
$$-x_1 + 2x_3 - x_5 + A_1 = 4$$
$$x_1 - x_2 + 2x_3 + A_2 = 4$$

Canonical form after introducing Artificial Variable

$$w = A_1 + A_2$$

$$0x_1 + x_2 - 4x_3 + x_5 = -8 + w$$

→ Phase - I

DATE
PAGE

$W = 4 > 0$, Infeasible Soln

Cyclic → When One Basic Variable comes again (rarely happens)

→ Terminate our program

~~Can't~~ No Real Life Problems

Initial
≤ → Slack Variable Add
≥ → Surplus " " or -ve Surplus
=

Convert Standard to Canonical Form
≤ Slack Variables are Initial Basic Feasible Soln
≥ Add A.V. if there is surplus variable
= " " (Initial Basic Feasible Soln)

Termination
1. No Entering Variable
2. Alternate Optimum if (non-basic has Z = 0)
3. Unbounded Soln (exit variable not present) or $a_{ij} \le 0$
4. Infeasible Soln → When A.V. are present in Basic

DATE
PAGE

(5) Cyclic - {Basic Solⁿ Repeated}

We can also solve system of Eqⁿˢ using Simplex Method given all solⁿ ≥ 0
We can use Two-phase Method.
Solⁿ can be found using I-phase only when $w=0$, then the solⁿ will be system of Eqⁿ solution.

$w > 0$ → no solⁿ
$w = 0$ → Unique Solⁿ